WITTGENSTEIN ON LANGUAGE AND LOGIC

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Introduction:

When we talk about logic, the first question that we need to discuss is what is logic? Logic can be understood as an *instrument or organon* which evaluates the correctness of reasoning. According to Charles Peirce, there are more than a hundred definitions of it. So when we study logic, we study its method and principles that are used to distinguish *correct arguments from incorrect arguments*. So, the studies of logic will us to learn certain techniques through which one can be able *to test the validity of all arguments*. Logic can also be defined as the science of reasoning. Moreover, logicians are not concerned with this process of inference rather with the initial and endpoints of that process. Further traditional logic is conceived with the science of valid inference. In this regard, McGinn says logic "is concerned with the laws whereby we justifiably move from one judgement or assertion of truth to another."[88] Likewise, Wittgenstein says that these laws are nothing over and above propositions of language and *their truth-functional articulation*. Wittgenstein's conception of logic is mainly found in his books *Tractatus Logico Philosophicus* and *Notes on Logic*. In *Tractatus,* we find two important components: one is the elementary proposition and another is the molecular or complex proposition which is the truth-function of elementary ones. He again represents the principle of truth-functionality in his *Notes on Logic* in the form of *ab-function* as a new dimension of representing a truth-functional account of a proposition. While discussing this *ab*-function Wittgenstein goes on to discuss his *concept of entailment*. So, through the discussion of these concepts, we will also try to understand the logic hidden inside the language because Wittgenstein said, "My whole task consists in explaining the nature of the proposition." (NB: 39)

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[88] McGinn, M., *Elucidating the* Tractatus, *Wittgenstein's Early Philosophy of Logic and Language,* Oxford University Press, 2006, p. 54.

Bipartite Reading:

Wittgenstein in his book *Tractatus* remarks, "A proposition can be true and false in virtue of being a picture of reality."[89] Similarly, in his *Notebooks,* he remarked "Only in this way can the proposition be true or false: it can only agree or disagree with reality by being a picture of a situation." [NB: 2.10. 14] A proposition, for Wittgenstein, can be determined as either true or false only in respect of *a fact, pictorial relationship and pictorial form*. These three things make a proposition *intrinsically true or false*. That is why he says in *Tractatus*, "A picture agrees with reality or fails to agree; it is correct or incorrect, true or false."[90] He goes on to say, "The agreement or disagreement of its sense with reality constitutes its truth or falsity."[91]

According to Wittgenstein, propositions of logic ((tautologies, contradiction etc.) belong to the category of complex or molecular proposition which is nothing but the truth functions of the elementary proposition. Therefore, the truth value of these molecular propositions is determined by the truth value of their constituent's propositions. That is why Wittgenstein in his *Tractatus* says, "A proposition is a truth-function of elementary propositions."[92] Molecular propositions are nothing but the "... expression of agreement and disagreement with truth-possibilities of elementary propositions."[93] This agreement and disagreement can be shown by a truth-table in the following manner:

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

[89] Wittgenstein, L., *Tractatus Logico Philosophicus,* Pears and Mcguinnes (trans.), London, Routledge Classics & Kegan Paul, 1922, p. 27.
[90] Ibid., p. 12.
[91] Ibid., p. 12.
[92] Ibid., p. 43.
[93] Ibid., p. 38.

Here, on one hand, we have an elementary proposition that works as the pictorial representation and on the other hand, we have molecular propositions which are the truth-functions of them. These two accounts of the *Tractatus* are based on the principle known as the *bipartite* reading of the *Tractatus*. In this regard, Georg von Wright says, "Wittgenstein's *Tractatus* may be called a synthesis of the theory of truth-functions and the idea that language is a picture of reality."[94] Similarly, K. T. Fann says, "Wittgenstein's theory of language in the *Tractatus* has two components: the 'picture theory and 'truth-function theory."'[95] Now the question is, is bipartite reading enough to understand the *Tractatus*? I do not think so because there are many accounts of the *Tractatus* such as ethical, logical, metaphysical, and religious. So from this, I can say that bipartite reading can be justified as a valid reading from a particular perspective i.e., logical perspective but not overall perspective. Some philosophers also raise some objections against this bipartite reading which are as follows:

The first objection is made by Brain McGuinness. He says that "In the first part of the Tractatus [...] we seem to be told that the essence of a proposition is to be a picture, while in the later parts we are told that its essence is to be a truth-function [...]. [A] [...] serious difficulty is that the two accounts seem to be quite separate things, and, if this is so, cannot both be adequate accounts of what it is to be a proposition."96 Like B. McGuinness, Michael Morris complains that "There is a risk of understanding Wittgenstein's account of language [...] as falling into two completely unconnected parts: one which is appropriate to the conception of elementary sentences as models, the other which concerns the construction of other sentences out of elementary sentences."97 Here, as a reply to these above objections, it can be said that while Wittgenstein is talking about propositions he talks from both the background of pictures i.e., elementary proposition and molecular proposition which is truth-functions of them. Therefore, it is clear to us that the language of the

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[94] Wright, G. H., "Ludwig Wittgenstein. A Biographical Sketch". *The Philosophical Review,* Vol. 64, 1955, p. 533.
[95] Fann, K. T., *Wittgenstein's Conception of Philosophy*, Oxford, Basil Blackwell, 1969, p. 8.
[96] McGuinness, B., 'Pictures and Forms', *Approaches to Wittgenstein*, London, Routledge, 2002, pp. 65-66.
[97] Morris, M., *Wittgenstein and the Tractatus Logico-Philosophicus*, London, Routledge, 2008, pp. 234-35.

Tractatus contains both the pictorial structure and truth-functional structure. Therefore from the bipartite reading of the Tractatus, it can be said that there is an internal relation between propositional articulation and logical articulation and also a propositions ability to express the sense that it does. Here questions may be asked that how does Wittgenstein show that an account of the nature of the proposition can be said to be able to explain the nature and relation of propositional and logical articulateness? And how does picture theory give rise to a truth-functional account of the nature of the proposition? In reply to the first question, Wittgenstein says that the concept of propositional and logical articulation is understood or is thought of as function-argument articulation. As an example, Wittgenstein in his Tractatus says "Like Frege and Russell I construe a proposition as a function of the expressions contained in it."98 He further goes on to say, "I write elementary propositions as functions of names so that they have the form 'fx', '□(x, y)', etc. Or I indicate them by the letters 'p', 'q', 'r'."99 In prototractatus, he says that Generally in what follows I indicate elementary propositions by the letters p, q, r, s, t, or else (like Frege) I write them as functions of their objects in the form –'□(x)', '□(x, y)', etc.[PT: 4.2212]. In this regard, Elizabeth Anscombe says that "if the elementary proposition consists of names in immediate connection – if it is just a concatenation of names – then it is not reproduced, even if it can faithfully be represented by a formula consisting of some letters for names and some letters for functions."100 Thus, through this function argument articulation Wittgenstein gave the justification of propositional and logical articulation. Therefore, it is clear that the nature of logic is embedded in the pictorial character of the language. In this regards Wittgenstein in his Notebooks remarks, "All logical constants are already contained in the elementary proposition" [NB: 5.11.14]. Even in the book Tractatus he says the same as, "An elementary proposition really contains all logical operations in itself. For 'fa' says the same thing as '(∃x).fx.x=a'. Wherever there is compositeness, argument and functions are present, and where these are present, we already have all the logical constants."101 Therefore, based on the above analysis, it can be said that as soon as elementary proposition i.e., pictorial

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[98] Wittgenstein, L., *Tractatus Logico-Philosophicus,* op. cit., p. 18.
[99] Ibid., p. 36.
[100] Anscombe, G. E. M., *An Introduction to Wittgenstein's Tractatus*, Second edition, Hutchinson, 1963, p. 100.
[101] Wittgenstein, L., *Tractatus Logico-Philosophicus,* op. cit., p. 56.

character and compositeness are given, logical articulateness is also given there. So the explanation of the propositional articulation and logical articulation in terms of function-argument articulation amounts to the idea of the functional model. Now the question is what is this functional model? In this regard, Frege says that the conceptual content which is also called the content of a judgement or of a proposition is analysable into function and argument in which the former is the constant part and the letter is the replaceable part. Therefore it is said that conceptual content is nothing but the combination of argument and function. Regarding this, Frege further says that "If in an expression [...], a simple or complex symbol occurs in one or more places and we imagine it as replaceable by another (but the same one each time) at all or some of these places, then we call the part of the expression that shows itself invariant a function and replaceable part its argument." [CN: §9]. Frege further asserts that the functional model can be applied to any possible object of judgement or proposition. Frege shows the expression of a function in *Begriffsschrift* of the argument 'A' as '(A)'. Here he kept argument A in a bracket following parentheses is the symbol for the function. He gave another example of the two arguments such as argument 'A' and argument 'B'. He shows the function of these two arguments as '(A, B)'. Following Frege in 4.24 of the *Tractatus* Wittgenstein presented a similar view i.e., the characterisation or expressions of elementary propositions, for example, 'f(x), '□(X, Y)' have a functional structure. Here 'f(x)' represents the function of one argument i.e., 'x' and '□(X, Y)' presents a function of two arguments such as 'x' and 'y'. Therefore, from this, it can be said that the significance of the functional model or structure for Wittgenstein is that it indicates the idea that a proposition is said to be articulate, complex and hence has a structure.

Now in reply to the second question i.e., how does picture theory give rise to a truth-functional account of a proposition, it can be said that the answer to this question will be clear from the remarks given by Wittgenstein in the book *Tractatus*. His position is as follows:

Truth-possibilities of elementary propositions mean Possibilities of existence and non-existence of states of affairs [TLP: 4.3].

We can represent truth-possibilities by schemata of the following kind ('T' means 'true', 'F' means 'false'; the rows of 'T's' and 'F's' under the row of elementary propositions symbolize their truth-possibilities in a way that can easily be understood):[TLP: 4.31]

p	q	r
T	T	T
F	T	T
T	F	T
T	T	F
F	F	T
F	T	F
T	F	F
F	F	F

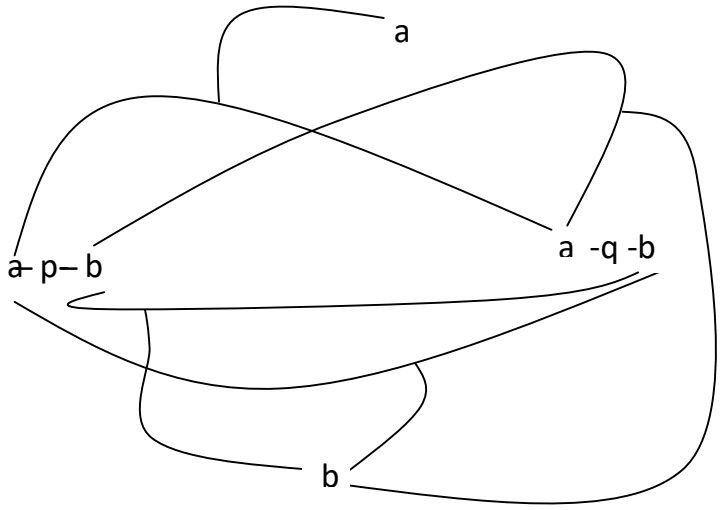
p	q
T	T
F	T
T	F
F	F

P
T
F

The above table says that truth-possibilities of elementary propositions are the conditions of the truth and falsity of propositions [TLP: 4.41]. Therefore, the truth possibilities of the elementary propositions show agreement and disagreement with reality. That means if it agrees with reality then the proposition pictures a fact and if it does not agree then the proposition doesn't picture a fact. Therefore, the elementary proposition which is pictorial in character gives rise to a truth-functional account of a proposition.

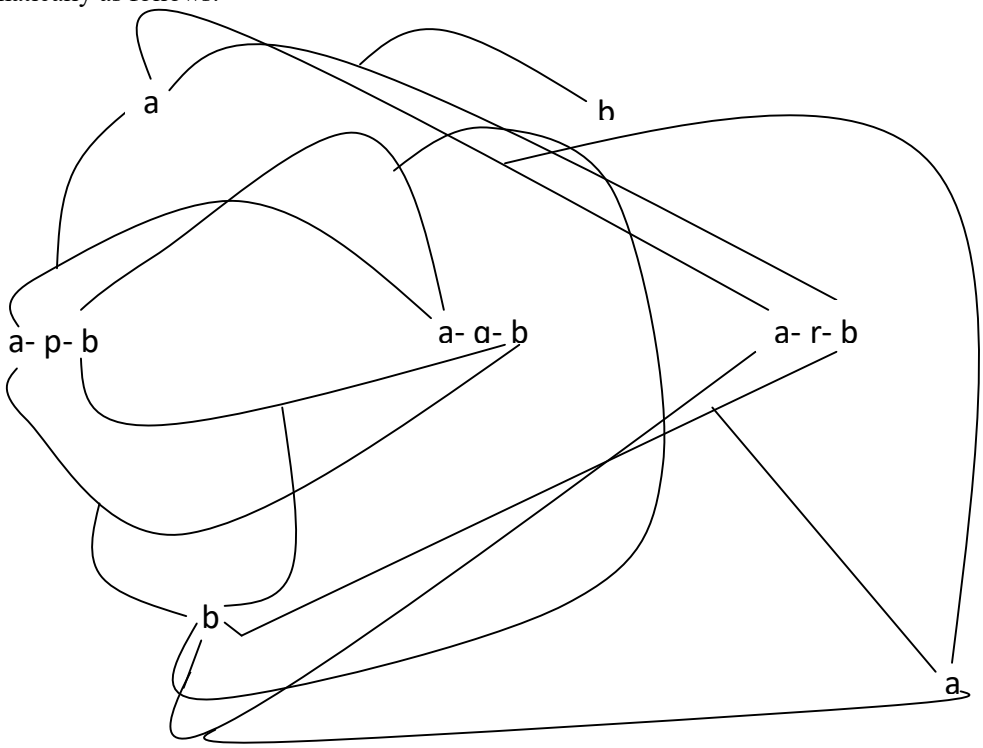
Bipolarity (*ab-function*):: In this section, I would like to discuss Wittgenstein's concept of *bipolarity* i.e., his notion of *ab-function* mentioned in the *Notes on Logic* and thereby I shall try to show the logic behind it. Now the question may be asked why does Wittgenstein develop the *ab-function*? How does Wittgenstein explain the sense of a proposition in the light of the *ab*-function? Wittgenstein said that every proposition bears two senses: one is true and another is false. Similarly in the *Notes on Logic*, he says, "Every proposition is essentially true-false: to understand it, we must know both what must be the case if it is true, and what must be the case if it is false. Thus a proposition has two poles, corresponding to the case of its truth and the case of its falsehood. We call this the sense of a proposition." [NL: 98-99] This remark given by Wittgenstein indicates the idea that a proposition has truth poles that he indicated with the later *a* and *b*. Here, *'a'* indicates to 'true' pole and *'b'* indicates to 'false' pole. Therefore a proposition with its truth poles for Wittgenstein appears to be as *'a-p-b'*. In *Notes on Logic*, Wittgenstein asserts molecular propositions which are the truth-functions that were made possible based on elementary propositions are

called *ab-functions*. So poles are important to understand Wittgenstein's account of molecular propositions. As we know that for Wittgenstein the proposition *p* is rendered with the help of truth poles as *a-p-b*. So the proposition *'~p'* is a molecular proposition with its true-false poles will be appearing as *'b-a-p-b-a'*. It seems to mean that, here the symbol '~' (negation) reversing the truth-conditions of the proposition *p*. That is turning the *a-pole* into the position of the *b-pole* and the *b-pole* seats in the place of *a-pole* and thereby it appears as *'b-a-p-b-a'* for the proposition of not p. After showing this, Wittgenstein now goes on to give another example which is devised with the help of two propositions such as proposition *p* and proposition *q*. To show this example of two propositions in his notion of *ab*-notation Wittgenstein developed his devising truth diagrams in the following manner. Here the truth diagram that Wittgenstein developed will show the case of a conjunctive proposition i.e., p & q. As we know that a conjunctive proposition is true if both its conjuncts are true and in other cases it is false. The diagram is as follows:



Here, the diagram shows that the outer '*a-pole'* is situated outside the diagram i.e., in the above of the diagram is the symbol of 'true pole' and the *'b-pole'* which has taken place outside the diagram i.e., in the bellow of the diagram is called 'the false pole'. There are two propositions on the two sides of the diagram. On the left side, we have the proposition *p* with its true poles appears as *a-p-b*. Here *a-pole* of the left side proposition *p* indicates true pole and *b-pole* indicates false pole. Similar is the case of the right side proposition *q*. In the diagram *a*-pole of the proposition *p* and *a*-pole of the proposition *q* are connected by a line and thereby we

have the result of the outer '*a-pole'* which is *'true pole'*. This is so because as we know a conjunctive proposition is true if both its conjuncts are true. So, here is the connection between *a-pole* i.e., the true pole of the proposition *p* and *a-pole* i.e., also the true pole of the proposition *q* making this conjunctive proposition *p & q* true. The outer '*b-pole'* which is a symbol of a false pole, is connected with all other connections of the poles of the proposition *p* and *q* showing the false cases of the conjunctive proposition. Therefore from the above diagram, we can clearly say that Wittgenstein's conception of *ab-function* which was made possible through elementary propositions i.e. *p* and *q* have pole '*ab' (true/false)* and hence is bipolar like elementary propositions which are clear from the above truth diagram. This truth diagram also shows that how a molecular proposition which is the true function of the elementary propositions, get its *ab-poles* through correlations with the *ab-poles* of the original elementary propositions. That is why Wittgenstein in the *Notes on Logic* remarks, "As the ab-function of atomic propositions is bipolar propositions, again we can perform ab-operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions." [NL: 94] Another example of the three propositions i.e., (p & q) ⊃ r can be represented diagrammatically as follows:



In this diagram first, we have connected the proposition *p* and *q* and thereby we get the proposition *p&q* (description of this *p&q* proposition has already been mentioned in the first diagram). Then we connect *'p&q'* with the proposition '*r'* through *ab-function* to make the *ab-poles* of the complex or molecular proposition *(p & q) ⊃ r*. As we know that an implicative proposition is false if its left-hand side is true and its right-hand side is false and in other cases it is true. Here in the diagram, we see that the outer '*a (true) pole'* of the proposition *p&q* is connected with the *b (false) pole* of the proposition *r* and thereby we have the result of the outer *'b-pole'* i.e., false pole which makes this molecular proposition i.e., *(p & q) ⊃ r* false. It is so because as we know that an implicative proposition is false if its left-hand side is true and the right-hand side is false. The outer '*a-pole'* which makes the proposition '*(p & q) ⊃ r'* true, is connected with all other connections of the poles of the proposition '*p&q'* and the proposition *'r'* showing the true cases of the above implicative proposition. Therefore, from this, we can say that molecular propositions or *ab-functions* do not introduce anything that was not already provided by the elementary propositions occurring as arguments in them. In other words, molecular propositions do not add anything to the elementary propositions, based on which they are constructed. The thing they only can do is that they just-make a rearrangement of the elementary propositions *ab-poles*. That is why Wittgenstein says, "Molecular propositions contain nothing beyond what is contained in their atoms; they add no materials information above that contained in their atoms. All that is essential about molecular functions is their T-F schema i.e., the statement of the cases when they are true and the cases when they are false." [NL: 98] In 1913, when Wittgenstein was busy with his study on logic, he wrote a letter to Moore and in that latter, he gave an example of the proposition of p equivalent p or it says 'p if and only if p' by his ab-function as follows:

As we know that the above proposition is called *bio-conditional* proposition which will be true if it's both left hand and right hand are true and if it's both left hand and right hand are false. Here in this diagram, the outer *'a'* pole is called true pole has taken place at the bellow the diagram, which is made possible through the

connection between *a-poles* of the atomic proposition left hand/right hand *'p'* and also through the connection between *b-poles* of the atomic proposition i.e., left hand/right hand *'p'*. On the other hand, the outer *'b'-pole* is called false pole which is made possible through the connection between the *a-pole* of the first occurrence of *'p'* and the *b-pole* of the second occurrences of *'p'* and again with the connection between the *b-pole* of the first occurrence of *p* and the *a-pole* of the second occurrence of *p*. After drawing this diagram, Wittgenstein says that the above two connections are not possible because the proposition *'p'* cannot both be true and false at the same time. Therefore, for Wittgenstein, the outer *'b'* pole is connected through an impossible link and hence cannot indicate a genuine possibility, must be ignored. So, we have now only outer *'a'*-pole i.e., true pole and hence always true and indicates the notion of tautology. Therefore, for Wittgenstein, the significance of *ab-function* is that the propositions of logic (tautologies, contradiction etc.) can be presented by using the same method that we used to establish the molecular propositions out of the elementary proposition. That is why he says that like molecular propositions, propositions of logic are nothing but the *ab-functions* of elementary propositions. The *ab-function* (bipolar in nature i.e., true/false pole) is a new notation by which he determines the sense of a proposition.

The Concept of Entailment: In this section, I would like to discuss the concept of entailment in terms of Wittgenstein. The concept of entailment is also known as a logical consequence for Wittgenstein. Wittgenstein in his book *Tractatus* defines the concept of entailment through the following remarks. As he says: "If all the truth-grounds that are common to a number of propositions are at the same time truth-grounds of a certain proposition, then we say that the truth of that proposition follows from the truth of the others." [TLP: 5.11] He further says, "In particular, the truth of a proposition 'p' follows from the truth of another proposition 'q' if all the truth-grounds of the latter are the truth-grounds of the former." [TLP: 5.12]

The truth-grounds of the one are contained in those of the other: p follows from q [TLP: 5.121].

Therefore, for Wittgenstein, the concept entailment says that if all the circumstances in which some propositions are true then we can say that those circumstances are also the circumstances in which a certain proposition is true then

we can say that the truth of the latter proposition entailed by or follows from the truth of the former proposition. Let's take an example to understand it. The following example will show how the one proposition i.e., *'~p'* is entailed by the propositions as *'p ⊃ q'* and *'~q'*.

p	q	p ⊃ q	~q	~p
T	T	T	F	F
F	T	T	F	T
T	F	F	T	F
F	F	T	T	T

The above truth table shows that the true possibilities of the propositions *p* and *q* in the first two columns. After that, we have determined the truth-conditions of the rest of the propositions such as *p⊃q, ~q, and ~p.* Now, here it is clearly visible that the proposition '*~p'* follows from the propositions *p⊃q,* and *~q,* it is so because in all the cases in which the latter propositions are both true, the proposition '~p' is also true. So, we can say that the truth grounds of the propositions *p⊃q,* and *~q,* are the same as the truth grounds of the proposition *'~p'.* In this regard Wittgenstein remarks in his book *Tractatus*, "When the truth of one proposition follows from the truth of others, we can see this from the structure of the propositions."[102] He goes on to say, "If the truth of one proposition follows from the truth of others, this finds expression in relations in which the forms of the propositions stand to one another: nor is it necessary for us to set up these relations between them, by combining them with one another in a single proposition; on the contrary, the relations are internal, and their existence is an immediate result of the existence of the propositions."[103] Therefore, Wittgenstein here is trying to say that the concept entailment is nothing but a structural relation between propositions. The structural relation for Wittgenstein is also called internal relation. It is so because the structure shows that the truth of one proposition follows from the truth of others. So, for Wittgenstein, the concept entailment can be understood with the help of the truth-conditions of propositions.

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[102] Op. Cit., p.46.
[103] Ibid., p. 46.

Relation between entailment and inference: It has been said that the concept of entailment given by Wittgenstein is intermingled with the nature of inference in *Tractatus*. Now the question is why does Wittgenstein talk about the internal relation of entailment in terms of its being the justification for an inference? In reply to this question, Wittgenstein says in the *Tractatus* that he seems to base the inference on the entailment. It is so because he thinks that justification of an inference lies in the presence of a relation of entailment between two propositions. In this regard, Wittgenstein writes, "If p follows from q, I can make an inference from q to p, deduce p from q. The nature of the inference can be gathered only from the two propositions. They themselves are the only possible justification of the inference. 'Laws of inference', which are supposed to justify inferences, as in the works of Frege and Russell, have no sense, and would be superfluous."[104] The above relation of inference and entailment is also found in the lectures that Wittgenstein gave in Cambridge in 1930-1932. In these lectures, in terms of entailment and inference, he says: *Inference is the transition from one proposition to another, a transition which we justify by saying that e.g. q follows from p. This relation is entirely determined when the two propositions are given [WLC: 56]*. He further says that: *Inference is justified by an internal relation which we see; the only justification of the transition is our looking at the two terms and seeing the internal relation between them [WLC: 57]*. Therefore for Wittgenstein, the relation between inference and entailment is that the latter provides the relevant justification for the former. So if we discuss inference in the light of the above view then inference can be taken as a justified inference i.e. followed from the internal relation of entailment between two propositions.

Concluding Remarks:

From the aforesaid discussion, I can say that the claim that an elementary proposition contains all logical constants in itself is justified by the sense of a proposition for Wittgenstein. In the book, *Tractatus* the sense of a proposition lies in the picture of a state of affairs that makes it either true or false. So from this, I can say that instead of *ab-function* Wittgenstein in the *Tractatus*, represents the truth conditions of molecular propositions with the help of truth tables. Whereas in the

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[104] Ibid., p. 46.

Notes on Logic, he shows how does one can get molecular propositions out of elementary propositions. Moreover, he also represents the truth conditions of molecular propositions with the help of his *ab-function*. The new notation also makes clear in what sense propositions of logic i.e. tautologies, contradictions etc. are the products of truth-functionally constructing molecular propositions. That is why Wittgenstein says that: Among the possible groups of truth-functions there are two extreme cases. In one of these cases, the proposition is true for all the truth-possibilities of the elementary propositions. We say that the truth conditions are *tautological.* In the second case, the proposition is false for all the truth-possibilities: the truth-conditions are *contradictory* [TLP: 4.46].

Here I want to divulge another point i.e., to understand Wittgenstein's concept of logic we need to understand the transcendence of logic after Wittgenstein. Regarding this Wittgenstein said *logic is transcendental.* Now the question is, in what sense logic is transcendental? Wittgenstein in *Tractatus* talks about ethical and logical transcendence. But there is a difference between the transcendence of ethics, religion and logic. Ethics, religion etc. are transcendental in the sense that they do not picture anything in the world, hence lie outside the world or transcends the world. Regarding the transcendence of ethics, we can say that as values lie outside the world so ethics transcendence the world. On the other hand, logic is transcendental in the sense that logic is associated with the structure of the world as a whole. It seems to mean that as soon as the structure of the world as a whole is given, logic is also given. That means for Wittgenstein logic is associated with the totality of facts. In this sense logic is transcendental and this transcendentality of logic is different from the transcendentality of ethics, religion etc. They transcend the facts of the world so they treat the world as a whole. Therefore, at last from the above discussion, following Wittgenstein, I conclude by saying that as soon as elementary propositions (language) are given, logic as a whole is also given.