INTRODUCTION

The study of wave and vibration phenomena has a distinguished history of hundred years. The first mathematician to describe the vibrations of pendulums, the resonance phenomenon and the vibrations of strings was Galileo. Some pioneer workers in the field of wave propagation are Cauchy, Poisson, Ostrogradsky, Green, Lame, Stokes, Navier, Clebsch and Christoffel.

Some of the major developments in the area of wave propagation are given below in chronological order :

1678 : Robert Hooke formulated the law of proportionality between stress and strain for elastic bodies. This law forms the basis for the static and dynamic theory of elasticity.

1821 : Navier investigated the general equations of equilibrium and vibration of elastic solids. Although not all of the developments of the work met with complete acceptance, it represented one of the most important developments in mechanics.

1822 : Cauchy developed most of the aspects of the pure theory of elasticity including the dynamical equations of motion for a solid.

1828 : Poisson investigated the propagation of waves through an elastic solid. He found that the two wave types, longitudinal and transverse, could exist.

1862 : Clebsch founded the general theory for the free vibration of solid bodies using normal modes.

1872 : J. Hopkinson performed the first experiments on plastic waves propagation in wires.

1883 : Saint Venant summarized the work on impact of earlier investigators and presented his results on transverse impact.

1887 : Rayleigh investigated the propagation of surface waves on a solid.

1904 : Lamb made the first investigation of pulse propagation in a semi-infinite solid.

1911 : Love developed the theory of waves in a thin layer overlying a solid and showed that such waves accounted for certain anomalies in seismogram records.

1949 : Davies published an extensive theoritical and experimental study on waves in bars.

1955 : Pekeris presented the solution to Lamb's problem of pulse propagation in a semi-infinite solid.

During the first three decades of this century the subject was not given so much importance by Mathematicians or Physicists. But in the later part of the 19th century interest in the study of waves in elastic solids attracted the researchers because of applications in the field of geophysics. Since that time in seismology the wave propagation has remained an interesting area because of the need for details information on earthquake phenomena, prospecting techniques and the detection of nuclear explosions. Bullen (1963), Ewing et al (1957), Cagniard (1962) and Pilant (1979) have discussed about seismic waves in their books.

During last 30-40 years the developement of theory of wave propagation in elasticity has been characterized by a detailed investigation of the classical methods of mathematical analysis and the trends to obtain specific results.
The solutions of many of the problems in elastodynamics, which are frequently encountered in practice, have made a significant contribution to the development of the theory of wave propagation as a whole. While earlier investigations in the theory of elasticity were essentially reduced to the construction of particular solutions, the invention of computer technology has led to the developement of general and quite universal methods of solving the problems of this theory, namely, the boundary value problems and initial boundary value problems for systems of differential equations having partial derivatives of a definite structure.

In an unbounded isotropic solid, two types of elastic waves may be propagated. These are dilatational wave and distortional wave. When a solid medium is deformed, both distortional and dilatational waves will normally be produced, and when a wave of either type impinges on a boundary of the solid, waves of both types are generated. In addition to these two types of wave which can travel through an extended solid medium, elastic waves may be propagated along the surface of a solid; these are known as Rayleigh waves, and the disturbances associated with them decay exponentially with depth. Since these waves spread only in two dimensions, they fall off more slowly with distance than the other types of elastic wave. They are of importance in seismic phenomena.

The propagation of waves in solids may be divided roughly into three categories. The first is elastic waves, where the stresses in the material obey Hooke's law. The two other main categories, visco-elastic waves, where viscous as well as elastic stresses act, and plastic waves in which the yield stress of the material is exceeded.

With regard to other works specially dealing with the propagation of waves in elastic solids we mention the books by Kolsky (1963), Brekhovskikh (1960), Achenbach (1975), Graff (1973) and Hudson (1980).

In recent years problems of diffraction of elastic wavs are of considereable importance in view of their application in Seismology and Geophysics. These types of problems can mainly be classifed into two categories. Firsty, diffraction of waves by semi-infinite plane barriers or cracks that are present in the medium and secondly, the diffraction in the presence of inclusions like wedges rigid strips, cones, cylinders, spheres, spheroids, ellipsoids or obstacle of any arbitrary shape. In bonding two materials with different mechanical elastic properties, very often it is not possible to obtain a homogeneous perfect bond due to the existence of entraped imperfections, for example, in the joints involving ceramics and metals used in manufacturing electronic devices and variety of reinforced composites. In nature the stratification of the earth is another example of bodies consisting of layered strucure. Indeed in geophysical stratifications, faults occur at the interface while in manufactured laminates imperfections occur at the interface of the adjoining layers.

The study of diffraction problems are associated with mixed boundary value problems.

We describe briefly the background of mixed boundary value problems below :

We consider a deformed elastic body occupying an open region D, whose boundary surface is S. It is assumed that S is piecewise smooth and the closure of D is $\overline{D} = D + S$. The surface S is usually considered to be closed and bounded, having the region D internal or external to it. Also S may be taken as open and extended to infinity or lying entirely at infinity.

The deformation and the state of stress within D and on S characterise the solution of the statical problem of elasticity. We can obtain an elasticity field $E = (u,e,T)$ where the elements in the parenthesis are the displancement field, the strain field and the stress field respectively. To ensure the uniqueness of the solution, we have to prescribe on the surface S one from each group of the following :

$$(u_1, \overset{n}{T}_1) \;,\; (u_2, \overset{n}{T}_2) \;,\; (u_3, \overset{n}{T}_3)$$

where u_i (i=1,2,3,) are components of the displacement u, $\overset{n}{T}_i$(i = 1,2,3) are the components of the stress vectar $\overset{n}{T}$ and n is the outward unit normal at an elementary area of the surface S. Thus we have a class of problems called boundary value problems in the theory of elasticity (Knops and Payne ,1971).

The boundary value problems generally can be classified into three types .

A. Traction boundary value problems.

Values of the stress components are prescribed on the boundary surface S.

B. Displacement boundary value problems.

values of the displacement components are prescribed on the boundary suface S

C. Mixed boundary value problems.

various types of mixed boundary conditions may be specifed on S or on different parts of S. Some possible combinations of these conditions are as follows :

(i) Tangential component of the stress and the normal component of the displacement are prescribed on S.

(ii) Displacement field is prescribed on a portion S_1 of the surface S and the stress is prescribed on the remaining part S_2 of S where $S_1 U S_2$ = S.

(iii) Tangential component of displacement and the normal component of the stress are prescribed on S.

(iv) The linear combination $\tau_{ij}n_j + \beta u_i$ is prescribed on S, where τ_{ij} are the components of the stress tensor and β is a non-nagative function prescribed on S. This is the case of so-called elastic support condition.

(v) A mixed-mixed type boundary value problem may be formulated under the conditions :

the shear stress component and the normal displacement component being prescribed on S_1 and the normal stress component and the shear stress component being prescribed on S_2.

For problems on the half spaces, besides the conditions (i) - (v) one has to impose the conditions at infinity as follows :

(a) that the difference of any two stress distributions is bounded therein, together with the condition - (i)

(b) that the difference of any two displacement fields is bounded therein, together with the conditions (ii)-(v).

The boundary value problems stated above correspond to the elastostatic case. In elastodynamics it is also required to specify the displacement and the velocity of the points throughout the region D at time $t = t_o$ i.e., at the commencement of straining. This additional requirement together with the conditions A,B or C constitute initial boundary value problems.

Boundary value problems are intimately connected with the theory of existence and uniqueness of solutions consistent with the laws of elasticity and geometrical configuration of the region D.

We now discuss a certain type of mixed boundary value problems which are known as contact problems in the theory of elasticity. The contact problem is formulated as a problem about the influence of a rigid body or an elastic body. As a rule, the initial contact takes place at one point and a contact surface is formed only when contacting bodies become nearer to each other. Generally, this contact surface increases in size. Therefore, we naturally introduce a restriction having a physical meaning : the stresses along the contour encircling the contact surface are finite.

Let us assume that a surface bounding a rigid or an elastic body is piecewise smooth. In this case, the size of the contact surface may increase only within the limits of the smooth region, right up to its edges. Consequently, for a sufficiently large value of compressive force, we can find the contact surface and this leads to the mixed boundary value problem. Naturally, the values of stress at such pints of the contact surface lying on the edge may be unbounded. In all cases with the sole exception of the case of complete coupling, it should be remembered that the contact pressure must be compressive. Otherwise, a cavity is formed between an elastic and a rigid body, leading to quite apparent modifications in the formulation of the problem.

There are two distinct classes of problems relating to indentation by a frictionless punch. In the first kind of indentation, called complete penetration, there is complete contact between the punch and the half-plane over a specified contact region, in the sense that the normal displacement of the half-plane at the boundary matches the profile of the punch. Such problems are charaterised by a contact pressure which has a singularity (square root) at the ends of the contact region. In the second kind, called incomplete penetration, the extent of the contact region, i.e. the extent of the region over which the normal displacement of the half plane matches the profile of the punch, is initially unknown. Cases of incomplete penetration are characterized by a contact pressure which is zero at the ends of the contact region.

Contact problems for the elastic half-plane, i.e. problems in which one body is a punch and the other an elastic half-plane, fall into the class of problems treated by the classical theory of elasticity. This theory was largely developed in the 19th centuryand is fully described with historical references in Love's (1944) treatise.

The literature on contact problems has been reviewed by a number of authors. Shtaerman (1949), Galin (1961), Ufliand (1965), Rvachev (1967) and Abramian (1971) are concerned mostly with the Soviet literature. Muskhelishvili's treatise (1953) is the basis of much of the Russian work, particularly that using complex variable methods.

There are number of areas of research which fall within the general scope of 'classical' contact problems. Some of these are listed below briefly.

1. Problems associated with loading over a region of the surface of the half-space which is neither circular nor elliptic. Foremost here is the work of Kalker (1972), (1977) on "elastic line" contact. This relates to loading over a long slender region, and has practical importance for rolling contact. Panek and Kalker (1977), in particular, present a simple approximate solution for the deformation produced by a narrow rectangular punch with rounded ends.
   This problem is related to those for a strip-shaped punch discussed by Borodachev (962) and Protsenko (1974).

2. The contact problem for a flat -faced punch of rectangular cross-section, with particular reference to the nature of the singularities on the edges and at the corners, has been studied by Borodachev (1976). Extensive analytical and numerical results for both frictionless and adhesive cases may be found in Brothers, Sinclair and Segedin (1977).

3. Contact problems for an elastic rectangle have been treated by Abramian (1957), Prasad and Chatterjee (1973), Dundurs, Kiattikomol and Keer (1974), Prasad and Dasgupta (1975).

4. Problems associated with the compression of a rigid or

elastic body between two half-planes, strips, half-spaces or slabs have been considered by Okubo (1951), Alblas (1974).

5. Contact problems for a sphere or a spherical shell. This is the subject of many research papers, among which we mention following. Abramian, Arutiunian and Babloian (1964), Goodman and Keer (1965).

6. Problems associated with a block or cylinder embedded in a semi-infinite elastic medium, for which the references are Poulos and Davis (1968), Dhaliwal, singh and Sneddon (1979).

Another type of contact problems that is encountered in practice is the study of the dynamic response of an elastic solid to moving loads or to oscillation of rigid punch and inclusions. The moving load or moving punch problems which have been studied may be put into three categories :

(i) Steady wave motion due to a load or punch moving with constant velocity for all time.

(ii) Transient wave motion due to a load or punch which begins to act at certain instant and then moves with constant velocity.

(iii) Transient wave motion due to a load or punch which begins to act at certain instant and then moves in some direction with nonunifom speed.

The steady motion of a line load on the surface of an elastic half-space was studied by Sneddon (1952), Cole and Huth (1958), Adams (1978), J.M.Golden (1982), Sve and Keer (1969), Alblas and Kuipers (1971,1971), Suhubi (1972).

The transient problem of a line load, which suddenly appears on the surface and then moves with constant velocity is of type (ii) and has been studied by Ang (1960).

As a representative of the third kind of problem we refer to the study of Freund (1972). An analytic technique was developed by Freund (1972) which made it possible to obtain an exact solution of a particular problem in category (iii).

Vibratory motion of a body on an elastic half-plane was treated by Karasudhi, Keer and Lee (1968). They considered the vertical, horizontal and rocking vibrations of a body on the surface of an otherwise unloaded half-plane. The problems were formulated so that shearing stress vanishes over the entire surface, and on oscillating displacement is prescribed in the loaded region. The problems were mixed with respect to the prescribed displacement and the remaining stress. Each case led to a mixed boundary value problem represented by dual integral equations which were reduced to a single Fredholm integral equation.

Wickham (1977) studied the problem of the forced two dimensional oscillations of a rigid strip in smooth contact with a semi-infinite elastic solid. He reduced the mixed boundary value problem with the help of Green's function to Fredholm integral equation of the first kind involving displacement boundary conditions. Using Noble's (1962) method, this equation was reduced to Fredholm integral equation of the second kind with a kernel which was small in the low frequency limit. Then applying the method of iteration, a simple explicit long-wave asymptotic formula for the normal stress in terms of the prescribed displacement and dimensionless wave number K was rigorously derived.

Rocking motion of a rigid strip on a semi-infinite elastic medium has been studied by Ghosh and Ghosh (1985) by using a different technique. The forced rocking of the strip about the horizontal axis has been reduced to a solution of a dual integral equation. Following Tranter's (1968) method the dual integral equation was solved for low frequency oscillations by reducing the equation to a system of linear algebraic equations.

In case of low frequency oscillations Noble's (1963) method of solving dual integral equations, Tranter's (1968) technique for solving dual integral equations, Matched Asymptotic Expassion, and variational principle are found to be very useful techniques.

Suppose that a mixed boundary value problem is formulated by suitable integral transform so as to be governed by a set of dual integral equations of the form

$$\int_0^\infty x^{-1}\left[1+K(x)\right]S(x)J_\nu(rx)\,dx = f(r) \quad , \quad 0\le r<a$$

$$\int_0^\infty S(x)J_\nu(rx)\,dx = g(r) \quad , \quad r>a$$

where the functions K(x), f(r) and g(r) are known.
According to Noble (1963), when $\nu > -\frac{1}{2}$

$$S(x) = \sqrt{\frac{2x}{\pi}}\left\{\int_0^a t^{1/2}\theta(t)J_{\nu-1/2}(xt)dt + \int_a^\infty t^{\nu+1/2}G(t)J_{\nu-1/2}(xt)dt\right\}$$

where $\theta(t)$ satisfies the Fredholm integral equation

$$\theta(t) + \frac{1}{\pi}\int_0^a M(\tau,t)\theta(\tau)d\tau = t^{-\nu}F(t) - H(t) \qquad (0 \le t < a) \qquad (1)$$

in which $M(\tau,t) = \pi\sqrt{\tau t}\int_0^\infty xK(x)J_{\nu-1/2}(\tau x)J_{\nu-1/2}(tx)dx$

$$F(t) = \frac{d}{dt}\int_0^t f(r)r^{\nu+1}(t^2-r^2)^{-1/2}dr$$

$$H(t) = t^{1/2}\int_0^\infty xK(x)J_{\nu-1/2}(xt)dx\int_a^\infty \xi^{\nu+1/2}G(\xi)J_{\nu-1/2}(x\xi)d\xi$$

$$G(\xi) = \int_\xi^\infty g(r)r^{-\nu+1}(r^2-\xi^2)^{-1/2}dr .$$

The integral equation (1) can be solved for $\theta(t)$ and consequently S(x) can be determined.

The problem of diffraction of normally incident plane acoustic wave by two coplanar, infinite, parallel, perfectly soft or rigid strips was considered by Jain and Kanwal (1972). The problem was solved by integral equation methods. The problem for the soft strips led to a Fredholm integral equation of first kind while that of rigid strips gave an integro-differential equation. Those equations were solved by the regular perturbation technique (Kanwal, 1971).

At the same time Jain and Kanwal (1972) solved the problem of diffraction of normally incident longitudinal and antiplane shear elastic waves by two parallel and coplanar rigid strips of finite width embedded in an infinite, isotropic and homogeneous elastic medium. The mixed boundary value problem was reduced to a set of dual integral equations with trigonometrical kernel. The solutions were obtained by using an integral equation perturbation technique (Kanwal, 1971) and Hilbert transform (Srivastava and Lowengrub, 1968). Using the theorem (Tricomi, 1951),

if $p \in L_2(a,b)$, then the equation

$$\frac{1}{\pi}\int_a^b \frac{h(x)}{x-y}\,dx = p(y) \quad , \quad y \in (a,b)$$

has the solution

$$h(x) = -\frac{1}{\pi}\left(\frac{x-a}{b-x}\right)^{1/2}\int_a^b \left(\frac{b-y}{y-a}\right)^{1/2}\frac{p(y)}{y-x}\,dy + \frac{C}{\sqrt{(x-a)(b-x)}}$$

where C is an arbitrary constant and the first term belongs to the class $L_2(a,b)$, Srivastava and Lowengrub (1968) found that the solution of the integral equation

$$\frac{1}{\pi}\int_a^b \frac{2th(t^2)}{t^2-y^2}\,dt = p(y) \quad , \quad y \in (a,b)$$

(provided that p satisfies the conditions of the above theorem) is given by

$$h(t^2) = -\frac{1}{\pi}\left(\frac{t^2-a^2}{b^2-t^2}\right)^{1/2}\int_a^b \left(\frac{b^2-y^2}{y^2-a^2}\right)^{1/2}\frac{2yp(y)}{y^2-t^2}\,dy + \frac{D}{\sqrt{(t^2-a^2)(b^2-t^2)}}$$

where D is an arbitrary constant.

Tait and Moodie (1981) have studied the problem of dynamic response of an elastic strip and that of pair of punches moving along the lateral boundaries of the strip and opening a crack along the mid surface. The problems were sloved in closed form by complex variable methods.

Diffraction of elastic waves by a rigid circular disc was considered by Mal, Ang and Knopoff (1968).
Low frequency diffraction by a elliptic disc have been studied by Sleeman (1967) and Roy and Sabina (1983).
Stallybrass and Scherer (1976) considered the problem of forced vertical vibration of a rigid frictionless elliptical disc on the surface of an elastic half-space. The mixed boundary value problem was reduced to a (two-dimensional) itegral equation and an approximation was obtained for the displacement of the disc by using variational procedure.

Arobinda Roy (1968) studied the dynamic response of an elliptical footing in frictionless contact with a homogeneous elastic half-space. Both vertical and horizontal vibrations were treated. Now we discuss the contact between rigid axisymmetric punch and an elastic half-space. Such contact problems may be classifed according to the type of punch, i.e. whether it is circular or annulur and whether its face is flat or curved; the type of indentation, i.e., whether it is a rotation or a translation about one or other of the axes, the type of contact, i.e., whether it is frictionless, adhesive, or in limiting friction, and if frictionless whether the contact is complete or incomplete.

Following Gladwell (1968) it may be assumed that the type of indentation, specified bythe displacement of the punch, is either

1. a rotation about Z-axis
2. a translation in the Z-direction
3. a rotation about the Y-axis
4. a translation in the X-direction.

Each of cases (2)-(4) gives rise to two distinct extreme problems in which the contact is assumed to be either completely adhesive or completely frictionless. The frictionless version of (1) is trivial in the sense that the half-space is not deformed at all.

England (1961) considered the axially symmetric indentation of a transversely isotropic layer resting on a rigid foundation. The problem of oscillations of a semi-infinite elastic solid by a smooth rigid circular disc on the free surface, performing small oscillations normal to its plane, without loosing contact with the surface of the solid has been studied by S.K. Bose (1967). The method of solution consists in introducing Hankel transforms and reduction to dual integral equations which have been solved by Tranter's method. Using Noble's (1963) method, Gladwell (1968) solved the problem of tangential and rotatory vibration of a rigid circular disc on a semi-infinite solid.

All the axisymmetric contact problems may be solved by using Hankel transforms and they then reduce to the solution of a number of sets (or pairs) of dual integral equations. To solve these dual integral equations there are various methods one of which is Tranter's method. Here we discuss briefly the method of Tranter (1986) method.

The solution of certain physical problems involving axisymmetric contact can be reduced to the determination of F(p) from so called dual integral equations of the form

$$\int_0^\infty G(p)F(p)J_\nu(rp)\,dp = f(r) \quad , \quad 0<r<1$$

$$\int_0^\infty pF(p)J_\nu(rp)\,dp = 0 \quad , \quad 1<r<\infty \qquad (2)$$

where G(p) and f(r) are known functions.

A solution F(p) of the above integral equations as a series of Bessel functions can be found by setting

$$F(p) = p^{-k}\sum_{m=0}^{\infty} a_m J_{\nu+2m+k}(p) \qquad (3)$$

where k is at present an arbitrary parameter, and proceeding as follows.

Substituting from (3) and changing the order of integration and summation, one gets

$$\int_0^\infty pF(p)J_\nu(rp)dp = \sum_{m=0}^\infty a_m \int_0^\infty p^{1-k} J_\nu(rp)J_{\nu+2m+k}(p)dp \qquad (4)$$

Provided $\nu>-1$ and $k>0$, the formula

$$I(\nu,\mu,\lambda,a,b) = \int_0^\infty \frac{J_\nu(at)J_\mu(bt)}{t^\lambda} dt = \frac{b^\mu \Gamma(\frac{\nu}{2}+\frac{\mu}{2}-\frac{\lambda}{2}+\frac{1}{2})}{2^\lambda a^{\mu-\lambda+1}\Gamma(\mu+1)\Gamma(\frac{\lambda}{2}+\frac{\nu}{2}-\frac{\mu}{2}+\frac{1}{2})}$$

$$\times \; {}_2F_1\left(\frac{\nu+\mu-\lambda+1}{2}, \frac{\mu-\lambda-\nu+1}{2} ; \mu+1 ; \frac{b^2}{a^2} \right)$$

shows that all the integrals on the right of (4) vanish when $r>1$ (because of the factor $\Gamma(-m)$ in the denominator of the term multiplying the hypergeometric function) and hence the series in (3) automatically satisfies the second of the dual equations (2). The coefficients a_m have now to be chosen so that the series in (3) satisfies the first of the dual equations (2). For this purpose we need the result

$$p^{-k} J_{\nu+2n+k}(p) = \frac{\Gamma(\nu+n+1)}{2^{k-1}\Gamma(\nu+1)\Gamma(n+k)} \int_0^1 r^{\nu+1}(1-r^2)^{k-1}F_n(k+\nu,\nu+1,r^2) \times$$

$$\times J_\nu(pr)dr \qquad (5)$$

where n is a positive integer or zero and

$$F_n(\alpha,\gamma,x) = {}_2F_1(-n,\alpha+n;\gamma;x) \tag{6}$$

is Jacobi's polynomial.

Substituting from (3) in the first of (2), multiplication by

$$r^{\nu+1}(1-r^2)^{k-1}F_n(k+\nu,\nu+1,r^2),$$

integration with respect to r between 0 and 1, interchange of the order of integrations and use of (5) give

$$\sum_{m=0}^{\infty} a_m \int_0^{\infty} G(p)p^{-2k}\, J_{\nu+2m+k}(p)J_{\nu+2n+k}(p)dp = E(\nu,n,k)$$

where

$$E(\nu,n,k) = \frac{\Gamma(\nu+n+1)}{2^{k-1}\Gamma(\nu+1)\Gamma(n+k)} \int_0^1 f(r)r^{\nu+1}(1-r^2)^{k-1}F_n(k+\nu,\nu+1,r^2)dr \tag{8}$$

Equation (7) with n=0,1,2,3,....... gives a set of simultaneous equations for the determination of the coefficients a_m. These simultaneous equations can be rewritten in a more convenient form by making use of the formula

$$\int_0^{\infty} p^{-1}\, J_{\nu+2m+k}(p)J_{\nu+2n+k}(p)dp = \begin{cases} 0 \;, & m\neq n \\ (2\nu+4n+2k)^{-1} \;, & m=n \end{cases} \tag{9}$$

this being the form taken by equation

$$\int_0^{\infty} \frac{J_\nu(at)J_\mu(at)}{t}\, dt = \frac{\Gamma(\frac{\nu}{2}+\frac{\mu}{2})}{2\Gamma(1+\frac{\nu}{2}-\frac{\mu}{2})\Gamma(1+\frac{\nu}{2}+\frac{\mu}{2})\Gamma(1-\frac{\nu}{2}+\frac{\mu}{2})}$$

$$= \frac{2}{\pi}\,\frac{\sin\frac{1}{2}(\mu-\nu)\pi}{\mu^2-\nu^2} \tag{10}$$

when μ and ν are replaced respectively by $\nu+2n+k$, $\nu+2m+k$ and when 'at' is replaced by p. We find in this way

$$a_n + \sum_{m=0}^{\infty} L_{m,n}\, a_m = (2\nu+4n+2k)E(\nu,n,k) \tag{11}$$

where

$$L_{m,n} = (2\nu+4n+2k)\int_0^{\infty} \left[G(p)p^{1-2k} - 1\right]p^{-1}\, J_{\nu+2m+k}(p)J_{\nu+2n+k}(p)dp \tag{12}$$

The iterative solution of the simultaneous equations (11) is

$$a_n = E_n - E_n' + E_n'' - \ldots\ldots. \tag{13}$$

where

$$E_n = (2\nu+4n+2k)E(\nu,n,k)$$

$$E_n' = \sum_{m=0}^{\infty} L_{m,n}\, E_m \quad , \quad E_n'' = \sum_{m=0}^{\infty} L_{m,n}\, E_m' \tag{14}$$

and so on.

Equations (3), (13), (14), (12) and (8) provide a theoretical solution of the dual integral equations. For a practical solution it is necessary to be able to choose the parameter k so that the expression $\left[G(p)p^{1-2k} - 1\right]$, which occurs in the formula (12) for $L_{m,n}$, is fairly small.

Now we consider another axisymmetric contact problem involving annular punch and torsion of an elastic half-space. These type of problems are three part boundary value problems. Triple integral equation method may be used to solve these problems. Some references are Tranter (1960).Cooke (1963), W.E.Williams (1963) and sneddon (1966). Gubenko and Mossakovskii (1960) solved the annular punch problem by using different technique. Olesiak (1965) attempted to solve the punch problem by reducing it to a series of dual integral equations and solved by successive approximations.

B.M. Singh, T.B. Moodie and J.B. Haddow (1980) considered also the problem of torsion by an annular disk of an elastic cylinder embedded in and bonded to an elastic half-spce. The problem was reduced to the solution of a Fredholm integral equation which was then analyzed by the method of Williams (1963). D.P. Thomas (1965) discussed the problem of diffraction of a general acoustic wave by a soft annular disc. Torsional oscillations of an elastic half-spce due to an annular disk has been studied by Jain and Kanwal (1970). Following Williams (1963) the problem was converted to a set of integral equations which were solved by iterative schemes when the outer radius of the disk is much larger than the inner radius.

A general formulation was given for the first time to the nonaxisymmetric annular punch problem by V.I. Fabrikant (1991). The problem was reduced to a two dimensional Fredholm integral equation with an elementary kernel which was solved numerically.

We describe here briefly the solution of a general class of boundary value problem by Williams's (1963) method.

Consider the general type of integral equation

$$\int_b^a f(t)\left[\, K(\rho,t) + K_1(\rho,t) \,\right] dt = \phi(\rho) \quad , \quad b<\rho<a \tag{15}$$

Integral equations of the general form of equation (15) occur in boundary value problems which can in some sense be regarded as perturbations on the electrostatic problem for the annulus. Examples of such problems are the diffraction of an acoustic wave by a soft annulus and the electrostatic problem for an annulus in a circular cylinder.

Assuming $\qquad \phi(\rho) = \sum_{n=-\infty}^{\infty} a_n \, \rho^n$

we can write

$$\phi_1(\rho) = \sum_{n=0}^{\infty} a_n \, \rho^n \quad , \quad \phi_2(\rho) = \sum_{n=-1}^{-\infty} a_n \, \rho^n$$

The integral equation (15) is thus equivalent to the pair of equations

$$\int_0^{\infty} f_1(t)K(\rho,t)dt = \phi_1(\rho) - \int_0^{\infty} f_1(t)K_1(\rho,t)dt \quad ; \quad 0<\rho<a \tag{16}$$

$$\int_0^{\infty} f_2(t)K(\rho,t)dt = \phi_2(\rho) - \int_0^{\infty} f_2(t)K_1(\rho,t)dt \quad , \quad \rho>b \tag{17}$$

where $\qquad f(\rho) = f_1(\rho) + f_2(\rho) \quad , \quad b<\rho<a$

and $\qquad f_1 + f_2 = 0 \quad , \quad 0<\rho<b \quad , \quad \rho>a \qquad (18)$

Further the integral equations (16) and (17) can be reduced to

$$\rho^{2n} S_1(\rho) = g_1(\rho) - \frac{1}{4}\int_0^\infty S_1(t)M_1(\rho,t)dt \quad , \quad 0<\rho<a \qquad (19)$$

$$\rho^{-2n} S_2(\rho) = g_2(\rho) - \frac{1}{4}\int_0^\infty S_2(t)M_2(\rho,t)dt \quad , \quad \rho>b \qquad (20)$$

where S_1 and S_2 are defined by

$$S_1(\rho) = \int_\rho^\infty \frac{t^{-n} f_1(t)dt}{(t^2-\rho^2)^{1/2}} \quad , \quad 0<\rho<a$$

$$S_2(\rho) = \int_0^\rho \frac{t^{n} f_2(t)dt}{(\rho^2-t^2)^{1/2}} \quad , \quad \rho>b$$

The functions M_1 and M_2 are defined by the relationships

$$K_1(\rho,t) = \frac{1}{(\rho t)^n}\int_0^\rho\int_0^t \frac{M_1(u,v)dudv}{(\rho^2-u^2)^{1/2}(t^2-v^2)^{1/2}} \qquad (21)$$

$$K_2(\rho,t) = (\rho t)^n\int_\rho^\infty\int_t^\infty \frac{M_2(u,v)dudv}{(u^2-\rho^2)^{1/2}(v^2-t^2)^{1/2}} \qquad (22)$$

Next, assuming two new functions $h_1(\rho)$ and $h_2(\rho)$ as

$$h_1(\rho) = \int_\rho^\infty \frac{t^{-n} f_2(t)dt}{(t^2-\rho^2)^{1/2}} \quad , \quad \rho>a$$

$$h_2(\rho) = \int_0^\rho \frac{t^{n} f_1(t)dt}{(\rho^2-t^2)^{1/2}} \quad , \quad 0<\rho<b$$

and using (18) the equations (19) and (20) finally can be written as

$$\rho^{2n} S_1(\rho) = g_1(\rho) - \frac{1}{4}\int_0^a S_1(t)M_1(\rho,t)dt + \frac{1}{4}\int_a^\infty h_1(t)M_1(\rho,t)dt \quad , \quad 0<\rho<a \qquad (23)$$

$$\rho^{-2n} S_2(\rho) = g_2(\rho) + \frac{1}{4}\int_0^b h_2(t)M_2(\rho,t)dt - \frac{1}{4}\int_b^\infty S_2(t)M_2(\rho,t)dt \quad , \quad \rho>b \qquad (24)$$

It can be shown that h_1 and h_2 will satisfy the equations

$$h_1(\rho) = \frac{n!}{\sqrt{\pi}\,\Gamma(n+\frac{3}{2})}\left[\rho^{-2n}\int_0^b \frac{wh_2(w)}{(\rho^2-w^2)} F\left(\frac{1}{2}, n, n+\frac{3}{2}, \frac{w^2}{\rho^2}\right)dw + \right.$$
$$\left. + \frac{d}{d\rho}\left\{\rho\int_b^\infty \frac{wg_2(w)}{(\rho^2+w^2)^{n+1}} F\left(\frac{n+1}{2}, \frac{n}{2}+1, n+\frac{3}{2}, \frac{4\rho^2w^2}{(\rho^2+w^2)^2}\right)dw\right\}\right] \quad , \quad \rho>a \qquad (25)$$

and

$$h_2(\rho) = \frac{n!\rho^{2n+1}}{\sqrt{\pi}\,\Gamma(n+\frac{3}{2})}\left[\int_a^\infty \frac{h_1(w)}{(w^2-\rho^2)} F\left(\frac{1}{2}, n, n+\frac{3}{2}, \frac{\rho^2}{w^2}\right)dw - \right.$$
$$\left. - \frac{d}{d\rho}\left\{\rho\int_0^a \frac{w^{2n}g_1(w)}{(\rho^2+w^2)^{n+1}} F\left(\frac{n+1}{2}, \frac{n}{2}+1, n+\frac{3}{2}, \frac{4\rho^2w^2}{(\rho^2+w^2)^2}\right)dw\right\}\right] \quad , \quad 0<\rho<b \qquad (26)$$

respectively provided g_1 and g_2 in these equations are now replaced by S_1 and S_2 respectively. These equations together with equations (23) and (24) thus give a set of four Fredholm integral equations for the functions h_1, h_2, S_1, S_2.

Now we pass on to the most interesting branch of Elastodynamics i.e. the diffraction of elastic waves by cracks which is of recent interest. Cracks are present in essentially all structural materials, either as natural defects or as a result of fabrication processes. In many cases, the cracks are sufficiently small so that their presence does not significantly reduce the strength of the material. In other cases, however, the cracks are large enough, or theymay become large enough through fatigue, stress corrosion cracking, etc., so that they must be taken into account in determining the strength. The body of knowledge which has been developed for the analysis of stresses in cracked solids is known generally as fracture mechanics. In recent years problems of diffraction of elastic waves by cracks are of considerable importance in view of their application in seismology and geophysics. Indeed in geophisical stratifications, faults occur at the interfaces while in manufactured laminates imperfections occur at the interface of the adjoining layers. This stress singularity near the edge of finite crack at the bimaterial interface is important in view of its practical application. Also the results of research on dynamic crack propagation are relevant in numerous applications. For example, a primary objective in engineering structures is to avoid a running fracture, or at least to arrest a running crack once it is initiated. Indeed there are several kinds of large engineering structures in which rapid crack growth is a definite possibility. In earth science, study of the elastic field near about the propagating fault is also important from the stand point of earthquake engineering.

Within the framework of a continuum model, such as the homogeneous, isotropic linearly elastic continuum, the classic analytical problem of fracture mechanics consists of the computation of the fields of stress and deformation in the vicinity of the tip of a crack, together with the application of a fracture criterion. In a conventional analysis inertia (or dynamic) effects are neglected and the analytical work is quasi-static in nature.

Because of loading conditions and material characteristics, however, there are many fracture mechanics problems which cannot be viewed as being quasi-static and for which the inertia of the material must be taken into account. Also inertia effects become of importance if the propagation of the crack is so fast, as for example in essentially brittle fracture, that rapid motions are generated in the medium. The label "dynamic loading" is attached to the effects of inertia on fracture due to repidly applied loads.

There are two broad classes of fracture mechanics problems that have to be treated as dynamic problems. These are concerned with

1. cracked bodies subjected to rapidly varying loads,
2. bodies containing rapidly propagating cracks.

In both cases the crack tip is an environment disturbed by wave motions.

Inpact and vibration problems fall into the first class of dynamic problems. In the analysis of such problems it is often found that

at inhomogeneities in a body the dynamic stresses are higher than the stresses computed from the corresponding problem of static equilibrium. This effect occurs when a propagating mechanical disturbance strikes a cavity. The dynamic stress "overshoot" is especially pronounced if the cavity contains a sharp edge. For a crack the intensity of the stress field in the vicinity of the crack tip can be significantly affected by dynamic effects. In view of the dynamic amplification, it is conceivable that there are cases for which fracture at a crack tip does not occur under a gradually applied system of loads, but where a crack does indeed propagate when the same system of loads is rapidly applied, and gives rise to waves which strike the crack tip.
The second class of problems is equally important. Indeed, there are several kinds of large engineering structures in which rapid crack growth is a definite possibility. Examples are gas transmission pipelines, ship hulls, aircraft fuselages and nuclear reactor components. Dynamic effects affect the stress fields near rapidly propagating cracks, and hence the conditions for further unstable crack propagation or for crack arrest. Another area to which the analysis of rapidly propagating cracks is relevant is the study of earthquake mechanisms.

There have been a number of comprehensive review articles in the general area of elastodynamic fracture mechanics. Some references are Achenbach (1972), Freund (1975), Achenbach (1976), Freund (1976) and Kanninen (1978).

At present, dynamic fracture mechanics solutions are largely confined to conditions where linear elastic fracture mechanics (LEFM) is valid. These are appropriate when the plastic deformation attending the crack tip is small enough to be dominated by the elastic field surrounding it. Problems of crack growth initiation under impact loads and of rapid unstable crack propagation and arrest can be treated with LEFM by using dynamically computed fields of stress and deformation. Engineering structures requiring protection against the possibility of large-scale catastrophic crack propagation are, however, generally constructed of ductile, tough materials. For the initiation of crack growth, LEFM procedures can give only approximately correct predictions for such materials. The elastic-plastic treatments required to give precise results have not yet been developed in a completely acceptable manner, even under static conditions.

The shapes of the cracks which have been studied uptil now are as follows :

(i) Semi-infinite plane cracks.

(ii) Finite Griffith cracks.

(iii) Penny shaped and annular cracks.

(iv) Non-planar cracks.

A transient problem involving the sudden appearance of a semi-infinite crack in a stretched elastic plate was considered by Maue (1954). Baker (1962) studied the problem of a semi-infinite crack suddenly appearing and growing at constant velocity in a stretched elastic body. The mixed boundary value problem is solved by transform methods including the Weiner-Hopf and Cagniard techniques. Atkinson and List (1978) considered the steady state semi-infinite crack propagation into media with spatially varying elastic properties. The quasi-static problem of an infinite elastic medium, weakened by an infinite number of semi-infinite, rectilinear, parallel and equally spaced cracks which are subjected to identic loads satisfying the conditions of antiplane state of strain was solved by Matczynski (1973). Sarkar, Ghosh and Mandal (1991) studied the problem of scattering of horizontally polarized shear wave by a semi-infinite crack running with uniform velocity along the interface of two dissimilar semi-infinite elastic media.

The powerful technique to solve the diffraction problem of semi-infinite crack is the Wiener-Hopf (1958) technique.

The in-plane problem of finite Griffith crack propagating at a constant velocity under a uniform load was first solved by Yoffe (1951). Sih (1968) has also provided a Riemann-Hilbert formulation of the same problem where both in-plane extensional and antiplane shear loads were considered.
Other reference treating elastodynamic problem involving a single finite Griffith crack are Loeber and Sih (1967), Ang and Knopoff (1964), Mal (1970, 1972) Chang (1971), Kassir and Bandyopadhyay (1983), Kassir and Tse (1983). Loeber and Sih (1967) solved the

problem of diffraction of antiplane shear waves by a finite crack by using integral transform method. Kassir and Bandyopadhyay (1983) considered the problem of impact response of a cracked orthotropic medium. Laplace and Fourier transforms were employed to reduce the transient problem to the solution of standard integral equation in the Laplace transform plane and was solved by Laplace inversion technique (Krylov et al, 1957); Miller and Gyuy, 1966).

The problems of finite Griffith crack lying at the interface of two dissimilar elastic media have been studied by Srivastava, Palaiya and Karaulia (1980), Nishida, Shindo and Atsumi (1984) and Bostrom (1987). Bostrom (1987) used the method of Krenk and Schmidt (1982) to solve the two-dimensional scalar problem of scattering of elastic waves under antiplane strain from an interface crack between two elastic half-spaces. Sih and Chen (1980) analyzed the dynamic response of a layered composite containing a Griffith crack under normal and shear impact.

The Problems of diffraction of elastic waves become more complicated when boundaries are present in the medium. Chen (1978) considered the problem of dynamic response of a central crack in a finite elastic strip. The crack was assumed to appear suddely when the strip is being stretched at its two ends. The problem was solved by Laplace and Fourier transform technique. Some other references are Srivastava, Gupta and Palaiya (1981), Srivastava, Palaya and Karaulia (1983), Shindo, Nozaki and Higaki (1986), De and Patra (1990).

Different techniques have been applied by many authors to tackle these type of crack problems. From these stand point, these problems may be divided into two categories : one for low frequency oscillation of the source or long wave scattering or transmission and the other for high frequency oscillation or short wave scattering or transmission in the medium. The term long and short are used in comparison to the region of the source of disturbance or the size of the crack or strip etc. inside the medium to the wave length of disturbance. The useful techniques for low frequency scattering due to Noble (1963) at Tranter (1968) have been discussed earlier. In case of high frequency oscillations Wiener-Hopf (Noble, 1958) technique and Keller's (1958) geometrical theory are found to be most suitable. Here we briefly discuss the methods.

Wiener-Hopf Method :

The typical problem obtained by applying Fourier transforms to partial differential equations is the following. One shall have to find unknown functions $\Phi_+(\alpha)$, $\Psi_-(\alpha)$ satisfying

$$A(\alpha)\Phi_+(\alpha) + B(\alpha)\Psi_-(\alpha) + C(\alpha) = 0 \qquad (27)$$

Where this equation holds in a strip $\tau_- < \tau < \tau_+$, $-\infty < \sigma < \infty$ of the complex α-plane, Φ_+ is regular in the half-plane $\tau > \tau_-$,

$\Psi_-(\alpha)$ is regular in $\tau < \tau_+$, and certain information which will be specified later is available regarding the behaviour of these functions as α tends to infinity in appropriate half-planes. The functions $A(\alpha)$, $B(\alpha)$, $C(\alpha)$ are given function of α, regular in the strip. For simplicity let us assume that A, B are also non-zero in the strip.

The fundamental step in the Wiener-Hopf procedure for solution of this equaion is to find $K_+(\alpha)$ regular and non-zero in $\tau > \tau_-$, $K_-(\alpha)$ regular and non-zero in $\tau < \tau_+$, such that

$$A(\alpha)/B(\alpha) = K_+(\alpha)/K_-(\alpha) \tag{28}$$

Use (28) to rearrange (27) as

$$K_+(\alpha)\Phi_+(\alpha) + K_-(\alpha)\Psi_-(\alpha) + K_-(\alpha)C(\alpha)/B(\alpha) = 0 \tag{29}$$

Decompose $K_-(\alpha)C(\alpha)/B(\alpha)$ in the form

$$K_-(\alpha)C(\alpha)/B(\alpha) = C_+(\alpha) + C_-(\alpha) \tag{30}$$

where $C_+(\alpha)$ is regular in $\tau>\tau_-$, $C_-(\alpha)$ is regular in $\tau<\tau_+$.

With the help of (30) rearrange (29) so as to define a function $J(\alpha)$ by

$$J(\alpha) = K_+(\alpha)\Phi_+(\alpha) + C_+(\alpha) = - K_-(\alpha)\Psi_-(\alpha) - C_-(\alpha) \tag{31}$$

So far this equation defines $J(\alpha)$ only in the strip $\tau_- < \tau < \tau_+$. But the second part of the equation is defined and is regular in $\tau > \tau_-$, and the third part is defined and is regular in $\tau < \tau_+$. Hence by analytic continuation $J(\alpha)$ must be regular in the whole α-plane. Then by the extended form of Liouville's theorem $J(\alpha)$ is a polynomial $p(\alpha)$

$$K_+(\alpha)\Phi_+(\alpha) + C_+(\alpha) = p(\alpha)$$

$$K_-(\alpha)\Psi_-(\alpha) + C_-(\alpha) = -p(\alpha) \qquad (32)$$

These equations determine $\Phi_+(\alpha)$, $\Psi_-(\alpha)$ to within the arbitrary polynomial $p(\alpha)$, i.e. to within a finite number of arbitrary constants which must be determined otherwise.

Keller's geometrical method :

Keller's theory of geometrical diffraction applied to elastodynamics states that the two conical surfaces of diffracted rays are generated when an incident ray strikes an edge. The surface of the inner cone consists of rays of longitudinal motion, while the surface of the outer cone is composed of rays of transverse motion. The half-angles of the cones are related by Snell's law. Fig.1 shows the cones generated by an incident longitudinal ray. For this case the diffracted longitudinal rays make the same angle ϕ_L with the tangent to the edge as the incident ray, and the diffracted rays of transverse motion are under an angle ϕ_T with the edge, where $C_L \cos\phi_T = C_T \cos\phi_L$.
For a straight diffracting edge, and an incident longitudinal ray, the diffracted displacement fields are related quantitatively to the incident field by

$$\vec{u}_d^L = e^{i\omega S_1/C_L} \left[S_1(1+S_1/R_i) \right]^{-1/2} D_L \hat{i}_L^d A e^{i\omega(S_0/C_L - t)}$$

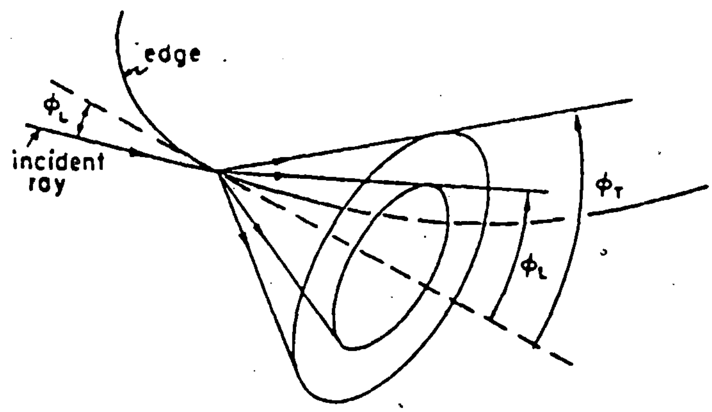

FIG. 1. Cones of diffracted longitudinal and transverse rays for an incident longitudinal ray.

$$\vec{u}_d^T = e^{i\omega S_2/C_T} \left[S_2(1+S_2/R_d)\right]^{-1/2} D_T \, \hat{i}_T^d \, A \, e^{i\omega(S_o/C_L - t)}$$

Here Aexp $[i\omega(S_o/C_L-t)]$ defines the amplitude and the phase of the incident field at the point of diffraction, and D_L and D_T are diffraction coefficents which relate the diffracted field to the incident field. Also S_1 and S_2 are the smaller of the principal radii of curvature of the diffracted wave front, or equivalently the distances along the diffracted rays from the points of diffraction to the observation point. The unit vectors $\hat{i}_L^d$ and $\hat{i}_T^d$ relate the directions of displacement of the diffracted field to the direction of displacement of the incident field. For a straight diffracting edge R_i is the radius of curvature at the point of diffraction of the curve formed by the intersection of the incident wave front and the plane which contains the incident ray and the edge, and

$$R_d = R_i \frac{\sin\phi_T \tan\phi_T}{\sin\phi_L \tan\phi_L}.$$

Papers involving the diffraction of elastic waves by two coplanar Griffith cracks are very few. Researches have been restricted to those of a single crack, because of the severe mathematical complexity encountered in finding solutions for two or more cracks. At first Jain and Kanwal (1972) overcame the difficulty and presented the solution for the diffraction problem of normally incident longitudinal and antiplane shear waves by two symmetrical coplanar Griffith cracks located in an infinite, isotropic and homogeneous elastic medium. However, they presented an approximate solution which is valid for low-frequency. Itou (1978) also studied the dynamic problem for an infinite elastic medium weakened by two coplanar Griffith cracks in which a self-equilibrated system of pressure is varied harmonically with time. To solve this problem, the author has expanded the surface displacement in a series of functions which is automatically zero outside the cracks and has used the Schmidt method. Itou (1980,1980) also solved two different problems involving two finite cracks. The problem of determining the transient stress distribution in an infinite elastic medium weakened by two coplanar Griffith cracks has been reduced to the following integral equation

$$\sum_{n=1}^{\infty} c_n(s)\left[-\frac{4c_L^3}{k^2 s^2 b}\int_0^{\infty} g(s,\xi)\sin\left(\frac{a+b}{2}\xi - \frac{n\pi}{2}\right)J_n\left(\frac{b-a}{2}\xi\right)\cos(\xi x)\,d\xi\right]$$

$$= -Pc_L(bs)\,, \quad a<x<b \tag{33}$$

with

$$g(s,\xi) = \frac{\left[\xi^2+k^2s^2/(2c_L^2)\right]^2 - \xi^2\gamma_1\gamma_2}{\xi\gamma_1} \tag{34}$$

where locations of the cracks are $a\le|x|\le b$, $|y|<\infty$, $z = 0$,

$c_L = \left(\frac{\lambda+2\mu}{\rho}\right)^{1/2}$, $c_T = \left(\frac{\mu}{\rho}\right)^{1/2}$, $k = c_L/c_T$ and $c_n(s)$ are the unknown coefficients.

To determine the coefficients $c_n(s)$ by Schmidt's method (1958) equation (33) can be rewritten as

$$\sum_{n=1}^{\infty} c_n(s)E_n(s,x) = -u(s,x) \quad , \quad a<|x|<b \tag{35}$$

where $E_n(s,x)$ and $u(s,x)$ are known functions and the coefficients $c_n(s)$ are unknown.

A set of functions $P_n(s,x)$ which satisfy the orthogonality condition

$$\int_0^b P_m(s,x)P_n(s,x)dx = N_n \,\delta_{mn} \quad , \quad N_n = \int_a^b P_n^2(s,x)dx \tag{36}$$

can be constructed from the function, $E_n(s,x)$, such that

$$P_n(s,x) = \sum_{n=1}^{\infty} \frac{M_{in}}{M_{nn}} E_i(s,x) \tag{37}$$

where M_{in} is the cofactor of the element d_{in} of D_n , which is defined as

$$D_n = \begin{vmatrix} d_{11} & d_{12} & \cdots\cdots & d_{1n} \\ d_{21} & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ \cdot & & & \\ d_{n1} & \cdots\cdots & \cdots & d_{nn} \end{vmatrix} \tag{38}$$

$$d_{in} = \int_a^b E_i(s,x)E_n(s,x)\,dx \;.$$

Using equations (35) and (36) one can obtain

$$c_n(s) = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{39}$$

with

$$q_j = \frac{-1}{N_j} \int_a^b u(s,x) P_j(s,x)\, dx \tag{40}$$

Recently, Verma and Jain (1990) have studied the problem of diffraction of obliquely incident longitudinal waves by two equal, parallel, and coplanar Griffith cracks located in an infinite, isotropic and homogeneous elastic medium. Using Green's functions the solution of this problem was first reduced to that of a pair of similar Fredholm integral equations of the first kind. When the wavelengths are large compared to the distance between the two outer edges of the two cracks, each integral equation of this pair was transformed to a set of Fredholm integral equations of the first kind with a simple kernel, which was solved by the inversion formula (Lowengrub and Srivastava, (1968).

Problems involving more than two finite cracks i.e. periodic array of coplanar finite cracks have been studied by Angel and Achenbach (1985), De Sarkar (1983), Garg (1981), Parihar and Lalitha (1987), Parton and Morozov (1978).

Another type of crack called cruciform crack has been studied by Brock and Deng (1985), Ong and Srivastava (1985).

Now we discuss the diffraction problem due to penny-shaped or annular cracks.

The transient stress and displacement fields around an embedded crack in the shape of a circle were first investigated by Embley and Sih (1971) for extensional impact and by Sih and Embley (1971) for torsional impact. Their method of solution involves isolating the singular portion of the dynamic stresses in the Laplace transform domain such that the dynamic stress intensity factor can be obtained by direct application of the Laplace inversion theorem. A penny shaped crack with its plane normal to the stretched direction of the elastic solid expanding at a constant velocity was considered by a number of investigators, namely Craggs (1966), Kostrov (1964) and Atkinson (1968). Sih and Loeber (1969) solved the problem of normal compression and radial shear waves impinging on a penny shaped crack. Other references are Mal (1970), Krenk and Shmidt (1982), Arin and Erdogan (1971), Ueda, Shindo and Atsumi (1983).

Krenk and Schmidt (1982) solved the problem of scattering of waves by a circular crack in an elastic medium by a direct integral equation method. The solution method was based on expansion of stresses and displacements on the crack surface in terms of trigonometric functions and orthogonal polynomials.

The general problem of two semi-infinite elastic media with different properties bonded to each other along a plane and containing a series of concentric ring shaped flat cavities was considered by Erdogan (1965), Using Green's functions for the semi-infinite plane, the problem was formulated as a system of simultaneous singular integral equations having cauychy type singularities. Shindo (1979,1981,1981) has studied different types of problem in elastodynamics involving flat annular crack. The problem of diffraction of normally incident torsional waves by a flat annular crack embedded in an infinite, isotropic, and homogeneous elastic medium was studied by Shindo (1979). The problem was reduced to that of solving the following triple integral equation :

$$\int_0^\infty \alpha^2 A(\alpha) J_1(\alpha r) d\alpha = \int_0^\infty \alpha g(\alpha) A(\alpha) J_1(\alpha r) d\alpha + (P_{30}/\mu)(r/b) \quad , \quad a<r<b$$

$$\int_0^\infty \alpha A(\alpha) J_1(\alpha r) d\alpha = 0 \quad , \quad 0 \le r \le a \ , \ b \le r \qquad (41)$$

Where $g(\alpha) = \alpha - \gamma(\alpha)$ in which the function $g(\alpha)$ has the order α^{-1} for large α. It is convenient to write the integral transform $A(\alpha)$ in terms of the finite integral given by

$$\alpha A(\alpha) = - \int_a^b t \phi(t) J_2(\alpha t) dt \qquad (42)$$

where J_2 is the second-order Bessel function of the first kind. On substitution of (42) in (41) yields the following singular integral equation of the first kind :

$$\frac{1}{\pi}\int_a^b \frac{1}{t}\,\phi(t)\left[\frac{b}{t-r} - \frac{3b}{2r}\log\left|\frac{2(t-r)}{(1-a_o)b}\right| + m_o(r,t) + m_1(r,t)\right]dt$$

$$= -P_{3o}/\mu\ , \quad a<r<b \qquad (43)$$

in which the Fredholm Kernels $m_o(r,t)$ and $m_1(r,t)$ are bounded in closed interval $a \le r,\ t \ge b$ and are given by :

$$m_o(r,t) = b\left[\left(\frac{(r/t)}{r+t} - \frac{2}{r}\right)E(t/r) + \frac{(r/t)E(t/r) - 1}{t-r} + \frac{4K(t/r)}{r} + \right.$$

$$\left. + \frac{3}{2r}\log\left|\frac{2(r-t)}{(1-a_o)b}\right|\right]\ , \qquad t < r$$

$$= b\left[\left(\frac{1}{t+r} - \frac{2t}{r^2}\right)E(r/t) + \frac{E(r/t) - 1}{t-r} + \frac{2tK(t/r)}{r^2} + \right.$$

$$\left. + \frac{3}{2r}\log\left|\frac{2(r-t)}{(1-a_o)b}\right|\right]\ , \qquad t > r \qquad (44)$$

$$m_1(r,t) = -\frac{\pi t^2 b}{r}\int_0^\infty g(\alpha)J_2(\alpha t)J_1(\alpha r)d\alpha \qquad (45)$$

Here K and E are the complete elliptic integrals of the first and second kind respectively, and $a_o = a/b$ is the radius ratio of the annular crack. From the second equation of (41) and the definition (42) it is clear that the integral equation must be solved under the following single valuedness condition :

$$\int_a^b \frac{1}{t}\,\phi(t)dt = 0 \qquad (46)$$

Substituting

$$R = r/b = \frac{1}{2}\,(1-a_o)s + \frac{1}{2}\,(1 + a_o)$$

$$T = t/b = \frac{1}{2}(1-a_o)r + \frac{1}{2}(1 + a_o) \tag{47}$$

$$P = bp/cT$$

$$\Phi(t) = \phi(t)/\left[(t/b)(p_{30}/\mu)\right]$$

the revised singular integral equation to the first kind (43) and single valuedness condition (46) are shown as

$$\frac{1}{\pi}\int_{-1}^{1} \Phi(\tau)\left[\frac{1}{\tau-s} - \frac{3(1-a_o)}{4R}\log|\tau-s| + \frac{1-a_o}{2}\left(M_o(s,\tau) + M_1(s,\tau)\right)\right]d\tau$$

$$= -1^{\circ} \tag{48}$$

$$\int_{-1}^{1} \Phi(\tau)\, d\tau = 0 \tag{49}$$

in which the Fredholm Kernels $M_o(s,\tau)$ and $M_1(s,\tau)$ are obtain by substituting (47) in (44) and (45) respectively. Thus the problem was reduced to the solution of the singular integral equation (48) under additional condition (49).

Following Erdogan (1973, 1963) the equations (48) and (49) has been solved by assuming

$$\Phi(\tau) = \frac{1}{(1-\tau^2)^{1/2}}\left[A_o + \sum_{n=1}^{\infty} A_n T_n(\tau)\right] \tag{50}$$

where $T_n(\tau)$ are Chebyshev polynomial of the first kind and A_n(n=0, 1, 2, ...) are unknown constants.

Bostrom and Olsson (1987) treated the problem of scattering of elastic waves by non-planar cracks. The method employed was a modification of the null field approach (T matrix method) where a fictitious surface was added to the surface of the crack so as to obtain a closed surface that should preferably be as sphere-like as possible.

Like elastic waves, diffraction of viscoelastic waves by crack or by inclusions are of considerable importance in view of their application of Seismology and Geophysics. Also the problems involving the motion of a punch on the surface of a viscoelastic half-space or on the free boundaries of long strips are extremely important in view of their application in road construction technology. Considerable studies had been made in the case of homogeneous media. But natural or artificial materieals are generally inhomogeneous. In addition, if the materials be dissipative, that effect can well be taken into account by considering the material to be viscoelastic.

Interest in the propagation of mechanical disturbances in viscoelastic media is comparatively recent in origin. The behaviour of extended anelastic structures under conditions of dynamic stressing is a development of the last fifty years. This situation contrasts with that obtained in the related field of linear elasticity in which technological requirements (e.g. the behaviour of brigdes under moving loads, the stresses involved in reciprocating mechanisms and the response of metals to shock loading) has stimulated much research into dynamical problems throughout the last hundred years. Perhaps the most important reason for the failure of a formal mathematical theory of viscoelasticity to develope stems from the lack of just such a practical stimulus. Traditionally, engineering design has made use of materials whose properties are adequately described in the working range by the laws of classical elasticity. However, with the gradual introduction of engineering compnents fabricated from the new synthetic plastic materials, it seems probable that the study of dynamic viscoelasticity will become a subject of increasing importance.

Other pertinent reasons contributing to the lack of a formal theory of dynamic viscoelasticity are the relative complexity of the equations describing the fundamental mechanical properties of anelastic solids and a lack of knowledge of these properties for many of the common materials. Fortunately, with the introduction of integral transform techniques by Gross (1947), a major simplification has been effected in handling the mathematical aspects of the subject. At the same time much experimental work, mostly dating from the publication of a paper by Alexandrov and Lazurkin (1940) on the mechanical properties of rubber, has been devoted to elucidating and classifying the behaviour of many of the rubber and polymer type materials. Here we give some references of papers on experimental investigation of some solids. Nolle (1949) (work on rubbers), Zener (1948) (metals), Sherby and Dorn (1958) (perpex) and Leadermann (1943)(silk, rayon, nylon). A generalized viscoelastic solid is specified by the existence of a functiuonal equation of state connecting stress (σ), strain (ε), time (t) and temperature (T).

$$F(\sigma,\varepsilon,t,T) = 0.$$

Presupposition of the existence of such a relation which may include time differential and integral operators of arbitrary order, immediately excludes problems associated with the plastic deformation of metals and the fracture of solids.

The simplest examples of linear viscoelastic solids are well known, e.g. Voigt solid, Maxwell solid, standard linear solid, Burgers solid, Newtonian fluid, etc. Diagrams of some models are shown in figure 2.

Recently and extensive study on boundary value problems in linear viscoelasticity has been made by Golden and Graham (1988) in their book. The problem of a rigid cylinder rolling on the surface of a viscoelastic half-space has been solved by Hunter (1961). The contact problem of rigid cylinder rolling slowly on a thin viscoelastic layer has been treated by Alblas and Kuipers (1970) assuming that the layer thickness is small compared to the width of the contact region of the cylinder. some contact punch problems in viscoelastic medium have been studied by Golden (1977, 1979,

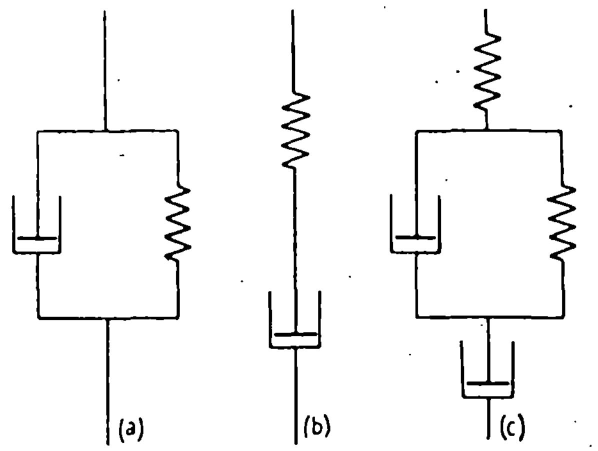

FIG. 2. Models of visco-elastic solids. (a) Voigt solid; (b) Maxwell solid; (c) more general solid.

1982). Crack propagation in viscoelastic medium has been studied by Willis (1972), Atkinson and List (1972), coussy (1987) and others. Willis (1972) considered steady-state Mode III crack propagation for a standard linear solid under general type of loading on the crack surfaces. Atkinson and List (1972) studied nonsteady SH-wave type crack propagation starting at t = 0 and moving with a constant velocity in the 'Maxwell solid' or using the viscoelastic model suggested by Achenbach and Chao. Sills and Benveniste (1981) and Coussy (1987) studied steady state crack propagation of SH-type at the interface between two visco-elastic media.

Recently, the transient elastodynamic stress intensity factor was determined for a cracked linearly viscoelastic body under impact by Georgiadis, Theocaris and Mouskos (1991). The body was considered to be infinite containing a finite crack. The solution was obtained by correspondence principle and the use of the Dubner-Abate-Crump Laplace-transform inversion technique.

In the thesis presented here we have studied some mixed boundary value problems in elastodynamics involving punches, inclusions and cracks. The work has been presented in three chapters. The first two chapters I and II deals with diffraction problems in elastic medium and the third chapter deals with diffraction problems in viscoelastic medium. Here we give the summeryof the thesis chapter wise.

In chapter-I, problem-1 contains vertical vibration of two rigid strips in smooth contact with a semi-infinite elastic medium. It is assumed that motion is forced by prescribed displacement distribution $v_o e^{-i\omega t}$ normal to the two strips located in the region $-a \le x \le -b$, $b \le x \le a$, $y = 0$, $|Z| < \infty$, where v_o is constant. The resulting mixed boundary value problem is reduced to the solution of a triple integral equation which has further been reduced to the solution of an integro-differential equation. Finally iterative solution valid for low frequency has been obtained. The integral equation was solved in a manner similar to that employed by Lowengrub and Srivastave (1968) in solving static problems for two caplanar cracks in an infinite elastic medium. From the solution of the integral equation, we have found out stresses just below the strips and also the vertical displacement at point outside the strips on the free surface. Low frequency solution due to antiplane motion of two strips on a semi-infinite elastic medium has also been derived.

In paper-2, we have considered the problem of diffraction of elastic waves by a apair of coplanar rigid strips between two homogeneous elastic half spaces for the case of antiplane strain. The resulting triple integral equation has been reduced to the solution of an integro differential equation and approximate solution has been obtained. These solutions have been used to obtain approximate values of the displacement field and also the stress intensity factors at the edges of the strips.

Making the distance between the inner edges of the strips tend to zero, the diffraction problem for a single rigid strip has been obtained. Even this result of the limiting case appears to have been presented here for the first time.

In third paper of this chapter we have studied the two dimensional problems of diffraction of elastic waves by four coplanar parallel rigid strips moving steadily on the free surface of a semi-infinite isotropic elastic medium. By Fourier transform the five part mixed boundary value problem has been reduced to the solution of a set of four integral equations. Following the technique, developed by Srivastava and Lowengrub (1970), the quadruple integral equations have been solved. The normal stress under the strips and displacement outside the strips are derived in closed form. The effect of stress intensity factors at the edges of the strips is shown by means of graphs. Also letting the strip velocity tend to zero the results for statical problem have been presented in this paper as a particular case.

In the last problem, i.e., paper-4 of chapter-I, we investigated the diffraction of torsional wave by a rigid annular disc at the interface of two bonded dissimilar elastic media. Here we have assumed that an antiplane shear wave given by $\Omega_2 r e^{ik_2(z-c_2t)}$, where Ω_2 is a constant, $k_2=\omega/c_2$ and $c_2=\surd(\mu_2/\rho_2)$, the shear wave velocity in medium 2, be incident normally on the annular rigid disc of inner and outer radii b and a respectively. Applying the method developed by Williams (1963) and used subsequently by Thomas (1965) and Jain et al (1970), the three part mixed boundary value problem has been reduced to the solution of a set of integral equations. The solutions of these integral equations are obtained iteratively for low frequency and small values of the ratio of the inner and outer radii of the disc. These solutions are used to determine the jump in stresses across the annular disc and stress intensity factors at both the edges of the disc. Torque and far field amplitudes in both the media have also been deduced. The effect of normalized frequency, material properties and geometric parameters in stress intensity factors and far field amplitude are shown graphically.

First problem of chapter-II deals with the interaction of normally incident time harmonic elastic waves with a periodic array of coplanar Griffith cracks in an infinite orthotropic medium. Due to geometrical symmetry the problem has been reduced to the solution of the problem of a single crack in a strip whose boundaries are shear free and constrained in a way not to permit normal displacement. Fourier transform has been used to reduce the problem to the solution of dual integral equations. By the application of Abel's integral the dual integral equations finally has been converted to a Fredholm integral equation. Stress intensity factor at the tip of the crack and crack opening displacement have been derived in closed form. To display the influence of the material orthotropy numberical values of stress intensity factor and crack opening displacement have been derived in closed form. To display the influence of the material orthotropy numerical values of stress intensity factor and crack opening displacement have been found out after solving the Fredholm integral equation numerically and plotted against dimensionaless frequency, distance respectively for three sets of orthotropic materials.

In the second paper of chapter II we have studied the diffraction of normally incident SH-waves by a Griffith crack situated in an infinitely long inhomogeneous elastic strip. The shear modulus (μ) and the density (ρ) of the material have been assumed to vary both in horizontal and vertical directions. Applying Fourier transform the mixed boundary value problems has been converted to the solution of dual integral equations. The dual integral equations finally has been ruduced to a Fredholm integral equation of second kind by applying Abel transform. Expressions for stress intensity factor and crack opening displacement have been derived. The numerical values of stress intensity factor and crack opening displacement have been depicted by means of graphs to show the effect of material inhomogeneity.

In chapter-III, first paper deals with the analysis of the stress and displacement field produced by a long punch moving on the boundary of a semi-infintie viscoelastic medium and producing Horizontal Shear waves. Two types of viscoelastic models viz. Maxwell Solid and Standard Linear Solid have been considered and loading in assumed to be such that Mode III conditions prevail. The mathematical technique which is employed here consists of the application of integral transforms and the solution of the resulting Wiener-Hopf equations for the transformed unknown variables. Both the steady and nonsteady solutions of the problem have been derived. Displacement and stress on the free surface and at points below the punch have been derived analytically and the nature of their variations with the velocity of the moving punch has been shown by means of graphs.

The last paper of the this chapter contains the analysis of steady and nonsteady cases of Mode III crack propagation in an inhomogeneous viscoelastic medium. Two types of viscoelastic models, namely Maxwell solid and Standard Linear solid have been considered. Material properties have been assumed to vary exponentially in the direction perpendicular to the direction of crack propagation. The problem has been solved by using Wiener-Hopf technique. We have studied how the material inhomogeneity affects the stress intensity factor and also the crack opening displacement when a Mode III type crack propagates through the inhomogeneous viscoelastic medium.

With this much of introduction, we now present the thesis chapterwise. References given in the thesis do not include all the previous workers in this line. But attempt has been made to include most of them.