CHAPTER – 5

Set theory and similar concepts in Indian Logic

We use the word 'set' in such a way that a set is completely determined when its members are given; i.e., if A and B are sets which have exactly the same members, then A = B. Thus we write:

The set of equilateral triangles = the set of equiangular triangles, for something belongs to the first set if and only if it belongs to the second, since a triangle is equilateral if and only if it is equiangular. This general principle of identity for sets is usually called the principle of extensionality for sets; it may be formulated symbolically thus:

$$(1)\ A = B \leftrightarrow (x)(x \in A \leftrightarrow x \in B).$$

Sometimes one finds it convenient to speak of a set even when it is not known that this set has any members. A geneticist may wish to talk about the set of women whose fathers; brothers and husbands are all hemophiliacs, even though he does not know of an example of such a women. And a mathematician may wish to talk about maps, which cannot be colored in fewer than five colors, even though he cannot prove that such maps exist. Thus it is convenient to make our usage of the term 'set' wide enough to include empty sets, i.e., sets which have no members.

It is clear that if A is a set which has no members, then the following statement is true, since the antecedent is always false:

$$(2)\quad (x)(x \in A \rightarrow x \in B).$$

And, correspondingly, if B is empty, i.e., has no members, then it is true that:

$$(3)\quad (x)(x \in B \rightarrow x \in A).$$

From (1), (2) and (3) we conclude that if two sets A and B are empty, then:

$$A = B;$$

That is to say, there is just one empty set; for given two empty sets, it follows from the principle of extensionality for sets that the two sets are identical. Hence we shall speak of the empty set, which we denote by a capital Greek lambda:

A is the set such that for every x, x does not belong to A ; that is, symbolically : (x)-(x ∈ A), and we abbreviate '- (x ∈ A)' to 'x ∉A', and write: (x)(x ∉A).

We shall find it convenient in general to use the notation '∈' to indicate that something does not belong to a set.

Often we shall describe a set by writing down names of its members, separated by commas, and enclosing the whole in braces. For instance, by:

{Roosevelt, Parker}

We mean the set consisting of the two major candidates in the 1904 American Presidential election. By:

$$\{1, 3, 5\}$$

We mean the set consisting of the first three odd positive integers. It is clear that

{1, 3, 5} = {1, 5, 3} (for both sets have the same members : the order in which we write down the members of a set is of no importance). Moreover, {1, 1, 3, 5} = {1, 3, 5} (for we do not count an element of a set twice).

The members of a set can themselves be sets. Thus a political party can be conceived as a certain set of people, and it may be convenient to speak of the set of political parties in a given country. Similarly we can have sets whose members are sets of integers: for instance, by:

$$\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$$

We mean the set which has just three members, namely, $\{1, 2\}$, $\{3, 4\}$ and $\{5, 6\}$. By:

$$\{\{1, 2\}, \{2,3\}\}$$

We mean the set whose two members are $\{1, 2\}$ and $\{2, 3\}$. By:

$$\{\{1, 2\}, \{1\}\}$$

We mean the set whose two members are the sets $\{1, 2\}$ and $\{1\}$.

A set having just one member is not to be considered identical with that member. Thus the set $\{\{1, 2\}\}$ is not identical with the set $\{1, 2\}$: this is clear from the fact that $\{1, 2\}$ has two members, whereas $\{\{1, 2\}\}$ has just one member (namely, $\{1, 2\}$). Similarly,

{Elizabeth II} $\neq$ Elizabeth II, for Elizabeth II is a woman, while {Elizabeth II} is a set.

Ordinarily it is not true that a set is a member of itself. Thus the set of chairs is not a member of the set of chairs : i.e., the set of chairs is not itself a chair. This remark illustrates the very great difference between identity and membership: for the assertion that $A = A$ is always true, whereas that $A \in A$ is usually false.

The relation of membership also differs from the relation of identity in that it is not symmetric: from $A \in B$ it does not follow that $B \in A$. For instance, we have:

$$2 \in \{1, 2\}, \text{ but:}$$
$$\{1, 2\} \notin 2.$$

Moreover, the relation of membership is not transitive: from $A \in B$ and $B \in C$ it does not follow that $A \in C$. Thus, for example, we have:

$$2 \in \{1, 2\} \text{ and:}$$
$$\{1, 2\} \in \{\{1, 2\}, \{3, 4\}\} \text{ but:}$$

$2 \notin \{\{1, 2\}, \{3, 4\}\}$, for the only members of $\{\{1, 2\}, \{3, 4\}\}$ are $\{1, 2\}$ and $\{3, 4\}$ and neither of these sets is identical with 2.

It should be noticed that if, for instance, $\{a, b\}$ is any set with two members, then, for every x, $x \in \{a, b\}$ if and only if either $x = a$ or $x = b$, that is, symbolically:

$$(x)(x \in \{a, b\} \leftrightarrow (x = a \vee x = b).$$

Similarly, if $\{a, b, c\}$ is a set with three members, then $x \in \{a, b, c\}$ if and only if either $x = a$ or $x = b$ or $x = c$. It is for this reason that we just said that $2 \notin \{\{1, 2\}, \{3, 4\}\}$; for if $x \in \{\{1, 2\}, \{3, 4\}\}$, then either $x = \{1, 2\}$ or $x = \{3, 4\}$; and since $2 \neq \{1, 2\}$ and $2 \neq \{3, 4\}$, it follows that $2 \notin \{\{1, 2\}, \{3, 4\}\}$.

It should also be noticed that there is a close relationship between saying that something has a property and saying that it belongs to a set : a thing has a given property if and only if it belongs to the set of things having the property. Thus to say that 6 has the property of being an even number amounts to saying that 6 belongs to the set of even numbers.

The principle of the identity of indiscernible in terms of properties. Expressed in terms of membership the principle becomes: If y belongs to every set to which x belongs, then $y = x$. Put in this form, the principle has perhaps a more obvious character than it has when put in terms of properties. For $x \in \{x\}$ (i.e., x belongs to the set whose only member is x), and hence, if y belongs to every set to which x belongs, we conclude that $y \in \{x\}$, so that $y = x$.

Inclusion : If A and B are sets such that every member of A is also a member of B, then we call A a subset of B, or say that A is included in B. We often use the sign '$\subseteq$' as an abbreviation for 'is included in'. Thus we can write, for instance:

The set of Indians is a subset of the set of men, or:
The set of Indians is included in the set of men, or simply:
The set of Indians $\subseteq$ the set of men. Symbolically we have:

(1) $A \subseteq B \leftrightarrow (x)(x \in A \rightarrow x \in B)$.

It is clear that every set is a subset of itself; i.e., for every set A we have: $A \subseteq A$. Moreover, the relation of inclusion is transitive; i.e., if $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$ (for if every member of A is a member of B, and every member of B is a member of C, then every member of A is a member of C). The relation of inclusion is not symmetric, however; thus $\{1, 2\} \subseteq \{1, 2, 3\}$, but it is not the case that $\{1, 2, 3\} \subseteq \{1, 2\}$.

It is intuitively obvious that identity, membership, and inclusion are distinct and different notions, but it is still somewhat interesting to observe that their distinction may be inferred simply from considering the questions of symmetry and transitivity. Thus inclusion is not the same as identity, since identity is symmetric while inclusion is not. And inclusion is not the same as membership, since inclusion is transitive while membership is not. And we have seen earlier that identity is not the same as membership, since identity is both symmetric and transitive, while membership is neither. In every day language all three notions are expressed by the one overburdened verb 'to be'. Thus in everyday language we write:

Elizabeth II is the present Queen of England,
Elizabeth II is a woman,
Women are human beings.
But in the more exact language being developed here:
Elizabeth II = the present Queen of England,
Elizabeth II $\in$ the class of women,
The class of women $\subseteq$ the class of human beings.

When $A \subseteq B$, the possibility is not excluded that $A = B$; it may happen also that $B \subseteq A$, so that A and B have exactly the same members, and hence are identical.

The Empty Set : As mentioned earlier, the empty set, A, is characterized by the property that, for every x, $x \notin A$.

Although nothing belongs to the empty set, the empty set can itself be a member of another set. Thus if we speak of the set of all subsets of the set {1, 2}, we are speaking of the set {{1, 2}, {1}, {2}, A} which has four members; the three-member set {{1, 2}, {1}, {2}} on the other hand, is the set of all non-empty subsets of {1, 2}.

We recall the fact that a set A is a subset of a set B if and only if every member of A is also a member of B, i.e., if and only if: for every x, if $x \in A$, then $x \notin B$. In particular, the empty set A is a subset of a set B if and only if: for every x, if $x \in A$, then $x \in B$. Since always $x \notin A$, however, it is always true that if $x \in A$. then $x \in B$. Thus, for every set B, we have:

$$A \subseteq B.$$

That is, the empty set is a subset of every set. In addition, the empty set is the only set which is a subset of the empty set; for if $B \subseteq A$, then, since we also have: $A \subseteq B$, we can conclude that $B = A$.

An empty set is to be accepted as a set and because through the acceptance of its setness it can be admitted distinguished from other set. Moreover, an empty set is also a subset of another empty set, because if something is susbstracted from another, it remains in the same status and hence it is taken as subset of another.

Operations on Sets : If A and B are sets, then by the intersection of A and B (in symbols: $A \cap B$) we mean the set of all things which belong both to A and to B. Thus, for every x, $x \in (A \cap B)$ if and only if $x \in A$ and $x \in B$; that is, symbolically:

(1) $(x)(x \in A \cap B \leftrightarrow x \in A \,\&\, x \in B)$.

If A is the set of all Indians, and B is the set of all blue-eyed people, then $A \cap B$ is the set of all blue-eyed Indians.

If A is the set of all men, and B is the set of all animals which weigh over ten tons, then A ∩ B is the set of all men who weigh over ten tons. In this case we notice that A ∩ B is the empty set (despite the fact that A ≠ A and B ≠ A, since some whales weigh more than ten tons). When A ∩ B = A, we say that A and B are mutually exclusive.

Our use of the term 'intersection' is similar to its use in elementary geometry, where by the intersection of two circles, for instance, we mean the points, which lie on both circles. Some authors use, instead of '∩', the dot '.' Which is used in algebra for multiplication; such authors often speak of the "product" of two sets, instead of their intersection.

If A and B are sets, then by the union of A and B (in symbols: A U B) we mean the set of all things which belong to at least one of the sets A and B. Thus, for every x, x ϵ (A U B) if and only if either x ϵ A or x ϵ B.

Symbolically:

One may intend to consider the union of two sets, however, even when they are not mutually exclusive. For instance, if A is the set of all human adults, and B is the set of all people less than 40 years old, then A U B is the set of all human beings.

If A and B are two sets, then by the difference of A and B (in symbols: A ~ B) we mean the set of all things which belong to A but not to B. Thus, for every x, x ϵ A ~ B if and only if x ϵ A and x ∉ B; that is, symbolically:

2. (x)(x ϵ A ~ B ↔ x ϵ A & x ∉ B).

If A is the set of all human beings, and B is the set of all human …, then A ~ B is the set of all human males. One often desires to consider the difference of two sets A and B, however, even when B is not a subset of A. For instance, if A is the set of human beings, and B is the set of all female animals, then A ~ B is still the set of all human males, and B ~ A is the set of all female animals which belong to a non-human species.

Domains of Individuals : Often one is interested, not in all possible sets, but merely in all the subsets of some fixed set. Thus in sociology, for instance, it is quite natural to say mostly about sets of human beings; and to speak with the understanding that when a set was mentioned it was to be taken to be a set of people, if an explicit statement to the contrary was not made. In such discourse one might say, for example, 'the set of albinos', and it would be understood that one was referring only to the set of albino people, and not also to albino monkeys, albino mice, and other albino animals.

Similarly, in some geometrical discourse the word 'set' to mean 'set of points' is used. Sometimes in mathematics people press into service in some specialized sense some of the various words mentioned above as being here taken to be synonymous with 'set': a geometrician might, for example, adopt the convention of speaking of set of points, classes of sets of points, and aggregates - or perhaps families - or geometrical curves.

When a fixed set D is taken as given in this way, and one confines himself to the discussion of subsets of D, we shall call D the domain of individuals, or sometimes the domain of discourse. Thus the domain of individuals of the sociological discussion mentioned above is the set of all human beings.

We shall denote the domain of individuals by 'V'. It is important to remember that though 'A', stands for a uniquely determined entity (the empty set), the symbol 'V' is interpreted differently in different discussions. In one context 'V' may stand for the set of all human beings, in another for the set of points of space, and in another for the set of positive integers.

When dealing with a fixed domain of individuals V, it is convenient to introduce a special symbol for the difference of V and a set A:

$$\sim A = V \sim A.$$

We call $\sim A$ the complement of A. More generally, the difference $B \sim A$ of B and A is called the complement of A relative to B; so the complement of a set is simply its complement relative to the given domain of individuals.

Translating Everyday Language : This part is concentrated to the problem of translating sentences of everyday language into the symbolism that we have been developing. It should clearly be borne in mind that the usage of everyday language is not so uniform that one can give unambiguous and categorical rules of translation. In everyday language we often use the same word for essentially different notions ('is', for example, for both '$\in$' and '$\subseteq$'); and, sometimes for literary elegance, we often use different words for the same notion ('is', 'is a subset of', and 'is included in', for example, for '$\subseteq$').

We consider here only those sentences, which can be translated into a symbolism consisting just of letters standing for sets, parentheses, and the following symbols:

$$\cap, \cup, \sim, \Lambda, =, \neq, \subseteq$$

Such a symbolism can handle statements involving one-place predicates very well, but it is not adequate to many-place predicates. This symbolism is essentially equivalent to the language of the classical theory of the syllogism it is important to note that we are not using here the notion of membership; we restrict ourselves to sets all of which are on the same level-subsets of some fixed domain of individuals.

An English statement of the form 'All ... are ...', where the two blanks are filled with common nouns such as 'men' or 'Indians' or 'philosophers', means, of course, that the set of things described by the first noun is a subset of the set of things described by the second noun. Thus, for example:

(1) All Indians are philosophers Means:

The set of Indians $\subseteq$ the set of philosophers, or, using 'A' as an abbreviation for 'the set of Indians', and 'P' as an abbreviation for 'the set of philosophers':

$$I \subseteq P$$

We can also express the meaning of this statement in other, equivalent, ways:

$$A \cup P = P, \text{ or: } A \cap \sim P = A, \text{ etc,}$$

and these other modes of expression often turn out to be useful.

We use the same mode of translation of statements of the form 'All … are …' also when the second blank is filled with an adjective. For example, we take:

(2) All Indians are mortal to mean:

The class of Indians $\subseteq$ the class of mortal beings, or, using obvious abbreviations:

$$A \subseteq M.$$

Sometimes, however, in contexts of this sort people suppress the word "all"- writing, for instance:

Tyrants are mortal instead of:

(3) All tyrants are mortal, or : Women are fickle instead of :

(4) All women are fickle, which we should translate, respectively, by:

$$T \subseteq M \text{ and:}$$

$$W \subseteq F$$

One must be on guard when translating statements of this kind, however; for ordinary language uses the same form also to express essentially different ideas. Thus, as we have seen before:

(5) Men are numerous does not mean:

The set of men $\subseteq$ the set of numerous things (i.e., that every man is numerous) but rather, letting M be the set of men and N be the set of sets which have numerous members:

$$M \in N.$$

Similarly:

(6) The apostles are twelve Means that the set of apostles belongs to the set of sets having just twelve members.

Corresponding to the distinction, which we have made between membership and inclusion, the older logic made a distinction between the "distributive" "collective" applications of the predicate to the subject. Using this terminology, one says that in (1), (2), (3), and (4) the predicate is applied to the subject distributively, and that in (5) and (6) it is applied to the subject collectively.

An English statement of the form 'Some…are…', where the blanks are filled b common nouns means that there exists something which is described by both terms: i.e., that the intersection of the two corresponding sets is not empty. Thus, for instance:

(7) Some Indians are philosophers Means that there exists at least one person who is both an Indians and a philosopher, and is accordingly translated:

$$I \cap P \neq I.$$

Although a statement of the form of (7) implies that the sets corresponding to subject and predicate are not empty, no such inference is to be drawn from a statement of the form of (1). Thus, for example, it is true that

All three-headed, six-eyed men are three-headed men, but it is not true that some three-headed, six-eyed men are three-headed men.

An English statement of the form 'No...are...' (where as before, the blanks are filled b common nouns) means that nothing belongs both to the set corresponding to the first noun, and to the set corresponding to the second noun: i.e., that the intersection of these two sets is empty. For instance, the sentence:

(8) No Americans are philosophers is translated:

$$A \cap P = A.$$

Thus (2) has the same meaning as:

No Americans are immortal since both can be translated:

$$A \cap \sim M = A.$$

An English statement of the form 'Some ... are not ...' (where the blanks are filled by common nouns) means that there exists something which belongs to the set corresponding to the first noun, and does not belong to the set corresponding to the second noun: i.e., that the intersection of the first set with the complement of the second is not empty. The sentence:

Some Americans are not philosophers is translated:

$$A \cap \sim P \neq A.$$

We turn now to the problem of translating some statements of a more complicated sort. The word 'and' often corresponds to the intersection of sets. Thus:

All Americans are clean and strong is translated (using obvious abbreviations):

$$A \subseteq C \cap S.$$

The same applies to the word 'but': thus:

Freshmen are ignorant but enthusiastic is translated:

$$F \subseteq I \cap E.$$

The situation is quite different, however, when the 'and' occurs in the subject rather than in the predicate. Thus:

(9) Fools and drunk men are truth tellers is translated, not by:

(10) $(F \cap D) \subseteq T$ but rather by:

(11) $(F \cup D) \subseteq T$.

For (9) means that both the following statements are true:

(12) All fools are truth tellers and:

(13) All drunk men are truth tellers; and (12) and (13) are translated, respectively, by:

(14) $F \subseteq T$ and:

(15) $D \subseteq T$; and (14) and (15) are together equivalent to (11). (It should be noticed that (10) says less than (11); for:

> $F \cap D \subseteq F \cup D$ is true for every F and D – and hence (10) is true whenever (11) is true – while a statement of the form (10) can be true even when the corresponding statement of the form (11) is false.

Often the statement to be translated does not contain any form of the verb 'to be' at all. Thus the statement:

Some Frenchmen drink wine can be translated:

$F \cap W \neq A$, if we think of 'F' as standing for the set of Frenchmen and 'W' as standing for the set of wine drinkers. The statement:

Some Americans drink both coffee and milk can be translated:

$A \cap C \cap M \neq A$, where 'A' stands for the set of Americans, 'C' for the set of people who drink coffee, and 'M' for the set of people who drink milk. Here we have adopted the practice, which is frequently employed, of suppressing parenthesis in representing the intersection of three or more sets, writing simply:

$A \cap C \cap M$ instead of: $A \cap (C \cap M)$; we shall sometimes adopt a similar practice in connection with the representation of the union of three or more sets. Still more complicated examples are possible. If we consider:

(16) Some Americans who drink tea do not drink either coffee or milk. The general form of this statement is: Some S are not P. The subject is translated:

$A \cap T$, where T = the set of tea drinkers, and the predicate is translated : $C \cup M$.

The whole sentence (16) is then translated :

$(A \cap T) \cap \sim (C \cup M) \neq A$, which is also equivalent to:

$A \cap T \cap \sim C \cap \sim M \neq A$, since, corresponding to De Morgan's laws for the sentential connectives, we have:

(17) $\sim (C \cup M) = \sim C \cap \sim M$.

Venn Diagrams : In studying sets and relations between them, it is sometimes helpful to represent the sets diagrammatically: one draws a rectangle to represent the domain of individuals, and then draws circles, or other figures, inside the rectangle – thinking of the points inside the various figures as corresponding to the members of the sets being represented by the figures. Thus sets A and B, for instance, mutually exclusive, can be represented by the following diagram:

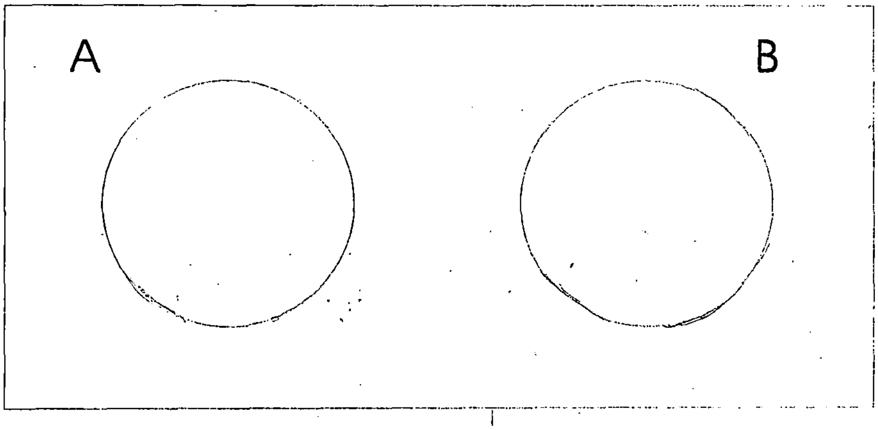

FIGURE 1

If we know that $A \subseteq B$, we can represent the situation by Figure 2.

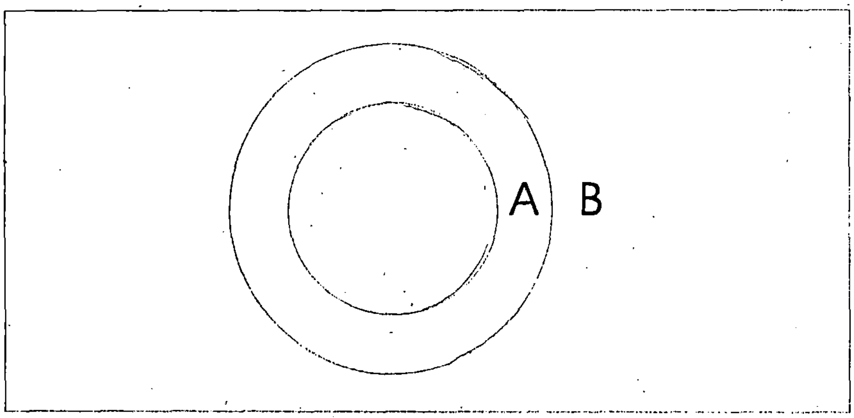

FIGURE 2

A more traditional way of describing Figure 2 is to say that all A are B, i.e., all members of A are members of B.

If we know of three sets A, B and C that $A \subseteq B$ (all A are B) and $B \cap C = A$, (i.e., no B are C), we can represent the situation by Figure 3.

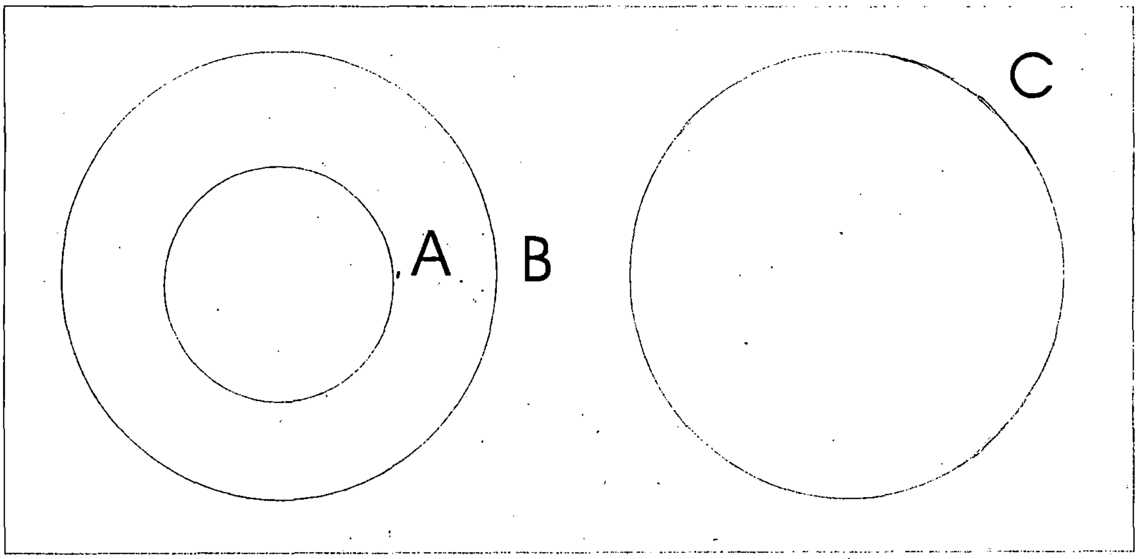

FIGURE 3

Sometimes, instead of trying to incorporate the given information into the diagram simply by drawing the circles in an appropriate manner, it is convenient to draw the figures in a rather arbitrary way (so that they will divide the interior of the rectangle into a maximum number of parts) and then get the information into the figure by other methods, such as the shading of areas. Having decided, let us say, to indicate by *horizontal shading* that an area corresponds to the empty set, we indicate that $A \cap B = A$ by Figure 4; and that $A \subseteq B$ by Figure 5; for to say that A is a subset of B means that no part of A lies outside B.

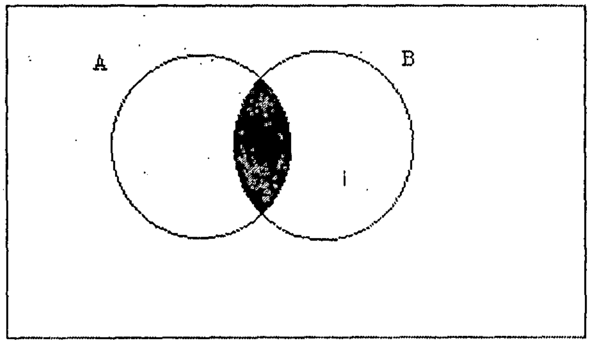

FIGURE 4

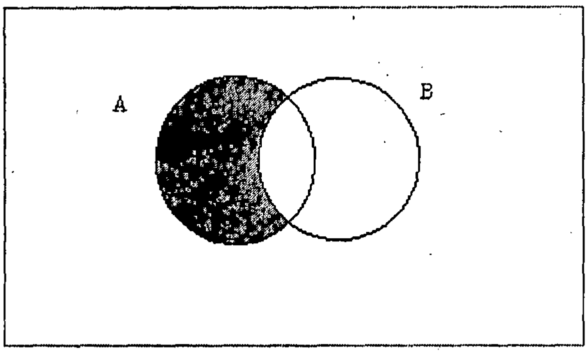

FIGURE 5

With these diagrams, it is often easy to show what conclusions can be drawn from given information about two or more sets. Thus suppose, for example, that it given, of two sets A and B both that $A \cap B = A$, and that $A \subseteq B$. The first statement (as indicated in Figure 4) means that the common part of A and B is to be shaded; and the second statement (as indicated in Figure 5) means that the part of A which is outside of B is to be shaded. Thus we obtain Figure 6, where we notice that all of A is shaded. Thus we see that the two given statements jointly imply that A is the empty set.

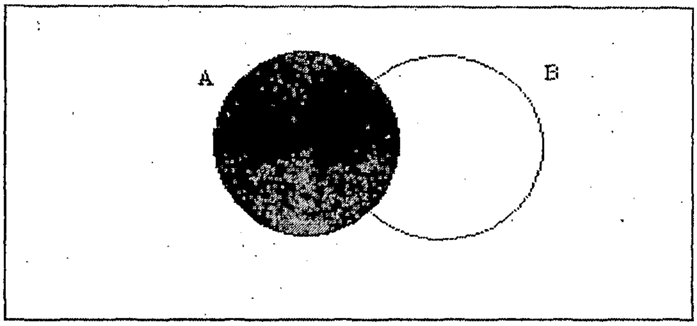

FIGURE 6

Horizontal shading is used to indicate emptiness of a region. Another kind of symbol is needed for non-emptiness. We shall use a device of linked crosses. Thus if $A \cap B \neq A$ (some A are B) we represent this situation by Figure 7; the cross indicates that the region common to A and B is not empty. We represent the more complicated situation :

$A \cap (B \cup C) \neq A$, (some A are either B or C) by Figure 8.

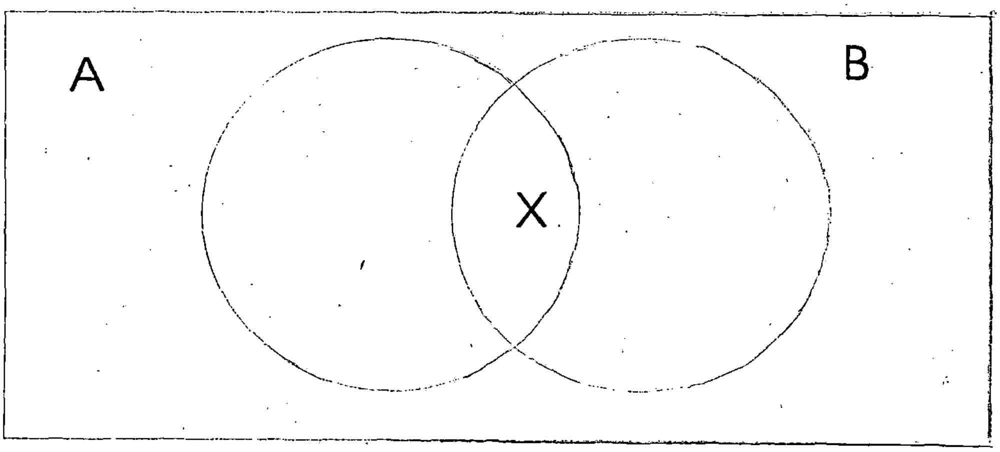

FIGURE 7

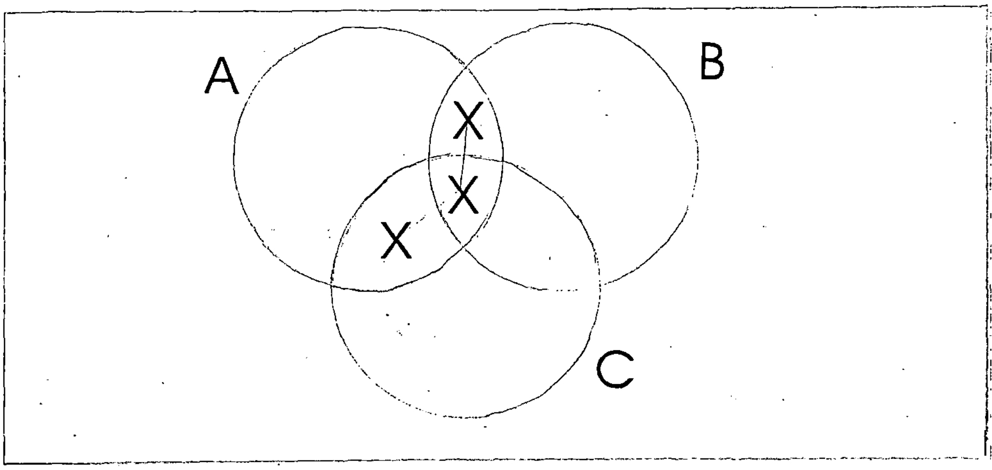

FIGURE 8

The three crosses in Figure 8 are linked to show that at least one of the three small regions is non-empty. If the linkage had been omitted in Figure 8, the figure would represent much more than that $A \cap (B \cup C) \neq A$. If the linkage were omitted, we could infer:

(1) $(A \cap B) \sim C \neq A$ (the top cross)

(2) $A \cap (B \cap C) \neq A$ (the middle cross)

(3) $(A \cap C) \sim B \neq A$ (the bottom cross)

Obviously, any one of the assertions (1) – (3) implies that $A \cap (B \cup C) \neq A$ and more. Without the linkage Figure 8 would say far too much.

The situation described by

$A \cup B \neq A$ (Something is either A or B)

$A \cup \sim C \neq A$ (Something is either A or not C) is represented by Figure 9. Note the two separate linkages, one for each of the two existential statements.

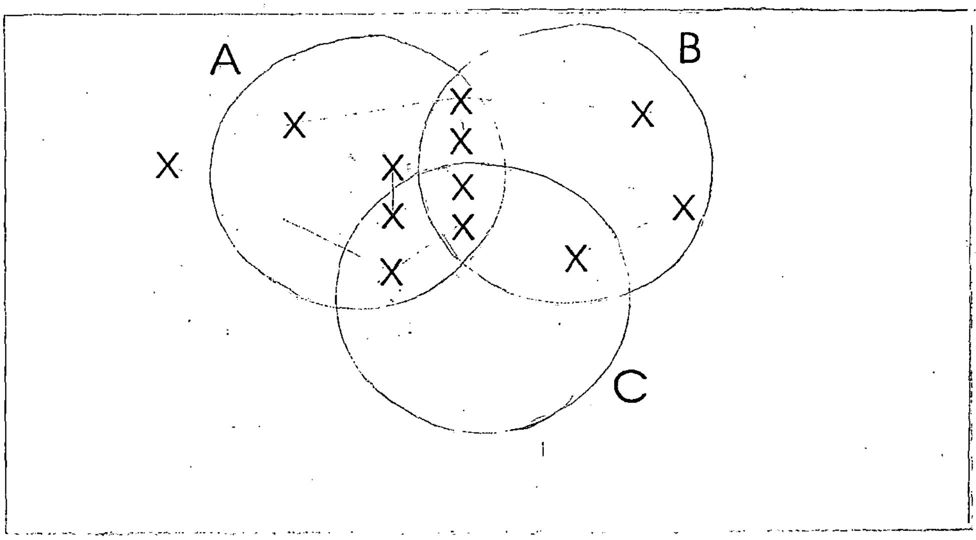

FIGURE 9

What should be an interpretation of a diagram in which a cross and shading occur in the same region? Suppose, for example, that we have:

(4) $A \cap C \neq A$ (Some A are C)

(5) $C \subseteq B$ (All C are B)

We obtain Figure 10, in which the part of C which is outside of B has been shaded horizontally, to show that it is empty and linked crosses have been placed in the two parts of the common region of A and C, to show that it is not empty. The problem of interpretation centres around Region (1). Consideration of (4) and (5) clearly urges the stipulation that shading dominates a cross, and hence Region (1) is empty. We are thus able to conclude that Region (2) is not empty, that is, (4) and (5) imply that $A \cap (B \cap C) \neq A$ (some A are B and C).

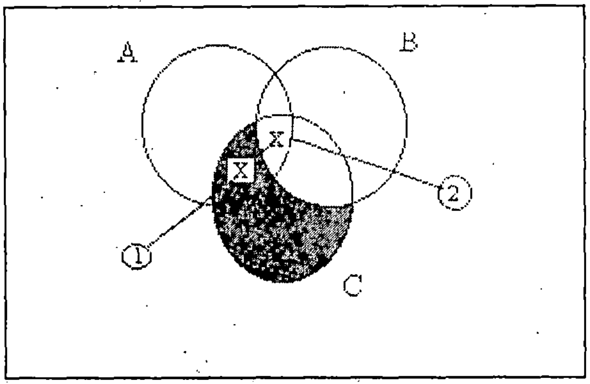

FIGURE 10

There is one set of circumstances in which we do not want to say simply that shading dominates a cross. When every cross in a linkage of crosses is "covered" by shading we must conclude that the diagram is inconsistent rather than that the linked regions are completely empty, for a linkage of crosses means that at least one of the regions linked is non-empty. We may in fact use these circumstances to investigate by use of Venn diagrams the consistency of a set of conditions imposed on sets. Thus suppose, for example, that it is given of three sets A, B and C that:

(6) $A \subseteq C$ (All A are C)

(7) $A \cap C = A$ (No A are C)

(8) $A \cap B \neq A$ (Some A are B)

This situation is represented by a Venn diagram in Figure 11.

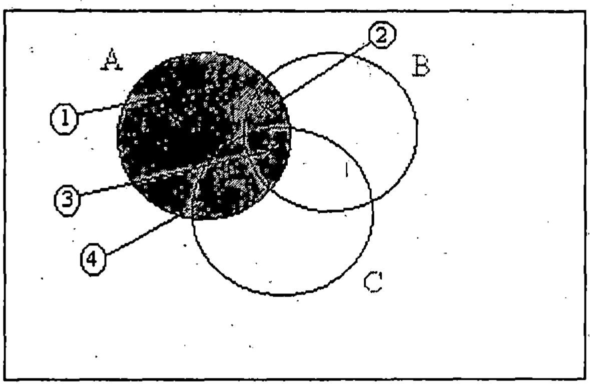

FIGURE 11

Assumption (6) leads us to shade Regions (1) and (2); assumption (7) leads us to shade Regions (3) and (4); and assumption (8) leads us to place two linked crosses in Regions (2) and (3). Thus the given assumptions imply that Regions (2) and (3) are both empty and non-empty, which is a contradiction.

There is, of course, a very great difference between saying that certain conditions on sets are inconsistent and saying merely that they imply that some set is empty. Thus assumptions (6) and (7) above imply that A is empty, but these two assumptions by themselves are not inconsistent.

With the notation for Venn diagrams now complete, it is of some interest to show how the apparatus may be used to establish the validity of classical syllogisms. As an example, consider the syllogism:

(9) No B are C

(10) All A are B

(11) Therefore no A are C

Premises (9) and (10) are represented by Figure 12.

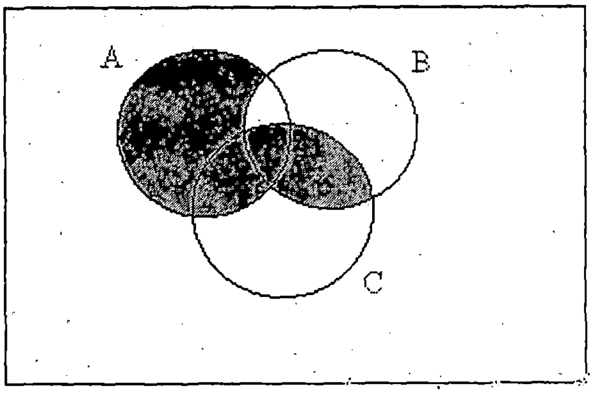

FIGURE 12

We now examine the diagram to see if it implies that no A are C. We have seen at once that the region common to A and C is horizontally shaded, and we conclude that the conclusion of the syllogism is valid. All other valid syllogisms may be tested in the same way, but there is no need to restrict the use of Venn diagrams to testing the validity of those arguments, which have the classical syllogistic form. Venn diagrams may be used to represent any argument, which does not involve more than three sets. Moreover, by a careful use of ellipses in place of circles relations among four sets can be represented diagrammatically, but relations among five or more sets can often not be represented by any simple diagrammatic device.[1]

The relation between set and subset as found in the western logic has affinities with Indian concepts of *parā* and *aparā sāmānya* (i.e., greater and comparatively smaller universal). The universal (*sāmānya*) is defined as a property which, being eternal, is inhered in many individuals (*nityatve sati anekasamavctatvam*) as per the view of the Naiyāyikas. To them the properties like 'cowness' etc. are eternal in nature and remain in all the individual cows through relation of inherence. In other words, it can also be said that the property 'animality' (*prāṇitva*) remains in all individual cases of animal through relation of inherence and hence it is also a case of universal.[2]

The Naiyāyika defines the universal as a character, which is *nitya* (eternal) and *anekasamaveta* (inheres in many particular instances). Therefore, according to Naiyāyikas, the relation between a universal and its particular instance is the relation of inherence. Further, the universal is an eternal character inhering in more than one particular instance. Therefore, where there is only one instance of a thing, its distinguishing character is not a logical universal, e.g., according to the Naiyāyika, there is only one *ākāśa* or ether. Therefore ether ness is just a distinguishing character and not a logical universal – an *upādhi* and not a *jāti*. Again when a character or feature which is related to the substrate which it characterises by some relation other than the relation of *samavāya* or inherence, it is no logical universal in the strict sense, e.g., negativity or *abhāvatva* is a common character of such particular *abhāvas* or negations as *ghaṭābhāva*, *paṭābhāva*, etc. But since the relation of *samavāya* holds only between positive objects of experience or *bhāvapadārthas*, and not between positive and negative objects, nor between one negative object and another, the relation of *samavāyatva* does not hold between *abhāvatva* or negativity and the particular negatives in which it is found as a common character. Thus *abhāvatva* or negativity, as not admitting of the relation of *samavāya*, is not a logical universal. The Naiyāyika also rejects overlapping universals as not being logical universals in the strict sense, e.g., *bhūtatva* or the character of being an element is common to the five elements earth, water, air, fire and ether and *mūrtatva* or the character of moving is common to the five moving substances, viz. earth, water, air, fire and mind. Thus both these characters have earth, water, air and fire as their common substances while 'the character of being an element' applies to *ākāśa* and not to mind, and 'the character of moving' applies to mind and not to *ākāśa.* Therefore, if 'the character of being an element is conceived as a universal, it will apply to the four *bhūtas* – earth, water, air and fire which are moving thing as well. And then the universal *bhūtatva* will coincide with the universal *mūrtatva* in respect of these four substances and ought therefore to apply to the other *mūrta,* viz., mind though it does not. And the same objection will hold in respect of *mūrtatva* which should apply to *ākāśa* though it does not. Further, the four substances, earth, water, air and fire, will have to be regarded as instances of two difference universals, which is like saying that some animals are both cows and buffaloes which is absurd. This is why character with partially overlapping denotation are not admitted by Naiyāyikas to be logical universals.

Another negative condition of a logical universal, according to the Naiyāyika, is *regressus ad infinitum*. Where the acceptance of a character as a universal will land one into an infinite regress, no logical universal is admissible according to the Naiyāyika. This is why the Naiyāyikas do not recognise universals of universals, e.g., 'horseness', 'cowness' and 'dogness' are three universals, and since each of these is a universal, universality is a character common to these universals. If universality is, therefore, to be regarded as a fourth higher universal, and 'horseness', 'cowness' and 'dogness' as particular instances of it, then, in so far as this higher universal is a fourth universal, one must conceive a still higher universal of these four universals, namely, 'horseness', 'dogness', 'cowness' and 'universality'. In the same way we shall have to go from a fourth to a fifth universal, from a fifth to a sixth and so on *ad infinitum.*

The fifth negative condition of a logical universal, according to the Naiyāyika, is *rūpahāni*. By this the Naiyāyika means that where recognition of a character as universal contradicts the intrinsic nature or *rūpa* of a thing, it is not admissible as a logical universal, e.g., *antya viśeṣa*, the ultimate differential, is an individuating principle inherent in every eternal substance. Each eternal substance is a unique individual because of the presence in it of this ultimate differential or *viśeṣa*. Each eternal substance has thus a *viśeṣa* inhering in it which differentiates it from all other objects of experience. *Viśeṣatva* or differentiating character is thus a character common to different *viśeṣas* inhering in different eternal substances. Why not then accept *viśeṣatva* as a universal, common character of the different *viśeṣas* of the innumerable eternal substances? The Naiyāyika answer is in the negative as the admission of *viśeṣatva* as a universal destroys the very nature of *viśeṣa* (*rūpahāni*). *Viśeṣa is* that which is unique, uncommon and if a common character of the uncommon be admitted it will destroy the very nature of the uncommon as uncommon.

A sixth negative condition also laid down by the Naiyāyikas is that no separate second universal can be admitted where the difference between two universals is a difference in name only, e.g., between *kalaśatva* and *kumbhatva*.

It may be noted that while Naiyāyikas repudiate universal of universals, they yet recognise a gradation of universals into higher and lower reaching up to one highest universal (*parājāti*) which is *sattā* or being. Thus according to the Naiyāyika, the universal of 'being' or *sattā* is the most comprehensive universal (*parājāti*) applying to all particulars while lower universals (*aparājāti*) apply to some particulars and do not apply to other particulars, e.g., *dravyatva*, substantiality, or substanceness, is a character of every *dravya* or substance, but not of a *guṇa* (quality) or a *karma* (motion). Similarly *guṇatva* holds of every *guṇa* or quality, but not of any *karma* or *dravya*. Thus, *dravyatva* is both *anuvṛttilakṣanạ* and *vyāvṛttilakṣaṇa*, both inclusive and exclusive. *Dravyatva*, e.g., is inclusive of *dravyas* and exclusive of *karmas* and *guṇas*. *Guṇatva* is inclusive of *guṇas* and exclusive of *dravyas* and *karmas*. But *sattā* or being is true of all *dravyas*, *guṇas* and *karmas*, i.e., it includes all and excludes nothing. In this sense *sattā* or 'being' is the highest universal or *parājāti* while other universals are lower in rank.

It is obvious from the above that what the Naiyāyika means by the gradation of universals into lower and highest reaching up to one *parājāti* or highest universal, viz., *sattā* is their grading in respect of extent or denotation, the highest being higher as possessing a wider or more extensive denotation and the lower being lower as possessing a narrowing or less extensive denotation and the highest being highest as possessing the most extensive denotation of all. The Naiyāyika does not mean a connotative subsumption of one universal under another and that is why he repudiates universals of universals as leading to infinite regress.[3]

The Nyāya theory of universals is not without its difficulties as both Buddhists and Advaitins have pointed out a universal is both eternal and an inherent character of its particular instances, then how does the Naiyāyika account for the appearance of a universal in a newborn instance of it? And how does he account for its disappearance, when it ceases to be? When a new jug is made out of a jump clay, does the eternal jug-ness (*ghaṭatva*) come suddenly into being in the newly made jug, or, when the jug is broken does the eternal jugness cease to be so far as the broken jug is concerned? Suppose the species we call 'cow' becomes extinct in course of evolution so that not a single individual is anywhere left on the earth. Where will the eternal 'cowness' go? Will it wander about like a floating adjective, an abstract universal without a particular locus? Further, when the universal inheres in a particular instance of it, does it inhere in it in its entirety, or does only a part of it inhere in the par particular instance? If it inheres in its entirety, then nothing of it will be left to inhere in other particular instances, so that if there be one individual cow there will be no other cows. And if it inheres only partially in a particular instance of it, then we are landed in the absurdity that an individual cow is only partly a cow and partly some other animal such as a buffalo. It may be noted that the Buddhists repudiate the Nyāya view of universals and offer instead their own theory known as *Apohavāda*. According to them, the so-called positive common character is a myth. Universality is only *anyavyāvṛtti*. It is common exclusion rather than common inclusion that constitutes universality. When we say X is a cow, we do not mean that it is one particular instance of the universal 'cowness' which X has in common with other cows as its inherent character. All that we mean is that it is not a horse, not a dog, not a man, etc. Further, according to Naiyāyikas, 'existence' (*sattā*) is the *parājāti*, highest universal and is an inherent common character of all *dravyas*, *guṇas* and *karmas*, substances, qualities and actions. Therefore, in so far as a cow or a horse or a chair or a table is a substance, it has existence or *sattā* as its inherent character. Therefore, the negative judgment 'a chair is not' or 'a table is not' or 'a horse is not' or 'a cow is not' amounts to a manifest self-contradiction, for this is the same as saying that the cow which is inherently existent does not exist. Contrariwise, when we say that the cow exists, our judgment becomes a tautology, for it amounts to saying that the inherently existent exists, or, that 'that to which existence belongs as an eternal inherent character exists'.[4]

Further, if the universal, as the Naiyāyika says, be an inherent eternal character of its particular instances, then in so far as one and the same particular is an instance of two or more universals, e.g., in so far as a cow is an instance of the universal of substance (*dravyatva*) and again an instance of the universal of *sattā* or being and also an instance of the universal 'cowness' (*gotva*) it becomes the seat of several universals, i.e., a case of overlapping universals or *jāti sankara*.

The Buddhists have accepted the reality of an object in terms of its casual efficacy (*arthakriyā-kāritva*). All objects that have got casual efficacy are momentary in nature.

It has been argued by the Vaiśeṣikas that the meaning of the term '*Sattva*' (existence or being) seems to be vague to them. The term '*sattva*' means an object's association with *sattā*, *sāmānya* or *jāti* and hence possessing this eternal generic property can be momentary.

In response to this Buddhists rejoin that they do not accept that an object possessing *sattā sāmānya* is existent. It is so, the existence would have to be admitted in substance, quality and action due to accepting *sattā sāmānya* there. To the Vaiśeṣikas *Sāmānya*, *Viśeṣa* and *Samavāya* do not possess existence or *Sattā* due to the problem of infinite regress. If *sattā* or *sāmānya* is accepted in *Sāmānya* or *Viśeṣa* etc. there would arise the question of accepting another *Sāmānya* in it i.e. *Sāmānyatva*, *Viśeṣatva* etc. and in this way the defect of infinite regress cannot be avoided. In fact, Vaiśeṣikas have accepted the *Sāmānya* etc. as *sat* as they are revealed as such, but this is not *Sāmānya* in the technical sense. If the Vaiśeṣikas accept *sattā* in the form of *astitva* in *Sāmānya* etc., and *sattā* in the form of *sattā Sāmānya* in substance etc. there would be *gaurava*, in determining the criterion of apprehending the *Sat* object. Moreover, another problem would crop up. There would arise common apprehension (*anugatapratyaya*) in the substance etc. due to having the same *sattājāti* in *Sāmānya* etc. and hence there would also arise the common apprehension, which is not observable.

It has been accepted by the Nyāya - Vaiśeṣikas philosophers that *Sattā* or *Jāti* exists in different *loci* bearing same shape and size through relation of inherence. In this connection, the Buddhists ask that, if *Sāmānya* exists in many things bearing same size, how do they admit *sāmānya* or *sattājāti* in different objects bearing different shapes and sizes like substance, quality and action and also between mustered seeds and mountain? To the Vaiśeṣikas *sattā sāmānya* exists in substance etc. through the relation of inherence (*sāmānya*). If it is taken for granted, the Buddhists argue how the usage of differentiation between a man and a cow in the form: 'This is a cow and this is a man' can be made. If it is said that the universals like humanity, cowness etc. pervading in a man and a cow are the causes of the usage of the differentiation between man, it is not tenable because the concept of universal as propounded by them is under consideration.

It is enquired by the Buddhists whether the universal exists in all objects or in all individuals belonging to a particular class. In the case of former, all objects would be of a same type due to the existence of same universal in them. If the universal 'humanity' existing in a human being remains in horse etc., the horse etc. would have to be considered as man due to having humanity in them, which is not possible. Moreover, it will go against the established thesis of the Naiyāyikas. If the latter is taken for consideration, it will also create some difficulties. That universal exists in all individuals belonging to the same class is admitted by Praśastapāda. If this line is accepted, it will lead to some philosophical difficulties as follows:

The universal 'Jarness' did not exist in a piece of mud before the origination of a jar but it is produced just after the origination of the same. It is asked by the Buddhists whether the universal 'jarness' existing in a jar situated in other place is related to this jar existing in a different place or not. If it is so, whether this universal is related to a particular individual after coming from other places or without coming from there. In the case of former the universal would have to be designated as substance as it possesses the action in the form of movement. In the case of latter there would arise the difficulty in apprehending the relation. For, how can the relation of one object to another be established without accepting the action or movement.

It cannot also be said that the jar ness etc. existing in a jar etc. is related to a jar existing in a different place through its self-extension. For, self-extension is possible for an object having parts (*Sāvayava*). As jarness etc. have no parts (*niravayava*) the extension of it is not possible.

Moreover, when a jar is destroyed, the problem is whether the jarness existing in it remains in it or is destroyed or goes elsewhere. The first alternative is not correct as universal cannot remain without its substratum i.e., an individual. Moreover, universal always remains only in the objects other than the eternal ones. If the second alternative is taken into account, it will lead us to accept the antithesis i.e., the eternity of the universal as accepted by the Naiyāyikas. The acceptance of the third alternative leads to accept another undesired situation. For, universal can go elsewhere if there is movement. If the existence of movement is accepted in *sāmānya*, it would turn into a *dravya* or substance but not *sāmānya* due to having movement in it.

To the Vaiśeṣikas universal exists in substance etc. through the relation of inherence. If it is so, the Buddhists argue that the ground on which a jar exists also contains the jarness existing in a jar as the lower part of a jar is connected with the upper surface of the ground. If jarness remains on the ground, the ground would also be taken as a jar, which is not possible. Moreover, jarness cannot pervade a jar existing on the ground without keeping it associated with the ground.

Considering all these defects the Buddhists do not accept *Sāmānya*. To them, *Sattva* is not in the form of *Sāmānya* but in the form of causal efficacy (*arthakriyā-kāritva*).

If *Sāmānya* is not accepted, how is the common knowledge (*anugatapratyaya*) among various individuals of the same class possible. To the Buddhists it is not true that cow is differentiated from other animals like horse etc., with the help of *Sāmānya*, but cow is known as distinct in terms of the knowledge of 'non – cow' (*agovyāvṛtti*). In the same way, a jar is known in terms of the knowledge of non – jar (*aghaṭavyāvṛtti*). This is type of negative way of knowing is called *apoha*. *Apoha* is that which can differentiate a particular object from others (*svetaravyāvṛttirūpa*).[5] The distinctness of a jar from other object (*ghaṭetarabheda*) which remains in all individual jars leads us to the apprehension in the form: 'This is a jar but not a cloth' and through this similar cognition among all individuals belonging to the same class is established the derivative meaning of the term '*apoha*' is as follows:

That which differentiates something from others (*apohanam*) is *apoha*. As it differentiates a particular object from others, it is called *anyāpoha*. Ratnakīrti has opined that the verbal usage in the form 'This is a cow' is originated from the apprehension in the form 'This cow is different from non-cow' (*agovyāvṛtti*). Hence, the phenomenon of *anyāpoha* is the cause of similar apprehension (*anugatapratyaya*) and hence there is no necessity of *Sāmānya*. In other words, the similar cognition of all individuals of the same class is due to an object's unique character (*svalakṣaṇāt*) which is possible through its distinct nature from other objects.[6]

Now the question of empty set or null set comes. A set having no members is to my opinion, not a set at all. A set having no members is contradictory in terms. In fact, the collection of some members makes a set in the ordinary sense of the term. If there is a set having no members at all, it is not to be said to be a set. Hence a set without any member is contradictory in terms.

In Indian logic the null set or empty set is found in the terms like 'sky-flower' (*khapuṣpa*), *hare's* horn (*śaśaśṛnga*) etc. The Naiyayīkas think that these words are meaningless as they have no references. As there is no object or referend of the terms, they are absurd entities. An object which is referred to by this word does not belong to the category of real. Hence it is neither substance nor quality nor action nor universal nor particularity nor absence. It is not even absence because for being an absence there should be an awareness of its absentee (*pratiyogī*). The cognition of absence always depends on that of an absentee (*pratiyogijñānasāpekṣam abhāvajñānam*). If an absentee (*pratiyogī*) is not known, its absence can also be known. The entities like *khapuṣpa* etc. do not come under the absence also, because these (the counter positives) are not known at all. The absence in the form: *'khapuṣpam nāsti'* is also non-sensical, as the absentee called *khapuṣpa* is not known at all or not knowable at all. An entity which cannot be categorised as substance, quality, action etc. is taken as non-entity (*apadārtha*). As the Naiyayikas are realists entities which have no existence at all. The following *kārika* carries the examples of empty terms: "*Eṣo bandhyāsuto yāti khapuṣpakṛta - śekharaḥ/ kūrmakṣīracaye snātaḥ Śaśaśṛngadhanurdharaḥ*". (That is, here goes the barren woman's son with a crown made of sky-flowers, who has bathed in a pool of tortoise – milk and carries a bow built of rabbit – horn).

Western logicians have accepted such null classes or empty classes in spite of knowing that there is not a simple member in it, because null or empty class is a notion, which is opposed to non-empty class. To prove that there is a non-empty class it is to be known what empty class is as per the principle – 'the cognition of an absence presupposes that of the absentee'. Hence empty set is to be admitted as a real set only to understand a non-empty set.

The grammarians in Indian tradition also contribute to this view.

The statements mentioned above – *bandhyāputra etc.* involve a lot of empty terms, yet they communicate some thought to the hearer and the discovery of its incompatibility with the world of facts makes him laugh at the speaker. Why should one find a statement amusing if one grasps nothing at all? The thought directly conveyed by an expression is looked upon by the Pāṇinians as a conceptual existence, which may or may not find a corresponding external counterpart in the realm of realities. Thus according to these grammarians a meaning, as a conceptual existence, is *independent of external existence*. A *referent* is not a must for a meaningful word. *Buddhisattā* or the conceptual existence, as it is understood by the Pāṇinians, is a near approach to what is described as 'Being' by Russell in *The Principles of Mathematics* where it is shown that even the words like *chimera* and *unicorn* have Being in spite of their having no *Existence*. If the words like *śaśaśṛnga* ('rabbit-horn') conveyed no sense how could they be used as *prātipadika* (i.e., stem), which, by definition, is bound to have a meaning (*arthavat*)[6]? The *Vaiyākaraṇa* concepts of *buddhisattā* and *bahiḥsattā* can be roughly replaced by '*Being*' and '*Existence*' respectively as they are employed by Russell in *The Principles of Mathematics*. Inspired by Patañjali's comment on the *matup-sūtra* (i.e., the Pāṇinian rule 05/02/94) – '*na sattām padārtho vyabhicarati*' (which literally means 'a word-meaning is not without an existence')[7]. Bhartṛhari, Kaiyaṭa, Helārāja and Nāgeśa have developed their doctrine of *buddhisattā*. Actually speaking, the word *sattā* in Patañjali's remark means *buddhisattā* or a conceptual existence, i.e., *Being*, and not *bahiḥsattā* or *Existence* proper. There is no such meaning of a word as has no conceptual existence – this is the purport of Patañjali's observation. Conceptual existence is no existence proper (i.e., *mukhyāsattā* according to Bhartṛhari); it is only an *imposed existence* (*aupacārikī sattā*), which is nothing external to human consciousness. Helārāja's description of *aupacārikī sattā* or *buddhisattā* as *bhāvābhāvasādhāraṇa* is highly significant.[8] It shows that conceptual existence is commonly extended to both the existent (*bhāva* or *sat*) and the non-existent (*abhāva* or *asat*). It is the thought or sense (an approximate equivalent, not an exact equivalent of *buddhisattā*), which every word must have. Hence, there is no word or expression which may be labelled as 'nonsensical'.

The Pāṇinian theory of meaning aspires after solving many a problem which one finds difficult to explain in terms of the naïve referential theory advanced by the Naiyāyikas. It provides an early but convincing answer to the plaguing problem – 'Is Existence a Predicate'? That which does not exist can never be referred to. As soon as we employ a term as the subject of a proposition we assume the existence of what it means. Predicting existence of something existent in sentences like '*vṛkṣaḥ asti*' ('the tree exists') runs the risk of being a mere 'referential tautology', an example of what we call *siddha-sādhana* ('establishing the established') in Indian Philosophy. Again denying existence of that which is existent, apparently sounds contradictory as we may find in sentences like '*vṛkṣaḥ nāsti*' ('the tree does not exist'). The Pāṇinians try to avoid this risk of either 'referential tautology' or 'referential contradiction' by saying that in the affirmative categorical statement we assert that our subject which is thought to have a conceptual existence has a factual existence too, whereas in a negative sentence the subject is not so fortunate as to have a factual existence apart from the assumed one.[8a] One may contend that even if the words '*asti*' and '*nāsti*' are supposed to have a conceptual existence for their meaning, the same difficulties will continue unabated. Let us in this context reproduce some significant observations from a thought provoking article by Prof. V.N. Jha :

'...... according to the *bauddhārtha-vādins*, all words express *bauddhārtha* and so when *ghaṭa* expresses "pot having conceptual existence"., the word asti should also express conceptual existence (*bauddhasattā*) and in that case the use of the verb asti becomes redundant since the "existence" is already conveyed by the word *ghaṭa* (*uktārthānam aprayogaḥ*). Similarly, there will be a contradiction in the negative sentence since *ghaṭa* conveys existence and *nāsti* conveys non-existence. Thus if *bauddhapadārtha* is accepted, the difficulty is unavoidable".[9]

But, to do full justice to the grammarians, we should clarify a bit further. In the affirmative sentence '*ghaṭaḥ asti*' ('the pot exists') the subject term conveys the thought or conceptual existence of a tree we may find in reality and the predicate term does that of a real existence. Our predicating '*asti*' of '*vṛkṣah*' amounts to the affirmation of the factual existence in relation to what is conveyed by the subject term. *Thought* is the vehicle through which this affirmation takes place. In the negative sentence, '*nāsti*' conveys the thought of the negation of existence. Our predicating '*nāsti*' *through the medium of thought* conveys the denial of the referent of '*asti*' (i.e., existence) in relation to the subject and shows that the latter has no existence apart from the assumed one. It is the *bauddha artha*, which is directly expressed by '*asti*' or '*nāsti*' at first, then it stretches out to the factual level and makes the sentence relevant by pointing to the existence and the negation of the subject respectively. The Pāṇinians are quite conscious that the word '*ghaṭaḥ*' is *apparently* sufficient, since as soon as we utter something we assume its existence. This is evident from Bhartṛhari's assertion that our uttering '*ghaṭa*' assumes that the pot has existence in the form of sustaining itself.[10] Existence or sustaining itself (*ātmadhāraṇa*) is the very nature of a thing. Nāgeśa hints at the same when he remarks – '*loke*' *styartham vinā śuddhapadārthānavagateśca*'.[11] [It is because a person cannot grasp a pure entity without grasping its existence.] Yet, the predicate term *asti* is of importance, if the context demands it. Suppose somebody expresses doubt about the existence of the pot and you then put a special emphasis on the word '*asti*' to dissolve his doubt. Here the assertion is of an additional significance; through the vehicle of *thought* it confirms the existence of the subject. If again, one is mistaken that a pot is there; you want to dispel that wrong conception. You then, through the thought-medium, confirm the negation of the existence and thus correct the erring person. The Vaiyākaraṇas attach paramount importance to the context and the speaker's will which often determine the use of a particular word. Herein lies the significance of Nāgeśa's remark that in their opinion the expressions 'does exist' and 'does not exist' are employed in order to intimate the 'existence' and 'non-existence' respectively of the subject concerned.[12]

The Pāṇinians adopt a similar line of thinking to account for the subject-predicate relation in the expression '*ankuro jāyate*' ('the sprout is being born'). *Ankura*, the subject, is taken as the nominative agent (*kartṛkāraka*) of the act of being born. The *kārakas* are the causal conditions, which combine to produce an action – "*samagrīsādhyatvāt kriyāṇām sarve kārakatvam kriyāniṣpattiviṣayam abhedena pratipādyante*"[13] as Heliārāja describes the matter. An action is a process, which awaits completion (*sādhya*), but those, which perform it, must be pre-established (*siddha*) and the nominative agent plays the major role in this production process. The subject (i.e., the nominative agent) having its existence already established, it is absolutely superfluous to speak of its birth. How can one assert the birth of something, which is already born and existent? However, this is no problem for the Pāṇinians according to whom, through the above sentence one relates the thought of a real birth to the sprout the existence of which is already assumed. Thus there is an obvious journey from what is imposed to what is real. Bhartṛhari, in his *Sādhanasamuddeśa*, arrives at the same conclusion from another angle. To the probable question how something which is not yet born can be treated as the nominative agent, Bhartṛhari puts forward the reply –

utpatteḥ prāg asadbhāvo buddhyavasthānibandhanaḥ/
aviśiṣṭaḥ satānyena kartā bhavati janmanaḥ//[14]

That which has no (factual) existence before its birth owes an intellectual existence to the speaker's intention and thereby has got an (*assumed*) efficacy to perform the action in question. The sentential meaning settles down to predicating external factual birth to that of which the existence is caught in intellect, and which has thus an intellectually grasped capacity to do some job *("buddhyā niścitasattvasya kriyāsiddhāvupagṛthītasāmarthyasya bāhyena rūpeṇa janmeti vākyartho" vatiṣṭhate.)*[15]

The Naiyāyikas may, on the contrary, count upon a 'possible reality' and argue that the referent of the word '*ankura*', though not existent at present, is to come into being in the near future. Only that thing can be called unreal or non-existent which does not exist in the three phases of time – past, present and future. A sprout is not unreal in that sense. Through the sentence '*ankuro jāyate*' one asserts the birth, a physical phenomenon, of something real which, however, does not exist at the present segment of time.

The Pāṇinians on the other hand, though speaking of a *superimposed apparent reality*, do not make allowances for any possible reality. There is no such thing as *past* or *future existence* (*bhūta-bhaviṣyat-sattā*). 'To exist' means 'to be present'.[16] A reality is only that which *is* and *not what may be* or *will be* or *will cease to be*. Otherwise how can we declare 'a thing has ceased to exist' or 'a thing will come to exist' which simply means that the thing does not exist? When one says 'the jar will exist' (i.e., at the time of its pre-absence or *prāgabhāva*) or 'it will be raining', one has only a conception or thought of the jar or of the rain which has not yet come into being, may be the prior perceptions of some existent jars or rain have worked behind this thought. One cannot deny that the particular jar which is conceived or thought of does not exist at the time of that conception. It is quite understandable why Nāgeśa has drawn our notice to a rule of Gautama, the Nyāyasūtrakāra himself, which states that the non-existent effect, is, however, established in our intellect ('*buddhisiddham tu tadasat*').[17] The weaver who knows which specific type of material is required for a specific type of product has a thought of the individual cloth, which he is going to produce. He does not mean to produce which is already existent. The non-existing individual then exists in the intellect alone; nevertheless his conception is nourished by the individuals he has seen. Nāgeśa makes it a point that even the referentialists and the realists like the Naiyāyikas cannot do away with '*buddhisattā*'. He has for his support an observation of another idealist, the author of the *Bhāmatī* on the *Śānkarabhāṣya* under the *Brahmasūtra* 1/1/2 ('*janmādyasya yataḥ*'), who states that a person makes a 'yet-to-be-produced' (*nirvartya*) non-existing jar the object of his production on the strength of a conceptual determination (*antaḥsamkalpātmanā*).[18]

Or, consider the situation when a person informs that he wants to cook. The particular act of cooking is not performed as yet; if it were so one could not wish to perform it again. One cannot perform what has been performed already. To find out a rationale behind the informative sentence 'I want cooking' you must say that here in this expression 'cooking' does not have the referent, i.e., the real *individual* act of cooking for its meaning. The word 'cooking' conveys just a thought or conception which awaits materialisation. A desire, just like a knowledge, should have an object of which it is the desire. In this sense, the object is one of the causal conditions that produce a desire. A desire is an inner phenomenon and so, Nāgeśa argues, the direct object which causes this desire should also be an inner one.[19] It is true that the desire works behind the actual happening of the act in the world of facts. But it is equally true that a pre-conception of that 'yet-to-materialise' object prompts the person to act. This is the import of the following comment of Durbalācārya, the learned commentator of Nāgeśa's *Laghumañjūṣā* – "*siddhe pākādāvicchāder virahād asiddhasyaiva pākāder icchām prati hetutāyāvācyatayā buddhāveva hetu-hetumatoḥ sāmānādhikaraṇyam upapādanīyam*".[20]

[Since, if the cooking is done there is no scope for the desire, 'yet-to-be-established' cooking etc. will be considered the cause; hence an equilocativity between the cause and what is caused is to be found out in (the plane of) intellect alone.][21]

Again, suppose somebody narrates that Kṛṣṇa is killing Kamsa, or one may refer to a particular scence of a drama and say 'Kamsa is being killed'. Now, if one believes in the Purāṇa story, Kamsa was slain long ago; how can he be the object of killing again? Patañjali's *Mahābhāsya* on Pāṇini's rule '*Hetumati ca*' (3/1/26) shows the way how one can account for the meaning cognition of the above statements – "*sato buddhiviṣayān prakāśayanti*".[22] [The narrators communicate the conceptually existent meanings.] Bhartṛhari, Kaiyaṭa and Nāgeśa – all agree that this is the only way in which Kamsa etc., the objects of imaginative intellect, are conveyed by words and thereby treated as different agents (*kārakas*) of an action which seems to be enacted before our eyes –

Śabdopahitarūpāmśca buddherviṣayatām gatān/
pratyakṣamiva kamsādīn sādhanatvena manyate//[23]

Nāgeśa reminds us of another case where acceptance of *bauddha artha* comes to the rescue. Somebody asks you – "who is Devadatta?" You, in reply, direct your index finger towards a particular person and say 'This person with armlets, earrings, a broad chest and round arms – Devadatta is like this.' Here Devadatta is a single person who is referred to. But whenever we talk of a resemblance we suggest that there is some other individual resembling the one who is placed before us. All this talk of resemblance is meaningless when only a single person is concerned. There cannot be different persons in one individual ('*ekavyaktau bhedābhāvāt*') who is referred to. To make the word '*īdṛśa*' (meaning 'like this') meaningful you have to admit that the word '*īdṛśa*', in spite of its having no other referent apart from the one which the name 'Devadatta' refers to, has a distinct thought for its meaning. Unlike the words '*go*' (meaning a cow) etc. where the etymological sense is fully sacrificed for the sake of a conventionally established meaning, the word '*īdṛśa*' retains the derivative sense 'like this'. For the Pāṇinians, however, there is nothing inconsistent in applying the word '*īdṛśa*' which is inserted to assert that the thought expressed by the different words, both the proper noun and its adjectives together, finds its confirmation in a real counterpart, the object of identification.[24]

The realists cannot justify the statements like '*vināśī śabdaḥ*' ('sound is destructible') until and unless they admit a conceptual or intellectual existence of the destruction, which is yet to materialise. The expression conveys a qualifier – qualificand relation between destruction and the entity, which is destroyed. Destruction is posterior absence (*dvaṁsābhāva*) and an absence or negation cannot be coeval with the negatum of which it is the negation. So how can there be a relation between the two? Yet one conjures up a non-existent relation in one's intellect; otherwise where is the scope for stating *vināśī śabdaḥ* or *vināśī ghaṭaḥ* ('The jar is destructible')? Even Jayantabhaṭṭa, the Naiyāyika of great repute, is apprehensive of this linguistic predicament and finds a wav-out by admitting that while stating *vināśī śabdaḥ*, the speaker connects through the thought-medium the existing sound with only a conceptually or intellectually existent destruction.[25]

Let us also take note of some faulty syllogisms (*hetvābhāsas*), which project a non-existent relation. Take an example of *bādhita hetvābhāsa* – '*vahnir anuṣṇaḥ dravyatvāt*'

('Fire is non-hot, since it is a substance'). Factually speaking, there can never be a relation between *sādhya* or probandum (i.e., 'being non-hot' or *anuṣṇatva)* and *pakṣa* or the subject, fire. Yet a thought of that relation is communicated through the above statement.[25a] There are of course phenomena like a fire and 'being non-hot', yet their relation is a non-entity. The same can be asserted with reference to a svarūpāsiddha hetvābhāsa which records that sound is impermanent, since it is visible (*'śabdaḥ anityaḥ cākṣuṣatvāt'*). Despite the fact that 'visible' and 'sound' have their respective referents, the thought expressed by the statement, which brings their relationship into the focus has no referent to fall back upon. Or consider the major premises involved in a *viruddha hetvābhāsa* – 'All products are eternal' (*'yet kṛtakam tan nityam'*) which iterates the contradictory of what really is. If the above statements are described as focussing only a new arrangement of the referents of the words, then one cannot help admitting that at least a non-existent arrangement is caught in the intellect. The referential status is lost beyond recovery in the case of an *āśrayāsiddha hetvābhāsa*, e.g., 'the sky-lotus is fragrant, since it is a lotus' (*'gaganāravindam surabhi, aravindatvāt'*) wherein the very subject is an empty term. Even the Naiyāyikas are doubtful if a truth-value can be assigned to the above statement. Gaṅgeśa himself seems to be in two minds when he remarks that the subject 'sky-lotus', if taken in its totality, makes the statement non-significant (*apārthaka*) which is considered a point of defeat (*nigrahasthāna*) for the arguer concerned.[26] But we have already discussed that the fate of not only *āśrayāsiddha*, but of many other *hetvābhāsas* as false assertions hangs in the balance, if one does not accept a thought of some non-existing relationship or arrangement to be conveyed by the statements concerned. In other words, many of our *hetvābhāsas* have got to be dismissed as purely *apārthaka* or nonsensical if we do not recognise a conceptual existence as the direct meaning of an expression.

Examine a debate (*vāda*), which consists of contradictory statements.[27] Does not the proponent grasp the meaning of what his opponent says? Had there been no meaning conveyed by the opponent's statement, why should the proponent strain himself by an uncalled for attempt to refute him? Why should he not ignore that statement as some sort of delirium? – *kiñcaivam sati vāde prativādiśabdasyābodhakatve tatkhaṇḍanakathocchedaḥ.*[28] One of the contradictory statements must be true and the other false. Should we then say that a false statement expresses a false fact? But there is *no false fact* at all.

Hence the grammarians believe that the empty set comprising of *bandhyāputra* etc. is associated with meaning, which is similar to the mathematicians. But for the Naiyāyikas such expressions are meaningless leading to the non-acceptance of null set, because even the absence of *bandhyāputra* etc. cannot be predicted due to the absence of the absentee as a category.

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7. Nāgeśa, *Laghumañjūṣā*, Vol. I, p. 343.

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13 Ibid; p. 240.

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15. *Vākyapadīya*, Sādhanasamuddeśa – 105.

16. Ibid. *Prakīrṇaprakāśa*; *Laghumañjūṣā*, Vol. I, p. 240; Pradīpa on *Mahābhāṣya* under the rule 5/2/94.

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18. Gautama Akṣpāda, Nyāyasūtra 4/1/50.

19. *Laghumañjūṣā*, Vol. I, p. 247.

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21. Durbalācārya, Kuñcikā on *Laghumañjūṣā*, Vol. I, p. 240.

22. Also see *Laghumañjūṣā*, Vol. I, '*icchādīnām antaḥakaraṇaniṣṭh atayā tatra viṣayasya sāmānādhikaraṇyenaiva kāraṇatvaucityena bauddhapadārthasattāvaśyakī*/p. 240.

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