

## DECLARATION

I declare that the thesis entitled **STUDIES ON VERY COMPACT STARS** has been written by me under the supervision of **Prof. S. Mukherjee**, Retired Professor, Department of Physics, the University of North Bengal. No part of the thesis has formed the basis for the award of any degree or fellowship previously.

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# CERTIFICATE

I certify that **Shibsankar Karmakar** has written the thesis entitled **STUDIES ON VERY COMPACT STARS**, for the award of Ph.D. degree of the University of North Bengal, under my supervision. He has carried out the work in the Department of Physics, the University of North Bengal.

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# Preface

The research work presented in the thesis is carried out at the University of North Bengal, District: Darjeeling, Pin-734013, West Bengal, India. The thesis is the result of my own research work, as well as the outcome of scientific collaborations. The thesis is organised as follows:

1. Chapter-1 gives the background of the research topics and its relevance in the study of very compact cold stars. A brief discussion of the methodology used and the results obtained have been given.

2. Chapter-2 is based on the paper entitled:

**“Maximum Mass of a Class of Compact Stars”**

R. Sharma, S. Karmakar and S. Mukherjee, *Int. J. Mod. Phys. D* **15**, 405 (2006)

3. Chapter-3 is based on the paper entitled:

**“Role of Pressure anisotropy on the Maximum mass of Cold Compact Stars”**

S. Karmakar, S. Mukherjee, R. Sharma and S. D. Maharaj, *Pramana J. Phys.* **68**, 881 (2007)

4. Chapter-4 is based on

i) revised version of the paper entitled:

**“Relativistic Models of Strange Stars with Anisotropy”**

B. C. Paul, P. K. Chattopadhyay, S. Karmakar and R. Tikekar, *Mod. Phys. Lett. A* **26**, 8 (2011)

ii) and the paper entitled:

**“Strange Star with Density Dependent Bag Parameter and Color Superconductivity”**

S. Karmakar, and S. Mukherjee. (*Communicated*)

5. Chapter-5 is based on the modified version of the paper entitled:

**“Relativistic Anisotropic Star and its Maximum Mass in Higher Dimension”**

B. C. Paul., P. K. Chattopadhyay and S. Karmakar, *Astrophys Space Sci.* **356**, 327 (2015)

6. Chapter-6 gives a discussion of the results obtained and some concluding remarks.

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*Dedicated to my late parents.....*

*SATISH CHANDRA KARMAKAR and NARAYANI KARMAKAR*