PART - I

CHAPTER - II

ON THE VIABILITY OF A FLAT-SPACE-TIME THEORY OF GRAVITATION

2.1 INTRODUCTION

In Special Relativity (SR) the laws of dynamics and the electromagnetic field equations refer to *global* inertial frames. On the other hand the assumption that all effects of a gravitational field are locally identical to the effects of uniform acceleration of the coordinate system led Einstein to believe in curved space time and hence to formulate his celebrated theory of gravitation commonly known as the General Theory of Relativity (GR). According to GR matter introduces non-flat space time, the metric of which being the measure of the produced gravitational field. Thus in presence of matter the concept of a global inertial frame is discarded once and for all. However from the pedagogical standpoint one may reasonably ask if it were absolutely necessary to deal with curved space-time in order to understand gravitation and whether it was not at all possible for certain to extend Newtonian Gravity (NG) in the frame work of global inertial frames to suit the requirements of special relativity (electromagnetism for example did not pose any such difficulty in this respect). In other words one may enquire whether historically all options had been explored in this regard before one was forced to discard such endeavours. It may sound highly unprofessional and out of culture to raise such questions after all these years, however, it will be rather interesting to note that nearly three quarters of a century have elapsed since the discovery of GR but still today we witness numerous articles which strive to understand gravity in the frame work of SR. We feel that no matter whether some specific efforts are rewarding or not they should not be lightly dismissed in general.

Apart from the pedagogical question there is still another reason which may prompt one to look for other theories of gravitation. It is true that the GR is very elegant and rich in its structure and is endowed with interesting philosophical imports. But despite these attributes, it suffers from a serious drawback in that the experimental support of GR is very limited in number and for that too one has to make use of a rather cavalier approximation of GR (Biswas, 1988). But since the issue of Physics is distinct from that of the elegance of a formalism, the meagreness of empirical support for GR clearly leaves room for a plethora of other possible theories.

In a recent paper Biswas (1988) tried to introduce SR into the theory of NG in a systematic manner to obtain a flat space-time theory of gravitation which was claimed to have produced correct values for (1) the perihelion advance of Mercury's orbit, (2) the bending of light near the Sun and (3) the gravitational redshift. The earlier simple minded attempts (Bergmann 1969; Bagge 1981; Phipps 1986, 1987) to fit SR into classical gravity in order to predict correct value for (1) did not succeed (Bergmann 1969; Peters 1987; Ghosal et al 1987). Some rigorous attempts are however available due to Thirring (1961), Feynmamm (1971) and Narlikar and others (1982, 1985) but we leave them out of the present scope since in the present text we are more interested in a simple minded approach of Special Relativistic extension of Newtonian Gravity (SRNG). However, in the background I of chapter I (sec. 1.2) we discussed all the simple minded earlier attempts in detail. These earlier works dealt with the equation of motion alone and made no reference to any possible gravitational field equations and that they failed to give results is not surprising since the perihelion advance is commonly known to be the major observational test of the non-linear terms of the field equations (Peters 1987). In this perspective Biswas's Paper (BP) seems to be a bit more promising and therefore deserves close scrutiny.

The scheme of presentation is as follows. In section **2.2**, we discuss briefly some salient features of BP. In section **2.3**, we develop heuristically a covariant formulation for the equation of motion. Planetary precession and bending of light have been obtained in section 2.4. In section 2.5 we propose to develop heuristically the tensorial field equations and section 2.6 provides a guideline along which the redshift can consistently be treated in the framework of flat-space-time theories of gravitation in general. Section 2.7 contains some concluding remarks.

2.2 SOME SALIENT FEATURES OF BP

According to BP gravitational field is represented by a tensor potential $h_{\mu\nu}$ in a global pseudo Euclidean space-time. The proposed non-linear field equation is given by

$$\Box^2 h_{\mu\nu} = (2\pi G/c^2)\rho_{\mu\nu} + K\partial_\lambda h_{\mu\alpha}\partial^\lambda h^\alpha_\nu \qquad \text{----(2.1)}$$

where the second term on the r.h.s of (2.1) with undetermined factor K is added to include the contribution of the energy momentum tensor of the produced field. $\rho_{\mu\nu}$ denotes the energy momentum tensor for matter.

Applying eqn.(2.1) [with $\rho_{\mu\nu}$=0] to the static spherically symmetric situation, Biswas obtained the following pair of equations:

$$\nabla^2 h_{00} = -K\ (\nabla h_{00})^2 \qquad \text{-----(2.2)}$$

$$\nabla^2 h_{11} = +K\ (\nabla h_{11})^2 \qquad \text{-----(2.3)}$$

where h_{00} & h_{11} , which are functions of r alone, are the only non-zero components of $h_{\mu\nu}$ in spherical coordinates. The free field solutions of the above are taken to be

$$h_{00} = \frac{1}{K} \ln \left[1- \frac{2KGM}{r}\right] \qquad \text{-----(2.4)}$$

$$h_{11} = -\frac{1}{K} \ln \left[1+ \frac{2KGM}{r}\right] \qquad \text{-----(2.5)}$$

In order to obtain the equation of motion Biswas started from the free particle mass shell equation

$$\eta_{\mu\nu}\ p^{\mu}\ p^{\nu} + m^2c^2 = 0 \qquad \text{-----(2.6)}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, p^{μ} & m are the four momenta and mass of the particle respectively. For a charged particle in an electromagnetic field (A^{μ}), p^{μ} in equation (2.6) is replaced by $p^{\mu}+A^{\mu}$. Similarly it is argued that for motion under gravity $\eta_{\mu\nu}$ is to be replaced by $a_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ in equation (2.6). The equation of motion then follows from the three Hamiltonian cp_0 given by

$$cp_0 = c\ [-(P_a\ a^{ab}\ P_b + m^2c^2)/a^{00}]^{1/2} = H \qquad \text{-----(2.7)}$$

It is thus demonstrated that the solutions (2.4) & (2.5) with K=2 when put in canonical equations for the Hamiltonian (2.7) correctly explains two major tests of GR, namely the perihelion precession of Mercury's orbit and the bending of light.

Unfortunately it has been recently demonstrated by Peters (1990) that while eqn. (2.2) is correct, equation (2.3) does not follow from the equation (2.1). The field equation (2.1) when written in a general coordinate system should read

$$h_{\mu\nu;\alpha}{}^{;\alpha} = \frac{2\pi G}{c^2} \rho_{\mu\nu} + K\, h_{\mu\alpha;\beta}\, h_{\nu}^{\alpha;\beta} \qquad \text{-----(2.8)}$$

and by correctly applying equation (2.8) [with $\rho_{\mu\nu}$=0] to case of tensors resolved in spherical coordinate system gives the unique solution for h_{11}

$$h_{11} = 0 \qquad \text{-----(2.9)}$$

which is different from Biswas' result [equation (2.5)]. However there is no change in equation (2.4). Since Biswas' results depend on (2.4) and (2.5) and the latter being incorrect, the claims of Biswas fall through.

However, if this is the only error of BP, it appears that some minor amendments to (2.8) could provide a viable flat-space-time theory of gravitation. In our opinion the error in BP as pointed out by Peters is not the only weak point in the theory. We observe that there are other drawbacks, which are to be removed before one hopes to obtain a viable relativistic theory of gravitation. Firstly, in order to obtain the equation of motion Biswas resorted to a non-covariant Hamilton's formulation. Equations are thus not manifestly covariant. One therefore needs to develop heuristically a fully covariant formulation for this purpose.

The other shortcoming of BP refers to the claim by Biswas that his theory correctly predicts red-shift. Though no supporting calculations have been given on grounds of simplicity! We feel that the matter of gravitational red-shift cannot be treated lightly in connection with any flat-space-time theory what-so-ever. Indeed, before one hopes to explain the observed gravitational red-shift according to any theory of gravitation in the framework of global inertial frames one will have to first contest the commonly held thesis that " the existence of gravitational red-shift shows that a consistent theory of gravity cannot be constructed within the framework of special relativity" (Misner 1973). Any attempt in this regard is clearly absent in BP. In section 2.6, we will provide a guideline along which the red-shift can consistently be treated in the framework of flat-space-time theories of gravitation in general.

2.3 EQUATIONS OF MOTION

We have already noted that Biswas' formulation of the equation of motion starts with the introduction of a 3-Hamiltonian, therefore the resulting canonical equations were not manifestly covariant. Besides, in view of the cherished parallelism between gravitation and electromagnetism in BP, it is not intelligible why in the mass shell equation (2.6) $\eta_{\mu\nu}$ and not p^{μ} would undergo change in presence of gravity.

In order to guess a correct equation of motion of a test object we shall rather resort to the covariant Lagrangian formulation. The covariant Lagrangian of a charged particle in an electromagnetic field is composed of two parts ($L = L_f + L_c$). The free particle Lagrangian L_f is proportional to $m_0\eta_{\mu\nu}u^\mu u^\nu$ and the coupling term L_c is proportional to $qA_\alpha u^\alpha$ where m_0, q, and u^μ are the inertial mass, charge and the 4-velocities of the test particle and A_μ are the components of the four vector potential of the electromagnetic field (Goldstein 1964). With this analogy, for a tensor potential $h_{\mu\nu}$ of the gravitational field one may reasonably guess a minimal coupling term to be $m_g h_{\mu\nu}u^\mu u^\nu$ where the gravitational charge m_g may be assumed to be $-m_0$ in view of equivalence principle and the attractive nature of gravitational force.

Thus the Lagrangian may be believed to be given by

$$L \propto m_0\, \eta_{\mu\nu}\, u^\mu u^\nu - m_0\, h_{\mu\nu}\, u^\mu u^\nu$$

It may be noted here that any other possible proportionality constant (including the sign) in the 2nd term above may be dumped into $h_{\mu\nu}$ without any consequence. One may thus assume

$$L = m_0(\eta_{\mu\nu} - h_{\mu\nu})\, u^\mu u^\nu$$

However, m_0 may be dropped and one may write

$$L = a_{\mu\nu}u^\mu u^\nu \qquad \text{------(2.10)}$$

where $$a_{\mu\nu} = \eta_{\mu\nu} - h_{\mu\nu} \qquad \text{------(2.11)}$$

The above arguments may thus provide some justification for the assumed transition $\eta_{\mu\nu} \rightarrow \eta_{\mu\nu} - h_{\mu\nu}$ in the free particle mass shell equation [equation (2.6)]. We also take this cue but instead of taking the Lagrangian form (2.10) we shall choose

$$L = (a_{\mu\nu} u^{\mu} u^{\nu})^{1/2} \qquad \text{-----(2.12)}$$

which also reduces to the free particle Lagrangian $L_f = (\eta_{\mu\nu} u^{\mu} u^{\nu})^{1/2}$ if $h_{\mu\nu}$ is assumed to be zero. The form (2.10) for the Lagrangian is rejected because one of the first integrals of motion which follows from $\mathcal{L}$ is

$$a_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} = \text{const.} \qquad \text{-----(2.13).}$$

This clearly contradicts the definition of proper time interval :

$$d\tau^2 = - \eta_{\mu\nu} dx^{\mu} dx^{\nu} \qquad \text{-----(2.14)}$$

No such problem arises out of the Lagrangian L given in equation (2.12).

To obtain the equation of motion we start with the Hamilton's principle. The action integral may be written as

$$\int L \, d\tau = \int (a_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau})^{1/2} d\tau = \int (a_{\mu\nu} \frac{dx^{\mu}}{ds} \frac{dx^{\nu}}{ds})^{1/2} ds$$

where s may be any scalar parameter of motion. Therefore the Lagrangian may also be taken as

$$L' = (a_{\mu\nu} \bar{u}^{\mu} \bar{u}^{\nu})^{1/2} \qquad \text{-----(2.15)}$$

$$\text{where } \bar{u}^{\mu} = \frac{dx^{\mu}}{ds}$$

The Euler-Lagrange equation corresponding to (2.15) becomes greatly simplified if the parameter 'ds' is taken to be

$$ds = (a_{\mu\nu}\, dx^{\mu}\, dx^{\nu})^{1/2} \qquad \text{------(2.16)}$$

In this case the equation of motion can conveniently be written as

$$\frac{1}{2}\,(a_{\mu\lambda,\nu} + a_{\nu\lambda,\mu} + a_{\mu\nu,\lambda})\,\frac{dx^{\mu}}{ds}\,\frac{dx^{\nu}}{ds} + a_{\mu\nu}\,\frac{d^2x^{\mu}}{ds^2} = 0 \qquad \text{---(2.17)}$$

The form (2.16) looks like the metric in GR. Indeed the equation of motion (2.17) as obtained above formally looks like that of a geodesic in a curved space-time defined by the line element (2.16). However this has nothing to do with geodesics since the present theory is in flat space-time and raising and lowering of indices is done by $\eta_{\mu\nu}$ (and not $a_{\mu\nu}$).

2.4 PLANETARY PRECESSION AND BENDING OF LIGHT

To obtain the orbit equation however we shall not use the equation (2.17) directly, instead it will be advantageous to start from the first integrals that are suggested by the symmetry of the problem. Note that here we are interested only in the static spherically symmectric fields. We first restrict ourselves to θ = $\pi/2$ plane as usual. We have two first integrals of motion at our disposal. Since L' does not contain (in a static spherically symmetric situation) $x^0(=t)$ and $x^3(=\phi)$ explicitly the corresponding generalized momenta are constants of motion. Therefore,

$$p_0 = \frac{\partial L'}{\partial \bar{u}^0} = \frac{a_{00}\bar{u}_0}{(a_{\mu\nu}\bar{u}^{\mu}\,\bar{u}^{\nu})^{1/2}} = \varepsilon \qquad \text{-----(2.18)}$$

$$p_3 = \frac{\partial L'}{\partial \bar{u}^3} = \frac{a_{33}\bar{u}_3}{(a_{\mu\nu}\bar{u}^{\mu}\,\bar{u}^{\nu})^{1/2}} = p_{\phi} \qquad \text{------(2.19)}$$

where ε & p_ϕ are constants.

However the equations (2.18) & (2.19) are supplemented with a constraint condition that follows from the definition of the parameter ds given by equation (2.16) :

$$a_{\mu\nu}\,\bar{u}^{\mu}\,\bar{u}^{\nu} = 1 \qquad \text{-----(2.20)}$$

Thus making use of (2.20) we can rewrite the two first integrals (2.18) & (2.19) as

$$a_{00}\,\bar{u}^{0} = \varepsilon \qquad \text{-----(2.21)}$$

$$a_{33}\,\bar{u}^{3} = p_\phi \qquad \text{-----(2.22)}$$

In spherical coordinate system both $\eta_{\mu\nu}$ & $h_{\mu\nu}$ are diagonal and for spherically symmetric $h_{\mu\nu}$ the only non-zero components of $h_{\mu\nu}$ can be h_{00} and h_{11} and so $a_{\mu\nu}$ takes the form (for the chosen orbit plane)

$$a_{\mu\nu} = \text{diag}\ (a_{00}\ ,\ a_{11}\ ,\ r^2, r^2).$$

Equation (2.20) can be written explicitly as

$$a_{00}(\bar{u}^{0})^2 + a_{11}(\bar{u}^{1})^2 + r^2(\bar{u}^{3})^2 = 1.$$

Using (2.21) and (2.22) we can write

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\ (dr/ds)^2 + p_\phi^2\ /\ r^2 = 1 \qquad \text{-----(2.23)}$$

Now since

$$\frac{dr}{ds} = \frac{dr}{d\phi}\,\frac{d\phi}{ds} = \frac{dr}{d\phi}\,u^{-3}$$

the equation (2.23) reduces to the orbit equation

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\,(dr/d\phi)^2\,(p_\phi^2\,/r^4) + p_\phi^2\,/\,r^2 = 1 \qquad \text{-----(2.24)}.$$

The usual substitution $u = \frac{1}{r}$ in equation (2.24) produces

$$\frac{\varepsilon^2}{a_{00}} + a_{11}\,(du/d\phi)^2\,p_\phi^2 + p_\phi^2\,u^2 = 1 \qquad \text{-----(2.25)}.$$

With the assumed solution of h_{00} & h_{11} of BP with K=1 [to obtain correct precession Biswas had to put K = 2] and using the flat metric in spherical coordinates as $\eta_{\mu\nu}$= dig $(-1,1,r^2,r^2\sin^2\theta)$

$$a_{00} = -\,[1+\ln(1-2GM/r)] \simeq -\,(1-2GM/r) \qquad \text{-----(2.26)}$$

$$a_{11} = [1+\ln(1+2GM/r)] \simeq (1+2GM/r) \qquad \text{-----(2.27)}$$

rearranging and differentiating (2.25) with respect to ϕ and using (2.26) and (2.27) the orbit equation takes the approximate form

$$\frac{d^2u}{d\phi^2} + u = 3GMu^2 + GM/S^2 \qquad \text{-----(2.28)}$$

where we have put $S^2 = -\,p_\phi^2$, S being the angular momentum of the test object per unit mass.

It is well known that the differential equation

$$d^2u\,/\,d\phi^2 + u = nGMu^2 + GM/S^2$$

gives precession

$$\omega \simeq n \times 14'' \text{ /century}$$

for the planet Mercury. Equation (2.28) therefore, correctly gives precession of about $3 \times 14''$/century for Mercury.

In order to obtain the bending of starlight grazing the Sun one can also use the orbit equation (2.28) but in that case the last term must be dropped since S, the angular momentum per unit mass for a zero rest mass particle is infinite. This is because for a photon, for example, the linear momentum and hence the angular momentum (when its motion is not directed towards the origin of the coordinate system) are always finite. In equation (2.28) the term S was interpreted as the angular momentum per unit mass for a massive particle. Here we extend this meaning of S to the case of a massless photon as well.

The resulting equation

$$d^2u / d\phi^2 + u = 3GMu^2 \qquad \text{-----(2.29)}$$

gives correct values for the bending of light (Bose 1980).

2.5 THE FIELD EQUATION

The field equations of Biswas

$$\Box^2 \; h_{\mu\nu} = 2\pi G\rho_{\mu\nu} + k\partial_\lambda h_{\mu\alpha}\partial^\lambda h^\alpha_\nu \qquad \text{-----(2.1)}$$

written in covariant notation yields

$$h_{\mu\nu;\alpha}{}^{;\alpha} = 2\pi G\rho_{\mu\nu} + kh_{\mu\alpha;\beta}\, h^{\alpha\;;\beta}_{\nu} \qquad \text{-----(2.8)}$$

or, $h_{\mu\nu;\alpha}{}^{;\alpha} = k h_{\mu\alpha;\beta} h_{\nu}^{\alpha;\beta}$ -----(2.30)

in absence of matter ($\rho_{\mu\nu} = 0$).

For the spherically symmetric static fields

$$h_{\mu\nu} = \text{diag} [h_{00}(r), h_{11}(r), 0, 0].$$

From (2.30) Biswas erroneously arrived at the pair of equations

$$\nabla^2 h_{00} = -k(\nabla h_{00})^2 \qquad \text{-----(2.2)}$$

$$\nabla^2 h_{11} = +k(\nabla h_{11})^2 \qquad \text{-----(2.3).}$$

Peters (1990) has shown that the latter equation (2.3) is incorrect. It was further shown that if equation (2.30) is correctly applied in a spherical coordinate system the only consistent solution for h_{11} is

$$h_{11} = 0 \qquad \text{-----(2.9).}$$

However, h_{00} can be taken as given by Biswas

$$h_{00} = \frac{1}{k} \ln [1-\frac{2kGM}{r}] \qquad \text{-----(2.4).}$$

It will be immediately seen that when these solutions [(2.4) and (2.9)] are used in equation (2.25) the orbit equation becomes approximately

$$d^2u / d\phi^2 + u = GMu^2 + GM (du/d\phi)^2 + GM/S^2 \qquad \text{-----(2.31).}$$

Surprisingly the above equation is also the path equation that follows from Bagge's ponderomotive force law of gravitation (Bagge 1981; Ghosal et al 1987). It was shown elsewhere (Ghosal et al 1987) that this equation yields only about 14" /century for Mercury's precession.

Thus to have correct results we need to modify the field equations of Biswas. In order to do this we shall follow the following guidelines :

(1) The new equations should be such that it would formally look like equation (2.8). This is required by the fact that the typical non-linear form of Biswas' equations result from a simple and well considered heuristic arguments and we feel therefore that one should not disturb it.

(2) Since the field equations are expected to be of the second order, the solutions for each component will have two arbitrary constants in general. In the spherically symmetric case there will be four such constants in total. Two of them may be made to vanish from the requirement that h_{00} & h_{11} tend to zero as $r \rightarrow \infty$. The one remaining constant in h_{00} may be set by using the far field approximation and correlating the result with that of Newtonian Gravity. The question now is how to fix the other constant in h_{11}. For this, one needs to have an additional postulate. Biswas had to make such a postulate in order to fix the arbitrary constant in his erroneous solution of h_{11} [vide section (1v) of BP]. We will rather do it differently. Our field equation will be such that in the spherically symmetric situation it automatically would display one built-in constraint so that the problem of fixing the arbitrary constant in h_{11} will be done away with.

Now we give *our* version of the field equations after the following steps.

(1) Define a tensor D^{μ}_{ν} which is diagonal and has non-zero components $D^{1}_{1} = -D^{0}_{0} = 1$ in spherical polar coordinates.

(2) Construct a scalar

$$\psi = \frac{1}{2} D^{\mu}_{\nu} h^{\nu}_{\mu} \qquad \text{-----(2.32).}$$

If the r.h.s of (2.32) is resolved in spherical polar coordinates (Note that since ψ is a scalar, the value of the same is independent of the coordinate system) we obtain

$$\psi = \frac{1}{2} [h_{00}(r) + h_{11}(r)] \qquad \text{-----(2.33)}$$

where $h_{00}(r)$ & $h_{11}(r)$ are the values of the field components in the same coordinate system.

(3) Define another tensor $\Lambda_{\mu\nu}$ whose only non-zero component in spherical polar coordinate is $\Lambda_{11} = 1$.

Now we postulate the field equations

$$H_{\mu\nu;\alpha}{}^{;\alpha} = k\, H_{\mu\alpha;\beta}\, H^{\alpha\ ;\beta}_{\nu} + 2\pi G\rho_{\mu\nu} \qquad \text{-----(2.34)}$$

where

$$H_{\mu\nu} = h_{\mu\nu} - \psi\, \Lambda_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} D^{\alpha}_{\beta} h^{\beta}_{\alpha} \Lambda_{\mu\nu} \qquad \text{-----(2.35).}$$

Though equation (2.34) contains the tensors $\Lambda_{\mu\nu}$ and $D_{\mu\nu}$ which have simple forms in spherical polar coordinates, it does not mean that the equation is adapted to spherical coordinates alone. Indeed, the equation (2.34) represents the general field equations which can be used in any coordinate system.

In the matter free region we have

$$H_{\mu\nu;\alpha}{}^{;\alpha} = k\, H_{\mu\alpha;\beta}\, H_{\nu}^{\alpha\ ;\beta} \qquad \text{-----}(2.36).$$

From equation (2.35) it is evident that in spherical polar coordinates and assuming spherical symmetry in $h_{\mu\nu}$, the only non-zero components of $H_{\mu\nu}$ are H_{00} and H_{11}. Therefore following Peters we have

$$H_{00} = \frac{1}{k} \ln [1- \frac{2kGM}{r}] \qquad \text{-----}(2.37)$$

and

$$H_{11} = 0 \qquad \text{-----}(2.38).$$

However, from equation (2.35),

$$H_{00} = h_{00}(r) \qquad \text{-----}(2.39)$$

and

$$H_{11} = h_{11}(r) - h_{00}(r) \qquad \text{-----}(2.40).$$

From (2.38) and (2.40) follows the constraint condition

$$h_{11}(r) = h_{00}(r)$$

Thus we have

$$h_{00} = \frac{1}{k} \ln [1- \frac{2kGM}{r}] \qquad \text{-----}(2.41a)$$

$$h_{11} = \frac{1}{k} \ln [1- \frac{2kGM}{r}] \qquad \text{-----}(2.41b)$$

One can verify that these solutions also give under approximations the same path equation that was obtained in section

(2.4) using Biswas' incorrect solutions [equations (2.4) and (2.5)]. The result of course is not surprising since the solutions (2.41b) and (2.5) are the same in their first order. Thus we see that after some modifications in the field equations and the ponderomotive force law of Biswas it is possible to obtain correct precession of Mercury's orbit and bending of light. It is also evident that our Lagrangian (2.12) representing the motion of a test object will also give the correct result for the radar eco delay experiment.

If for some reasons one wishes to adhere to Biswas' incorrect solutions for $h_{11}(r)$ [equation (2.5)] and is prepared to sacrifice the guidelines mentioned earlier in this section, there is another version of the field equation

$$h_{\mu\nu;\alpha}{}^{;\alpha} - \phi\, \Lambda_{\mu\nu;\alpha}{}^{;\alpha} = k\,(h_{\mu\alpha;\beta}\, h^{\alpha\;;\beta}_{\nu} - \phi\, \Lambda_{\mu\alpha;\beta}\, \Lambda^{\alpha\;;\beta}_{\nu}) \qquad \text{--(2.42)}$$

where the scalar,

$$\phi = \eta^{\alpha\beta}\, h_{\alpha\gamma}\, \Lambda^{\gamma}_{\beta}$$

One may resolve the equation in spherical polar coordinates and verify the following non trivial equations :

$$1/r^2 \frac{d}{dr} (r^2\, dh_{00}/dr) = -k\, (dh_{00}/dr)^2 \qquad \text{-----(2.43a)}$$

$$1/r^2 \frac{d}{dr} (r^2\, dh_{11}/dr) = k\, (dh_{11}/dr)^2 \qquad \text{-----(2.43b)}$$

Solutions of which (dropping the additive constants)

$$h_{00} = \frac{1}{k} \ln [1 - \frac{k}{Ar}] \qquad \text{-----(2.44a)}$$

$$h_{11} = -\frac{1}{k} \ln [1 - \frac{k}{Br}] \qquad \text{-----(2.44b)}$$

and

with the additional postulate $B = -A$ result in equations (2.4) and (2.5) used by Biswas.

2.6 ENERGY EQUATION AND REDSHIFT

Biswas claimed that the results of the gravitational red-shift [Pound-Rebka Experiment (1960)] matches with his theory . However, it was not worked out on the ground of simplicity. We thus have no means to understand what precisely was in the author's mind regarding the explanation of gravitational red-shift and we think that the matter of red-shift in the context of a flat-space-time theory requires some deliberations. Being essentially a special relativistic theory, BP deals with standard clocks and scales which should remain unaffected by the field. Therefore the common general relativistic interpretation that red-shift is the result of gravitational retardation of clocks (Weinberg 1972) is hence no longer valid in special relativistic theory of gravitation. There is another heuristic interpretation according to which energy of a photon must decrease when it climbs out of a gravitational field (Misner 1973; Einstein 1923). An important corollary that follows from the above interpretation is that the gravitational red-shift must imply the non-existence of global Lorentz frames (Misner 1973; schield 1960). Thus any effort to consolidate (globally) special relativity and gravitation seems to fall through unless red-shift is viewed from a different perspective. Below we shall see how gravitational field should bring in changes into the atomic (or neuclear) processes itself so that similar atoms (or neuclei) at two different spatial locations would behave differently. This fact will provide an explanation for the red-shift according to BP or any other flat-space-time theories for that matter.

To understand this let us first calculate the energy of a test object of rest mass m_0 undergoing radial motion.

We start from the equation (2.20). For radial motion we can write

$$a_{00}\,(\bar{u}^0)^2 + a_{11}\,(\bar{u}^1)^2 = 1$$

or $$a_{00}(dt/ds)^2 + a_{11}(dr/dt)^2(dt/ds)^2 = 1 \qquad \text{-----(2.45).}$$

Substituting $\bar{u}^0 = dt/ds = \varepsilon/a_{00}$ (vide 2.21) in equation (2.45) and multiplying it with m_0 on both sides we have

$$E = \frac{m_0\,(-a_{00})^{1/2}}{[\,1 + (a_{11}/a_{00})v^2\,]^{1/2}} \qquad \text{-----(2.46)}$$

where we have put $-m_0^2\varepsilon^2 = E^2$. The quantity E can be interpreted as the total energy of the particle in a gravitational field, since, in the far field the right hand side reduces to $m_0/(1-v^2)^{1/2}$ which is the so-called relativistic energy.

Consider that an atomic or neuclear transition (as an example one may consider the 14.4 kev transition of a Fe^{57} nucleus which was used in the Pound Rebka experiment) takes place between the two of its energy states E_1 and E_2 with the corresponding rest masses m_1 & m_2 of the atom / nucleus respectively [Note that according to SR an atom (or a nucleus) will have different rest masses for its different energy states]. Then from (2.46) we have

$$E_2 - E_1 = (m_2 - m_1)\,\frac{(-a_{00})^{1/2}}{[1 + (a_{11}/a_{00})v^2]^{1/2}} = h\nu \qquad \text{-----(2.47)}$$

where ν is the frequency of the emitted photon. If we wish to compare this transition with that of a similar atom with two energy states corresponding to the same rest masses m_1 & m_2 at infinity ($a_{00} \rightarrow -1$, $a_{11} \rightarrow 1$) we have similarly

$$E_2' - E_1' = (m_2 - m_1) \frac{1}{(1-v^2)^{1/2}} = h\nu' \qquad \text{-----(2.48)}$$

where the prime denotes the corresponding quantities at ∞ .

If the transition takes place when the atoms are at rest (v=0) at their respective positions,we have from equations (2.47) and (2.48) the frequencies of the emitted photons, given by

$$h\nu = \Delta m \ (-a_{00})^{1/2} = \Delta m \ [1 + \ln \ (1-2GM/r)]^{1/2} \qquad \text{-----(2.49)}$$

and $$h\nu' = \Delta m \qquad \text{-----(2.50)}$$

where $\Delta m = m_2 - m_1$

or combining (2.49) and (2.50)

$$\nu = \nu' \ [1 + \ln \ (1-2GM/r)]^{1/2} \qquad \text{-----(2.51).}$$

The equation (2.51) represents the general redshift formula according to our theory. Retaining the first term in the expansion of the logarithmic term in (2.51) we have

$$\nu \simeq \nu' \ (1 - 2GM/r)^{1/2} \qquad \text{-----(2.52).}$$

From the last expression (which is the typical general relativistic expression) it is evident that the present theory can correctly account for the observed red-shift.

2.7 SUMMARY

Among several attempts to obtain a Special Relativistic extension of Newtonian Gravity (SRNG), a recent one proposed by T.Biswas seems to be promising. However, there are several shortcomings, some modifications have been suggested in order to remove these shortcomings. In section 2.3 we have heuristically developed a covariant Lagrangian formulation for the equation of motion of test objects. Secondly in section 2.5 we have developed heuristically a tensorial field equations and the equations are found to be non-linear, the vacuum spherically symmetric solutions for which when put in the proposed equation of motion produces correct values for the advance of the perihelion of Mercury's orbit and bending of light near the Sun. The prospect of accommodating gravitational red-shift in context of flat-space-time theories in general is discussed in section 2.6.

All this has been done just to show that it is possible to obtain a Lorentz covariant theory of gravitation, similar to that of electromagnetism, which can duplicate all the verifiable general relativistic effects. Among other things this article discusses the pedagogical importance of the present flat-space-time theory of gravitation and in no way intends to project it as a viable alternative to GR.

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