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Emerging Spacetime from Quantum Field Theory

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This article provides some glimpses of recent developments towards understanding the emergence of spacetime from quantum many-body dynamics. The idea goes under the name of Gauge/Gravity or Gauge/String duality. In this article, I review two recent developments that took place during the past decade, namely, the quantum entanglement and quantum complexity in strongly coupled Quantum Field Theories and their interplay with the gravitational dynamics in one higher dimension.

I. INTRODUCTION

It has been more than three decades by now since the discovery of the celebrated correspondence between Quantum Gravity (or more firmly the String Theory) living in $d + 1$ spacetime dimensions and generic classes of (non-Abelian) Quantum Field theory living in d spacetime dimensions [1, 2]. This correspondence is based on a sacred principle of nature, often called the "holographic principle" which states that the duality principle is a non-perturbative statement, meaning that it should be valid for all ranges of couplings. For a nice and comprehensive review on the subject, see [3, 4]. Until now, this duality has been tested with remarkable precision, revealing astonishing matches that are based on the underlying symmetry algebra and the computation of the correlation functions on either side of the duality [5]. Although the duality conjecture is a non-perturbative statement, in reality, it is often useful to test its consequences in certain limits of the coupling. This limit corresponds to setting the Newton's constant G_N small enough such that all the quantum gravity loop corrections are significantly small, and hence only tree level scattering on the gravitational counterpart is important, which is referred to *classical* theory of general relativity.

On the field theory counterpart, the classical gravitational limit corresponds to taking a large $N_c \to \infty$ limit, where N_c corresponds to the number of colors in gauge theory. One of the profound realizations of duality is the identification of the gravitational coupling with the number of colors in the gauge theory, that is, $G_N \sim \frac{1}{N_c}$ in some appropriate units of length. The duality involves another set of couplings, namely the string scale, defined as $\alpha' \sim l_s^2$, which typically characterizes the size of the tiny little string moving in a spacetime which has an inherent length scale L. One can therefore introduce a dimensionless coupling constant, which we define as the t'Hooft coupling $\lambda = \frac{L^4}{\alpha'^2}$. On the gravitational side of the duality, the parameter λ controls the stringy corrections to the classical geometry. These corrections are important when the string size (l_s) is comparable to the size (L) of the spacetime it is propagating, namely $l_s \sim L$. Such corrections are negligible when $l_s \ll L$, which is sometimes referred to as the particle or the

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classical limit of the string. In this limit, all higher curvature (or stringy effects) are negligible, and one is left with the classical Einstein gravity of the form :

$$S_{gravity} = \frac{1}{16\pi G_N} \int \sqrt{-g}R + S_{matter} + S_{GH} + S_{ct}. \tag{1}$$

On the field theory counterpart, the classical string limit corresponds to a domain in which the gauge theory becomes strongly coupled $\lambda \gg 1$. We will further elaborate on this. On the gauge theory side, we have two parameters (each of which is associated with a perturbative expansion), namely the number of colors N_c and the t'Hooft coupling λ. Setting $N_c \to \infty$ switches off all the surfaces of higher genus corrections that appear with various powers of $1/N_c$, leaving us only diagrams in the sphere. All of these Feynman graphs appear at an order N_c^2, and are therefore diagrams of a classical theory or the theory at the tree level. These graphs can be further reorganized in the powers of the t'Hooft coupling (λ), which serves as a parameter of a perturbation series in the standard sense of a Quantum Field Theory.

In summary, classical Einstein gravity in $d+1$ dimensions serves as an equivalent description of a strongly coupled ($\lambda \gg 1$) (non-Abelian) gauge theory in d dimensions, in the limit of a large number of colors $N_c \to \infty$. For the rest of our discussion, we shall restrict ourselves to the above limit of the correspondence where the string theory, or equivalently the gravity in the bulk, is purely classical while the gauge theory is non-perturbative in the sense of the t'Hooft coupling (λ). In this particular limit, the identification of the partition functions on both sides of the duality becomes quite intuitive. For example, considering the fact that the gravitational theory lives in a $d+1$ dimensional manifold $\mathcal{M}$ with a d-dimensional boundary $\partial\mathcal{M}$, we have a precise mapping of the two partition functions, namely

$$Z_{gravity}\Big|_{\partial\mathcal{M}} = Z_{QFT}. \tag{2}$$

Given the identification (2), the correlation functions of the QFT living on the boundary can be computed from the gravitational partition function in the bulk by taking appropriate boundary limits of various fields propagating in the bulk $d+1$ dimensional manifold ($\mathcal{M}$).

With the above introduction to the subject, we are now in a position to discuss the potential applicability of the gauge/string duality that allows us to unveil various non-perturbative phenomena in a Large N_c QFT that are inaccessible in the standard perturbative expansion of the partition function. In particular, we discuss two important developments in QFTs that have happened in the last decade, namely the Entanglement Entropy [6] and the Quantum Complexity [7]. The goal of our discussion would be how to compute these entities in a strongly correlated gauge theory living in d-dimensions, which allows for a dual description in terms of classical gravitational dynamics in $d+1$ dimensions.

II. QUANTUM ENTANGLEMENT AND EMERGENCE OF SPACETIME

Consider the d-dimensional boundary $\partial\mathcal{M}$ of the $d+1$-dimensional spacetime $\mathcal{M}$, where quantum degrees of freedom are distributed. For our purpose, we take a spatial ($t=0$) slice as shown in Fig.1. We divide the spatial region into two sub-regions, namely A and B. The total Hilbert space is a direct product space of the form $\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$. Given the above structure, we consider a pure state $|\Psi\rangle \in \mathcal{H}$. Our goal is to understand how the degrees of freedom in the region A are

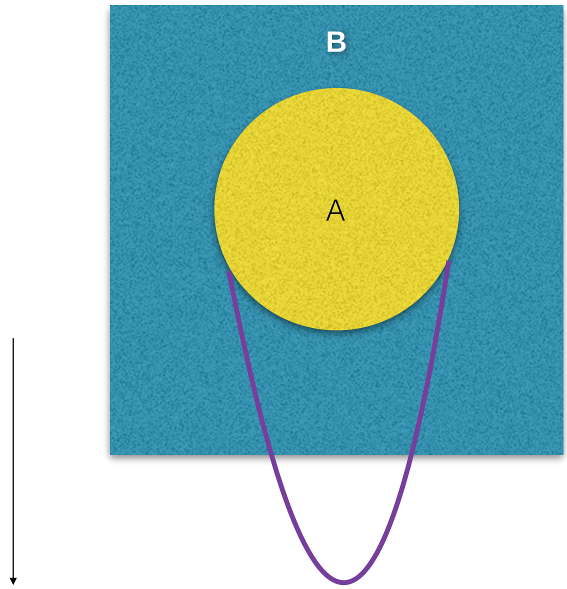

FIG. 1. The sub-regions in the boundary theory. The extremal surface (purple line) is extended along the bulk radial direction, which is shown by the arrow, adding a third dimension to the picture. This extra dimension emerges from the quantum entanglement between sub-regions A and B.

entangled with the rest of the degrees of freedom that are distributed in the region B. To quantify the degree of entanglement one introduces the notion of entanglement entropy, which we define below.

The first step towards defining the entanglement entropy is to introduce the reduced density matrix $\rho_A = \text{Tr}_B(|\Psi\rangle\langle\Psi|)$, which is obtained by tracing out the degrees of freedom pertaining to region B from the total density matrix. The amount of quantum entanglement between A and B is quantified in terms of the von Neumann entropy, which is also known as the entanglement entropy [6, 8, 9]

$$S_{EE} = -\text{Tr}_A(\rho_A \log \rho_A). \tag{3}$$

Computing entanglement entropy in Large N QFTs is, in general, a non-trivial task. In local QFTs, one often encounters UV divergences due to short-range correlations across the boundary ∂A of the spatial region A.

A remarkable proposal to compute quantum entanglement in large N strongly correlated QFTs has been proposed by the authors in [6]. Their proposal invokes the idea of the extremal surface that extends along the bulk radial direction whose boundary coincides with ∂A. In other words, we have an emerging extra dimension as a result of quantum entanglement in QFTs living at the boundary ($\partial\mathcal{M}$) of the spacetime ($\mathcal{M}$), see Fig. 1. The compelling evidence behind their proposal comes from a simple computation of the extremal surface in three-dimensional Anti-de Sitter

(AdS_3) space whose metric can be expressed as

$$ds^2 = \frac{1}{z^2}(-dt^2 + dx^2 + dz^2) \tag{4}$$

where z is the emergent radial direction that gives rise to the three-dimensional bulk. The field theory dual to background (4) is a conformal field theory (CFT) in two dimensions. Here (t, x) are the boundary directions in which the CFT lives.

The extremal surface is obtained by setting $t = 0$, which yields spatial intervals along the real line $\mathbb{R}$ and, thus, choosing an embedding $x = x(z)$. The resulting induced metric can be expressed as

$$ds^2_{ind} = \frac{1}{z^2}(x'(z)^2 + 1)dz^2. \tag{5}$$

The extremal surface can be obtained by knowing the equation of motion for $x(z)$ and thereby substituting it into the definition of the area functional $\mathcal{A}_{ext}$. The holographic principle states that the entanglement entropy in the Large N_c 2d CFT can be computed by knowing the area of the extremal surface [6, 8, 9]

$$S_{EE} = \frac{\mathcal{A}_{ext}}{4G_N}. \tag{6}$$

These ideas have recently been extended to *timelike* separated intervals in the QFTs, where one considers different timelike slices in the boundary theory and the associated measure of quantum entanglement between these intervals [10–14]. This gives rise to the notion of timelike entanglement entropy, which is typically associated with a complex density matrix commonly known as the transfer matrix [12]. In the dual gravitational counterpart, the extremal surface is parametrized by the choice $t = t(z)$ and $x = x(z)$. The corresponding induced metric characterizing the surface is given by

$$ds^2_{ind} = \frac{1}{z^2}(-t'(z)^2 + x'(z)^2 + 1)dz^2. \tag{7}$$

The extremal surface ($\mathcal{A}_{ext}$) is obtained by substituting the solutions to the equations of motion pertaining to $t(z)$ and $x(z)$. Notice that the results of spatially separated events can be obtained as a special choice $t =$ constant. On the other hand, setting $x =$ constant, one obtains a timelike entanglement between purely timelike separated events [11]. It appears that the corresponding extremal surface is complex and is associated with a complex saddle characteristic of a timelike separation between intervals. A field-theoretic interpretation of timelike entanglement is underway and represents an exciting future direction of investigation.

III. QUANTUM COMPLEXITY AND GAUGE/GRAVITY DUALITY

Complexity has emerged as one of the most intriguing ideas of modern quantum field theory and has recently gained renewed attention in the context of quantum information and quantum computing. In quantum computation, the idea simply translates into the fact of how many fundamental/basic operations are needed to perform to accomplish a complicated algorithm in a quantum simulation. In the context of circuit complexity, the entity measures how many elementary gates are required to build a complicated electronic network. In QFTs, the notion of complexity measures the number of degrees of freedom in a theory, which is thereby related to the central charge in a conformal field theory. There have been several proposals in gauge/gravity duality that eventually compute some form of complexity in a Large N_c gauge theory at strong coupling. These go under the name of the proposals "complexity=volume" [15], "complexity=area" [16] and "complexity=anything" [17]. It would be nice to have a consensus among these approaches, which has yet to be achieved. The complexity we will be describing here is different from the one above and goes by the name Krylov or spread complexity. Generally speaking, this measures the rate of spread of a state $|\psi(t)\rangle$ or the operator $O(t)$ in a subspace of the full Hilbert space, which is denoted as the Krylov basis [18–21]. In the following, we elaborate more on this. We begin by considering the spread complexity of states. To understand K-complexity growth, we introduce a one-dimensional chain (which we denote as a Krylov chain) that is spanned by a discrete set of lattice points, which defines a Krylov basis $\{|K_n\rangle\}$. Krylov basis emerges naturally in a Hamiltonian time evolution of a state

$$\begin{aligned} |\psi(t)\rangle &= e^{-iHt}\,|\psi(0)\rangle \\ &= (1 - itH + \frac{(it)^2}{2!}H^2 + \cdots)\,|\psi(0)\rangle \end{aligned} \tag{8}$$

which can be summarized by considering an expansion in a Krylov basis of the form

$$|\psi(t)\rangle = \sum_n \psi_n(t)\,|K_n\rangle\,. \tag{9}$$

The Krylov basis can be identified as

$$|K_n\rangle = \{|0\rangle\,, |1\rangle\,, \cdots\} = \{|\psi(0)\rangle\,, H\,|\psi(0)\rangle\,, \cdots\}. \tag{10}$$

Physically, Eq. (9) translates into the fact that one starts with a state at ($t = 0$), the zeroth lattice site characterized by a state $|0\rangle$, which spans over the n-th site characterized by $|n\rangle$ at time t. In other words, Krylov complexity measures the rate of spread of the wavefunction over discrete lattice points characterized by Krylov basis elements $|K_n\rangle$. The coefficients $\psi_n(t)$ produce the probability distribution $p_n(t) = |\psi_n(t)|^2$ that satisfies a discrete Schrodinger Eq. [21]

$$i\partial_t\psi_n = a_n\psi_n + b_n\psi_{n-1} + b_{n+1}\psi_{n+1}, \tag{11}$$

where a_n and b_n are the Lanczos coefficients. Finally, we define Krylov complexity as the average position on the 1d Krylov chain

$$\mathcal{C}(t) = \langle n\rangle = \sum_n n p_n(t). \tag{12}$$

Next, we discuss the Krylov complexity associated with the growth of the local operator $O(t)$.

Local operators are typically associated with a time evolution of the form

$$\begin{aligned} O(t) &= e^{iHt}O(0)e^{-iHt} \\ &= (1 + it[H,] + \frac{(it)^2}{2!}[H, [H,]] + \cdots)O(0). \end{aligned} \tag{13}$$

By introducing a (super) Liouville operator $\mathcal{L} = [H, \cdot]$, one can rewrite Eq. (13) as a linear combination

$$O(t) = \sum_n \frac{(it)^n}{n!}\mathcal{L}^n O(0) = \sum_n i^n \varphi_n(t) O_n. \tag{14}$$

Like before, we have a one-dimensional Krylov chain spanned by a discrete set of orthonormal bases $O_n \sim \mathcal{L}^n O(0)$ generated through nested commutators. Physically, the above relation (14) describes the spread of operator growth (over a certain domain of time) starting with its initial value $O(0)$. The coefficients $\varphi_n(t)$ are associated with the probability distribution $p_n(t) = |\varphi_n(t)|^2$, which satisfy an equation similar to that of Eq. (11). Finally, we define the Krylov complexity as the average on the n-th site

$$\mathcal{C}(t) = \sum_n n|\varphi_n(t)|^2. \tag{15}$$

In a gauge/gravity setup, the notion of complexity of states corresponds to the fact that the dual geometry is evolving with time. On the other hand, the complexity associated with the growth of operators is related to the trajectory of a massive probe particle along the geodesic, where the increasing momentum of the particle is related to the rate of growth of complexity in the dual quantum theory [7, 21]. This idea has been recently established based on a set of precise calculations in the AdS_3/CFT_2 setup and can be expressed as [21]

$$\partial_t C(t) = -\frac{P}{\epsilon} \tag{16}$$

where P is the proper radial momentum of the particle and ϵ is the UV cut-off. The actual computation of Krylov complexity is a tedious task, as one has to simulate over n lattice points to solve the wave function(s) ψ_n or φ_n, depending on the type of complexity in which one is interested. The final goal of these computations is to figure out the Lanczos coefficients (a_n and b_n) that save the day for us. These computations become extremely simple in the presence of an underlying conformal symmetry, for example, an $SL(2, R)$ symmetry of a conformal Krylov chain [21]. The other miraculous way out is offered by gauge/gravity duality mentioned above in Eq. (16).

IV. SUMMARY AND CONCLUSIONS

For the past three decades, we have witnessed overwhelming evidence behind the celebrated correspondence between the theory of quantum gravity and quantum field theory. The theory of quantum gravity (or string theory) lives in one dimension higher than the quantum field theory. This is in accordance with the NO-GO theorem due to Weinberg and Witten [22], which states that in a local and "renormalisable" quantum field theory with a covariant energy momentum tensor, there cannot be any form of a composite or elementary particle with spin greater than one. In other words, the graviton with $s = 2$ must live in spacetime dimensions higher than gauge bosons carrying spin one.

The duality has opened up a vast landscape of possibilities, of which only two have been reviewed here, leaving the rest untouched. The most remarkable fact about this duality is that it offers a platform to compute various observables in the non-perturbative domain of a generic quantum field theory, where the standard perturbative techniques are unavailable. At a deeper level, the duality offers an understanding of the non-perturbative nature of the theory of quantum gravity, which might offer a dual description in terms of some Large N_c non-Abelian gauge theories that are tractable in the sense of standard perturbative techniques.

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