P R E F A C E

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Structural members such as plates and shells are frequently used in various machine parts and the scope of study of bending properties of such members is sufficiently broad. Within elastic limit, various plate and shell problems have been dealt with by numerous eminent Scientists and research workers. All these problems may be classified as

(a) Static Problems,

(b) Dynamic Problems and

(c) Thermal Problems.

Any elastic behaviour (whether 'Static' or 'dynamic' or 'Thermal') of plates and shells are influenced by the following factors :-

i) Material properties defined by Young's Modulus 'E' and Poisson's Ratio 'ν' , both of which may be constants or variables.

ii) Geometry of the plates and shells : Geometry of the plate may be simple such as circular, Ractangular, Triangular, Elliptic etc., or may be complicated like Parabolic, Polygonal or Skewed one. Shells may be cylindrical, spherical, conical etc.

iii) Thickness (h) of the plate or shell may be constant or variable as well.

iv) Types of loading such as Transverse (or Lateral) Loading, In-plane Loading, Concentrated Loading, Edge Loading, Combined Loading etc.

and

v) Nature of support, i.e., edge conditions of the plates and shells may be 'Simply-Supported' or 'Clamped' having 'Movable' or 'Immovable' states.

THEORY OF PLATES :

Whenever the deflection 'w' of a thin plate is small in comparison with its thickness 'h', a very approximate but satisfactory theory of bending of the plate by lateral load can be developed by making the following assumptions :-

i) there is no deformation in the middle plane of the plate and hence this plane remains 'Neutral' during bending,

ii) points of the plate lying initially on a normal to the middle plane of the plate remain on the normal to the middle surface of the plate after bending,

iii) the stresses normal to the middle plane of the plate, arising from the applied loading, can be disregarded.

These assumptions constitute the simplest and widely used "Classical Small Deflection Theory" or "Linear Theory", developed by Joseph Louis Lagrange in 1811. According to this theory all the stress components can be expressed by the deflection 'w' of the plate, which is a function of the two co-ordinates in the plane of the plate. This function has to satisfy a linear partial differential equation which together with the boundary conditions, completely define 'w'. Thus the solution of this equation gives all the necessary information for calculating stresses at any point of the plate. Following the above-mentioned linear theory, numerous works on the bending of thin plates have been carried out by many research workers and the Bibliography of these works has been nicely incorporated in the book, "Theory of Plates and Shells" by S.Timoshenko and S. Woinowski - Krieger (1962).

In most of these cases, bending of a plate is accompanied by strain in the middle plane, but, careful calculations show that the corresponding stresses in the middle plane are negligible, if the deflections of the plate are small in comparison with its thickness. If the deflections are not small in comparison with the plate - thickness, these supplementary stresses in the middle plane of the plate must be taken into account in deriving the governing differential equations of the plate. The differential equations so obtained become non-linear in character and their solutions are much more complicated.

With the advent of modern technology and systems exposed to oppressive operation conditions, the linear hypothesis could no longer be 'retained'. Whenever Forces, Deformations, Velocities, Temperatures and other factors become excessive, non-linear effects come into play and their influences can no longer be ignored. This situation occurs also in the particular field of Applied mechanics involving plates and shallow shells. These elements, when used in modern structures, such as "High-Speed Aeroplanes", "Missiles" and "Space-Vehicles", are often subject to large transverse deflections and reveal clearly non-linear response.

METHODS OF ATTACK OF THE NON-LINEAR PROBLEMS OF THIN PLATES :

So far there has been a wide application of the three types of differential equations for the non-linear analyses of thin plates, thick plates and sandwich plates. They are

I) Von-Kármán's Equations,

II) Berger's Equations and

III) Banerjee's Equations.

I) The first formulation of the theory of plates with a stretching of the middle plane and moderately large deflections was developed by Gehring (1860) and was improved upon by G. Kirchhoff (1883). The potential energy per unit area of the plates was written as a sum of a quadratic function of the quantities defining the Extension of the middle plane and a quadratic function of the quantities defining Bending. The equations of motion with finite displacements were deduced by the Principle of Virtual work. The stress function satisfying the in-plane force equilibrium equations was introduced by A. Foppl (1907). The currently popular form of the two governing equations, in the rectangular cartesian co-ordinates, was given by T. Von-Karman (1910). The equations of Von-Kármán are in the coupled form and involve Transverse Deflection and the Membrance Stress Function as unknown functions. These are difficult to solve. Several numerical methods have been employed by different investigators for solutions of Von-Kármán's equations.

INTERESTING WORKS ON VON-KÁRMÁN'S EQUATIONS :

Among the authors who initially treated the non-linear analysis of plates, the works of S.Timoshenko (1937) and S.Way (1934, 1938) need special mention. Since then many authors investigated the different plate problems using Von-Kármán's Equations. Useful works in this field are due to S. Levy (1942), S. Levy and S.Greenman(1942), W. Z. Chien (1947), Chi-Teh Wang (1948), H. N. Chu and G. Herrmann (1955, 1956), N. A. Weil and N. M. Newmark (1956), S.J.Medwadowski (1958), W. A. Nash (1959), W. A. Nash and I. D. Cooley (1959), N. Yamaki (1964), J. L. Nowinski (1962, 1963, 1964), A.M.Alwan(1964), J. L. Nowinski and I. A. Ismail (1965), G. Z. Harris and E. H. Mansfield (1967), J. B. Kennedy and Simon Ng.(1967), Robert Schmidt (1968), H. F. Bauer (1968), J. M. Whitney and A. W. Leissa (1969), M. Sathyamoorthy and K. A. V. Pandalai (1972), Richard Bolton (1972), J. L. Nowinski and H. Ohnabe (1973), K. Kanakeraju and C. R. V. Rao (1976), S. Dutta (1976, 1978), M.A.Sayed and R.Schmidt (1977),

B. M. Karmakar (1978, 1979), M. Sathyamoorthy (1978, 1979), M. Sathyamoorthy and C. Y. Chia (1979, 1980), B. Banerjee and S.Dutta (1980), J. N. Reddy and W. C. Chao (1981), J. N. Reddy and C. L. Huang (1981), B. Banerjee (1982, 1983, 1984), S. K. Chaudhuri (1982, 1984), S. Das (1984), K. Kanakaraju and C. R. V. Rao (1986), P. C. Dumir (1988), G. L. Ostiguy and N. Nguyen (1988), Yeh Kai-Yuan, Zheng Xiao - Jing and Zhou You-he (1989), M. Gorji (1989), H. Kobayashi and K. Sonoda (1989), K. M. Liew and K. Y. Lam (1990), J. W. Zhang (1991), B. Singh and S. Chakravorty (1991), H.Kobayashi and K. Sonoda (1991), U. S. Gupta R. Lal and S. K. Jain (1991), M. Ganapathi, T. K. Varadan and B. S. Sarma (1991), G. B. Chai (1991), and D. J. Gorman (1991).

All these workers solved the coupled form of Van-Kármán's equations by different numerical methods which are elegant but laborious.

II) H. M. Berger (1955) offered an approximate method for non-linear analyses of thin elastic plates by neglecting the so-called second strain invariant in the expression for the total potential energy of the system. He investigated the large deflections of circular and rectangular plates both for clamped <u>immovable</u> and simply-supported <u>immovable</u> edges. Although no physical justification of his assumption was given in the paper, the results obtained in both the cases are in very good agreement with other known results. The speciality of Berger's equations is that the equations have been decoupled so that the solutions of the differential equations have been simplified and obtained in the closed form. Actually Berger's equations are linear in character. The essential non-linearity depends on the coupling parameter 'α'. These specialities made Berger's equation very popular and many useful works on plates of immovable edges have been done following Berger's equations.

USEFUL WORKS ON BERGER'S EQUATIONS :

The comprehensive works on Berger's equations which need special mention are due to T. Iwinski and J. L. Nowinski (1957), M. L. Williams (1958), N. A. Nash and J. R. Modeer (1959, 1960), S. Basuli (1961), J. E. Hassert and J. L. Nowinski (1962), T. Wah (1963), S. N. Sinha (1963), J. L. Nowinski and T. A. Ismail (1965), M. C. Pal (1967), B. Banerjee (1967, 1968, 1969), Cheng-Ih-Wu and T. R. Vinson (1968, 1969), M. C. Pal (1969, 1970, 1973), M.Sathya-moorthy and K. A. V. Pandalai (1970, 1973, 1974), J. Mazumdar and R. Jones (1974), P. Biswas (1975), S. Dutta (1975, 1976), M. M. Banerjee (1976), N. Kamiya (1976), B. M. Karmakar (1977), J. Mazumdar and R. Jones (1977), S. Dutta (1976, 1977), M.Sathyamoorthy (1977, 1978), N. Kamiya (1978), B. Banerjee and S. Dutta (1979), B. Banerjee (1982), S. Das (1984), T. Das (1986), R. Bera and G. C. Sinharay (1992) and R. Bera and B. Mukhopadhyay (1994).

It is to be noted that Berger's equations are meaningful for immovable edge conditions only. It leads to meaningless results for movable edge conditions. This has been pointed out by J. L. Nowinski and H. Ohnabe (1972) in an excellent research work.

III) In 1981 B. Banerjee and S. Dutta offered a modified strain energy expression to investigate non-linear problems of thin elastic plates. A set of decoupled differential equations are obtained under this modified strain energy expression. The authors have tested the accuracy of their equations by solving different non-linear plate problems. They have obtained sufficiently accurate results both for movable and immovable edge conditions. The equivalent hypothesis of Banerjee's equations is that the radial stretching of the plate is proportional to $(dw / dr)^2$. This is resonable because under any type of loading and under any boundary condition the extra strain imposed by bending is represented by the term $(dw / dr)^2$. It is believed that Banerjee's equations are more welcome from the practical point of view because unlike Von-Kármán's equations, they are uncoupled and unlike Berger's equations they give reasonable results both for movable and immovable edge conditions.

IMPORTANT WORKS ON BANERJEE'S EQUATIONS :

So far many useful works have been carried out on Banerjee's equations. Important works are due to B. Banerjee and S. Dutta (1981), B. Banerjee (1984), B. Banerjee and G. Sinharay (1985, 1986) and S. K. Ghosh (1991) who have utilised Banerjee's equations in different non-linear thin plate problems under different types of loading and achieved satisfactory results.

Employing Banerjee's hypothesis on thick plates, R.Bhattacharjee and B. Banerjee (1988) have achieved satisfactory results towards non-linear behaviours of thick plates under different types of loading.

Utlising Banerjee's equation on skewed plates, excellent results were also obtained by A. Roy, B. Banerjee and B.Bhattacharjee (1991, 1992, 1993, 1994) for rhombic plates under different types of loading. Numerical results obtained in the above cases are interesting and useful from the practical point of view.

USEFUL WORKS ON SANDWICH PLATES :

Investigations on finite deformations of sandwich plates are gaining importance day by day due to its wide applications in modern designs. Outstanding research workers who carried out interesting investigations in this field are Reissner (1948), Alwan (1964) and Nowinski and Ohnable (1973).

Reissner proposed the basic differential equations for finite transverse deflections of sandwich plates under the assumptions that the plate consists of a core layer and two face layers of such construction that the face parallel stresses in the core and the variation of the face stresses over the thickness of the face layers are negligible. The resultant equations permit the analysis of the effect of transverse shear stress deformation and transverse normal stress deformation in the core on the overall behaviour of the plate. It is shown that, in general, the effect of the transverse normal stresses in the core is negligibly small compared with the effect of the transverse shear stresses. The equations, when simplified by the omission of the transverse normal stress terms, are brought into a form suitable for the solution of rectangular-plate problems. It was further shown that the range of deflections for which the linear theory is adequate decreases in accordance with a simple explicit formula as the core is made softer relative to the faces. He used the finite deflection equations to obtain plate-buckling equations that include the effect of transverse shear stress deformation on buckling loads. This work is superb.

Alwan proposed differential equations for large deflection of sandwich plates with orthotropic core in terms of the Airy stress function. This paper is very interesting and useful for modern design.

Nowinski and ohnabe presented a generalization of Reissner equations for the isotropic plates to the case when the material of both the core and the faces is isotropic and of differing elastic characteristics. They derived a system of four governing equations involving the deflection w, stress function ϕ, and two functions, α and β, characterizing the shearing of the core. A transformation enables one to reduce the system to two equations involving two functions, w and ϕ only. For the isotropic faces the system reduces to two equations obtained by Alwan for large deflections of plates and to the equations of Cheny (1968) for small deflections. The integral analysis based on the application of Reissner - Hellinger variational principles is simpler than the five equations differential approach of Wempner and Baylor (1965). The authors obtained an explicit solution for a simply supported rectangular sandwich plate prevented from inplane motions on the contour by means of Galerkin's procedure. Numerical calculations using UNIVACE 1108 computer was performed for various aspect ratios and the thickness of the plate as well as for several contrasting sets of elastic constants of orthotropic and isotropic materials. This paper is an attractive work on orthotropic sandwich plates. All these investigations involve non-linear partial differential equations in the coupled form. These equations are difficult to solve analytically.Different numerical techniques have been employed to get desired solutions.

To get over the difficulties in the solutions of coupled differential equations for sandwich plates, N. Kamiya (1976) has employed Berger's well known techniques to solve non-linear problems of sandwich plates. The author has offered a new set of governing equations by using Berger's approximation to study the non-linear static behaviours of sandwich plates. He has analysed in detail the case of rectangular sandwich plates. The accuracy of his method depends on a correction factor F (b/a). Thus although Kamiya has been able to simplify the coupled differential equations of sandwich plates proposed by earlier authors, his attempt has been restricted to a particular plate geometry due to introduction of this correction factor. More over Berger's method fails completely for movable edge conditions.

Later Karmakar (1978) employed Berger's line of thought to investigate free vibrations of rectangular sandwich plates. The numerical results obtained in this study are interesting.

Large deflection analysis of heated sandwich plates is very important in modern design. Unfortunately work in this area has not received much attention so far. An attractive work in this field is due to Kamiya (1978), who has quite elegantly analysed the large deflections of rectangular sandwich plates under thermal loading by using Berger-type equations. The author's analysis is useful. He has compared the results with those obtained from conventional governing equations solved by the Ritz-Galerkin method. These results are approximate due to Berger's approximation which is fairly accurate for immovable simply supported edges and fails for movable edges. Ritz-Galerkin's method also involves approximations.

Dutta and Banerjee (1993), in reply to Berger's equations on sandwich plates, proposed a new set of uncoupled differential equations in rectangular cartisian co-ordinate system to analyse the non-linear behaviours of sandwich plates. Accuracy of the results obtained by the authors for different sandwich plates both for movable as well as immovable edge condition has been tested under different types of loading and found to be in excellent agreement with other known results.

THEORY OF SHELLS :

The main suppositions of the theory of thin plates also form the basis for the usual theory of the thin shells. There exists, however, a substantial difference in the behavior of plates and shells under the action of external loading. The static equilibrium of a plate element under a lateral load is only possible by action of bending and twisting moments, usually accompanied by shearing forces, while a shell, in general, is able to transmit the surface load by "membrane" stresses which act parallel to the tangential plane at a given point of the middle surface and are distributed uniformly over the thickness of the shall. This property of shells makes them, as a rule, a much more rigid and a more economical structure than a plate would be under the same conditions.

In principle, the membrane forces are independent of bending and are wholly defined by the conditions of static equilibrium. The methods of determination of these forces represent the so-called "membrane theory of shells". However, the reactive forces and deformation obtained by the use of the membrane theory at the shell's boundary usually become incompatible with the actual boundary conditions. To remove this discrepancy the bending of the shell in the edge zone has to be considered, which may affect slightly the magnitude of initially calculated membrane forces. This bending, however, usually has a very localized character and may be calculated on the basis of the same assumptions which were used in the case of small deflections of thin plates. But there are problems, especially those concerning the elastic stability of shells, in which the assumption of small deflections should be discontinued and the "Large-deflection Theory" should be used.

Non-linear analyses of shells are receiving considerable attention from design engineers for their wide application in modern engineering design. Interesting works in this field are due to Nash and Modeer (1959), W. Nowacki (1962), Nowinski and Ismail (1964), Basuli (1964), Bhattacharya (1976), Biswas (1978, 1980), Ramachandran (1976), Sinharay and Banerjee (1985) and Bera (1993).

Nash and Modeer studied the non-linear behaviour of shallow spherical shells by using Berger's method. W. Nowacki offered some interesting works on thermal deflections and stresses of cylindrical and spherical shells based on the linear as well as non-linear theory in thermoelasticity. Nowinski and Ismail used Berger's line of thought and investigated large deflections in cylindrical shell panels. The authors' analyses also are of interest from a practical point of view. An interesting paper on large deflection of shell panel by Basuli needs special mention where the author studied elegantly cylindrical shell panel following Berger approximation. Bhattacharya's analysis is also based on Berger's method. He studied non-linear dynamic behaviours of cylindrical shells. Large deflection analysis of a heated cylindrical shell has been carried out by Biswas where he applied Berger's approximation. In another paper the author studied the large deflection of heated orthotropic cylindrical shallow shell. Ramachandran, on the other hand, used Marguerre's shallow shell equations to analyze successfully the large amplitude vibrations of shallow spherical shells. Sinharay and Banerjee have offered a modified strain energy expression to study the non-linear behaviour of spherical and cylindrical shells. The authors have obtained significant results both for movable as well as immovable edge conditions.

Non-linear analysis of sandwich shell is attractive to design engineers for its wide application in space industry. Useful works in this field are due to R. K. Bera (1996) and Bhattacharya and Banerjee (1996). Bera studied the non-linear behaviour of a shallow unsymatrical sandwich shell of double curvature following a new approach while Bhattacharya and Banerjee studied large deflection of sandwich shells following displacement formulations.

A suitable analysis of large deflection of heated spherical shell has been given by R. Sircar (1978) where he used wellknown Berger's method. Behaviours of cylindrical shells with circular holes subject to uniform impulsive pressure have been studied by L. V. Andreev, A. V. Antsiferov, M. E Maslov and I. D. Pavlenko (1989). The authors' analysis is noteworthy. M. M. Suleimanova and O. P. Panova (1990) offered a suitable analysis on stress and strain of non-linear notched shell. This paper is also interesting. Non-linear deformation of cylindrical shell with local heat shock has been carried out by N. I. Obodan and N. B. Makarenko (1990). P. Kumar, M. D. Olson and D. L. Anderson (1991) studied the large deflection elastic-plastic analysis of cylindrical shells by using the finite strip method. All these papers are useful in designs.

Another interesting paper on vibrations of shallow shells could be located where Bera (1993) has successfully investigated non-linear vibrations of shallow shells of arbitrary shape using constant deflection contour lines theory. A useful paper on snaping of a thin spherical cap by Paul and Bera (1993) can be located where non-linear analysis of spherical cap has been studied following a new approach.

Shells of non-uniform thickness and non-homogeneous material are sometimes encountered in the design of machine parts and their stress analyses are imperative to design engineers. As far as it is known, works in this field are very rare. Only one paper could be located in this area, where Sinharay and Banerjee (1986) investigated large amplitude free vibrations of spherical and cylindrical shells of nonuniform thickness. They employed a modified strain energy expression in their investigation and, although their results are interesting, some amount of approximation has enterd because of the use of modified strain energy expression. Another interesting paper may also be located in this area where Mukhopadhyay and Bera (1995) studied quite elegantly non-linear thermal vibration of nonhomogeneous elastic shell.

AIM OF THE PRESENT PROJECT :

The aim of the present project is to investigate one non-linear problem of heated sandwich plate and a few non-linear problem on shells under different types of loading and under different edge conditions. The shell problems have been solved by displacement formulations. The thesis is divided into three Chapters as follows :

The first Chapter contains only one paper devoted to large deflection analysis of heated sandwich plates. A new set of uncoupled differential equations of sandwich plates has been proposed under thermal loading by using Banerjee's hypothesis (1991). The analysis of a simply supported rectangular sandwich plates with movable as well as imnovable edge has been carried out in detail. The proposed differential equations are uncoupled hence results can be obtained with minimum computational effort. These differential equations seen to be sufficiently accurate for practical purpose. They yield useful results both for movable as well as immovable edge conditions.

It is noteworthy that previous investigations by Kamiya (1976) depends upon a correction factor which varies according to the plate geometry. But the present study does not depend on any correction factor. Thus the proposed differential equations can be utilized to study the non-linear behaviours of sandwich plates of any shape with proper choice of the normal displacement. Moreover, Kamiya utilized Berger's approximation which fails completely for movable edge conditions.

To test the accuracy of the proposed differential equations the classical equations for heated sandwich plates have been solved by displacement formulations. The solutions have been obtained by solving the differential equations for the inplane displacements completly.

Results obtained from the present study have been plotted graphically. Some of the results are given in the tables. The numerical results obtained from the classical equations are also shown side by side for comparison. The numerical results for the maximum deflections of a rectangular sandwich plate are shown in tabular form. Comparative study of the results obtained from the present analysis and from the classical equations shows that, non-linear behaviours of sandwich plates with movable as well as immovable edges under thermal loading can be predicted with ease and accuracy by using the proposed differential equations.

The second Chapter is concerned with the non-linear behaviours of different kinds of shells under different types of loading. This Chapter contains three papers.

The first one deals with the non-linear analysis of clamped cylindrical shell under static, dynamic and thermal loadings. In each case the displacement formulations are used. The differential equations for in-plane displacements have been solved completely. The final equation is solved by Galerkin's error minimizing technique.

In the case of static loading numerical results showing central deflection against normal loading are given in tabular forms, and compared with other known results.

In the case of dynamic loading a comparative study of the ratio of non-linear and linear frequencies with the amplitude parameter for various geometries ($\frac{a^2}{Rh}$ = 0.25, 1.25 etc.) has been shown in tabular forms.

For the thermal loading two cases $\nabla^2 T$ = Constant and $\nabla^2 T \neq$ constant have been considered which are useful in practical design. The numerical results obtained from the present study showing central deflections against thermal loadings are given in tabular forms.

All the numerical results obtained from the present study are in good agreement with those obtained from open literature. The analysis seems to be more accurate because the differential equations for the in-plane displacements have been solved completely. Thus the present study yields sufficiently accurate results both for movable as well as immovable edge conditions with ease and accuracy.

The Second paper of this Chapter contains non-linear analysis of heated cylindrical and spherical shells under simplysupported edge conditions. The displacement formulations are used to solve the problem. The numerical results of the present study are given in tabular as well as in graphical forms and compared with other known results.

For cylindrical shells the deflection parameter $\left(\frac{w}{h}\right)$ have been plotted against the load function Ω. Results by Berger's method and by Sinharay and Banerjee's methods have been also shown side by side for comparison.

In the case of spherical shells also central deflection parameters $\left(\frac{w}{h}\right)$ against load function Ω have been shown and compared with those of knownliterature.

The Third paper of this Chapter is devoted to the non-linear analysis of a heated orthotropic thin cylindrical shell under thermal loading. Attempts have been made to solve this non-linear problem by using displacement formulations. The numerical results obtained from the present study showing central deflection parameter versus thermal loading have been given in the tabular form and compared with other known results.

The Third and the last Chapter of this thesis deals with one problem on large amplitude free vibrations of spherical shells of variable thickness. As far as it is known, works in this field are very rare. Only one paper could be located in this area where Sinharay and Banerjee (1989) investigated large amplitude free vibrations of spherical and cylindrical shells of non-uniform thickness by employing a modified strain energy expression. Their results are interesting but involve some amount of approximation.

The present paper is an attempt to study the large amplitude free vibrations of spherical shells of variable thickness by using displacement formulations. The differential equation for the in-plane displacement has been completely solved. This study seems to be more accurate than earlier works, because it never utilizes approximation except in obtaining the final equations of time function. A comparative study of the ratio of non-linear and linear frequencies with the amplitude parameter 'A' for various geometries under different thickness variation parameters has been made.