CHAPTER - II

A SUMMARY OF RESULTS ON ELECTRON LATERAL DISTRIBUTION IN AIR SHOWERS

2.1 Introduction

The coordinates of shower core and the age parameter are usually determined by fitting the observed electron densities at various points in a shower by using a theoretical distribution function. The shower size is then calculated using the known core coordinates and shower age. The assumed distribution function can introduce errors in the estimation of size in the fitting procedure. The most extensively used lateral distribution function given by Greisen[2] is a modification of the form given by Nishimura and Kamata[11].

2.2 Nishimura-Kamata-Greisen (NKG) function

According to cascade theory for a pure electromagnetic shower, the local density of electrons is written as

$$\Delta(r) = \frac{N_e f(r/r_0, s)}{r_0^2}$$

where N_e (shower size) is the total number of electrons, r_0 is known as Moliere unit of distance and s is known as the shower age parameter.

The expression for $f(r/r_0, s)$ is given by Bethe (quoted in Williams[72]) to represent approximately the Moliere distance for small values of r/r_0 as

$$\left(\frac{r}{r_0}\right) f\left(\frac{r}{r_0}\right) = 0.45\left(1+4\frac{r}{r_0}\right) \exp\left(-4\left(r/r_0\right)^{2/3}\right)$$

Greisen[2] gave the following empirical formula which fits the Nishimura-Kamata distribution for showers of maximum development and for showers of ages upto 1.4.

$$f(r/r_0, s) = C(s)\left(\frac{r}{r_0}\right)^{s-2}\left(1+\frac{r}{r_0}\right)^{s-4.5} \quad \ldots (2.1)$$

where s is given by

$$s = 3t/\left[t + 2\ln(E/t_0)\right]$$

where E is the energy of a primary at a depth t measured in units of radiation length from the point of production of the shower, ϵ_0 is the critical energy of the electron in air which is 0.084 GeV and C(s) is the normalisation constant.

The expressions of C(s) used for actual evaluation are as follows:

$$C(s) = 0.443\ s^2\ (1.90 - s) \quad \text{for } s < 1.6$$

$$= 0.366\ s^2\ (2.07 - s)^{5/4} \quad \text{for } s < 1.8$$

2.3 Modification of NKG function

It was indicated by Hara et al[26] that the lateral distribution of electrons of EAS did not fit to a NKG function with a single age parameter over a whole range of core distance. Moreover, they find that the lateral distribution of electrons is flatter beyond 1 Moliere unit and the age of the lateral distribution of electrons for showers of size greater than 3.16×10^5 when fitted to NKG function lies roughly between 1.3 - 1.5 within experimental errors.

But for smaller shower sizes, this type of flattening is difficult to observe for individual showers and hence on the average they express the lateral distribution of electrons as

$$f(X) = C_1 \; X^{s-2} \, (1+X)^{s-4.5} \, (1+C_2X^d), \; X = r/r_0$$

... (2.2)

where r is the distance from core, r_0 is Moliere unit and C_1 is a normalisation factor and is given in terms of a beta-function by

$$C_1 = \frac{1}{2\pi}\left[\beta(s, 4.5-2s) + C_2\,\beta(s+d, 4.5-d-2s)\right]^{-1}$$

Fixing $d = 1.6$, the value of C_2 increases with the size. Some values of C_2 are shown in Table 2.1.

Table 2.1

Size	C_2
1.0×10^5	0.0
3.16×10^5	0.1
1.0×10^6	0.1
3.16×10^6	0.1 - 0.2
1.0×10^7	0.2 - 0.3

Since the lateral distribution of electrons in EAS was found to be steeper near the core (Tien Shan experiment) than NKG distribution, Dedenko et al[13] gave the corrected NKG function by introducing a constant k for better agreement with the experimental data as

$$f(r) = C(s) \left(\frac{r}{kr_0}\right)^{s-2} \left(1+\frac{r}{kr_0}\right)^{s-4.5} \qquad \ldots (2.3)$$

where r_0 is Moliere radius and $k = 0.6$.

Linsley[27] also generalised the NKG formula to fit large showers at Volcano Ranch $(5.10^7 < N_e < 2.10^8)$ as follows:

$$\Delta_{VR} = \frac{N_e}{r_0^2}\, C(\alpha,\eta) \left(\frac{r}{r_0}\right)^{-\alpha} \left(1+\frac{r}{r_0}\right)^{-(\eta-\alpha)} \qquad \ldots (2.4)$$

which is normalised to N_e by setting

$$C(\alpha,\eta) = \Gamma(\eta-\alpha) \Big/ 2\pi\, \Gamma(2-\alpha)\, \Gamma(\eta-2)$$

where r_0 is Moliere length.

Kaneko et al[15] observed the lateral distribution of electrons at Mt. Chacaltaya (5,200 m.a.s.l). They find that the lateral distribution of electrons is flatter than the NKG formula at distances above $1r_0$ (r_0 = 155m). They include an extra factor in the NKG formula to allow for an increased spread and flattering near the core of real showers as follows:

$$\Delta(r, N_e) = \frac{C_1 N_e}{2\pi r_0^2}\left(\frac{r}{r_0}\right)^{s-2}\left(1+\frac{r}{r_0}\right)^{s-4.5}\left[1 + C_2\left(\frac{r}{r_0}\right)^{2.0}\right] \quad \ldots (2.5$$

where $C_1 = 1 \Big/ \left[\frac{\Gamma(s)\,\Gamma(4.5-2s)}{\Gamma(4.5-s)} + C_2\,\frac{\Gamma(2.0+s)\,\Gamma(2.5-2s)}{\Gamma(4.5-s)}\right]$

and $C_2 = 0.100 + 0.125\ (\mathrm{Sec}\,\theta - 1)$

for $3 \times 10^6 < N_e < 3 \times 10^9$, $0 < \theta < 60^\circ$,

$0.1r_0 < r < 3r_0$ and r_0 = 155m.

2.4 Calculation of Capdevielle et al.

As pointed out by Capdevielle et al[25], it is difficult to describe the lateral distribution of electrons in EAS with a single age parameter. For showers of all sizes, the formula derived by Capdevielle et al is given by

$$\Delta_e(r) = \frac{N_e}{m^2 r_0^2}\, f_{HG}\left(r/mr_0\right) \quad \ldots (2.6)$$

with

$$f_{HG}(r/mr_0) = C_{HG}(s)\left(\frac{r}{mr_0}\right)^{s-2}\left(\frac{r}{mr_0}+1\right)^{s-4}\left(1+d\,\frac{r}{mr_0}\right)^{2.7-s} \quad \ldots (2.7)$$

where $d = 0.026$, $m = 0.5$, r_0 is the Moliere radius and N_e is the size.

A possible parametrization for $C_{HG}(s)$ is given by

$$C_{HG}(s) = 0.3265 \exp\left[-0.5\left(\frac{s-1.125}{0.499}\right)^2\right], \quad s \leq 1.4$$

$$C_{HG}(s) = 0.2854s^2 - 1.385s + 1.66, \quad s > 1.4.$$

2.5 Calculation of Lagutin et al.

The problem of applicability of NKG formula near the shower core and correct accounting of the primary and secondary particle threshold energies was considered by Lagutin et al[24]. In particular, the rising of secondary threshold energy leads to some narrowing of lateral distribution. It is observed by Lagutin et al[24] that the dependence on primary energy E is important at smaller distances $r \leq 2m$ from the shower axis and in the region $r \gtrsim 80m$ where density increases with the increase of E. Such behaviour is in agreement with the analytical theory by Nishimura[73]. If E is in the range $10^3 - 10^4$ GeV, the distribution does not depend on E for $0.5 \leq r \leq 300$m. But for higher E, the NKG function does not describe the lateral distribution. They find that the better agreement with the experimental results is given by

$$f(r) = m^{-2} \quad f_{NKG}\,(r/m) \qquad \ldots (2.8)$$

where $m \approx 0.78 - 2.21s$ for $0.6 \leq s \leq 1.6$.

2.6 Monte Carlo calculations of Hillas et al.

Hillas and Lapikens[23] have simulated electron initiated cascade showers in the atmosphere and developed a lateral distribution function taking into account various energy dependence of electron and photon interaction processes down to an energy less than 1 MeV. This is the most detailed simulation of cascades by electron and photon primaries upto 100 GeV. The calculated electron density as a function of core distance is represented by the following distribution function

$$f(r) = C(s) \left(\frac{r}{r_0}\right)^{a_1 + a_2(s-1)} \left(1 + \frac{r}{r_0}\right)^{b_1 + b_2(s-1)} \qquad \ldots (2.9)$$

where C(s) is the normalisation constant and the parameters are chosen for best fit of the experimental results as

$$r_0 = 24m, \; a_1 = -0.53, \; a_2 = 1.54, \; b_1 = -3.39, \; b_2 = 0.$$

A comparison of the various distribution is indicated in figure 2.1.

2.7 Experimental observations on lateral distribution of electrons.

The recent measurements of electron lateral distribution in the size range 10^4 to 10^6 particles have been reported among others by Hara et al[74], Abdrashitov et al[34], Khristiansen et al[75], Danilova et al[76], Kaneko et al[15]. A representative lateral distribution measured by Hara et al[74] is shown in figure 2.2. They find that the lateral density of electrons cannot be represented by standard NKG function at all core distances with the single age parameter and that the transition effect in 50 mm thick scintillator is negligibly small in showers of size 10^5 particles. The other groups find that the experimental distribution are different from those expected from NKG function at smaller and larger core distances.

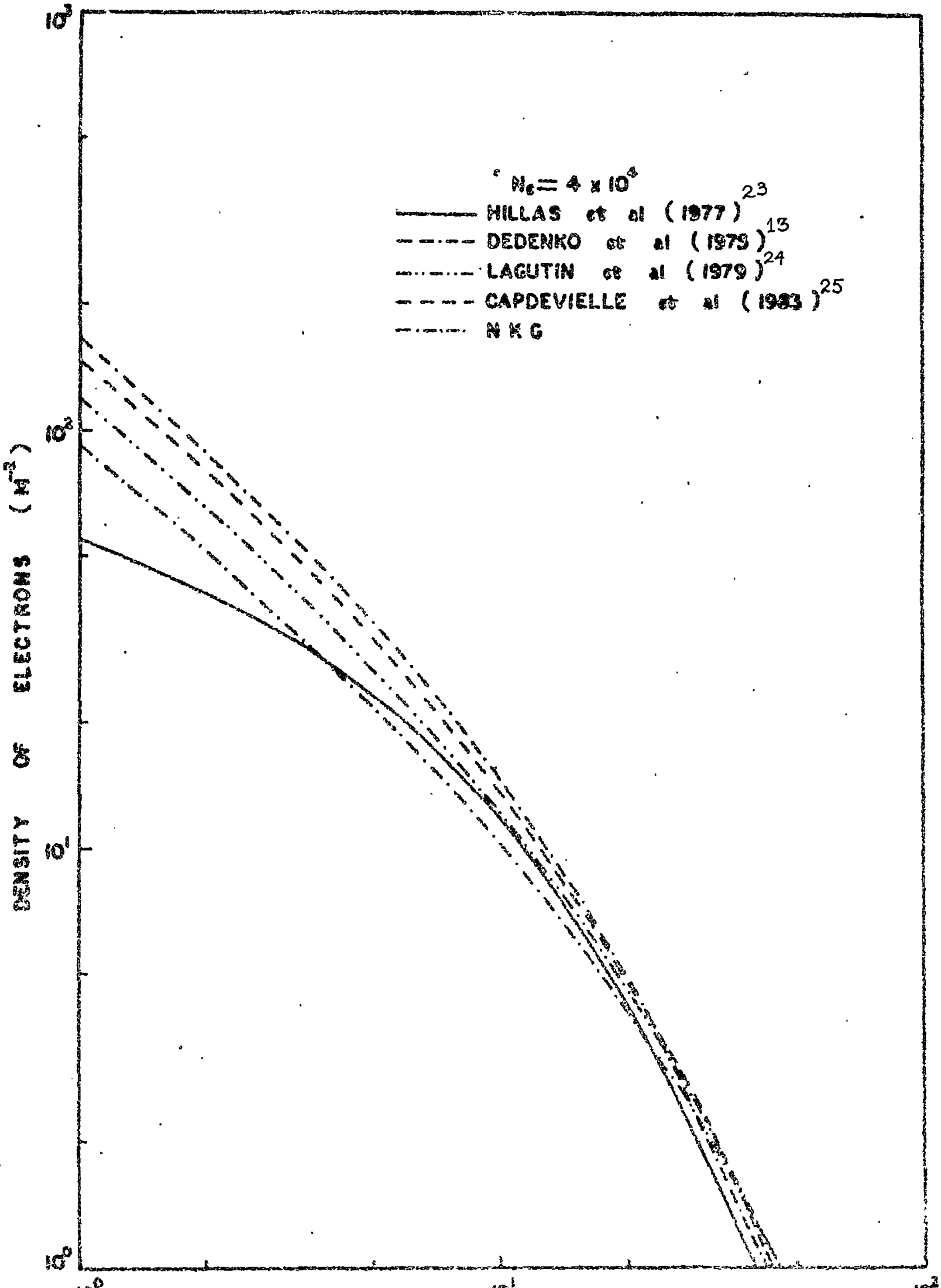

FIG. 2.1. A COMPARISON OF LATERAL DISTRIBUTION FOR $N_e = 4 \times 10^4$ AND $s = 1.2$.

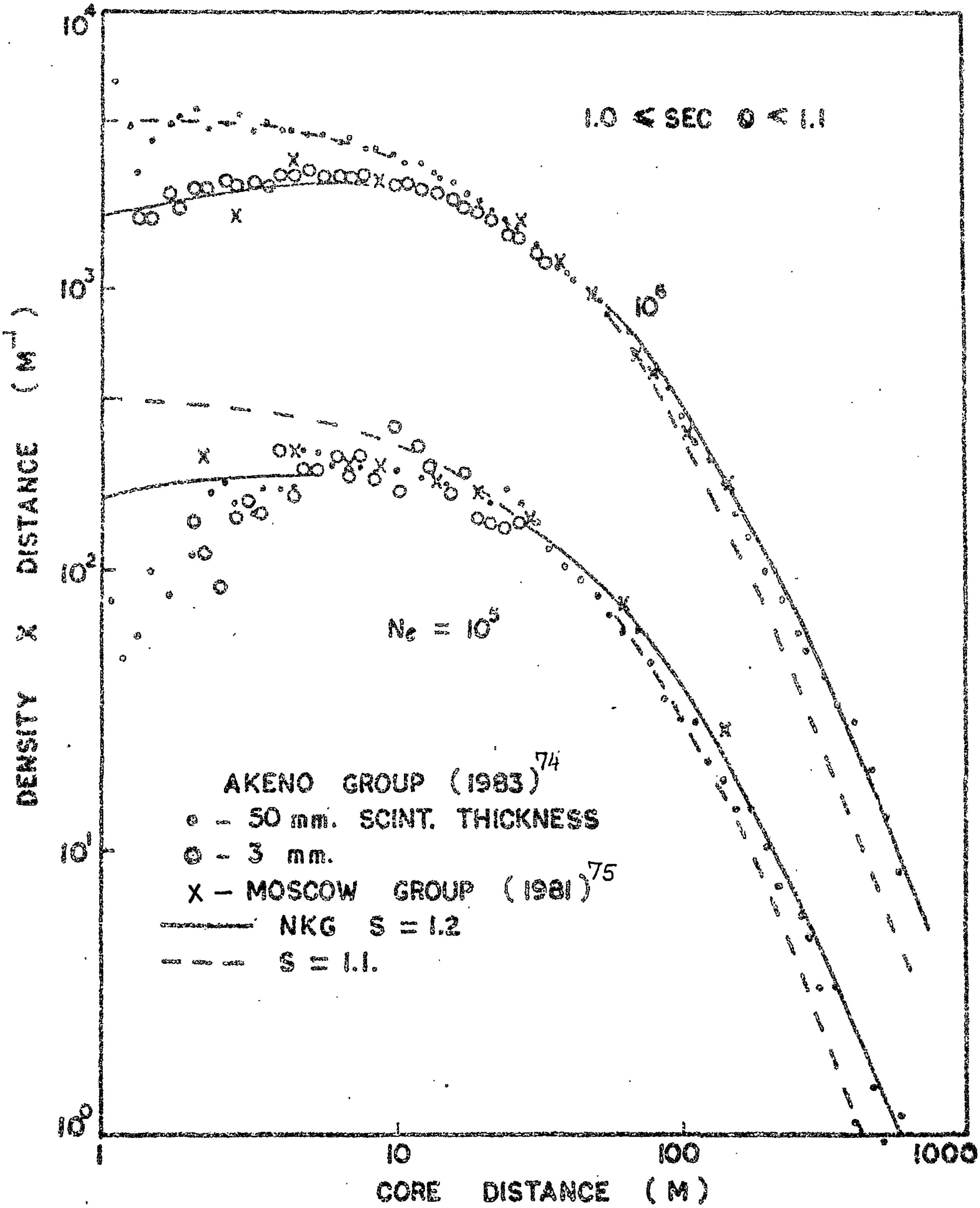

FIG. 2.2. AN EXAMPLE OF ELECTRON LATERAL DISTRIBUTION.