CHAPTER - 1

INTRODUCTION

INTRODUCTION :

A pencil of radiation traversing a medium will be weakened by its interaction with matter. If the specific intensity I_ν *therefore becomes* $I_\nu + dI_\nu$ after traversing a thickness ds in the direction of its propagation, then

$$dI_\nu = -k_\nu \rho \, I_\nu \, ds \qquad (1.1)$$

where ρ is the density of the material. The quantity k_ν introduced in this manner defines the *mass absorption coefficient* for radiation of frequency ν. Now, it should not be assumed that this reduction in intensity, which a pencil of radiation experiences while passing through matter, is necessarily lost to the radiation field. For it can very well happen that the energy lost from the incident pencil may all reappear in other directions as *scattered radiation.* In general, I may however expect that only a part of the energy lost from an incident pencil will reappear as scattered radiation in other directions and that the remaining part will have been 'truly 'absorbed in the sense that it represents the transformation of radiation into other forms of energy (or even of radiation of other frequencies). I shall therefore have to distinguish between *true absorption* and *scattering.* Considering first the case of scattering, I say that a material is charecterised by a *mass scattering coefficient* k_ν if from a pencil of radiation incident on an element of mass of cross-section $d\sigma$ and height ds, energy is scattered from it at the rate

$$K_\nu \rho \, ds \times I_\nu \, \mathrm{Cos}\vartheta \, d\nu \, d\sigma \, d\omega \tag{1.2}$$

in all directions. Since the mass of the element is

$$dm = \rho \, \mathrm{Cos}\vartheta \, d\sigma \, ds \tag{1.3}$$

I can also write

$$K_\nu \, I_\nu \, dm \, d\nu \, d\omega \tag{1.4}$$

It is now evident that to formulate quantitatively the concept of *scattering* we must specify in addition the angular distribution of the scattered radiation (1.4).

I shall therefore introduce a *phase function* $P(\cos\theta)$ such that

$$K_\nu \, I_\nu \, P(\mathrm{Cos}\theta) \, \frac{d\omega'}{4\pi} \, dm \, d\nu \, d\omega \tag{1.5}$$

gives the rate at which energy is being scattered into an element of solid angle $d\omega'$ and in a direction inclined of an angle θ to the direction of incidence of a pencil of radiation on an element of mass dm . Accordingly the rate of loss of energy from the incident pencil due to scattering in all directions is

$$K_\nu \, I_\nu \, dm \, d\nu \, d\omega \int P(\mathrm{Cos}\theta) \, \frac{d\omega'}{4\pi} \, ; \tag{1.6}$$

this agrees with (1.4) if $\int P(\mathrm{Cos}\theta) \, \frac{d\omega'}{4\pi} = 1$ (1.7)

i.e. if the phase function is normalised to unity .

In the general case when both scattering and true absorption are present, I shall still write for the scattered energy the same expression (1.5). But in this case the total loss of energy from the incident pencil must be less than (1.5), accordingly

$$\int P(\cos\theta)\ \frac{d\omega'}{4\pi} = \omega_o \le 1 \qquad (1.8)$$

Thus the general case differs from the case of pure scattering only by the fact that the phase function is not normalised to unity.

It is evident from our definitions that ω_o represents the fraction of the radiation lost from an incident pencil due to scattering, while $(1 - \omega_o)$ represents the remaining fraction which has been transformed into other forms of energy. I shall refer to ω_o as the *albedo for single scattering.* A radiation field is said to be isotropic at a point, if the radiation is independent of direction at that point. And if the intensity is the same at all points and in all directions the radiation field is said to be homogeneous and isotropic. Moreover , when $\omega_o = 1$, I shall say that I have a conservative case of perfect scattering. when $\omega_o \ne 1$ I shall say that I have a non conservative case of scattering .

Next to the isotropic scattering greatest interest is attached to Rayleigh's scattering which is an example of conservative anisotropic scattering.

1.1 Introduction to scattering problems

1.11 Coherent and Non-Coherent Scattering.

When the radiation is emitted in the frequency in which it was absorbed the atom is said to scatter coherently. On the other hand, when frequency of the emitted radiation differs from that of the absorbed radiation I call it the case of non-coherent scattering. Non-coherent scattering is sometimes used to mean that the scattering involves not only a change in frequency but also a complete redistribution in frequency i.e. scattering in which the frequency of re-emission is in correlation with the frequency absorbed. From practical point of view, strictly coherent scattering does not exist in astrophysics (vide, Edmonds [1955]). I designate the scattering as Coherent and Non-coherent according to our theoretical consideration of the problem when an atom absorbs energy of certain frequency, ν, the probability that the energy will be re-emitted in the same frequency will be maximum if

(i) the atom is at rest .

(ii) the atom is in the lowest quantum state

(iii) in a weak radiation field.

Departure from any of the above three conditions will cause non-coherent scattering .

1.12 Coherent Scattering Problems.

Chandrasekhar [1960] applied the method of discretes ordinate to solve the transfer equation for coherent scattering in stellar atmosphere with Planck's function as a linear function of optical depth, viz.,

$$B_\nu(T) = b_o + b_1\tau. \tag{1.9}$$

The equation of transfer for coherent scattering has also been solved by Eddington's method (where η_ν, the ratio of line to the continuum absorption coefficient, is constant) and Stromgren method (when η_ν , has small but arbitrary variation with optical depth (vide, Woolley and Stibbs, 1953). Dasgupta [1977] applied the method of Laplace transform and Wiener-Hopf technique to find an exact solution of the transfer equation for coherent scattering in stellar atmosphere with Planck's function as a sum of elementary functions

$$B_\nu(T) = b_o + b_1\tau + \sum_{r=2}^{n} b_r E_r(\tau) \tag{1.10}$$

by use of a new representation of the H-function obtained by Dasgupta [1977]. Extensive study has been made on coherent scattering by various authors thereafter and before.

1.13 Noncoherent Scattering Problems.

In stars having high temperature and high energy density, the induced transition-probabilities at lower frequencies increase sufficiently. The ground state or the lower state in case of a sub ordinate line then possesses a finite width and the frequency of the absorbed and the emitted radiation differ from each other introducing a noncoherency in the formation of absorption lines. Though in a single scattering there is a change in frequency giving rise to either a loss or a gain in energy of the atom, in a number of scattering the total loss of energy balances with the total gain in energy. In the case of interlocking without redistribution, if radiation in one line flows from centre to the wings then then in another line it flows from wings back to the centre. The doppler broadening introduces, another important type of non-coherent scattering. If a moving atom absorbs radiation from one direction and emits it in another, the frequencies of the absorbed and emitted radiation will differ even if the process is coherent in the atom's rest frame. Another type of non-coherent scattering is that due to pressure broadening which is the simplest and at the same time most important case in stellar atmosphere. Let an electron, due to absorption of energy, jumps to a higher level where there is a perturbing atom or ion. Now if the perturbing atom goes away before the electron suffers downward transition, the atom may absorb some amount of energy from the electron and the electron will consequently emit radiation of frequency quite different from that of the absorption. This type of scattering gives rise to the process known as Stark Effect, e.g. Hydron lines in stellar atmosphere are broadened by Stark Effect. Impact broadening becomes important when the velocity of the perturbation is large. In case of lines widened by impact broadening the scattering is partly coherent and partly non-coherent. Domke and Staude (1973) considered the formation of a Zeeman-multiplets by noncoherent scattering and true absorption in a M-E atmosphere. The solution of the line formation problem is obtained (vide, Domke and Staude, 1973) for an exponential form of the Planckian source function.

1.14 Interlocking Problems.

Interlocking of multiplets is another type of non-coherent scattering. When the lower state possesses a common upper state by absorption from any of the lower sub-state, the re-emission will be controlled by the transition probability of the various lines regardless from a certain sub-state of the lower state in a certain frequency ν has a non-zero probability of returning to another lower sub-state emitting in a frequency different from ν giving rise to non-coherent scattering. Similar case will arise when the number of upper sub-states will possess a common lower state. This type of non-coherency has a special name interlocking of lines without redistribution . Woolley and Stibbs [1953] considered the problem of interlocking without redistribution in details and gave an appropriate solution applying Eddington's method. Busbridge and Stibbs [1953] applied the principle of invariance to solve the same problem and calculated three hypothetical line profiles for doublets. However Busbridge and Stibbs [1953] did not attempt calculation of the line profiles for triplets because they feared any such attempt would have involved considerable labour. Karanjai [1968a] profitably applied his approximate form for the H-function [1968b] to minimize to a great extent the labour of such computations. Dasgupta and Karanjai [1972] applied Sobolev's probabilistic method to solve the transfer equation for the case of interlocking without redistribution.

Another exact solution of the equation of transfer has been given by Dasgupta [1956] by his modified form of Wiener-Hopf technique. Karanjai and Barman [1981] applied the extension of the method of discrete ordinate to find an exact solution of the problem of line formation by interlocking in the M-E model. Karanjai and Karanjai [1985] used the method of Laplace transform and Wiener-Hopf technique to solve the equation of interlocked lines taking the Planck function as a nonlinear function of optical depth. Karanjai [1982] has calculated Mg b line contours with the help of the solution obtained by Dasgupta and

Karanjai [1972] and showed that his calculated lines have a good agreement with the observation. Dasgupta [1978] obtained an exact solution of the transfer equation for non-coherent scattering arising from interlocking of principal lines without redistribution of the H-function obtained by Dasgupta [1977]. While solving the transfer equation Dasgupta considered the Planck's function to be linear in τ (Optical depth) (equation (1.9)).

Karanjai and Karanjai [1985] considered two non-linear form of Planck function Viz;

$$\text{(a)} \quad B_\nu(T) = B(t) = b_o + b_1 e^{-\beta\tau} \tag{1.11}$$

in an exponential atmosphere (vide, Degl 'Innocenti, 1979) where β, b_o and b_1 are positive constants.

$$\text{(b)} \quad B_\nu(T) = B(t) = b_o + b_1\tau + E_2(\tau); \tag{1.12}$$

in an atmosphere considered by Busbridge [1955]. Roymondal, Biswas and Karanjai [1988] solved the equation of transfer for non-coherent scattering by F_n method. Recently, Basak and Karanjai [1995] solved the transfer equation for interlocked multiplets in anisotropically scattered atmosphere.

1.15. Anisotropic Scattering Problems.

The equation of transfer for plane parallel Rayleigh's scattering phase function can be put in the form

$$\mu\frac{dI(\tau,\mu)}{d\tau} = I(\tau,\mu) - \frac{3}{16}\left[(3-\mu^2)\int_{-1}^{+1} I(\tau,\mu')d\mu' + \right.$$

$$+ (3\mu'^2 - 1) \int_{-1}^{+1} I(\tau,\mu')\mu'^2 \, d\mu' \Big] \tag{1.13}$$

According to Chandrasekhar [1960] the solution of the equation of transfer (1.13) for Rayleigh scattering can be put in the form

$$J(\tau) = \frac{3}{16}\Big[\int_0^{\infty} (3E_1 - E_3)_{|t-\tau|} J(t)dt + \int_0^{\infty} (3E_3 - E_1)_{|t-\tau|} \times$$

$$\times k(t)\, dt \Big] \tag{1.14}$$

and $$k(\tau) = \frac{3}{16}\Big[\int_0^{\infty} (3E_3 - E_5)_{|t-\tau|} J(t)\, dt +$$

$$+ \int_0^{\infty} (3E_5 - E_3)_{|t-\tau|} \times k(t)\, dt \Big] \tag{1.15}$$

where $$J(t) = (1/2) \int_{-1}^{+1} I(\tau,\mu)d\mu \tag{1.16}$$

$$k(t) = (1/2) \int_{-1}^{+1} I(\tau,\mu)\mu^2 \, d\mu \tag{1.17}$$

$$E_n(y) = \int_1^{\infty} \frac{dx}{x^n} e^{-xy} \tag{1.18}$$

Equation (1.14) and (1.15) represents a pair of integral equations for J and K. The linear integral equation which replace the equation of transfer (1.13) become increasingly of higher order. According to Rayleigh phase function

$$p(\mu,\phi\ ;\mu',\phi') = (3/4)\Big[\ 1 + \mu^2\mu'^2 + (1-\mu^2)(1-\mu'^2)\cos^2 \times$$
$$\times\ (\phi-\phi') + 2\mu\mu'(1-\mu^2)^{1/2}(1-\mu'^2)^{1/2}\cos(\phi-\phi')\Big] \quad (1.19)$$

the scattering function can be expressed in the form

$$S(\mu,\phi;\mu_o,\phi_o) = \frac{3}{8}\Big[\ S^{(0)}(\mu,\mu_o) - 4\mu\mu_o(1-\mu^2)^{1/2}(1-\mu_o^2)^{1/2} \times$$
$$\times\ S^{(1)}(\mu,\mu_o)\cos(\phi_o-\phi) + (1-\mu^2)(1-\mu_o^2)\times$$
$$\times\ S^{(2)}(\mu,\mu_o)\cos2(\phi_o-\phi)\Big] \quad (1.20)$$

(vide, Chandrasekhar, 1960). The law of darkening for the problem with a constant net flux and for Rayleigh phase function has been expressed in the form (vide, Chandrasekhar, 1960)

$$I(0,\mu) = \frac{3}{4}F\left\{\mu + \frac{3}{16}H(\mu)\int_0^1 \mu'^2 H(\mu')\left[\frac{3-\mu'^2}{\mu+\mu'} + \mu' - c\right]d\mu'\right\} \quad (1.21)$$

Consequently the axially symmetric problem in semi-infinite plane parallel atmosphere with a constant net flux in the total intensity ($I_l + I_r$) is one which is physically significant. The transfer of radiation in the atmosphere of early type stars with surface temperature exceeding 15,000 ^oK is predominantly controlled by the scattering by free electrons.

Chandrasekhar [1960] discussed the equations of Radiative transfer for an electron scattering atmosphere and gave the solution of the equation by discrete ordinate method (Chandrasekhar, 1960). Sweigert [1970] solved the integral equation of Radiative transfer numerically for both conservative and non- conservative cases in which scattering is governed by the Rayleigh phase function. The polarisation produced by Rayleigh scattering was neglected. Solution were tabulated over a wide range of optical depths and for varying amounts of absorption measured by the albedo for single scattering. These numerical results may prove useful in the interpretation of planetary reflectiveness, particularly in the ultraviolet where the importance of Rayleigh scattering increases appreciably due to the λ^{-4} dependence of the scattering cross-sections. Sweigert [1970] presented numerical solution to the integral equation for both finite and infinite atmosphere according to the Rayleigh phase function with absorption. Abhyankar and Fymat [1970a] discussed the imperfect Rayleigh scattering in a semi-infinite atmosphere. The extinction of radiation in a coherent scattering gaseous medium is caused partly by true absorption, which result in a loss of incident photons from the radiation field and partly by scattering , which simply modifies the paths of the photons without actually removing them from the field . In other words, the medium exhibits imperfect scattering. The reflection matrix $\phi(\mu,\phi,;\mu_0,\phi_0)$ for a semi-infinite plane parallel stratified homogeneous atmosphere, scattering in accordance with the conservative Rayleigh phase matrix was obtained by Chandrasekhar [1960]. The corresponding solution for a non-conservative Rayleigh atmosphere in which the albedo for single scattering Ω is constant, but different from unity, are presented for some representative values of Ω. They showed that the reduction in value of the albedo increases the absolute degree of polarization and brings the Babinet and Brewster neutral points closer to the Sun; the points even coalesce with the Sun for vary small albedo values. Abhyankar and Fymat [1970b] discussed the theory of radiative transfer in inhomogeneous atmospheres. Here in the case where the phase matrix corresponding to azimuth independent term of the radiation field scattered by an inhomogeneous plane-parallel atmosphere, is separable in the form

$$p^{(o)}(\mu,\mu') = M(\mu).\ M^{+}(M) \qquad (1.22)$$

(Where the sign + stands for simple transaction) is simplified matrix equation of the problem are treated by the perturbation method of Fymat and Abhyankar. In this connection they have studied the regions of convergence in the case of a Rayleigh scattering. The regions of convergence in the case of Rayleigh scattering law are delimited when the solution for conservative Rayleigh scattering is taken as the reference. It has been shown that the region of convergence for Rayleigh scattering is slightly smaller than that of convergent for all optical depth when the maximum value of Ω is less than about 0.945 ; for higher values of Ω there is apparently no convergence for large optical depths.

Fuzhong Weng [1992] applied a multi-layer discrete ordinate method for vector Radiative transfer in a vertically inhomogeneous, emitted and scattering atmosphere. In that work , the up welling radiance from the vector radiance transfer model, established is compared with Chandrasekhar's analytical solutions for a conservative Rayleigh Scattering atmosphere.

While the solution for conservative Rayleigh scattering is known in all details of intensity and state of polarization for a wide range of optical thickness, the corresponding solution for non-conservative Rayleigh scattering , often dealt in planetary atmosphere , are not available. The perturbation method, developed by Fymat and Abhyankar [1970a

,1970b] and its present extension enable to derive such solution for homogeneous atmosphere with albedo for single scattering different from unity. Fymat and Abhyankar [1970c] also discussed the theory of radiative transfer of partially polarised radiation through an inhomogeneous semi-infinite atmosphere. They solved it by the application of matrix perturbation method by introducing a matrix N-function to a semi-infinite atmospheres in the form of a Newmann series. The region of convergence of this series solution is delimited for Rayleigh law of scattering. An iteration scheme for computing the solution was discussed and as an illustration, sample computations were presented in which the N-functions for homogeneous Rayleigh non-conservative atmosphere with albedo for single scattering Ω = 0.25 and 0.75 were derived for the N_o-function for a reference homogeneous atmosphere with Ω_o=0.5.

Fymat and Abhyankar [1970a,1970b] linearized the nonlinear singular integral equations for the radiative transfer in inhomogeneous plane-parallel atmosphere of arbitrary stratification by using a perturbation technique (vide , Fymat and Abhyankar,1970a) which has also been applied (vide , Fymat and Abhyankar,1970b) successfully to a semi-infinite plane parallel atmosphere. Fymat and Abhyankar (1970b) also dealt with diffuse reflection by a semi-infinite non-conservative Raleigh atmosphere. Pomraning [1970] consi-

-dered the classical problem of computing the albedo from a half-space and showed that one can derive an appropriate variational principle for this problem and that the variational estimates of the albedo based upon asymptotic trial functions are remarkably accurate . Further , it is shown that the albedo is insensitive for the descriptions namely

(1) An isotropic phase function averaged over polarization
(2) The Rayleigh phase function averaged over polarization.
(3) Rayleigh scattering properly accounting by the Rayleigh scattering law averaged over polarization, the equation of transfer is (Chandrasekhar, 1960)

$$\mu \frac{\partial I(z,\mu)}{\partial z} + I(z,\mu) = (c/2)\left[\int_{-1}^{+1} I(z,\mu')d\mu' + \right.$$

$$\left. + (1/2)\, P_2(\mu)\int_{-1}^{+1} P_2(\mu')\, I(z,\mu')\, d\mu' \right] \quad (1.23)$$

where $$P_2(\mu) = (3\mu^2 - 1)/2 \quad (1.24)$$

z being the spatial co-ordinate measured in optical distance , μ, the cosine of the angle between the photon flight direction and an inward normal intensity and c the ratio of the scattering coefficient to the collision coefficient. Pomraning suggested that in a certain work on radiative transfer the complexities introducing by accounting for polarization effects and the anisotropy of the Rayleigh phase function can be avoided. It may be sufficient depending upon the accuracy required to assume an isotropic phase function averaged over polarization.

Casti, Kagiwada and Kalaba [1970] discussed about external radiation fields for isotropically scattering finite atmospheres bounded by a Lambert law Reflection. Casti, Kagiwada and Kalaba [1070] provided formulae for obtaining the diffusely transmitted and reflected radiation fields for a planetary isotropically scattering atmosphere of finite thickness in terms of the solution to the problem with no planetary surface .

From numerical result they showed that these reflected and transmitted fluxes are essentially the same whether isotropic or Rayleigh scattering laws are assumed.

Kagiwada and Kalaba [1971] derived all the basic equations of the Cauchy system mathematically from the basic integral equation for the source function $\mathcal{J}$ for the atmospheres bounded by Lambert's law Reflector.

The problem of the determination of radiation fields in finite ,conservative, isotropically scattering media bounded by a Lambert's law Reflector, has been reduced by Kalaba (1970) to a Cauchy system involving auxiliary functions of merely one angular argument.

Buell , Casti , Kalaba, and Ueno [1970] discussed exact solution of a family of matrix integral equations for multiple scattered, partially polarised radiation. In the theory of multiple scattering of partially polarized radiation, a key role is played by the integral equation ,

$$J(t,x,z) = I\, e^{-(x-t)/z} + \int_0^x K(|\, t-y\,|)J(\, y,x,z\,)dy \quad (1.25)$$

$$0 \leq t \leq x \leq x_1 \;, \quad 0 \leq z \leq 1 \quad (1.26)$$

where J and K stand for n $\times$ n square matrices; I is the unit n $\times$ n matrix, and the matrix kernel k can be represented in the form

$$K(r) = \int_0^1 e^{-r/z'} W(z')dz' \;, \quad r>0 \quad (1.27)$$

where W is a square n $\times$ n matrix. It is shown that this family of matrix integrals can be transformed into a Cauchy problem. The Cauchy system solves the integral equation for the matrix J. The theory is for general phase function.

Hulst and Grossman [1968] discussed multiple light scattering in planetary atmosphere. The diffuse reflection and transmission by plane , homogeneous atmospheres consisting of particles with an anisotropic scattering was discussed for various phase functions.

It has been shown that "the doubling method" can be purformed most conveniently with great accuracy from very thin to very thick layers. The accuracy obtained with various integration schemes in depth and in angle was discussed in some detail .

Kagiwada and Kalaba [1967] estimated the local anisotropic scattering function on the basis of multiple scattering properties for the general phase function . The phase function is expanded in a series of Legendre polynomials i. e.,

$$p(\cos\alpha) = \sum_{m=0}^{m} C_m P_m (\cos\alpha) \tag{1.28}$$

and the coefficients are determined so as to best explain diffuse reflection measurements .

Busbridge [1960] discussed the anisotropic scattering with general phase function :

$$p(\mu,\mu') = \sum_{\nu=0}^{N} \omega_\nu P_\nu(\mu) P_\nu(\mu') \tag{1.29}$$

$$\text{where} \quad -1 \le \mu \le 1 \ , \quad -1 \le \mu' \le 1 \tag{1.30}$$

Busbridge [1960] discussed the solution of the homogeneous equation given by

$$J(\tau,\mu) = \Lambda_{\tau,\mu} \left\{ J(t,\mu') \right\} \qquad (1.31)$$

which are atmost $O(\tau)$ as $\tau \longrightarrow \infty$, for conservative and non-conservative cases.

The solution for 'The auxiliary equation ' given by

$$(1-\Lambda)_{\tau,\mu} \left\{J(t,\mu',\mu_o)\right\} = p(\mu, -\mu_o) \qquad (1.32)$$

$$\exp(-\tau/\mu_o) \quad \text{where} \quad 0<\mu_o<1 , \quad -1\leq \mu \leq 1 \qquad (1.33)$$

has also been discussed (vide, Busbridge, 1960) in terms of H-function. Finally,the law of diffuse reflection has been worked out .

Horak and Chandrasekhar [1970] considered the the problem in radiative transfer, parallel light of flux density πF_o is incident on a plane-parallel, semi-infinite atmosphere which scatters light in accordance with the phase function

$$p(\cos\vartheta) = \omega_o + \omega_1 P_1(\cos\vartheta) + \omega_2 P_2(\cos\vartheta) \qquad (1.34)$$

where $\omega_o \leq 1$ and ω_o (the albedo), ω_1 , ω_2 are constants and P_1 and P_2 are Legendre polynomials. They have found out the exact and the details of the solution for the emergent radiation field by using the invariance principle method.

The diffuse reflection of light by a semi-infinite atmosphere scattering with phase function

$$1 + \omega_1 P_1(\cos\vartheta) + \omega_2 P_2(\cos\vartheta) \qquad (1.35)$$

has been delt by Horak and Janowsek [1965].

Orchard [1967] obtained the reflection and transmission of light by thick atmosphere of pure scattering with the same phase function. To obtain these Orchard (1967) applied exact radiative transfer theory to the case of a parallel light incident from an arbitrary direction on the non-absorbing plane parallel atmosphere of large optical thickness.

Busbridge and Orchard [1968] applied the same theory to find reflection and transmission of light by thick atmospheres of pure scattering with a phase function

$$1 + \sum_{n=1}^{N} \omega_n P_n(\cos\vartheta) \qquad (1.36)$$

Kolesov and Sobolev [1969] and Kolesov and Smoktii [1972] applied the general theory of anisotropic scattering developed by Sobolev to solve the problem of diffuse reflection and transmission of light by a semi-infinite atmosphere with a three and four term scattering indicatrix.

Kolesov [1971] discussed about H-function for some scattering indicatrices with different values of the asymmetry factor.

The asymptotic solution for the phase function $(1 + \omega \cos\vartheta)$ has been found out by Piotrowski [1955, 1956] using the method of discrete ordinates as developed by Chandrasekhar [1960]. At the same time, Piotrowski has found out the asymptotic value of the transmittance in the case of the phase function

$$\sum_{n=0}^{N} \omega_n P_n (\cos \vartheta) \tag{1.37}$$

but he was unable to obtain the limit of this, as the norm of the partition used for the Gauss quadrature tended to zero. Usugi and Irvine [1970a] computed reflection function for conservative isotropic scattering by the method of successive scattering . By the same method, Usugi and Irvine [1970b] derived basic formulae for the computation of line profiles and equivalent width of an absorption line. Usugi and Irvine [1968] showed that the absorption spectra can be computed in a model planetary atmosphere using the Newmann series solutions.

Uesegi, Irvine and Kawata [1971] showed that the diffuse reflection may be computed for arbitrary single scattering albedo if the reflection functions in the conservative case are known.

Mullikin [1964a] studied the transfer of radiation in homogeneous plane parallel atmosphere of finite and semi-infinite thickness for three different types of phase functions and computed the X– , Y– equations by additional linear constraints so that a unique pair of functions is specified by the requirement of analyticity in a half plane and transforemed the linear singular equations and linear constraints into suitable form for numerical computations.

For the semi-infinite atmosphere, Fredholm equations are solved exactly (vide, Mullikin, 1964a) to give a determination of the H-function in terms of simple quadratures.

Burniston and Siewert [1970] discussed a matrix version of the classical Riemann-Hilbert problem defined on an open contour. Finally as an illustration linear integral equation for Chandrasekhar's function $H_l(\mu)$ and $H_r(\mu)$ are established in a form enumarable to solution by numerical iteration. Bond and Siewert [1970] have computed the first twenty two moments of Chandrasekhar's function $H_l(\mu)$ and $H_r(\mu)$ related to the scattering of polarized light.

Carlstedt and Mullikin [1966] obtained equations needed to determine the X– and Y– functions firstly studied by Busbridge. Carlstedt and Mullikin [1966] also obtained asymptotic formulae for thick atmospheres uniformly valid for various Characteristic functions. All these equations are also applicable to Rayleigh phase functions. Domke [1971, 1972] solved radiative transfer equation with conservative Rayleigh scattering for both finite and semi-infinite atmosphere, based on Sobolev's method for arbitrary distribution of primary sourses.

Mullikin [1966a] has studied extensively and analytically and numerically the complete Rayleigh scattered field within a homogeneous plane-parallel atmosphere. The solution to this problem at any optical depth has been expressed in terms of scalar function for which there already exists an efficient and accurate computer programme. Various asymptotic formulae of a relatively simple form have been obtained from this solution.

Steady state multiple scattering problems for homogeneous plane parallel atmospheres have been extensively studied [Mullikin, 1966b] by means of the principle of invariance of Ambertsumian and Chandrasekhar. The purpose of that was to report on the results obtained from a fruitful combination of the linear and nonlinear theories. This analysis is applied to Rayleigh polarization scattering .

A study is made of the existence and uniqueness problems

[Mullikin, 1964b] for Chandrasekhar ψ_l and ϕ_l equations for radiative transfer in homogeneous atmosphere's with anisotropic scattering .

Mullikin [1963] reported on some recent mathematical studies concerning the uniqueness of solutions to Chandrasekhar's mathematical formulation of principle of invariance in the theory of Radiative Transfer . The uniqueness question for his ψ_l^m and ϕ_l^m equations has been studied.

Siewert and Burniston [1972] showed that a solution to the system of singular integral equations and the linear constraint which define mathematically the H-matrix relevant to the scattering of polarizated light can exist and are unique. Finally, Siewert and Burniston (1972) gave an explicit analytical result for the appropriate canonical matrix for conservative Rayleigh scattering. Hulst (1970) reduced the problems of radiative transfer with a general anisotropic phase functions completely to H-functions and two sets of polynomials known as the Kuščer polynomials and the Busbridge polynomials.

Hulst [1969] discussed some problems of anisotropic scattering in planetary atmospheres. Here the similarity rules to compare atmospheres with anisotropic and isotropic scattering were reviewed .

With the aid of the invariant imbedding technique, Bellman, Kagiwada, Kalaba and Ueno [1967] derived a complete set of integro-differential equations for the dissipation functions of an inhomogeneous finite slab with anisotropic scattering.

Siewert [1968] presented a new approach to develope Chandrasekhar's scattering matrix for a semi-infinite Rayleigh scattering atmosphere which can be used to determine the emergent angular distribution for any of the standard half space problems.

Siewert and Fralay [1967] solved the conservative Rayleigh scattering problem in a semi-infinite atmosphere by the application of the singular eigen function expansion technique. Bond and Siewert [1971] have studied the non-conservative equation of transfer for a combination of Rayleigh and isotropic scatter scattering.

Wallance [1972] presented a discussion on Rayleigh and Raman scattering by pure H_2 in a planetary atmosphere. Kuzmina [1970a, 1970b] discussed Milne's problem for polarized radiation scattered according to conservative and non-conservative Rayleigh's law.

Sobolev [1969a] investigated on diffuse reflection and transmission of light by an atmosphere with anisotropic scattering. Sobolev [1969b, 1970] also discussed on anisotropic light scattering in an atmosphere of finite optical thickness.

Kolesov and Sobolev [1969] discussed on some asymptotic formulae in the theory of anisotropic light scattering. Grinin [1971] discussed on the theory of non-stationary radiation transfer for anisotropic scattering by the application of the modified Sobolev's probability method. Pomraning (1969) formulated the modified Eddington's approximation proposed earlier for isotropic scattering for a general scattering law.

Stokes and De Marcus [1971] used variational principle for calculating line profiles of inhomogeneous planetary atmosphere.

Sekera and Ashburn [1953] , and Sekera and Blauch [1954] gave tables relating to Rayleigh scattering of light in the atmosphere. The extensive numerical results based on Chandrasekhar's analysis have been obtained for Rayleigh atmospheres with optical thickness ranging up to 1 (vide, Sekera, 1956, 1967 and vide, Sekera and Viezee, 1961).

Case and Zweifel [1967] treated isotropic scattering and some simple example of anisotropic transfer , based on the work of Mika and others . Formulations for general anisotropic scattering were presented by McCormik and Kuscer [1966] and in practical form by Shultis and Kaper [1969] and in full detail by Kaper, Shultis and Veninga [1970].

Chandrasekhar [1960] has considered the problem of radiative transfer with general anisotropic scattering in the Milne-Eddington model to obtain the exact form of emergent intensity from the bounding face and nth approximate intensity at any optical depth by discrete ordinates procedure assuming Planck's function to be linear in the optical depth. Das [1973] obtained an exact solution of this problem using the Laplace transform and Wiener-Hopf technique.

Das [1978,1980] has solved various problems of radiative transfer in finite and semi-infinite atmosphere using a method involving Laplace transform and linear singular operators.

Sobolev [1956] dealt with the one dimensional problem of time-dependent diffuse reflection and transmission by a probabilistic method.

Diffuse reflection of time-dependent parallel rays by a semi-infinite atmosphere was treated by Ueno [1962] on the basis of the principle of invariance. Bellman et al [1962] obtained an integral equation governing diffuse reflection of time dependent parallel rays from the lower boundary of a finite inhomogeneous atmosphere .

In recent years Karanjai and Talukdar (1991, 1992), Karanjai and Biswas (1992, 1993) and Roy Choudhury and Karanjai (1995a, 1995b) solved radiative transfer problems in anisotropically scattering media by spherical harmonic method using different approximate forms for the intensity. Ueno [1965] also obtained this equation by probabilistic method. Matsumoto [1967a] derived functional equations in the internal radiation field due to time-dependent incident radiation allowing for the time dependence given by Dirac's δ-function and Heaviside unit step function Matsumoto [1967b] also derived a complete set of functional equations for the scattering (S) and transmission (T) functions which govern the laws of diffuse reflection and transmission of time-dependent parallel rays by a finite , inhomogeneous,plane parallel, non-emitting and isotropically scattering atmosphere with incident radiation governed by Dirac's δ-function and Heaviside's unit step-function. A formulation of time-dependent H-function was accomplished by means of the Laplace transform in the time-domain. Numerical evaluation of the H-function based on numerical inversion of the Laplace transform presented by Bellman et.al [1966] was made.

Recently Karanjai and Biswas [1988] derived the time-dependent X- and Y-functions for homogeneous, plane parallel non-emitting and isotropic atmosphere of finite optical thickness using the integral equation method developed by Rybicki [1971]. Biswas and Karanjai [1990a] have derived the time-dependent H-, X- and Y- functions in a homogeneous atmosphere scattering anisotropically with Dirac's δ-function and heaviside unit step-function type time-dependent incidence. Biswas and Karanjai [1990b] have also derived the solution of diffuse reflection and transmission problem for homogeneous isotropic atmosphere of finite optical depth. The problem of the time-independent scattering and transmission of radiation in plane parallel atmosphere of two layers was treated first by Van de Hulst [1963], (vide, Tsujita, 1968). Hawking [1961] dealt with the problem analytically starting with Milne's integral equation. Gutshabad [1957] formulated the problem as solutions of simultaneous integral equations. So far as his equations are solvable , the scattering and transmission functions required are given exactly for two layers of different albedo and different large optical thickness.

In the theory of radiative transfer for homogeneous plane parallel stratified finite atmosphere the X- and Y- functions of Chandrasekhar [1960], play a central role. These equations satisfy a system of coupled non-linear integral equations. Busbridge [1960] has demonstrated the existence of the solutions of these coupled nonlinear integral equations in terms of a particular solution of an auxiliary equation. Busbridge [1960] has obtained two coupled linear integral equations for X(z) and Y(z) which defined the meromorphic extensions to the complex domain |Z| of the real valued solution of the coupled non-liner integral equations for X- and Y- functions are the solutions of the coupled linear integral equations. Mullikin [1964c] has proved that all solutions of coupled nonlinear integral equations are solutions of the coupled linear integral equation but there exists a unique solution of the coupled linear integral equations with some linear constraints. Finally Mullikin (1964c) has obtained the Fredholm equations of X- and Y- functions which are easy for iterative computations. Das [1979] has obtained a pair of the Fredholm equations with Wiener-Hopf technique from the coupled linear integral equations with coupled linear constraints. The transport equation for the intensity of radiation in a semi-infinite atmospheres with no incident radiation and scattering according to the planetary phase function $\omega(1 + x \cos \vartheta)$ has been considered. This equation has been solved by Chandrasekhar [1960] using his principle of invariance to get the emergent radiation.

The singular eigen function approach of Case [1960] is also applied to get the intensity of radiation at any optical depth. Boffi [1970] has also applied the two sided Laplace transform to get the emergent intensity and the intensity at any optical depth. Das [1979] solved exactly the equation of transfer for scattering albedo $\omega < 1$ using Laplace transform and the Wiener-Hopf technique and also deduced the intensity at any optical depth by inversion.
In the study of the time-dependent radiative transfer problem in finite homogeneous plane-parallel atmosphere, it is convenient to introduce X- and Y- functions [1960]. These functions satisfy non-linear coupled integral equations . Due to their important role in solving transport problems , it is advantageous to simplify the equations satisfied by them. Lahoze [1989] did this andobtained exact linear and decoupled integral equations satisfies by the time-independent X- and Y- functions .

1.2 SUMMARY OF WORK DONE.

The present thesis is concerned with the solution of some scattering problems of Radiative Transfer. The work presented in chapter 2 is concerned mainly with the solution of scattering problems by the method based on " Laplace Transform and Wiener-Hopf technique " and " Principle of Invariance " .

The transport equation for the intensity of radiation in a semi-infinite atmosphere with no incident radiation and scattering according to the planetary phase function $\omega(1 + x \cos\theta)$ has been solved exactly by a method based on the use of laplace transform and Wiener-Hopf technique. in section 2.2. The exact solution of the transfer equation with three-term scattering indicatrix in an exponential atmosphere is obtained by the same method in section 2.3. The matrix transform equation for a scattering which scatters radiation in accordance with the phase matrix obtained from a combination of Rayleigh and isotropic scattering in a semi-infinite atmosphere has been solved in section 2.5 by the same method . The basic matrix equation is subject to the Laplace transform to obtain an integral equation for the emergent intensity matrix. On application of the Wiener-Hopf technique this matrix integral equation gives the emergent intensity matrix in terms of a singular H-matrix and an unknown matrix. The unknown matrix has been obtained by equating the asymptotic solution of the boundary condition at infinity.

The equation of transfer for a semi-infinite plane parallel atmosphere with no incident radiation and for the scattering according to the conservative anisotropic phase function has been solved by the method of " Principle of Invariance " and using the law of diffuse reflection in **section 2.4.** In **section 2.6** the nonlinear integral equations for X- and Y-functions (vide , Chandrasekhar, 1960) for anisotropically scattering atmosphere has been derived. The anisotropy is represented by means of a phase function which can be expressed in terms of finite-order Legendre Polynomials.

The principle of invariance is applied to derive the functional equations for time-dependent diffuse reflection and transmission function. Next I consider the time dependent diffuse reflection and transmission of plane parallel rays by a slab consisting of two homogeneous anisotropically scattering layers, whose scattering and transmission functions are known

In chapter 3 the equation of transfer has been solved by different methods Viz.,

(i) Eddington's Method (Sec-3.2).

(ii) Laplace transform and Wiener-Hopf technique (Sec-3.3).

(iii) Busbridge's Method (Sec-3.4).

(iv) Discrete Ordinates (Sec-3.5).

in an isotropic coherently scattering atmosphere with exponential Planck function (equation (1.11)).

In chapter 4 the equation of transfer for interlocked multiplets, has been solved by the discrete ordinate method and by the method used by Busbridge and Stibbs [1954] using Planck function as an exponential function of optical depth in sections 4.2 and 4.3 respectively. Four approximate forms of H-function (vide, Karanjai and Sen, 1970, 1971) has been used to calculate the residual intensities for doublets and triplets in section 4.4. and the concerned results has been shown in both tables and figures.

In chapter 5 the one sided Laplace transform together with the theory of linear singular operators has been applied to solve the transport equation which arises in the problem of a finite atmosphere having ground reflection according to Lambert's Law taking the Planck's function as an exponential function of optical depth (Sec-5.2).

The time-dependent X- and Y- functions (Biswas and Karanjai, 1990) which gives rise to a pair of the Fredholm equations with the application of the Wiener-Hopf technique has been obtained in section 5.3. These Fredholm equations define time-dependent X-functions in terms of time-dependent Y-functions and vice-versa. These representations are unique with respect to the coupled linear constraints defined by Mullikin (1964a). An exact linearized and decoupled integral equation satisfied by Time-Dependent X- and Y- function has been obtained using the method used by Lahoz (1989) in section 5.4.

REFERENCES

Abhyankar, K. D. and Fymat, A. L .: 1970a, Astron. and Astrophys. Vol.4, pp.101

Abhyankar, K. D. and Fymat, A. L .: 1970b, Ap.J.Vol. 159, pp.1009.

Abhyankar, K. D. and Fymat, A. L .: 1970c, Ap.J. Vol.159, pp.1019.

Basak, S. and Karanjai, S .: 1995, Submitted to Vidya Sagar University , Jr. of Physical Sciences.

Bellman , R., Kagiwada, H., Kalaba, R., and Ueno, S.: 1967 , J.Mathe.Phys. Vol.8(10), pp.2137.

Bellman, R., Kalaba, R. and Ueno, S.: 1962, Icras, Vol.1, pp.191.

Bellman, R., Kalaba, R. and Lockett, A.: 1966, Numerical Inversion of The Laplace Transform (American Elsevier, New York) Chap.2.

Biswas, G. and Karanjai, S.: 1990a, Astrophys. Space Sci. Vol. 165 , pp. 119.

Biswas, G. and Karanjai, S.: 1990b, Astrophys. Space Sci. Vol.164, pp.15

Boffi, V.C.: 1970, J.Math.Phys. 11,267.

Bond, G. R. and Siewert, C. E .: 1970, Ap. J. Vol. 159, pp.731

Bond G. R. and Siewert, C. E .: 1971, Ap.J. Vol.164, No.97

Buell, J.,Casti, J., Kalaba, R. and Ueno, S .: 1970, J.

Mathe. Phys. Vol. 11(5), pp.1678.

Burniston, B. E. and Siewert, C. E .: 1970, J. Math.Phys. Vol.11 (No.10), pp.3091.

Busbridge, I. W. and Stibbs, D. W. N.: 1953, Monthly Notices Roy.Astron.Soc. Vol.113, pp.52

Busbridge, I. W. and Stibbs, D. W. N.: 1954, Monthly Notices Roy.Astron.Soc. Vol.114, pp.2

Busbridge, I.W.: 1960, The Mathematics of Radiative Trans. Cambridge, Cambridge University Press.

Busbridge I. W. and Orchard. S. E .: 1968, Ap.J. Vol.154, pp.729

Carlstedt,J.L. and Mullikin, T.W.: 1966, Ap. J. Vol. 143 (No.113), pp.618

Case, K.M.: 1960, Ann. Phys. Vol.9 , pp.1

Case K. M. and Zweifel, P. F .: 1967, Linear Transport Theory. Addison Wesley Publ. Co. Reading .

Casti, J., Kagiwada, H.and Kalaba, R.: 1970, J. Q. R. S. T. Vol.10 pp.637

Chandrasekhar, S .: 1960, Radiative Transfer, Dover, New York.

Das, R.N.: 1978, Astrophys. Space Sci. Vol. 57, pp. 419.

Das, R.N.: 1979, Astrophys. Space Sci. Vol. 60, pp. 49.

Das, R.N.: 1980, Astrophys. Space Sci. Vol. 67, pp. 335.

Dasgupta, S. R.: 1956, Science Cult. Vol. 22, pp. 177

Dasgupta, S. R. and Karanjai, S .: 1972, Astrophys. Space

Sci. Vol.18,pp.246

Dasgupta, S.R.: 1977a, Astrophys. Space Sci. Vol. 50 pp. 187

Dasgupta, S.R.: 1977b, Phys. Letters 64A, 342

Dasgupta, S. R .: 1978, Astrophys. Space Sci. Vol. 56, PP.13

Domke, H and Staude, J.: 1973, Solar Physics Vol 31, pp. 279

Domke, H.: 1971, Soviet Astron. — AJ. Vol.15(2), pp. 266 Sept.-Oct. [Actpoh, *,48(2), 341,1971]

Domke, H.: 1972, Soviet Astron. — AJ. Vol.15(4), pp. 616. Jan-Feb. [Actpoh *, 48(4). 777, 1971]

Edmonds, F.N.: 1955 , Ap.J. vol.121, pp. 418

Fuzhong Weng.: 1992, J.Q.S.R.T. Vol. 47(1), pp. 19-33

Fymat, A. L. and Abhyankar, K. D.: 1970a, Ap.J. Vol. 158, pp.315

Fymat, A.L. and Abhyankar, K.D.: 1970b, Ap.J. Vol.158,pp.325

Grinin, V.P.: 1971, Astrofizika, Vol.7 pp.203

Gutshabad , S.D.: 1957, Vestnik Lenin. University. Vol.12, pp.158.

Hawking, F.M.: 1961, Ap. J. Vol.134, pp.28

Horak, H.G. and Janowsek, A.L.: 1965, Ap. J. suppl. (No-101) Vol.11, pp.277.

Hulst, H.C.Van de.: 1970, Astron.and Astrophys. Vol.9,pp.359

Kalaba, R .: 1970, Supported by the National Inst. of Health under Grand No. GM-16437-02.

Karanjai, S.: 1968a, Publ. Astron. Soc. of Japan, Vol. 20, pp.173

Karanjai, S .: 1968b, Indian Jr. of Th. Physics. Vol. 16, pp.85

Karanjai, S and Barman, S. K .: 1981, Astrophys. Space Sci. Vol.77, pp.271.

Karanjai, S.: 1982, North Bengal University. Review, Vol. 77 pp.57.

Karanjai, S. and Karanjai, M.: 1985, Astrophys. Space Sci, Vol.115, pp. 295

Karanjai, S. and Biswas, G .: 1988, Astrophys. Space Sci, Vol.149, pp.29.

Karanjai, S. and Biswas, L .: 1992, Astrophys. Space Sci, Vol. 192, pp. 151.

Karanjai, S. and Biswas, L .: 1993, Astrophys. Space Sci, Vol. 207 , pp. 221.

Karanjai, S. and Sen M.: 1970, Publ. Astron. Soc. Japan, Vol. 22, pp.235

Karanjai, S. and Sen M.: 1971, Astrophys. Space Sci, Vol. 13, pp.267

Karanjai, S. and Talukdar, M .: 1991, Proceedings of National Seminar on Mathematics, Calcutta. pp. 103.

Karanjai, S. and Talukdar, M .: 1992, Astrophys. Space Sci, Vol. 197, pp. 309.

Kagiwada, H. G. and Kalaba, R. E .: 1967, J.Q.S.R.T. Vol. 7, pp.295

Kagiwada, H.G. and Kalaba, R.E.: 1971 , J.Q.R.S.T. Vol. 11, pp.1101.

Kalaba, R .: 1970, A new method for determining radiation fields infinite, conservative, isotropically scattering media bounded by Lambert's law Reflector (Supported by the National Inst. of Health under Grand No.GM-16437-02).

Kapar, H. G. and Shultis, J. K. and Vaninga J. G .: 1970, J.Comp.Phys. Vol.6(2), pp.288.

Kolesov, A. K.: 1971, Trudy Astron. Obs. Leningrad, Vol.28,pp.3

Kolesov, A. K. and Sobolev, V. V .: 1969a, Trudly Astron. Obs. Leningrad. Vol.26, pp.3

Kolesov, A.K. and Sovolev , V. V .: 1969b, Astrofizika Vol.5, pp.175.

Kolesov, A. K. and Smoktii, O. I .: 1972, Sov. Astron. A. J, Vol.15(5),802

Kuzmina, M.G.: 1970, Dokl. Akad. : Nauk SSSR Ser.Mat. Fiz: Vol.193, pp.30.

Kuzmina, M.G.: 1970, Dokl. Akad. : Nauk SSSR Ser.Mat. Fiz: Vol.201, pp.809

Lahoz L.W.: 1989, J.Q.S.R.T. Vol.42, pp.563.

McCormik, N. J. and Kuscar I. J .: 1966, Math. Phys. Vol. 7, pp.2036.

Matsumoto,M:1967a, Publ. Astr. Japan., Vol.19, pp. 163.

Matsumoto,M:1967b, Publ. Astr. Japan., Vol.19, pp. 434.

Mullikin, T. W .: 1963, Uniqueness problems in the mathematics of Multiple scattering (Re-printed from I.C.E.S. Pergamon Press pp.559)

Mullikin, T. W .: 1964a , Trans. American. Math. Soc. Vol.113 (No.2) pp.316

Mullikin, T.W.: 1964b, Ap.J. Vol.139(4), pp.1267.

Mullikin, T.W.: 1964c, Ap.J. Vol.139, pp.379.

Mullikin, T.W.: 1966a, Ap.J. Vol.145(3), pp.886

Mullikin, T.W.: 1966b, Multiple scattering in homogeneous plane parallel atmosphere (Project Rand - Contract No.65 - 362 , pp.697)

Orchard, S.E.: 1967, Ap.J. Vol.149,pp.665.

Piotrowski, S.L.:1956, Acta.Astron. C,6,61

Piotrowski, S.L.: 1955, Bull, Acad. Pol.Sci.Vol. 111, pp.303

Pomraning, G.C.: 1969, J.Q.S.R.T. Vol. 9, pp.1011.

Pomraning, G.C.: 1970, Ap. J. Vol.159, pp.119.

Roy Chowdhury A. and Karanjai, S.: 1995a, Submitted to Vidya Sagar University Jr. of Physical Sciences.

Roy Chowdhury A. and Karanjai,S.: 1995b, Submitted to Bulletin of the Gauhati University Mathematical Association.

Roymondal R. C. , Biswas, G. and Karanjai, S .: 1988 , Astrophys. Space Sci. Vol.146 PP. 311.

Rybicki, G.B.: 1971, J.Q.S.R.T. Vol.11, pp.827.

Sekera, Z and Ashburn, E.V.: 1953, NAVORD, Rept. 2061, U. S. Naval Ord.Test. Station. Inyokern, Calif.

Sekera, Z and Blauch, G.: 1954, Sci.Rept. No.3. Contract AF.

19(122)-239. Air Force. Cambridge Research Centre.

Sekera, Z .: 1956, Advances in Geophysics Vol.3 ed. H. E. Landberg.N.Y. Acad. Press. pp.43-104.

Sekera, Z and Viesu, W.: 1961, RAND Corporation, R-389-PR.

Shultis, J.K. and Kapar, H.G.: 1969, Astron. Astrophy. Vol.3 pp.110

Siewert, C.E. and Burniston, E . E .: 1972, Ap.J. Vol.174 pp.629

Siewert, C.E. and Fralay, S.K.: 1967 , Ann.Phys. New York. Vol.43, pp.338.

Siewert, C.E.: 1968 , Ap.J. Vol.152,pp.835.

Sobolev, V.V.: 1969a, Astrofizika, Vol.5,pp.5

Sobolev, V.V.: 1969b, Astrofizika, Vol.5,pp.343.

Sobolev , V. V .: 1956, Transport of Radiant Energy in Stellar and Planetary Atmospheres, Nauka, Moscow.

Sobolev, V.V.: 1970, Astron. Zhurn.Akad.Nauk, SSSR, 47, 246.

Stokes, R.A. and De Marcus, W.C. Icarus.: 1971, Vol. 14. pp.307.

Sweigert, A.V.: 1970a, Ap. J. Suppl. (No.182) Vol. 22. pp.1

Sweigert, A. V .: 1970b, Ap. J. Suppl. (No. 182) Vol. 161. pp.797.

Tsujita,J.: 1968, Publ.Astron.Soc.Japan.Vol.20, pp.270.

Ueno, S.: 1962, J. Math. Anal. Appl. Vol.4, pp.1

Ueno, S.: 1965, J.Math.Anal.Appl. Vol.11, pp.11.

Uesugi, Akira and Irvine, William M.: 1969, J. Atmos. Sci.

Vol.26(No.5) pp.973

Uesugi,Akira and Irvine, William M.: 1970a, Ap. J. Vol. 159 ,pp.127.

Uesugi,Akira and Irvine, William M.: 1970b, Ap.J. Vol. 161 ,pp.243

Uesugi, Akira., Irvine, William M. and Kawata, Yoshiyuki .: 1971 , J.Q.S.R.T. Vol.11 pp.797.

Van De Hulst, H.C.: 1963, Tech. Repo.Goddard Ins. for Space Stud. NASA,88

Van de Hulst, H.C.and Grossman, K.: 1968, Reprinted from "The Atmospheres of Venus and Mars" Edited by John, C. Brandt and Michel, B. Mc. Elroy, pp.35

Wallance, L.: 1972, Ap.J. Vol.176, pp.249

Woolley, R.v.d.R. and Stibbs, D.W.N.: 1953, The Outer Layers of a Star, Oxford University Press, London.