Chapter 5

Quark-Diquark Stars

5.1 Introduction

One of the most interesting features of a compact star is its mass to radius ratio. The X-ray pulsar Her X-1 is a compact object whose mass-radius constraint has attracted considerable attention in recent years. Her X-1, one of the best studied X-ray pulsars, has a pulse period of 1.24 s, has a strong magnetic field ($\sim 3 \times 10^{12}$ G), and its average luminosity is $\sim 2 \times 10^{37}$ $erg\ s^{-1}$, assuming its distance to be 5 kpc [40]. Based on the observational data (e.g., luminosities, pulse periods, spin up rates etc.), Wasserman and Shapiro [135] constructed semi-empirical mass-radius relations for two X-ray pulsars, Her X-1 and 4U 0115 + 63. Later Li *et al* [40] determined the mass-radius relation of Her X-1 semi-empirically from the observations of its spin variation and cyclotron spectral lines. The estimated mass and radius of Her X-1 are 0.98 ± 0.12 $M_{\odot}$ and 6.7 ± 1.2 km, respectively. If this semi-empirical mass-radius relation for Her X-1, derived by Li *et al* [40], is correct, the question of the composition of Her X-1 becomes very interesting since standard neutron star EOS cannot give this compactness. Schertler *et al* [136] showed that a quark phase softens the equation of state (EOS) of neutron star matter at high densities leading to a more compact equilibrium configuration.

Considering MIT bag model to describe the quark phases, Li *et al* [40] suggested that Her X-1 might be a strange star. But Madsen [128] pointed out that strange matter is unstable for a bag constant $B^{1/4}$ above 164 Mev, considered in [40]. Very recently, Horvath and Pacheco [72] have made the suggestion that Her X-1 might be a mixture of free quarks and diquarks, the latter having a self-interaction described by an effective $\frac{\Lambda}{4}|\Phi|^4$ potential. Kastor and Traschen [76] computed an approximate EOS for a quark-diquark mixture and showed that at a comparatively low density this has the form of a polytrope $p(\rho) = K\rho^\Gamma$, with adiabatic index $\Gamma = 2$, where p and ρ denote pressure and energy density of the star and K is a dimensional quantity. The maximum mass and radius obtained by them for a star having a quark-diquark core surrounded by a less dense envelope of nuclear matter were 1.79 $M_\odot$ and 11.4 km, respectively. Her X-1, however, is a more compact object. Considering all these aspects, the suggestion of Horvath and Pacheco [72] merits a detailed investigation.

Here, we study the problem of Her X-1 from a different angle. Considering the estimated mass and radius of Her X-1 as input parameters in the model outlined in chapter 2, we find the EOS of the star for $\lambda = 100$, which agrees with the relevant equation of state, given by Horvath and Pacheco [72]. However, the conclusion that can be drawn from our study differs from that of Horvath and Pacheco [72].

5.2 Combined boson-fermion star

The EOS obtained by Horvath and Pacheco [72] was based on earlier works by Ruffini and Bonazzola [57] and Colpi *et al* [58], who studied the equilibrium configuration of a system of massive scalar field. The possibility that scalar fields present in the early Universe could condensate to form stars, known as boson stars, have been studied first by Kaup [56] and Ruffini and Bonazzola [57]. Boson stars are gravitationally bound, spherically symmetric configurations of complex scalar fields minimally coupled to gravity as given in general relativity. Theories of inflation of the Universe prefer a flat Universe which requires non-baryonic dark matter. Boson stars may provide a considerable fraction of the non-baryonic part of dark matter within the halo of galaxies [55]. The physical nature of the spin 0-particle out of which a boson star is formed is still an open issue. Until now, no fundamental elementary scalar particle has been found in accelerator experiments which could serve as the main constituent of the boson stars. One of the first attempts to incorporate a scalar field in the theory of gravity were made by Brans and Dicke [137]. Colpi *et al* [58] proved that the existence of a self-interaction in the bosonic field Lagrangian could yield higher values for the masses of the configurations. The self-interaction term has the common form $\frac{\Lambda}{4}|\phi|^4$. Since then, many papers have been published on boson stars (e.g, [138] & [139]). A review of these models can be found in [60]. Gleiser [139] investigated the dynamical stability of boson stars against small radial oscillations. Henriques *et al* [59] and Jetzer [60] presented a new model for cold stellar objects composed of both bosons and fermions, parametrized by the central densities of the bosons and fermions. A more realistic description of the system were introduced by de Sousa and Tomazelli [140] who introduced an effective coupling between bosons and fermions. Sahkamoto and Shiraishi [141] gave an exact solution for a non-rotating boson star in $(2+1)$ dimensional gravity with a negative cosmological constant and later extended the work to the case of a boson-fermion star. They also found solutions for rotating boson stars in $(2+1)$ dimensional gravity [142]. There exists a decisive difference between self-gravitating objects made of fermions or bosons: For a many fermion system the Pauli exclusion principle forces the typical fermion into a state with very high quantum number, whereas many bosons can coexist all in the same ground state (Bose-Einstein condensation). This feature is exhibited in the critical particle number N_{crit} for a stable configuration; for fermions $N_{crit} \simeq (\frac{m_{pl}}{m})^3$, whereas, for massive bosons without self-interaction $N_{crit} \simeq (\frac{m_{pl}}{m})^2$, where m_{pl} is the Plank mass.

We assume that boson-fermion stars are static and spherically symmetric with no magnetic field. We consider the standard line element for such stars,

$$ds^2 = -e^{2\gamma(r)}dt^2 + e^{2\mu(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{5.1}$$

The Lagrangian for the bosonic field may be chosen as

$$L_{eff} = \frac{1}{2}\left(\partial_\mu\Phi^*\partial_\nu\Phi - m^2\Phi^*\Phi\right) - \frac{\Lambda}{4}\left(\Phi^*\Phi\right)^2. \tag{5.2}$$

Here, m is the bosonic mass and Λ is a dimensionless coupling constant.

Energy-momentum tensor for the bosonic field has the form

$$T^B_{\mu\gamma} = \frac{1}{2}\left(\Phi^*_\mu\Phi_\gamma + \Phi_\mu\Phi^*_\gamma\right) - \frac{1}{2}g_{\mu\gamma}\left[g^{\rho\sigma}\Phi^*_\rho\Phi_\sigma + m^2|\Phi|^2 + \frac{1}{2}\Lambda|\Phi|^4\right] \tag{5.3}$$

Energy-momentum tensor for the fermionic field can be written as

$$T^F_{\mu\gamma} = \left(\rho^F + p^F\right)u_\mu u_\gamma + g_{\mu\gamma}p^F \tag{5.4}$$

where, u^μ is the 4-velocity of the fluid. The energy-momentum tensor for a mixed boson-fermion configuration is given by

$$\tau_{\mu\gamma} = T^B_{\mu\gamma} + T^F_{\mu\gamma}. \tag{5.5}$$

The form of the scalar field, giving a spherically symmetric matter distribution, is assumed to be

$$\Phi(r,t) = \phi(r)e^{-i\omega t}, \tag{5.6}$$

which ensures the possibility of a static solution.

For the line element (5.1), the field equations now lead to (introducing $8\pi G$)

$$\frac{(1-e^{-2\mu})}{r^2} + \frac{2\mu' e^{-2\mu}}{r} = 4\pi G\left[2\rho_f + \left(\omega^2 e^{-2\gamma} + m^2\right)\phi^2 + \phi'^2 e^{-2\mu} + \frac{1}{2}\Lambda\phi^4\right] \tag{5.7}$$

$$\frac{2\gamma' e^{-2\mu}}{r} - \frac{(1-e^{-2\mu})}{r^2} = 4\pi G\left[2p_f + \left(\omega^2 e^{-2\gamma} - m^2\right)\phi^2 + \phi'^2 e^{-2\mu} - \frac{1}{2}\Lambda\phi^4\right] \tag{5.8}$$

$$e^{-2\mu}\left(\gamma'' + \gamma'^2 - \gamma'\mu' + \frac{\gamma'}{r} - \frac{\mu'}{r}\right) = 4\pi G\left[2p_f + \left(\omega^2 e^{-2\gamma} - m^2\right)\phi^2 - \phi'^2 e^{-2\mu} - \frac{1}{2}\Lambda\phi^4\right]; \tag{5.9}$$

The scalar wave equation

$$\Phi^{\mu}\Phi_{\mu} - m^2\Phi - \Lambda|\Phi|^2\Phi = 0, \tag{5.10}$$

now takes the form

$$\phi'' + \left(\frac{2}{r} + \gamma' - \mu'\right)\phi' + \left[(\omega^2 e^{-2\gamma} - m^2) - \Lambda\phi^2\right]e^{2\mu}\phi = 0. \tag{5.11}$$

From eqnuations (5.8) & (5.9), we find that the pressure is not isotropic. The pressure anisotropy essentially depends on the self-interaction. But if ϕ' is very small, i.e., if we can neglect ϕ', we get,

$$\gamma'' + \gamma'^2 - \gamma'\mu' - \frac{\gamma'}{r} - \frac{\mu'}{r} - \frac{(1 - e^{2\mu})}{r^2} = 0. \tag{5.12}$$

This equation has already been taken up in chapter 2. Assuming

$$e^{2\mu} = \frac{1 + \lambda r^2/R^2}{1 - r^2/R^2}, \tag{5.13}$$

we get the solution of this equation given by Mukherjee *et al* [67],

$$\psi(z) = e^{\gamma} = A\left[\frac{\cos[(n+1)\zeta + \delta]}{n+1} - \frac{\cos[(n-1)\zeta + \delta]}{n-1}\right]. \tag{5.14}$$

Thus if the scalar field is almost homogeneous, the Vaidya-Tikekar [63] type of model may be used to study the stellar structure. It may so happen that the scalar field is essentially homogeneous inside the star. To see this we rescale our parameters as

$$\Lambda_* = \frac{\Lambda}{4\pi G m^2}, \qquad r_* = \frac{rm}{\sqrt{\Lambda_*}}$$

$$\sigma = \sqrt{\Lambda_*}\phi, \qquad \rho_*^F = \frac{4\pi G\Lambda_*\rho^F}{m^2}$$

$$p_*^F = \frac{4\pi G\Lambda_* p^F}{m^2}, \qquad \Omega = \frac{\omega}{m}$$

In the case large self-interaction, i.e., $\Lambda_* >> 0$, we may neglect terms of $O(\Lambda_*{}^{-1})$ and rewrite the equations (5.7)-(5.9) and (5.11) in terms of dimensionless quantities as

$$\frac{(1-e^{-2\mu})}{r_*^2}+\frac{2\mu' e^{-2\mu}}{r_*}=\left[2\rho_*^F+\left(\Omega^2 e^{-2\gamma}+1\right)\sigma^2+\frac{1}{2}\sigma^4\right] \tag{5.15}$$

$$\frac{2\gamma' e^{-2\mu}}{r_*}-\frac{(1-e^{-2\mu})}{r_*^2}=\left[2p_*^F+\left(\Omega^2 e^{-2\gamma}-1\right)\sigma^2-\frac{1}{2}\sigma^4\right] \tag{5.16}$$

$$e^{-2\mu}\left(\gamma''+\gamma'^2-\gamma'\mu'+\frac{\gamma'}{r_*}-\frac{\mu'}{r_*}\right)=\left[2p_*^F+\left(\Omega^2 e^{-2\gamma}-1\right)\sigma^2-\frac{1}{2}\sigma^4\right] \tag{5.17}$$

$$\left[(\Omega^2 e^{-2\gamma}-1)-\sigma^2\right]e^{2\mu}\sigma=0. \tag{5.18}$$

Here a prime denotes differentiation with respect to r_*.

The scalar wave equation (5.18) has two solutions:

- $\sigma(r_*)=0$, which we choose as the exterior solution.
- $\sigma^2=\left(\Omega^2 e^{-2\gamma(r_*)}-1\right)$, which gives the interior solution.

At the boundary of the star, we must have $\sigma=0$ which determines the value of Ω. Earlier workers have treated the scalar wave equation as an eigen value problem for Ω and assuming some specific values of the wave function at the centre with certain boundary conditions, solved the problem. But, in our case, for a large self-interaction, if we consider the solution of Mukherjee *et al* [67], this becomes a simple exercise. The value of Ω in our model has the form $\Omega=e^{\gamma(b)}$, where b is the radius of the star.

Restoration of dimensional parameters and the coordinate variable r gives,

$$\frac{(1-e^{-2\mu})}{r^2}+\frac{2\mu' e^{-2\mu}}{r}=8\pi G\left[\rho^F+\frac{m^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma}-1\right)\left(3\Omega^2 e^{-2\gamma}+1\right)\right] \tag{5.19}$$

$$\frac{2\gamma' e^{-2\mu}}{r}-\frac{(1-e^{-2\mu})}{r^2}=8\pi G\left[p^F+\frac{m^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma}-1\right)^2\right]. \tag{5.20}$$

The above set of equations clearly show that the total energy density and pressure of the star can be decomposed as

$$\rho^{Tot}=\rho^F+\rho^B, \tag{5.21}$$

$$p^{Tot}=p^F+p^B, \tag{5.22}$$

where, bosonic contribution for the energy density and pressure are given, respectively by

$$\rho^B = \frac{m^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma} - 1\right)\left(3\Omega^2 e^{-2\gamma} + 1\right) \tag{5.23}$$

$$p^B = \frac{m^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma} - 1\right)^2. \tag{5.24}$$

From chapter 2, we can write the total energy-density and pressure as,

$$\rho^{Tot} = \frac{1}{R^2(1-z^2)}\left[1 + \frac{2}{(\lambda+1)(1-z^2)}\right] \tag{5.25}$$

$$p^{Tot} = -\frac{1}{R^2(1-z^2)}\left[1 + \frac{2z\psi_z}{(\lambda+1)\psi}\right]. \tag{5.26}$$

Since the metric coefficients γ and μ are known, for a star of given mass and radius, we can calculate ρ^{Tot} and p^{Tot}, for a fixed value of λ, the parameter characterising the EOS of the star.

We shall employ this technique to the case of a quark-diquark star and address the problem of Her X-1, to see if it is a quark-diquark star.

5.3 Quark-diquark EOS

The possibility of diquarks was first put up by Gell-Mann [143]. Since then, over 500 papers on diquarks have been published (see Anselmino *et al* [144]). Ida and Kobayaski [145] and Lichtenberg and Tassie [146] introduced diquarks in order to describe a baryon as a composite state of two particles, a quark and a diquark. Using the model of Colpi *et al* [58], Horvath *et al* [147] formulated an EOS for diquarks and discussed its various features.

Donogue and Sateesh [148] explored the possibility that pairs of quarks might form diquark clusters in the density regime above the deconfinement transition for hadronic matter at finite density. It is believed that hadronic matter at high enough supernuclear density may undergo a transition to a deconfined state of quark and gluons. It has been suggested that this deconfinement occurs through an intermediate stage, in which nucleons are dissociated but quarks are correlated in spin-singlet pairs called diquarks.

In general, any two-quark system is a diquark. Quarks are color-triplet, spin $\frac{1}{2}$ objects. The possible states of a pair of quarks are thus given by $((M, J) = (\bar{3}, 0), (\bar{3}, 1), (6, 0)$ & $(6, 1))$, where M and J denote the color $SU(3)$ quantum number and spin, respectively. The spin-spin interaction energy between a pair of quarks is given by [149]

$$H_s = -C \sum_{i \neq j} b_i^\dagger \sigma^a \lambda^B b_i b_j^\dagger \sigma^a \lambda^B b_j \tag{5.27}$$

where, i, j labels the interacting quarks, $b_i^\dagger$ is the creation operator for the ith quark, σ^a are the Pauli spin matrices and λ^B-s are Gell-Mann matrices for color $SU(3)$.

This interaction generates a $N - \Delta$ mass difference:

$$< \Delta | H_s | \Delta >= 16C$$
$$< N | H_s | N >= -16C$$

which gives the mass difference, $m_\Delta - m_N = 32C = 300\ MeV$, from which we can calculate the value of C.

Amongst the possible states of a pair of quarks, $(\bar{3}, 0)$ state gives the maximum binding energy as can be seen below.

$$< \bar{3}, 0 | H_s | \bar{3}, 0 >= -16C$$
$$< 6, 1 | H_s | 6, 1 >= -\frac{8C}{3}$$
$$< \bar{3}, 1 | H_s | \bar{3}, 1 >= \frac{16C}{3}$$
$$< 6, 0 | H_s | 6, 0 >= 8C.$$

Henceforth, by the term diquark we will mean only the $(\bar{3}, 0)$ state, which is most strongly bound. It will be represented by a scalar field Φ^α, where α is the color index ($\alpha = 1, 2, 3$).

The mass of the diquark has been derived from the $N - \Delta$ mass difference [148] as

$$m_D \simeq \frac{2}{3}\left[\frac{m_\Delta + m_N}{2}\right] - \left[\frac{m_\Delta - m_N}{2}\right] = 575\ MeV. \tag{5.28}$$

The stability of the diquark as a bound state has been discussed in [149]. However, it has recently been reported that a computer simulated lattice analysis gives for the diquark a mass of around 700 MeV, in vacuum, and it seems to be unbound [151]. However as pointed out in [151], the mass m_D appearing in the effective Lagrangian need not be equal to the vacuum mass. We, in our calculation, will stick to the value of 575 MeV, in accordance with the calculations in [72].

The limitations of QCD perturbative theory has generated many phenomenological models for treating problems in high density physics. MIT bag model [48] is one them. In a bag model, it is assumed that the quarks remain confined in a bag and the confinement is caused by a Universal pressure B on the surface of the bag [48]. One of the many advantages of bag model is that it is fully relativistic. To describe a quark-diquark system we shall assume the bag model.

Let us assume that the X-ray pulsar Her X-1 is a bag containing u and d quarks and diquarks. We choose the diquark field as

$$L_{eff} = \frac{1}{2}\left(\partial_\mu\Phi^*\partial^\mu\Phi - m_D^2\Phi^*\Phi\right) - \frac{\Lambda}{4}\left(\Phi^*\Phi\right)^2, \tag{5.29}$$

where, the bosonic diquark mass m_D and the dimensionless coupling constant Λ have been estimated, from the calculation of Jaffe and Low [150], to be $575 MeV$ and 111.2, respectively [148].

The equilibrium of the system demands:

$$\mu_u + \mu_e = \mu_d,$$

$$\mu_D = \mu_u + \mu_d,$$

where μ specifies the chemical potential of the species. In section (5.2), we have showed that for large self-interaction ($\Lambda_* \gg 0$), the scalar field is essentially homogeneous inside the star. Gleiser [139] also pointed out that pressure anisotropy decreases with the increasing value of Λ. In case of diquarks,

$$\Lambda_* = \frac{\Lambda}{4\pi G m_D^2} \sim 10^{30},$$

where we substituted, $\Lambda = 111.2$, $m_D = 575\ MeV$ and $G = m_{Pl}^{-2} \approx 2.2 \times 10^{-5}\ gm$. This shows that we may well neglect the terms involving $O({\Lambda_*}^{-1})$ and take up the model discussed in chapter 5.2 to describe a quark-diquark configuration.

From section 5.2, we get the total energy-density and pressure due to diquark field as

$$\rho^D = \frac{m_D^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma} - 1\right)\left(3\Omega^2 e^{-2\gamma} + 1\right) \tag{5.30}$$

$$p^D = \frac{m_D^4}{4\Lambda}\left(\Omega^2 e^{-2\gamma} - 1\right)^2 \tag{5.31}$$

Following [72] we write the total energy-density and pressure of the system as

$$\rho^{Tot} = m_q(n_u + n_d) + \frac{3}{10}\frac{\pi^{4/3}\hbar^2}{m_q}(n_u^{5/3} + n_d^{5/3}) + \rho_D + B \tag{5.32}$$

$$p^{Tot} = \frac{1}{5}\frac{\pi^{4/3}\hbar^2}{m_q}(n_u^{5/3} + n_d^{5/3}) + p_D - B \tag{5.33}$$

Here B is the bag constant, m_q is the mass of quarks and n_u, n_d and n_D are the number densities of up quark, down quark and diquark, respectively. The first term on the right hand side of equation (5.32) denotes the rest mass energy density of the quarks. Following [72] we take, $B = 57\ MeV fm^{-3}$, $m_q = 360\ MeV$ and $m_D = 575\ MeV$.

Baryon number density of the system is given by

$$n_B = \frac{2}{3}n_D + \frac{1}{3}(n_u + n_d). \tag{5.34}$$

The mixture state has to be electrically neutral, which gives the condition,

$$\frac{1}{3}n_D + \frac{2}{3}n_u - \frac{1}{3}n_d = 0. \tag{5.35}$$

Table 5.1: Values of various parameters for a star of mass $M = 0.88\ M_{\odot}$, and $b = 7.7\ km$, for different values of λ (ρ_c and ρ_b denote central and surface density of the star, respectively).

λ	$R(km)$	δ	A	$\rho_c(GeVfm^{-3})$	$\rho_b(GeVfm^{-3})$
2	20.2238	2.2337	0.9333	0.6637	0.4374
5	27.5447	2.4153	1.6913	0.7156	0.4182
10	36.6273	2.4974	2.9926	0.7419	0.4094
20	50.0726	2.5445	5.6114	0.7579	0.4044
50	77.4878	2.5751	13.4818	0.7686	0.4012
100	108.7790	2.5858	26.6039	0.7723	0.4000
200	153.2640	2.5912	52.8501	0.7743	0.3994

5.4 Results

For a given mass and radius and for a particular choice of the parameter λ, total energy density ρ^{Tot} and pressure p^{Tot} can be calculated using the model discussed in Chapter 2. We assume that the mass and radius of Her X-1 are given by 0.88 $M_{\odot}$ and 7.7 km, respectively (values within the experimental ranges of mass and radius of Her X-1). Values of related parameters for different choices of the parameter λ are shown in Table 5.1.

We find that for $\lambda = 100$ the EOS obtained in this model agrees accurately with the EOS obtained by Horvath and Pacheco [72] for a quark-diquark mixture as shown in Fig.5.1. The predicted EOS is almost linear in both the cases. The variation of $\frac{dp}{d\rho}$ with different λ in our model is shown in Table 5.2.

Equations (5.32)-(5.35) can be solved numerically to get the number densities of quarks and diquarks in equilibrium. In (5.32) and (5.33) ρ^{Tot} and p^{Tot} can be determined from the solutions given by Mukherjee *et al* [67].

Table 5.2: Variation of $\frac{dp}{d\rho}$ for different values of λ for a star of $M = 0.88\ M_{\odot}$ and $b = 7.7\ km$.

λ	$\frac{dp}{d\rho}\vert_{r=b}$	$\frac{dp}{d\rho}\vert_{r=0}$
2	0.3518	0.3418
5	0.2847	0.2714
10	0.2634	0.2479
20	0.2513	0.2362
50	0.2446	0.2292
100	0.2424	0.2268
200	0.2413	0.2256

The value of Ω, however, depends on the choice of the boundary condition and our choice differs from that of Horvath and Pacheco [72]. It may be noted that a boson star has no well defined boundary. It describes an infinite exponentially decreasing atmosphere. Actually, most of the boson star calculations (e.g.,[60], [72]) are based on the boundary condition, $\sigma(r) \rightarrow 0$ as $r \rightarrow \infty$ and the total mass of a pure boson star is defined as $M = \lim_{r\rightarrow\infty} M(r)$, where $e^{2\gamma(r)} = 1 - \frac{2M(r)}{r}$.

But in the case where there is a fermionic component, which has an almost classical distribution, the reasonable boundary condition should be $\sigma(r = b) = 0$, where b is the radius of the star. We have noted that the imposition of this boundary condition does not allow a large contribution to the energy density by the bosonic component. This is consistent with the observation made by Horvath in an earlier paper [149] that diquark contribution becomes important only below a certain critical density $\rho_* = 7 \times 10^{14}\ gm\ cm^{-3}$.

At the boundary of the star, using equation (5.18) we get, $\Omega = e^{\gamma(b)}$. For $\lambda = 100$, this gives, $\Omega \sim 0.81416$. In Table 5.3 we give the total energy density and pressure as

Table 5.3: Energy-density and pressure contribution from diquarks and fermions

r(km)	ρ^D $(GeV\ fm^{-3})$	p^D $(GeV\ fm^{-3})$	ρ^{Tot} $(GeV\ fm^{-3})$	p^{Tot} $(GeV\ fm^{-3})$
0	0.0552	0.0038	0.7723	0.0885
2	0.0494	0.0031	0.7308	0.0790
4	0.0345	0.0017	0.6263	0.0546
6	0.0155	0.0004	0.5000	0.0243
7.7	0	0	0.4000	0

a function of radius and the contributions from quarks and diquarks.

In Fig.5.1, although we find that the agreement between a quark-diquark EOS and the EOS obtained by using the solution of Mukherjee *et al* [67] spectacular, one should not conclude immediately that Her X-1 is a quark-diquark star. In our calculation, energy density of the quark-diquark star at the boundary is $\sim 0.4\ GeVfm^{-3}$. If one uses the boundary condition, $\sigma(b) = 0$, almost all the contribution comes from the fermionic part. The diquark contribution vanishes at the boundary, while at the centre of the star ($r = 0$), $\rho^D \approx 0.055\ GeV\ fm^{-3}$, $p^D \approx 0.004\ GeV\ fm^{-3}$ where $\rho^{Tot} \approx 0.77\ GeV\ fm^{-3}$ and $p^{Tot} \approx 0.088\ GeVfm^{-3}$. In Fig5.2, we have shown the variation of bosonic scalar field inside the star.

Further, to be realistic, one should consider a core-envelope model, where one considers a core region containing a quark-diquark mixture while the envelope is made of normal neutron matter. This is necessary because in our model $\rho(b) \approx 0.4\ GeV\ fm^{-3}$, which is less than the density required for deconfinement, viz. $\rho_{dc} \approx 0.6\ GeV\ fm^{-3}$ [152]. The junction of the core-envelope region may then be thought of as a layer where the transition from quark phase to the confined hadronic phase occurs. In our model, the matching of the core and the envelope can be achieved easily by choosing for the two regions appropriate of values of λ and R. This, of course, presumes that the matter content in both the regions can be described by this model with appropriate values of λ. This will be taken up in chapter 8.

5.5 Discussions

We have found that for a large value of λ, our model provides a simple description of a class of compact stars like Her X-1. The EOS in this model agrees very well with the EOS obtained in [72] for a quark-diquark mixture. However, a correct imposition of the boundary condition may require a modification of the parameters given in [72] and hence whether Her X-1 is a quark-diquark star still remains inconclusive. It may be noted that we have not considered a possible rotation of Her X-1. Li *et al* [40] have pointed out that the rotation of Her X-1 is very slow compared to the critical angular velocity $\Omega_{cr} = \sqrt{\frac{GM}{b^3}}$, and hence the effect of this rotation on the $(M - b)$ relation will be negligible. Also, considering the rotational effect up to Ω^2 (where Ω is the angular velocity observed at infinity) in case of a slowly rotating boson-fermion star, de Sousa and Silveira [90] observed that field equations remain the same as in the case of a boson-fermion star with no rotation. Hence, they concluded that properties like mass and radius and the total number of particles of a static model do not differ from that of a slowly rotating boson-fermion star. Thus, although we assumed a static model for Her X-1, error in our results due to rotation, if any, should be small.

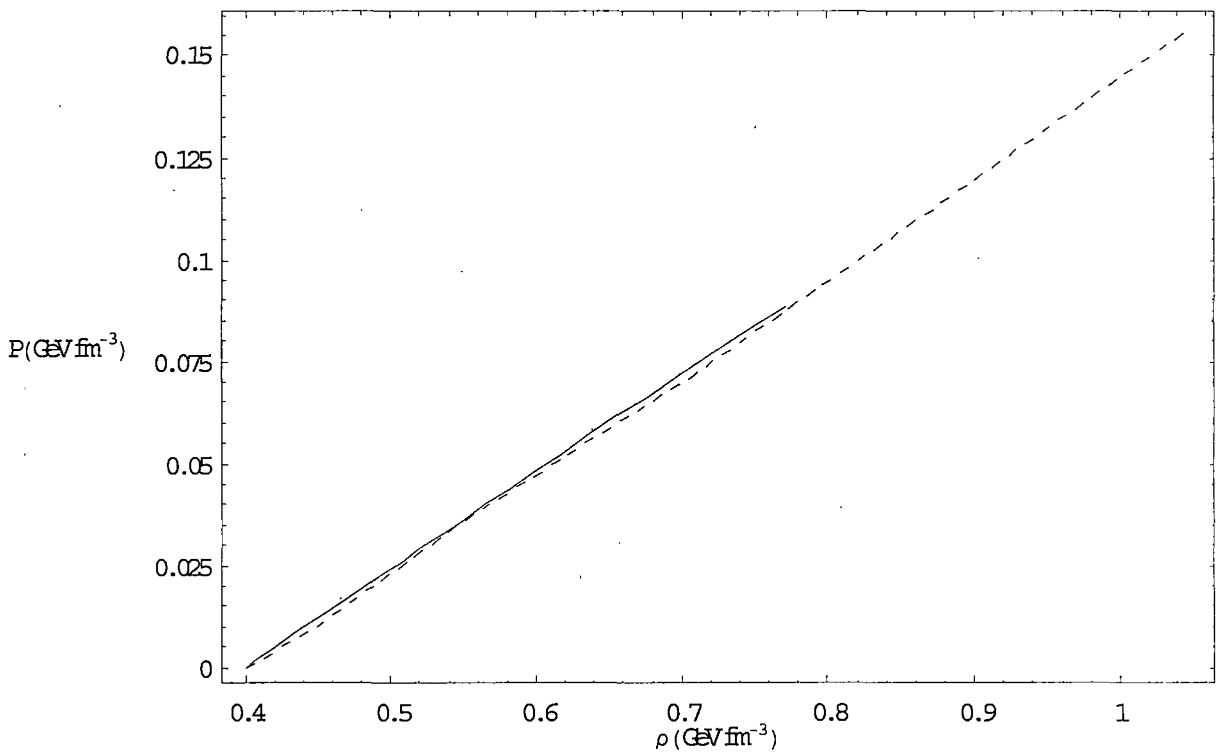

Figure 5.1: (a) EOS of a star of mass $M = 0.88\ M_\odot$ and radius $b = 7.7\ km$ in our model (solid line), (b) EOS obtained by Horvath & Pacheco [72] for a quark-diquark mixture state (dashed line).

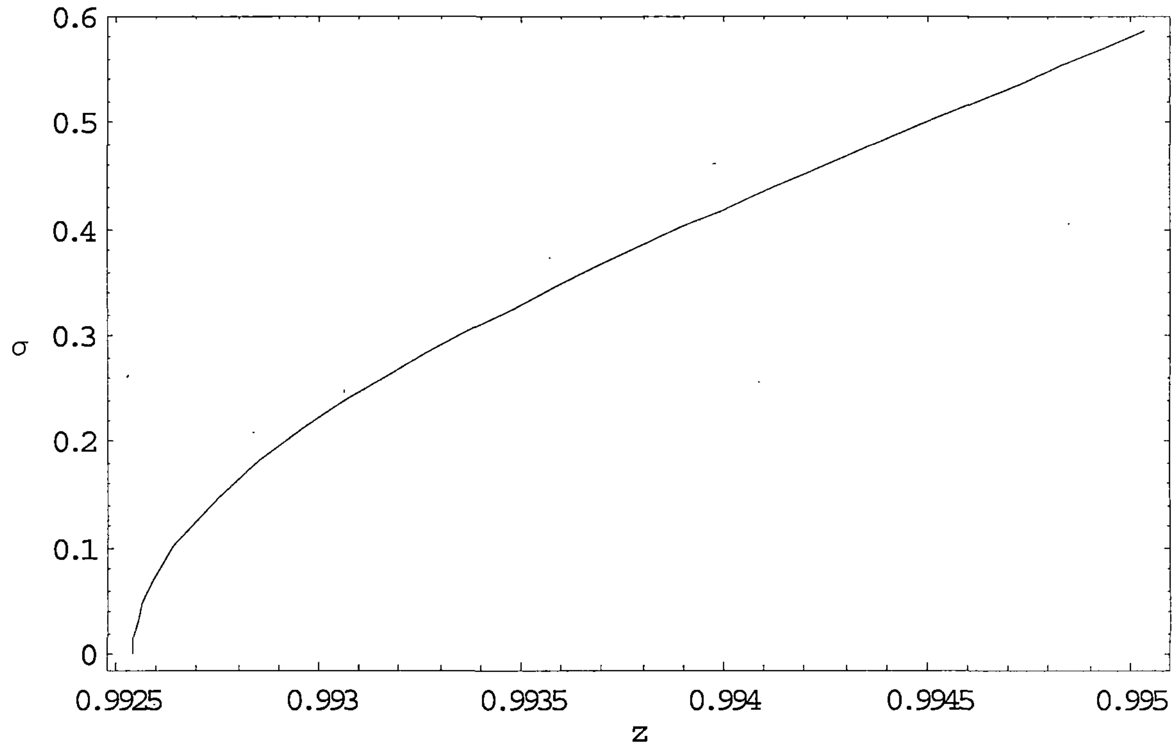

Figure 5.2: Scalar wave function σ plotted against the variable z for a star of mass $M = 0.88\ M_{\odot}$, radius $b = 7.7\ km$ with $\lambda = 100$.