

CHAPTER - 1

The Concepts of Logic and Mathematical Logic

At the outset a question may be raised : 'What is Logic ?' Charles Pierce thinks - 'Nearly a hundred definitions of it have be given'.¹ But Pierce goes on to write : 'It will, however, generally be conceded that its central problem is the classification of arguments, so that all those that are bad are thrown into one division, and those which are good into another...' Logic is the science of reasoning which is derived from mathematics.'

The study of logic, involves the study of the methods and principles used in distinguishing correct (good) from incorrect (bad) arguments. This definition is not intended to imply, of course, that one can make the distinction only if he has studied logic. But the study of logic will associate one with the capacity to distinguish between correct and incorrect arguments, and it will do so in several ways. First of all, the proper study of logic will approach it as an art as well as a science, and the student will do exercises in all parts of the theory already known. Here, as anywhere else, practice will help to make perfect. In the second place, the study of logic, especially mathematical symbolic logic, like the study of any other exact science, will tend to increase one's proficiency and perfection in reasoning. And finally, the study of logic will give the student certain techniques for testing the validity of all arguments, including his own. This knowledge is of value because when mistakes are easily detected they are less likely to be made.

Logic has also been defined as the science of reasoning. That definition, although it gives a clue to the nature of logic, is not I think, quite accurate. Reasoning is a kind of inferring certain fact on the basis of some supporting evidence. Reasoning is that special kind of thinking called inferring, in which conclusions are drawn from premises. As

thinking, however, it is not the special province of logic, but part of the psychologist's subject matter as well. Psychologists who examine the reasoning process find it to be extremely complex and highly emotional, consisting of awkward trial and error procedures illuminated by sudden flashes of insight. These are all of importance to psychology. But the logician is not interested in the actual process of reasoning. He is concerned with the correctness of the completed process. His question is always : is the conclusion reached drawn from the premises used or assumed ? If the premises provide adequate grounds or supporting evidences for accepting the conclusion to be true also, then the reasoning is correct. Otherwise it is incorrect. The logician's methods and techniques have been developed primarily for the purpose of making the distinction clear. The logicians in general and mathematicians in particular are interested in all reasoning, regardless of its subject matter, but only from this special point of view.

Inferring which is a form of reasoning is an activity in which one proposition is affirmed on the basis of one or more other propositions accepted as the starting point of the process. The logician is not concerned with the *process* of inference, but with the propositions that are the initial and end points of that process, and the relationships between them.

Propositions are either true or false, and they differ from questions, commands and exclamations. Grammarians classify the linguistic formulations of propositions, questions, commands and exclamations as declarative, interrogative, imperative and exclamatory sentences, respectively. These are familiar notions. It is customary to distinguish between declarative sentences and the propositions which are assertive in nature. The distinction is brought and the propositions they may be uttered to assert. The distinction is brought out clearly indicating that a declarative sentence is always part of a language, in which it is spoken or written, whereas propositions are not peculiar to any of the languages in which they may be expressed. Another difference between them is that the same sentence may be uttered in different contexts to assert different propositions. (For example, the sentence 'I am happy' may be uttered by different persons to make different assertions). The same sort of distinction can be

drawn between sentences and *statements*. The same statement can be made using different words, and the same sentence can be uttered in different contexts to make different statements.

Corresponding to every possible inference is an *argument*, and with these arguments logic is chiefly concerned. An argument may be defined as any group of propositions or statements of which one is claimed to follow logically from the others, which are regarded as grounds for the truth of that one. In ordinary usage the word 'argument' also has other meanings, but in logic it has the technical sense, we use the word 'argument' also in a derivative sense to refer to any sentence or collection of sentences in which an argument is formulated or expressed. When undertaken we will be presupposing that the context is sufficiently clear to ensure that unique statements are made or unique propositions are asserted by the utterance of those sentences.

Every argument has a formal and material structure, in the analysis of which the terms 'premiss' and 'conclusion' are usually employed. The *conclusion* of an argument is that proposition which is affirmed on the basis of the other propositions of the argument, and these other propositions which are affirmed as providing grounds or reasons for accepting the conclusion are the *premises* of that argument or the grounds of inference in the sense of deduction.

We note that 'premiss' and 'conclusion' are relative terms, in the sense that the same proposition can be a premise in one argument and conclusion in another. Thus the proposition *All men are mortal* is premises in the argument :

All men are mortal;
Socrates is a man.
Therefore, Socrates is mortal.

and conclusion in the argument

All animals are mortal.

All men are animals.

Therefore all men are mortal.

Any proposition can be either a premiss or a conclusion, depending upon its context and place occupied. It is a premiss when it occurs in an argument in which it is assumed for the sake of proving some other proposition. And it is conclusion when it occurs in an argument, which is claimed to prove it, or to the basis of other propositions, which are assumed.

A distinction can be made between *deductive* and *inductive* arguments. All arguments involve the claim that their premises provide some grounds for the truth of their conclusions, but only a *deductive* argument involves the claim that its premises provide *absolutely conclusive* grounds. The technical terms 'valid' and 'invalid' are used in place of 'correct' and 'incorrect' in characterizing deductive arguments. A deductive argument is *valid* when its premises and conclusion are related in such a way that it is absolutely impossible for the premises to be true unless the conclusion is true also. The task of deductive logic is to clarify the exact feature of the relationship which holds between premises and conclusion in a valid argument, and to provide techniques for discriminating the valid from the invalid.

Inductive arguments involve the claim only that their premises provide some grounds for their conclusions. Neither the term 'valid' nor its opposite 'invalid' is properly applied to inductive arguments. Inductive arguments differ among themselves in the degree of probability, which their premises confer upon their conclusions, and are studied in inductive logic. But in this section i.e., mathematical logic we shall be concerned only with deductive arguments, and shall use the word 'argument' to refer to deductive arguments exclusively.

Truth and Validity :

Truth and falsity characterize propositions or statements, and may derivatively be said to characterize the declarative sentences in which they are formulated. But arguments are not properly characterized as being either true or false. On the other hand, validity and invalidity characterize arguments rather than propositions or statements.² There is a connection between the validity or invalidity of an argument and the truth or falsehood of its premises and conclusions, but the connection is by no means a simple one.

Some valid arguments contain true propositions only, as for example,

All bats are mammals.
All mammals have lungs.

But an argument may contain false propositions exclusively, and be valid nevertheless, as, for example.

All trout are mammals.
All mammals have wings.
Therefore all trout have wings.

But an argument is valid because *if* its premises were true its conclusion would have to be true also, even though in fact they are all false. These two examples cited above show that although some valid arguments have true conclusions, not all of them do. The validity of an argument does not guarantee the truth of its conclusion.

Let us consider the following statement :

If I am President then I am famous.
I am not President.
Therefore I am not famous.

In the above case we can see that although both premises and conclusions are true, it is invalid. Its invalidity is made obvious by comparing it with another argument of the same form :

If Rockefeller is President then he is famous.

Rockefeller is not President.

Therefore, Rockefeller is not famous.

This argument is clearly invalid on account of the fact that its premises are true but its conclusion false. The two latter examples show that although some invalid arguments have false conclusions, not all of them do. The falsehood of its conclusion does not guarantee the invalidity of an argument. But the falsehood of its conclusion does guarantee that *either* the argument is invalid *or* at least one of its premises is false.

There are two conditions that an argument must satisfy to establish the truth of its conclusion. It must be valid, and all of its premises must be true. The logician is concerned with only one of those conditions. To determine the truth or falsehood of premises is the task of scientific inquiry in general, since premises may deal with any subject matter at all. But determining the validity or invalidity of arguments is the special province of deductive logic. The logician is interested in the question of validity even for arguments whose premises might happen to be false.

It might be suggested that we should confine our attention to arguments having true premises only. But it is often necessary to depend upon the validity of arguments whose premises are not known to be true. Modern scientists investigate their theories by deducing conclusions from them which predict the behaviour of observable phenomena in the laboratory or observatory. The conclusion is then tested directly by observation, and if it is true, this tends to confirm the theory from which it was deduced, whereas if is false, this disconfirms or refutes the theory. In either case, the scientist is vitally

interested in the validity of the argument by which the testable conclusion is deduced from the theory being investigated, for if that argument is invalid his whole procedure is without point. The foregoing is an oversimplified account of scientific method, but it serves to show that questions of validity are important even for arguments whose premises are not true.

So far we have seen that logic is concerned with arguments and that these contain propositions or statements as their premises and conclusions. The conclusions are not linguistic entities, such as declarative sentences, but rather what declarative sentences are typically uttered to assert. However, the communication of propositions and arguments requires the use of language, and this complicates our problem. Arguments formulated in English or any other natural language are often difficult to appraise because of the vague and equivocal nature of the words in which they are expressed, the ambiguity of their construction, the misleading idioms they may contain, and their pleasing but deceptive metaphorical style. The resolution of these difficulties is not the central problem for the logician, however, for even when they are resolved, the problem of deciding the validity or invalidity of the argument remains.

To avoid the peripheral difficulties connected with ordinary language, researchers in the various sciences have developed specialized technical terminology. The scientist economizes the space and time required for writing his reports and theories by adopting special symbols to express ideas which would otherwise require a long sequence of familiar words to formulate. This has the further advantage of reducing the amount of attention needed, for when a sentence or equation grows too long its meaning is more difficult to grasp. The introduction of the exponent symbol in mathematics permits the expression of the equation.

Logic, too, has had a special technical notation developed for it. Aristotle made use of certain abbreviations to facilitate his own investigations, and modern symbolic logic has grown by the introduction of many more special symbols. The difference between the

old and the new logic is one of degree rather than of kind, but the difference in degree is tremendous. Modern symbolic logic has become immeasurably powerful a tool through which the analysis and deduction through the development of its own technical language is done. The special symbols of modern logic permit us to exhibit with greater clarity and precision the logical structures of arguments, which may be obscured by their formulation in ordinary language. It is an easier task to divide arguments into the valid and the invalid when they are expressed in a special symbolic language, for in the peripheral problems of vagueness, ambiguity, idiom, metaphor etc. do not arise. The introduction and use of special symbols serve not only to facilitate the appraisal of arguments, but also to clarify the nature of deductive inference. All these tools are employed to disintegrate language and come to a logical conclusion.

The logician's special symbols are much better adapted than ordinary language to the actual drawing of inferences. Their superiority in this respect is comparable to that enjoyed by Arabic numerals over the older Roman kind for purposes of computation. It is easy to multiply 148 by 47, but very difficult to compute the product of CXLVIII and XLVII. Similarly, the drawing of inferences and the evaluation of arguments is greatly facilitated by the adoption of a special logical notation. To quote Alfred North Whitehead, an important contributor to the advance of symbolic logic :

... by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain.³

Like the Western Logicians the Indians also developed some tools of reasoning and some technical terms for clarity and precision.

The Indian Logicians also believe that reasoning is the backbone of all theories and in the principles called good reasoning (*sutarka*) and bad reasoning (*kutarka*). The Naiyayikas also feel that reasoning is a special kind of thinking called inferring. To them in each and every piece of knowledge there is inference as it is substantiated through reasoning.

Knowledge is a kind of lamp by which the nature of an object is revealed. According to the Naiyāyikas, cognition is of two types : presentative cognition (*anubhava*) and recollection (*smṛiti*)⁴ A presentative cognition may be valid (*yathārtha*) and invalid (*ayathārtha*). A valid presentative cognition which is called *Pramā* is of four types : *Pratyakṣa*, *Anumiti*, *Upamiti* and *Śābda*. The uncommon causes (*Karaṇa*) of these four types of knowledge are called Perception (*Pratyakṣa*), Inference (*Anumāna*) comparison (*Upamāna*) and verbal testimony (*Śābda*) and they are special sources of attaining valid knowledge (*Pramāṇas*).⁵

Perception is the knowledge, which arises out of the contact (*sannikarsa*) of the sense-organs (*indriya*) with objects (*artha*). This knowledge will be indescribable (*avyapadeśya*) i.e. indeterminate (*nirvikalpaka*) non-deviated (*avyabhicāri*) definite (*vyavasāyātmakam*) i.e., determinate (*Savikalpaka*). This definition of perception given by older logicians has been rejected by Gangeśa, as it does not cover God's perception. According to Gangeśa, perception is the knowledge of which the knowledge is not the uncommon cause having operative process (*Karaṇa*).⁶ The perceptual knowledge of an object is independent in the sense that it does not depend on the knowledge of other objects and hence it is immediate (but not mediate).

Generally, when a man apprehends with the help of his sense organs must be true if, of course, there is no defect (in perception). No man questions about the truth of the cognition which is attained through sense organs unless anything contradictory to it is found.

Perception is the basis of all kinds of knowledge. Without taking recourse to perception other sources of valid knowledge i.e., inference, comparison and verbal testimony are not possible.

Inference consists in making an assertion about an object on the strength of the knowledge of the probans, which is invariably connected with it. The word '*anumāna*' literally means the cognition, which follows from other knowledge. Here the prefix

'anu' means 'after' and 'māna' means 'knowledge'. From this literal meaning it follows that the perceptual knowledge of the probans gives rise to the inferential knowledge. One can infer the existence of fire, for example, after perceiving the smoke, which has got an uninterrupted connection with the surface of the mountain. The knowledge of invariable concomitance (*vyāpti*) is the key for having inferential knowledge. This knowledge of *vyāpti* is not possible without the help of perception. *Vyāpti* is nothing but an invariable co-existence between probans and probandum.⁷ The knowledge of the *probandum* as related to the subject of inference (*pakṣa*) depends on the previous knowledge of the *probans* as related to the subject and at the same time as invariably related to the *probandum*. One can infer fire on the mountain by virtue of the fact that one perceives smoke on it and has observed it as invariably accompanied by fire. In both the cases the necessity of perception cannot be denied.

Perception and inference are equally important sources of valid knowledge. Perception is independent in respect of the knowledge of other objects while inference is dependent on the previous knowledge. Perception can reveal those objects that are within the range of our sense organs, i.e., it can give us the knowledge of the present objects that are within the reach of our sense organs in a normal way.⁸ But inference can give the knowledge of those objects that are not connected with the sense organs.

Though perception is the fundamental basis of all kinds of knowledge yet inference is by far the important source of knowledge in our society. Hence inference as a special source of valid knowledge (*pramāṇa*) is accepted by the philosophers of all schools of Indian Philosophy with the single exception of the materialistic school (the *Cārvāka* school). But it should be clearly borne in mind that the philosophers of the *Cārvāka* school also do not deny the existence of inference as such. They only hold that inference cannot be accepted as a special source of valid knowledge or *Pramāṇa*. There can at most be the knowledge of probability through inference, but not definite valid knowledge.⁹ So the dispute with *Cārvākas* in regard to inference is limited only to the question of its having the nature of the special source of valid knowledge or otherwise.

The *Cārvākas* hold that inference by virtue of having the capacity of producing the knowledge of probability cannot produce definite valid knowledge.

The philosophers of other schools, theist or atheist, strongly oppose the stand point of *Cārvākas* that inference has got no capacity to produce definite valid knowledge. The idea behind the strong opposition is that inference has got tremendous utility in our day to day life, and unless it can produce definite valid knowledge, it cannot satisfactorily be an instrument to serve us in meeting the diverse needs of our life. It is, of course, to be concluded, they say that though the knowledge of probability also can serve us to a certain extent to meet the requirements of our life, particularly in respect of guiding us in the field of activity yet it can never serve our purpose in every respect and in all cases. Definite valid knowledge of a particular object alone can guide us invariably towards action, and this definite valid knowledge can certainly be produced by inference in most of the cases.

In a society the help of an inference is taken almost in every step, but generally we are unaware of the fact that we are inferring some objects. In most of the cases inference is drawn spontaneously. Illiterate persons are found to be guided by inference, not to speak of the literate. Cultivators are seen to infer some object after seeing some sign or mark (*linga*). The Naiyāyikas are of the opinion that even a child also infers.¹⁰ A child attains inferential knowledge spontaneously without being aware of the inferential procedure. The inherent process of inference of a child can be shown in the following manner.

A child comes to know the primary relation (*Samketa*) of a term with its meaning at first from the verbal usages of the old person (*Vrddhasya śabdādhīnavyavahārādeva*). When a man also is aware of the meaning of a term (*vyutpanna*) asks another man who also knows the meaning of the same term to bring a cow, the person who has been asked to bring a cow by the senior person (*uttamavrddha*) brings it after hearing the words of the senior and realising the meaning of it. On observing the performance of the man who has been asked to bring a cow, child draws the inference in the form : 'This bringing of

a cow is the result of the inclination, the object of which is the bringing of a cow as it has got effortness in it, as in the case of my inclination so suck mother's breast'. (*idam gavānayanam svagocarapravṛttijanyam, ceṣṭātvāt madīyastanapānādivat*)'.¹¹ Then he comes to infer the state or condition of being produced by the knowledge of the feasibility (by one's effort) of which the bringing of a cow has become qualificand (*gavānayanadharmikakāryatājñānajanyatvam*)¹² in respect of the inclination with the help of the syllogistic argument in the form : 'That inclination to bring a cow is produced by the knowledge of the feasibility (by one's effort) of which the inclination the same has become qualificand, as it has got the generic property existing in inclination, as in the case of my own (inclination). Here inclination towards a particular action has become qualificand to the knowledge of the feasibility by one's effort. Any type of inclination presupposes this type of knowledge of feasibility. Then the child again forwards the syllogistic argument in the form : 'The knowledge of the feasibility (by one's effort) of which the bringing a cow has become qualificand, has an uncommon cause, as it is an effort having effortness in it as in the case of a jar. (*gavānayanagocaratajñānam asādhāraṇahetukam kāryatvāt, ghaṭavat*). Any type of effect has not its special cause and hence, the effect in the form of bringing a cow needs some special cause. After drawing such inference the child comes to know that the knowledge of the verbal usages of the old persons (*vṛddhavyarahāra*) is the uncommon cause (*asādhāraṇakāraṇa*) of the knowledge mentioned above.¹³ A child attains this type of inferential knowledge being completely unaware of the abovementioned inferential procedure.

In a society no man believes in a statement which is baseless. In other words, a statement, which is not properly grounded, cannot impress other beings. If our neighbours or relatives are advised to do something or not to do something, they should be convinced with the help of arguments in favour of our statements. In every sphere of our life, we are going on saying something depending on some arguments in as much as the groundless speech will fall flat upon others,¹⁴ which is also a form of inference.

The valid inferential knowledge guides us in innumerable walks of our life beginning with the dealings with our fellow people in our everyday life. Our life becomes thoroughly impracticable.¹⁵ unless our fellow-beings are properly and satisfactorily dealt with and this can never be done unless we definitely and rightly understand the mind of people around us. This understanding of others' mind depends on inference in most of the cases.¹⁶

Moreover, from the red-colour of a mango it is inferred that it is ripen. In the like manner, the past rain is inferred from the muddy current of the river. In the same way, the mental states like pleasure, pain etc. existing in a man can be inferred from their different types of expressions and gestures. Sometimes, the exact place or country where a man resides can be inferred after observing his dress or his particular language carefully. Thus innumerable instances of the knowledge based on inference in our every day life can be shown.

A man is desirous of doing these types of works by which his purpose might be served and hence, it can be said that the end-in-view (*Prayojana*) inspires him to do some activities.¹⁷ In order to get or get rid of something a man engages himself in activity.¹⁸ Man's desire is related to result in the form of pleasure or the absence of pain and to the means of it. The longing for the result of some action presupposes the knowledge of it.¹⁹ Hence the desire for the result is due to the existence of the knowledge of it, which is also a form of inference. The cause of desire for the means (of the result) is the knowledge of its conduciveness to the object, which is desirable (*iṣṭasādhanatājñāna*). This knowledge of its conduciveness to that which is desirable is considered as a *Hetu* to the desire for the means also.²⁰ Again, the knowledge of the feasibility through one's effort (*kṛtisādhyatājñāna*) and the knowledge of its conduciveness to that which is desirable (*iṣṭasādhanatājñāna*) are considered as the reasons behind the desire for doing something. Nobody thinks to do action without having the knowledge of its feasibility through one's effort, the cause of desire.²¹ This can also be taken as an instance of inference in our day to day life. In the same way, the knowledge of its being productive of what is extremely unpleasant (*dviṣṭasādhanatājñāna*) is the cause of aversion (*dveṣa*),

the object of which causes pain. Here aversion towards an object is inferred on the strength of its *dviṣṭa-sādhanatājñāna*.²²

The existence of the imperceptible objects like *Ātman*. God etc. can easily be proved with the help of inference only.²³ Hence, the logicians prove the existence of *Ātman*, as a locus of the attributes like desire, aversion, effort etc.²⁴ In other words, soul-ness (*ātmatva*) is inferred as the limiter (*avacchedaka*) of this inherent causness of pleasure, pain etc. Again, that which imparts consciousness of pleasure, pain etc. Again, that which imparts consciousness in the sense organs and also in the body is *Ātman*. Though the contentness (*Viśayatva*) of the perceptions like 'I am unhappy' etc. remain in *Ātman*, it would not be possible at first to make a person (bearing doubt about it) convinced that *Ātman*, the object of the above-mentioned perception, is different from body etc. Hence, another strong argument is to be forwarded. As no result is produced from the cutting instrument like an axe etc. without being guided by an agent, the eyes, etc., the sense organs, cannot produce any result without being guided by an agent. That is why, the agent in the form of impeller of the sense organs is *Ātman*.²⁵ The syllogistic argument regarding the existence of *Ātman* existing in others' body is as follows : "The body of Devadatta is endowed with *Ātman*, as it is associated with the condition of being qualified by inclination like a chariot."

(*Debadattaśarīram ātmavat pravṛttimattvāt rathavat*)

It is a fact that a considerable number of people in our society believes in the existence of God even in this modern age, but a very few of them have realised Him. This belief in God is based on some grounds, but *not* on blind faith. The Naiyāyikas have taken pain to highlight the existence of the Divine with the help of some grounds or arguments that are inferential in nature. These syllogistic arguments are as follows.

(a) As the effects like jar etc. are caused by an agent, the earth (*kṣiti*), dyads (*ankura*)²⁶ etc. must have caused by an agent. The agentness of it, being not possible in persons like us having limited knowledge and power, remains in God. Hence, God is inferred as the cause or agent of earth etc.²⁷ (b) the activity in which dyad becomes a

promoter (*prayojaka*) at the time of initial creation is caused by an effort (*prayatnajanya*), as it is an activity. This world is originated from the combination of atoms. These atoms cannot be combined with each other automatically (without being guided by a conscious being) due to their inanimate character. This Conscious Being is nothing other than God.²⁸ (c) The absence of the coming downwards of weighty substances (*gurutvavatām*) is caused by an effort, which becomes an obstacle to the coming down of a substance, as it is endowed with steadiness, as in the case of the absence of falling of a bird (*pakṣipatanābhāvavat*)²⁹ This world having weight is not coming down due to having some power in the form of effort, which is God. (d) The destruction of the universe presupposes the existence of an effort, as it is a destruction in character as in the case of the destruction of a jar.³⁰ This effort from which the destruction of the universe follows is in the form of God. (e) The initial verbal usage, as in the case of the usages of the scripts introduced in modern age. This independent person is God.³¹ (f) The *Vedas* are introduced by a being who is other than an individual who entangles in the worldly affairs, as it has the property of being the Veda (*Vedatvāt*). That which is not of this type would not be of this type, as in the case of a piece of literature.³² The *Asamṣārī Puruṣa* is God. (g) The *Vedas* are introduced by a *Puruṣa* (*Pauruṣeya*) as they possess sentences as in the case of the *Mahābhārata* etc. This *Puruṣa* is God.³³

The imperceptible objects like atom, *Ākāśa Kāla* etc. are admitted by the Naiyayikas with the help of inference. The Naiyayikas have explained the origination of the whole universe in terms of the combination of atoms. This theory would have been meaningless if the existence of atom were not proved through inference. The syllogistic argument is as follows : If the whole of an object has an endless series of parts, there would arise the contingency of equality in respect of size between mountain and a mustard seed. If the whole has some parts, the parts also have some other parts in which there are other parts and so on. In this way, there would arise 'Infinite Regress' (*Anavasthā*). As there is no final unit of a definite size, we cannot add these up to make different sizes. Hence there would arise the contingency of equality in dimension between very big and small objects; *as in mathematics anything multiplied by zero is*

zero.³⁴ So this process of division must be stopped anywhere. If the limit is taken as non-eternal, it would be taken into account that a positive effect may be produced even when there is the absence of inherent cause. If the limit is considered as non-eternal, it must be taken as an effect which remains in it through the relation of inherence (*samavāya*). As there are no parts in it, it can be said that it is a positive effect having no parts. As a positive effect having no parts is not possible, it would be taken as an eternal object.³⁵ As the gradation of the medium dimension (*Mahatparimāṇatārātmya*) has a limit in *Ākāśā* etc., the gradation of the atomic dimension (*aṇuparimāṇa*) has limit somewhere. Where there is limit is an atom.³⁶ It cannot be said that the limit of atomic dimension is a triad (*Trasareṇu*). That a triad possesses its parts (*avayava*) can be established with the help of the syllogistic argument in the form : “A triad possesses its parts, as it is a substance capable of being perceived like a jar” (*Trasareṇuḥ sāvayavavaṃ cākṣuṣadravyatvāt ghaṭavāt*). That the parts of a triad (i.e., dyads) possess their own parts can also be established by another inferential argument in the form : “The parts of a triad possess their own parts, as they become the producer of an object of medium dimension, as in the case of *kapāla* i.e., upper part of the jar (*Trasarenoravayavāḥ sāvayavāḥ mahādārambhakatvāt kapālavat*). A part of a dyad, one of the parts of a triad, is called Atom.³⁷

In the same way *Ākāśa* is inferred as the locus of sound.³⁸ The existence of *Kāla* is inferred from its general causeness to the objects that are produced (*janya*), from its being the locus of this universe,³⁹ and from its being an uncommon cause (*karāṇa*) of this knowledge of priority (*Paratva*) and posteriority (*Aparatva*).⁴⁰

The philosophers of the theistic school specially take recourse to inference as a means to go above the sphere of grief and sorrow and attain fulfilment in the form of attaining salvation or *Mokṣa*. The Upanisadic injunction that realisation of the soul should be attained through hearing, thinking and constantly meditating upon the nature of the soul is accepted by the philosophers of the theist school as a supreme gospel, by obeying which a man can rise above the sphere of sorrow and grief and can attain salvation. ‘Thinking’ in the injunction is nothing other than inference of the soul as distinct from

other worldly objects (*Mananam cātmanah itarabhinnatvena anumānam*). This inference of the true nature of the soul should be attained through frequent practice of inference. Hence, it can easily be understood what great importance has been attached to inference by the theistic philosophers of our country having regards to the utility of inference in the matter of attaining the supreme goal of life.

Each and every object of this world can be inferred as distinct from other worldly objects. As for example, a jar can be inferred as distinct from the objects other than the jar i.e., pot etc. In this way, a pot can be inferred as distinct from the objects other than pot etc. i.e., jar etc. The *Hetu* of the inference of some object as distinct from others is the definition of that object. As for example, a cow is distinct from the animals other than cow, as it is the locus of the dewlap etc (*Gauḥ gavetarabhinnā sāsṅādimattvāt*).

From the above discussions, it can be conducted that inference has great utility in each and every sphere of our life. The particular kind of conditions, both positive and negative, cannot guide or control our activities in the majority of cases of our life for the simple reason that the capacity of those conditions to produce definite valid knowledge is limited compared to that of the procedure of inference.

Though it can be argued that procedure of inference is extremely complicated and the highly educated persons can only successfully apply it, trained in the art of drawing right inference, it can also be equally emphasized that the drawing of inference from some given data is not a so difficult proposition. It has already been said that even the illiterate persons also are found to be spontaneously drawing inference from circumstances and controlling their activities accordingly. It is, of course, very difficult to give scholarly analysis of the procedure of inference, but to draw inference from some given data is not at all difficult, rather it is, to a great extent, spontaneous.

Keeping therefore, in view this practical aspect of inference, it can safely be concluded that Inference has got great utility in our everyday life.



Though the Naiyāyikas and Buddhists follow the form of inference like *Pratijñā*, *hetu*, *udāharaṇa* etc, they believe in the material truth also. In symbolic logic an inference is taken as true, if it is true formally irrespective of its material truth. In this connection it may be said that both in Indian and Western Philosophy inference which is constituted with premise and conclusion occupies a prominent role. Though the formal structure of inference as found in Western Mathematic Logic is derived from the mathematical principles directly, the Indian Systems such inferential methods are not directly *taken from Mathematics*. In fact the Indian Philosophers had some intuitive power, which may correspond to the Mathematical intuition.

REFERENCES

- 1 'Logic', in Dictionary of Philosophy, edited by James Mark Baldwin, New York, The Macmillan Company, 1925.
- 2 Some logicians use the term 'valid' to characterize statements, which are logically true. For the present, however, we apply the terms 'valid' and 'invalid' to arguments exclusively.
- 3 *An Introduction to mathematics* by A.N. Whitehead. Oxford, Eng., Oxford University Press, 1911.
- 4 “ ... Buddhistu dvividhā matā
Anubhūtiḥ smṛtiśca syādanubhutiścaturvidhā.”
Bhāṣāpariccheda, Verse No. 51.
Sā dvividhā Smṛtiranubhavaścaturvudhah.”
... yathārthānubhavaścaturvidhah.”
Tarkasamgraha, Chowkhamba Sanskrit Santhan, pp. 32-33.
- 5 ‘Etāsām catasṛṇām karaṇāni catvāri
‘Pratyakṣānumānomānaśabdāḥ pramāṇāni sūtroktāni veditavyāni’
Siddhāntamuktāvalī on Verse 51.
‘Tat karanamapi caturvidhaṁ pratyṣ -
kānumānopanānaśabdabhedāt’
Tarkasamgraha, pp. 34-35, (Same ed.).
- 6 “Indriyārthasannikarṣotpannam jñānam
avyapadeyśamavyabhicāri vyavasāyātmakam pratyakṣam” –
“Athavā jñānākaraṇakam jñānam pratyakṣam”
Siddhāntamuktāvalī on Verse, 51.
- 7 “Yatra dhūmastatrāgniriti, sāhacaryaniyamo vyāptiḥ.”
Tarkasamgraha, p. 49 (Same Edition).
- 8 Here the phrase ‘in a normal way’ has been used in order to exclude the super-normal perception alaukika pratyakṣa through which objects of all times can be perceived.

- 9 “Avalāvālagopālāhālika pramukhā api buddhyante niyatādarthādarthāntaramasamsāyam.
Nyāyamañjarī, p. 110, Chowkhamba Ed.
- 10 Ibid.
- 11 “Prathamam padeṣu-samketagraho vṛddhasya vyutpannasyaśabdādāhīnavyavahārādeva
vākyaṇām, Tathāhi, gāmānayeti kenacinnipuṇena niyuktaḥ kaścana vyutpannastadvākyaṭo’rtham
pratitya gavānayanam karoti taccopalabhamāna vāla idam gavānayanam
svagocarapravṛttijanyam caṣṭātvāt madīyastanapānā divadityanumāya ...”
Śabdaśaktiprakāśikā Nāmaprakaraṇa, Prose portion on Verse No. 20; p. 116 (Jayccandra Śarmā
Ed.).
- 12 “Svaviśayadharmiketi pravṛttiviśayaviśeṣyaketyarthaḥ. Kāryatā kṛtisādhyatā”.
Commentary on, Prose portion of verse no. 20 of Śabdaśaktiprakāśikā, p. 116, Ed. by
Jayaccandra Śarmā.
- 13 “... Sā gavānāyanapravṛtṭiḥ svavi
ṣayadharmikakāryaṭajñānājanyā, pravṛtṭitvānnijapravṛttivaditi
pravṛtṭergavānayanadharmikakāryaṭajñānājanyatvam prasādhyā
gavānayanagocaratajñānamasādhāraṇahetukam kāryatvād ghaṭavadityevam anuminavānaḥ ...
śrutam vṛddhavākyaṃeva tadasādhāraṇakāraṇatvenāvadhārayati.”
Śabdaśaktiprakāśikā (Nāmaprakaraṇa) Prose portion on Verse No. 20, p. 116, Edited by
Jayaccandra Śarmā.
- 14 “Aśīraskavacanopanyāse sādhyāsiddheḥ. Ekākinī pratijñā hi pratijñātam na sādhyāet”.
Bauddhadarśana, Sarvadarśanasamgraha.
- 15 “Anumānāpalāpe tu pratyakṣādapi durlabhā lokayātreṭi lokāḥ syurlikhitā iva niścalāḥ.”
Nyāyamañjarī, p. 110, Chowkhamba.
- 16 “Pramāṇāntarasāmānyastheteḥ anyadhiyogateḥ, pramāṇāntarasadbhāyrah ...”
Bauddhadarśana, Sarvadarśanasamgraha.
- 17 “Yamarthamadhikṛtya pravartate tatprayojanath.”
Nyāyasūtra 1.1.24.
“Yena prayuktaḥ pravartate tat prayojanam
Yamarthamabhipsanjihāsan vā karmārabhate tenānena
Sarve prāṇinaḥ sarvāni karmāṇi sarvāśca vyāptāt.”
Nyāyabhāṣya on Sūtra 1.1. 24.

- 18 “Yamarthamāptavyam hātavyamavasāya tadāptihānopāyamanutiṣṭhati tat prayojanam, tadveditavyam pravṛttihetutvāt.” Ibid.
- 19 “Icchā hi phalaviṣyīṇī upāyaviṣyīṇīca. Phalam tu sukhārtho duḥkhābhāvaśca. Tatra phalecchām prati phalajñānam kāraṇam”. Siddhāntamuktāvalī on verse 146.
Nirduḥkhatve sukhe cecchā tajjñānādeva jñāyate”.
Bhāṣāparicchedaḥ, Verse 146.
- 20 Icchā tu tadupāye syādiṣṭopāyatvadhīryadi”.
Bhāṣāparicchedaḥ, Verse 146.
- 21 “Cikirsā kṛtisādhyatvaprakārecchā tu yā bhavet, Taddhetuḥ kṛtisādhyeṣṭasādhanatvamatirbhavet”.
Bhāṣāparicchedaḥ, Verse 147.
- 22 Dviṣṭasādhanābuddhirbhavet dveṣasya kāraṇam.”
Bhāṣāparicchedaḥ, Verse 14.
- 23 Ātman, God etc. can be known through *yogaja pratyakṣa*, which is not at all easy task. Hence inference is the easy method through which common men can be convinced as to the existence of Self, God etc.
- 24 “IcchādveṣaprayatnasukhaduḥkhajñānānyĀtmano lingamiti.”
Nyāyasūtra, I. 1. 10.
“Ātmatvajātistu sukhaduhkādisamavāyākāraṇatāvachedakatayā sidhyati”.
Siddhāntamuktāvalī on Verse 47.
“Jñānādhikaraṇamātmā”.
Tarkasamgraha, p. 19 (Chowkhamba).
- 25 Indriyāṇam śārīrasya ca paramparayā caitanyasampādakaḥ yapyapy Ātmani aham sukhi aham duḥkhītyādipratyakṣaviṣayatvamastyēva, tathāpi vipratipannam prati prathamata evaśarīrādibhinnastatpratitigocara iti pratipādayitum na śakyate ityataḥ pramāṇāntaram darśayati kāraṇamiti. Vāsyādīnām chidādikāraṇāṇām, kartāramantareṇa phalānupadhānam drṣṭam, evam cakṣurādīnām jñānakaraṇāṇāmapi phalopadhānam kartāramantareṇa nopapadyata ityatīriktaḥ kartā kalpyate.”
Siddhāntamuktāvalī on Verse 47.

- 26 Here the term 'Ankura' means dyad or dvyaṅka. In the *Kiranāvalī* commentary on *Siddhāntamuktāvalī* it is said that, just as the object which is seen at first as a promoter of a tree arising out of the seed is called *Ankura*, the object which is the promoter of the world-tree (*Samsārataru*) arising from two atoms has got resemblance with *Ankura* and hence 'dvyaṅka' is to be understood by the term 'Ankura'. Here 'dvyaṅka' is metaphorised as 'Ankura' and world as tree. The original commentary runs as follows : “*Ankureti = yathā vijādupannasya vrkṣaprayojakasya prāthamikadarśanaviśayasyānkuratvam tāthā paramānūbhyaṃmutpannasya samsārataruprayojakasya dvyaṅkasyānkurasāmyāt ankuraśabdena dvyaṅkam lakṣyate*”.
Kiranāvalī on *Siddhāntamuktāvalī*, p. 16. (Edited by Krishnaballava Acharya).
- 27 “Yathā ghaṭādikāryam katrjanyam tathā kṣityamkurādikamapi”
Siddhāntamuktāvalī, on Verse I.
- 28 “Sargādyakālinadvyaṅkaprayojakām karma prayatnajanyam karmatvāt”
Dinakari on *Siddhāntamuktāvalī*, Verse No. 1, p. 20, (Chowkhamba).
- 29 “Gurutvavatām patanābhāvaḥ patanapratibandhakaprayatnaprayuktaḥ dṛtitvāt, pakṣipatanābhāvavat”. *Ibid.*
- 30 Brahmāṇḍanaśaḥ prayatnajanyah nāśatvāt, ghaṭanāśavat”. *Ibid.*
- 31 “Ghaṭādivyavahāraḥ svatantrapuruṣaprayojyaḥ vyavahāratvāt, ādhunikakalpitalipyādivyavahāratvāt”. Dinakarī on *Siddhāntamuktāvalī*, Verse I, p. 29. (Chowkhamba).
- 32 “Vedaḥ asamsāripuruṣapraṇītaḥ. Vedatvāt, yannaivam tannaivam yathā kāvyamiti”, *Ibid.*
- 33 “Vedaḥ pauruṣeyaḥ vākyatvāt bhāratādivat”. *Ibid.*
- 34 *Bhāṣāpariccheda*, p. 44. Edited by Swami Madhavananda.
- 35 “Teṣām cāvayavadhārāyā anantatve merusaṣpayorapi sāmyaprasangaḥ, ataḥ kvacid viśrāmo vācyāḥ yatra tu viśrāmaḥ tasyānityatve’ sambhaveta (bhāva) kāryotpattiprasangāt tasya nityatvam”.
Siddhāntamuktāvalī on Verse 37.
- 36 “Mahatparimāṇatāratamyasya gaganādaḥ viśrāntatvamivāṇuparimāṇatāratamayasāpi kvacidviśrāntatvamiti tasya paramāṇutvasuddhiḥ.” – *Ibid.*

- 37 “Na ca trasareṇāveva viśrāno’stīti vācyam.
Trasareṇuḥ sāvayavaḥ cāksuṣadravyatvāt
Ghatavadityanumānena tadavayavasidhau, trasareṇoravayavāḥ sāvayavāḥ mahadārambhakatvāt,
Kapātavadityanumānena tadavayavasiddheḥ.”
Sidhhāntamuktāvalī on Verse 37,
“Tryaṇukāvayovo’pi sāvayavaḥ mahadārambhakatvāt kapālavat, yo dvyāṇukāvayavaḥ sa
paramāṇuḥ.
Dīpikā on *Tarkasamgraha*, p. 190, Chowkhamba with seven commentaries.
- 38 Śabdagaṇakamākāśam.
Tarkasamgraha, p. 1718 (Chowkhamba).
- 39 “Janīyānām janakaḥ kālah jagatāmāśrayo matah”.
Bhāṣāpariccheda, Verse No. 45.
- 40 “Paratvāparatvabuddhirasādhāraṇam nimittam kāla eva.”
Ibid.