SOME MATHEMATICAL PRINCIPLES IN INDIAN PHILOSOPHY : A PHILOSOPHICAL STUDY

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TO WHOM IT MAY CONCERN

This is to certify that the thesis entitled: ***'Some Mathematical Principles in Indian Philosophy: A Philosophical Study'*** *written by Sm. Sudeshna Bhaumik (Mitra) is the result of her analytic and critical thinking on the subject. So far as I know, thesis is not submitted before for the award of the PhD degree or any other degree of the University of North Bengal or any other Universities. As the thesis bears the evidence of her original thinking and intellectual labour, I recommend its submission for evaluation in connection with her PhD degree of the University of North Bengal.*

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PREFACE

The present work entitled : "*Some Mathematical Principles in Indian Philosophy : A Philosophical Study*" is the result of intensive study on the mathematical principles and their application in the field of Indian Logic. Though, the principles involved in Western Mathematical Logic do not have any influence on the Indian Philosophical Systems, a thorough research on this reveals that there are some striking resemblances between two traditions inspite of having temporal and cultural differences between them. Some of the mathematical concepts like *Reductio-ad-absurdum* zero, set-sub-set relationship, inferential deduction etc. will find their echo in Indian counterparts like the concepts of *Tarka*, *śūṇya*, *para* and *apara*, *sāmānya* etc., though there was no evidence of interaction of the Indian thinkers with the western scholars. In fact, intuition or wisdom does not have any spatial or racial or temporal differences.

An effort has been made to undertake a comparative study on these principles in a very systematic manner so that a scholar belonging to the discipline of Mathematics may gather an idea about the great mathematical thinking available in the ancient India. A traditional Sanskrit Scholar having no proper background of Western Mathematics can have a glimpse of it through this comparative study. The study is not limited to the comparative estimate only, but it is an attempt to make a critical evaluation of both the tradition. How far I am successful in doing so will be judged by the scholars in the field. Any comment, constructive or destructive, will enlighten me and enable me to reconsider the conclusions I have arrived at this thesis, which is also a mark of the philosophical thesis.

Dated : 10th Day of September, 2004

Sudeshna Bhaumik (Mitra)
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CHAPTER - 1

The Concepts of Logic and Mathematical Logic

At the outset a question may be raised : 'What is Logic ?' Charles Pierce thinks - 'Nearly a hundred definitions of it have be given'.[1] But Pierce goes on to write : 'It will, however, generally be conceded that its central problem is the classification of arguments, so that all those that are bad are thrown into one division, and those which are good into another...' Logic is the science of reasoning which is derived from mathematics.'

The study of logic, involves the study of the methods and principles used in distinguishing correct (good) from incorrect (bad) arguments. This definition is not intended to imply, of course, that one can make the distinction only if he has studied logic. But the study of logic will associate one with the capacity to distinguish between correct and incorrect arguments, and it will do so in several ways. First of all, the proper study of logic will approach it as an art as well as a science, and the student will do exercises in all parts of the theory already known. Here, as anywhere else, practice will help to make perfect. In the second place, the study of logic, especially mathematical symbolic logic, like the study of any other exact science, will tend to increase one's proficiency and perfection in reasoning. And finally, the study of logic will give the student certain techniques for testing the validity of all arguments, including his own. This knowledge is of value because when mistakes are easily detected they are less likely to be made.

Logic has also been defined as the science of reasoning. That definition, although it gives a clue to the nature of logic, is not I think, quite accurate. Reasoning is a kind of inferring certain fact on the basis of some supporting evidence. Reasoning is that special kind of thinking called inferring, in which conclusions are drawn from premises. As thinking, however, it is not the special province of logic, but part of the psychologist's subject matter as well. Psychologists who examine the reasoning process find it to be extremely complex and highly emotional, consisting of awkward trial and error procedures illuminated by sudden flashes of insight. These are all of importance to psychology. But the logician is not interested in the actual process of reasoning. He is concerned with the correctness of the completed process. His question is always : is the conclusion reached drawn from the premises used or assumed ? If the premises provide adequate grounds or supporting evidences for accepting the conclusion to be true also, then the reasoning is correct. Otherwise it is incorrect. The logician's methods and techniques have been developed primarily for the purpose of making the distinction clear. The logicians in general and mathematicians in particular are interested in all reasoning, regardless of its subject matter, but only from this special point of view.

Inferring which is a form of reasoning is an activity in which one proposition is affirmed on the basis of one or more other propositions accepted as the starting point of the process. The logician is not concerned with the *process* of inference, but with the propositions that are the initial and end points of that process, and the relationships between them.

Propositions are either true or false, and they differ from questions, commands and exclamations. Grammarians classify the linguistic formulations of propositions, questions, commands and exclamations as declarative, interrogative, imperative and exclamatory sentences, respectively. These are familiar notions. It is customary to distinguish between declarative sentences and the propositions which are assertive in nature. The distinction is brought and the propositions they may be uttered to assert. The distinction is brought out clearly indicating that a declarative sentence is always part of a language, in which it is spoken or written, whereas propositions are not peculiar to any of the languages in which they may be expressed. Another difference between them is that the same sentence may be uttered in different contexts to assert different propositions. (For example, the sentence 'I am happy' may be uttered by different persons to make different assertions). The same sort of distinction can be drawn between sentences and *statements*. The same statement can be made using different words, and the same sentence can be uttered in different contexts to make different statements.

Corresponding to every possible inference is an *argument*, and with these arguments logic is chiefly concerned. An argument may be defined as any group of propositions or statements of which one is claimed to follow logically from the others, which are regarded as grounds for the truth of that one. In ordinary usage the word 'argument' also has other meanings, but in logic it has the technical sense, we use the word 'argument' also in a derivative sense to refer to any sentence or collection of sentences in which an argument is formulated or expressed. When undertaken we will be presupposing that the context is sufficiently clear to ensure that unique statements are made or unique propositions are asserted by the utterance of those sentences.

Every argument has a formal and material structure, in the analysis of which the terms 'premiss' and 'conclusion' are usually employed. The *conclusion* of an argument is that proposition which is affirmed on the basis of the other propositions of the argument, and these other propositions which are affirmed as providing grounds or reasons for accepting the conclusion are the *premises* of that argument or the grounds of inference in the sense of deduction.

We note that 'premiss' and 'conclusion' are relative terms, in the sense that the same proposition can be a premise in one argument and conclusion in another. Thus the proposition *All men are mortal* is premises in the argument :

All men are mortal;
Socrates is a man.
Therefore, Socrates is mortal.

and conclusion in the argument

All animals are mortal.
All men are animals.
Therefore all men are mortal.

Any proposition can be either a premiss or a conclusion, depending upon its context and place occupied. It is a premiss when it occurs in an argument in which it is assumed for the sake of proving some other proposition. And it is conclusion when it occurs in an argument, which is claimed to prove it, or to the basis of other propositions, which are assumed.

A distinction can be made between *deductive* and *inductive* arguments. All arguments involve the claim that their premises provide some grounds for the truth of their conclusions, but only a *deductive* argument involves the claim that its premises provide *absolutely conclusive* grounds. The technical terms 'valid' and 'invalid' are used in place of 'correct' and 'incorrect' in characterizing deductive arguments. A deductive argument is *valid* when its premises and conclusion are related in such a way that it is absolutely impossible for the premises to be true unless the conclusion is true also. The task of deductive logic is to clarify the exact feature of the relationship which holds between premises and conclusion in a valid argument, and to provide techniques for discriminating the valid from the invalid.

Inductive arguments involve the claim only that their premises provide some grounds for their conclusions. Neither the term 'valid' nor its opposite 'invalid' is properly applied to inductive arguments. Inductive arguments differ among themselves in the degree of probability, which their premises confer upon their conclusions, and are studied in inductive logic. But in this section i.e., mathematical logic we shall be concerned only with deductive arguments, and shall use the word 'argument' to refer to deductive arguments exclusively.

Truth and Validity :

Truth and falsity characterize propositions or statements, and may derivatively be said to characterize the declarative sentences in which they are formulated. But arguments are not properly characterized as being either true or false. On the other hand, validity and invalidity characterize arguments rather than propositions or statements.[2] There is a connection between the validity or invalidity of an argument and the truth or falsehood of its premises and conclusions, but the connection is by no means a simple one.

Some valid arguments contain true propositions only, as for example,

All bats are mammals.
All mammals have lungs.

But an argument may contain false propositions exclusively, and be valid nevertheless, as, for example.

All trout are mammals.
All mammals have wings.
Therefore all trout have wings.

But an argument is valid because *if* its premises were true its conclusion would have to be true also, even though in fact they are all false. These two examples cited above show that although some valid arguments have true conclusions, not all of them do. The validity of an argument does not guarantee the truth of its conclusion.

Let us consider the following statement :

If I am President then I am famous.
I am not President.
Therefore I am not famous.

In the above case we can see that although both premises and conclusions are true, it is invalid. Its invalidity is made obvious by comparing it with another argument of the same form :

> If Rockefeller is President then he is famous.
> Rockfeller is not President.
> Therefore, Rockfeller is not famous.

This argument is clearly invalid on account of the fact that its premises are true but its conclusion false. The two latter examples show that although some invalid arguments have false conclusions, not all of them do. The falsehood of its conclusion does not guarantee the invalidity of an argument. But the falsehood of its conclusion does guarantee that *either* the argument is invalid *or* at least one of its premises is false.

There are two conditions that an argument must satisfy to establish the truth of its conclusion. It must be valid, and all of its premises must be true. The logician is concerned with only one of those conditions. To determine the truth or falsehood of premises is the task of scientific inquiry in general, since premises may deal with any subject matter at all. But determining the validity or invalidity of arguments is the special province of deductive logic. The logician is interested in the question of validity even for arguments whose premises might happen to be false.

It might be suggested that we should confine our attention to arguments having true premises only. But it is often necessary to depend upon the validity of arguments whose premises are not known to be true. Modern scientists investigate their theories by deducing conclusions from them which predict the behaviour of observable phenomena in the laboratory or observatory. The conclusion is then tested directly by observation, and if it is true, this tends to confirm the theory from which it was deduced, whereas if is false, this disconfirms or refutes the theory. In either case, the scientist is vitally interested in the validity of the argument by which the testable conclusion is deduced from the theory being investigated, for if that argument is invalid his whole procedure is without point. The foregoing is an oversimplified account of scientific method, but it serves to show that questions of validity are important even for arguments whose premises are not true.

So far we have seen that logic is concerned with arguments and that these contain propositions or statements as their premises and conclusions. The conclusions are not linguistic entities, such as declarative sentences, but rather what declarative sentences are typically uttered to assert. However, the communication of propositions and arguments requires the use of language, and this complicates our problem. Arguments formulated in English or any other natural language are often difficult to appraise because of the vague and equivocal nature of the words in which they are expressed, the ambiguity of their construction, the misleading idioms they may contain, and their pleasing but deceptive metaphorical style. The resolution of these difficulties is not the central problem for the logician, however, for even when they are resolved, the problem of deciding the validity or invalidity of the argument remains.

To avoid the peripheral difficulties connected with ordinary language, researchers in the various sciences have developed specialized technical terminology. The scientist economizes the space and time required for writing his reports and theories by adopting special symbols to express ideas which would otherwise require a long sequence of familiar words to formulate. This has the further advantage of reducing the amount of attention needed, for when a sentence or equation grows too long its meaning is more difficult to grasp. The introduction of the exponent symbol in mathematics permits the expression of the equation.

Logic, too, has had a special technical notation developed for it. Aristotle made use of certain abbreviations to facilitate his own investigations, and modern symbolic logic has grown by the introduction of many more special symbols. The difference between the old and the new logic is one of degree rather than of kind, but the difference in degree is tremendous. Modern symbolic logic has become immeasurably powerful a tool through which the analysis and deduction through the development of its own technical language is done. The special symbols of modern logic permit us to exhibit with greater clarity and precision the logical structures of arguments, which may be obscured by their formulation in ordinary language. It is an easier task to divide arguments into the valid and the invalid when they are expressed in a special symbolic language, for in the peripheral problems of vagueness, ambiguity, idiom, metaphor etc. do not arise. The introduction and use of special symbols serve not only to facilitate the appraisal of arguments, but also to clarify the nature of deductive inference. All these tools are employed to disintegrate language and come to a logical conclusion.

The logician's special symbols are much better adapted than ordinary language to the actual drawing of inferences. Their superiority in this respect is comparable to that enjoyed by Arabic numerals over the older Roman kind for purposes of computation. It is easy to multiply 148 by 47, but very difficult to compute the product of CXLVIII and XLVII. Similarly, the drawing of inferences and the evaluation of arguments is greatly facilitated by the adoption of a special logical notation. To quote Alfred North Whitehead, an important contributor to the advance of symbolic logic :

... by the aid of symbolism, we can make transitions in reasoning almost mechanically by the eye, which otherwise would call into play the higher faculties of the brain.[3]

Like the Western Logicians the Indians also developed some tools of reasoning and some technical terms for clarity and precision.

The Indian Logicians also believe that reasoning is the backbone of all theories and in the principles called good reasoning (*sutarka*) and bad reasoning (*kutarka*). The Naiyayikas also feel that reasoning is a special kind of thinking called inferring. To them in each and every piece of knowledge there is inference as it is substantiated through reasoning.

Knowledge is a kind of lamp by which the nature of an object is revealed. According to the Naiyāyikas, cognition is of two types : presentative cognition (*anubhava*) and recollection (*smṛiti*)[4] A presentative cognition may be valid (*yathārtha*) and invalid (*ayathārtha*). A valid presentative cognition which is called *Pramā* is of four types : *Pratyakṣa, Anumiti, Upamiti* and *Śābda*. The uncommon causes (*Karaṇa*) of these four types of knowledge are called Perception (*Pratyakṣa*), Inference (*Anumāna*) comparison (*Upamāna*) and verbal testimony (*Śabda*) and they are special sources of attaining valid knowledge (*Pramāṇas*).[5]

Perception is the knowledge, which arises out of the contact (*sannikarsa*) of the sense-organs (*indriya*) with objects (*artha*). This knowledge will be indescribable (*avyapadeśya*) i.e. indeterminate (*nirvikalpaka*) non-deviated (*avyabhicāri*) definite (*vyavasāyātmakam*) i.e., determinate (*Savikalpaka*). This definition of perception given by older logicians has been rejected by Gangeśa, as it does not cover God's perception. According to Gangeśa, perception is the knowledge of which the knowledge is not the uncommon cause having operative process (*Karaṇa*).[6] The perceptual knowledge of an object is independent in the sense that it does not depend on the knowledge of other objects and hence it is immediate (but not mediate).

Generally, when a man apprehends with the help of his sense organs must be true if, of course, there is no defect (in perception). No man questions about the truth of the cognition which is attained through sense organs unless anything contradictory to it is found.

Perception is the basis of all kinds of knowledge. Without taking recourse to perception other sources of valid knowledge i.e., inference, comparison and verbal testimony are not possible.

Inference consists in making an assertion about an object on the strength of the knowledge of the probans, which is invariably connected with it. The word '*anumāna*' literally means the cognition, which follows from other knowledge. Here the prefix '*anu*' means 'after' and '*māna*' means 'knowledge'. From this literal meaning it follows that the perceptual knowledge of the probans gives rise to the inferential knowledge. One can infer the existence of fire, for example, after perceiving the smoke, which has got an uninterrupted connection with the surface of the mountain. The knowledge of invariable concomitance (*vyāpti*) is the key for having inferential knowledge. This knowledge of *vyāpti* is not possible without the help of perception. *Vyāpti* is nothing but an invariable co-existence between probans and probandum.[7] The knowledge of the *probandum* as related to the subject of inference (*pakṣa*) depends on the previous knowledge of the *probans* as related to the subject and at the same time as invariably related to the *probandum*. One can infer fire on the mountain by virtue of the fact that one perceives smoke on it and has observed it as invariably accompanied by fire. In both the cases the necessity of perception cannot be denied.

Perception and inference are equally important sources of valid knowledge. Perception is independent in respect of the knowledge of other objects while inference is dependent on the previous knowledge. Perception can reveal those objects that are within the range of our sense organs, i.e., it can give us the knowledge of the present objects that are within the reach of our sense organs in a normal way.[8] But inference can give the knowledge of those objects that are not connected with the sense organs.

Though perception in the fundamental basis of all kinds of knowledge yet inference is by far the important source of knowledge in our society. Hence inference as a special source of valid knowledge (*pramāṇa*) is accepted by the philosophers of all schools of Indian Philosophy with the single exception of the materialistic school (the *Cārvāka* school). But it should be clearly borne in mind that the philosophers of the *Cārvāka* school also do not deny the existence of inference as such. They only hold that inference cannot be accepted as a special source of valid knowledge or *Pramāṇa*. There can at most be the knowledge of probability through inference, but not definite valid knowledge.[9] So the dispute with *Cārvākas* in regard to inference is limited only to the question of its having the nature of the special source of valid knowledge or otherwise.

The *Cārvākas* hold that inference by virtue of having the capacity of producing the knowledge of probability cannot produce definite valid knowledge.

The philosophers of other schools, theist or atheist, strongly oppose the stand point of *Cārvākas* that inference has got no capacity to produce definite valid knowledge. The idea behind the strong opposition is that inference has got tremendous utility in our day to day life, and unless it can produce definite valid knowledge, it cannot satisfactorily be an instrument to serve us in meeting the diverse needs of our life. It is, of course, to be concluded, they say that though the knowledge of probability also can serve us to a certain extent to meet the requirements of our life, particularly in respect of guiding us in the field of activity yet it can never serve our purpose in every respect and in all cases. Definite valid knowledge of a particular object alone can guide us invariably towards action, and this definite valid knowledge can certainly be produced by inference in most of the cases.

In a society the help of an inference is taken almost in every step, but generally we are unaware of the fact that we are inferring some objects. In most of the cases inference is drawn spontaneously. Illiterate persons are found to be guided by inference, not to speak of the literate. Cultivators are seen to infer some object after seeing some sign or mark (*linga*). The Naiyāyikas are of the opinion that even a child also infers.[10] A child attains inferential knowledge spontaneously without being aware of the inferential procedure. The inherent process of inference of a child can be shown in the following manner.

A child comes to know the primary relation (*Samketa*) of a term with its meaning at first from the verbal usages of the old person (*Vṛddhasya śabdādhīnavyavahārādeva*). When a man also is aware of the meaning of a term (*vyutpanna*) asks another man who also knows the meaning of the same term to bring a cow, the person who has been asked to bring a cow by the senior person (*uttamavṛddha*) brings it after hearing the words of the senior and realising the meaning of it. On observing the performance of the man who has been asked to bring a cow, child draws the inference in the form : 'This bringing of a cow is the result of the inclination, the object of which is the bringing of a cow as it has got effort ness in it, as in the case of my inclination so suck mother's breast'. (*idam gavānayanam svagocarapravṛttijanyam, ceṣṭātvāt madīyastanapānādivat*)'.[11] Then he comes to infer the state or condition of being produced by the knowledge of the feasibility (by one's effort) of which the bringing of a cow has qualificand (*gavānayanadharmikakāryatājñānajanyatvam*)[12] in respect of the inclination with the help of the syllogistic argument in the form : 'That inclination to bring a cow is produced by the knowledge of the feasibility (by one's effort of which the inclination the same has become qualificand, as it has got the generic property existing in inclination, as in the case of my own (inclination). Here inclination towards a particular action has become qualificand to the knowledge of the feasibility by one's effort. Any type of inclination presupposes this type of knowledge of feasibility. Then the child again forwards the syllogistic argument in the form : 'The knowledge of the feasibility (by one's effort) of which the bringing a cow has become qualificand, has an uncommon cause, as it is an effort having effortness in it as in the case of a jar. (*gavānayanagocaratajjñānam asādhāraṇahetukam kāryatvāt, ghaṭavat*). Any type of effect has not its special cause and hence, the effect in the form of bringing a cow needs some special cause. After drawing such inference the child comes to know that the knowledge of the verbal usages of the old persons (*vṛddhavyarahāra*) is the uncommon cause (*asādhāraṇakāraṇa*) of the knowledge mentioned above.[13] A child attains this type of inferential knowledge being completely unaware of the abovementioned inferential procedure.

In a society no man believes in a statement which is baseless. In other words, a statement, which is not properly grounded, cannot impress other beings. If our neighbours or relatives are advised to do something or not to do something, they should be convinced with the help of arguments in favour of our statements. In every sphere of our life, we are going on saying something depending on some arguments in as much as the groundless speech will fall flat upon others,[14] which is also a form of inference.

The valid inferential knowledge guides us in innumerable walks of our life beginning with the dealings with our fellow people in our everyday life. Our life becomes thoroughly impracticable.[15] unless our fellow-beings are properly and satisfactorily dealt with and this can never be done unless we definitely and rightly understand the mind of people around us. This understanding of others' mind depends on inference in most of the cases.[16]

Moreover, from the red-colour of a mango it is inferred that it is ripen. In the like manner, the past rain is inferred from the muddy current of the river. In the same way, the mental states like pleasure, pain etc. existing in a man can be inferred from their different types of expressions and gestures. Sometimes, the exact place or country where a man resides can be inferred after observing his dress or his particular language carefully. Thus innumerable instances of the knowledge based on inference in our every day life can be shown.

A man is desirous of doing these types of works by which his purpose might be served and hence, it can be said that the end-in-view (*Prayojana*) inspires him to do some activities.[17] In order to get or get rid of something a man engages himself in activity.[18] Man's desire is related to result in the form of pleasure or the absence of pain and to the means of it. The longing for the result of some action presupposes the knowledge of it.[19] Hence the desire for the result is due to the existence of the knowledge of it, which is also a form of inference. The cause of desire for the means (of the result) is the knowledge of its conduciveness to the object, which is desirable (*iṣṭasādhanatājñāna*). This knowledge of its conduciveness to that which is desirable is considered as a *Hetu* to the desire for the means also.[20] Again, the knowledge of the feasibility through one's effort (*kṛtisādhyatājñāna*) and the knowledge of its conduciveness to that which is desirable (*iṣṭasādhanatājñāna*) are considered as the reasons behind the desire for doing something. Nobody thinks to do action without having the knowledge of its feasibility through one's effort, the cause of desire.[21] This can also be taken as an instance of inference in our day to day life. In the same way, the knowledge of its being productive of what is extremely unpleasant (*dviṣṭasādhanatājñāna*) is the cause of avertion (*dveṣa*), the object of which causes pain. Here avertion towards an object is inferred on the strength of its *dviṣṭa-sādhanatājñāna.*[22]

The existence of the imperceptible objects like *Ātman*. God etc. can easily be proved with the help of inference only.[23] Hence, the logicians prove the existence of *Ātman*, as a locus of the attributes like desire, avertion, effort etc.[24] In other words, soul-ness (*ātmatva*) is inferred as the limitor (*avacchedaka*) of this inherent causness of pleasure, pain etc. Again, that which imparts consciousness of pleasure, pain etc. Again, that which imparts consciousness in the sense organs and also in the body is *Ātman*. Though the contentness (*Viṣayatva*) of the perceptions like 'I am unhappy' etc. remain in *Ātman*, it would not be possible at first to make a person (bearing doubt about it) convinced that *Ātman*, the object of the above-mentioned perception, is different from body etc. Hence, another strong argument is to be forwarded. As no result is produced from the cutting instrument like an axe etc. without being guided by an agent, the eyes etc., the sense organs, cannot produce any result without being guided by an agent. That is why, the agent in the form of impeller of the sense organs is *Ātman.*[25] The syllogistic argument regarding the existence of *Ātman* existing in others' body is as follows : "The body of Devadatta is endowed with *Ātman*, as it is associated with the condition of being qualified by inclination like a chariot."

(*Debadattaśarīram ātmavat pravṛttimattvāt rathavat*)

It is a fact that a considerable number of people in our society believes in the existence of God even in this modern age, but a very few of them have realised Him. This belief in God is based on some grounds, but *not* on blind faith. The Naiyāyikas have taken pain to highlight the existence of the Divine with the help of some grounds or arguments that are inferential in nature. These syllogistic arguments are as follows.

(a) As the effects like jar etc. are caused by an agent, the earth (*kṣiti*), dyads (*ankura*)[26] etc. must have caused by an agent. The agentness of it, being not possible in persons like us having limited knowledge and power, remains in God. Hence, God is inferred as the cause or agent of earth etc.[27] (b) the activity in which dyad becomes a promoter (*prayojaka*) at the time of initial creation is caused by an effort (*prayatnajanya*), as it is an activity. This world is originated from the combination of atoms. These atoms cannot be combined with each other automatically (without being guided by a conscious being) due to their inanimate character. This Conscious Being is nothing other than God.[28] (c) The absence of the coming downwards of weighty substances (*gurutvavatām*) is caused by an effort, which becomes an obstacle to the coming down of a substance, as it is endowed with steadiness, as in the case of the absence of falling of a bird (*pakṣipatanābhāvavat*)[29] This world having weight is not coming down due to having some power in the form of effort, which is God. (d) The destruction of the universe presupposes the existence of an effort, as it is a destruction in character as in the case of the destruction of a jar.[30] This effort from which the destruction of the universe follows is in the form of God. (e) The initial verbal usage, as in the case of the usages of the scripts introduced in modern age. This independent person is God.[31] (f) The *Vedas* are introduced by a being who is other than an individual who entangles in the worldly affairs, as it has the property of being the Veda (*Vedatvāt*). That which is not of this type would not be of this type, as in the case of a piece of literature.[32] The *Asamṣārī Puruṣa* is God. (g) The Vedas are introduced by a *Puruṣa* (*Pauruṣeya*) as they possess sentences as in the case of the *Mahābhārata* etc. This *Puruṣa* is God.[33]

The imperceptible objects like atom, *Ākāśa Kāla* etc. are admitted by the Naiyayikas with the help of inference. The Naiyayikas have explained the origination of the whole universe in terms of the combination of atoms. This theory would have been meaningless if the existence of atom were not proved through inference. The syllogistic argument is as follows : If the whole of an object has an endless series of parts, there would arise the contingency of equality in respect of size between mountain and a mustard seed. If the whole has some parts, the parts also have some other parts in which there are other parts and so on. In this way, there would arise 'Infinite Regress' (*Anavasthā*). As there is no final unit of a definite size, we cannot add these up to make different sizes. Hence there would arise the contingency of equality in dimension between very big and small objects; *as in mathematics anything multiplied by zero is* *zero.*[34] So this process of division must be stopped anywhere. If the limit is taken as non-eternal, it would be taken into account that a positive effect may be produced even when there is the absence of inherent cause. If the limit is considered as non-eternal, it must be taken as an effect which remains in its through the relation of inherence (*samavāya*). As there are no parts in it, it can be said that it is a positive effect having no parts. As a positive effect having no parts is not possible, it would be taken as an eternal object.[35] As the gradation of the medium dimension (*Mahatparimāṇatāratamya*) has a limit in *Ākāśā* etc., the gradation of the atomic dimension (*aṇuparimāṇa*) has limit somewhere. Where there is limit is an atom.[36] It cannot be said that the limit of atomic dimension is a triad (*Trasareṇu*). That a triad possesses its parts (*avayava*) can be established with the help of the syllogistic argument in the form : "A triad possesses its parts, as it is a substance capable of being perceived like a jar" (*Trasareṇuḥ sāvayavavaṃ cākṣuṣadravyatvāt ghaṭavāt*). That the parts of a triad (i.e., dyads) possess their own parts can also be established by another inferential argument in the form : "The parts of a triad possess their own parts, as they become the producer of an object of medium dimension, as in the case of *kapāla* i.e., upper part of the jar (*Trasarenoravayavāh sāvayavāḥ mahadārambhakatvāt kapālavat*). A part of a dyad, one of the parts of a triad, is called Atom.[37]

In the same way *Ākāśa* is inferred as the locus of sound.[38] The existence of *Kāla* is inferred from its general causeness to the objects that are produced (*janya*), from its being the locus of this universe,[39] and from its being an uncommon cause (*karaṇa*) of this knowledge of priority (*Paratva*) and posteriority (*Aparatva*).[40]

The philosophers of the theistic school specially take recourse to inference as a means to go above the sphere of grief and sorrow and attain fulfilment in the form of attaining salvation or *Mokṣa*. The Upanisadic injunction that realisation of the soul should be attained through hearing, thinking and constantly meditating upon the nature of the soul is accepted by the philosophers of the theist school as a supreme gospel, by obeying which a man can rise above the sphere of sorrow and grief and can attain salvation. 'Thinking' in the injunction is nothing other than inference of the soul as distinct from other worldly objects (*Mananam cātmanaḥ itarabhinnatvena anumānam*). This inference of the true nature of the soul should be attained through frequent practice of inference. Hence, it can easily be understood what great importance has been attached to inference by the theistic philosophers of our country having regards to the utility of inference in the matter of attaining the supreme goal of life.

Each and every object of this world can be inferred as distinct from other worldly objects. As for example, a jar can be inferred as distinct from the objects other than the jar i.e., pot etc. In this way, a pot can be inferred as distinct from the objects other than pot etc. i.e., jar etc. The *Hetu* of the inference of some object as distinct from others is the definition of that object. As for example, a cow is distinct from the animals other than cow, as it is the locus of the dewlap etc (*Gauḥ gavetarabhinnā sāsṇādimattvāt*).

From the above discussions, it can be conducted that inference has great utility in each and every sphere of our life. The particular kind of conditions, both positive and negative, cannot guide or control our activities in the majority of cases of our life for the simple reason that the capacity of those conditions to produce definite valid knowledge is limited compared to that of the procedure of inference.

Though it can be argued that procedure of inference is extremely complicated and the highly educated persons can only successfully apply it, trained in the art of drawing right inference, it can also be equally emphasized that the drawing of inference from some given date is not a so difficult proposition. It has already been said that even the illiterate persons also are found to be spontaneously drawing inference from circumstances and controlling their activities accordingly. It is, of course, very difficult to give scholarly analysis of the procedure of inference, but to draw inference from some given data is not at all difficult, rather it is, to a great extent, spontaneous.

Keeping therefore, in view this practical aspect of inference, it can safely be concluded that Inference has got great utility in our everyday life.

Though the Naiyāyikas and Buddhists follow the form of inference like *Pratijñā*, *hetu*, *udāharaṇa* etc, they believe in the material truth also. In symbolic logic an inference is taken as true, if it is true formally irrespective of its material truth. In this connection it may be said that both in Indian and Western Philosophy inference which is constituted with premise and conclusion occupies a prominent role. Though the formal structure of inference as found in Western Mathematic Logic is derived from the mathematical principles directly, the Indian Systems such inferential methods are not directly *taken from Mathematics*. In fact the Indian Philosophers had some intuitive power, which may correspond to the Mathematical intuition.

REFERENCES

1 'Logic', in Dictionary of Philosophy, edited by James Mark Baldwin, New York, The Macmillan Company, 1925.

2 Some logicians use the term 'valid' to characterize statements, which are logically true. For the present, however, we apply the terms 'valid' and 'invalid' to arguments exclusively.

3 *An Introduction to mathematics* by A.N. Whitehead. Oxford, Eng., Oxford University Press, 1911.

4 " ... Buddhistu dvividhā matā
Anubhūtiḥ smṛtiśca syādanubhutiścaturvidhā."
Bhāṣāpariccheda, Verse No. 51.
Sā dvividhā Smṛtiranubhavaścaturvudhah."
... yathārthānubhavaścaturvidhah."
Tarkasamgraha, Chowkhamba Sanskrit Santhan, pp. 32-33.

5 'Etāsām catasṛṇām karaṇāni catvāri
'Pratyakṣānumānomānaśabdāḥ pramāṇāni sūtroktāni veditavyāni'
Siddhāantamuktāvalī on Verse 51.
'Tat karanamapi caturvidhaṁ pratyṣ -
kānumānopanānaśabdabhedāt'
Tarkasamgraha, pp. 34-35, (Same ed.).

6 "Indriyārthasannikarṣotpannam jñānam
avyapadeyśamavyabhicāri vyavasāyātmakam pratyakṣam" –
"Athavā jñānākaraṇakam jñānam pratyakṣam"
Siddhāntamuktāvalī on Verse, 51.

7 "Yatra dhūmastatrāgniriti, sāhacaryaniyamo vyāptiḥ."
Tarkasamgraha, p. 49 (Same Edition).

8 Here the phrase 'in a normal way' has been used in order to exclude the super-normal perception alaukika pratyakṣa through which objects of all times can be perceived.

9 "Avalāvālagopālahālika pramukhā api buddhyante niyatādarthādarthāntaramasamśayam. *Nyāyamañjari*, p. 110, Chowkhamba Ed.

10 Ibid.

11 "Prathamam padeșu-samketagraho vṛddhasya vyutpannasyaśabdādhīnavyavahārādeva vākyānām, Tathāhi, gāmānayeti kenacinnipuṇena niyuktaḥ kaścana vyutpannastadvākyato'rtham pratitya gavānayanam karoti taccopalabhamāna vāla idam gavānayanam svagocarapravṛttijanyam caṣṭātvāt madīyastanapānā divadityanumāya ..." Śabdaśaktiprakāśikā Nāmaprakaraṇa, Prose portion on Verse No. 20; p. 116 (Jayccandra Śarmā Ed.).

12 "Svaviṣayadharmiketi pravṛttiviṣayaviśeṣyaketyarthaḥ. Kāryatā kṛtisādhyatā". Commentary on, Prose portion of verse no. 20 of Śabdaṣaktiprakāśikā, p. 116, Ed. by Jayaccandra Śarmā.

13 "... Sā gavānāyanapravṛttiḥ svavi ṣayadharmikakāryatajñānajanyā, pravṛttitvānnijapravṛttivaditi pravṛttergavānayanadharmikakāryatājñānajanyatvam prasādhya gavānayanagocaratajjñānamasādhāraṇahetukam kāryatvād ghaṭavadityevam anuminavānaḥ ... śrutam vṛddhavākyameva tadasādhāraṇakāraṇatvenāvadhārayati." Śabdaśaktiprakāśikā (Nāmāprakarana) Prose portion on Verse No. 20, p. 116, Edited by Jayaccandra Śarmā.

14 "Aśiraskavacanopanyāse sādhyāsiddheḥ. Ekākinī pratijñā hi pratijñātam na sādhyāet". Bauddhadarśana, Sarvadarśanasamgraha.

15 "Anumānāpalāpe tu pratyakṣādapi durlabhā lokayātreti lokāḥ syurlikhitā iva niścalāh." *Nyāyamañjarī*, p. 110, Chowkhamba.

16 "Pramāṇāntarasāmānyastheteḥ anyadhiyogateḥ, pramāṇāntarasadbhāyraḥ ..." Bauddhadarśana, Sarvadarśanasamgraha.

17 "Yamarthamadhikṛtya pravartate tatprayojanath." *Nyāyasūtra* 1.1.24.
"Yena prayuktaḥ pravartate tat prayojanam
Yamarthamabhipsanjihāsan vā karmārabhate tenānena
Sarve prāṇinaḥ sarvāni karmāṇi sarvāśca vyāptāt."
Nyāyabhāṣya on Sūtra 1.1. 24.

18 "Yamarthamāptavyam hātavyamavasāya tadāptihānopāyamanutiṣthati tat prayojanam, tadveditavyam pravṛttihetutvāt." Ibid.

19 "Icchā hi phalaviṣyiṇī upāyaviṣayiṇīca. Phalam tu sukhārtho duḥkhābhāvaśca. Tatra phalecchām prati phalajñānam kāraṇam". Siddhāntamuktāvalī on verse 146.
Nirduḥkhatve sukhe cecchā tajjñānādeva jñāyate".
Bhāṣāparicchedaḥ, Verse 146.

20 Icchā tu tadupāye syādiṣṭopāyatvadhiryadi".
Bhāṣāparicchedaḥ, Verse 146.

21 "Cikirsā kṛtisādhyatvaprakārecchā tu yā bhavet,
Taddhetuḥ kṛtisādhyeṣṭasādhanatvamatirbhavet".
Bhāṣāparicchedaḥ, Verse 147.

22 Dviṣtasādhantābuddhirbhavet dveṣasya kāraṇam."
Bhāṣāparicchedaḥ, Verse 14.

23 Ātman, God etc. can be known through *yogaja pratyakṣa*, which is not at all easy task. Hence inference is the easy method through which common men can be convinced as to the existence of Self, God etc.

24 "IcchādveṣaprayatnasukhaduḥkhajñānānyĀtmano lingamiti."
Nyāyasūtra, 1. 1. 10.
"Ātmatvajātistu sukhaduhkādiṣamavāyakāranatāvacchedakatayā sidhyati".
Siddhāntamuktāvalī on Verse 47.
"Jñānādhikaraṇamātmā".
Tarkasamgraha, p. 19 (Chowkhamba).

25 Indriyāṇam śārīrasya ca paramparayā caitanyasampādakaḥ yapyapy Ātmani aham sukhi aham duḥkhītyādipratyakṣaviṣayatvamastyeva, tathāpi vipratipannam prati prathamata evaśarīrādibhinnastatpratitigocara iti pratipādayitum na śakyate ityataḥ pramāṇāntaram darśayati kāraṇamiti. Vāsyādīnām chidādikāranāṇām, kartāramantareṇa phalānupadhānam dṛṣṭam, evam cakṣurādinām jñānakaraṇānāmapi phalopadhānam kartāramantareṇa nopapadyata ityatīriktaḥ kartā kalpyate."
Siddhāntamuktāvalī on Verse 47.

26 Here the term '*Ankura*' means dyad or dvyaṇuka. In the *Kiranāvalī* commentary on *Siddhāntamuktāvalī* it is said that, just as the object which is seen at first as a promoter of a tree arising out of the seed is called *Ankura*, the object which is the promoter of the world-tree (*Samsārataru*) arising from two atoms has got resemblance with Ankura and hence '*dvyaṇuka*' is to be understood by the term '*Ankura*'. Here 'dvyaṇuka' is metaporised as 'Ankura' and world as tree. The original commentary runs as follows : "*Ankureti = yathā vijādutpannasya vṛkṣaprayojakasya prāthamikadarśanaviṣayasyānkuratvam tāthā paramāṇūbhyāmutpannasya samsārataruprayojakasya dvyaṇukasyānkurasāmyāt ankuraśabdena dvyaṇukam lakṣyate*". Kiranāvalī on *Siddhāntamuktāvalī*, p. 16. (Edited by Krishnaballava Acharya).

27 "Yathā ghaṭādikāryam katṛjanyam tathā kṣityamkurādikamapi" *Siddhāntamuktāvalī*, on Verse I.

28 "Sargādyakālinadvyaṇukaprayojakam karma prayatnajanyam karmatvāt" Dinakari on *Siddhāntamuktāvalī*, Verse No. 1, p. 20, (Chowkhamba).

29 "Gurutvavatām patanābhāvaḥ patanapratibandhakạprayatnaprayuktaḥ dṛtitvāt, pakṣipatanābhāvavat". *Ibid.*

30 Brahmāṇḍanaśaḥ prayatnajanyah nāśatvāt, ghaṭanāśavat". *Ibid.*

31 "Ghaṭādivyavahāraḥ svatantrapuruṣaprayojyaḥ vyavahāratvāt, ādhunikakalpitalipyādivyavaḥāravat". Dinakarī on *Siddhāntamuktāvalī*, Verse I, p. 29. (Chowkhamba).

32 "Vedaḥ asamsāripuruṣapraṇītaḥ. Vedatvāt, yannaivam tannaivam yathā kāvyamiti", *Ibid.*

33 "Vedaḥ pauruṣeyaḥ vākyatvāt bhāratādivat". *Ibid.*

34 *Bhāṣāpariccheda*, p. 44. Edited by Swami Madhavananda.

35 "Teṣām cāvayavadhārāyā anantatve merusarṣapayorapi sāmyaprasangaḥ, ataḥ kvacid viśrāmo vācyaḥ yatra tu viśrāmaḥ tasyānityatve' sambhaveta (bhāva) kāryotpattiprasangāt tasya nityatvam". *Siddhāntamuktāvalī* on Verse 37.

36 "Mahatparimāṇatāratamyasya gaganādau viśrāntatvamivāṇuparimāṇatāratamayasāpi kvacidviśrantatvamiti tasya paramāṇutvasuddhiḥ." – *Ibid.*

37 "Na ca trasareṇāveva viśrāno'stīti vācyam.

Trasareṇuḥ sāvayavaḥ cāksuṣadravyatvāt

Ghatavadityanumānena tadavayavasidhau, trasareṇoravayavāh sāvayavāḥ mahadārambhakatvāt,

Kapātavadityanumānena tadavayavasiddheḥ."

Sidhhāntamuktāvalī on Verse 37,

"Tryaṇukāvayovo'pi sāvayavah mahadārambhakatvāt kapālavat, yo dvyaṇukāvayavaḥ sa paramāṇuḥ.

Dīpikā *on Tarkasamgraha*, p. 190, Chowkhamba with seven commentaries.

38 Śabdaguṇakamākāśam.

Tarkasamgraha, p. 1718 (Chowkhamba).

39 "Janyānām janakaḥ kālah jagatāmāśrayo matah".

Bhāṣāpariccheda, Verse No. 45.

40 "Paratvāparatvabuddhirasādhāraṇam nimittam kāla eva."

Ibid.

CHAPTER - 2

Tools for Logical Analysis

So far we have seen the role of reasoning/inference in Western Mathematical Logic. Now let us turn our attention to some characteristic features of Mathematical Logic, which is very much essential for philosophising or for knowing the truth.

All statements can be divided into two kinds, simple and compound. A *simple* statement is one, which does not contain any other statement as a component part, whereas every *compound* statement does contain another statement as a component part. For example, 'Unethical practice will be eradicated or this society will become uninhabitable' is a compound statement that contains as its components the two simple statements 'Unethical practice will be eradicated' and 'this society will become uninhabitable'. The component parts of a compound statement may themselves be compound, of course. We turn now to some of the different ways in which statements can be combined into compound statements.

The statement 'Roses are red and violets are blue' is a *conjunction*, a compound statement formed by inserting the word 'and' between two statements. Two statements so combined are called *conjuncts*. The word 'and' has other uses, however, as in the statement 'Castor and Pollux were twins', which is not compound, but a simple statement asserting a relationship. We introduce the dot '·' as a special symbol for combining statements conjunctively. Using it, the preceding conjunction is written 'Roses are red · violets are blue'. Where p and q are any two statements whatever, their conjunction is written p·q.

Every statement is either true or false, so we can speak of the *truth value* of a statement, where the truth value of a true statement is *true* and the truth value of a false statement is *false*. There are two broad categories into which compound statements can be divided, according to whether or not there is any necessary connection between the truth value of the compound statement and the truth values of its component statements. The truth value of the compound statement 'Smith believes that lead is heavier than zinc' is completely independent of the truth value of its component simple statement 'lead is heavier than zinc', for people have mistaken as well as correct beliefs. On the other hand, there is a necessary connection between the truth value of a conjunction and the truth values of its conjuncts. A conjunction is true if both its conjuncts are true, but false otherwise. Any compound statement whose truth value is completely determined by the truth values of its component statements is a *truth – functionally* compound statement. The only compound statements we shall consider here will be truth – functionally compound statements.

Since conjunctions are truth-functionally compound statements, our symbol is a truth-functional connective. Given any two statements p and q there are just four possible sets of truth values they can have, and in every case the truth value of their conjunction p·q is uniquely determined. The four possible cases can be exhibited as follows:

in case p is true and q is true, p·q is true;
in case p is true and q is false, p·q is false;
in case p is false and q is true, p·q is false;
in case p is false and q is false, p·q is false.

Representing the truth values true and false by the capital letters 'T' and 'F', respectively, the way in which the truth value of a conjunction is determined by the truth values of its conjuncts can be displayed more briefly by means of a *truth table* as follows:

p	q	p·q
T	T	T
T	F	F
F	T	F
F	F	F

Since it specifies the truth value of p·q in every possible case, this truth table can be taken as *defining* the dot symbol. Other English words such as 'moreover', 'furthermore', 'but', 'yet', 'still', 'however', 'also', 'nevertheless', 'although', etc., and even the comma and the semicolon, are also used to conjoin two statements into a single compound one, and all of them can be indifferently translated into the dot symbol so far as truth values are concerned.

The statement in the form : 'It is not the case that water is colder than ice' is also compound, being the *negation* (or *denia*l or *contradictory*) of its single component statement 'water is colder than ice'. We introduce the symbol '~', called a *cur*l to symbolize negation. There are often alternative formulations in English of the negation of a given statement. Thus where L symbolizes the statement 'water is colder than ice', the different statements 'it is not the case that water is colder than ice', 'it is false that water is colder than ice', 'it is not true that water is colder than ice', 'lead is not heavier than gold' are all indifferently symbolized as ~L. More generally, where p is any statement whatever, its negation is written ~p. Since the negation of a true statement is false and the negation of a false statement is true, we can take the following truth table as defining the curl symbol:

p	~q
T	F
F	T

When two statements are combined disjunctively by inserting the word 'or' between them, the resulting compound statement is a *disjunction* (or *alternation*), and the two statements so combined are called *disjuncts* (or *alternatives*). The word 'or' has two different senses, one of which is clearly intended in the statement 'Premiums will be waived in the event of sickness or unemployment'. The intention here is obviously that premiums are waived not only for sick persons and for unemployed persons, but also for persons who are both sick and unemployed. This sense of the word 'or' is called *weak* or *inclusive*. Where precision is at a premium, as in contracts and other legal documents, this sense is made explicit by use of the phrase 'and /or'.

A different sense of 'or' is intended when a restaurant lists 'tea or coffee' on its menu list, meaning that for the stated price of the meal the customer can have one or the other, but *not both*. This second sense of 'or' is called *strong* or *exclusive*. Where precision is at a premium and the exclusive sense of 'or' is intended, the phrase 'but not both' is often added.

A disjunction which uses the inclusive 'or' asserts that at *least one disjunct is true*, while one which uses the exclusive 'or' asserts that at *least one disjunct is true but not both are true*. The *partial common meaning*, that at least one disjunct is true, is the whole meaning of an inclusive disjunction, and a part of the meaning of an exclusive disjunction.

In Latin, the word '*vel*' expresses the inclusive sense of the word 'or', and the word '*aut*' expresses the exclusive sense. It is customary to use the first letter of 'vel' to symbolize 'or' in its inclusive sense. Where p and q are any two statements whatever, their weak or inclusive disjunction is written pvq. The symbol 'v', called a *wedge* (or a *vee*), is a truth - functional connective, and is defined by the following truth table:

p	q	p v q
T	T	T
T	F	T
F	T	T
F	F	F

An obviously valid argument containing a disjunction is the following Disjunctive Syllogism:

> The United Nations will be strengthened or there will be a third world war.
> The United Nations will not be strengthened.
> Therefore there will be a third world war.

It is evident that a Disjunctive Syllogism is valid on *either* interpretation of the word 'or', that is, regardless of whether its first premiss asserts an inclusive or exclusive disjunction. It is usually difficult, and sometimes impossible, to discover which sense of the word 'or' is intended in a disjunction. But the typical valid argument that has a disjunction for a premiss is, like the Disjunctive Syllogism, valid on either interpretation of the word 'or'. Hence we effect a simplification by translating any occurrence of the word 'or' into the logical symbol 'v' – *regardless of which sense of 'or' is intended.* Of course where it is explicitly stated that the disjunction is exclusive, by use of the added phrase 'but not both', for example, we do have the symbolic apparatus for symbolizing that sense, as will be explained below.

The use of parentheses, brackets, and braces for punctuating mathematical expressions is familiar. No number is uniquely denoted by the expression '6 + 9 ÷ 3', although when punctuation makes clear how its constituents are to be grouped, it denotes either 5 or 9. Punctuation is needed to resolve ambiguity in the language of symbolic logic too, since compound statements may themselves be combined to yield more complicated compounds. Ambiguity is present in p·q v r, which could be either the conjunction of p with q v r, or else the disjunction of p·q with r. These two different senses are unambiguously given by different punctuations: p· (q v r) and (p·q) v r. In case p and q are both false and r is true, the first punctuated expression is false (since its first conjunct is false) but the second punctuated expression is true (since its second disjunct is true). Here a difference in punctuation makes all the differences between truth and falsehood. In symbolic logic, as in mathematics, we use parentheses, brackets, and braces for punctuation. To cut down on the number of punctuation marks required, however, we establish the symbolic convention that in any expression the curl will apply to the smallest component that the punctuation permits. Thus the ambiguity of ~p v q, which might mean either (~p) v q or ~(p v q), is resolved by our convention to mean the first of these, for the curl can (and therefore by our convention does) apply to the first component p rather than to the larger expression p v q.

The word 'either' has a variety of different uses in English. It has conjunctive force in 'The Disjunctive Syllogism is valid on either interpretation of the word 'or'. It frequently serves merely to introduce the first disjunct in a disjunction, as in 'Either the United Nations will be strengthened or there will be a third world war'. Perhaps the most useful function of the word 'either' is to punctuate some compound statements.

Thus the sentence :

'More stringent anti-pollution measures will be enacted and the laws will be strictly enforced or the quality of life will be degraded still further'.

Can have its ambiguity resolved in one direction by placing the word 'either' at its beginning, or in the other direction by inserting the word 'either' right after the word 'and'. Such punctuation is effected in our symbolic language by parentheses. The ambiguous formula p·q v r discussed in the preceding paragraph corresponds to the ambiguous sentence considered in this one. The two different punctuations of the formula correspond to the two different punctuations of the sentence affected by the two different insertions of the word 'either'.

All types of conjunctions are not formulated by explicitly placing the word 'and' between complete sentences, as in 'Charlie's neat and Charlie's sweet'. Indeed the latter would more naturally be expressed as 'Charlie's neat and sweet'. And the familiar 'Jack and Jill went up the hill' is the more natural way of expressing the conjunction. 'Jack went up the hill and Jill went up the hill'. It is the same with disjunctions: 'Either Alice or Betty will be elected' expresses more briefly the proposition alternatively formulated as 'Either Alice will be elected or Betty will be elected'; and 'Charlene will be either secretary or treasurer' expresses somewhat more briefly the same proposition as 'Either Charlene will be secretary or Charlene will be treasurer'.

The negation of a disjunction is often expressed by using the phrase 'neither-nor'. Thus the disjunction 'Either Alice or Betty will be elected' is denied by the statement 'Neither Alice nor Betty will be elected'. The disjunction would be symbolized as A v B and its negation as either ~(A v B) or as (~A)·(~B). (The logical equivalence of these two formulae will be discussed in Section 2.4). To deny that at least one of two statements is true is to assert that both of the two statements are false.

The word 'both' serves various functions, one of which is a matter of emphasis. To say 'Both Jack and Jill went up the hill' is only to emphasize that the two of them did what they are asserted to have done by saying 'Jack and Jill went up the hill'. A more useful function of the word 'both' is punctuational, like that of 'either'. 'Both ... and - - - are not ---' is used to make the same statement as 'Neither ... nor - - - is ---'. In such sentences the *order* of the words 'both' and 'not' is very significant. There is a great difference between.

Alice and Betty will not both be elected.
and
Alice and Betty will both not be elected.

The former would be symbolized as ~(A·B), the latter as (~A)·(~B).

Finally, it should be remarked that the word 'unless' can be used in expressing the disjunction of two statements. Thus 'Our resources will soon be exhausted unless more recycling of materials is effected' and 'Unless more recycling of materials is effected our resources will soon be exhausted' can equally well be expressed as 'Either more recycling of materials are effected or our resources will soon be exhausted' and symbolized as M v E.

Since an exclusive disjunction asserts that at least one of its disjuncts is true but they are not both true, we can symbolize the exclusive disjunction of any two statements p and q quite simply as (p v q)· ~(p·q). Thus we are able to symbolize conjunctions, negations, and both inclusive and exclusive disjunctions. Any compound statement, which is built up out of simple statements by repeated use of truth-functional connectives, will have its truth value completely determined by the truth values of those simple statements. For example, if A and B are true statements and X and Y are false, the truth-value of the compound statement ~[(~A v X) v ~(B·Y)] can be discovered as follows. Since A is true, ~A is false, and since X is false also, the disjunction (~A v X) is false. Since Y is false, the conjunction (B·Y) is false, and so its negation ~(B·Y) is true. Hence the disjunction (~A v X) v ~(B·Y) is true, and its negation, which is the original statement, is false. Such a stepwise procedure, beginning with the inmost components, always permits us to determine the truth-value of a truth-functionally compound statement from the truth-values of its component simple statements. Here we find a very systematic expression of a statement through appropriate symbol.[1]

All these operatives are also available in Indian Philosophical systems, though linguistically (not symbolically). The term '*ca*' meaning 'and' is used to connect one or more than one entities. The grammarians have shown various meanings of *ca* like *samāhāra*, *anvācaya* etc. In the like the disjunction is expressed with the term '*vā*', or '*athavā*'. In the same way, the implication is expressed with the terms '*yadi*' or '*cet*' etc. Negation is so important in Indian Logic that it occupies the place of a category called *abhāva*. Though regarding the nature of absence and its way of meaning there is controversy among the thinkers within the Indian systems, it is accepted by all that negation is possible and it has got a prominent role in conveying meaning. But it should be carefully noted that the absence of something is one of the meanings of negation. It has got various other meanings also.

Lastly, the grammarians have accepted six meanings of negation of which 'absence' is one. It has been stated by the grammarians "*Tatsādṛsyamabhāvaśca tadanyatvaṁ tadalpatā, aprāśastyaṁ virodhaśca nañarthā ṣaṭprakīrtā*". That is, negative particle '*nañ*' can be used in the sense of similarity i.e., *Sādṛśya* (e.g., *abrāhmaṇa* meaning similar to Brāhmaṇas), in the sense of absence i.e., *abhāva* (i.e., *asat* meaning the absence of honesty), in the sense of mutual difference i.e. *anyatva* (e.g. *aghaṭa* meaning different from a jar), in the sense of less quantity i.e., *alpatā* (e.g. *akeśī* meaning less quantity of hair, in the sense of non-suitability (*aprāśastya*) (e.g. *asamaya* meaning improper time) and in the sense of enmity or contradiction (*virodha*) (e.g. *asura*) meaning the enmity with the duties (*suravīrodhi*). "*Tatsādrśyamobhāvaśca tadanyatvaṁ tadalpatā/aprāśastyaṁ virodhaśca nañarthāḥṣaṭ prakīrtitā*". Among these six meanings the second one can be applicable in the case of *Advaita* meaning the absence of duality as told earlier. Unless we have an idea of duality we cannot prove its absence. It is sensible to negate something, which really exists in this world. To negate something which does not exist in this world is non-sensical. It is very much a futile exercise to prove the non-existence of an object, which is absurd. This phenomenon is called *alīkapratiyogikābhāva* i.e., an absence, the absentee of which is an absurd entity. If phenomenal existence of duality is an absurd entity, the negation of it is meaningless leading to a futile exercise. To prove the absence of duality (*advaita*) 'duality' has to be accepted as an existent object. In order to know this duality the different means of knowing (*pramāṇa*) are very much relevant. The realisation of the absence of duality follows from the realisation of Brahman.

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CHAPTER - 3

Methods of *Reductio - ad - absurdum* and *Tarka*

'*Redictio - ad - absurdum*' is a mathematical concept, which has accepted as a philosophical method by the Indian and Western Philosephers. In the mathematical logic Copy has introduced this method, which is otherwise known as indirect proof. This indirect proof is nothing but the proof by *reductio - ad - absurdum*. It is said in the mathematical logic that the negation of the conclusion is deliberately taken and from this it is shown that, if it is taken for granted, it will lead to some contradiction or absurdity. Hence this assumed conclusion is abandoned and the original conclusion is established as true. "The method of indirect proof often called the method of proof by *reductio - ad - absurdum* is familiar to all who have studied elementary geometry. In deriving the theorems, Euclid often begins by assuming the opposite of what he wants to prove. If that assumption leads to a contradiction or reduces to an absurdity then the assumption must be false and so its negation – the theorem to be proved, must be true." - (Copy : *Symbolic logic* 5th edition). This method is also found in Indian logic where mainly the Naiyayikas (both earlier and latter) have adopted to prove certain conclusion. Though there was no interaction between two traditions, some striking resemblances are found between them. This method is described by the Naiyayikas as many ways like *Vipakṣavādhaka tarka*, *āhāryajñāna*, *aniṣṭaprasanga, aniṣṭa āpatti* etc. In the following pages we shall see the nature and logical flavour of this method as accepted in Indian Philosophical systems specially in *Nyāya*.

A problem may be raised how one can think of "knowledge produced through desire' (*icchājanyajñāna*). A solution to this problem may be offered in the following way. Let us look towards the exact nature of *āhāryajñāna*. The knowledge, which is produced out of one's own desire at the time when there is the contradictory knowledge, is called *āhāryajñāna*. (*Virodhijñānakālīnecchāprayojyajñānatvaṃ āhāryajñānatvam* or '*Vādhakālīnecchājanyam jñānam*')[1]. The word '*āhārya*' means 'artificial', which is found in the *Bhaṭṭikāvya* where the ladies are described as *āhāryaśobhārahitairamāyaiḥ*[2] (that is, free from artificial beauty). From this, it follows that the word *anāhārya* means 'natural' which is expressed by the term '*amāyaiḥ*'. When we talk of *āhārya*-konwledge, it has to be taken as an artificial knowledge on account of the fact that between two objects an object is *deliberately* thought as otherwise in spite of knowing the distinct character or real nature of these two objects. In these cases one's desire of thinking an object as otherwise acts as an instrument (*icchājanya*). It is to be borne in mind that the Navya Naiyāyikas have given much importance on *vivakṣā* (that is, will to say). Let us put forth some cases where we find a knowledge produced through the instrumentality of desire (*icchājanyajñāna*). One is allowed to say *sthālī pacati* (he cooks with clay-pot) with the nominative case ending to the pot instead of the correct expression '*sthālyā pacati*, with the instrumental case ending with the word *sthālī* if one so desires.

Apart from these there are a few cases where we find knowledge attained through the instrumentality of desire (*icchājanya*) as in the case of *pakṣatā*. If someone bears a strong desire to infer (*siṣādhayiṣā*), he can infer in spite of having *siddhi*. ('*siṣādhayiṣāsattve'numitirbhavatyeva*')[3]. It is permissible as the Naiyāyikas believe in the theory of *pramāṇasamplava* (that is, capability of applying various *pramāṇas*) to ascertain an object. According to this theory, 'fire' which is perceived can be inferred if someone so desires. That a cloth is completely different from a jar is completely known from the perception and hence there is not at all any necessity to infer a cloth as distinct from a jar. In spite of this one is found to infer : 'It (that is, a cloth) is endowed with the mutual absence of a jar, as it has got clothness' (*ghaṭānyonyābhāvavān paṭatvāt*). All these cases are supportable as an individual desires to do so and hence the role of *icchājanyatva* in the attainment of knowledge cannot be denied. But it should be clearly borne in mind that all *icchājanya* – inferences or knowledges – are not *āhārya*. The *icchājanya – jñāna* as found in the case of *rūpaka* and *tarka* are the instances of *āhāryajñāna*. From the above-mentioned cases it is proved that desire may act as the instrument of knowledge, which is called *icchājanyajñāna*.

Another problem may be raised how the concept of *āhāryajñāna* can be accommodated in *Nyāya* as the sentence conveying such cognition has no *yogyatā* or semantic competency. It may seem strange to us as to why such artificial nature of knowledge is at all essential in the context of *Nyāya*. Though there is no direct result of the deliberation of such artificial knowledge due to not having semantic competency (*yogyatā*), it plays a great role in pointing out the exact nature of an object *indirectly*.

The importance of accepting *āhāryajñāna* can be realized easily if we ponder over the importance of *tarka* as a philosophical method. *Tarka* is nothing but an *āhāryajñāna*, which is evidenced from the definition given in the *Nīlakanṭhaprakāśikā* on *Dīpikā* '*Āhāryavyāpyavattābhramajanya āhāryavyāpakavattābhramastarkaḥ*'[4] That is, *tarka* is an imposed (*āhārya*) erroneous cognition of the existence of a pervader (*vyāpaka*) which is produced by another imposed erroneous cognition of the existence of a *vyāpya*. If the knowledge in the form – 'There is fire in the lake' (*hrado vahnimān*) is produced out of one's desire at the time where there is the awareness of the contradictory knowledge in the form – 'there is the absence of fire in the lake' (*hrado vahnyabhāvavān*), it is called *āhārya*. In this case erroneous cognition is deliberate which is not found in ordinary illusion.

The main purpose of accepting *āhāryajñāna* is to ascertain the true nature of an object (*viṣayapariśodhaka*) and to remove the doubt of deviation (*vyabhicāraśamkānivartaka*). The *āhāryajñāna* existing in the former type – 'If it has no fire, it has no smoke (*Yadyam vahnimān na syāt tadā dluīmavān na syāt*) ascertains the existence of fire in a particular locus. In the same way, the Navya Naiyāyikas have accepted another form of *tarka*, which is also *āhārya* in order to eliminate one's doubt of deviation (*vyabhicāraśamkā*). If someone bears a doubt whether smoke and fire have an invariable relation or not, this doubt of deviation (*vyabhicāraśamkā*) can be dispelled by demonstrating the *āhārya* knowledge in the form : 'If smoke be deviated from fire, it will not be caused by fire', (*dhūmo yadi vahnivyabhicārī syāt tarhivahnijanyo na syāt*). From this it is indirectly proved that as smoke is caused by fire, it will not be deviated from fire.[5]

By virtue of being *āhārya* both the parts – the ground (*āpādaka*) and consequent (*āpādya*) are imaginary or hypothetical. If the first part is true, the second part would become automatically true. But it is a well – known fact that the second part is not true in so far as we do not get any smoke, which is not caused by fire. So, the doubt as to the deviation of fire with smoke can be removed by applying the *tarka* in the form of *āhārya*. It, being a kind of mental construction, is useful for removing doubt and hence it becomes promoter to *pramāṇas*. This *āhārya* cognition is otherwise called *aniṣṭāpatti* or *aniṣṭāprasanga*, that is, introduction of the undersired through which the desired one is established. This imposition of the undersired is of two types :

rejection of the established fact and the acceptance of the non-established object (*Syādaniṣṭam dvividham smṛtam prāmāṇikaparityāgastathe-taraparigrahaḥ*). If there is an *āhāryajñāna* in the form – 'water cannot quench thirst', there would arise an objection – 'If it is so, no thirsty people should drink water'. It is known from our experience that water is capable of quenching thirst, which is denied here and hence it comes under the first type of *aniṣṭa*.

If it is said that water causes burning, there would arise objection in the form – 'If it is so, the drinking of water would cause a burning sensation. The burning sensation from water is not an established fact, which is admitted here and hence it belongs to the second type of *aniṣṭa*. We often take recourse to *āhāryajñāna* even in our day-to-day debate. If an opponent says to a Naiyāyika that self is non-eternal (*anitya*), he may first agree with what the opponent says in the following manner – 'O.K., initially I agree with you that self is non-eternal'. This agreement for the time being is *āhārya* and the next step in the form – 'If self were non-eternal in nature, there would not have been the enjoyment of *karma*, rebirth or liberation due to the destruction of the self', is also *āhārya* which indirectly points to the eternality of self. In the same way, various expressions like 'If I were a bird, I would have flown from one place to another', 'If you were a firmament, I would have stretched my wings like a crane' (which reminds me of a Bengali song – *Tumi ākāś yadi hate āmi balākār mato pākhā meltām*) can be included under *āhāryajñāna*.

The accommodation of *āhāryajñāna* in Navya Nyāya is primarily to promote an indirect method through which truth is ascertained. In the indirect proof in symbolic logic the negation of the conclusion is deliberately taken which is also an *āhārya* and from this it is shown that, if this is taken as conclusion, it will lead to some contradiction or absurdity as said earlier. It the negation of P which is originally a conclusion is taken as a conclusion of *āhārya* – type and proved it as contradictory or absurd, it will automatically follow that the original conclusion, that is, P (*anāhārya*) is true. The method is also called the method of proof by *reductio ad absurdum*.[6]

In metaphorical expressions such *āhāryajñāna* bears a completely different import. *Rūpaka* remains in the representation of the subject of description, which is not concealed, as identified with another well-known standard (*rūpakam rūpitāropād viṣaye nirapahnave*.)[7] In the famous case of rūpaka – *mukhacandra* the *upameya* is 'face' which is identified with 'moon'. In this case, the distinction between these is not concealed in spite of having excessive similarity. Though the difference between them is not concealed yet there is the ascription of the identification between two objects (*atisāmyāt anapahnutabhedayoḥ upamānopameyayoḥ abhedāropaḥ*). In spite of knowing the distinction between *upamāna* and *upameya*, there is the hypothetical ascription of identity deliberately, which is also an *āhārya*.[8]

From the above discussions, it is known to us that the accommodation of the *āhāryajñāna* presupposes some intention of an individual. In the case of metaphor, *āhāryatva* is taken recourse to in order to show the extreme similarities between two objects. In the same way, *āhāryajñāna* is accepted by the logicians to ascertain the real nature of an object indirectly. Hence *āhāryajñāna* can be utilized as an accessory to a *pramāṇa* (*pramāṇānugrāhakarūpeṇa*). Though semantic competency (*yogyatā*), the criterion of the meaningfulness of a sentence, is not found in the sentences conveying *āhāryajñāna*, meaning of such sentences is easily understood by others. Had these been not understood at all, the absence of *yogyatā* cannot also be known. Moreover, as there is semantic incompetency, a search for other indirect or secondary meaning is permissible. As there is the absence of *yogyatā* in the expressions like *mukhacandra* and 'If I were a bird, I would have flown', etc., a thorough search for indirect meaning like extreme similarity (*atisāmya*) between face and moon, the absurdity of describing a man as bird, etc. have to be ascertained. It is to be kept in mind that the semantic competency is essential only in the case of direct meaning (*śakyārtha*) but not in implicative or suggestive meaning (*lakṣyārtha* or *vyaṅgyārtha*). In fact, an implicative or suggestive meaning is looked for if there is the incompetency among the words (*mukhyārthatādhe*). Hence the semantic incompetency paves way to the indirect meaning as found in the expressions like 'I am building castles in the air', etc. Following the same line it can be said the *āhāryajñāna* can communicate something to us indirectly in spite of not having the said competency.

Can we speak of *āhāryajñāna* existing in the pure music or *rāgas*, pure dance or abstract paintings that are new worlds created through imagination ? In response to this, the following suggestion can be made. Though *āhāryajñāna* is a product of imagination, all imagination cannot be taken as *āhāryajñāna*. The imaginary ideas as found in the fanciful stories or fairy tales, etc., are not *āhārya*. Some imagination is created out of one's own will (*icchāprayojya*) at the time when one is conscious of the contradictory knowledge (*virodhijñānakālīna*). In spite of being conscious of the fact that fire cannot stay in the lake, we imagine that the lake has fire out of our strong will. It is a case of *āhārya* as already mentioned. In the case of pure music, dance and abstract paintings, we are not aware of the contradictory knowledge (*virodhijñāna*) through which the imaginary states are sublated (*bādhita*). Though these are the cases of imagination having the characteristic of *icchāprayojyatva*, or *icchājanyatva*, they are not *āhāryajñāna* due to the lack of the other characteristic, that is, *virodhijñānakālīnatva* or *vādhakālīnatva*. In the case of *āhāryajñāna* both the characteristics should be taken as adjuncts of imaginations. An imaginary cognition associated with *icchāprayojyatva* or *icchājanyatva* and *virodhijñānakālīnatva* is called *āhārya*. Due to the absence of the second characteristic the charge of *avyāpti* of the definition of *āhāryajñāna* to the pure music, etc., does not stand on logic.

I propose to consider some problems relating to the concept of *Tarka*. As a mode of apogogic proof the concept has been ubiquitous in diverse philosophical persuasions in India. However I shall have my considerations focused on Vātsayanas's view on the matter. I will be defending in position of. I shall consider in the context the formūlations of the later thinkers of the Nyāya School.

Let me begin by quoting Gautama's definition of *Tarka* in translation. *Tarka* is a kind of knowledge or deliberation, which is applied for the purpose of determining the right knowledge of an object whose nature is known roughly, but not specifically after pointing out some causes in favour of it. ("*Avijñātetattve*'rthe *kāraṇopapattitaḥ tattvajñānārthamuhastarkaḥ*".)[9]

Tarka is intended to reveal the right knowledge of an object. It cannot be employed for revealing an object, which is unknown. That's why, the term '*avijñāta*' is not introduced in the definition. The '*tattva*' as conjoined with *avijñāta* points to the fact that the object which, though vaguely known, is not known as it really is (*tattva*) may be known through *Tarka*. In order to indicate that in case the *tattva* of an object is hitherto unknown or which is not known as such could be known though *Tarka*, the term '*artha*' is inserted in the definition. This deliberation must be supported with the justification or ground in favour of a particular conclusion (*kāraṇopapattiḥ*).

In respect of an object not known properly an inquiry arises in the cogniser. He may laterly became confused in seeing the existence of two contrary characteristics in the same object of inquiry. But finally he removes his doubt by ascertaining one of the characteristics of the object on the strength of some proofs favouring one of the alternatives. In other words, the individual knower has to get proof in favour of a particular alternative, which eliminates the other.[10]

Let us try to understand the process of reasoning following Vātsyāyana with the help of an example. A person desirous to know the real nature of the self or the knower may be in doubt expressible in the form 'Whether it possesses the properties of something which is produced or those of something which is not produced'. How to eliminate one of the alternatives ? In order to show the method Vātsyāyana indicates that the potential cogniser proceeds to eliminate one alternative by applying some arguments of the following form. He thinks that, if the self possesses the properties of something not produced, which is otherwise called eternal, it can enjoy the result of *karma* performed in the previous birth. In Nyāya it is held that among suffering, birth, inclination, evil and false knowledge each of the succeeding one causes the preceeding one and the cessation of the succeeding one leads to the cessation of the preceeding one, and this indeed is the state of liberation. Accordingly, the knower would have got into both transmigratory as well as liberated states.[11] If the self, on the other hand, is taken as possessing the properties of the produced, it will not have these. For, the knower after being produced becomes associated with the body, the senseorgans, happiness, and miseries etc. on account of which he does not have any scope for enjoying the result of *karma* done by him, as he is non-eternal in nature. The knower, of course, does not exist before his coming into being. The knower who did not pre-exist or who is completely annihilated at death is not capable of enjoying the fruits of his *karma*. As the knower is non-eternal, like other non-eternal objects, he has no existence before his coming into being and then he is completely annihilated at the destruction of his body. If this be the case, the relation of a knower with more than one body and the absolute cessation of body i.e., absolute cessation of birth would be impossible, leading to the impossibility of liberated and transmigratory states.

In other words, the non-eternal knower cannot be associated with more than one body for enjoying the remaining result of *karma* and he would not be free from being birth forever. If the knower were eternal, he would be associated with many bodies and would be able to enjoy the result of *karma*. For that matter he might be liberated after certain period. Hence, if the knower in the sense of self is taken as possessing effectual properties, he would never enjoy the result of *karma* and liberation. This alternative i.e., the self as possessing the causally brought about properties cannot be taken as granted due to the absence of the proper ground mentioned above. This type of argumentation or this method of elimination is called *Tarka.*[12]

Vātsāyana describes the method as a promoting to the ascertainment of right knowledge, but it is not right knowledge itself. Because, *Tarka*, after pointing out some grounds, asserts one of the alternatives, but it does not point out this alternative definitely as having such and such characteristics. In other words, though accessory to the attainment of right knowledge, it does not definitely assert a particular alternative in the form : 'This object is of such nature'. The main characteristic features of the object are not deliberated through this method and hence it is not right knowledge itself.[13]

It is accessory to the right knowledge because, it, after pointing out some grounds in favour of the ascertainment of the right knowledge of an object i.e. correct alternative, becomes promoter to the *Pramāṇas*. As *Pramāṇa* is associated with *Tarka*, its power is enhanced and thus the enhanced power becomes helpful for the revelation of the right knowledge (*tattva*).[14] In the context '*tattva*' means 'thatness' i.e., to know an object as it really is. In other words, the positivity of the positive and the negativity of the negative entity may be described as *tattva* i.e. the absolute sameness.[15] This real nature of an entity is revealed through *Pramāṇa* associated with *Tarka*, the promoter.

From the Vātsāyana's analysis it is found that *Tarka* is generally adopted by an individual who inquires into the nature of an object not known properly and who is in a confusion there being two contrary properties in an object (arising out of not having proper knowledge of the object). At this point Vātsāyana suggests that inquiry comes first and there arises confusion about the nature of the objects. The confusion as to its nature prompts an individual to employ *Tarka* so that the confusion may be removed by way of having *tattvajñāna*. A problem may arise in this regard. It is not always true that confusion follows from the inquiry into right knowledge. But sometimes inquiry into right knowledge follows as well from confusion. Inquiry about an object follows if the object is confusedly apprehended or if there be any necessity to know it. Vācaspati Mishra in his *Tātparyaṭīkā* and *Bhāmatī* on *Adhyāsabhāṣya* of Samkara has remarked that inquiry into an object is permissible if there is *sandigdhatva* (confusion) regarding the nature of an object and *saprayojanatva* (having the need) for knowing it. In other words, here the properties like '*asandigdhatva*' and '*saprayojanatva*' leave no room for inquiry (*jijñāsyatva*). The properties are called *vyāpakas* while *jijñāsyatva* is called *vyāpya* on account of the fact that the relation between them can be described as '*Vyāpya-Vyāpakabhāva*'. For where there is *sandigdhatva* and *saprayojanatva*, there is *jijñāsyatva*. If a jar is seen in broad daylight and if our senseorgans and mind are connected, further inquiry about the object is not permissible because it is *asandigdha* i.e. not subject to confusion. If somebody asks 'How many teeth a crow possesses', (*kākasya kati dantāḥ*) there should be no need for further inquiry as the case is one of '*saprayojanatva*'. Hence, any type of inquiry presupposes confusion regarding the nature of an object, apart from the need of knowing it.[16]

In connection with the explanation of *Tarka* in the *Nyāyasūtra* Vātsyāyana appears to give priority to inquiry, which precedes confusion.

The problem may be solved in the following manner. Vātsyāyana's position may be justified if we ponder over the theory. Though in most cases inquiry arises out of one's confusion, it cannot be denied that sometimes curiosity or inquiry may arise in one's mind spontaneously. Now curiosity may lead one into confusion as to the nature of the object to be known. That is, a man who aspires to know would endeavour to know the nature of an object. If the object apparently possesses two contrary features, he will be in confusion. This confusion prompts to apply the method of *Tarka*. At this point confusion again gives rise to a second order of inquiry in that form : 'This object is of this type or that type'. This sort of inquiry prompts the knower to resort to one more *Tarka*. Hence, Vātsyāyana need not be taken as contradicting the view of Vācaspati Miśra. What Vātsyāyana has been trying to point out is that there are two types of inquiry of which the second order one is the reason for the application of *Tarka*. If someone begins the process of knowing with confusion and then inquiry is of first order, Vātsyāyana makes the remark intentionally in order to include both first order and second order levels of inquiry as providing the reason for employing *Tarka*. This may again be substantiated by the fact that we may sometimes have the superficial knowledge of an object. But if the specific knowledge of the object is to be acquired, another type of inquiry becomes necessary, which may be designated as a second order inquiry. Any type of inquiry either of first order or of second order is the reason for application of the method of *Tarka*. Hence, Vacaspati's view that confusion gives rise to inquiry is not rejected by Vātsyāyana. Cognitive inquiry sometimes assumes the first order or sometime the second order form or status. Both of them are preceded by earlier state of confusion.

In connection with the explanation of the Vātsyāyana's view on *Tarka*, it would not be uncalled for if Vacaspati Misra's and Nāvya Naiyāyikas views are put forward for a better appreciation of the Vātsyāyana's statement regarding *Tarka*. Vacaspati says that one cannot know an object through *Pramāṇa* if there arises any doubt. *Pramāṇa* cannot be applied as long as the doubt of illusion is not dispelled through *Tarka* in the form of *aniṣṭāpatti* (imposition of the undesired). After the removal of doubt, *Pramāṇa* can reveal the object and hence *Tarka* is called as an accessory to *Pramāṇa*.

It has been stated in the *Bhāṣya* that *Tarka* is to be applied when object is confusedly apprehended as having the existence of two contrary characteristic features. Ultimately this doubt is removed through *Tarka*, which eliminates the other possibilities.

If such be the case, another problem may be raised. So far as the elimination of doubt through *Tarka* is concerned, we are adopting *Tarka* in each and every case of knowledge. We are going on eliminating one object from another following this process of elimination. When the knowledge of a cow is attained, the cow is eliminated, though unconsciously, from the 'non-cow'. We are unconsciously following the methodology of the *Tarka* in the form : "If this cow were horse etc., it would not have possessed such characteristics existing in a cow'. From this *aniṣṭāpatti* we draw our conclusion in the form : 'As this cow does not possess the characteristic features of a horse etc. this animal is cow'. In this way, each and every piece of knowledge is the outcome of *Tarka* though we are not always aware of the technicalities of the method. That is the reason why the Buddhists have laid greater emphasis on the concept of *apoha*. It may be recalled that Rāmanuja has explained the term '*apohana*' found in the *śloka* of the *Bhagavadgītā.*[17] as *Ūha* or *Tarka*. Venkaṭanātha in his *Nyāyapariśuddhi* has admitted the above-mentioned meaning of the term *Tarka* and has referred to Rāmanuja's view.[18]. From considerations as above it follows that *Tarka* has a wider perspective. It may be said to have a use in each and every case of knowledge, not alone the object in confusion. Why had Vātsyāyana laid so much of emphasis on the fact that *Tarka* is to be applied whenever an object is in some confusion (*avijñātatattva*)? From the foregoing analysis it is found that we are ever applying *Tarka* even when the object is known. In other words, it automatically comes to our mind that the known object i.e. 'jar' is different from 'non-jar'. That 'jar' is different 'non-jar' is known on the strength of the knowledge of the characteristic features of a jar as well as 'non-jar'. So Vātsyāyana's concept of *Tarka* surely have been much more wider. That *Tarka* is needed for revelation of the object about which we have no specific knowledge is to be taken as a restricted function of *Tarka*, as suggested by Vātsyāyana. That would be too inadequate.

In response to the above-mentioned problem one solution may be offered to strengthen Vātsyāyana's position. It is true that we go on eliminating when we attain knowledge of an object. Though it is done spontaneously, it would be too much to give justification for knowledge of an object, which is not at all in confusion. If it were not in confusion, what is the use of providing *Tarka* (in a demonstrative way) for the justification of its knowledge ? To provide justification or proof for the object which is already established gives rise to a logical defect called *Siddhasādhana*. Though this method of elimination is adopted unconsciously, the intellectual demonstration of the method gives rise to the defect mentioned above, as this attitude is nothing but an effort to prove the object already established. Keeping this in view Vātsyāyana has emphasized that *Tarka* is to be applied in an object which is not specifically known. This view of Vātsyāyana is strengthened when he says that argument is to be put forward in the case when the object is neither ascertained nor unknown (completely) but in confusion.[19] This theory is applicable in any type of argumentation, not only of *Tarka.*

The form of *Vyāpti* is : where there is deviation of fire, there is the negation of being a product of fire (*yatra yatra vahnivyabhicāritvam tatra tatra vahnijanyatvābhāvaḥ*). In this form of *Vyāpti* the first part is *Vyāpya* (pervaded) and the second one *Vyāpaka* (pervader). In the same way, it can be said that the *Āpādaka*-part is the pervader and *Āpādya*-part is pervaded. So invariable concommitance or *Vyāpti* is included in *Tarka.* In order to remove doubt about the existence of *Vyāpti* determined by *āpādya* and existing in *āpādaka* in the form : "whether *āpādaka* is pervaded by *āpādya* or not" (*Āpādaka āpādyavyāpyo na vā*) in this *Vyāpti*, the necessity of applying another *Tarka* will arise. In this *Tarka* there is another *Vyāpti.* In order to remove the doubt of the above-mentioned form existing in this *Vyāpti* also, another *Tarka* will have to be resorted to and in this way the defect called 'Infinite Regress' (*anavasthā*) would crop up.[20]

The above-mentioned view is not tenable. For the doubt of deviation does not arise in *Vyāpti* of a *Tarka*, for it would involve contradiction (*vyāghāta*) in respect of one's own activity and hence, the necessity of another *Tarka* does not arise at all.

One can doubt so long as there does not arise any contradiction in respect of one's own ptactical activity. A man is not permitted to bear any doubt about *Vyāpti* between smoke and fire, as he seeks fire in his practical life to get smoke without any hesitation. If he has a slightest doubt regarding *Vyāpti* between smoke and fire, he would not seek fire for having smoke. If there is any doubt, it will contradict his own activity. In this way, it can be said that a man takes food to satisfy his hunger and takes recourse to words to make others understand his desire etc. So, one's own activities indicate the absence of doubt in them. Moreover, if we go on doubting, our doubting would be subject of doubt. So, each and every case is not the subject of doubt.[21]

It is found that *Tarka* is to be applied when there is doubt of deviation, but not in all cases of inferences. In some cases inference is possible without any *Tarka*. The baby is found to move on to suck mother's breast without turning to other objects. The reason behind this inclination of a newborn baby is the knowledge of its conduciveness to the desired object (*iṣṭasādhanatājñāna*). The reason behind its absence of inclination to other objects is the knowledge of their conduciveness in gaining objects that are not desired (*aniṣṭasādhanatājñāna*). How does a baby come to know of the conduciveness to the desired object ? As the baby has got no scope for experiencing conduciveness to the desired object in this life, it is assumed that in the previous birth he had acquired the knowledge of *Vyāpti* in the form : "Where there is the means for the maintenance of my life, there is the means of attaining my desired object" (*yatra yatra majjīvanarakṣopāyatvam tatra tatra madiṣṭa-sādhanatvam*). The impression of the knowledge of *Vyāpti*, which was experienced in the previous birth, remains in the soul of a newborn baby. After the awakening of the impression i.e. *samskāra*, the baby attains the knowledge of *Vyāpti* which gives rise to the inference.[22] In this inference there is no scope for applying *Tarka* (*Reductio-ad-absurdum*), as he bears no doubt about the efficacy of sucking mother's breast and hence, there would not arise the defect of *Anavasthā* (*Infinite Regress*).[23]

It has been shown that as the child has got the knowledge of *Vyāpti* without the help of any *Tarka*, it (*Tarka*) cannot be the cause of ascertaining *Vyāpti*. As the knowledge of *Vyāpti* is possible without taking recourse to *Tarka*, there is the violation of the rule 'the method of agreement in absence.' Here, the effect i.e. the ascertainment of *Vyāpti* is present while the proposed cause (i.e. *Tarka*), is absent. So, *Tarka* cannot be the cause of ascertaining *Vyāpti*,[24] but it may help in removing the doubt of deviation existing in an inference.

In this way the Indian as well as Western Logicians apply the method of *Reductio-ad-absurdum* or *tarka* to determine the real cause on real nature of an object, which originally belongs to geomentry.

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[3] *Siddhāntamuktāvalī* on verse no. 70.

[4] *Nīlakaṇṭhaprakāśikā* on Dīpikā on Tarkasamgraha, p.376, edited by Satkari Sharma Bangiya, with seven commentaries, Chowkhamba, 1976.

[5] *Tattvacintāmaṇi* (Anumānakhanda), Gangeśa, Vyāptigrahopāyah chapter.

[6] *Symbolic Logic* (4th ed.), Irving M. Copi, Macmillan, London, 1973, p. 53.

[7] *Sāhityadarpaṇa*, Chapter X, edited by Haridās Siddhāntavagīśa, p. 630. 1875 (B.S.).

[8] *Kusumapratimā* on *Sāhityadarpaṇa*, Chapter X, edited by Haridās Siddhāntavagīśa, 1875 (B.S), p. 621.

[9] *Nyāyasūtra* 1. 1. 40.

[10] *Vātsyāyanabhāṣya* on 1. 1. 40.

[11] *Ibid.*

[12] *Ibid.*

[13] *Ibid.*

[14] *Ibid.*

[15] *Vātsyāyanabhāṣya* 1.1.1.

[16] *Bhāmatī* on *Adhyāsabhāṣya*.

[17] *Śrīmadbhagavadgītā* 15/16.

[18] Phanibhuṣan Tarkavāgīśa : *Nyāyadarśana* Vol. 1, P. 353 W.B.B.B.

[19] *Nyāyabhaṣya* on 1.1.1.

20 "Nanu tarko'pyavinābhāvamapekṣya pravartate, tato'navasthayā bhavitavyam," *Nyāyakusumāñjali*, Chowkhamba, p. 345, Henceforth, N.K. " Śamkāyā avadhistarkaḥ, tannivartakatvāt, Nanu tarko'pi vyāptimūlakatayā tarkāntarāpekṣāyāmanavasthā..."
Prakāśa Commentary on *Nyāyakusumāñjali*, Chowkhamba, p. 342 Henceforth Prakāśa on N.K.
Tarkasya vyāptigrahamūlakatvenānavastheti cet."
Tattvacintāmani (vyāpigrahopāya ch) henceforth : T.C.

21 "Tarkamūlavyāpatu svakriyāvyāghātena, vyabhicāraśamkaiva nodetīti na tatra tarkāpakṣetyarthaḥ." – *Prakāśa* on N.K., p. 342.

22 "Yadi hyanvayavyatirekāvadhṛtakāraṇabhāvam kāraṇam vinā kāryotpattim śamkyeta, tadā niyamena dhūmārtham vahnestṛptyarthamannasya parapratipattyartham śabdasyopādānam na kuryāt, tairvināpi teṣām sambhāvāt. Yasmāt tadupādānameva tādṛśaśāmkāpratibandhakam."
Prakāśa on N.K., p. 342
Raghunath Ghosh : "Certain ambiguities and clarifications in Professor Mohanty's 'Gangesa's Theory of Truth".
- *The Visvabharati Journal of Philosophy*, 1980-1982 (Combined Vol.).

23 Evam Śarīrasya caitanye bālakasya stanyapāne pravṛttirṇa syāt, iṣṭasādhanatājñānasya taddhetutvāt, tadānīmiṣṭasādhanatāsmārakābhāvāt. Manmate tu janmāntarānubhūteṣṭasādhanatvasya tadānīm smaraṇādevapravṛttiḥ, - *Siddhāntamuktāvalī* on Verse No. 48.

24 "Jātamātrasya pravṛttinivṛttihetutvānumitijanakavyāptijñānam tarkam vinaiva..."- T.C. (Vyāptigrahopāya Chapter).

CHAPTER - 4

The Concept of Śūnya or Zero

It is stated by L. Hogben that the intention of *śūnya* of 'Zero' liberated the human intellect from the prison bows of the counting frame. The invention of Zero is an wonderful thing in ancient Mathematical literature. Though there is the diversity of opinion regarding the date and time of the invention of '0' and agency of such invention, it is an established fact that the ancient thinkers were completely aware of its usages. Professor Hallsted said – "The importance of the creation of the zero mark can never be exaggerated. This giving to airy nothing, not merely a local habitation and a name a picture, a symbol, but helpful power is the characteristic of the Hindu race whence it sprang".

All the Western thinkers have agreed that the Indian thinkers had used it since a long time. In the fourth century B.C. Kautilya had used the term *śūnya* in his *Arthaśāstra*. The term like *śūnya-niveśana, śūnya-sthāna* etc. are found in use in his book. The Indian Mathematicians had been using the term *śūnya* in order to convince the 'unknown number' (*ajñātarāśi*).

Moreover, the use of *śūnya* is found in the literary pieces like *Vāsavadattā* of Subandhu, *Kādambarī* of Bāṇabhaṭṭa and in the *Naiṣadhacarita* of Śrīharsa etc.

Brahmagupta in his writing called *Brahmasphuṭasiddhānta* has given a clarified view on *śūnya*. It is stated by Āryabhaṭṭa that, if zero is conjoined with some number, the value of the number remains unchanged. This is also true in the case of subtraction. If some number multiplies zero, the result will be zero. It is said by Brahmagupta – "*Dhanayordhanamṛṇamṛṇayoranantaram śamakai kham/Ṛṇamaikyam ca dhanamṛṇa - dhanaśūṇyayoḥ śūṇyayoḥ śūṇyam//*" That is, the positive numbers if added with another positive numbers, the result would be positive. If otherwise, it would be negative. If zero is added to a positive number, the number will remain same or unchanged. If zero were added to a negative number, the result would be unchanged equally. If two zeros are added the result will be of zero. If this theory is explained in terms of modern mathematical idioms, it would be as follows :

$$a - a = 0; \qquad a + a = a; \qquad -a + -0 = -a; \qquad 0 + 0 = 0$$

In the case of multiplication there are some rules. If zero is multiplied with a positive number the result will be zero. If a negative number multiplies it then also the result is zero. If zero is multiplied by zero, it is also zero ($a \times 0 = 0$; $-a \times 0 = 0$; $0 \times 0 = 0$). The original *śūtra* runs as follows "*Śūṇyanayoḥ ṛdhanayoḥ ṛśūṇyayoh vā vadhaḥ śūṇyam.*"

Brahmagupta has laid some principles also. If zero is divided by zero, the result will be zero. If zero divides a positive or negative number, the result will be zero. The original *śloka* runs as follows

"*Dhanabhaktam dhanamṛṇahṛtamṛṇam*
dhanam bhavati kham khabhaktam kham/
bhaktamṛṇena dhanamṛṇam dhanena
hṛtamṛṇamṛṇam bhavati//"

Achārya Bhāskara thinks that if a number is divided by zero the value of the number will remain the same, which is technically called *kha-hara*, i.e. a/0 = *kha-hara*. If some number is added to zero, the result remains unchanged as admitted by Bhāskara also. We had hinted to great Indian theory in this context. He said that the origination and destruction of innumerable individual beings are occurring in god having infinite power of omnipotent. With this origination and destruction of innumerable beings the Omnipotent god is not affected. In other words, with this he remains unchanged. From this it is known that, if something is added or subtracted to the Almighty, He remains unchanged. It can be understood with the help of the metaphor of ocean, which is also unbound. If water of a jar is poured on the ocean, it remains unchanged, only because ocean is unbound and unmeasured, which is described by the *Upaniṣadic* seers as

'*Pūrṇa*' or 'full'. If something is full or *pūrṇa*, it is not possible to affect its holistic character. It is said in the Upaniṣad "*Oum pūrṇamadaḥ pūrṇamidaṁ pūrṇāt pūrṇamudacyate/Pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyate.*" (*Bṛhadāraṇyaka Upaniṣad, Shantimantra*). That is, something is added to *Pūrṇa* or subtracted from *Pūrṇa*, it remains as *Pūrṇa*. Even if *Pūrṇa* is taken away from *Pūrṇa*, the *Pūrṇa* entity remains unchanged or *Pūrṇa*. It can be symbolically represented as follows :

Pūrṇa + a = Pūrṇa; Pūrṇa – a = Pūrṇa; Pūrṇa - Pūrṇa = Pūrṇa.

From the above it can be said that almighty or omniscient or omnipotent being is *Pūrṇa* and hence all limited objects either added or subtracted to Him cannot affect Him at all. Moreover, can we really know the definite character of such *Pūrṇa* entity ? The entity, which is capable of being perceived or visualized, can be described. Something which is not capable of being seen or known and which has no colour, shape, size etc. can never be described, because it is not possible to get it's holistic character. If at all He is described, it either partial nature or a human being's imagination which has no reality at all. In this sense He may be described as *śūṇya*, because no description is adequate to catch hold of His real nature. That is why, the Buddhists argued that is true in any case of Reality, which is linguistically indescribable and hence each and every real object is empty or *śūṇya*. The Buddhists admit that each and every entity is relative and hence it is empty having no particular character of its own.

For the sake of attaining the state of abiding in the illimitable the Buddhists admonish that each and every person should realize the essenceless character of objects, it is which is technically called *Śūṇya*. When the knowledge of silver occurs in the place of a sea shell, it is sublated by the subsequent knowledge and what is revealed to us is that the silver was not at there. In a like manner, the awareness that no real silver etc. are met with the states of waking and dreaming counters the knowledge of silver etc. Had there been a piece of real silver, there would have been the true knowledge of the action of seen, the substratum of the illusory silver i.e., the shell referred to by the term 'this' (*idam*), the superimposed property i.e., silverness and the relation of inherence between silverness and shell ness.[1] Actually it is not taken as true. For, if the illusory silver were present, all these would have been real. All of us are of the opinion that the perception of silver of a seashell is illusory. It cannot be said that these things are partially real and partially unreal, because it is unthinkable to assume an object, which is real as well as unreal. In the case of the superimposed objects, locus of superimposition, their relation, action of seeing and viewer, if one is countered, others will also be the same. In other words, the knower, the object known and knowledge are mutually dependent. The reality of one object is dependent on the other. If one is considered to be illusory, others would also be similarly fated,[2] just the fatherhood of a man is false if it is proved that the case of his having children is false. The object, which is real, must be independent and should not depend on others for its origination and existence. But actually each and every object is found to be dependent on others and hence it is not real.

The Buddhist believes in the theory of momentariness on account of which they do not accept the permanent character of an object. Such impermanent character also exists in the feeling of pleasure, universal etc. This amounts to the acceptance of *Sarvaśūṇyavāda*. An object is known in four ways :

a) as existent;
b) as non-existent;
c) as both existent and non-existent; and
d) as different from existent and non-existent (*sadasadbhinna*).

The object, which is free from this four-fold ways of description, is called *Śūṇya*.[3] This may be illustrated in the following way. If 'existence' (*sattā*) becomes the essence of an object, the function of the instrumental causes (*Kārakavyāpāra*) for its manifestation would become useless. If the 'non-existence' (*asattā*) an accepted as the nature of objects like jar etc., the effort to produce it is useless due to its non-existent character. There cannot be the cause of the existent objects like space etc. While it is impossible to look for a cause of the non-existent objects like sky-flower etc. As the 'existence' is diametrically opposite to 'non-existence', there can be an object bearing both existence and non-existence character like sky-flower etc. As the 'existence' is diametrically opposite to 'non-existence', there cannot be an object bearing both existence and non-existence, and also an object devoid of both existence and non-existence.[4]

Śūṇyatā or voidness is the nature for this indeterminable, indescribable nature of things. Things appear to exist, but our intellect fails to draw out a real nature of their existence if we try to understand them. That is why, it is accepted that the apparent phenomenal world is perceived by us. Behind this phenomenal world there is a reality, which is not capable of being described.

It has been said in the *Lankāvatarasūtra* that the nature of objects conceived with the help of intellect is indeterminable. That is why, the objects are stated to be indescribable due to not having any essence. When it is said 'This is that object', it is only for verbal communication. But when an effort is made to conceive the object, it will be seen as lacking any essence. Just as an object seen in the dream is nothing but the work of imagination, the object seen in the waking state is also imaginary due to its relation with our ignorance.[5] This point is highlighted with the help of a metaphor. Just as a female body is imagined as a combination of flesh and bone, locus of desire and enjoyable edible object by a saint, amorous person and a dog respectively, an object also is similarly imagined in various predisposition and attitude.[6]

If the very nature of an object is imagined, there is no reality of it. Such an object is properly to be called indescribable as it dependent on other things. Nāgārjuna describes this fact about dependent origination as *Śūṇya.* In other words, it is the *Dharma* of a thing that it depends on other things for its origination. *Dharma* does not exist there if it is not *Śūṇya.* An object is *śūṇya,* which means an object has got conditional and changable character, which ultimately suggests its indescribability.[7]

If an object is always dependent on other conditions, it is called conditional on account of which an object is impossible to be described as either *sat,* or *asat* etc. For this reason it follows that each and every thing exists in relation to others and hence it is relative. So, the theory called *Śūṇyavāda* may be described as the theory of relativity. No phenomenal object or experience is absolute, or independent and hence, it is not absolutely or unconditionally true.

The Buddhist formulated the theory of impermanence, dependent origination etc. to be applied to the phenomenal world which enjoys only an apparent reality (*samvṛtisatyatā*). Apart from this there is a world where these theories do not apply. That world which is unconditional, unchanging, non-relative has got absolute reality (*paramārthasatyatā*). Each and every teaching of Buddha is to be understood in two ways : first through the light of phenomenal reality and secondly through that of absolute reality.[8]

It should be clearly borne in mind that the truth at the phenomenal level points to the attainment of the truth at the absolute level. An individual, after pondering over the impermanent and dependent nature of a thing, can transcend this world and attain the world of *Nirvāṇa*. The transcendent world may be described as possessing the characteristics opposite of those of the phenomenal world, though Buddha had never directly spoken of that state owing to its indescribable character. The state, which is not capable of being known through ordinary intellect, is indescribable. That is why, Buddha observed silence whenever he was asked about the transcendent state. This fact indicates that the truth of the transcendental experience cannot be described with the apparatus of ordinary intellect and descriptive capability. Moreover, any description through language, being *Kalpanā* or imposition, cannot disclose the reality.[9]

The object known through language has a secondary reality known as *Saṃvṛtisatyatā.* The transcendent reality, which we are talking about, cannot be expressed through any *Vikalpa* or *Kalpanā* as it possesses *paramārthasatyatā.*

This theory is known as *Mādhyamika* as it adopts the middle path (*madhyama*). It does not accept the extreme views i.e. absolute reality and absolute unreality of the things. To say that an object relatively real means that it is neither absolutely real nor absolutely unreal.[10]

In the present time suffering of mankind is found in a global scale due to violence in mind, body and speech. All individual beings have become disintegrated because they are suffering from this worldly disease (*bhavaroga*) due to the absence of right vision (*Samyag dṛṣṭi*) of the objects.

If the nature of an object is known as *śūṇya*, an individual may be free from the wrong notion of an object. The detachment towards the enjoyable objects is possible for a man if he realises that the nature of the known object is relative, conditional and apparent. The phenomenon of *upekṣā*, which is accepted as *Brahmavihāra*, is possible if the void character of an object is realized. Detachment towards an object gives rise to *Upekṣā* where a man can remain indifferent in loss and gain and in different to the ups and downs of life. The detachment and *Upekṣā* again are related to the understanding of the void character of an object in the sense as mentioned earlier.

Nāgārjuna has highlighted this point with the metaphor of mirage. According to him, an individual who, mistaking a mirage for water and then comes to know that it was not at all water is not a fool. In the same way, a man who considers this world as having existence just like a mirage and afterwards comes to know its absence is a real knower having no infatuation towards the external world.[11]

One who understands *Śūṇyatā* (voidness) can understand dependent origination (*pratītyasamutpāda*). The knower of dependent origination alone can realise four Noble Truths, which are the causes of the removal of thirst etc. Due to this an individual can know the real *Dharma* as well as the cause of it and its result leading to the knowledge of suffering. Those who know these can know the real nature of happiness and suffering, and also know the means of the attainment of happiness and removal of suffering.[12]

This state is known as right vision (*samyagdṛṣṭi*). Ignorance of the real nature of an object is the main cause of our suffering. Morality is possible only through the change of attitude towards the objects of enjoyment. The real or right knowledge of them can generate in us detachment and this in turn renders moral action possible. All other ways like right resolve (*samyak samkalpa*), right speech (*samyak vāk*), right conduct (*samyak karmānta*), right livelihood (*samyag ājīva*), right effort (*samyag vyāyāma*) right attention (*samyak smṛti*) and right concentration (*samyak samādhi*) follow from the right knowledge of the objects. All other moral action like *maitrī*, *karuṇā* and *muditā* apart from *upekṣā* are possible due to one's realizing the non-essential or void character of sensuous objects.

If we can understand that the objects known as pleasant etc. are not really such, and relative in nature, we shall not lust for enjoying them. This is the root of the possible change of attitude toward them and would lead one to the path of renunciation. Having renounced the world might bring us at the threshold of the transcendental wisdom, only then we shall be able to perform moral actions and enjoy the taste of abiding in the illimitable (*brahmavihāra*). Under this circumstance an individual is purged of his ignorance, the root of violence, jealously, exploitation etc. and be a model for others in society. That *Śūṇyavāda* a relevant point of view for restoring world-peace by removing the cause of violence, exploitation etc. is accepted by Nāgārjuna who has said that the person knowing *Śūṇyavāda* knows the meaning of all and otherwise, he does not know anything ("*Prabhavati ca śūṇyateyam yasya prabhanti tasya sarvarthāḥ/Prabhavati na tasya kiñcit na bhavati śūṇyatā yasya*").[13]

An object is essenceless (*śūṇya*) because it is relative. This sense of devoidness is a mathematical concept described as zero as mentioned earlier. The Bauddha Logician Nāgārjuna has successfully shown another dimension of *śūṇyata* which is used in the sense of relativity. If an object is known in terms of other objects, which are technically called by them as *apoha*, then the object known has no essence of its own. Had it been, it would have been known independent of other entities. As it is not possible, it is better to take it as void or *Śūṇya, which* is the property of mathematics.

REFERENCES

1 *"Svapne jāgaraṇe ca na mayā dṛṣṭamidam rajatāditi viśiṣṭaniṣedhaṣyopalambhāt. Yadi dṛṣṭam sat tadā tadviśiṣṭasya darśanaśyedantayā adhiśṭhānasya ca tasminnadhyastasya rajatatvādestatsambhandhasya ca samavāyādeḥ sattvartm syāt." – Sarvadarśanasamgraha* (Bauddha-darśana).

2 *Sarvadarśana samgrahah*, Bauddhadarśana.

3 *"Bhikṣupāpraśaraṇanyāyena kṣaṇạbhangādyabhidhānamukhena sthāyitvānukūlavedanīya-tvānugatatvasarva satyatvabhrma-vyāvartanena sarvaśūṇyatāyāmeta paryavasānam. Atastattm Sadasadubhayānubhayātmakacatuṣkoṭi vinirmuktaṁ śūṇyameva". Ibid.*

4 *Tathāhi yadi ghaṭādeh sattvam svabhāvastarhi kārakavyāpāravaiyarrthyam. Asatsvabhāva iti pakṣe prācina eva doṣa prāduḥ syāt. Yathoktam-na sataḥ kāraṇāpekṣa vyomāderiva yujyate. Kāryasyāsambhāvihetuḥ khapuṣpā-derivāsata iti." – Ibid.*

5 "Buddhyā vivicyamānānam svabhāvo nāvadhāryate ato nirabhilāpyaste niḥsvabhāvaśca darśitā iti, idam vastuvalayatam yadvadanti vipaścitaḥ, yathā yathārthaścintyante viśiryante tathā tatheti ca", *Ibid.*

6. *"Paribrạṭkāmukasunamekasya pramadatanou kuṇapaḥ kāminibhakṣya iti tisro vikalpana iti" Ibid.*

7 *Mādhyamikaśāstra, kārikā* 18.

8 *Ibid, kārikā* – 19.

9 Ibid., kārikā no. 8-9.

10 *"Kalpanāpodhamabhrāntam pratyakṣam"* Nyāyabindu with ṭīkā, Pratyakṣa pariccheda.

11 *Mūlamadhyamakārikā* No. 18.

12 *Ibid.* 56.

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CHAPTER – 5

Set theory and similar concepts in Indian Logic

We use the word 'set' in such a way that a set is completely determined when its members are given; i.e., if A and B are sets which have exactly the same members, then A = B. Thus we write:

The set of equilateral triangles = the set of equiangular triangles, for something belongs to the first set if and only if it belongs to the second, since a triangle is equilateral if and only if it is equiangular. This general principle of identity for sets is usually called the principle of extensionality for sets; it may be formulated symbolically thus:

$$(1)\ A = B \leftrightarrow (x)(x \in A \leftrightarrow x \in B).$$

Sometimes one finds it convenient to speak of a set even when it is not known that this set has any members. A geneticist may wish to talk about the set of women whose fathers; brothers and husbands are all hemophiliacs, even though he does not know of an example of such a women. And a mathematician may wish to talk about maps, which cannot be colored in fewer than five colors, even though he cannot prove that such maps exist. Thus it is convenient to make our usage of the term 'set' wide enough to include empty sets, i.e., sets which have no members.

It is clear that if A is a set which has no members, then the following statement is true, since the antecedent is always false:

$$(2)\quad (x)(x \in A \rightarrow x \in B).$$

And, correspondingly, if B is empty, i.e., has no members, then it is true that:

$$(3)\quad (x)(x \in B \rightarrow x \in A).$$

From (1), (2) and (3) we conclude that if two sets A and B are empty, then:

$$A = B;$$

That is to say, there is just one empty set; for given two empty sets, it follows from the principle of extensionality for sets that the two sets are identical. Hence we shall speak of the empty set, which we denote by a capital Greek lambda:

Λ is the set such that for every x, x does not belong to Λ ; that is, symbolically : (x)-(x ∈ Λ), and we abbreviate '- (x ∈ Λ)' to 'x ∉Λ', and write: (x)(x ∉Λ).

We shall find it convenient in general to use the notation '∈' to indicate that something does not belong to a set.

Often we shall describe a set by writing down names of its members, separated by commas, and enclosing the whole in braces. For instance, by:
{Roosevelt, Parker}

We mean the set consisting of the two major candidates in the 1904 American Presidential election. By:

$$\{1, 3, 5\}$$

We mean the set consisting of the first three odd positive integers. It is clear that

{1, 3, 5} = {1, 5, 3} (for both sets have the same members : the order in which we write down the members of a set is of no importance). Moreover, {1, 1, 3, 5} = {1, 3, 5} (for we do not count an element of a set twice).

The members of a set can themselves be sets. Thus a political party can be conceived as a certain set of people, and it may be convenient to speak of the set of political parties in a given country. Similarly we can have sets whose members are sets of integers: for instance, by:

$$\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$$

We mean the set which has just three members, namely, $\{1, 2\}$, $\{3, 4\}$ and $\{5, 6\}$. By:

$$\{\{1, 2\}, \{2,3\}\}$$

We mean the set whose two members are $\{1, 2\}$ and $\{2, 3\}$. By:

$$\{\{1, 2\}, \{1\}\}$$

We mean the set whose two members are the sets $\{1, 2\}$ and $\{1\}$.

A set having just one member is not to be considered identical with that member. Thus the set $\{\{1, 2\}\}$ is not identical with the set $\{1, 2\}$: this is clear from the fact that $\{1, 2\}$ has two members, whereas $\{\{1, 2\}\}$ has just one member (namely, $\{1, 2\}$). Similarly,

{Elizabeth II} ≠ Elizabeth II, for Elizabeth II is a woman, while {Elizabeth II} is a set.

Ordinarily it is not true that a set is a member of itself. Thus the set of chairs is not a member of the set of chairs : i.e., the set of chairs is not itself a chair. This remark illustrates the very great difference between identity and membership: for the assertion that $A = A$ is always true, whereas that $A \in A$ is usually false.

The relation of membership also differs from the relation of identity in that it is not symmetric: from $A \in B$ it does not follow that $B \in A$. For instance, we have:

$2 \in \{1, 2\}$, but:

$\{1, 2\} \notin 2$.

Moreover, the relation of membership is not transitive: from $A \in B$ and $B \in C$ it does not follow that $A \in C$. Thus, for example, we have:

$2 \in \{1, 2\}$ and:

$\{1, 2\} \in \{\{1, 2\}, \{3, 4\}\}$ but:

$2 \notin \{\{1, 2\}, \{3, 4\}\}$, for the only members of $\{\{1, 2\}, \{3, 4\}\}$ are $\{1, 2\}$ and $\{3, 4\}$ and neither of these sets is identical with 2.

It should be noticed that if, for instance, $\{a, b\}$ is any set with two members, then, for every x, $x \in \{a, b\}$ if and only if either $x = a$ or $x = b$, that is, symbolically:

$$(x)(x \in \{a, b\} \leftrightarrow (x = a \vee x = b).$$

Similarly, if $\{a, b, c\}$ is a set with three members, then $x \in \{a, b, c\}$ if and only if either $x = a$ or $x = b$ or $x = c$. It is for this reason that we just said that $2 \notin \{\{1, 2\}, \{3, 4\}\}$; for if $x \in \{\{1, 2\}, \{3, 4\}\}$, then either $x = \{1, 2\}$ or $x = \{3, 4\}$; and since $2 \neq \{1, 2\}$ and $2 \neq \{3, 4\}$, it follows that $2 \notin \{\{1, 2\}, \{3, 4\}\}$.

It should also be noticed that there is a close relationship between saying that something has a property and saying that it belongs to a set : a thing has a given property if and only if it belongs to the set of things having the property. Thus to say that 6 has the property of being an even number amounts to saying that 6 belongs to the set of even numbers.

The principle of the identity of indiscernible in terms of properties. Expressed in terms of membership the principle becomes: If y belongs to every set to which x belongs, then $y = x$. Put in this form, the principle has perhaps a more obvious character than it has when put in terms of properties. For $x \in \{x\}$ (i.e., x belongs to the set whose only member is x), and hence, if y belongs to every set to which x belongs, we conclude that $y \in \{x\}$, so that $y = x$.

Inclusion : If A and B are sets such that every member of A is also a member of B, then we call A a subset of B, or say that A is included in B. We often use the sign '$\subseteq$' as an abbreviation for 'is included in'. Thus we can write, for instance:

The set of Indians is a subset of the set of men, or:
The set of Indians is included in the set of men, or simply:
The set of Indians $\subseteq$ the set of men. Symbolically we have:

(1) $A \subseteq B \leftrightarrow (x)(x \in A \rightarrow x \in B)$.

It is clear that every set is a subset of itself; i.e., for every set A we have: $A \subseteq A$. Moreover, the relation of inclusion is transitive; i.e., if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ (for if every member of A is a member of B, and every member of B is a member of C, then every member of A is a member of C). The relation of inclusion is not symmetric, however; thus $\{1, 2\} \subseteq \{1, 2, 3\}$, but it is not the case that $\{1, 2, 3\} \subseteq \{1, 2\}$.

It is intuitively obvious that identity, membership, and inclusion are distinct and different notions, but it is still somewhat interesting to observe that their distinction may be inferred simply from considering the questions of symmetry and transitivity. Thus inclusion is not the same as identity, since identity is symmetric while inclusion is not. And inclusion is not the same as membership, since inclusion is transitive while membership is not. And we have seen earlier that identity is not the same as membership, since identity is both symmetric and transitive, while membership is neither. In every day language all three notions are expressed by the one overburdened verb 'to be'. Thus in everyday language we write:

Elizabeth II is the present Queen of England,
Elizabeth II is a woman,
Women are human beings.
But in the more exact language being developed here:
Elizabeth II = the present Queen of England,
Elizabeth II $\in$ the class of women,
The class of women $\subseteq$ the class of human beings.

When $A \subseteq B$, the possibility is not excluded that $A = B$; it may happen also that $B \subseteq A$, so that A and B have exactly the same members, and hence are identical.

The Empty Set : As mentioned earlier, the empty set, A, is characterized by the property that, for every x, $x \notin A$.

Although nothing belongs to the empty set, the empty set can itself be a member of another set. Thus if we speak of the set of all subsets of the set {1, 2}, we are speaking of the set {{1, 2}, {1}, {2}, A} which has four members; the three-member set {{1, 2}, {1}, {2}} on the other hand, is the set of all non-empty subsets of {1, 2}.

We recall the fact that a set A is a subset of a set B if and only if every member of A is also a member of B, i.e., if and only if: for every x, if $x \in A$, then $x \notin B$. In particular, the empty set A is a subset of a set B if and only if: for every x, if $x \in A$, then $x \in B$. Since always $x \notin A$, however, it is always true that if $x \in A$. then $x \in B$. Thus, for every set B, we have:

$$A \subseteq B.$$

That is, the empty set is a subset of every set. In addition, the empty set is the only set which is a subset of the empty set; for if $B \subseteq A$, then, since we also have: $A \subseteq B$, we can conclude that $B = A$.

An empty set is to be accepted as a set and because through the acceptance of its setness it can be admitted distinguished from other set. Moreover, an empty set is also a subset of another empty set, because if something is susbstracted from another, it remains in the same status and hence it is taken as subset of another.

Operations on Sets : If A and B are sets, then by the intersection of A and B (in symbols: $A \cap B$) we mean the set of all things which belong both to A and to B. Thus, for every x, $x \in (A \cap B)$ if and only if $x \in A$ and $x \in B$; that is, symbolically:

(1) $(x)(x \in A \cap B \leftrightarrow x \in A \ \& \ x \in B)$.

If A is the set of all Indians, and B is the set of all blue-eyed people, then $A \cap B$ is the set of all blue-eyed Indians.

If A is the set of all men, and B is the set of all animals which weigh over ten tons, then A ∩ B is the set of all men who weigh over ten tons. In this case we notice that A ∩ B is the empty set (despite the fact that A ≠ A and B ≠ A, since some whales weigh more than ten tons). When A ∩ B = A, we say that A and B are mutually exclusive.

Our use of the term 'intersection' is similar to its use in elementary geometry, where by the intersection of two circles, for instance, we mean the points, which lie on both circles. Some authors use, instead of '∩', the dot '.' Which is used in algebra for multiplication; such authors often speak of the "product" of two sets, instead of their intersection.

If A and B are sets, then by the union of A and B (in symbols: A U B) we mean the set of all things which belong to at least one of the sets A and B. Thus, for every x, x ϵ (A U B) if and only if either x ϵ A or x ϵ B.

Symbolically:

One may intend to consider the union of two sets, however, even when they are not mutually exclusive. For instance, if A is the set of all human adults, and B is the set of all people less than 40 years old, then A U B is the set of all human beings.

If A and B are two sets, then by the difference of A and B (in symbols: A ~ B) we mean the set of all things which belong to A but not to B. Thus, for every x, x ϵ A ~ B if and only if x ϵ A and x ∉ B; that is, symbolically:

2. $(x)(x \in A \sim B \leftrightarrow x \in A \,\&\, x \notin B)$.

If A is the set of all human beings, and B is the set of all human ..., then A ~ B is the set of all human males. One often desires to consider the difference of two sets A and B, however, even when B is not a subset of A. For instance, if A is the set of human beings, and B is the set of all female animals, then A ~ B is still the set of all human males, and B ~ A is the set of all female animals which belong to a non-human species.

Domains of Individuals : Often one is interested, not in all possible sets, but merely in all the subsets of some fixed set. Thus in sociology, for instance, it is quite natural to say mostly about sets of human beings; and to speak with the understanding that when a set was mentioned it was to be taken to be a set of people, if an explicit statement to the contrary was not made. In such discourse one might say, for example, 'the set of albinos', and it would be understood that one was referring only to the set of albino people, and not also to albino monkeys, albino mice, and other albino animals.

Similarly, in some geometrical discourse the word 'set' to mean 'set of points' is used. Sometimes in mathematics people press into service in some specialized sense some of the various words mentioned above as being here taken to be synonymous with 'set': a geometrician might, for example, adopt the convention of speaking of set of points, classes of sets of points, and aggregates - or perhaps families - or geometrical curves.

When a fixed set D is taken as given in this way, and one confines himself to the discussion of subsets of D, we shall call D the domain of individuals, or sometimes the domain of discourse. Thus the domain of individuals of the sociological discussion mentioned above is the set of all human beings.

We shall denote the domain of individuals by 'V'. It is important to remember that though 'Λ', stands for a uniquely determined entity (the empty set), the symbol 'V' is interpreted differently in different discussions. In one context 'V' may stand for the set of all human beings, in another for the set of points of space, and in another for the set of positive integers.

When dealing with a fixed domain of individuals V, it is convenient to introduce a special symbol for the difference of V and a set A:

$$\sim A = V \sim A.$$

We call $\sim A$ the complement of A. More generally, the difference $B \sim A$ of B and A is called the complement of A relative to B; so the complement of a set is simply its complement relative to the given domain of individuals.

Translating Everyday Language : This part is concentrated to the problem of translating sentences of everyday language into the symbolism that we have been developing. It should clearly be borne in mind that the usage of everyday language is not so uniform that one can give unambiguous and categorical rules of translation. In everyday language we often use the same word for essentially different notions ('is', for example, for both '$\in$' and '$\subseteq$'); and, sometimes for literary elegance, we often use different words for the same notion ('is', 'is a subset of', and 'is included in', for example, for '$\subseteq$').

We consider here only those sentences, which can be translated into a symbolism consisting just of letters standing for sets, parentheses, and the following symbols:

$$\cap, \cup, \sim, \Lambda, =, \neq, \subseteq$$

Such a symbolism can handle statements involving one-place predicates very well, but it is not adequate to many-place predicates. This symbolism is essentially equivalent to the language of the classical theory of the syllogism it is important to note that we are not using here the notion of membership; we restrict ourselves to sets all of which are on the same level-subsets of some fixed domain of individuals.

An English statement of the form 'All ... are ...', where the two blanks are filled with common nouns such as 'men' or 'Indians' or 'philosophers', means, of course, that the set of things described by the first noun is a subset of the set of things described by the second noun. Thus, for example:

(1) All Indians are philosophers Means:

The set of Indians $\subseteq$ the set of philosophers, or, using 'A' as an abbreviation for 'the set of Indians', and 'P' as an abbreviation for 'the set of philosophers':

$$I \subseteq P$$

We can also express the meaning of this statement in other, equivalent, ways:

$$A \cup P = P, \text{ or: } A \cap \sim P = A, \text{ etc,}$$

and these other modes of expression often turn out to be useful.

We use the same mode of translation of statements of the form 'All ... are ...' also when the second blank is filled with an adjective. For example, we take:

(2) All Indians are mortal to mean:

The class of Indians $\subseteq$ the class of mortal beings, or, using obvious abbreviations:

$$A \subseteq M.$$

Sometimes, however, in contexts of this sort people suppress the word "all"- writing, for instance:

Tyrants are mortal instead of:

(3) All tyrants are mortal, or : Women are fickle instead of :

(4) All women are fickle, which we should translate, respectively, by:

$$T \subseteq M \text{ and:}$$

$$W \subseteq F$$

One must be on guard when translating statements of this kind, however; for ordinary language uses the same form also to express essentially different ideas. Thus, as we have seen before:

(5) Men are numerous does not mean:

The set of men $\subseteq$ the set of numerous things (i.e., that every man is numerous) but rather, letting M be the set of men and N be the set of sets which have numerous members:

$$M \in N.$$

Similarly:

(6) The apostles are twelve Means that the set of apostles belongs to the set of sets having just twelve members.

Corresponding to the distinction, which we have made between membership and inclusion, the older logic made a distinction between the "distributive" "collective" applications of the predicate to the subject. Using this terminology, one says that in (1), (2), (3), and (4) the predicate is applied to the subject distributively, and that in (5) and (6) it is applied to the subject collectively.

An English statement of the form 'Some...are...', where the blanks are filled b common nouns means that there exists something which is described by both terms: i.e., that the intersection of the two corresponding sets is not empty. Thus, for instance:

(7) Some Indians are philosophers Means that there exists at least one person who is both an Indians and a philosopher, and is accordingly translated:

$$I \cap P \neq I.$$

Although a statement of the form of (7) implies that the sets corresponding to subject and predicate are not empty, no such inference is to be drawn from a statement of the form of (1). Thus, for example, it is true that

All three-headed, six-eyed men are three-headed men, but it is not true that some three-headed, six-eyed men are three-headed men.

An English statement of the form 'No...are...' (where as before, the blanks are filled b common nouns) means that nothing belongs both to the set corresponding to the first noun, and to the set corresponding to the second noun: i.e., that the intersection of these two sets is empty. For instance, the sentence:

(8) No Americans are philosophers is translated:

$$A \cap P = A.$$

Thus (2) has the same meaning as:

No Americans are immortal since both can be translated:

$$A \cap \sim M = A.$$

An English statement of the form 'Some ... are not ...' (where the blanks are filled by common nouns) means that there exists something which belongs to the set corresponding to the first noun, and does not belong to the set corresponding to the second noun: i.e., that the intersection of the first set with the complement of the second is not empty. The sentence:

Some Americans are not philosophers is translated:

$$A \cap \sim P \neq A.$$

We turn now to the problem of translating some statements of a more complicated sort. The word 'and' often corresponds to the intersection of sets. Thus:

All Americans are clean and strong is translated (using obvious abbreviations):

$$A \subseteq C \cap S.$$

The same applies to the word 'but': thus:

Freshmen are ignorant but enthusiastic is translated:

$$F \subseteq I \cap E.$$

The situation is quite different, however, when the 'and' occurs in the subject rather than in the predicate. Thus:

(9) Fools and drunk men are truth tellers is translated, not by:

(10) $(F \cap D) \subseteq T$ but rather by:

(11) $(F \cup D) \subseteq T$.

For (9) means that both the following statements are true:

(12) All fools are truth tellers and:

(13) All drunk men are truth tellers; and (12) and (13) are translated, respectively, by:

(14) $F \subseteq T$ and:

(15) $D \subseteq T$; and (14) and (15) are together equivalent to (11). (It should be noticed that (10) says less than (11); for:

$F \cap D \subseteq F \cup D$ is true for every F and D – and hence (10) is true whenever (11) is true – while a statement of the form (10) can be true even when the corresponding statement of the form (11) is false.

Often the statement to be translated does not contain any form of the verb 'to be' at all. Thus the statement:

Some Frenchmen drink wine can be translated:

$F \cap W \neq A$, if we think of 'F' as standing for the set of Frenchmen and 'W' as standing for the set of wine drinkers. The statement:

Some Americans drink both coffee and milk can be translated:

$A \cap C \cap M \neq A$, where 'A' stands for the set of Americans, 'C' for the set of people who drink coffee, and 'M' for the set of people who drink milk. Here we have adopted the practice, which is frequently employed, of suppressing parenthesis in representing the intersection of three or more sets, writing simply:

$A \cap C \cap M$ instead of: $A \cap (C \cap M)$; we shall sometimes adopt a similar practice in connection with the representation of the union of three or more sets. Still more complicated examples are possible. If we consider:

(16) Some Americans who drink tea do not drink either coffee or milk. The general form of this statement is: Some S are not P. The subject is translated:

$A \cap T$, where T = the set of tea drinkers, and the predicate is translated : $C \cup M$.

The whole sentence (16) is then translated :

$(A \cap T) \cap \sim(C \cup M) \neq A$, which is also equivalent to:

$A \cap T \cap \sim C \cap \sim M \neq A$, since, corresponding to De Morgan's laws for the sentential connectives, we have:

(17) $\sim(C \cup M) = \sim C \cap \sim M$.

Venn Diagrams : In studying sets and relations between them, it is sometimes helpful to represent the sets diagrammatically: one draws a rectangle to represent the domain of individuals, and then draws circles, or other figures, inside the rectangle – thinking of the points inside the various figures as corresponding to the members of the sets being represented by the figures. Thus sets A and B, for instance, mutually exclusive, can be represented by the following diagram:

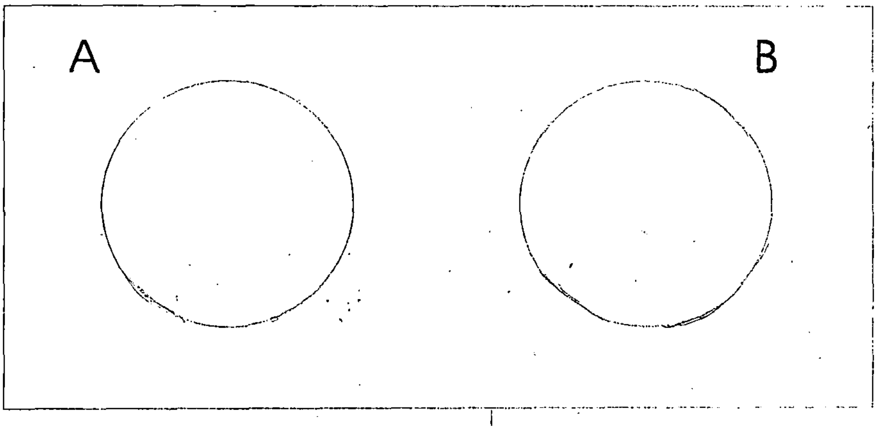

FIGURE 1

If we know that $A \subseteq B$, we can represent the situation by Figure 2.

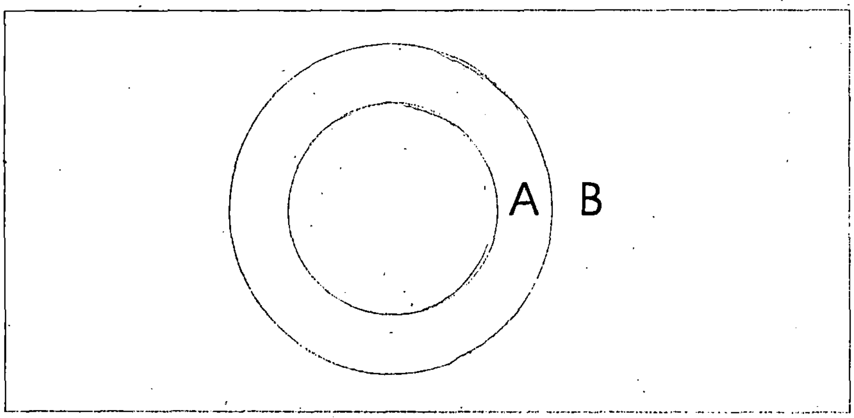

FIGURE 2

A more traditional way of describing Figure 2 is to say that all A are B, i.e., all members of A are members of B.

If we know of three sets A, B and C that $A \subseteq B$ (all A are B) and $B \cap C = A$, (i.e., no B are C), we can represent the situation by Figure 3.

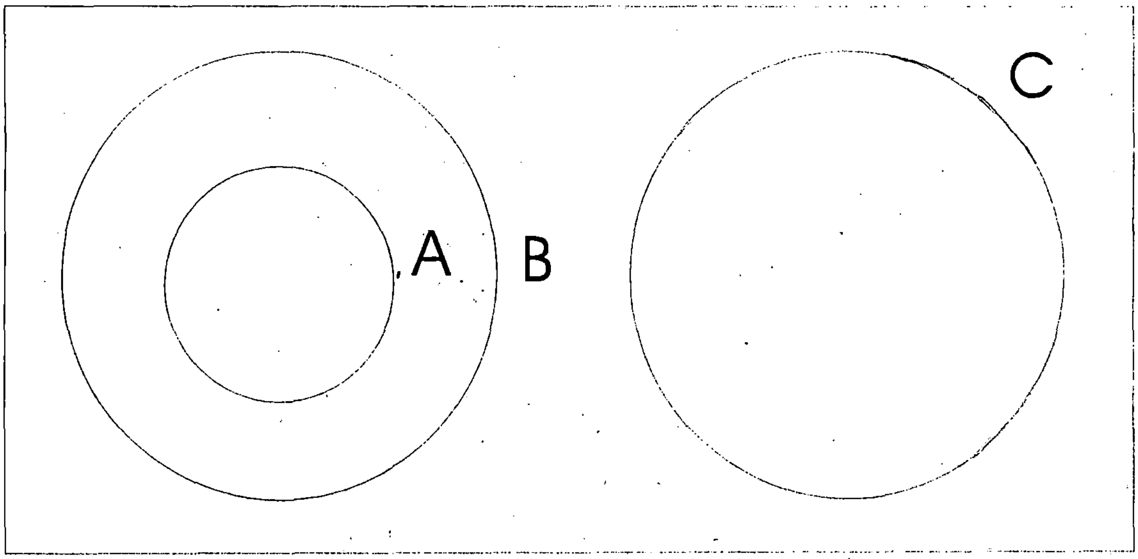

FIGURE 3

Sometimes, instead of trying to incorporate the given information into the diagram simply by drawing the circles in an appropriate manner, it is convenient to draw the figures in a rather arbitrary way (so that they will divide the interior of the rectangle into a maximum number of parts) and then get the information into the figure by other methods, such as the shading of areas. Having decided, let us say, to indicate by *horizontal shading* that an area corresponds to the empty set, we indicate that $A \cap B = A$ by Figure 4; and that $A \subseteq B$ by Figure 5; for to say that A is a subset of B means that no part of A lies outside B.

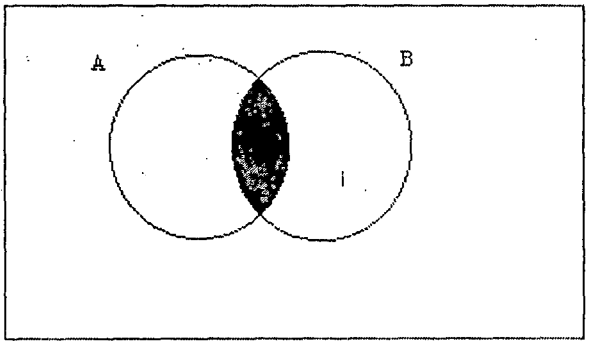

FIGURE 4

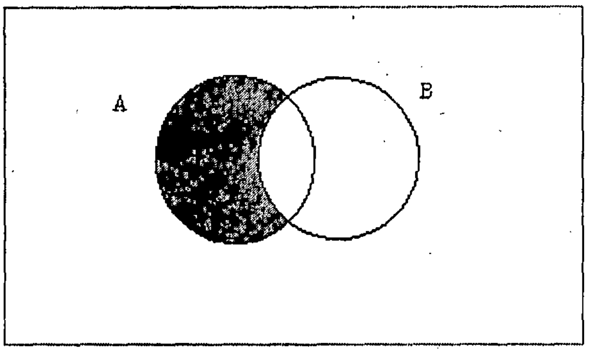

FIGURE 5

With these diagrams, it is often easy to show what conclusions can be drawn from given information about two or more sets. Thus suppose, for example, that it given, of two sets A and B both that $A \cap B = A$, and that $A \subseteq B$. The first statement (as indicated in Figure 4) means that the common part of A and B is to be shaded; and the second statement (as indicated in Figure 5) means that the part of A which is outside of B is to be shaded. Thus we obtain Figure 6, where we notice that all of A is shaded. Thus we see that the two given statements jointly imply that A is the empty set.

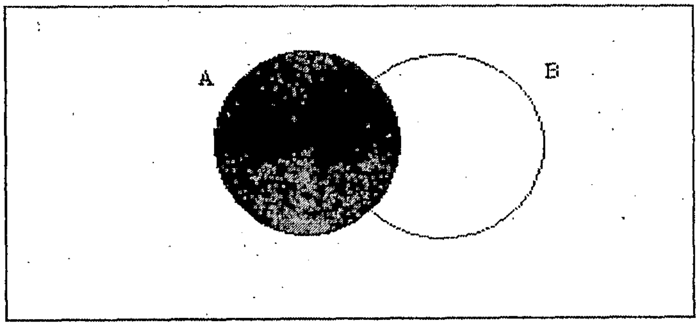

FIGURE 6

Horizontal shading is used to indicate emptiness of a region. Another kind of symbol is needed for non-emptiness. We shall use a device of linked crosses. Thus if $A \cap B \neq A$ (some A are B) we represent this situation by Figure 7; the cross indicates that the region common to A and B is not empty. We represent the more complicated situation :

$A \cap (B \cup C) \neq A$, (some A are either B or C) by Figure 8.

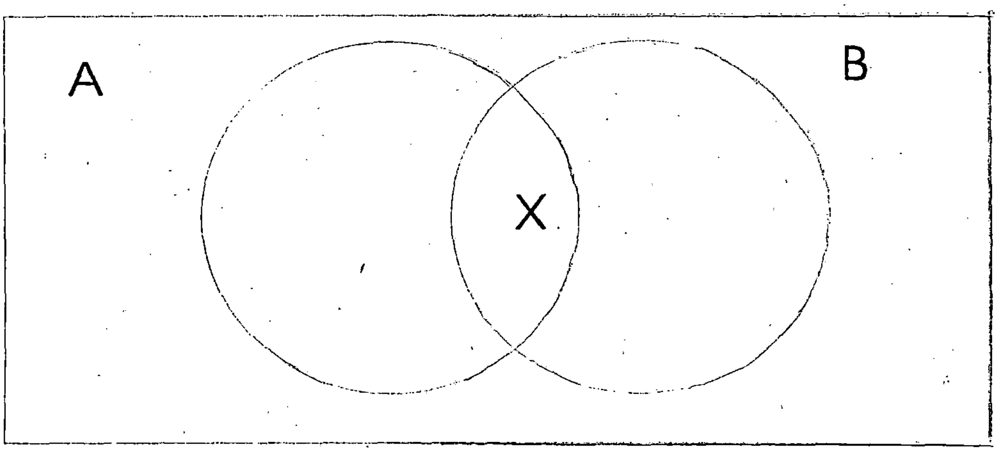

FIGURE 7

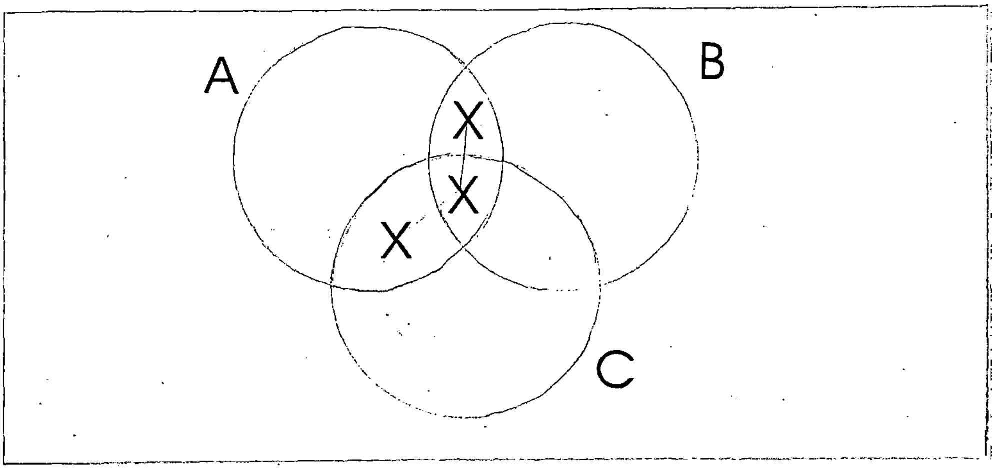

FIGURE 8

The three crosses in Figure 8 are linked to show that at least one of the three small regions is non-empty. If the linkage had been omitted in Figure 8, the figure would represent much more than that $A \cap (B \cup C) \neq A$. If the linkage were omitted, we could infer:

(1) $(A \cap B) \sim C \neq A$ (the top cross)

(2) $A \cap (B \cap C) \neq A$ (the middle cross)

(3) $(A \cap C) \sim B \neq A$ (the bottom cross)

Obviously, any one of the assertions (1) – (3) implies that $A \cap (B \cup C) \neq A$ and more. Without the linkage Figure 8 would say far too much.

The situation described by

$A \cup B \neq A$ (Something is either A or B)

$A \cup \sim C \neq A$ (Something is either A or not C) is represented by Figure 9. Note the two separate linkages, one for each of the two existential statements.

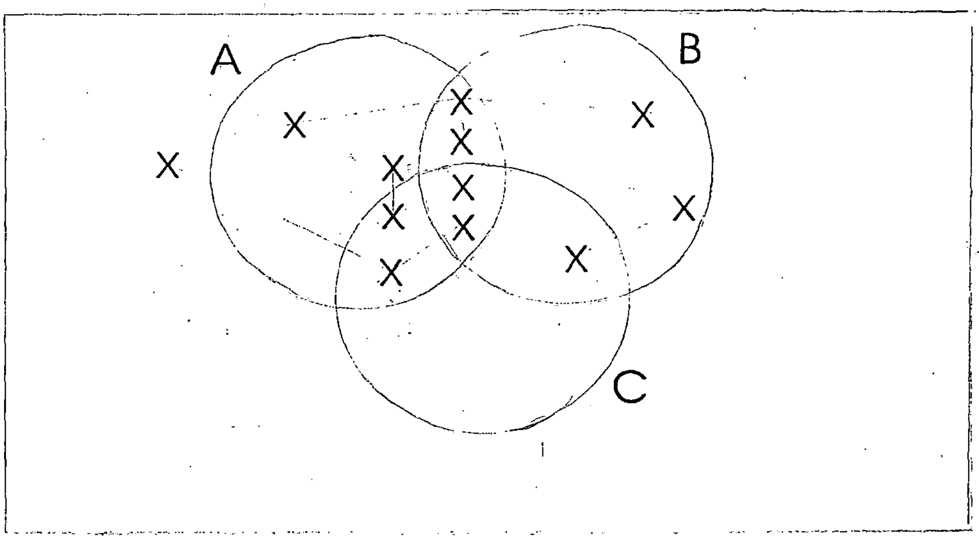

FIGURE 9

What should be an interpretation of a diagram in which a cross and shading occur in the same region? Suppose, for example, that we have:

(4) $A \cap C \neq A$ (Some A are C)

(5) $C \subseteq B$ (All C are B)

We obtain Figure 10, in which the part of C which is outside of B has been shaded horizontally, to show that it is empty and linked crosses have been placed in the two parts of the common region of A and C, to show that it is not empty. The problem of interpretation centres around Region (1). Consideration of (4) and (5) clearly urges the stipulation that shading dominates a cross, and hence Region (1) is empty. We are thus able to conclude that Region (2) is not empty, that is, (4) and (5) imply that $A \cap (B \cap C) \neq A$ (some A are B and C).

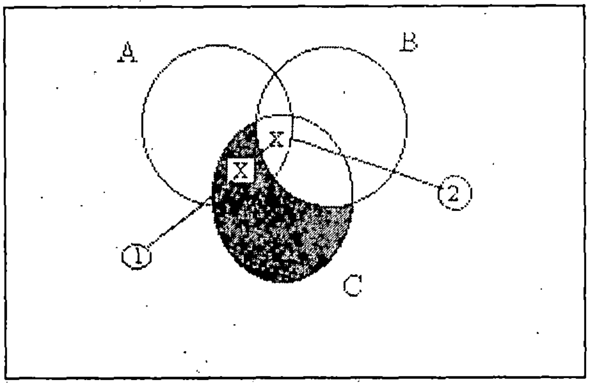

FIGURE 10

There is one set of circumstances in which we do not want to say simply that shading dominates a cross. When every cross in a linkage of crosses is "covered" by shading we must conclude that the diagram is inconsistent rather than that the linked regions are completely empty, for a linkage of crosses means that at least one of the regions linked is non-empty. We may in fact use these circumstances to investigate by use of Venn diagrams the consistency of a set of conditions imposed on sets. Thus suppose, for example, that it is given of three sets A, B and C that:

(6)	$A \subseteq C$	(All A are C)
(7)	$A \cap C = A$	(No A are C)
(8)	$A \cap B \neq A$	(Some A are B)

This situation is represented by a Venn diagram in Figure 11.

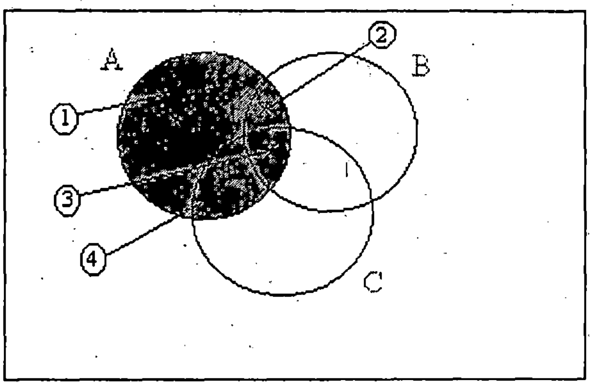

FIGURE 11

Assumption (6) leads us to shade Regions (1) and (2); assumption (7) leads us to shade Regions (3) and (4); and assumption (8) leads us to place two linked crosses in Regions (2) and (3). Thus the given assumptions imply that Regions (2) and (3) are both empty and non-empty, which is a contradiction.

There is, of course, a very great difference between saying that certain conditions on sets are inconsistent and saying merely that they imply that some set is empty. Thus assumptions (6) and (7) above imply that A is empty, but these two assumptions by themselves are not inconsistent.

With the notation for Venn diagrams now complete, it is of some interest to show how the apparatus may be used to establish the validity of classical syllogisms. As an example, consider the syllogism:

(9) No B are C

(10) All A are B

(11) Therefore no A are C

Premises (9) and (10) are represented by Figure 12.

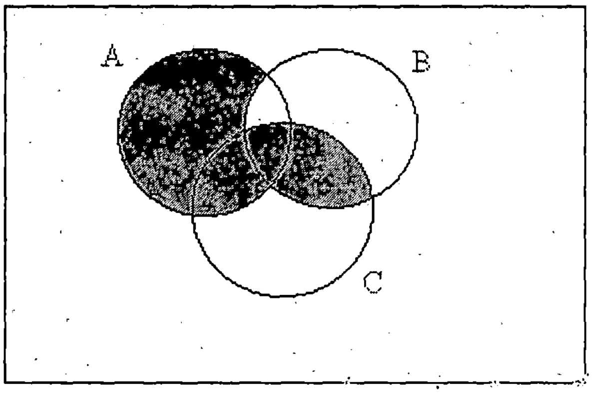

FIGURE 12

We now examine the diagram to see if it implies that no A are C. We have seen at once that the region common to A and C is horizontally shaded, and we conclude that the conclusion of the syllogism is valid. All other valid syllogisms may be tested in the same way, but there is no need to restrict the use of Venn diagrams to testing the validity of those arguments, which have the classical syllogistic form. Venn diagrams may be used to represent any argument, which does not involve more than three sets. Moreover, by a careful use of ellipses in place of circles relations among four sets can be represented diagrammatically, but relations among five or more sets can often not be represented by any simple diagrammatic device.[1]

The relation between set and subset as found in the western logic has affinities with Indian concepts of *parā* and *aparā sāmānya* (i.e., greater and comparatively smaller universal). The universal (*sāmānya*) is defined as a property which, being eternal, is inhered in many individuals (*nityatve sati anekasamavctatvam*) as per the view of the Naiyāyikas. To them the properties like 'cowness' etc. are eternal in nature and remain in all the individual cows through relation of inherence. In other words, it can also be said that the property 'animality' (*prāṇitva*) remains in all individual cases of animal through relation of inherence and hence it is also a case of universal.[2]

The Naiyāyika defines the universal as a character, which is *nitya* (eternal) and *anekasamaveta* (inheres in many particular instances). Therefore, according to Naiyāyikas, the relation between a universal and its particular instance is the relation of inherence. Further, the universal is an eternal character inhering in more than one particular instance. Therefore, where there is only one instance of a thing, its distinguishing character is not a logical universal, e.g., according to the Naiyāyika, there is only one *ākāśa* or ether. Therefore ether ness is just a distinguishing character and not a logical universal – an *upādhi* and not a *jāti*. Again when a character or feature which is related to the substrate which it characterises by some relation other than the relation of *samavāya* or inherence, it is no logical universal in the strict sense, e.g., negativity or *abhāvatva* is a common character of such particular *abhāvas* or negations as *ghaṭābhāva*, *paṭābhāva*, etc. But since the relation of *samavāya* holds only between positive objects of experience or *bhāvapadārthas*, and not between positive and negative objects, nor between one negative object and another, the relation of *samavāyatva* does not hold between *abhāvatva* or negativity and the particular negatives in which it is found as a common character. Thus *abhāvatva* or negativity, as not admitting of the relation of *samavāya*, is not a logical universal. The Naiyāyika also rejects overlapping universals as not being logical universals in the strict sense, e.g., *bhūtatva* or the character of being an element is common to the five elements earth, water, air, fire and ether and *mūrtatva* or the character of moving is common to the five moving substances, viz. earth, water, air, fire and mind. Thus both these characters have earth, water, air and fire as their common substances while 'the character of being an element' applies to *ākāśa* and not to mind, and 'the character of moving' applies to mind and not to *ākāśa.* Therefore, if 'the character of being an element is conceived as a universal, it will apply to the four *bhūtas* – earth, water, air and fire which are moving thing as well. And then the universal *bhūtatva* will coincide with the universal *mūrtatva* in respect of these four substances and ought therefore to apply to the other *mūrta*, viz., mind though it does not. And the same objection will hold in respect of *mūrtatva* which should apply to *ākāśa* though it does not. Further, the four substances, earth, water, air and fire, will have to be regarded as instances of two difference universals, which is like saying that some animals are both cows and buffaloes which is absurd. This is why character with partially overlapping denotation are not admitted by Naiyāyikas to be logical universals.

Another negative condition of a logical universal, according to the Naiyāyika, is *regressus ad infinitum*. Where the acceptance of a character as a universal will land one into an infinite regress, no logical universal is admissible according to the Naiyāyika. This is why the Naiyāyikas do not recognise universals of universals, e.g., 'horseness', 'cowness' and 'dogness' are three universals, and since each of these is a universal, universality is a character common to these universals. If universality is, therefore, to be regarded as a fourth higher universal, and 'horseness', 'cowness' and 'dogness' as particular instances of it, then, in so far as this higher universal is a fourth universal, one must conceive a still higher universal of these four universals, namely, 'horseness', 'dogness', 'cowness' and 'universality'. In the same way we shall have to go from a fourth to a fifth universal, from a fifth to a sixth and so on *ad infinitum.*

The fifth negative condition of a logical universal, according to the Naiyāyika, is *rūpahāni*. By this the Naiyāyika means that where recognition of a character as universal contradicts the intrinsic nature or *rūpa* of a thing, it is not admissible as a logical universal, e.g., *antya viśeṣa*, the ultimate differential, is an individuating principle inherent in every eternal substance. Each eternal substance is a unique individual because of the presence in it of this ultimate differential or *viśeṣa*. Each eternal substance has thus a *viśeṣa* inhering in it which differentiates it from all other objects of experience. *Viśeṣatva* or differentiating character is thus a character common to different *viśeṣas* inhering in different eternal substances. Why not then accept *viśeṣatva* as a universal, common character of the different *viśeṣas* of the innumerable eternal substances? The Naiyāyika answer is in the negative as the admission of *viśeṣatva* as a universal destroys the very nature of *viśeṣa* (*rūpahāni*). *Viśeṣa is* that which is unique, uncommon and if a common character of the uncommon be admitted it will destroy the very nature of the uncommon as uncommon.

A sixth negative condition also laid down by the Naiyāyikas is that no separate second universal can be admitted where the difference between two universals is a difference in name only, e.g., between *kalaśatva* and *kumbhatva*.

It may be noted that while Naiyāyikas repudiate universal of universals, they yet recognise a gradation of universals into higher and lower reaching up to one highest universal (*parājāti*) which is *sattā* or being. Thus according to the Naiyāyika, the universal of 'being' or *sattā* is the most comprehensive universal (*parājāti*) applying to all particulars while lower universals (*aparājāti*) apply to some particulars and do not apply to other particulars, e.g., *dravyatva*, substantiality, or substanceness, is a character of every *dravya* or substance, but not of a *guṇa* (quality) or a *karma* (motion). Similarly *guṇatva* holds of every *guṇa* or quality, but not of any *karma* or *dravya*. Thus, *dravyatva* is both *anuvṛttilakṣanạ* and *vyāvṛttilakṣaṇa*, both inclusive and exclusive. *Dravyatva*, e.g., is inclusive of *dravyas* and exclusive of *karmas* and *guṇas*. *Guṇatva* is inclusive of *guṇas* and exclusive of *dravyas* and *karmas*. But *sattā* or being is true of all *dravyas*, *guṇas* and *karmas*, i.e., it includes all and excludes nothing. In this sense *sattā* or 'being' is the highest universal or *parājāti* while other universals are lower in rank.

It is obvious from the above that what the Naiyāyika means by the gradation of universals into lower and highest reaching up to one *parājāti* or highest universal, viz., *sattā* is their grading in respect of extent or denotation, the highest being higher as possessing a wider or more extensive denotation and the lower being lower as possessing a narrowing or less extensive denotation and the highest being highest as possessing the most extensive denotation of all. The Naiyāyika does not mean a connotative subsumption of one universal under another and that is why he repudiates universals of universals as leading to infinite regress.[3]

The Nyāya theory of universals is not without its difficulties as both Buddhists and Advaitins have pointed out a universal is both eternal and an inherent character of its particular instances, then how does the Naiyāyika account for the appearance of a universal in a newborn instance of it? And how does he account for its disappearance, when it ceases to be? When a new jug is made out of a jump clay, does the eternal jug-ness (*ghaṭatva*) come suddenly into being in the newly made jug, or, when the jug is broken does the eternal jugness cease to be so far as the broken jug is concerned? Suppose the species we call 'cow' becomes extinct in course of evolution so that not a single individual is anywhere left on the earth. Where will the eternal 'cowness' go? Will it wander about like a floating adjective, an abstract universal without a particular locus? Further, when the universal inheres in a particular instance of it, does it inhere in it in its entirety, or does only a part of it inhere in the par particular instance? If it inheres in its entirety, then nothing of it will be left to inhere in other particular instances, so that if there be one individual cow there will be no other cows. And if it inheres only partially in a particular instance of it, then we are landed in the absurdity that an individual cow is only partly a cow and partly some other animal such as a buffalo. It may be noted that the Buddhists repudiate the Nyāya view of universals and offer instead their own theory known as *Apohavāda*. According to them, the so-called positive common character is a myth. Universality is only *anyavyāvṛtti*. It is common exclusion rather than common inclusion that constitutes universality. When we say X is a cow, we do not mean that it is one particular instance of the universal 'cowness' which X has in common with other cows as its inherent character. All that we mean is that it is not a horse, not a dog, not a man, etc. Further, according to Naiyāyikas, 'existence' (*sattā*) is the *parājāti*, highest universal and is an inherent common character of all *dravyas*, *guṇas* and *karmas*, substances, qualities and actions. Therefore, in so far as a cow or a horse or a chair or a table is a substance, it has existence or *sattā* as its inherent character. Therefore, the negative judgment 'a chair is not' or 'a table is not' or 'a horse is not' or 'a cow is not' amounts to a manifest self-contradiction, for this is the same as saying that the cow which is inherently existent does not exist. Contrariwise, when we say that the cow exists, our judgment becomes a tautology, for it amounts to saying that the inherently existent exists, or, that 'that to which existence belongs as an eternal inherent character exists'.[4]

Further, if the universal, as the Naiyāyika says, be an inherent eternal character of its particular instances, then in so far as one and the same particular is an instance of two or more universals, e.g., in so far as a cow is an instance of the universal of substance (*dravyatva*) and again an instance of the universal of *sattā* or being and also an instance of the universal 'cowness' (*gotva*) it becomes the seat of several universals, i.e., a case of overlapping universals or *jāti sankara*.

The Buddhists have accepted the reality of an object in terms of its casual efficacy (*arthakriyā-kāritva*). All objects that have got casual efficacy are momentary in nature.

It has been argued by the Vaiśeṣikas that the meaning of the term '*Sattva*' (existence or being) seems to be vague to them. The term '*sattva*' means an object's association with *sattā*, *sāmānya* or *jāti* and hence possessing this eternal generic property can be momentary.

In response to this Buddhists rejoin that they do not accept that an object possessing *sattā sāmānya* is existent. It is so, the existence would have to be admitted in substance, quality and action due to accepting *sattā sāmānya* there. To the Vaiśeṣikas *Sāmānya*, *Viśeṣa* and *Samavāya* do not possess existence or *Sattā* due to the problem of infinite regress. If *sattā* or *sāmānya* is accepted in *Sāmānya* or *Viśeṣa* etc. there would arise the question of accepting another *Sāmānya* in it i.e. *Sāmānyatva*, *Viśeṣatva* etc. and in this way the defect of infinite regress cannot be avoided. In fact, Vaiśeṣikas have accepted the *Sāmānya* etc. as *sat* as they are revealed as such, but this is not *Sāmānya* in the technical sense. If the Vaiśeṣikas accept *sattā* in the form of *astitva* in *Sāmānya* etc., and *sattā* in the form of *sattā Sāmānya* in substance etc. there would be *gaurava*, in determining the criterion of apprehending the *Sat* object. Moreover, another problem would crop up. There would arise common apprehension (*anugatapratyaya*) in the substance etc. due to having the same *sattājāti* in *Sāmānya* etc. and hence there would also arise the common apprehension, which is not observable.

It has been accepted by the Nyāya - Vaiśeṣikas philosophers that *Sattā* or *Jāti* exists in different *loci* bearing same shape and size through relation of inherence. In this connection, the Buddhists ask that, if *Sāmānya* exists in many things bearing same size, how do they admit *sāmānya* or *sattājāti* in different objects bearing different shapes and sizes like substance, quality and action and also between mustered seeds and mountain? To the Vaiśeṣikas *sattā sāmānya* exists in substance etc. through the relation of inherence (*sāmānya*). If it is taken for granted, the Buddhists argue how the usage of differentiation between a man and a cow in the form: 'This is a cow and this is a man' can be made. If it is said that the universals like humanity, cowness etc. pervading in a man and a cow are the causes of the usage of the differentiation between man, it is not tenable because the concept of universal as propounded by them is under consideration.

It is enquired by the Buddhists whether the universal exists in all objects or in all individuals belonging to a particular class. In the case of former, all objects would be of a same type due to the existence of same universal in them. If the universal 'humanity' existing in a human being remains in horse etc., the horse etc. would have to be considered as man due to having humanity in them, which is not possible. Moreover, it will go against the established thesis of the Naiyāyikas. If the latter is taken for consideration, it will also create some difficulties. That universal exists in all individuals belonging to the same class is admitted by Praśastapāda. If this line is accepted, it will lead to some philosophical difficulties as follows:

The universal 'Jarness' did not exist in a piece of mud before the origination of a jar but it is produced just after the origination of the same. It is asked by the Buddhists whether the universal 'jarness' existing in a jar situated in other place is related to this jar existing in a different place or not. If it is so, whether this universal is related to a particular individual after coming from other places or without coming from there. In the case of former the universal would have to be designated as substance as it possesses the action in the form of movement. In the case of latter there would arise the difficulty in apprehending the relation. For, how can the relation of one object to another be established without accepting the action or movement.

It cannot also be said that the jar ness etc. existing in a jar etc. is related to a jar existing in a different place through its self-extension. For, self-extension is possible for an object having parts (*Sāvayava*). As jarness etc. have no parts (*niravayava*) the extension of it is not possible.

Moreover, when a jar is destroyed, the problem is whether the jarness existing in it remains in it or is destroyed or goes elsewhere. The first alternative is not correct as universal cannot remain without its substratum i.e., an individual. Moreover, universal always remains only in the objects other than the eternal ones. If the second alternative is taken into account, it will lead us to accept the antithesis i.e., the eternity of the universal as accepted by the Naiyāyikas. The acceptance of the third alternative leads to accept another undesired situation. For, universal can go elsewhere if there is movement. If the existence of movement is accepted in *sāmānya*, it would turn into a *dravya* or substance but not *sāmānya* due to having movement in it.

To the Vaiśeṣikas universal exists in substance etc. through the relation of inherence. If it is so, the Buddhists argue that the ground on which a jar exists also contains the jarness existing in a jar as the lower part of a jar is connected with the upper surface of the ground. If jarness remains on the ground, the ground would also be taken as a jar, which is not possible. Moreover, jarness cannot pervade a jar existing on the ground without keeping it associated with the ground.

Considering all these defects the Buddhists do not accept *Sāmānya*. To them, *Sattva* is not in the form of *Sāmānya* but in the form of causal efficacy (*arthakriyā-kāritva*).

If *Sāmānya* is not accepted, how is the common knowledge (*anugatapratyaya*) among various individuals of the same class possible. To the Buddhists it is not true that cow is differentiated from other animals like horse etc., with the help of *Sāmānya*, but cow is known as distinct in terms of the knowledge of 'non – cow' (*agovyāvṛtti*). In the same way, a jar is known in terms of the knowledge of non – jar (*aghaṭavyāvṛtti*). This is type of negative way of knowing is called *apoha*. *Apoha* is that which can differentiate a particular object from others (*svetaravyāvṛttirūpa*).[5] The distinctness of a jar from other object (*ghaṭetarabheda*) which remains in all individual jars leads us to the apprehension in the form: 'This is a jar but not a cloth' and through this similar cognition among all individuals belonging to the same class is established the derivative meaning of the term '*apoha*' is as follows:

That which differentiates something from others (*apohanam*) is *apoha*. As it differentiates a particular object from others, it is called *anyāpoha*. Ratnakīrti has opined that the verbal usage in the form 'This is a cow' is originated from the apprehension in the form 'This cow is different from non-cow' (*agovyāvṛtti*). Hence, the phenomenon of *anyāpoha* is the cause of similar apprehension (*anugatapratyaya*) and hence there is no necessity of *Sāmānya*. In other words, the similar cognition of all individuals of the same class is due to an object's unique character (*svalakṣaṇāt*) which is possible through its distinct nature from other objects.[6]

Now the question of empty set or null set comes. A set having no members is to my opinion, not a set at all. A set having no members is contradictory in terms. In fact, the collection of some members makes a set in the ordinary sense of the term. If there is a set having no members at all, it is not to be said to be a set. Hence a set without any member is contradictory in terms.

In Indian logic the null set or empty set is found in the terms like 'sky-flower' (*khapuṣpa*), *hare's* horn (*śaśaśṛnga*) etc. The Naiyayīkas think that these words are meaningless as they have no references. As there is no object or referend of the terms, they are absurd entities. An object which is referred to by this word does not belong to the category of real. Hence it is neither substance nor quality nor action nor universal nor particularity nor absence. It is not even absence because for being an absence there should be an awareness of its absentee (*pratiyogī*). The cognition of absence always depends on that of an absentee (*pratiyogijñānasāpekṣam abhāvajñānam*). If an absentee (*pratiyogī*) is not known, its absence can also be known. The entities like *khapuṣpa* etc. do not come under the absence also, because these (the counter positives) are not known at all. The absence in the form: *'khapuṣpam nāsti'* is also non-sensical, as the absentee called *khapuṣpa* is not known at all or not knowable at all. An entity which cannot be categorised as substance, quality, action etc. is taken as non-entity (*apadārtha*). As the Naiyayikas are realists entities which have no existence at all. The following *kārika* carries the examples of empty terms: "*Eṣo bandhyāsuto yāti khapuṣpakṛta - śekharaḥ/ kūrmakṣīracaye snātaḥ Śaśaśṛngadhanurdharaḥ*". (That is, here goes the barren woman's son with a crown made of sky-flowers, who has bathed in a pool of tortoise – milk and carries a bow built of rabbit – horn).

Western logicians have accepted such null classes or empty classes in spite of knowing that there is not a simple member in it, because null or empty class is a notion, which is opposed to non-empty class. To prove that there is a non-empty class it is to be known what empty class is as per the principle – 'the cognition of an absence presupposes that of the absentee'. Hence empty set is to be admitted as a real set only to understand a non-empty set.

The grammarians in Indian tradition also contribute to this view.

The statements mentioned above – *bandhyāputra etc.* involve a lot of empty terms, yet they communicate some thought to the hearer and the discovery of its incompatibility with the world of facts makes him laugh at the speaker. Why should one find a statement amusing if one grasps nothing at all? The thought directly conveyed by an expression is looked upon by the Pāṇinians as a conceptual existence, which may or may not find a corresponding external counterpart in the realm of realities. Thus according to these grammarians a meaning, as a conceptual existence, is *independent of external existence*. A *referent* is not a must for a meaningful word. *Buddhisattā* or the conceptual existence, as it is understood by the Pāṇinians, is a near approach to what is described as 'Being' by Russell in *The Principles of Mathematics* where it is shown that even the words like *chimera* and *unicorn* have Being in spite of their having no *Existence*. If the words like *śaśaśṛnga* ('rabbit-horn') conveyed no sense how could they be used as *prātipadika* (i.e., stem), which, by definition, is bound to have a meaning (*arthavat*)[6]? The *Vaiyākaraṇa* concepts of *buddhisattā* and *bahiḥsattā* can be roughly replaced by '*Being*' and '*Existence*' respectively as they are employed by Russell in *The Principles of Mathematics*. Inspired by Patañjali's comment on the *matup-sūtra* (i.e., the Pāṇinian rule 05/02/94) – '*na sattām padārtho vyabhicarati*' (which literally means 'a word-meaning is not without an existence')[7]. Bhartṛhari, Kaiyaṭa, Helārāja and Nāgeśa have developed their doctrine of *buddhisattā*. Actually speaking, the word *sattā* in Patañjali's remark means *buddhisattā* or a conceptual existence, i.e., *Being*, and not *bahiḥsattā* or *Existence* proper. There is no such meaning of a word as has no conceptual existence – this is the purport of Patañjali's observation. Conceptual existence is no existence proper (i.e., *mukhyāsattā* according to Bhartṛhari); it is only an *imposed existence* (*aupacārikī sattā*), which is nothing external to human consciousness. Helārāja's description of *aupacārikī sattā* or *buddhisattā* as *bhāvābhāvasādhāraṇa* is highly significant.[8] It shows that conceptual existence is commonly extended to both the existent (*bhāva* or *sat*) and the non-existent (*abhāva* or *asat*). It is the thought or sense (an approximate equivalent, not an exact equivalent of *buddhisattā*), which every word must have. Hence, there is no word or expression which may be labelled as 'nonsensical'.

The Pāṇinian theory of meaning aspires after solving many a problem which one finds difficult to explain in terms of the naïve referential theory advanced by the Naiyāyikas. It provides an early but convincing answer to the plaguing problem – 'Is Existence a Predicate'? That which does not exist can never be referred to. As soon as we employ a term as the subject of a proposition we assume the existence of what it means. Predicting existence of something existent in sentences like '*vṛkṣaḥ asti*' ('the tree exists') runs the risk of being a mere 'referential tautology', an example of what we call *siddha-sādhana* ('establishing the established') in Indian Philosophy. Again denying existence of that which is existent, apparently sounds contradictory as we may find in sentences like '*vṛkṣaḥ nāsti*' ('the tree does not exist'). The Pāṇinians try to avoid this risk of either 'referential tautology' or 'referential contradiction' by saying that in the affirmative categorical statement we assert that our subject which is thought to have a conceptual existence has a factual existence too, whereas in a negative sentence the subject is not so fortunate as to have a factual existence apart from the assumed one.[8a] One may contend that even if the words '*asti*' and '*nāsti*' are supposed to have a conceptual existence for their meaning, the same difficulties will continue unabated. Let us in this context reproduce some significant observations from a thought provoking article by Prof. V.N. Jha :

'...... according to the *bauddhārtha-vādins*, all words express *bauddhārtha* and so when *ghaṭa* expresses "pot having conceptual existence"., the word asti should also express conceptual existence (*bauddhasattā*) and in that case the use of the verb asti becomes redundant since the "existence" is already conveyed by the word *ghaṭa* (*uktārthānam aprayogaḥ*). Similarly, there will be a contradiction in the negative sentence since *ghaṭa* conveys existence and *nāsti* conveys non-existence. Thus if *bauddhapadārtha* is accepted, the difficulty is unavoidable".[9]

But, to do full justice to the grammarians, we should clarify a bit further. In the affirmative sentence '*ghaṭaḥ asti*' ('the pot exists') the subject term conveys the thought or conceptual existence of a tree we may find in reality and the predicate term does that of a real existence. Our predicating '*asti*' of '*vṛkṣah*' amounts to the affirmation of the factual existence in relation to what is conveyed by the subject term. *Thought* is the vehicle through which this affirmation takes place. In the negative sentence, '*nāsti*' conveys the thought of the negation of existence. Our predicating '*nāsti*' *through the medium of thought* conveys the denial of the referent of '*asti*' (i.e., existence) in relation to the subject and shows that the latter has no existence apart from the assumed one. It is the *bauddha artha*, which is directly expressed by '*asti*' or '*nāsti*' at first, then it stretches out to the factual level and makes the sentence relevant by pointing to the existence and the negation of the subject respectively. The Pāṇinians are quite conscious that the word '*ghaṭaḥ*' is *apparently* sufficient, since as soon as we utter something we assume its existence. This is evident from Bhartṛhari's assertion that our uttering '*ghaṭa*' assumes that the pot has existence in the form of sustaining itself.[10] Existence or sustaining itself (*ātmadhāraṇa*) is the very nature of a thing. Nāgeśa hints at the same when he remarks – '*loke*' *styartham vinā śuddhapadārthānavagateśca*'.[11] [It is because a person cannot grasp a pure entity without grasping its existence.] Yet, the predicate term *asti* is of importance, if the context demands it. Suppose somebody expresses doubt about the existence of the pot and you then put a special emphasis on the word '*asti*' to dissolve his doubt. Here the assertion is of an additional significance; through the vehicle of *thought* it confirms the existence of the subject. If again, one is mistaken that a pot is there; you want to dispel that wrong conception. You then, through the thought-medium, confirm the negation of the existence and thus correct the erring person. The Vaiyākaraṇas attach paramount importance to the context and the speaker's will which often determine the use of a particular word. Herein lies the significance of Nāgeśa's remark that in their opinion the expressions 'does exist' and 'does not exist' are employed in order to intimate the 'existence' and 'non-existence' respectively of the subject concerned.[12]

The Pāṇinians adopt a similar line of thinking to account for the subject-predicate relation in the expression '*ankuro jāyate*' ('the sprout is being born'). *Ankura*, the subject, is taken as the nominative agent (*kartṛkāraka*) of the act of being born. The *kārakas* are the causal conditions, which combine to produce an action – "*samagrīsādhyatvāt kriyāṇām sarve kārakatvam kriyāniṣpattiviṣayam abhedena pratipādyante*"[13] as Heliārāja describes the matter. An action is a process, which awaits completion (*sādhya*), but those, which perform it, must be pre-established (*siddha*) and the nominative agent plays the major role in this production process. The subject (i.e., the nominative agent) having its existence already established, it is absolutely superfluous to speak of its birth. How can one assert the birth of something, which is already born and existent? However, this is no problem for the Pāṇinians according to whom, through the above sentence one relates the thought of a real birth to the sprout the existence of which is already assumed. Thus there is an obvious journey from what is imposed to what is real. Bhartṛhari, in his *Sādhanasamuddeśa*, arrives at the same conclusion from another angle. To the probable question how something which is not yet born can be treated as the nominative agent, Bhartṛhari puts forward the reply –

utpatteḥ prāg asadbhāvo buddhyavasthānibandhanaḥ/
aviśiṣṭaḥ satānyena kartā bhavati janmanaḥ//[14]

That which has no (factual) existence before its birth owes an intellectual existence to the speaker's intention and thereby has got an (*assumed*) efficacy to perform the action in question. The sentential meaning settles down to predicating external factual birth to that of which the existence is caught in intellect, and which has thus an intellectually grasped capacity to do some job *("buddhyā niścitasattvasya kriyāsiddhāvupagṛthītasāmarthyasya bāhyena rūpeṇa janmeti vākyartho" vatiṣṭhate.)*[15]

The Naiyāyikas may, on the contrary, count upon a 'possible reality' and argue that the referent of the word '*ankura*', though not existent at present, is to come into being in the near future. Only that thing can be called unreal or non-existent which does not exist in the three phases of time – past, present and future. A sprout is not unreal in that sense. Through the sentence '*ankuro jāyate*' one asserts the birth, a physical phenomenon, of something real which, however, does not exist at the present segment of time.

The Pāṇinians on the other hand, though speaking of a *superimposed apparent reality*, do not make allowances for any possible reality. There is no such thing as *past* or *future existence* (*bhūta-bhaviṣyat-sattā*). 'To exist' means 'to be present'.[16] A reality is only that which *is* and *not what may be* or *will be* or *will cease to be*. Otherwise how can we declare 'a thing has ceased to exist' or 'a thing will come to exist' which simply means that the thing does not exist? When one says 'the jar will exist' (i.e., at the time of its pre-absence or *prāgabhāva*) or 'it will be raining', one has only a conception or thought of the jar or of the rain which has not yet come into being, may be the prior perceptions of some existent jars or rain have worked behind this thought. One cannot deny that the particular jar which is conceived or thought of does not exist at the time of that conception. It is quite understandable why Nāgeśa has drawn our notice to a rule of Gautama, the Nyāyasūtrakāra himself, which states that the non-existent effect, is, however, established in our intellect ('*buddhisiddham tu tadasat*').[17] The weaver who knows which specific type of material is required for a specific type of product has a thought of the individual cloth, which he is going to produce. He does not mean to produce which is already existent. The non-existing individual then exists in the intellect alone; nevertheless his conception is nourished by the individuals he has seen. Nāgeśa makes it a point that even the referentialists and the realists like the Naiyāyikas cannot do away with '*buddhisattā*'. He has for his support an observation of another idealist, the author of the *Bhāmatī* on the *Śāṅkarabhāṣya* under the *Brahmasūtra* 1/1/2 ('*janmādyasya yataḥ*'), who states that a person makes a 'yet-to-be-produced' (*nirvartya*) non-existing jar the object of his production on the strength of a conceptual determination (*antaḥsamkalpātmanā*).[18]

Or, consider the situation when a person informs that he wants to cook. The particular act of cooking is not performed as yet; if it were so one could not wish to perform it again. One cannot perform what has been performed already. To find out a rationale behind the informative sentence 'I want cooking' you must say that here in this expression 'cooking' does not have the referent, i.e., the real *individual* act of cooking for its meaning. The word 'cooking' conveys just a thought or conception which awaits materialisation. A desire, just like a knowledge, should have an object of which it is the desire. In this sense, the object is one of the causal conditions that produce a desire. A desire is an inner phenomenon and so, Nāgeśa argues, the direct object which causes this desire should also be an inner one.[19] It is true that the desire works behind the actual happening of the act in the world of facts. But it is equally true that a pre-conception of that 'yet-to-materialise' object prompts the person to act. This is the import of the following comment of Durbalācārya, the learned commentator of Nāgeśa's *Laghumañjūṣā* – "*siddhe pākādāvicchāder virahād asiddhasyaiva pākāder icchām prati hetutāyāvācyatayā buddhāveva hetu-hetumatoḥ sāmānādhikaraṇyam upapādanīyam*".[20]

[Since, if the cooking is done there is no scope for the desire, 'yet-to-be-established' cooking etc. will be considered the cause; hence an equilocativity between the cause and what is caused is to be found out in (the plane of) intellect alone.][21]

Again, suppose somebody narrates that Kṛṣṇa is killing Kamsa, or one may refer to a particular scence of a drama and say 'Kamsa is being killed'. Now, if one believes in the Purāṇa story, Kamsa was slain long ago; how can he be the object of killing again? Patañjali's *Mahābhāsya* on Pāṇini's rule '*Hetumati ca*' (3/1/26) shows the way how one can account for the meaning cognition of the above statements – "*sato buddhiviṣayān prakāśayanti*".[22] [The narrators communicate the conceptually existent meanings.] Bhartṛhari, Kaiyaṭa and Nāgeśa – all agree that this is the only way in which Kamsa etc., the objects of imaginative intellect, are conveyed by words and thereby treated as different agents (*kārakas*) of an action which seems to be enacted before our eyes –

> *Śabdopahitarūpāmśca buddherviṣayatām gatān/*
> *pratyakṣamiva kamsādīn sādhanatvena manyate//*[23]

Nāgeśa reminds us of another case where acceptance of *bauddha artha* comes to the rescue. Somebody asks you – "who is Devadatta?" You, in reply, direct your index finger towards a particular person and say 'This person with armlets, earrings, a broad chest and round arms – Devadatta is like this.' Here Devadatta is a single person who is referred to. But whenever we talk of a resemblance we suggest that there is some other individual resembling the one who is placed before us. All this talk of resemblance is meaningless when only a single person is concerned. There cannot be different persons in one individual ('*ekavyaktau bhedābhāvāt*') who is referred to. To make the word '*īdṛśa*' (meaning 'like this') meaningful you have to admit that the word '*īdṛśa*', in spite of its having no other referent apart from the one which the name 'Devadatta' refers to, has a distinct thought for its meaning. Unlike the words '*go*' (meaning a cow) etc. where the etymological sense is fully sacrificed for the sake of a conventionally established meaning, the word '*īdṛśa*' retains the derivative sense 'like this'. For the Pāṇinians, however, there is nothing inconsistent in applying the word '*īdṛśa*' which is inserted to assert that the thought expressed by the different words, both the proper noun and its adjectives together, finds its confirmation in a real counterpart, the object of identification.[24]

The realists cannot justify the statements like '*vināśī śabdaḥ*' ('sound is destructible') until and unless they admit a conceptual or intellectual existence of the destruction, which is yet to materialise. The expression conveys a qualifier – qualificand relation between destruction and the entity, which is destroyed. Destruction is posterior absence (*dvaṁsābhāva*) and an absence or negation cannot be coeval with the negatum of which it is the negation. So how can there be a relation between the two? Yet one conjures up a non-existent relation in one's intellect; otherwise where is the scope for stating *vināśī śabdaḥ* or *vināśī ghaṭaḥ* ('The jar is destructible')? Even Jayantabhaṭṭa, the Naiyāyika of great repute, is apprehensive of this linguistic predicament and finds a wav-out by admitting that while stating *vināśī śabdaḥ*, the speaker connects through the thought-medium the existing sound with only a conceptually or intellectually existent destruction.[25]

Let us also take note of some faulty syllogisms (*hetvābhāsas*), which project a non-existent relation. Take an example of *bādhita hetvābhāsa* – '*vahnir anuṣṇaḥ dravyatvāt*'

('Fire is non-hot, since it is a substance'). Factually speaking, there can never be a relation between *sādhya* or probandum (i.e., 'being non-hot' or *anuṣṇatva)* and *pakṣa* or the subject, fire. Yet a thought of that relation is communicated through the above statement.[25a] There are of course phenomena like a fire and 'being non-hot', yet their relation is a non-entity. The same can be asserted with reference to a svarūpāsiddha hetvābhāsa which records that sound is impermanent, since it is visible (*'śabdaḥ anityaḥ cākṣuṣatvāt'*). Despite the fact that 'visible' and 'sound' have their respective referents, the thought expressed by the statement, which brings their relationship into the focus has no referent to fall back upon. Or consider the major premises involved in a *viruddha hetvābhāsa* – 'All products are eternal' (*'yet kṛtakam tan nityam'*) which iterates the contradictory of what really is. If the above statements are described as focussing only a new arrangement of the referents of the words, then one cannot help admitting that at least a non-existent arrangement is caught in the intellect. The referential status is lost beyond recovery in the case of an *āśrayāsiddha hetvābhāsa*, e.g., 'the sky-lotus is fragrant, since it is a lotus' (*'gaganāravindam surabhi, aravindatvāt'*) wherein the very subject is an empty term. Even the Naiyāyikas are doubtful if a truth-value can be assigned to the above statement. Gangeśa himself seems to be in two minds when he remarks that the subject 'sky-lotus', if taken in its totality, makes the statement non-significant (*apārthaka*) which is considered a point of defeat (*nigrahasthāna*) for the arguer concerned.[26] But we have already discussed that the fate of not only *āśrayāsiddha*, but of many other *hetvābhāsas* as false assertions hangs in the balance, if one does not accept a thought of some non-existing relationship or arrangement to be conveyed by the statements concerned. In other words, many of our *hetvābhāsas* have got to be dismissed as purely *apārthaka* or nonsensical if we do not recognise a conceptual existence as the direct meaning of an expression.

Examine a debate (*vāda*), which consists of contradictory statements.[27] Does not the proponent grasp the meaning of what his opponent says? Had there been no meaning conveyed by the opponent's statement, why should the proponent strain himself by an uncalled for attempt to refute him? Why should he not ignore that statement as some sort of delirium? – *kiñcaivam sati vāde prativādiśabdasyābodhakatve tatkhaṇḍanakathocchedaḥ.*[28] One of the contradictory statements must be true and the other false. Should we then say that a false statement expresses a false fact? But there is *no false fact* at all.

Hence the grammarians believe that the empty set comprising of *bandhyāputra* etc. is associated with meaning, which is similar to the mathematicians. But for the Naiyāyikas such expressions are meaningless leading to the non-acceptance of null set, because even the absence of *bandhyāputra* etc. cannot be predicted due to the absence of the absentee as a category.

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22. Also see *Laghumañjūṣā*, Vol. I, '*icchādīnām antaḥakaraṇaniṣṭh atayā tatra viṣayasya sāmānādhikaraṇyenaiva kāraṇatvaucityena bauddhapadārthasattāvaśyakī*/p. 240.

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CHAPTER - 6

Some Astronomical Principles : An Indian Perspective

Two types of knowledge: *aparāvidyā* (inferior knowledge) and *parāvidyā* (superior or spiritual knowledge) have been accepted in Indian tradition. An act of worship done with a specific worldly desire was considered an inferior form of worship and was popular with the kings. *Aparāvidyā* enables man to attain material progress, enrichment and fulfilment of life and *parāvidyā* ensures attainment of self-realization or salvation in life (*Chānd. Up.* 7.1.7; *Muṇḍa. Up.* 1.2.4-5). The Vedic people, in general, made interesting synthesis by adopting *nitya* (perpetual or daily) and *kāmya* (optional for wish fulfilment) sacrifices or offerings. The first was supposed to bring happiness to the family and second was for material progress. The perpetual daily sacred fires and the optional fires were placed on altars of various shapes. As to the reasons, which might have induced the ancient Indians to devise all these strange shapes, the *Ṛg-Veda* (1.15.12) says, 'He who desires heaven, may construct falcon shaped altar, for falcon is the best flyer among the birds'. These may appear to be superstitious fancies but led to important contributions in geometry and mathematics because of their conviction in social value systems.

To find the right time for religious, agricultural, new year and other social festivals gave the motivation for recording of recurrence of repeated events from seasons, stars, movement of planets, moon etc. This helped to develop many a framework for mapping of movements of heavenly bodies with reference to East, West, North and South points, *nakṣatras*, calendar, *yuga*, *mahāyuga* and movement of planets for mean and true positions of planets. Various mathematical and trigonometrical tables were also formulated for better and better results. The *Vedānga Jyotiṣa*[1] mentions as under:

Having saluted Time with bent head, as also Goddess Saraswati, I shall explain the lore of Time, as enunciated by sage Lagadha. As the crests on the head of the peacocks, the jewels on the serpents, so is the (*jyotiṣa gaṇitam*) held at the head of all lores among all *vedāṅga śāstras*.

Gaṇita is a variant reading for *jyotiṣa* meaning computation, which is the essence of this science.

Another tradition, which has enriched specialized activities in mathematics and astronomy is the *guru-śiṣya-paramparā* (teacher-student tradition). Different *recensions* of Vedic schools, *śulbakāras*, *jyotiṣkaras* (Varahāmihira names twenty scholar before him), Kusumpura school, schools of Ujjain and *Asmakadeśa*, and Jain and Buddhist schools are also well-known in this connection.

Beside these, there were commercial and other problems, which were tackled for *lokavyavahārārtha*, used for common people. The restriction and emphasis were also assured on social use and value systems, which helped people to take up different activities for commerce, education and other areas. These helped undoubtedly to concretise knowledge resulting in original contributions to mathematics and astronomy.

Construction of altars and nature of knowledge :

The ritual connection of Indian geometry, as elaborated by Thibaut and Burk, has been intensively discussed by Datta, Seidenberg, Sen and Bag, and others.[2] The ceremonies were performed on the top of altars built either in the sacrificer's house or on a nearby plot of ground. The altar is a specified raised area, generally made of bricks for keeping the fire. The fire altars were of two types. The perpetual fires (*nitya agni*) were constructed on a smaller area of one Sq. *puruṣa* and optional fires (*kāmya agni*) were constructed on a bigger area of 7½ sq. *puruṣas* or more, each having minimum of five layers of bricks. The perpetual fires had twenty-one bricks and optional fires had two hundred bricks in each layer in the first construction with other restrictions. For optional fire altars, the whole family of the organizer had to reside by the construction site of optional fire altar, for which another class of structure known as *mahāvedī* and other related *vedīs* were made. However, a summary of these types of altars with ground shapes are grouped below:

(a) Perpetual fire altars (area coverage: 1 sq. *puruṣa*): *āhavanīya* (square), *gārhapatya* (circle or square), *dakṣiṇāgni* (semicircle).

(b) Optional fire altars (area coverage: 7-½ square *puruṣa*): *Caturasraśyenacit* (hawk bird with square body, sq uare wings, square tail), *kankacit* and *alajacit* (bird with curbed wings and tail), *prauga* (triangle), *ubhayata prauga* (rhombus), *rathacakracit* (circle), *draṇacit* (trough), *śmaśānacit*, (isosceles trapezium), *kūrmacit* (tortoise) etc.

(c) Vedis: *mahāvedī* or *saumikī vedī* (isosceles trapezium), *sautrāmani vedī* (isosceles trapezium, and also one-third of the *mahāvedī*), *paitṛkī vedī* (isosceles trapezium or a square, area one-third of *sautrāmani vedī*), *prāgvaṃśa* (rectangle).

One can guess the nature of knowledge, which could originate from such altar constructions. However, the *Śulba-Sūtras* of Baudhāyana, Āpastamba and other schools have summarized this knowledge as available from the Saṃhitas and Brāhmaṇas. Both Baudhāyana and Āpastamba belonged to different schools but followed a similar pattern, which also suggests that these schools inherited the knowledge from older schools. While giving details, the *Śulba-Sūtras* use the word *vijñāyate* (known as per traditions), *vedervijñāyate* (known as per Vedic tradition) etc. very often. A summary of this knowledge will be of great interest.

Baudhāyana gives various units of linear measurements viz., 1 *pradeśa* = 12 *angulas*; 1 *pada* = 15 *ang*; *iṣa* = 188 ang; 1 *akṣa* = 104 ang; 1yuga = 86 ang; 1 *janu* = 32 ang; 1 *śamyā* = 36 ang; 1 *bāhu* = 36 ang; 1 *prakarma* = 2 *padas*; 1 *aratni* = 2 *prādeśas*; 1 *puruṣa* = 5 *aratnis*; 1 *vyāyama* = 4 *aratnis*; 1 ang = 34 *tilas* = ¾ inch (approx.).

Knowledge of rational numbers like 1, 2, 3 ... 10, 11 ... 100 ... 1000, ½, ⅓, ¼, ⅛, 1/16, 3/2, 5/12, 7½, 8½, 9½ etc. were used in decimal word notations and their fundamental operations like addition, subtraction, multiplication and division were carried without any mistake.

Baudhāyana had knowledge of square, rectangle, triangle, circle, isosceles, trapezium and various other diagrams and transformation of one figure into another and *vice-versa*. Methods of construction of square by adding two squares or subtracting two squares were known. The areas of these figures were also calculated correctly. That the length, breadth and diagonal of a right triangle maintain a unique relationship, $a^2 + b^2 = c^2$ (where a = length, b = breadth and c = hypotenuse) or in other words (1, 1, 2) (2, 1, 3) formed important triplets thereby forming an important basis for number line, and were used for construction of bricks and geometrical figures. For easy verification, Śulbakaras suggested triplets expressed in rational and irrational numbers like (3, 4, 5), (12, 5, 13), (15, 8, 17), (7, 24, 25), (12, 35, 37), (15, 36, 39), (1, 3, $\sqrt{10}$), (2, 6, $\sqrt{40}$), (1, $\sqrt{10}$ $\sqrt{11}$), (188, 78⅓, 203⅔), (6, 2½, 6½), (10, 41/6, 105/6) and so on. A general statement on 'Theorem of Square on Diagonal' was also enunciated thus: 'The areas (of the squares) produced separately by the length and the breadth of a rectangle together equals the area (of the square) produced by the same diagonal'. This has been wrongly referred to Pythagoras who was associated with the triplet (3, 4, 5) and the theorem is known as the Pythagorian theorem. Indian knowledge is based on rational and irrational arithmetical facts and geometrical knowledge of transformation of area from one type to the other and its importance was perhaps correctly and perfectly realised. How the Babylonians, Egyptians, Chinese and the Greeks came upon the knowledge of triplets but not the general statement is equally important for an interesting study.[3]

The *Śulba-Sūtra* tradition vanished. Only a limited commentary from a late period is available. Whether the tradition has been lost or the elements have been absorbed in temple architecture is still to be investigated and may be the part of other studies.

Decimal scale, decimal place-value, numerical symbols and zero :

'Our numerals and the use of zero', observes Sarton[4] (1955), 'were invented by the Hindus and transmitted to us by the Arabs (hence the name Arabic numerals which we gave them)'. The study of Sachs, Neugebauer on Babylonian tablets, Kaya and Carra de Vaux on Greek sciences, Needham on Chinese sciences and study of Mayan culture have many interesting issues. The study of scholars likes Smith and Karpinski, Datta, Bag and Mukherjee[5] have analysed Indian contributions, but still there is a need for a comprehensive volume. However, the salient points may be of interest: The Indians had three-tier system of word-numerals starting from the Samhitās as follows:

(a) *Eka* (1), *dvi* (2), *tṛ* (3), *catur* (4), *pañca* (5), *ṣaṭ* (6), *sapta*(7), *aṣṭa* (8),and *nava* (9).

(b) *Daśa* (10), *vimśati* (2 x 10), *trimśat* (3 x 10), *catvārimśat* (4 x 10), *pañcāśat* (5 x 10), *ṣaṣṭhī* (6 x 10), *saptati* (7 x 10), *aśiti* (8 x 10) and *navati* (9 x 10).

(c) *Eka* (1), *daśa* (10), *śata* (10^2), *sahasra* (10^3), *ayuta* (10^4), *niyuta* (10^5), *prayuta* (10^6), *arbuda* (10^7), *nyarbuda* (10^8), *samudra* (10^9), *Madhya* (10^{10}), *anta* (10^{11}) and *parārdha* (10^{12}).

The names and their order have been agreed upon by almost all the authorities for (a) and (b), whereas there is a variation in (c) where mostly one or two terms have been added later. The numbers below 100 were expressed with the help of (a) and (b) sometimes following additive or substractive principles e.g., *trayodaśa* (3 + 10 = 13), *unavimśati* (20 – 1 = 19), while for numbers above hundred, groups (a), (b) and (c) were used. For example, *sapta śatāni vimsati* = (720), *ṣaṣthim śahasrā navatim nava* = (60, 099).

One feature of the application of the scale is that it has been used in higher to lower order (*sahasra, śata, daśa* and lastly the *eka*). Real problem started when the numerical symbols began to appear. The *aṣṭakarṇī* or *aṣṭamṛdam* fairly indicates that Vedic people identified eight marks but whether they identified other symbols is not known. The

Mahābhārata (III. 132-134) narrates a story in which it says that 'The signs of calculation are always only nine in number[2]. The *aṣṭādhāyī* of *Pāṇini* (450 BC) used the word *lopa*, and Patañjali the word *śunya* in connection to metrical calculations. When Brāhmī and Kharoṣṭhi numerals/alphabets appeared on the scene, there were lot of confusions creating more problems for ordinary business people and the mathematicians and astronomers as to how to use the numerical symbols and adjust with the existing decimal system. The early inscriptions show the number system was additive and did not use decimal scale. Moreover numerical symbols were many in the beginning and it was difficult to decipher the correct meaning.

First attempt of a synthesis of the Vedic decimal system with the prevalent situation was possibly made by the Jains. The *Anuyogadvārsūtra* (100 BC) has described the numerals as *anka* as we find in Bengali to mean mathematics and describes decimal scale as decimal places (*gaṇanāsthāna*) and their numeral vocabulary was analogous to, that of the Brahmanic literature. They have enlarged these places to 29 and beyond, and we find more clear statement in mathematics cum astronomical texts from *Āryabhaṭta* onwards in expressions like *sthānāsthānam daśaguṇam syāt* (from one place to next it should be ten times) and *daśaguṇottarāḥ samjñāḥ* (the next one is ten times the previous one). This indicates that the scale was merged with the places, and the system became very simple. For example, the Vedic numbers:

sapta śatāni vimśati and *ṣaṣṭhim sahasrā navatim nava* reduces to:

sahasra (10^3),	*śata* (10^2),	*daśa* (10),	*eka* (1)	Places
	7	2	0	= 720
60	0	9	9	= 60,099

The order of the Vedic scale was from higher to lower (*sahasra, śata, daṣa* and *eka*). But later, the order of the scale was changed from left to right i.e., *eka, daśa, śata* etc. This is obvious when we think that in the Vedic system all words were spoken, and in the latter system the scale obviously followed the written style (that is from left to right) and the place values were from *eka* to higher order. Moreover, the symbols were not standardized and interpreted differently in different regions. To avoid this problem, experts coined synonymous words and used them as symbols in decimal place-value in lower to higher order and the actual number was obtained by reversing the number. For example: *śunya* (0), *dvi* (2), *pañca* (5), *yama* (2) was actually 2520.

The Kashmiri *Atharva-Veda* also uses similar symbols.

Association of decimal scale with place-value was so popular in Indian tradition that it was not even commented upon. The popularity went to deep that even Śankaracārya (c. AD 800), the great social reformer, pointed out that the same numerical sign if placed in unit, tenth, and hundredth places becomes 1, 10, 100. The men in business or in elementary schools (*pāthśalā*) used wooden board (*pāṭī*), hence the name *pāṭīgaṇita*, for quick calculation, in which dust was spread and finger or hard materials were used for calculation. The system also moved to Java, Malaya, and other East. Indian colonies along with the business people, which is evidenced from some available inscriptions. The use of decimal place-value in lower order with word-numerals and higher to lower order with numerical symbols was in practice. For calculation on a *pāṭī*, numerical symbols were used, but for writing or copying a manuscript, final results were written in word numerals to avoid confusion in decoding a symbol and also to keep rhythm in verses in which it was written. The zero in many places of *Bakshālī. Ms* (AD 400) has been used as a round symbol (*śunya*, 0). It also came out as dot (·), may be, that thick tip of pen used for circle became dot in the process. This is distinctly visible in *Bakshālī Ms* and the Kashmiri *Atharva-Veda.* Alberuni (c. 1020 AD) has incidentally referred to two systems of notation of numbers, namely alphabetic (*abjd*) system (*Huruf al-jummal* or *HiSab al-jummal*) and the Indian numerals (*al-Arqam al-Hind*). He has recorded Indian numerals of nine symbols, and zero as dot in the *Kitab al-Tafhim* (the book of instruction in the art of astrology). He also referred to circular symbol (O) of the Indian. *Al-Khwārizmiī* (825 AD), another Central Asian scholar, writes about Indian numerals[6] thus, 'The beginning of the order is on the right side of this writer, and this will be the first of them consisting of unity. If instead of unity they wrote X, it stood in the second digit and their figure was that of unity, they needed a figure of ten similar to the figure of unity so that it became known that this was X, and they put before it one digit and wrote it in a small cicle "0" so that it would indicate that the place of unity is vacant.' The Indian name *śunya* was taken over by the Arabs as *aṣ-ṣifr*. This was subsequently changed to *zephirum* (1202, Fibonacci), *tziphra* (1340, Planudes) and *Zenero*, *zepiro* (sixteenth century, Italy).

Astronomical Features :

The fire sacrifices including construction of fire altars appearing in the *Saṃhitās*, *Brāhmaṇas* and the *Śrauta-Sūtras* clearly indicate that *agnihotṛs* were concerned mainly with directions for laying the sacrificial fires, new and full moon, the season and accurate calculation of the times of the year etc. The fixation of east-west and north-south lines was considered very important for the construction of altars. *Kātyāyana* says′ that eastern and western shadow-points of a central pole in a circle on equinoctial day fix the east-west line and the line perpendicular to that gives the north-south line. The *Śatapatha-Brāhmaṇa* reports the *Kṛttikā* never deviated from the east. R.N. Apte, the well-known Vedic scholar feels that east point was verified by the rising point of *Kṛttikā* in the Vedic period. The Vedic people took Sun as the sole light-giver of the universe, the cause of the seasons, winds, controller and the lord of the world. The Moon was described as *sūryaraśmi*, one which shines by the sun's light. Different phases of the moon viz., *rākā* (full-moon day), *anumati* (preceding full-moon day), *kuhu* (next full-moon day), *śinivāli* (preceding new-moon day) etc. was known. The *Taittirīya-Brāhmaṇa* gives a full list of name of 15 days of the light half (*pūrva pakṣa*) and also of dark half (*aparapakṣa*). The day was called *vāsava* or *aha* and reckoned from sunrise to sunset. The day was further divided into different parts. The period from one moonrise to the next or from one moonset to the next was known as *tithi*, which is somewhat different from the present concept of *tithi* of fixed time. That the phenomenon of new and full-moon is related to moon's elongation from the sun was also correctly guessed. The invisibility of the Moon on the new-moon day is explained by its being swallowed by the Sun and its appearance by its being released by the Sun.

Nakṣatras, Months, Names of Seasons and New Year :

The *nakṣatras* or the group of stars near which the moon could be seen are the conventional division or marker of the ecliptic, (the path) which is followed by Moon, Sun and the Planets. The Moon returns to the same position in more than 27 days but less than 28 days. There is mention of 28 *nakṣatras* early *saṃhitās*, *Atharva-Veda* and others, but the numbers were reduced to 27 from the time of *Vedānga Jyotiṣa*. The names of these *nakṣatras* with their presiding deities are enumerated in the *Yajur-Veda* beginning with *Kṛttikā*. The lunar (or synodic month) was measured from full-moon to full-moon or from new-moon to new-moon (*TS.* 7.5.6). The *Taitt*, *saṃhitā* (5.6.7) refers to 12 or 13 lunar months of a year and calls the 13th (intercalary) month by the names *Saṃsarpa* and *Aṃhaspati* (*TS.* 1.4.14). The *six ṛtus* in solar year with names of 12 tropical months are in *Taitt. Sam*, (4.4.11.1) and *Vājasaneyī saṃ* (13.14). Studies have already been made on *Rohiṇi*, *Kṛttikā*, *Bharaṇī* and *Aśvinī* legends by Tilak, Shamaśāstry, Sengupta, Dikshit, Kupannaswamy Śāstry which indicate that the reference point on the ecliptic on equinoxial day (when day and night were equal) shifted to asterism *Rohiṇi*, *Kṛttikā* (Saṃhitā period), *Bharaṇī* (*Vedānga Jyotiṣa* period), *aśvinī* (*Sūrya Siddhānta* period) in course of time. A seal from the Mohenjo-daro (*M.* 2430) corroborates the *Kṛttikā* legend[7] which supports that the new year started from *Agrahāyanī* after the full moon at *Kṛttikā*. By the time of *Kauśitakī-Brāhmaṇa* (19.8) the new year started with the winter solstice (shortest day) and it was on the new moon day of Māgha. This indicates that the purnimānta system (from the termination of full moon) changed to amānta system (from the termination of new moon) with time and the effort went on to synchronize lunar month with the beginning of the tropical year and seasons.

	Nakṣatras Taitt-Saṃ 4.4.10 Athar. Ved. 19.10	Lunar Months	Solar Months & Seasons Taitt-saṃ 4.4.11 Vāj. Saṃ 13.14	
1.	*Kṛttikā* (Alcyon, 360 long)	Kārtika	Urja	Śārada (Autumn)
2.	Rohini (Aldebaran, 460)			
3.	Mṛgaśirsa (λ Orionis, 600)	Agrahāyanī	Saha	Haimanta (Dewy)
4.	Ārdrā (Betelguese, 650)			
5.	Punarvasu (Pollux, 900)			
6.	Puṣyā (Cancrii, 1050)	Pauṣa	Sahasya	
7.	Āśleṣa (Hydras, 1090)			
8.	Maghās (Regulas, 1260)	Māgha	Tapa	Śiśira (Winter)
9.	(Pūrva) Phalguni (δ Leonis, 1380)	Phalguni	Tapasya	
10.	(Uttara) Phalguni (Denebola, 1480)			
11.	Hastā (δ Corvi, 1700)			
12.	Citrā (Spica, 1800)	Caitra	Madhu	Vasanta Spring
13.	Svātī (Arcturus, 1810)			
14.	Viśākhe (Centauri, 2160)	Vaiśakha	Mādhava	
15.	Anurādhās (Scorpi, 2190)			
16.	Jyeṣṭhā (Antares, 2260)	Jaiṣṭha	Śukra	Grīṣma (Summer)
17.	Mūlā (λ Scorpi, 2410)			
18.	(Pūrva) Āṣādhās Sagittarii, 2510)	Āṣāḍa	Śuci	
19.	(Uttara) Āṣādhās Sagittarii, 2590)			
20.	Abhījit (Vega, 2620)			
21.	Śravanā (Altair, 2780)	Śrāvaṇa	Nabha	Varṣā (Rainy)
22.	Śraviṣṭhās (β Delphini, 2930)			
23.	Satabhisā (λ Aquarii, 3180)			
24.	(Pūrva) Proṣṭhapadās (Markob, 3300)	Bhadrapadā	Nabhasya	
25.	(Uttara) Proṣṭhapadās (√ Pegasi, 3460)			
26.	Revatī (ℓh Piscium, 3560)			
27.	Aśvaujau (β Arietis, 100)	Aśvina	Isa	Śārada (Autumn)
28.	Bharaṇīs (41Arietis, 250)			

Luni-solar adjustments and time units :

The natural means of measuring a year originated from the experience of periodic recurrence of climatic seasons. Likewise, the natural means of measuring a day was the period between two full moons. The return of the Sun to the same position with respect to the fixed star might have appeared to be much more reliable than the slow seasonal variation of the length of day light. There appears to be a constant attempt at adjusting the lunar month with the season. The *Taittirīya-Saṃhitā* (7.2.6) mentions how eleven days ceremony (*ekādaśarātra*) were performed after lunar year of 354 days to make up with the seasons (*ṛtus*) i.e., with the sidereal year of 365.25 days. The idea of intercalating a month at regular intervals of time or of adding of five or six days in one month or more months was thus developed. Naturally, three units of time measurement viz., the solar day, the lunar month and the solar year are involved. Consequently the luni-solar adjustments depended on the problem of finding the integers x, y, z which satisfy the relation, x years = y months = z days. The *Ṛg-Veda* gives two years (*samvatsara, parivatsara*), *Taittirīya-Brāhmaṇ* (1.4.10) mentions four years (*samvatsara, parivatsara, idavatsara* and *ānuvatsara*). Shamśāstry also believed in a four year cycle, first three years of 360 days and fourth year = 146¼ = 365¼ days. The *Vedanga Jyotiṣa* gives (both *Ṛgvedic* recension – *ārca jyotiṣa*, 36 verses and *Yajurvedic* recension - *yājuṣa jyotiṣa*, 43 verses) a five-year cycle, the number of sunrising, moon rising, *tithi*, *nakṣatra*, the new-moon, full-moon, account for solstices, seasons, Sun's northward and southward journey, increase and decrease of day lengths during these journeys, rules for determining the beginning of the season, etc. It prescribes five solar years = 62 sidereal months (moon's revolution) = 67 lunar months (synodic) months. In this cycle of five years, solar days = 5x12x30 = 1800, civil days (solar rising_ = 1830, sidereal days (solar rising plus solar cycle, earth's rotation) = 1835. A five year was conceived as *yuga* in *Vedānga Jyotiṣa*. The conception of *caturyuga*, and *kalpa* were later developments. The *Vedānga Jyotiṣa* (*Y-VJ*, 24) also gives time units (measured by water clock), 1 *āḍikā* = 50 *palas*, 1 *kuḍava* = 1/16 *āḍikā* = 3⅛ *palas*, 1 *nāḍikā* = 200 palas – 3/16 *āḍakas* = 4 *āḍakas*-3/16 *āḍakas* = 6 1/16 *āḍakas* = 61 *kuḍavas*, 1 *nāḍika* = 10 1/20 *kalās*, 60 *nāḍikās* = *muhūrtas* = 1 day.[8]

Astrological tradition :

The *Atharva Jyotiṣa* (in 14 chapters, 162 verses) attached to *Atharva Veda* tradition deals with the *muhūrta* branch of astrology. *Atharva Veda* says that it was taught by Pitāmaha to Kaśyapa. The *muhūrta* as a time unit comes first in the *Śatapatha Brāhmaṇa* (12.3.2.5) which gives 1 *divasa* = 30 *muhūrtas*, 1 *muhūrtas* = 15 *kṣipra*, 1 *kṣpra* = 15 *itarhis*, 1 *itarhi* = 15 *idānis*, 1 *idāni* = 15 *prāṇas*. The *Atharva Jyotiṣa* gives the unit as *ahorātra* (whole day), *muhūrta*, *truṭi*, *kalā*, *lava*, former being 30 times the next instead of 15 times. The last unit is *nimeśa* (1 *lava* = 12 *nimeśa*). Reference to 7 planets, sun, moon and five planets are found. The *Atharva-Veda* (19.9) writes, 'May the Gandramāsa and Āditya *graha* along with Rāhu prove auspicious to you'. The *Atharva Jyotiṣa* also gives names of seven week days with names of planets and describing planets as 'Lord of the days'. The names are *āditya* (sun), *śoma* (moon), *bhauma* (mars), *budha* (mercury), *bṛhaspati* (Jupiter), *bhārgava* (Venus) and *śani* (Saturn). In other places it has also used names of planets as being applicable to planets like *Sūrya* (Sun), *Lohitānga* (Moon), *Śomasuta* (Mercury), *Devaguru* (Jupiter), *Bhṛgu*, *Śukra* (Venus), *Muryasuta* (Saturn). This also contains the seeds of prediction of *Jātaka* astrology expressing fears, woes and horrors when certain *nakṣatras* are accompanied by planets, meteors (*ulkā*) etc. The *Aśvālayana Sutra* contains instructions like fields should be ploughed on the *uttara proṣṭhapāda*', (*Gṛhya-Sūtra*, 2.10.3), 'thread ceremony should be performed on auspicious *nakṣatras*' (*Gṛhya-Sūtra*, 1.4.1) etc. The words, *muhūrta* and *kṣana* also appear in the '*nirukta*'. The *Yājñavalka Smṛti* gives the names of nine planets (seven planets, *Rāhu* and *Ketu* (*Ācāradhyāya*) and twelve parts of the ecliptic (*rāśis*) in order to find proper times (*Śūrya-saṃkarama*) for performing the *śrādha*. The *Yavana Jātaka* of Sphujidhvaja[9] (AD 269) is considered as a major work carrying Greek influence. But it has also used (last chapter) time units viz. *palas*, *kuḍavas*, *liptās*, *nāḍikā*, *kāla*, *muhūrta*, (or *kṣaṇa*) same as *Vedānga Jyotiṣa*, with the exception that it has used 1 *nāḍikā* = 10 *kālas* instead of 10 1/20 *kālas* used by *Vedānga Jyotiṣa*, possibly to avoid fraction. It has conceived a cycle of 165 years instead of 5 years and has given the number of solar months, solar days, civil days, intercalary months, *tithes*, omitted *tithes*, sidereal months and other elements for this cycle. The *Bṛhajjātaka* (or *Bṛhat Jātaka*) is the next most important work on astrology by Varāhamihira (c. AD 505) who also compared *Bṛhat Jātaka* (horoscopic work) and three smaller works on marriage and *yātrā* (journey) on astrology, besides collection of five important works on astronomy (*Pañcasiddhāntikā*). Varāhamihira referred to Parāsara (twice, *Māṇḍavya* and *ācaryas* like *Satya*, *Maya*, *Yarana*, *Manithya*, *Jivasarma*, *Viṣṇugupta* (identified as Cāṇakya *Viṣṇugupta*, the minister of Candragupta by the commentator). *Bhaṭṭotpala* (c. AD 966), the commentator on the *Bṛhat Jātaka* has given astrological quotations from the works of *Gārgī*, *Badarāyana*, Yājñavalkya and Māṇḍyavya. The *Bṛhat Jātaka* contains 36 Greek words namely, the 12 signs – *kriyā*, *taburi*, *jituma*, *kulir*, *leya*, *pāthena*, *jūka*, *kaurpya*, *tauksika*, *akokera*, *Hridroga*, *Ittham* and twenty four other words including the words, *liptā*, *horā*, *dreskāna*, *Kendra*, *koṇa* etc. On the basis of these terms, Weber and Kern believe that there was Greek influence on Indian astrological works. Large number of late astrological works on *jātakas* (*Kālacakra-Jātaka*, *Gauri-Jātaka*, *Mīnarāja-Jātaka*, *Jātakasāra* of Nṛhari, Sārāvalī-*Jataka Paddhati* etc.) dealing with misery and woe based on ascendants and descendents of planets, *horās* (by Vaśiṣṭha, Garga, Parāśara, Varāha, Lalla, Nārada etc.) dealing with whether an event will happen at all, if so, when and how on the basis of horoscope cast, *muhūrtas* (*tithes*, *nakṣatras* prohibited for auspicious ceremony, taboos etc.), *pāśaka vidyā* (questions answered according to the casting of dice), *tājik* (forecasting as per ascendance and descendence of planets) etc. were in vogue.

Yuga, Kalpa and Mahāyuga :

The *Vedāṅga Jyotiṣa* had a cycle of five years in a *yuga*. The *Yavana Jātaka* had 165 years cycle. The *Pāṇini*, though refers to *yuga* and *kali*, it is *Manusmṛti* which gives four *yugas* (*Kṛta*, *Tretā*, *Dvāpara* and *Kali*) = 12000 divine years = 4320000 ordinary human years, since 1 Divine year = 360 ordinary years. The *Mahābhārata* also had the same *yuga* and time units as that of *Manusmṛti*. The Siddhāntic astronomy, unlike Greek astronomy, has established on epoch when all planets were in zero longitude. Āryabhaṭṭa I considered on epoch when the Sun, the Moon, Mars, Mercury, Jupiter, Venus and Saturn were last in zero longitude at sunrise in Lankā (a hypothetical place at the intersection of the equator and the meridian of Ujjain) on Feb. 18, Friday, 3102 BC. The period of one such epoch to the next, according to Āryabhaṭṭa I is 1,080,000 years. When the Moon's apogee and the Moon's ascending node are included in the list of the planets, the above mentioned period (*mahāyuga*) becomes 4,320,000 years, which is defined as the duration of *yuga*. The *yuga* is a period of time which begins and ends when the Sun, Moon, Mars, Mercury, Jupiter, Venus, Saturn, the Moon's apogee and the moon's ascending node are in zero longitude. It consists of 4 periods, *Kṛta*, *tretā*, *dvāpara* and *kali*, the current quarter *yuga* is the current *kaliyuga* which is assumed to have begun at sunrise at Lankā on Friday, 18 Feb., 3102 BC. A bigger period than the *yuga* is called *kalpa*. According to Āryabhaṭṭa I, *kalpa* consists of 1008 *yugas*, and 459¾ *yugas* had elapsed at the beginning of current *kaliyuga* since the beginning of current *kalpa*. The main difference in Āryabhaṭṭa, Sūrya-Siddhānta and Brahmana Siddhānta School is that the length of the year and motion of planets in a *kalpa* and *mahāyuga* is different and other elements are same. There is also difference of opinion as to the starting of *kali* era on 18 Feb., Friday, 3102 BC, which needs more careful scrutiny. The Hindu astronomical works called *Siddhānta* adopt the terms of creation as the epoch of calculation whereas those called *kaliyuga* as the epoch of calculation.

Old Siddhāntic Tradition :

The Jains did positive contributions to mathematics. A few works like *Surya Prajñnapti*, *Candra Prajñapti*, *Jamboo Divīpa Prajñapati*, *Sthānānga Sūtra*, *Bhagavatī Sūtra*, *Anuyogadvāra Sūtra* are available to us. It deals with problems dealing with circle, chord, circumference, p (=10), diameter, arc, segment, big numbers, infinity, laws of indices, symbols, operations etc. Varāhamihira was born in AD 505 in the village of Kapithaka (Farrukabad district of Uttar Pradesh) and moved to Ujjain. His forefathers migrated to India from Maga country in Persia and settled in the Kapithaka. He quoted Āryabhaṭṭa I several times and compiled *Pañcasiddhāntikā* i.e., five siddhānta works, namely, *auliśa*, *romaka*, *vaśiṣṭha*, *saura* and *paitāmaha* besides astrological works. Colebrooke (1807-17), Whitney and Burgess (1860), Kern (1865), Thibaut (1890) and a few other European scholars passed judgement on the relative importance and origin of Indian astronomy. Thibaut in his introduction to the *Pañcasiddhāntikā* observes that the *Paitāmaha Siddhānta* (c. AD 80) is the oldest and carries pre-scientific stage of astronomical knowledge. The *Vaśiṣṭha Siddhānta,* written prior to AD 269 is more advanced. The *Romaka* and *Pauliśa* have Greek influence. The *Saura Siddhānta* only contains new features. During the early centuries of the Christian era the Indians were in touch with the Greeks, Romans and other scholars and those of Babylonian and Greek knowledge may have been available to them.

The scholars like Dikshit, Sengupta, Ganguly, Kuppannaswamy and Shukla[11] testified that the refinements introduced by Ptolemy (AD 150) and even Hipparchus (150 BC) remained unknown to India. Whatever Greek influences are there, they are all of pre-Ptolemaic period and possibly of pre-Hypparchus time. The extent and nature of contact were through conferences or direct borrowal through translation of texts is still to be investigated. Neugebaur has shown that the *Vaśiṣṭha* and *Pauliśa* were inspired by Babylonian linear astronomy.

The *Pañcasiddāntikā* (five *siddhāntas*) were known in India from first century AD to fifth century AD. By this time, the Indians had already acquired the knowledge of zero and decimal place-value, eight fundamental operations of arithmetic addition, subtraction, multiplication, division etc., rule of three, inverse rule of three, knowledge of combinations of six savours (a, b, c, d, e, f), 2 at a time, c (6,4) – ab, ac, ad, ae, af, bc, bd, be bf, cd, ce, cf, de, df, ef – fifteen in all), 3 at a time c (6,3), 4 at a time c (6,4) was known. Likewise, the knowledge of binomial expansion for calculating the shortcomings in metrical rhythm of music based on *long* (a) and short (b) sounds were known. Or in other words binomial expansion like $(a + b)^2 = 1.a^2 + 2a.b + 1.\ b^2$, $(a + b)^3 = 1.a^3 + 3.a^2b + 3.ab^2 + 1.b^3$, $(a + b)^4 = 1.a^4 + 4.a^3b + 6\ a^2b^2 + 4ab^3 + 1.b^4$ and achieved various other mathematical results. These undoubtedly brought great change in the Indian scenario in the field of mathematics and astronomy. The development of algebraic and trigonometric tools also revolutionized the calculations and methods in astronomy.[10] A series of writings came in with Āryabhaṭṭa II Āryapakṣa school, Lāṭadeva, the student of Āryabhaṭṭa I and author of revised (*Sūryasiddhānta Sūryapakṣa* school), Brahmagupta (Brahmapakṣa school), Bhāskara I (AD 628), student of Āryabhaṭṭa School and a host of other scholars namely Lalla (c. AD 749), Vaṭeśvara (AD 904), Āryabhaṭṭa II (AD 950), Śrīpati (AD 1039), Bhāskara II (AD 1150) and others. Āryabhaṭṭa I, an *āsmakīya* (Keralian) lived in Magadha (modern Bihar) and wrote his *Āryabhaṭṭa*. Magadha in ancient time was a great centre of learning and is well-known for the famous university at Nalanda (situated in the modern district of Patna). There was a special provision for study of astronomy in this university. Āryabhaṭṭa I is referred to as *kulapa* (= *kulapati* or Head of a University) by the commentator.

Āryasiddhānta and Āryabhaṭīya of Āryabhaṭai (b, ad 496) :

The *Āryasiddhānta* of Āryabhaṭṭa is only known from the quotations of Varāhmihira (AD 505), Bhāskara I (AD 600) and Brahmagupta (AD 628) in which the day begins at midnight at Lankā. The *Āryabhaṭṭa* begins the day with sun-rise on Sunday *caitra-kṛṣṇādi*, *Śaka* 421 (AD 499). A summary of contents of *Āryabhaṭṭa* will give an idea how the knowledge exploded. Under arithmetic, it discusses alphabetic system of notation and place-value including fundamental operations like squaring, square root, cubing and cube root of numbers. The geometrical problems deal with area of triangle, circle, trapezium, plane figures, volumes of right pyramid, sphere, properties of similar triangle, inscribed triangles and rectangles. Theorem of square on the diagonal, application of the properties of similar triangles. The algebra has concentrated in finding the sum of natural numbers (series method), square of n-natural numbers, cubes of n-natural numbers, formation of equation, use of rule of three for application (both direct and inverse rule), solution of quadratic equation, solution of indeterminate equation [by = ax + c, X = (by – c) / a] where solution of x and y were obtained by repeated division (*kuṭṭaka kuṭ* means to pulverize) etc. In trigonometry, *jyā* (R Sine) is defined, and 28 *jyā* table at an interval of 3^0 45' (R = 3438') was constructed, the value of $\pi = 3.1416$ was found to be the correct to 34 places of decimals. Āryabhaṭṭa I's value of p = 62832 / 20000 = 3 + 1/7 + 1/17 + 1/11. Successive convergents are 3, 22/7, 355/113, 3927/1250 which were used by later astronomers. In astronomy, three important hypothesis were made viz.,

(1) The mean planets revolve in geocentric circular orbits,
(2) The true planets move in epicycles or in eccentric,
(3) All planets have equal linear motion in their respective orbits.

The knowledge of indeterminate equations played a significant role. The method of indeterminate equation was a successive method of division. The same method is possibly used for value of, solution of first degree and second-degree indeterminate equation. It was also used to determine the mean longitude of plane for mean longitude = (R x A)/C, where R = revolution number of planets, A = *ahargaṇa* = no. of days since the epoch and C = no. of days in a *yuga* or *kalpa*. Large number of astronomical problems of Bhāskara I are changed to (ax – c)/b = y where x = *ahargaṇa* any y = Sun's mean longitude.

Astronomical corrections and astronomical instruments :

The geocentric longitude of a planet is derived by the mean longitude by the following corrections.

(1) Correction for local longitude (*deśāntara* correction).
(2) Equation of the centre (*bāhuphala*).
(3) Correction of the equation of time due to eccentricity of the ecliptic.
(4) Correction of local latitude (*cara*) in case of sun and moon, and an additional correction (*śighraphala*) in case of other planets.

Besides these, Vaṭeśvara (904) gave lunar correction, which gives deficit of the moon's equation of centre and evection. Bhāskara II (1150) gave another correction, variation.

Mañjulā (932) used a process of differentiation in finding the velocity of planet. All siddhāntic astronomy gave method and time of eclipse, along with *tithi*, *nakṣatra*, *kāraṇa*, *yuga*, since these had an important bearing on religious observations.

A large number of astronomical instruments were referred to and used. To cite a few from Lalla's *Śiṣyadhivṛddhida*[11] (eighth – eleventh centuries), these are (1) air and water instrument, (2) *golayantra*, (3) man with a rosary of beads, (4) self-rotating wheel, and self-rotating spheres, (*cakra yantra* circle), (6) *dhanur yantra* (semi-circle), (7) *kartari yantra*, the scissors, (8) *kapāla yantra* (set horizontally on the ground and its needle vertical), (9) *bhagaṇa yantra*, (10) *ghaṭī yantra* and conversion of observed *ghaṭīs* into time only, (11) *śanku yantra*, (12) *śalākā yantra*, needle, (13) *śakaṭa yantra* (for *tithi* observation), (14) *yaṣṭhī yantra* and graduated tube (for altitude, zenith distance and *bāhu*) and others.

The extension of knowledge by Kerala astronomers :

The knowledge of *jyā* (or *jiva*), *kojyā* and *śara* for a planet in a circle of known radius (*trijyā*) was used. The scholars used gradually improved value of (*trijyā*) where Sinus totus = 24^{th} *jyā* = R = 3438' (Āryabhaṭṭa I), 120' (*Pañcasiddhānta*), 3270' (Brahmagupta), 3437'44"19" (Govindasvāmī, AD 850), 3437'44"48"' (Mādhava c. AD 1400). From the relation $C = 2\pi R$, where C = circumference and R = radius of the circle, R was calculated. The value of C was taken as C = 360 degrees = 21600 minutes and $\pi = 3.1416$ (Āryabhaṭṭa I). Mādhava (c. AD 1400) used a value of π correct to 11 places of decimals. Mādhava used knowledge of series to approximate the value of *jīva* $= s = s^3 / 3lr^2$, and *śara* $= s^2/2lr$ for an arc s of radius r and applied them repeatedly.

Important trigonometrical relations were also found by scholars from Āryabhaṭṭa I onwards. The successive approximation[12] of Mādhava and other Kerala astronomers lead, for $s = rx$, to the discovery of

$$\sin x = x - x^3/3! + x^5/5! -- \ldots$$
$$\cos x = 1 - x^2/2! + x^4/4! -- \ldots$$

These were investigated and rediscovered later in Europe.

For the study of history it is important to find out the date or at least a period of any incident in the history of any country. This aspect of fixing the dates of Indian history came to importance when European scholars began their studies of Indian history. The European scholars had no concept of measurement of time with the help of astronomy. They used to count the days and years from a particular king or event. Such records are not present in the ancient history of India. Therefore nobody was able to fix the dates of the ancient Indian history.

It made clear that the Indian authors have given details of time by the astronomical records and by proper studies we can easily fix the dates of various incidents in the ancient Indian history accurately.

We think that the modern scientific calendar is very good because it is easy but it is not good. If a calendar from our wall is lost we are unable to find the date and month. If we happen to go to some isolated island after wrecking our ship then in four to five days we will forget the date and month and then will never be recalled. But a man like me who has studied the Indian calendar can easily tell *Tithi*, *Nakṣatra*, *Pakṣa,* Lunar months by looking at the night sky. By looking at the rising sun *Ṛtu* and *Ayana* can be told. Counting of the days can be done by *Tithi* and *Nakṣatra*. *Tithi* depends on the size and shape of the Moon. *Nakṣatra* is the star near which the Moon is seen. Every day the Moon changes its *Nakṣatra* and completes a round in 28 days. Thus the Indian calendar is eternal.

Nakṣatra tells the position of the Moon among the stars. *Tithi* tells the distance of the Moon from the Sun, because in one *Tithi* the Moon goes 12 degrees away from the Sun. Lunar month is named after the *Nakṣatra* near which the full-moon resides. It is also the *Nakṣatra* which rises at the Sunset and remains visible throughout the night. From the name of the Lunar month the Moon's position is fixed and from it the position of the Sun can be easily calculated. The sun is exactly opposite at the full-moon while both are together at *Amāvasyā.*

Ṛtus are six in number, three in each *Ayana. Ayanas* are two, *Uttarāyana* and *Dakṣināyana. Uttarāyana* means the Sun's northern journey. It begins with *Shishira Ṛtu* at the Winter Solstice on 22nd December of the modern calendar. *Dakshināyana is the southern journey of the Sun which begins with the Varṣā Ṛtu* at the Summar Solstice on 22nd June. *Varṣā, Sarad, Hemanta* are the three *Ṛtus* of *Dakshināyana,* while *Shishira, Vasanta* and *Greeshma* are the three *Ṛtus* of *Uttarāyana.* Each *Ṛtu* consists of two solar months. Nabha and Nabhasya are the two solar months of *Varṣā Ṛtus, which* correspond with 22nd June to 21st July and 22nd July to 21st August. *Iṣā* and *Urjā* are the two months of *Sharad. Isa* corresponds to 22nd August to 21st September. A period from 23rd September to 22nd October means *Urja.* 23rd September of the beginning of *Urja* is the Autumnal Equinox having equal day and night. *Hemanta ṛtu* consists of *Saha* (22 October to 21st November) and S*ahasya* (22nd November to 21st December). 22nd December is the Winter Solstice when *Tapa,* the first month of *Ṛtu Śiśira* begins. The second month of *Śiśira* i.e. *Tapasyā* extends from 22nd January to 21st February. *Mādhu,* the first month of *Vasanta Ṛtu* begins on 22nd February and ends with the Vernal Equinox on 21st March. Then *Mādhava,* the second month of *Vasanta* begins which ends on 21st April. From 22nd April *Greeshma* begins, the first month of which is *Ṣukra.* Its second month *Suchi* begins on 22nd May and ends on 21st June. 22nd June is the Summer Solstice when Nabha and Varṣā begin. Thus the solar seasonal months of *Ṛtus* correspond with the modern seasonal months (*Taittirīya Samhitā* 4.4.1. Also *Viṣhṇu Purāṇa Ansha* 2, *Adhyāya 8, Śloka* 83).

Viṣṇu Purāṇa 2.8. 70 clearly speak about two solar months forming *Ṛtu.*

Ancient authors are particular in give *Tithi, Nakṣatra,* lunar month and *Ṛtu.* Sages are particular to mention the *Nakṣatra* of the Sun at the Winter Solatice or the Summer Solstice or the equinoxes. From these records it is evident that the ancient sages knew of the precession of Equinoxes due to which the Sun recedes in the *Nakṣatra Cakra.* If we know the rate of the precession of equinoxes as one degree in 72 years or one *Nakṣatra* in 960 years or one *Rashi* i.e. 30 degrees in 2160 years, we can calculate the year or the period of any incident about which these records are available. We get the data for calculations if the *Ṛtu* and lunar month is given, because the lunar month indicates the Sun's position in *Nakṣatras* while *Ṛtu* indicates the Earth's position in relation to the Sun. Let us now see how precise is the method to fix the dates or years or period of some incidents.

Pañcānga means five important points regarding the time. These are *Tithi, Nakṣatra, Māsa, Ṛtu* and *Ayana.* In addition to these five, the sages were particular to record the positions of the planets, which pinpoint the time.

In the epic *Mahābhārata*, Maharshi Vyāsa gives the positions of all the planets in Bhishma Parva, *Adhyāya* 2,3, thus – Saturn in *Pūva*, Jupiter in *Śārvaṇa*, *Rāhu* in *Uttara Āṣāḍha* and Mars in *Anuradhā.* All these planets have a fixed rotational period. Saturn completes one rotation in 29.4545832 years. Jupiter takes 11.863013 years for one rotation. *Rāhu* takes 18.5992 years, Mars takes 1.88089 years per rotation. Taking the respective positions of each of these planets in any year we can calculate backwards to find out when the planets were in the said positions. I found the year to be 5561 years before Christ. When Bhishma died it was Winter Solstice and he had fought for ten days and had lied on the arrow bed for 58 days. 68 days earlier than 22nd December is 16th October when the Great War must have commenced. Thus we could fix the date of the *Mahābharata* war exactly as 16th October 5561 B.C. Three more planets are found described by Vyāsa in the *Mahābhārata* under the names of *Shveta*, *Shyama* and *Teevra.* These are Uranus, Neptune and Pluto. Calculating back we can find that the respective positions get confirmed on the above date. In addition Vyāsa describes two consecutive *Amāvasyās*, one *Kṣaya Pakṣa,* two eclipses lunar and solar in one month's time and a big comet. All these evidences show the same date 16th October 5561 B.C. Eighteen mathematical points converge on this date and many other evidences point to the same date. Therefore we have to accept it as the date of the beginning of the *Mahābhārata* war.

Accepting this date we can fix the dates of about sixty incidents from the *Mahābhārata* including the date of exile 4th September 5574 B.C. and exposure of Arjuna on 16th April 5561 B.C.

Except Astronomy no other method can give such details and exact dates. In the case of the *Mahābhārata* we have to solve some riddles put forth by the sage Vyāsa deliberately, but in another books no such problem arises.

Ṛtus and lunar months as described in the *Mahābhārata* point to the same time of 5561 B.C. The planet Saturn had been in the arms of Rohini according to the *Mahābhārata.* According to the calculations of late Shri Dixit S.B. this phenomenon took place for many centuries before 5294 B.C.

At the time of Rāma's birth five planets were exalted according to *Vālmīki Ramāyaṇa.* The exalted positions well-known in astrology are thus : Sun 10° in *Meṣa*, Mars 28° in *Makara*, Jupiter 5° in *Karka*, Venus 27° in *Meena*, Saturn 20° in *Tulā.* For calculations of thousands of years only the slow moving planets are useful and only by two planets Saturn and Jupiter we cannot fix the exact date. At least three planets are essential. In the *Vālmīki Rāmāyaṇa,* it is stated at *Ayodhya Kāṇḍa* 4/18 that the Sun, Mars and Rāhu were aspecting *Dasaratha's Nakṣatra* at the time of Rāma's coronation on *Caitra Śuddha* 9th, when Rāma completed 17 years of his life. In the *Caitra Māsa* Sun resides in *Meṣa* Rāśi, so Rāhu also must have been *Meṣa* or just opposite in *Tulā* Rāśi. Rāhu completes one rotation in 18.5992 years, hence 17 years earlier Rāhu must have been in

Kanyā i.e. Virgo. Calculations on this data show the date of Rāma's birth as 4th December 7323 years B.C.

Instead of coronation Rāma had to leave for forest in *Hemanta Ṛtu* on *Caitra Śuddha* 9th. At this occasion Vālmīki states the star positions at Ayodhyā 41/10, 11. These tally with the modern mathematics. Rāma went to the forest life on 29th November 7306 B.C. Rāma was married to Sitā on 7th April 73-7 B.C., in *Bhadrapada Śuddha* 3rd on *Uttarā Phālgunī Nakṣatra.*

When Hanumāna returned after burning Lankā, Vālmīki describes the sky vividly, which indicates the date as *Pouṣa* Krishna 1st or 3rd September 7292 B.C. The war was fought from 3rd November to 15th November 7282 B.C.

I have fixed almost 40 dates of various important events from the *Rāmāyaṇa*, showing that the astronomical records are helpful in fixing dates in history.

Vedic literature records abundant astronomical data, which can fix the periods. Ṛigveda 10161-13 states – "Who awakened Rubhus ? The Sun replied, "The Dog, because today is the end of the year." The Dog means Canis major or *Mṛga Nakṣatra*. Rubhus means clouds. Clouds were awakened by *Mṛga* means rains began with *Mṛga*. At present rainy season begins with *Mṛga*. Thus a cycle of 27 *Nakṣatras* must have been elapsed between the present age and the Vedic age. At the rate of 960 years per *Nakṣatra* 27 *Nakṣatras* have taken 25920 years. So the Vedic statement is 25920 years old. In other word the period of Ṛgveda is 23720 years B.C.

Taittirīya Samhitā 7-4-8 states that *Pūrva Phālgunī* is the last night of the year while *Uttara Phālgunī* is the first night of the new year, and *Vasanta* is the mouth of the year. This shows the Vernal equinox in the month of *Phālguna* at the full Moon. Naturally, the Sun was diagonally opposite in the *Bhādrapada Nakṣatra* at the vernal equinox showing a period of 23720 years B.C.

Ṛgveda 4-57-5 requests *Shunasirau* to shower the water made in heavens on the Earth. *Shuna* means god. *Sirau* means two heads. Two heads of dogs means Canis major and minorine *Mṛga Nakṣatra*. This suggests beginning of the rainy season at *Mṛga Nakṣatra* the period being 23720 years B.C.

The equinoxes recede backwards through *Nakṣatra*. So from *Mṛga* it must have gone to *Kṛttikā*. To show it there is an evidence in the names of the Stars *Kṛttikā*. The constellation *Kṛttikā* is formed of seven small stars, which are named as *Ambā*, *Dulā*, *Nitatni*, *Abhrayanti*, *Meghayani*, *Varṣāyanti*, *Cuputikā* (*Taittirīya Brāhmaṇa* 3-1-4). All these names indicate water. *Abhayanti* means bringing *Abhra* i.e., white clouds. *Meghayanti* means bringing water-loaded black clouds. *Varṣayanti* means bringing showers of rains. At the beginning of rainy season first white clouds of *Abhra* come, then *Megha* or water-loaded clouds come and then showers come. Thus these names definitely point to the rainy season. *Kṛttikā* was bringing the *Varṣā Ṛtu* during 21800 years B.C.

Between *Mṛga* and *Kṛttikā* there is *Rohiṇi*. The rainy season must have begun on *Rohiṇi* some time. It is reported in the *Mahābhārata Vana Parva* 230 wherein it is also stated that the fall of *Vega* or Abhijit began because of *Kṛttikā* went to water i.e. Summer Solstice when *Varṣā Ṛtu* began. This was the condition around 21800 years B.C. Before this, during 22760 years B.C. the Summer solstice was at the *Rohiṇi*.

Apabharaṇi is a Vedic name of *Bharaṇi Nakṣatra*. It was called as *Apabharaṇi* because it was filling water. How a *Nakṣatra* from the sky fill water on the earth? Of course by inducing rains. Thus this name was given around 20840 B.C.

The deity of *Mūla Nakṣatra* is mentioned as *Niruti*. *Nairutya* means southwest. The sages discovered that the rainy season in India comes from southwest winds. Therefore they fixed the deity when *Mūla* was at the summer solstice and introduced rains during 11240 B.C.

Before *Mūla, Pūrva Āṣāḍha* used to induce rains so that its deity is fined as water during a period of 12200 B.C.

Ṛgveda 1-164-19 states, "*O Indra and Soma, you two rotating like a yoked horse are supporting this work.*" These rotating deities are nothing else but the solstices. That time *Jyeṣṭhā* was at the summer solstice whose deity is Indra, and Mṛga whose deity is Soma was at the winter solstice. This period is 10280 B.C.

Taittirīya Brāhmaṇa 1-5-1-6.7 states that *Kṛttikā* to *Viśākhā* is the northern course of the Sun while *Anurādhā* to *Āpabharaṇi* was the southern course of the Sun. This clearly shows that the winter solstice was between *Kṛttikā* and *Bharaṇī* i.e. 26″ 40′ while the summer solstice was between *Viśākhā* and *Anurādhā* at 213″ 20′. On 22nd December 1995 the Sun is at 245″ 50′ 45″ in *Mūla Nakṣatra* on the winter solstice. It has come here from 26″ 40′. So total shift back is 140″ 49′ 15″. The rate of the precession of equinoxes is 50.2 per year. So it is seen that 10098. 704 years ago the statement is written. It comes to 8103 years B.C.

Āśvalāyana Gṛyhya Sūtra 2-3-5 advises to meditate on *Hemanta Ṛtu* on *Margasīrṣa Pūrṇimā*. Naturally, *Hemanta* was ending at *Māgha Pūrṇimā* on the winter solstice. The full Moon at *Māgha* shows the Sun to be at 307″ at winter solistice. At present the winter solstice occurs at 246″. It shows that the Sun has receded back by 61″. The precession rate is 72 years per degree. So 61X 72 = 4392 years ago *Āśvalāyana Gṛyhya sūtra* is written. It shows 2404 B.C. at least 1900 B.C. as its period.

ŚuśrūtaSamhitā 6-10 tells rainy season in *Bhādrapada* and *Āśvina Masas*. It was the condition around 400 B.C. The Samhitā also states that *Māgha Māsa* and *Śishira Ṛtu* began together. This indicates 2000 B.C. Thus it appears that the first edition of *ŚuśrūtaSamhitā* was formed in 4000 B.C. while the second edition was prepared at 2000 B.C.

Maitrāyaṇi Upaniṣad 6/14 records the summer solstice at the beginning of the *Māgha Nakṣatra* or at 120°. At present it is at 65° 40′ 42″. The difference is of 54° 19′ 18″ or

195558 seconds. The precession rate is 50.42″ per year. It shows that *Maitrāyaṇi Upaniṣad* is composed 3895,5776 years ago or 1909 B.C.

Vishnu Purāṇa tells at 2/8/66 to 76 that equal day and night come when the Sun entered *Meṣa* or *Tula Rāsi*. It shows a period not later than 1609 B.C.

Vedānga Jyotiṣa shows the winter solstice on *Dhaniṣṭhā* which cannot be later that 1640 B.C.

Parāśara tells Summer solstice at middle of *Aśleṣ*ā at 113° 20′. Its date comes to 1159 B.C.

Kauśitakī or *Śankhāyāna Brāhmaṇa* 1/3 tells that in the middle of the rainy season one should see at the *Pūnarvasu Nakṣatra* while offering oblation. But in these days in the first fortnight the Moon does not combine with *Punarvasu.* Therefore give oblation on *Amāvasyā,* which comes after *Āṣāḍha,* because on the *Amavasyā* there is *Purnavasu Nakṣatra.*

In the first fortnight of *Punarvasu* is invisible in the months of *Pauṣa* to *Āṣāḍha.* These are omitted. It shows that Jeshtha and *Āṣāḍha* were not the months of rainy season It is suggested to use *Amavasyā* coming after *Āṣāḍha.* Thus it appears that the rainy season used to begin from *Śrāvaṇa Māsa.* At present the rainy season begins from *Jeṣṭha Māsa.* The rainy season has shifted back by two months. It means that the Sun has receded 60°. At the rate of 72 years per degree the shift must have taken place in 4320 years. Thus *Kauśitakī Brāhmana* appears to be written around 2320 B.C.

Matsya Purāṇa 204/5 tells that if one oblates on the *Trayodaśi* with *Māgha Nakṣatra* during *Varṣā Ṛtu* he gets benefited. *Māgha Nakṣatra* on *Trayodaśi* comes in *Sārvaṇa* on *Bhadrapada māsa.* Thus these two were the months of *Varṣā Ṛtu* when *Mātsya Purāṇa* is composed. The period is around 200 B.C.

Shrimad Bhāgavata Purāṇa shows equal day and night when the Sun was in *Meṣa* or *Tula Rāśi*. It shows the time to be 1600 to 2000 B.C.

Kālidāsa describes *Varṣā Ṛtu* from *Āṣāḍha* first day. From that he appears to be at the beginning of the Christian era i.e. 2000 years ago since today.

Varāhmihira gives the summer solstice at *Karkadya* i.e. at 90°. So his date is 520 A.D.

Thus all the steps of one to two thousand years are shown by Astronomy. This cannot be done by Archaeology or linguistics of any other science or method. There is a gap from 12000 B.C. to 20000 B.C. which can be filled up by stray evidences such as the fall of *Vega* (Abhijit) mentioned in the *Mahābhārata* which took place around 13000 B.C. Vālmīki Rāmāyaṇa states that *Daityakula* had *Mūla* as their *Nakṣatra* and *Ikṣvaku Kula Nakṣatra* was *Viśākhā*. It means that *Daitya Kula* began when Vernal equinox was at *Mūla* i.e. 17000 B.C. *Ikṣvaku Kula* started with *Viśākhā* at the Vernal equinox around 15000 B.C. Thus all the steps of history of the ancient Indian culture are seen ranging from 25000 B.C. to 520 A.D. Only Astronomy can give such fineries about time.

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CHAPTER – 7

Mathematical principles in ordinary life

In this section an effort will be made to show that how mathematical principles regulate the life of an ordinary human being, particularly to the villagers. The mathematical sense of them was so prominent that they formulate a peculiar formūla to indicate something else. In fact, they are habituated in using a mathematical phrase on a mathematical code to describe something, which is very difficult to express in ordinary non-mathematical language. Sometimes mathematical principles serve the purpose of artistic expressions, which are normally done by the villagers. A particular number may be seen to be similar with the name of someone's dear one. If he or she wants to express it artistically, he or she will take recourse to the numbers or addition or multiplication of some numbers, the result of which is phonetically similar to his/her wife's or husband's name. Particularly among the villagers and tribal people such techniques taken from mathematics are normally used as modes of expression.

A fairly discernible trend in any venture seeking mathematical traditions in the Indian context seems to be obsessed with the contributions of great celebrities of mathematics in those days. It is uncriticality in such studies that hardly enables anybody to get at the evolution of mathematical thinking or ideas in periods of antiquity. This apparent lack of linkage, often blurred by religious beliefs, between different types of pursuits, doubtless, make such studies vulnerable to a total view which reckons necessarily societal, cultural, historical and, in some sense, philosophical underpinnings. On the other hand, a discipline like mathematics did not have, to speak in modern idioms, elitist adherents only but also practitioners of mathematics, to use modern jargon, at the grass root level. Extant studies on mathematical traditions in India have, by and large, identified areas of pursuits associated with dominantly theoretical studies but not specifically those out of which mathematics can emerge somewhat organically as a discipline oriented activity.

A cultural vis-à-vis anthropological view of mathematics or strictly speaking, mathematical way of thinking becomes a necessity and hence, some examples in the Indian context drawn from folk wisdom and practices are set forth. Next, we delve into conceptual issues on ethno mathematics and its possible offshoots. Environment can hardly be ignored in such contexts and hence, the linkage between ethno mathematics and environment is brought out. Following this, we swing back to the consideration of cultural vis-à-vis anthropological dimensions of mathematics. A matrix model is made use of to bring out relational features between entries on mathematics and those on cultural dynamics.

In the succeeding section, philosophical view of ethno mathematics is considered. Some remarks, as a part of winding up, are made in the context of a wider need for developing or perhaps, identifying an integrated trajectory of mathematical thinking against the backdrop of the vast landscape intermingling society, culture, philosophy, history, ethos and mathematics.

ETHNO MATHEMATICS : CONCEPT OF D'AMBROSIO

The image of mathematics that has come to stay is one of abstraction divorced from realities, but at the same, one that caters to usages of a diverse nature. The use of mathematics, for its own sake and for others, is being contested, and has remained within the confines of a select few; the bulk of society, as it were, is outside the group of the privileged few. This poses a threat to dynamics of culture and society. Hence, in a bid to set at nought possible disruption of societal equilibrium, there is now an effort to seek social and cultural roots of mathematics in the vast repository of human endeavour.

The shift in paradigm lies in looking at reality from the standpoint of 'perception' by individuals. Ubiratan D'Ambrosio, the architect of this view, has proposed a model of human endeavour which has connectivities, reality-individual-action-reality, obviously a cycle, as characteristic of human individuals, see, D'Ambrosio.[1] This does not run counter to hierarchial order of human endeavour, which allows the flow from individuals to collective (or social) to cultural behaviour and ultimately, to cultural dynamics. Knowledge fits in as action in the framework of cyclic model and this allows human endeavour to move from one level to another. The basic assumption, here, is action inherent in individuals, whatever be the dimensions. Action keeps on recasting the reality, which may be material; or may be purely a cognitive (intellectual, psychic and emotional) reality.

The concept of knowledge emerges properly if we look deep into several cultural contexts. Knowledge, in such contexts, does not make a distinction between actions to understand and to create and hence, science and technology, as they are understood, are well reckoned. The complementarities of science and artistic activities are also well assured.

The reality, as mentioned earlier, is taken to have essentially two dimensions; namely, environmental (including natural and artificial) and intellectual (emotional, psychic, cognitive) which constitute the intrinsic and intimate abstract reality of ideas. Thoughts then become part of the reality-affecting individuals along with their emotions. Reality is obviously social and hence, one has to accept the possibility of interconnectivities between environmental, abstract and social entities.

D'Ambrosio[2] builds up the concept of society out of cultural attitudes and diversities so that different groups of individuals behave in a similar way, on account of their modes of thought, interests, motivations, often myths etc. Societal groups exist with clearly defined cultural roots, modes of production, class structure, class conflicts, individual rights etc. Such groups develop, over the centuries, ways to count, to measure, relate and classify or in other words, ways to mathematize. These are in many ways different from the ways, which are done by other cultural groups. The question that now comes up is this: how do we relate mathematics with culture? What kind of culture is this? What kind of mathematics is this?

By culture, we ought to understand something that is added to the world as a result of human labour, creativity of an endeavour for survival; it becomes thus an acquisition to be shared and nurtured by (cultural) groups. The structured form of knowledge; it precedes all forms of scientific understanding of the world through recorded repository of endeavour of all civilizations.

The word *ethno mathematics* is used in such contexts, because the word *ethnos* has come to connote race/culture, necessarily implying specific cultural racial codes, symbols, values, attitudes and so on. D'Ambrosio succinctly puts this as:

Ethno mathematics is a concept resulting from perception of man as an animal in search of survival and continuation of species but with a plus over the other animal species. This plusis the drive to transcend one's won existence (the sense of the past and future, sense of religion and art, sense of explaining and understanding) and to transcend, by giving to it and extra dimension, the search for survival and procreation, which is *Homo Sapiens*, results from a different perception of the other, giving rise to senses of love, shareability, generosity, charity and the like. We prefer to call *mathema* the action of explaining and understanding in order to transcend and of managing and coping with reality in order to survive. Throughout all known life histories and throughout the history of mankind, *technes* (or tics) of mathematics have been developed in very different and diversified cultural environments i.e., in diverse *ethnos*. So, in order to satisfy drives towards survival and transcendence, human beings have developed and continue to develop in every new experience and in diverse cultural environments, their ethno mathematics. These are communicated vertically and horizontally in time and for the reason of being more or less effective, more or less potent and sometimes even for political reasons, these various tics have either lasted and spread (i.e., measuring) or confined themselves to restricted groups and even disappeared.[3]

According to him, ethno mathematics depends on new understanding of history and epistemology, 'essentially a new way of looking into the process of generation, transmission institutionalisation and diffusion of knowledge', see, D'Ambrosio.[4]

This approach should make us think afresh, particularly in relation to human understanding and creativity, whether there are socio-emotional and political elements and, perhaps, many more, which we know as arts, science, religion, culture etc. We should also make an effort to verify whether there was any semblance of historical entity or even existential entity, before such levels were designated. These ideas inevitably give rise to theoretical concerns as a part of the explanation of nature as a whole and of human endeavour in the environment. Human beings have intrinsic but unique characteristics of codifying and symbolizing these practices as a means for survival and then going beyond this, that is, to transcend. There does not seem to be any dearth of endeavour through cultural environments to explore the body-physique and its ailments, strengths, weaknesses etc. and there is a greater realization now than what it was ever before, see, Horacio.[5] The history of development of mathematical ideas show, in very succinct terms, evolution of the concept of time and also its importance. Even while going to the other extreme, one finds a counterpart in rural but culturally vibrant setting. For example, in Tamilnadu, vide, L.S. Saraswati,[6] the concern was to identify and measure time with, of course, the ulterior objective of performing some activities, mainly agricultural and, perhaps, some rituals of religion. D'Ambrosio draws upon this as follows:

What they do for reckoning time is to use a simple device for measuring shadow. They take a piece of straw of any length and divide it into sixteen equal parts by folding it in half, eight times. The straw is then bent like an 'L' and held on the ground with the vertical portion towards the sun, so that shadow of the upright portion falls on the horizontal portion kept on the ground. The vertical portion is adjusted in such a way that the length of its shadow is equivalent to the length of the horizontal portion. In this way the number of parts of the upright portion indicate the number of units of time that have passed since sunrise if it is forenoon or the number that have passed since noon if it is the afternoon. People in this community are reported to tell time and to reckon with the precision of minute.

One finds in human endeavour the kind of cycle, already mentioned, namely, 'reality-individual-actual-reality'. Thus, in terms of D'Ambrosian framework of ethno mathematics, the population of Tamilnadu has developed tics (technes) of *mathema* (action of explaining and understanding in order to cope with reality and, perhaps, to transcend). One can cite a host of such tics of *mathema* in wide and diverse cultural environment that is what D'Ambrosio calls diverse *ethnos*.

Ethno mathematics has, thus, come to stay as a concept that allows generation, transmission and diffusion of knowledge with an accent on socio-cultural environment. It is not just rituals or daily mundane activities that show semblances of mathematical enterprises. There is in *situ* reality of mathematical exercises well reflected through folk-culture, folk-lore, folk-rhymes, folk-proverbs, folk-riddles and folk-wisdom, see, Bhaumik and Sinha.[7] Let us now cite a few of them, as they occur in a dominantly Bengali setting.

We begin with few cases that appear as a part of folk-practice and folklore. The first one is about the number 'sixty' whose Bengali variant in a dialect is *shait*, as it is pronounced.

A Bengali tribal woman, who is not allowed to take her husband's name, in order to provide the name of her husband, puts it as follows:

Tin tero diyā bāro
Noi diyā milāni karo
Mor soāmir nāmti aei
Pār kore dāo bārit jai

This means: if you; multiply the number 'three' by 'thirteen' and add 'twelve' and 'nine' with the result, you see the answer is simply 'shait' i.e. sixty. Here the numerals *tin* (three), *tero* (thirteen), *bāro* (twelve), *noi* (nine) are used. And the number *shait* (sixty) comes out of the mathematical operation (i.e., 3x13+12+9=60).

There goes a similar riddle: 'Panchanan' is the other name of the God Siva, the husband of the Goddess Pārvatī. There runs a beautiful mathematical verse commented with the name of 'Panchanan'.

Tin tero madhye bāro
Chār diā puran karo
Āmār bāri nandigram
Aei āmār swāmir nām

If one multiplies 'three' with 'thirteen' and adds 'twelve' plus 'four' with the result, one gets the number 'fifty five' which corresponds to the Bengali version 'Panchanan'. Thus originates the name 'Panchanan'. In accordance with the accepted condition, the problem may be posed as follows: (3x13+12+4=55). All these show how the community can handle computational or operational exercises on numbers.

The following example, again, will reflect the concern for numeracy. While playing together in the field, farmer's boys count the number of players among themselves by reciting very interesting and peculiar rhymes, each word of which represents a particular number. For example,

Yākor byākor tyākor shāil
Kāil porshu Mongol bār
Kāri gone majumdār
Dhāner āgā nāler shish
Khāiā doba unish bish

Here the word *yākor* represents the number one, *byākor* two, *tyākor* three, *shāil* four, *kāil* five, *porshu* six, *mongol* seven, *bār* eight, *kāri* nine, *gone* ten, *majum* eleven, *dār* twelve, *dhāner* thirteen, *āgā* fourteen, *nāler* fifteen, *shish* sixteen, *khāiā* seventeen, *doba* eighteen, *unish* nineteen, *bish* twenty. Thus they are able to count all the numerals from one to twenty.

A few more examples regarding numbers may be given here.

Śunyo: The word śunyo implying mathematically 'zero' may be seen in the Bengali proverb: *Dusṭa gorur cheye śunyo goal bhalo*

It is better to have an empty (*Śunyo*) cowshed than a notorious cow.

Rām: Ram stands for the number 'One' in some places of Bengal for weighing things.

Duna: It means *double*. There is a well-known proverb in Bengali.

This means, double-strength may be gained taking less diet whereas more food brings danger.

Dera: This means 'one and a half'. There is a saying (*Kame Kura Bhojae Dera*). This is about one idle person, which takes one, and half times of his normal food.

Punke: It is a fraction and is one-sixteenth part of the whole. In the proverb *Punke satru baro apod. Punke* stands for a small fractional quantity. A small enemy may sometimes be harmful.

Kara and gaṇḍā: One *kara* is one-twentieth part of a *paisa* and four *aras* make a *gaṇḍā*. A *gaṇḍā* is one fifth of a *paisa*. A *Bāul* song regarding this *kara* and *gaṇḍā* may be quoted here:

Amike tai ami janlem na
Ami ami kari kintu, ami amar thik hailo na
Karai karai kari gani
Char karai ek ganda gaṇi
Kotha hoite elam ami, tare koi gani?

The central thought of the song that the Bāul sings is: 'I always keep account of my belongings but I have never tried to know myself'.

The following examples illustrate the use of terms for measuring the weights of things:

Poā: This is one-fourth of a *seer* and is an important unit of measurement of weight prevalent among the folk-life and culture. Four '*chhataks*' make a *poā*. A proverb regarding the *poā* goes like this:

Bhāt roche nā roche moā
Chria roche poā poā

This may be stated as follows: you don't like bread but you take delight in eating cakes and tasty crisps in *poās* (that is, in plenty).

Rek is equal to twenty '*chhataks*' and one-fourth of a *don*. A proverb associated with *rek* is as under:

Khābona khābona anichhey
Tin rek dāl ek uchhey

This means that even though I have no appetite, I may take three *rek dal*.

Ari is another unit in expressing the measurement of weight. The proverb connected with this is as follows:

Hā-bhāter ari āthāro seere

In fact, *ari* does not actually mean eighteen seers. One *ari* is equal to sixteen *seers*.

Here are some examples illustrating the use of shapes and sizes that are the basic rudiments of geometrical knowledge.

Ārāhi Penchi: It is an ornament with 'two and a half circles'. Its shape is pinpointed here in the following Bengali riddle.

Gol gol ārāhi penchi tār nām ki?

Sidha: It means straight. A well-known proverb in Bengali runs thus:

Sidhā āngule ghee othe nā

It is not possible to collect butter with a straight finger.

The conception of time is lying scattered in various folk-literature - in phrases, idioms, proverbs, lyrics and in all walks of rural life of Bengal. A proverb with time:

Samayer 'ek' phonr
Asamayer 'dash' phonr

The equivalent English proverb is 'a stitch in time saves nine'.

Prahar is an important unit in measuring time used by the villagers in their daily lives. This can be illustrated in the Bengali proverb:

Kukurer māār āarāi prahar

It means that the pangs of a dog do not last long and soon after, it responds to the person beating it. The inner significance of the proverb is that a shameless person commits the same nuisance after being insulted time and again. *Āarāi prahar* mentioned in the proverb signifies a very small period of time, as one *prahar* is one-eighth part of a day.

Thousands of such examples may be cited from the folk-practice and folk-culture.

All these are remarkable examples of ethno mathematics, which abounds in the community. If we look at them critically, we may be persuaded to a point of view that these stem form and, perhaps will continue to mark, the participation in what sometimes may be called ceaseless cultural dynamic.

SOCIAL AND CULTURAL FACETS

Given the premise that mathematics is a fairly deep-rooted element in the fabric of culture that keeps on flowing through the ages, we need to hark back to social and cultural history coupled with anthropology with a view to unearthing mathematical elements that have stood the test of time. By social history of mathematics, vide Mehrtens et al.,[8] one aims at understanding the interplay of socio-cultural, economic and political factors in the development of mathematics. There are few studies on this score, see Wilder,[9] Bhaumik and Sinha.[10] But that mathematics is embedded in cultural phenomena without members being aware of the same has been excellently brought out by Ascher.[11] One can cite a host of pedagogical studies which show, in an abundant measure, the mathematical stuff produced by many different historical cultures in their immense diversity, running to contemporary times, see, Ascher,[12] Frankestein,[13] Lave,[14] Millroy[15] and, certainly, Freire.[16] Of late, there is move to link up, in such contexts, cultural features and mathematical topics, in a one-to-one correspondence style. Rubenstein[17] attempts to develop a matrix in which there are entries for both the dimensions, cultural and mathematical. Indeed, cultural attributes are amenable to analysis in terms of few distinct categories, which/are, by no means, decisively comprehensive. For example, one can have categories with labels of languages, of history and geography, economics and politics, social aspects, aesthetics and recreation, see, Rubenstein;[18] it should be mentioned that the last five entries are side rubrics, covering elements that may not be strictly distinguishable in few cases. But, by and large, customs, traditions, habitats etc. are well reckoned as social features while aesthetics have dominantly fine arts, performing arts, handicrafts etc. as the leading components. Mathematical entries can have wide extents and, sometimes, depths depending on levels of capabilities and competences. But, if we agree to basic rudiments of mathematics, we can possibly agree to categories such as communication; reasoning, number and numeration, measurement, patterns, functions, and algebra, geometry, statistics and probability, and discrete mathematics, see, Rubenstein.[19] All these are designed as 'eight by three' matrices. For example, in a row of language and a column with a number and numeration, one may have an entry relating to comparison of strength to people, animals and gods so as to build, in ancient India, powers of ten or in a row for geometry and column of social features, one may seek the way of structurally similar elements in Taj Mahal and so on. Essentially it comes to examining closely some leading strand of cultures and also in cultural phenomena, mathematical entities, as Ascher[20] has mentioned: 'how people categorize things is one of the major differences between one culture and another'. The matrix should be so designed as to begin with a cultural context and to extract as much of the possible mathematics from it as one may can. Thus the multicultural matrix becomes crucial to the process of generation, transmission, dissemination and, perhaps, institutionalisation of apparently naïve ideas in mathematico-cultural terms and relationships which, indeed, form the bedrock of ethno mathematics.

PHILOSOPHICAL VIEW

Having disposed of cultural and social aspects interspersed with historical elements, let us explore if there are philosophical underpinnings of ethno mathematics. The epistemological approach to mathematics is sufficiently well-known but that, too, in an ethno mathematical framework is yet to be conceptualised to a fairly acceptable degree.

Our starting point ought to be the human element, rather, a view of human beings which is so fundamental to the idea of ethno mathematics. Unless one sets up the linkages between the idea of ethno mathematics and a view how people relate to other human beings and to the world, one can hardly build up possible philosophical moorings of ethno mathematics.

Let us have (see, Borba)[21] the basic concept on the view of humans. There is the well-known phenomenological approach in which a human can only be seen in relation with the world and if there is no world, human is not visible. Thus, there is an intrinsic linkage between the concepts 'human' and 'world', with a serious limitation that both of them have or acquire meanings because of the humans. There is, of course, the fundamental relation of each human to other humans depending essentially on abilities to understand to comprehend existing implications and also to go well beyond extant nuances. It, therefore, boils down to the existence of the location in the world each person in definite historical situations. To live in a world, one has to undergo experiences often without being consciously aware of the element of time. Freire[22] makes positive distinctions between one kind of consciousness and the other. Mathematically speaking, a transitive consciousness develops in a person, the ability to reflect deeper on the experiences, at least to the extent of distinguishing the current ones with earlier counterparts. There is obviously an element of criticality in transitive consciousness. But, unless one crosses this hump, one can hardly be active so as to embark upon interactions with other persons, say, in the form of dialogues. This is precisely what has been called dialogical process. A coupling of the two is essential to the subsequent development processes of each human in relation to the other. One should hasten to add that in the dialogue, words are not only elements to be communicated but the very wide variety of unconscious signs and symbols such as pauses, gestures which may even baffle lexicographic storage. There is every eventuality that signs and symbols may evolve from one cultural group to another in their implications, meanings and interpretations. A necessary concomitant for the dialogue should share a common concern and a perception. In other words, the so-called dialogue can hardly become a reality if there be no problems emanating out of the cross-fertilization of problems with different groups. According to Saviani : 'A problem, as any other aspect of human experience, has a subjective side and an objective one, closely connected by a dialectical unity.... The concept of problem implies a consciousness of a situation of necessity (subjective aspect) and a situation that puzzles his consciousness (objective aspect)'.[23]

Both the aspects, subjective and objective, of the definition of a problem have cultural limits, for these cannot but depend on cultural traditions of a person so much so that even the irritants having objective semblances are culturally bound, for the simple reason that an impediment in a given culture may not pose to be the same in another. Thus, one gets stuck and also grapples with it for own survival in the evolution of life-styles. A mathematical problem is bound to emerge because of this encounter and hence, the generation of mathematics. Language is definitely an important vehicle to express a way of knowing developed by a culturally sensitive group of human beings. If we accept that one way of knowing is mathematics, then according to Borba,[24] mathematical knowledge 'expressed in the language code of a given socio-cultural groups' should be termed ethno mathematics, with connotations for ethno mathematics set forth earlier.

CONCLUDING REMARKS

It should not be taken for granted that ethno mathematics as a distinctive area of pursuits has gone totally unscathed. The bulk of such exercises have ranged heavily on educational enterprises perhaps, often slurring over conceptual shortcomings. While the cultural context as a central theme is undeniably important, there is scepticism as to the intrinsic characteristics of ethno mathematics to be able to be transformed to the nature of mathematics which is dominantly intellectual. The social institution of mathematics which sociologists, historians and others pursue as an important area of investigation can hardly be glossed over.

Cobb[25] has looked upon ethno mathematics as self-generated mathematics, which as argued, is basically individualistic and anarchistic. Ethno mathematics, it is often alleged, addresses the question of how context affects and structures experience. There is an element of substance in the unassailable statement that mathematics which we do not directly experience can still have important consequences having direct impact on our life-styles. Critical thinking stems out of confrontation with problems as described above. Once critical thinking acquires a certain measure of communality in respect of sharing problems, 'self-generated mathematics' of Cobb should be inevitable to ensue. Mellin-Olsen[26] has gone to the extent of drawing upon the Freirian concept of 'conscientization' in that the people are made aware of their culture how their experiences are structured and conditioned. One may look for conceptual underpinnings of how the ethno mathematics produced could be affected by techniques, be made available and the value adopted by the social institutions of mathematics. One can even move on a bit of micro-level analysis so as to investigate that if structures of mathematical experiences have any influence on the generation of ethno mathematics, taking into account that these forms, basically part of subcultures and mathematical resources. This may be logically pushed to a conclusion that ethno mathematics can facilitate structuring of mathematical experiences of those outside the subculture which have generated it; but of course, that can again be the role of ethno mathematics once it gets a solid footing.

Ethno mathematics puts the accent on plurality of mathematics rather than a monocultural phenomenon. It often rakes up the notion that mathematics is only produced by mathematicians. It enables us to look for ways in which one can understand cultural ways of producing and expressing their mathematics. That there can be indigenous 'frozen' mathematics in diverse cultures of the country hardly requires any reiteration. The indigenous wisdom, if it is a correct term in the context of the International Year for Indigenous Populations (1993), provides the vast repository of ethno mathematical and ethno scientific concepts and ideas which need to be articulated as inter-connectivities discernible in the landscape of mathematical endeavours.

The Advaita Vedāntins have used a sentence 'daśamastvamasi' (i.e., you are the tenth) to explain the phenomenon of perception generated through testimony called *śābdajanyapratyakṣa*. An individual who is really tenth but did not count himself out of ignorance. When it is pointed out whether he has not counted himself and he is the tenth, he suddenly realises that he is a member occupying the number of 'ten'. This realisation is direct perception generated through testimony. In such cases the philosophers of different schools have used number to indicate a fact or person.

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CHAPTER – 8

Conclusive Remarks

We may cite some concluding remarks as discussed in earlier chapters regarding the implication and application of some mathematical principles in Indian philosophical tradition.

First, inference is taken as most fundamental means of knowing in Western Mathematical Logic as found in different fields of knowing. In fact, without inference no knowledge is possible. Such is the view held by the Naiyāyikas though to them the form of *anumāna* is somehow different. Though there are fundamental differences between the theory of inference and *anumāna* yet in both the systems it is accepted as very important way of knowing. One may enquire in this context that to the Western Logicians the following form of inference is a correct one :

1. $P \supset Q$
2. P

$\therefore$ Q

It is valid, because the conclusion is deduced under M.P. rule. Can it be applicable to the form of inference admitted in Nyāya ? It may be expressed in the following way :

1. Wherever there is *śimśapātva* (a kind of tree called *Śiśu*) there is *vṛkṣatva* (i.e., treeness).
2. There is *śimśapātva* (i.e., the property existing in a *Śiśu* tree).

Therefore, there is treeness.

The Nāiyayikas can explain the case as *anumāna* (inference) if it is desired strongly by them which is called *siṣādhayiṣā* (desire to infer). To them anything can be inferred from the related thing if there is strong desire to infer, even if it is a case of perception. In the previous example, that a *simśapā* is a tree is known through perception. In spite of this one, is allowed to infer 'treeness' from the *śimśapātva* if he has got strong desire to infer *(siṣādhayiṣā)*.

The rule of M.P. is directly applicable to the Buddhist notion of inference. To them 'treeness' is inferred on the strength of the property *śimśapātvā*. To the Buddhist any determinate cognition *(savikalpaka)* is inferential in nature. That which is expressed through language, universal etc. is called determinate cognition. The entity which is devoid of such language and momentary in nature is alone perceptual. When a tree is known as such through the fact that it is *śimśapā*, it is surely inferential. The first premiss is – 'Wherever there is *śimśapātva* there is treeness'. The second premiss is 'This has got *śimśapātva*'. The conclusion follows from these is : 'This has got treeness'. This is a clear case of Modus Ponence. In this way all the rules can be applied to Indian theories of inference.

Secondly, mathematical logic deals with the proof by *Reductio-ad-absurdum* or Indirect Proof as a method. We have shown already that the Naiy*āyikas* have made use of such method technically known as *Tarka* in connection with determining the nature of an object and with removing the doubt of deviation *(vyabhicāraśamkā). The Reductio-ad-absurdum* method is known as *vipakṣabādhakatarka* in Indian Logic.

Thirdly, we have shown the applicability of the Set theory in Indian Logic. The Set-Subset relationship is found among the Indian Logicians when they deal *para*, *apara* and *parāpara sāmāṇya*. In this connection an effort has been made to show that the concept of null set is not foreign to Indian systems. So far as the grammarians view is concerned, the terms conveying null set are well accepted by them. As they provide meaning. On the other hand the Naiyayikas do not admit such position. Because, cannot be included under seven accepted categories and hence nonsensical.

Fourthly, the concept of *śūṇya* (zero) is found in both in mathematics and Indian philosophical literature almost in the same sense. Sometimes, absence (*abhāva*) is expressed with the term *śūṇya* or zero. If someone wants to convey the absolute absence of money in his house, it may be conveyed with the term *śūṇya* or zero. It is said in the proverb used in Bengali language, which runs as follows : '*dustu garur ceye śūṇya goāl bhālo*' (i.e., it is better to have an empty cowshed than to have a notorious cow). Another example may be taken from the song of Kazi Nazrul Islam which runs as follow : *Śūṇya e buke pākhi mor phire āy*' (That is, O bird, come back to my chest which is empty). Here also the term *śūṇya* has been taken as empty. Another meaning of it is accepted by the Buddhists. To them *śūṇya* means *catuṣkoṭivinirmukta* (free from four-fold modes of expression). When an object is relative it cannot be expressed in language like *aśti*, *nāsti* etc. For this reason it is called relative. Moreover, the Upaniṣadic seers have to show an inner link between *śūṇya* and *pūraṇa,* as both of them have some connotation in certain stage as found in the Mantra – '*pūrṇasya pūrṇamādāya pūrṇamevāvaśiṣyati*'.

Lastly, the Sanskrit rendering of the term 'number' is *saṁkhyā*, which occupies a prominent role in Indian Philosophy as well as in Mathematics. Any type of calculation, pointing out, direction, identification etc. are done with the help of number. Hence in our day to day behaviour the use of number has a significant role. The number serves the purpose of identifying an object from others and hence it has the power of distinguishing an object from the rest (*itaravyāvarttaka*). In fact, through number we can 'see' the thing in its own nature and it is instrumental to the right cognition of an object, which is echoed in the derivative meaning of the term *Saṁkhyā*. The root *khyā* is prefixed with *saṁ*, which means *samyak jñāna* or right cognition. The term *samkhyā* means right cognition of an object in wider perspective. The literature dealing with the right cognition of the entities is colled *saṁkhyā*. There are two entities admitted in *Saṁkhyā*, which are *Prakṛti* and *Puruṣa*. The real cognition (*saṁkhyā*) of these lies in the fact of knowing that they are two in number, but *not* identical. In this way, the term *saṁkhyā* means right cognition of the discrimination between two objects. In other words, the cognition of *Prakṛti* and *Puruṣa* as two (but not one) is called *saṁkhyā*. In the same way, to realise the existence of only *one entity* in the whole world movable and immovable is also called right cognition according to the Advaita Vedānta. That is why, *saṁkhyā* is defined as the cause of the phenomenon of *counting* (*'gananāvyavahāre tu hetuḥ saṁkhyā bhidhiyate'* – *Bhāṣāpariccheda*, Verse No. 106). It is said in the commentary that a jar is known as different from a pot through the instrumentality of the number given to the jar and it is a case of perception. The number 'one' is identified with the particular jar and it cannot be realised elsewhere in another jar or cloth (*ghaṭādisvarūpasya ekatvasamkhyātve ghaṭāntare tatpratyayābhāvaprasaṅgāt*). The number '*one*' is the basic depending on which the property called '*twoness*' existing in both the entities can be known. That is why, the cognition of '*two*' and more than '*two*' is generated through the dependence to the comparative cognition (*"dvitvādayo vyāsajyavṛttisaṁkhyā apekṣābuddhijanyāḥ"* – Siddhāantamuktāvalī on Verse – 106). When someone realises '*two*', he attains it '*one*', and another '*one*' (*ayaṁ ekaḥ ayaṁ ekaḥ iti buddhiḥ*). From the number '*two*' to 'innumerable' this comparative cognition works and hence it is called *apekṣābuddhijanyaḥ*.

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