CHAPTER 1

INTRODUCTION

1.1 The EAS and the lateral distribution of electrons:

The cosmic rays discovered by Hess in 1911 by balloon borne pressurised ionisation chamber is now a vast subject. It has thrown light on the structure of this universe and even down to the interaction between the subnuclear particles . In this thesis a very narrow branch of this vast subject, the extensive air shower (EAS) will be studied. The secondary cosmic ray contains two kinds of components, viz. the soft component and the hard component. The soft component consists of mainly the electrons and the photons and are absorbed easily. The hard component contains the rest of the particles. They can penetrate lead block with lengths greater than 10 cms. We shall discuss only of the former particles.

It is known that the protons are the most abundant particles in the primary cosmic rays. The percentage of other nuclei decreases rapidly with increase in mass number. The EAS is produced by the interaction of the primary cosmic rays with the atmosphere of the earth. When a primary cosmic ray like proton or a heavy nucleus falls on the upper atmosphere it may collide with air nuclei producing hadrons (mainly pions) and possibly few direct photons and leptons . The incident proton after losing some energy will continue its motion. However because of the Coulomb scattering it changes its course a little. The angle of deflection is given by the famous Rutherford formula :

$$\tan \frac{\theta^{*2}}{2} = \frac{Z Z' e^2}{M v^{*2} b} \quad \text{-------} \quad (1.1)$$

Here the starred quantities are measured in the CM system. θ^* is the angular deviation, z and z′ are respectively the atomic number of the target and the projectile, e is the electronic charge, M is the reduced mass, v^* is the relative velocity and b is the impact parameter. As v^* is very large θ^* is very small. Photons are created by the decay of π-meson, and electrons are generated by the decay of μ-mesons.

These primary and secondary particles descend through the atmosphere and create new particles by ionisation, excitation bremsstrahlung, pair production, Compton effect, photoelectric effect There are also many more less important modes of particle production.

In the ionisation process one or more electrons and some photons are created by a particle. However in the excitation process only photons are generated. The average energy loss per gm/cm^2 of a charged particle passing through matter was calculated by Bethe and Bloch[1]

$$-\frac{dE}{dx}\bigg|_{ion} = \frac{Nz}{A}\,\frac{2\pi z'^2 e^4}{m v^2}\left[\ln\frac{2mv^2 W\gamma^2}{I^2(z)} - \beta^2\right] \dashrightarrow (1.2)$$

Here $\beta = \frac{v}{c}$ and $\gamma = (1-\beta^2)^{-1/2}$, z′ is the charge of the incident particles and z is the atomic number of the medium. The loss increases with increase in electron energy but the rise is not steep.

The bremsstrahlung cross-section using quantum-mechanical calculation was given by Bethe[2]

$$\sigma_\gamma(E,k)\,dk = 4z^2\alpha\, r_e^2 \frac{dk}{k}\, F(E,u) \dashrightarrow (1.3)$$

Here σ_γ represents the cross-section, $u = k/E$, the ratio of the energies of the emitted photon to the incident electron, z is the atomic number of the target, α is the fine structure constant. $\frac{e^2}{\hbar c}$ and $F(E, u)$ is a function that depends on a parameter ξ, where $\xi = \frac{100\, mc^2}{E} \cdot \frac{u}{1-u}\, z^{-1/3}$

The function F also depends on the screening effect. The average energy loss of an electron per gm/cm^2 in this process is

$$-\frac{dE}{dx} = \int_0^{E-mc^2} \frac{N}{A}\,\sigma_r(E,k)\,k\,dk \quad \text{---->} (1.4)$$

Here A is the atomic number of the material, N is the number of molecules per mole or simply the Avagadro number. Energetic electrons not only produce bremsstrahlung by deflecting around nucleus but around atomic electrons also. Taking this into account the mean free length for the radiation loss of an electron, x_o is defined by the relation

$$\frac{1}{X_o} = \frac{4NZ(1+Z)}{A}\,\alpha\, r_e^2 \ln\left(191\, Z^{-1/3}\right) \quad \text{---->} (1.5)$$

The quantity x_o is also known as the radiation length. In air it is about 37.1gm/cm^2 or is equivalent to 308 metres.

Bremsstrahlung increases with increase in the particle energy but the gradient is small.

The pair creation process is inverse to the bremsstrahlung process. Here electron-positron pair is created by a photon with interaction from matter. The cross-section is given by:

$$\sigma_{pair}(k) = 4\alpha Z^2 r_e^2 \begin{cases} \left[\frac{7}{9}\ln\frac{2k}{mc^2} - \frac{109}{54}\right] & \text{for } mc^2 \ll k \ll 137\, mc^2 Z^{-1/3} \\ \left[\frac{7}{9}\ln\left(191 Z^{-1/3}\right) - \frac{1}{54}\right] & \text{for } k \gg 137\, mc^2 Z^{-1/3} \end{cases} \quad \text{---->} (1.6)$$

Pair production cross section is small for low energy particles. However it increases rapidly and in the higher energy region it becomes the predominant source.

Here we must take into account the Landau-Pomeranchuk effect. In dense media bremsstrahlung and pair creation cross section gets smaller. This is due to the fact that the radiation process terminates when a radiating electron is scattered by nearby atoms. Pair creation may also take place directly by a charged particle. The virtual photon associated with the charged particle is responsible for the process. However this being a higher order process than bremsstrahlung, the cross section is very small. For massive particles the pair creation process may predominate as the process is independent of the mass of the particle. But still the energy loss in the former case is not large. This is because the impact parameter for the former case is $\hbar/mc$ and for the latter it is $\hbar/Mc$. The average energy loss x cross section for the pair creation is $z^2\alpha^2 r_e^2 (m/M) E$ and for bremsstrahlung it is $z^2 \alpha r_e^2 (\frac{m}{M})^2 E$. Here E is the energy of the incident particle. So energy loss by pair creation is greater only if $M > m/\alpha$.

Pair creation can also take place in the photon-photon interaction. In a head on collison two photons may anihilate to give to pair creation. The cross section is given by:

$$\sigma = \frac{\pi}{4} r_e^2 \left(\frac{mc^2}{k^*}\right)^2 \beta^* \left[2(\beta^{*2} - 2) + \frac{3-\beta^{*4}}{\beta^*} \ln\left(\frac{1+\beta^*}{1-\beta^*}\right)\right] \quad \text{---} \rightarrow (1.7)$$

Here k^* is the energy of a photon and the velocity of the electron in the CM system is $c\beta^*$. The electron-positron pair may also be created virtually giving rise to photon-photon scattering. However the scattering cross section is very small for these processes.

Photons in an anologous manner may be scattered by the Coulomb field and such scattering is known as the Delbruck scattering.

In the atmosphere low energy photons may give rise to photo-electric effect and Compton effect. For the photoelectric process the cross section for K-shell electrons is: $\sigma_K = \sigma_{th} \frac{64}{Z^2} \left(\frac{\hbar c}{e^2}\right) \left(\frac{E_K}{K}\right)^{7/2} g(\eta)$ -----> (1.8)

Here $\sigma_{th} = \frac{8\pi}{3} r_e^2 = 6.65 \times 10^{-25}$ cm^2 and is known as the Thomson scattering coefficient, E_K is the ionisation potential for the K-shell and $g(\eta)$ is the deviation from the Born approximation and is given by

$$g(\eta) = 2\pi \sqrt{\frac{E_K}{K}} \frac{\exp(-4\eta \cot^{-1}\eta)}{1-\exp(-2\pi\eta)} \quad \text{--->} (1.19)$$

$$\text{and } \eta = \sqrt{\frac{E_K}{K-E_K}} = \frac{Z_{eff}\, e^2}{\hbar v} \quad \text{--->} (1.10)$$

Here v is the velocity of the electron.

At relativistic energies for light elements under Born approximation the cross section is:

$$\sigma_K = \sigma_{th} \times \frac{3}{2} Z^5 \left(\frac{e^2}{\hbar c}\right)^4 \left(\frac{mc^2}{k}\right)^5 (\gamma^2-1)^{3/2} \times \left[\frac{4}{3} + \frac{\gamma(\gamma-2)}{\gamma+1} \left\{1 - \frac{1}{2\gamma\sqrt{\gamma^2-1}} \ln\left(\frac{\gamma+\sqrt{\gamma^2-1}}{\gamma-\sqrt{\gamma^2-1}}\right)\right.\right. \quad \text{--->} (1.11)$$

$$\text{where } \gamma = \frac{k+mc^2}{mc^2}$$

For $k \gg mc^2$ this relation reduces to :

$$\sigma_K = \sigma_{th} \times \frac{3}{2} Z^5 \left(\frac{e^2}{\hbar c}\right)^4 \frac{mc^2}{k} \quad \text{--->} (1.12)$$

For heavy elements however Born's approximation does not hold good. Equation(1.12) should then be multiplied by a correction factor: $g_{rel}(\eta) = \exp\left[-\pi\eta + 2\eta^2(1-\ln\eta)\right]$

At slightly higher energy Compton process becomes important. The total cross section for the Compton effect is:

$$\sigma_c = r_e^2 \frac{1}{q}\left[\left\{1 - \frac{2(q+1)}{q^2}\right\} \ln(2q+1) + \frac{1}{2} + \frac{4}{q} - \frac{1}{2(2q+1)^2}\right] \quad \text{--->} (1.13)$$

$$\text{where } q = \frac{k_0}{mc^2}$$

The cross section simplifies to:

$$\sigma_c = \sigma_{th}\left(1 - 2q + \frac{26}{5}q^2 + \cdots\right) \text{ for } q \ll 1 \text{ and } \sigma_c = \sigma_{th} \cdot \frac{3}{8} \cdot \frac{1}{q}\left(\ln 2q + \frac{1}{2}\right) \text{ for } q \gg 1$$

Thus the Compton effect decreases with energy. Besides these in an EAS Cerenkov radiation and transition radiation predicted by Cerenkov, Ginzbarg and Frank[4] may also take place. The former process appears when a charged particle acquires a velocity which exceedes the phase velocity of light in that medium. Thelater process arises when a charged particle crosses the boarder of two media with different dielectric constant .

In this discussion we have neglected the nuclear collisons. The protons or light nuclei make head on collisons with the air molecules and produce mesons(π and μ) apart from nuclear particles π-mesons after decay produce γ-rays and μ-mesons produce electrons.

In the developement of an EAS the secondary particles produced create tertiary process. Thus the number of particles gradually increase until the particles lose so much energy that they become unable to produce further particles. Again during the developement of shower some particles are lost by the processes described earlier. So at the begining of the shower the number of particles created is far greater than the particles lost. After traversing downwards the number of created particles become less and less while the number of lost particles increase rapidly. So there is a stage where these two numbers are equal. It is the position where the shower contains largest number of particles. After that the number gradually decreases.

The cascade process is a stochastic process and it can be solved both by Monte Carlo and the analytical method.

The analytical method starts with the diffusion equations set up by Landau[5]:

$$\left.\begin{aligned}\frac{\partial\pi}{\partial t} &= -A'\pi + B'\gamma + \frac{E_s^2}{4E^2}\left(\frac{\partial^2}{\partial\theta_x^2}+\frac{\partial^2}{\partial\theta_y^2}\right)\pi - \left(\theta_x\frac{\partial}{\partial x}+\theta_y\frac{\partial}{\partial y}\right)\pi + \epsilon_0\frac{\partial\pi}{\partial E}\\ \frac{\partial\gamma}{\partial t} &= C'\pi - \sigma_0\gamma - \left(\theta_x\frac{\partial}{\partial x}+\theta_y\frac{\partial}{\partial y}\right)\gamma\end{aligned}\right\} \quad (1.14)$$

To simplify the very complex problem of the EAS phenomenon two kinds of approximations are usually made.

In the so called approximation A, the loss of energy by ionisation and Compton effect are neglected. The asymptotic form of complete screening are used for radiation and pair production cross section.

In the approximation B the ionisation loss is included but its effect is assumed to be constant. These approximations do not include the processes like nuclear interactions, Compton effect direct pair productions by electrons etc. However as their cross sections are small they can be neglected.

Equation(1.14) is written under the approximation B. In the equation $\pi(E,x,y,\theta_x,\theta_y,t)$ and $\gamma(E,x,y,\theta_x,\theta_y,t)$ are the differential sprectrum of the electrons and protons respectively. The term $(\partial^2/\partial\theta_x^2 + \partial^2/\partial\theta_y^2)$ signifies the Coulomb scattering of the electron in thickness 'dt', while $(\theta_x\frac{\partial}{\partial x} + \theta_y\frac{\partial}{\partial y})$ represents the change of distribution.

$\epsilon_0 \frac{dE}{dt}$ is the ionisation loss of the electrons under approximation B. ϵ_0 is the critical energy, and finally A', B', C' and σ_0 are operators. By bremsstrahlung and pair creation there is constant loss or gain of particles. These operators represent them.

The coupled partial differential equations(1.14) do not yield any solution under ordinary method. Landau approximation which ensures small divergence is sometimes used. Even so the process is so cumbersome that we use one of its moments:

$$\langle \theta_n \rangle = \int \theta^n f(\theta) \sin\theta \, d\theta \quad \text{and} \quad \langle r^n \rangle = \int r^n f(r) \, 2\pi r \, dr$$

Again the diffusion equations set up by Landau are really expansion of a function upto the second term. The omissions of the higher terms directly affect the moments higher than the second order.

The Landau equations however contain some sources of errors, besides the abovementioned difficulty.

According to Moliere[6] the omission of the higher order terms actually means that we are neglecting primarily the large angle single scattering. It however includes the multiple small angle scattering. The single large angle scattering are found very few in the EAS and hence the omission does not pose any difficulty.

The second order term in the diffusion equation diverges logarithmetically. To overcome this a suitable cut off procedure is needed. This is to restrict the scattering at large angles. This may also be attributed to the finite radius of the nuclei The cut off is not known accurately.

The diffusion equations assume a constant density atmosphere. This has some serious effect on lateral distribution.

Following Belenky[7] and taking into consideration the absorption and reproduction of electrons and photons Dedenko has improved the diffusion equation by adding a few more terms. He then applied 'successive generation' method devised by Rosental and Zatsepin and 'step by step' method devised by himself. He has shown that for primary energies ≅ 10 Gev at shower maximum the mean square lateral deviation is about 35 m. The lateral distribution is yet to be calculated.(8)

The first calculation on the lateral distribution of electrons was given by Moliere(9) but he treated the shower under approx i-mation A, thus neglecting the ionisation loss and used an approximation which waswrong. As a result the number of particles near critical energy was overestimated and the particle with less energy were below the expected number.

Bethe following Moliere has suggested a lateral distribution function as follows:

$$x f(x) = 0.45 (1+4x) \exp(-4x^{2/3})$$

The lateral displacement of the electrons while passing near the air nuclei suffers displacement laterally. Only this effect was taken into account byMoliere and later by Nishimura and Kamata.

A charged particle of energy E after crossing a distance 'dt' in a medium suffers multiple Coulomb scattering. If the angle of scatteting is $\delta\Theta$ then $\langle \delta\Theta \rangle^2 = \left(\frac{E_s}{E}\right)^2 dt$ Where E_s =21.2 Mev. The natural unit of natural displacement

$$r_M = \frac{E_s}{\epsilon_0} x_0 = 9.50 \text{ gm/cm}^2 = \frac{73.5}{P} \cdot \frac{T}{273} \text{ metre}$$

Here P is measured in atmosphere unit of Pressure and T in degree kelvin. x_0 is the radiation unit=37.7 gm/cm^2. r_M Is often called the Moliere radius. In the equation given by Bethe the factor $x = \frac{r}{r_m}$, is the distance from the axis of the shower in terms of the Moliere radius.

The use of the Moliere radius unit to measure the lateral spread has several advantages. In this unit all high energy particles ie. particles having energy far greater than the critical energy scatter in an analogous manner.

Similarly for the measurement of the angle of scattering the unit of angle is $\theta_M = \frac{E_s}{\epsilon_0} = \frac{1}{4}$ radian

Nishimura and Kamata[10] have solved the diffusion equation without approximating the ionisation loss term. They have given graphs of lateral distribution for the shower maximum and for late showers. Two inferences can be drawn from these graphs. Firstly the Nishimura distribution at the shower maximum is greater than the Bethe formula upto a distance 25 m, but they are very close together. After that upto 120m the Bethe distribution is slightly higher and after that Nishimura distribution is again higher. In the last portion the distributions deviate from each other gradually. Secondly there are more particles near the core for early showers. For late showers there are more particles at larger distances. In other words the distribution for early showers is steep and for late showers it is flat. At nearly $r=r_m$ the distributions are more or less equal. It appears very much surprising that though Bethe formula contains some serious errors it gives good comparison with the experimental results. But the errors in the formula are such that they approximately compensate one another and hence the agreement with the experimental results occurs.

The curves given by Nishimura are given for homogeneous atmosphere. In practice this is not so. However there is no serious error in the curves for the above assumption if there is perfect equilibrium between the shower and the medium. The variation of atmospheric pressure with altitude changes only the value of r_m. The dependence of r_m with pressere is given earlier. In reality the equilibrium stated earlier is not very good and the lateral distribution at a certain height is influenced by the distribution at higher level at lower pressure. Grisen has studied this effect and observed that in the lower atmosphere the lateral spread can be compared with the spread at the uniform density atmosphere. It was shown that if P and P′ be the respective pressures of the medium then their relations are given by $P' = (p-0.07)$ atmosphere The unit of lateral displacement will then change to

$$r_m' = \frac{73.5}{P-0.07}\left(\frac{T}{273}\right) m$$

According to Nishimura and Kamata calculations the Landau equations were solved for some special cases for example when $E_0 = \infty$ ie. the primary has infinite energy the solution was obtained for any point. However for finite primary energy the solution was obtained for distribution only near the axis. In the process they introduced a new parameter known as the age parameter(S). It was given by the relation

$$S = \frac{3t}{t + 2\ln(E_0/\epsilon_0) + 2\ln(r/r_m)}$$

Here 't' is the thickness of the medium in terms of radiation lengths It can be seen that the age parameter 'S' is a slowly varying function of distance. In the lower atmosphere ($t \simeq 25$ and $\ln(E_0/\epsilon_0) \simeq 15$), if r/r_m changes a factor of 10, S changes by 15%. Thus Nishimura and Kamata though assumed that 'S' will remain constant throughout only if $\ln(E_0/\epsilon_0) \gg \ln\left(\frac{r_{max}}{r_{min}}\right)$

where R_{max} And r_{min} are the maximum and minimum ranges of r respectively over which the experiment is made. The shower age thus falls with distance.

Accordingto Nishimura and Kamata the distribution near the core is $f(x) \propto x^{s-2}$ which is in conformity with the results derived earlier by Pomeranchuk and Midgal.

The following formula gives the Nishimura- Kamata distribution and was suggested by Grisen (11)

$$f(x) = C(s)\, x^{s-2} (1+x)^{s-4.5} \quad \text{----}\rightarrow (1.15)$$

The formula is known as the famous NKG formula for the lateral distribution and was suggested by Grisen. This distribution is for the electrons in the EAS. Here $x = r/r_m$ and r is the distance of point considered from the core of the shower. The distribution was claimed to be valid for $0.6 \leq S \leq 1.8$ and $0.01 \leq r/r_M \leq 10$. The normalising factor depends on the value of the age parameter. The dependence is shown below.

S =	0.6	0.8	1.0	1.2	1.4
C(S)=	0.22	0.31	0.40	0.44	0.43

The distribution f(x) is so normalised that

$$\int_0^\infty 2\pi x\, f(x)\, dx = 1$$

The electron density is given by the following formula:

$$\rho(r) = C(s) \left(\frac{N}{r_M}\right)^2 \left(\frac{r}{r_M}\right)^{s-2} \times \left(1 + \frac{r}{r_M}\right)^{s-4.5} \quad \text{---}\rightarrow (1.16)$$

Here N stands for the total number of electrons in the EAS. The age parameter here is assumed to be constant throughout the distance. It indicates the status of the shower. At the begining it is zero. At the shower maximum it is 1 and for late showers it is greater than 1.

The NKG formula is a good formula and was used throughout the world until recently. However another type of lateral distribution computation has developed now-a-days with the introduction of the fast computers. The solution of diffusion equation calculation has lost some of its importance. The Monte Carlo method of simulating a shower inside the computer is now done in EAS work. The process is given below.

1.2 THE MONTE CARLO METHOD

The Monte Carlo method is a way of computing an unknown variable by using random numbers. A primary cosmic ray say a proton is considered to be incident on the upper atmosphere of the earth and collides with an air molecule. The product and their momentum distribution will be model dependent. Some of the existing models are the a)fireball model

b)scaling model

c)CKP model etc.

The collison will generate particles like pion,kions,nucleons, and anti-nucleons. All the particles produced are assigned some transverse momentum value in such a manner that the vector sum of the transverse momenta of all of them is zero. Analogously a longitudinal momentum and a rapidity is also assigned to them. They are also so chosen that the conservation laws remain valid. The simulation of such parameters are done by using random numbers between 0 and 1. These parameters have no definite value. They can have many sets of value obeying certain conservation laws. Hence the use of random numbers are suitable for the solution of the right set of value.

If we denote the square of the CM energy by q and if with that energy a projectile of energy E (laboratory) and mass m is incident on a target particle of mass m_p ,then the following relation holds good:-

$$q = m^2 + m_p^2 + 2 m_p E \quad \text{GeV}^2$$

In a hadron-hadron collison $(\pi^+, \pi^\circ, \pi^-)$, $(p, \bar{p})$, $(n, \bar{n})$, (K^+, K^-), $(K_0, \bar{K}_0)$ etc. particles are formed.

The multiplicity of $\pi^{\pm}$ and $K^{\pm}$ follow the relation given below.

$$\langle n(q)\rangle = a_1 + a_2(\ln q) + a_3/\sqrt{q} \quad \text{----} \rightarrow (1.16)$$

While multiplicity of the rest are governed by the relation

$$\langle n(q)\rangle = a_1 + (a_2 + a_3/\sqrt{q}) \ln q \quad \text{---} \rightarrow (1.17)$$

The values of a_1, a_2, a_3, a_4 are given by Antinucci et al[12]. Here multiplicity (n_c) means the number of particles produced. It is dependent on the energy of the incident particle. However the reduced multiplicity is energy dependent as shown by Koba et al[13]. And a polynomial of 5th order may be fitted with the distribution. But use of a variable $\eta = \dfrac{n_c(n_c-1)}{\langle n_c(n_c-1)\rangle}$ is suitable for the description of multiplicity determination[14]. The multiplicity is then given by e^{η}. A value of η from this distribution is randomly selected by using a random number R, by using the relation $\lambda = -\ln R$. The number of individual types of particles can be obtained from this. The Product particles are then followed. They either interact with other particles or gets absorbed or decays spontaneously or reaches the observational level. At the top of the atmosphere where the primary cosmic ray is incident is assumed to be the origin of the coodinate system. The distance after which it will collide with an air molecule is given by the mean free path.

If $\sigma_{p\text{-}air}$ represents the inelastic cross-section of the proton air molecule collison then $\lambda_{p\text{-}air} = 24500/\sigma_{p\text{-}air}$,where $\lambda_{p\text{-}air}$ is the mean free path. The probability that a particle will travel a depth t is given by $e^{-t/\lambda}$ .Thus the distance distance traversed by a particle between creation and interaction with other particles is governed by the relation

$$t = -\lambda \ln R \quad gm/cm^2$$

where R is a random number. These particles then move downwards to other points of similiar interaction. The pions and kions decay through one of its possible decay modes. Using random numbers we can determine the point of decay, the branching ratio and the decay time. Now if the interaction point is higher than the decay point then the particle will interact with the air molecule and if it is the reverse then the particle will decay. The co-ordinates of the decay and interaction point is determined by noting the angle of scattering during the previous interaction, the distance traversed between the two events. In the case of decay, the direction and velocity of the decay particles can be calculated by using decay kinematics. Thus their movements are monitored until they reach the level of observation or their energy fall below 5 Gev. After that the particle will be unable to produce further interaction The hadrons at any depth in the atmosphere can also be calculated by simply noting their number at that depth. γ -rays are produced either by the decay of π° or by bremsstrahlung. Similiarly they disappear by photoelectric effect or pair creation. The creation and interaction point is also determined by the random numbers. Usually the two γ -rays are assumed to carry equal energy. Though in practice their energies are unequal, we ignore them as the error introduced is within tolerable limits. Similiarly neutral kaons decay into two electrons of equal energy.

Thus at the level of observation we get the shower particles with co-ordinates. Now detectors of definite area are placed at the observational level. If some particles fall on it then the detectors will record it. Thus an artificial shower is simulated in a computer.

The Monte Carlo calculations are preffered to the solution of the diffusion equation now-a-days. This is because the exact solution of the diffusion equation is not possible. Moreover these equations do not include all types of interactions in the air with the shower particles. Either approximation A or B is needed to frame them. However the Monte Carlo calculations can include all types of known interactions. Of course we must know the distribution function of all collisons and the mean free paths the decay probability etc. with the introduction of high energy accelerators these data are available now upto a very high energy These data are extrapolated upto the domain of the energy of the cosmic rays particle energy. It seems possible that some errors might creep in these results because actual data at the superhigh energy is not available. But the results we are obtaining from thecalculations are comparable with the results from the laboratory The Monte Carlo calculations will be more popular in future.

The use of Monte Carlo calculations has raised some doubt about the accuracy of the NKG formula at all shower distances. Allan et al[15] from their Monte Carlo calculations noted that the widths of the showers are very much less than that of the NKG formula. The results of Hessel andCrawford[16] went wrong.

Hillas et al[17] carried on Monte Carlo calculations including energy dependent pair production, bremsstrahlung cross sections with detailed screening factors correct to the Born approximation, logarithmic rise in the ionisation loss varying with energy, Compton effect, Multiple Coulomb scattering and multiple scattering These factors are used all over the world, for Monte Carlo calculations, Hillas added several other factors in the above m mentioned process.

He considered knock on electrons above 1 Mev. Thus the ionisation loss should be reduced as previously such electrons were also considered among the ionisation electrons. Thus ultimately there were no logarithmic rise. He therefore used separately accurate expressions representing the ionisation and knock on electrons and positrons. In the pair productions or in the bremsstrahlung the produced electron or photon is assumed to follow the Sommerfield angular distribution with θ^{-4} tail. He used a more detailed treatment of single and multiple scattering without any approximation. He also considered two more modifications- the anihilation of positrons in flight and secondly the radiation length was reduced to 35 gm/cm^2 according to the suggestions of Gennant and Pilkuhn(18). At low energy they followed the electron in short track segments upto zero energy but photons could not be followed in such way. Only approximate calculations were done for photons below 1 Mev. They studied the EAS phenomenon initiated at intervals of 40 gm/cm^2. The axis of the shower was inclined 25° with the vertical. Primary photons and electrons of 1, 10, 100 Gev were considered. They found that the spread is narrower.

The lateral displacement obtained by them was fitted by the NKG type of formula

$$\rho(r) = \frac{C(S)\,N}{r_H^2} \cdot \left(\frac{r}{r_H}\right)^{a_1 + a_2(s-1)} \cdot \left(1 + \frac{r}{r_H}\right)^{b_1 + b_2(s-1)} \quad \text{-----} \rightarrow (1.18)$$

where $s \simeq \frac{3t}{(t+2u)}$ and $u = \ln(E_0/\epsilon_0)$

Here they have neglected the effect of the term $2\ln(r/r_m)$ in the denominator of the expressions for S. Thus the dependence of S with distance was neglected. Here r_H is a distance in terms of which the lateral spread is measured. In the NKG formula this unit of length for measuring the lateral spread was 80 m. But hereas the auther has claimed the distribution to be more steep, the unit r_H =24m. approximately ¼ of the Moliere unit. C(S), a_1, a_2, b_1, b_2 are constants to be fitted. From the calculations he found that these are flatter by about 0.5 in the exponent of r/r_H, compared to the NKG distribution, both at small and large distances. It was also noted that the age parameter of electron induced shower at a certain depth is greater than the age parameter of a photon induced shower.

For small age parameter they found excess density at the large distances and for $1.0 \leqslant S \leqslant 1.3$ half the shower particles remain within a radius of 13 to 28 metres. In all probability it never becomes flatter than $\left(\sim \frac{1}{r^4}\right)$.

Such effcts were also observed by Lagutin et al[19]. There also the lateral distribution was steeper than the NKG distribution. Hillas[20] again cited papers to claim that his simulation results agree well with the Lagutin et al. He simulated showers upto 1000 Gev and found that the distribution of the electron andphoton induced showers are different. The peak of the photon induced shower lags behind the peak of the electron induced shower slightly. The distribution is higher than the NKG distribution at first, then the Hillas distribution falls off more steeply than the NKG distribution. In the same paper he modified his former formula for very large distances from the axis of the shower. His distribution may be written as

$$f(x) = C(S)\left(\frac{r}{r_H}\right)^A\left(1+\frac{r}{r_H}\right)^B M(r) \quad ----\rightarrow (1.19)$$

The difference between the formula (1.18) and (1.19) is the presence of a factor M(r). This factor is inserted to give extra steepness at the tail end of the distribution.

$$M(r) = \left(1+\frac{r}{L}\right)/\left(1+\left(\frac{r}{L}\right)^3\right) \quad \text{with} \quad L=600\ m.$$

Here for photon induced showers R_0=(25+38) , A= 1.2S -0.2 and for electron showers R_0 =(29+38) , A= 1.25 S-0.2

In both the above type of shower B varies with energy and its value lies between $-2.91 \leq B \leq -3.45$ for photon shower and $-3.01 \leq B \leq -3.19$ for electron shower.

Here also the unit of the measurement of lateral spread r_H is lessthan the unit in the NKG distribution. But here it also depends on the age parameter, and $r_H \simeq 30$ m. The exponent B in the NKG distribution contains the age parameter. But in the Hillas distribution it is independent of S. For photon induced showers the value of B varies more widely . Thus we arrive at a very complex lateral distribution which carries primary particle and energy dependent constants. Still according to Hillas the results are unsatisfactory in some region.

In the NBU cosmic ray unit the research workers are using the Hillas distribution (1.18) satisfactorily for several years. For medium distance from the core the formula used here is given below :-

$$\rho(r) = \frac{C(S)\,N_e}{r_H^2}\left(\frac{r}{r_H}\right)^{a_1+a_2(S-1)}\left(1+\frac{r}{r_H}\right)^{b_1+b_2(S-1)}$$

Here N_e is the total number of electrons in the shower. r_H =24 m. a_1=-0.53, a_2=1.54, b1=-3.39, b_2=0.01

The NKG formula for the lateral distribution of electrons does not include the zenith angle variation . To take into account of this effect Linsley[21] has suggested a NKG type of formula by analysing the Volcano Ranch experiments like the follows :-

$$f(x) = C(s,q)\, x^{s-2} (1+x)^{-(q+s-2)} \quad \text{---} \rightarrow (1.21)$$

where $C(s,q) = \frac{\Gamma(q+s-2)}{2\pi\,\Gamma(s)\,\Gamma(q-2)}$ and $q \simeq 3$ for $N \simeq 10^9$

it increases with size but decreases with zenith angle increment.

There is another line of thinking regarding the accuracy of the NKG function. The deviation was observed by Kristiansen[22] in 1975 and Kawaguchi[23] in 1975 that at the large distance from the axis the distribution is flat but at small distance it is steep.

Miyake[24] in 1968, Kristiansen[25] in 1971 have noted that a single age parameter is insufficient to describe the lateral distribution of electrons. Linsley[26] , Aguirre[27], and Porter[28] have shown that the age parameter increases as the distance from the shower core is increased. Capdevielle[29] et al had considered this thing in greater detail. By simulation and from the experimental results of the Tien Shan experiment he concluded that the age parameter is not a single parameter but a complex function of distance:-

$$S(r) = \alpha \ln \beta\,(r/r_0) + \bar{S}_t$$

for r between 15 to 150 metres and within the size range 10^5 to 4.5×10^6 .The constants α and β are model dependent. In one model $\alpha(N_e) = 3.8\times10^{-2} + 0.326 \ln(10^6/N_e)$

and $\beta(N_e) = 2.55\,[\mathrm{th}\,(N_e/5\times10^5)]^{0.25}$

The general formula can be written as:-

$$\rho(r) = C(s)\,\frac{N_e}{r_M^2}\left(\frac{r}{r_M}\right)^{S(r)-2}\left(1+\frac{r}{r_M}\right)^{S(r)-4.5} \quad \text{------}\rightarrow (1.22)$$

Here S(r) is given by the formula above. For the shower size 3×10^5 to 4.5×10^5
$S(r) = \alpha \ln\left(\beta \frac{r}{r_M}\right) + \bar{S}_t$ with $\bar{S}_t = 1.285$, $\alpha = 0.0714$ and $\beta = 1.525$

S(r) varies with r appreciably between 20 m to 120 m. Beyond this limit the effect is not much pronounced.

Capdevielle had also discussed other experimental results. In the Tien Shan experiment for distance upto 10 m from the core S=1.2 for shower size 10^5. However for the same shower within the distance from 20 metres to 80 metres the age parameter S=0.8. Thus the age parameter decreases with distance. This is in contrary to the formula (1.22). In the Volcano Ranch experiments for a distance of 400 metres it was seen that the age parameter increases upto 1.4. The deviation is also large and formula(1.22) is unable to explain them.

Elsewhere the author tried to established other complicated dependenceof S on r like:-

$$S(r) = \alpha\xi^2 + \beta\xi + \gamma \quad \text{with } \xi = \ln\left(\frac{r}{r_M}\right)$$

But these results are too much complicated for computation and do not improve the general situation.

Indeed Lagutin[30] had also given another formula analogous to Hillas. The difference between the Hillas and the Lagutin distribution is such that in the Hillas formula the Moliere radius was reduced to half and in the Lagutin formula it is (0.78-0.21 S) r_m ie. close to 0.5 r_m and in the Hillas distribution the exponent of the term $(1+ r/ r_m)$ is independent of S but in the case of the Lagutin distribution it is not so. Van Der Walt[31] tried to estimated the shower parameters by maximum likelyhood method The data he used was from the ten detectors of the Petchefstroom University. He found from the error analysis that the distribution is steeper than the NKG distribution but the Lagutin distribution is even steeper. than the experimental curve. However it may be argued that Walt had so few detectors in his hand that his conclusion may not be true.

It can thus be observed that the basis of all the functions prescribed later is the NKG distribution. Others have only modif ed it. So it may be written without doubt that theNKG function is more or less exact. The error is not very large. But later experimental results indicate that the distribution is steeper than what is given in the NKG function. There is so much evidence that we cannot deny it. Then there are two lines of approach. One line is headed by Hillas and Lagutin. They mainly reduced the Moliere distance to a great extent. The second line was headed by Capdevielle et al. They argued that the age parameter is not constant for all distances from the core. So, they suggested a dependence of the age parameter with distance.

In this thesis these three distribution functions viz. NKG , Hillas , and Capdevielle are compared. The results will be given in the third c in the third chapter. If the Hillas distribution is more appropriate then the Moliere distance should be reduced. If the Capdevielle distribution is appropriate then the defect is with the age parameter. As the two types of defects are independent of each other and both have ample evidence in their support the author felt it justified to consider a new distribution function which incorporates both the defects . In this distribution the Moliere distance is reduced and on the other hand the dependence of the age parameter with distance was accepted. The distribution is like the following :-

$$f(x) = C(S)\, x^{a_1 + a_2(S(r)-1)} (1+x)^{b_1 + b_2(S(r)-1)}$$

with $a_1 = -0.5$, $a_2 = 1.54$, $b_1 = -3.39$, $b_2 = 0$, $x = \frac{r}{r_0}$, $r_0 = 30$ m

and $S(r) = \alpha \ln \beta (r/r_0) + \bar{S}_t$

where $x = r/r_0$, r_0 is a new unit of radius.

The performances of this equation alongwith the values of the parameters for the best fit are given in the 3rd chapter.

1.3 MUON DISTRIBUTION

Muons are generated by the decay of pions or other unstable particles mainly kaons. They decay into electrons or positrons and neutrinos or may interact with the air molecule. The surviving muons reach the observational level. From time to time different distribution functions are prescribed to describe the lateral distribution of muons.

The simplest one is given by the relation :-

$$\rho_\mu = A\, r^{-n}$$

where A and n are constants. This function is a very simple type of function and fits approximately with the experimental distribution. A slightly improved formula is :-

$$\rho_\mu(r) = K\, R^{-(n+ar)}$$

Here R is the distance from the core, K, n, a are constants. In the Cosmic Ray Research Centre NBU such formula wasf fitted with the experimental data and was found to be satisfactory. The result is given in chapter 3. In recent times Kristiansen et al[32] have suggested a new formula :-

$$\rho_\mu(r) = A\, r^{-n} \exp\left(-\frac{r}{R}\right)$$

where R = 80 m, the Moliere radius. A and n are constants.

1.4 THE LATERAL DISTRIBUTION OF HADRONS

The hadrons or the nuclear active particles include pions and kaons, nucleons and antinucleons. These particles are detected with considerable difficulty. At small distances from the core huge number of electrons mask them. The hadron distribution being very steep, only few nucleons are observed at larger distances. Thus due to the low density there they are difficult to detect. The total number of such particles is also very small. The lateral distribution function is given by the approximate relation :-

$$\rho_H(r) = A\, r^{-\alpha} \quad \text{with } 1.1 \leq \alpha \leq 2.0$$

Some of the groups those studied the distribution are the Moscow group, in which Abrosimov[33] et al. have obtained $\alpha = 1.1$ when the shower size was 8×10^4. The Pamir group[34] found $\alpha = 1.3$ when $N = 1 \times 10^5$ and $\alpha = 1.7$ when $N = 8 \times 10^5$

In the core of a shower it is observed that there is no flattening. This could only mean that most of the nucleons and mesons are contained within a circle of radius 0.5 m. The rapid fall of the curves representing the lateral distribution of the nucleons prove it. However α cannot remain constant with increasing r , because otherwise N would diverge. So somewhere α must be greater than 2 , so that the nucleon density becomes zero somewhere.

Delerotin et al[35] had estimated the total number of nuclear active particles in a shower. According to him in a shower of 10^5 particles there are 1000 hadrons, and they carry about 10% of the above energy.

1.5 THE LATERAL DISTRIBUTION OF THE ENERGY FLOW

Energy flow of the various components of the EAS was measured by many authors. The lateral distribution and the energy spectra was also discussed. It is known that the energy per electron is independent of the size of the EAS. It however depends on two factors : a) the age parameter , b)the distance from the core. As the age parameter will increase the particle will suffer more collisons and ionisation and excitation etc. with the air molecule. Thus their energy and hence energy/particle will diminish gradually. Again as the distance from the axis of the shower will be greater the age parameter will increase with distance and hence energy/particle will also diminish to some extent. Cerenkov detectors are used to determine the energy of the particles near the core. Thus the energy density can be obtained. The energy flow data can be used to find out the lateral distribution. For small distances a power law is sufficient to describe the lateral distribution. The distribution gives us some hints about the core structure.

Asakimori et al[36] have shown that there is a change in the core structure about $N \simeq 10^5$. The experiment was done in the EAS array of the Kobe university. He used nine Cerenkov detectors. Among them four counters were at the centre of the array and the five others were near the centre. The counters were calibrated by a single muon of vertical incidence and the observed values were compared with the values obtained. During the propagation of an EAS whose core hit the central detectors, the sum of the energies registered by the central four counters were determined ($\sum E$). He had shown that fluctuations of $\sum E$ in the size range $1 \times 10^5 - 2 \times 10^5$ is larger than the showers with size $> 2 \times 10^5$. He calculated the lateral distribution to eliminate the dependence on r, the distance from the shower axis. The exponent was evaluated. It was again noted that near the size 2×10^5 there is an abrupt change in the value of the exponent. Even in the value of the energy per particle there is some abrupt change in that size range. Below 2×10^5 size the energy per particle was found to be 0.65 Gev/particle while after that size the value is 1.02 gev/particle. These changes in the core structure and the energies of the particles in the vicinity of of the size 2 x 10^5 seems to indicate a change in the composition of the primary cosmic rays.

In the NBU air shower array in the absence of Cerenkov detectors a simplified procedure described later was followed In the above procedure the change noted by Asakimori was observed The details of the procedure and result is given in chapter 5.

1.6 CHARGE COMPOSITION OF THE COSMIC RAYS

Charge composition of the primary cosmic rays are important because they can threw some light about the magnetic field of our galaxy depending on the nature of primary it can be told 1) if there is any preferential mechanism of acceleration of the light and heavy nuclei inside our galaxy , 2)the energy at which the extra galactic component becomes appreciably and finally if 3) there is any rigidity cut off inside our galaxy due to the magnetic field there. Peter and Linseley has already suggested such a rigidity cut off near the region of 10^5 shower size region. Thus if there is such a cut off then the primary will get richer in heavier nuclei and around 3 x 10^7 Gev even the heaviest of the nuclei will not reach the earth. The primaries above that energy must then come from other galaxies and should be predominantly protons, because during the long flight collisons with photons will reduce them into single protons. It is expected that they should not deviate from the actual course. Thus by noting the direction of the primary we can infer about the galaxy from which they are coming. But upto 100 Gev ballon bourne detectors can be used to get valuable information about the primaries but beyond 10^5 Gev where our centre of object is we do not get any information by that way as the frequency becomes very small. The study of the EAS is therefore necessary.

Many ballon bourne experiments were conducted using scintillation andCerenkov counters and even emulsion stacks. Ginzburg et al[37] have shown that the composition of the average mass numbers 1,4,14,32,56 is about 49%,27%,12%,5%,7% respectively. Hillas[38] had also compiled such data available from differente sources. It is seen that his results tally favourably with the result obtained by Ginzburg. This result is for primaries below 100 Gev. However above 100 Gev the result can be inferred indirectly from the EAS consideration. One method is to simulate showers artificially by Monte Carlo calculations. However to do this we have to assume a model of interaction of the shower particles with the air molecule. The models are not well established. Hence we are not absolutely sure of the results coming out of these calculations. Now of the characteristics of the EAS most of them depend both on the nature of the primary and the model of interaction. Thus the results from the Monte Carlo calculations can not give us some definite clue about the primary composition.

The Kiel and the Sydney group[39] studied the core structure of the EAS. The change in the core structure was noted and the conclusion they arrived at is that around 10^6 -10^7 Gev the composition is mixed. It becomes richer in heavier primaries at higher energy. Similiar observations were found by Asakimori et al. Pamir and Chacaltaya[40] group by noting the fluctuation in the Cerenkov radiation.

Different other authors have also written on this topics. The Sydney group found a corelation between the shower size and the multicore events. There occurence is maximum around 10^7 Gev.

Bohm et al studied the energy spectrum of the leading particle and concluded that the mass number of the average primary mass is around 10 to 60. Measurement of energy flow can also throw some light in this respect. In this thesis a work in this respect will be given in chapter 5.

Again by studing the fluctuation in the muon content in the EAS we can also arrive at the same conclusion.

Thus all these factors taken together can give some clue about the composition of the primary. Theresult from a single consideration is not enough for definite conclusion because it may be attributed to other reasons like the interactions with the air molecules etc.