CHAPTER - 2

Tools for Logical Analysis

So far we have seen the role of reasoning/inference in Western Mathematical Logic. Now let us turn our attention to some characteristic features of Mathematical Logic, which is very much essential for philosophising or for knowing the truth.

All statements can be divided into two kinds, simple and compound. A *simple* statement is one, which does not contain any other statement as a component part, whereas every *compound* statement does contain another statement as a component part. For example, 'Unethical practice will be eradicated or this society will become uninhabitable' is a compound statement that contains as its components the two simple statements 'Unethical practice will be eradicated' and 'this society will become uninhabitable'. The component parts of a compound statement may themselves be compound, of course. We turn now to some of the different ways in which statements can be combined into compound statements.

The statement 'Roses are red and violets are blue' is a *conjunction*, a compound statement formed by inserting the word 'and' between two statements. Two statements so combined are called *conjuncts*. The word 'and' has other uses, however, as in the statement 'Castor and Pollux were twins', which is not compound, but a simple statement asserting a relationship. We introduce the dot '·' as a special symbol for combining statements conjunctively. Using it, the preceding conjunction is written 'Roses are red · violets are blue'. Where p and q are any two statements whatever, their conjunction is written p·q.

Every statement is either true or false, so we can speak of the *truth value* of a statement, where the truth value of a true statement is *true* and the truth value of a false statement is *false*. There are two broad categories into which compound statements can be divided, according to whether or not there is any necessary connection between the truth value of the compound statement and the truth values of its component statements. The truth value of the compound statement 'Smith believes that lead is heavier than zinc' is completely independent of the truth value of its component simple statement 'lead is heavier than zinc', for people have mistaken as well as correct beliefs. On the other hand, there is a necessary connection between the truth value of a conjunction and the truth values of its conjuncts. A conjunction is true if both its conjuncts are true, but false otherwise. Any compound statement whose truth value is completely determined by the truth values of its component statements is a *truth – functionally* compound statement. The only compound statements we shall consider here will be truth – functionally compound statements.

Since conjunctions are truth-functionally compound statements, our symbol is a truth-functional connective. Given any two statements p and q there are just four possible sets of truth values they can have, and in every case the truth value of their conjunction p·q is uniquely determined. The four possible cases can be exhibited as follows:

in case p is true and q is true, p·q is true;
in case p is true and q is false, p·q is false;
in case p is false and q is true, p·q is false;
in case p is false and q is false, p·q is false.

Representing the truth values true and false by the capital letters 'T' and 'F', respectively, the way in which the truth value of a conjunction is determined by the truth values of its conjuncts can be displayed more briefly by means of a *truth table* as follows:

p	q	p·q
T	T	T
T	F	F
F	T	F
F	F	F

Since it specifies the truth value of p·q in every possible case, this truth table can be taken as *defining* the dot symbol. Other English words such as 'moreover', 'furthermore', 'but', 'yet', 'still', 'however', 'also', 'nevertheless', 'although', etc., and even the comma and the semicolon, are also used to conjoin two statements into a single compound one, and all of them can be indifferently translated into the dot symbol so far as truth values are concerned.

The statement in the form : 'It is not the case that water is colder than ice' is also compound, being the *negation* (or *denial* or *contradictory*) of its single component statement 'water is colder than ice'. We introduce the symbol '~', called a *curl* to symbolize negation. There are often alternative formulations in English of the negation of a given statement. Thus where L symbolizes the statement 'water is colder than ice', the different statements 'it is not the case that water is colder than ice', 'it is false that water is colder than ice', 'it is not true that water is colder than ice', 'lead is not heavier than gold' are all indifferently symbolized as ~L. More generally, where p is any statement whatever, its negation is written ~p. Since the negation of a true statement is false and the negation of a false statement is true, we can take the following truth table as defining the curl symbol:

p	~q
T	F
F	T

When two statements are combined disjunctively by inserting the word 'or' between them, the resulting compound statement is a *disjunction* (or *alternation*), and the two statements so combined are called *disjuncts* (or *alternatives*). The word 'or' has two different senses, one of which is clearly intended in the statement 'Premiums will be waived in the event of sickness or unemployment'. The intention here is obviously that premiums are waived not only for sick persons and for unemployed persons, but also for persons who are both sick and unemployed. This sense of the word 'or' is called *weak* or *inclusive*. Where precision is at a premium, as in contracts and other legal documents, this sense is made explicit by use of the phrase 'and /or'.

A different sense of 'or' is intended when a restaurant lists 'tea or coffee' on its menu list, meaning that for the stated price of the meal the customer can have one or the other, but *not both*. This second sense of 'or' is called *strong* or *exclusive*. Where precision is at a premium and the exclusive sense of 'or' is intended, the phrase 'but not both' is often added.

A disjunction which uses the inclusive 'or' asserts that at *least one disjunct is true*, while one which uses the exclusive 'or' asserts that at *least one disjunct is true but not both are true*. The *partial common meaning*, that at least one disjunct is true, is the whole meaning of an inclusive disjunction, and a part of the meaning of an exclusive disjunction.

In Latin, the word '*vel*' expresses the inclusive sense of the word 'or', and the word '*aut*' expresses the exclusive sense. It is customary to use the first letter of 'vel' to symbolize 'or' in its inclusive sense. Where p and q are any two statements whatever, their weak or inclusive disjunction is written pvq. The symbol 'v', called a *wedge* (or a *vee*), is a truth - functional connective, and is defined by the following truth table:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

An obviously valid argument containing a disjunction is the following Disjunctive Syllogism:

> The United Nations will be strengthened or there will be a third world war.
> The United Nations will not be strengthened.
> Therefore there will be a third world war.

It is evident that a Disjunctive Syllogism is valid on *either* interpretation of the word 'or', that is, regardless of whether its first premiss asserts an inclusive or exclusive disjunction. It is usually difficult, and sometimes impossible, to discover which sense of the word 'or' is intended in a disjunction. But the typical valid argument that has a disjunction for a premiss is, like the Disjunctive Syllogism, valid on either interpretation of the word 'or'. Hence we effect a simplification by translating any occurrence of the word 'or' into the logical symbol 'v' – *regardless of which sense of 'or' is intended.* Of course where it is explicitly stated that the disjunction is exclusive, by use of the added phrase 'but not both', for example, we do have the symbolic apparatus for symbolizing that sense, as will be explained below.

The use of parentheses, brackets, and braces for punctuating mathematical expressions is familiar. No number is uniquely denoted by the expression '6 + 9 ÷ 3', although when punctuation makes clear how its constituents are to be grouped, it denotes either 5 or 9. Punctuation is needed to resolve ambiguity in the language of symbolic logic too, since compound statements may themselves be combined to yield more complicated compounds. Ambiguity is present in p·q v r, which could be either the conjunction of p with q v r, or else the disjunction of p·q with r. These two different senses are unambiguously given by different punctuations: p· (q v r) and (p·q) v r. In case p and q are both false and r is true, the first punctuated expression is false (since its first conjunct is false) but the second punctuated expression is true (since its second disjunct is true). Here a difference in punctuation makes all the differences between truth and falsehood. In symbolic logic, as in mathematics, we use parentheses, brackets, and braces for punctuation. To cut down on the number of punctuation marks required, however, we establish the symbolic convention that in any expression the curl will apply to the smallest component that the punctuation permits. Thus the ambiguity of ~p v q, which might mean either (~p) v q or ~(p v q), is resolved by our convention to mean the first of these, for the curl can (and therefore by our convention does) apply to the first component p rather than to the larger expression p v q.

The word 'either' has a variety of different uses in English. It has conjunctive force in 'The Disjunctive Syllogism is valid on either interpretation of the word 'or'. It frequently serves merely to introduce the first disjunct in a disjunction, as in 'Either the United Nations will be strengthened or there will be a third world war'. Perhaps the most useful function of the word 'either' is to punctuate some compound statements.

Thus the sentence :

'More stringent anti-pollution measures will be enacted and the laws will be strictly enforced or the quality of life will be degraded still further'.

Can have its ambiguity resolved in one direction by placing the word 'either' at its beginning, or in the other direction by inserting the word 'either' right after the word 'and'. Such punctuation is effected in our symbolic language by parentheses. The ambiguous formula p·q v r discussed in the preceding paragraph corresponds to the ambiguous sentence considered in this one. The two different punctuations of the formula correspond to the two different punctuations of the sentence affected by the two different insertions of the word 'either'.

All types of conjunctions are not formulated by explicitly placing the word 'and' between complete sentences, as in 'Charlie's neat and Charlie's sweet'. Indeed the latter would more naturally be expressed as 'Charlie's neat and sweet'. And the familiar 'Jack and Jill went up the hill' is the more natural way of expressing the conjunction. 'Jack went up the hill and Jill went up the hill'. It is the same with disjunctions: 'Either Alice or Betty will be elected' expresses more briefly the proposition alternatively formulated as 'Either Alice will be elected or Betty will be elected'; and `Charlene will be either secretary or treasurer' expresses somewhat more briefly the same proposition as 'Either Charlene will be secretary or Charlene will be treasurer'.

The negation of a disjunction is often expressed by using the phrase 'neither-nor'. Thus the disjunction `Either Alice or Betty will be elected' is denied by the statement 'Neither Alice nor Betty will be elected'. The disjunction would be symbolized as A v B and its negation as either ~(A v B) or as (~A)·(~B). (The logical equivalence of these two formulae will be discussed in Section 2.4). To deny that at least one of two statements is true is to assert that both of the two statements are false.

The word 'both' serves various functions, one of which is a matter of emphasis. To say 'Both Jack and Jill went up the hill' is only to emphasize that the two of them did what they are asserted to have done by saying 'Jack and Jill went up the hill'. A more useful function of the word 'both' is punctuational, like that of 'either'. 'Both ... and - · - are not ---' is used to make the same statement as 'Neither ... nor - · - is ---'. In such sentences the *order* of the words 'both' and 'not' is very significant. There is a great difference between.

Alice and Betty will not both be elected.
and
Alice and Betty will both not be elected.

The former would be symbolized as ~(A·B), the latter as (~A)·(~B).

Finally, it should be remarked that the word 'unless' can be used in expressing the disjunction of two statements. Thus 'Our resources will soon be exhausted unless more recycling of materials is effected' and 'Unless more recycling of materials is effected our resources will soon be exhausted' can equally well be expressed as 'Either more recycling of materials are effected or our resources will soon be exhausted' and symbolized as M v E.

Since an exclusive disjunction asserts that at least one of its disjuncts is true but they are not both true, we can symbolize the exclusive disjunction of any two statements p and q quite simply as (p v q)· ~(p·q). Thus we are able to symbolize conjunctions, negations, and both inclusive and exclusive disjunctions. Any compound statement, which is built up out of simple statements by repeated use of truth-functional connectives, will have its truth value completely determined by the truth values of those simple statements. For example, if A and B are true statements and X and Y are false, the truth-value of the compound statement ~[(~A v X) v ~(B·Y)] can be discovered as follows. Since A is true, ~A is false, and since X is false also, the disjunction (~A v X) is false. Since Y is false, the conjunction (B·Y) is false, and so its negation ~(B·Y) is true. Hence the disjunction (~A v X) v ~(B·Y) is true, and its negation, which is the original statement, is false. Such a stepwise procedure, beginning with the inmost components, always permits us to determine the truth-value of a truth-functionally compound statement from the truth-values of its component simple statements. Here we find a very systematic expression of a statement through appropriate symbol.[1]

All these operatives are also available in Indian Philosophical systems, though linguistically (not symbolically). The term '*ca*' meaning 'and' is used to connect one or more than one entities. The grammarians have shown various meanings of *ca* like *samāhāra*, *anvācaya* etc. In the like the disjunction is expressed with the term '*vā*', or '*athavā*'. In the same way, the implication is expressed with the terms '*yadi*' or '*cet*' etc. Negation is so important in Indian Logic that it occupies the place of a category called *abhāva*. Though regarding the nature of absence and its way of meaning there is controversy among the thinkers within the Indian systems, it is accepted by all that negation is possible and it has got a prominent role in conveying meaning. But it should be carefully noted that the absence of something is one of the meanings of negation. It has got various other meanings also.

Lastly, the grammarians have accepted six meanings of negation of which 'absence' is one. It has been stated by the grammarians "*Tatsādṛsyamabhāvaśca tadanyatvaṁ tadalpatā, aprāśastyaṁ virodhaśca nañarthā ṣaṭprakīrtā*". That is, negative particle '*nañ*' can be used in the sense of similarity i.e., *Sādṛśya* (e.g., *abrāhmaṇa* meaning similar to Brāhmaṇas), in the sense of absence i.e., *abhāva* (i.e., *asat* meaning the absence of honesty), in the sense of mutual difference i.e. *anyatva* (e.g. *aghaṭa* meaning different from a jar), in the sense of less quantity i.e., *alpatā* (e.g. *akeśī* meaning less quantity of hair, in the sense of non-suitability (*aprāśastya*) (e.g. *asamaya* meaning improper time) and in the sense of enmity or contradiction (*virodha*) (e.g. *asura*), meaning the enmity with the duties (*suravīrodhi*). "*Tatsādrśyamobhāvaśca tadanyatvaṁ tadalpatā/aprāśastyaṁ virodhaśca nañarthāḥṣaṭ prakīrtitā*". Among these six meanings the second one can be applicable in the case of *Advaita* meaning the absence of duality as told earlier. Unless we have an idea of duality we cannot prove its absence. It is sensible to negate something, which really exists in this world. To negate something which does not exist in this world is non-sensical. It is very much a futile exercise to prove the non-existence of an object, which is absurd. This phenomenon is called *alīkapratiyogikābhāva* i.e., an absence, the absentee of which is an absurd entity. If phenomenal existence of duality is an absurd entity, the negation of it is meaningless leading to a futile exercise. To prove the absence of duality (*advaita*) 'duality' has to be accepted as an existent object. In order to know this duality the different means of knowing (*pramāṇa*) are very much relevant. The realisation of the absence of duality follows from the realisation of Brahman.

REFERENCE

1 Irving M Copi : *Symbolic Logic*, 4th Edition, MacMillan, London 1973, 2nd Chapter.