APPENDIX II

6.2. The relation (3.96) of chapter 3

To establish the relation, (3.96), of Chapter 3, I consider

$$D_m(x) = (1 - \lambda)\sum_i \frac{a_i \mu_i^m}{1 + \mu_i x} =$$

$$= (-1)^m \quad (1 - \lambda)\sum_i \frac{a_i \mu_i^m}{1 - \mu_i x} \tag{6.6}$$

I can derive a single recursion formula for $D_m(x)$ .Then

$$D_m(x) = \frac{1}{x}\left[(1 - \lambda) \sum_i a_i \mu_i^{m-1} \left(1 - \frac{1}{1 + \mu_i x} \right) \right] =$$

$$= \frac{1}{x}\left[\psi_{m-1} - D_{m-1} \right] \tag{6.7}$$

where

$$\psi_m = (1 - \lambda) - \sum_i a_i \mu_i^m \tag{6.8}$$

From this formula I have

$$D_m(x) = \frac{\psi_{m-1}}{x} - \frac{\psi_{m-2}}{x^2} + \ldots\ldots + (-1)^{m-2}\frac{\psi_1}{x^{m-1}} +$$

$$+ \frac{(-1)^{m-1}}{x^m}\left[\psi_o - D_o(x) \right] \qquad (m = 0,1,\ldots 4n) \tag{6.9}$$

and

$$\psi_o = 2(1 - \lambda) \tag{6.10}$$

Moreover, let p_{2j} be the coefficient of μ^{2j} in the Legendre polynomial $P_{2n}(\mu)$, then

$$\sum_{j=0}^{n} P_{2j} \; D_{2j}\left(k_r \right) = (1-\lambda) \sum_i \frac{a_i}{1+\mu_i k_r} \left[\sum_{j=0}^{n} P_{2j} \mu_i^{2j} \right] \qquad (6.11)$$

Since μ_i's are the zeros of $P_{2n}(\mu)$, Equation (6.11) reduces to

$$\sum_{j=0}^{n} P_{2j} D_{2j} \left(k_r \right) = 0 \qquad (6.12)$$

Substituting for $D_{2j}\left(k_r \right)$ into equation (6.12) I get the characteristic equation as

$$\frac{P_{2n} \; \lambda}{2n} + \dots\dots\dots\dots + P_o = 0 \qquad (6.13)$$

From this equation it follows that

$$\frac{1}{\left(k_1 \; \dots \; k_n \right)^2} = \frac{(-1)^n \; P_o}{\lambda \; P_{2n}} =$$

$$= \frac{\left(\mu_1 \; \dots\dots \; \mu_n \right)^2}{\lambda} \qquad (6.14)$$

and $\mu_1 . \mu_2 \; \dots . \mu_n \quad k_1 . \dots \; k_n = (\lambda)^{1/2}$ (6.15)