CHAPTER - 5

INTERPRETATION OF EXPERIMENTAL RESULTS WITH THEORIES AND CONCLUSIONS.

In these chapter the experimental data are compared with theories in a new way. The experimental and theoretical results of differential coherent scattering cross sections are expressed in units of Thomson cross section $\left[\frac{r_0^2}{2}(1+\cos^2\theta \right]$ and displayed as a function of momentum transfer q in mc $\left[q = 2k\sin\theta/2 / 20.60744 \right.$, where K is the photon energy in $\left. mc^2 \right]$.

In the present work comparison of experimental results with different theories are divided into three parts. In the first part experimental results are compared with theories for photon energies in the range 25 KeV to 145 KeV, in the second part the photon energy range considered ~~is~~ from 100 KeV to 1 MeV while in the last part the γ energy range varied from 1 MeV to 2.754 MeV. In these comparisons we have tried to find out between the relation of experimental results and different theoretical predictions. And to do this we have included experimental results of some recent high precision experiments by other workers mentioned below together with the results obtained by the author.

(i) Schumacher et al[58,59], photon energies, 59.54 KeV, 412 KeV, 662 KeV, 889 KeV, 1.12 MeV and 2.754 MeV.

(ii) Tirsell et al[60], photon energies, 25.19, 36.84, 46.00, 55.57 and 74.96 KeV.

(iii) Nath et al[61], photon energy 145 KeV.

(iv) Hauser et al[62], photon energy 145 KeV.

(v) Hardie et al[17,18], photon energy 1.33 MeV.

(vi) Kahane et al[63], photon energy 6.84 MeV.

(vii) Barros et al[64], photon energy 145 KeV.

(viii) Kane et al[65], photon energy 1.33 MeV.

In the photon energy range 25 KeV to 145 KeV, Rayleigh scattering is important due to proximity of K-absorption edges of target atoms with incident photon energies. And to our knowledge only a couple of experiments have been performed else where using photon energies below 100 KeV. In one of the experiments Schumacher and Stoffregen[68] have compared their experimental data for photon energy of 59.54 KeV and target atoms of atomic numbers from Z = 30 through Z = 82 with theoretical results using form factor approximation. They have found differences with theories especially near photoeffect K-edges (for Z = 73). In the other experiment Tirsell et al[60] have reported experimental results for photon energy in the range

25-75 KeV and atomic numbers 50 $< Z <$ 79 at scattering angles 45° - 135°, finding poor agreement again with form factor approximation.

In the second region of photon energy from 100 KeV to 1 MeV, Rayleigh scattering dominates and there is more or less satisfactory agreements between different experiments[59,66] and theories. We find it interesting to include our small angle data in the comparison in this photon energy range with latest theoretical predictions to cover a wide range of momentum transfer. We have made the comparison for Lead (Z = 82) the most widely used target in this photon energy range.

And finally, the photon energy range from 1 MeV to 3 MeV is considered because of the importance of Delbruck scattering in the photon energy range.

5.1 25 KeV - 145 K

We begin our comparisons of experimental data for photon energy range 25 KeV - 145 KeV with the help of Figs. 9-12, where the dependence of coherent differential scattering cross section in units of Thomson cross section per electron on the momentum transfer q (in mc) is shown for each of several energies. When we examine the data points referred in sec. 4.6 in reference to the respective ratios $E_K/_E$ (in the range 0.55 - 3.20 for Au and 0.6 - 3.49 for

Pb at an interval of 0.1 near $E_K/E = 1$) of the K-edge energy to the incident photon energy E, in different regions of q distribution, we notice in Fig. 9 that at photon energy with $E_K/E > 1$ and in the range $0.05 < q < 0.1$, the data points show better agreement with values intermediate between the predictions according to RHF[10] form factor calculation and dispersion corrected CL[51] calculations. We also find some indications of explicit dependence on the photon energy beyond q = 0.1 (Fig. 11). The data points with $q < 0.1$ do not show such energy dependence (Fig. 9) as expected from the fundamental condition of form factor approximation. The data points with $E_K/E < 1.0$ agree with dispersion corrected CL calculations. The S-matrix calculation of Kissel and Pratt and the CL calculations are shown in Fig. 9 with the displaying of the data points with $E_K/E > 1.0$ and with $E_K/E < 1.0$. We note a close agreement between these two predictions which agree with the data points for several photon energies. At small momentum transfer q below 0.05 mc and for $E_K/E > 1.0$, the RHF form factor predictions in close agreement with those of CL calculations, appear to be the best approximation to KP predictions.

In order to exhibit the importance of the contribution of outer atomic shells beyond K-shells of heavy atoms we plot in Fig. 11 and Fig. 10 shell-wise KP predictions and

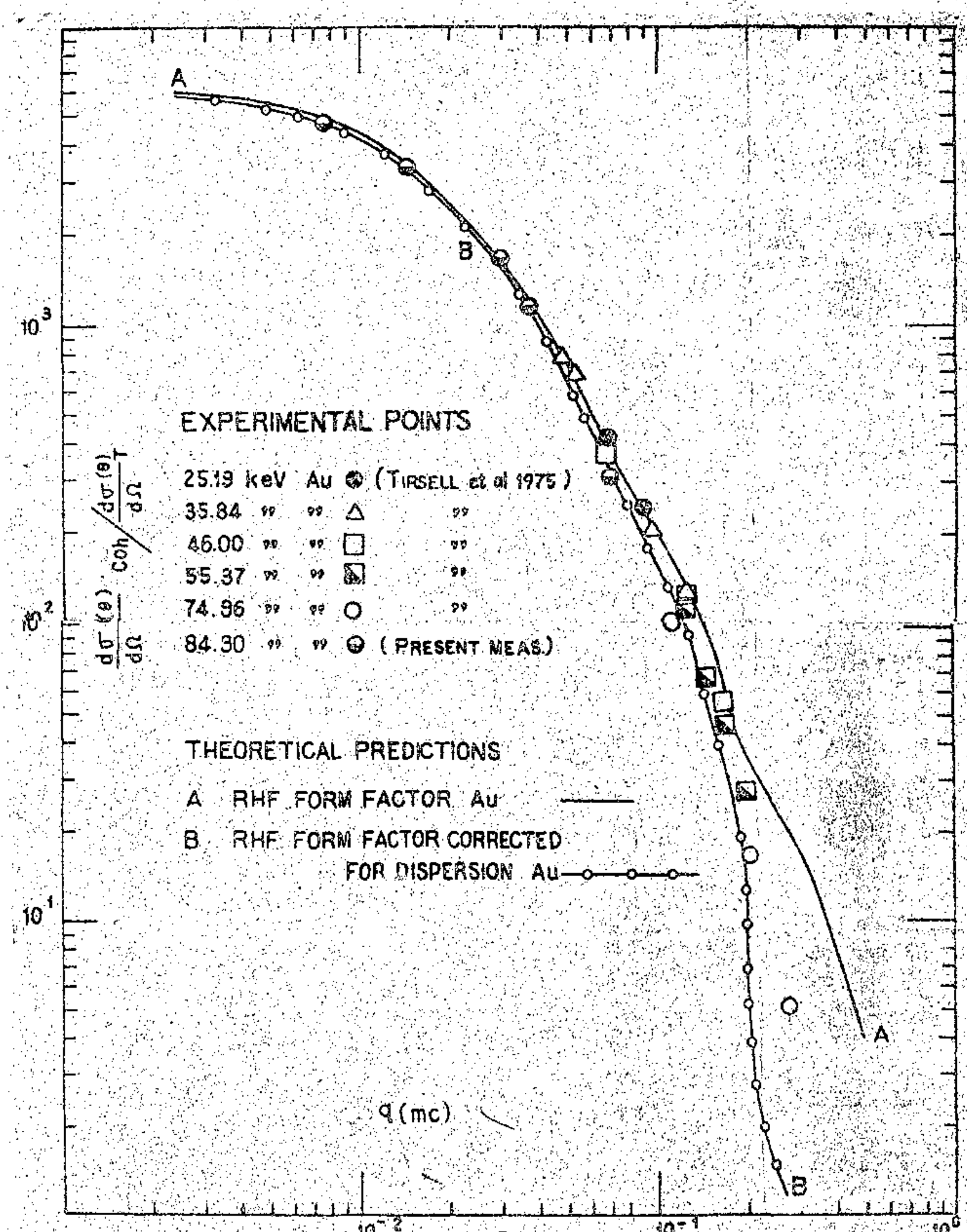

A
B
10³
10²
10¹
dσ(θ)/dΩ Coh / dσ(θ)/dΩ T
EXPERIMENTAL POINTS
25.19 keV Au ⊛ (TIRSELL et al 1975)
35.84 " " △ "
46.00 " " □ "
55.37 " " ▨ "
74.96 " " ○ "
84.30 " " ⊖ (PRESENT MEAS.)
THEORETICAL PREDICTIONS
A RHF FORM FACTOR Au
B RHF FORM FACTOR CORRECTED FOR DISPERSION Au
q (mc)
10⁻²
10⁻¹
10⁰

Fig. 9

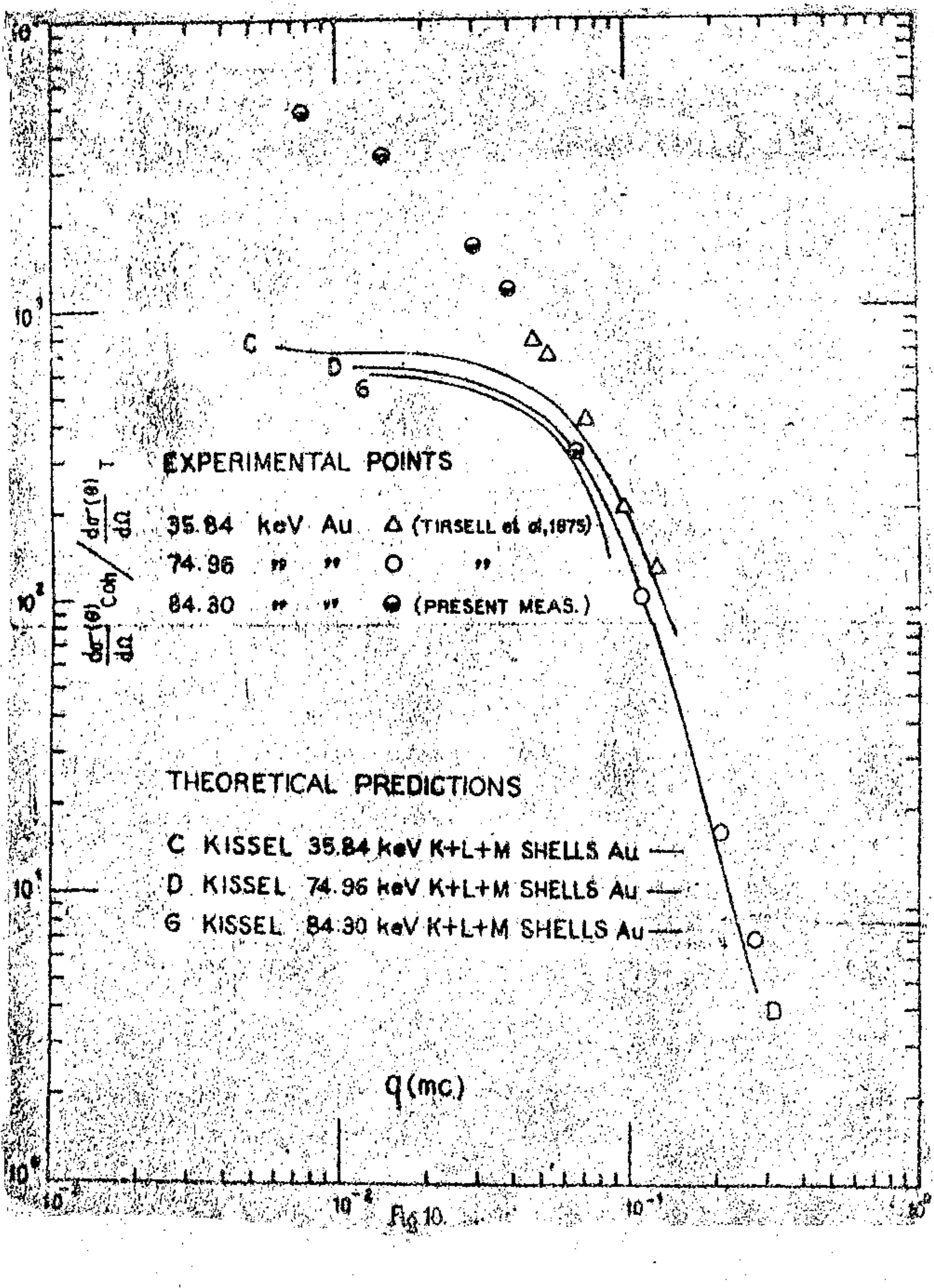
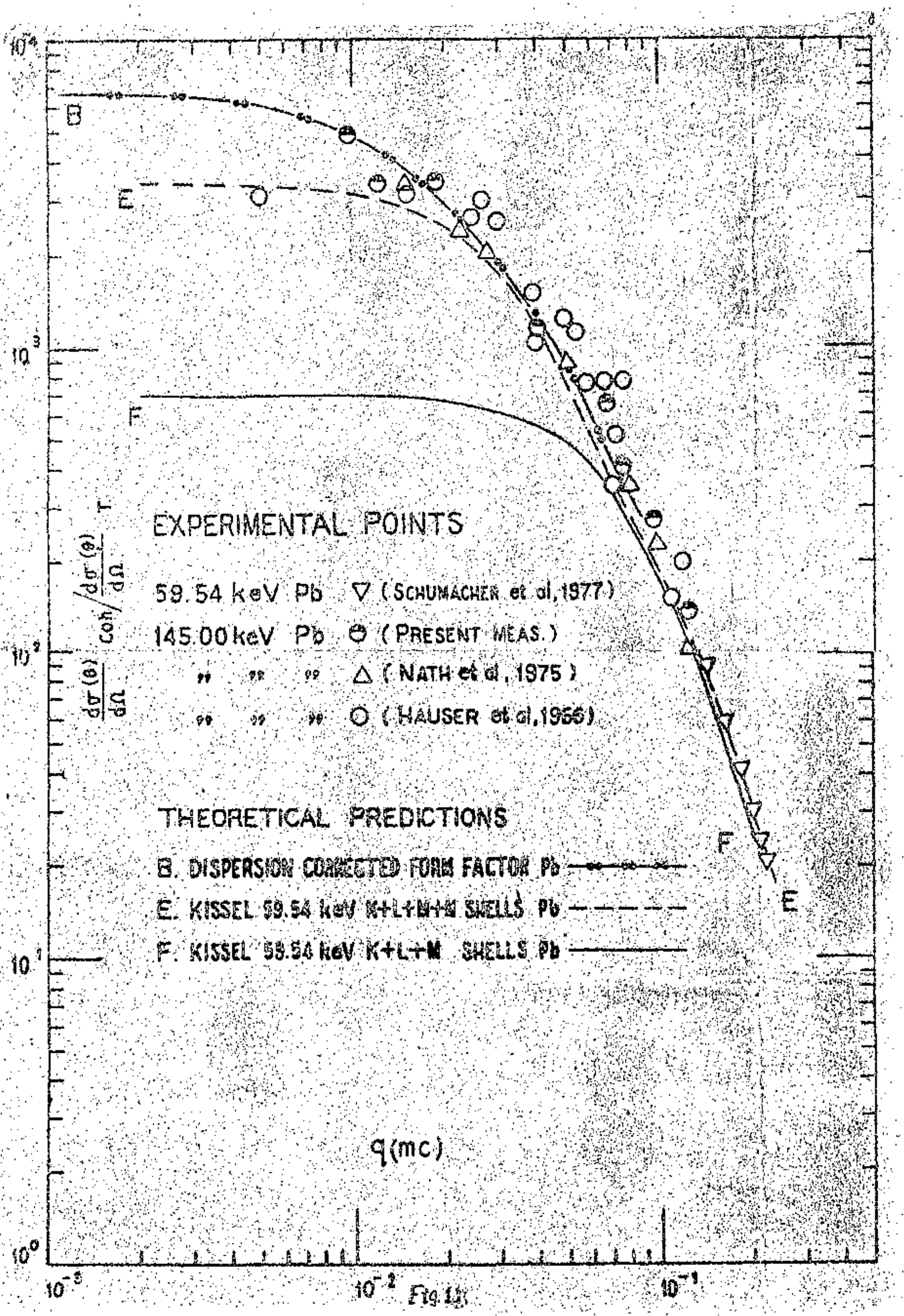
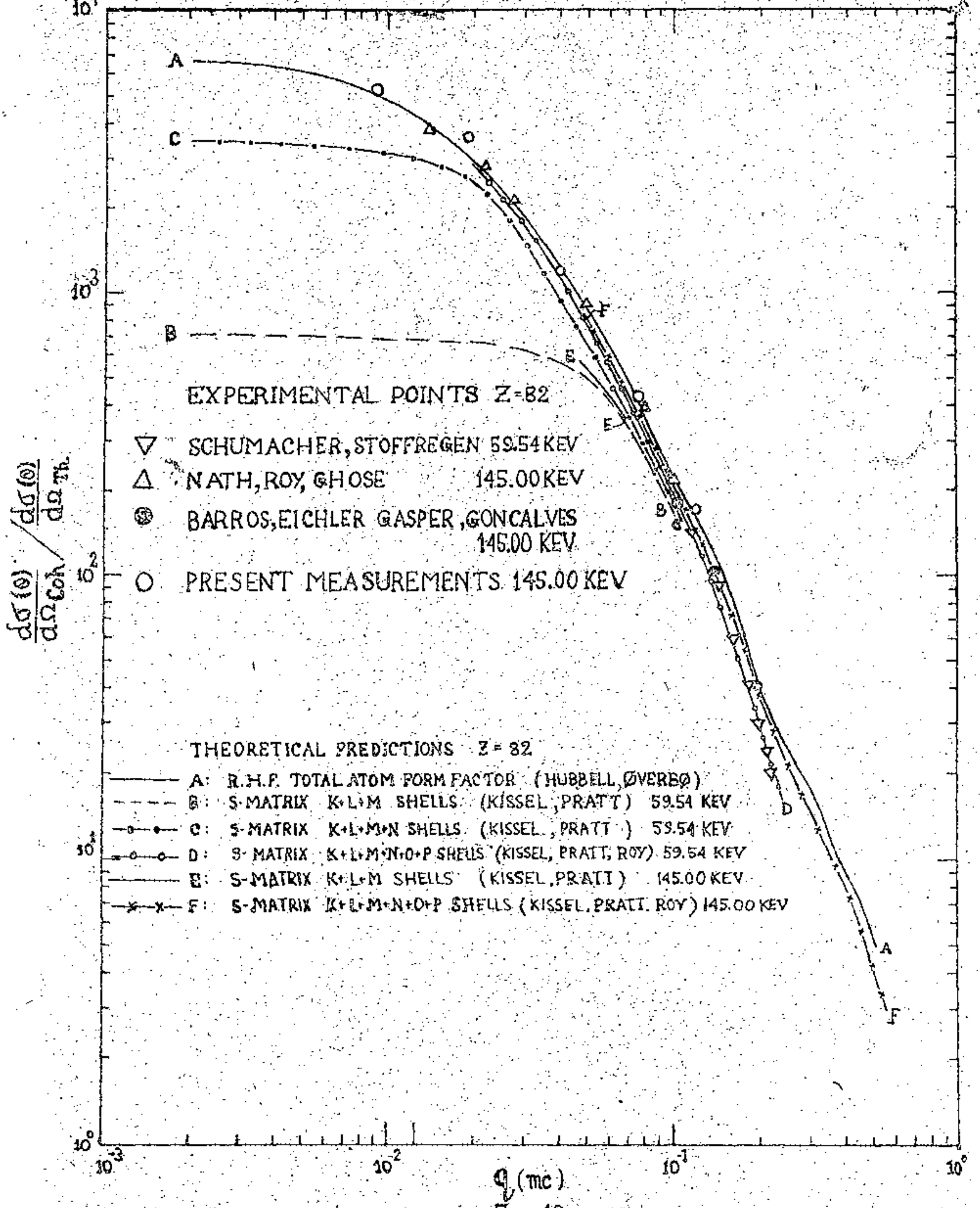

10⁴
10³
10²
10¹
10⁰
C
D
G
D
EXPERIMENTAL POINTS
35.84 keV Au △ (TIRSELL et al, 1975)
74.96 " " ○ "
84.30 " " ● (PRESENT MEAS.)
THEORETICAL PREDICTIONS
C KISSEL 35.84 keV K+L+M SHELLS Au —
D KISSEL 74.96 keV K+L+M SHELLS Au —
G KISSEL 84.30 keV K+L+M SHELLS Au —
dσ(θ)Coh/dΩ / dσ(θ)T/dΩ
q (mc)
10⁻³
10⁻²
10⁻¹
10⁰

Fig 10.



10^4
10^3
10^2
10^1
10^0
10^{-3}
10^{-2}
10^{-1}
B
E
F
F
E
$\frac{d\sigma(\theta)}{d\Omega}_{coh} \Big/ \frac{d\sigma(\theta)}{d\Omega}_T$
q(mc)
EXPERIMENTAL POINTS
59.54 keV Pb ▽ (SCHUMACHER et al, 1977)
145.00 keV Pb ⊖ (PRESENT MEAS.)
" " " △ (NATH et al, 1975)
" " " ○ (HAUSER et al, 1966)
THEORETICAL PREDICTIONS
B. DISPERSION CORRECTED FORM FACTOR Pb
E. KISSEL 59.54 keV K+L+M+N SHELLS Pb
F. KISSEL 59.54 keV K+L+M SHELLS Pb

Fig.12



10^4
10^3
10^2
10^1
10^0
10^{-3}
10^{-2}
10^{-1}
10^0
A
C
B
F
E
D
EXPERIMENTAL POINTS Z=82
▽ SCHUMACHER, STOFFREGEN 59.54 KEV
△ NATH, ROY, GHOSE 145.00 KEV
⊛ BARROS, EICHLER GASPER, GONCALVES 145.00 KEV
○ PRESENT MEASUREMENTS 145.00 KEV
THEORETICAL PREDICTIONS Z = 82
A: R.H.F. TOTAL ATOM FORM FACTOR (HUBBELL, ØVERBØ)
B: S-MATRIX K+L+M SHELLS (KISSEL, PRATT) 59.54 KEV
C: S-MATRIX K+L+M+N SHELLS (KISSEL, PRATT) 59.54 KEV
D: S-MATRIX K+L+M+N+O+P SHELLS (KISSEL, PRATT, ROY) 59.54 KEV
E: S-MATRIX K+L+M SHELLS (KISSEL, PRATT) 145.00 KEV
F: S-MATRIX K+L+M+N+O+P SHELLS (KISSEL, PRATT, ROY) 145.00 KEV
$\frac{d\sigma(\theta)}{d\Omega_{coh}} \Big/ \frac{d\sigma(\theta)}{d\Omega_{Th}}$
q (mc)

Fig. 12

the data at 59.54, 74.96 and 84.30 KeV on Au and at 145 KeV on Pb respectively. We see that below q = 0.06 mc outer shell contributions are significant and has to be included in an exact manner to obtain agreement with data for $E_K/E > 1.0$. Above q = 0.1 mc, the sum of the contributions upto M-shell by S-matrix method is adequate when $E_K/E > 1.0$ for Au and Pb atoms. We plot in Fig. 11 the cross section for Pb including the total atom (K+L+M+N+O+P shells) prediction of Kissel, Pratt and Roy[8] and observe that for both $E_K/E > 1.0$ (145 KeV) and $E_K/E < 1.0$ (59.54 KeV) there is definite improvement with respect to data points below q = 0.06 mc approaching RH.FO form factors. The contribution of O and P shell electrons to the Rayleigh amplitudes in the calculation[8] are estimated via modified form factor for the real parts and use of the forward angle ratio of outer-electron to inner-electron imaginary amplitudes predicted by optical theorem and the photo effect cross sections gives the contribution by the imaginary parts.

In absence of exact calculation of Rayleigh amplitudes from S-matrix formalism for other elements present experimental results at 84.30 and 145 KeV and for target atoms Z = 13,29,47,50,57,62,66 are compared. With RHF and NRHF total atom form factor predictions in Figs. 12,13,14,15,16 We note that for E/E > 1.0 both R.H.F. and N.R.H.F. form

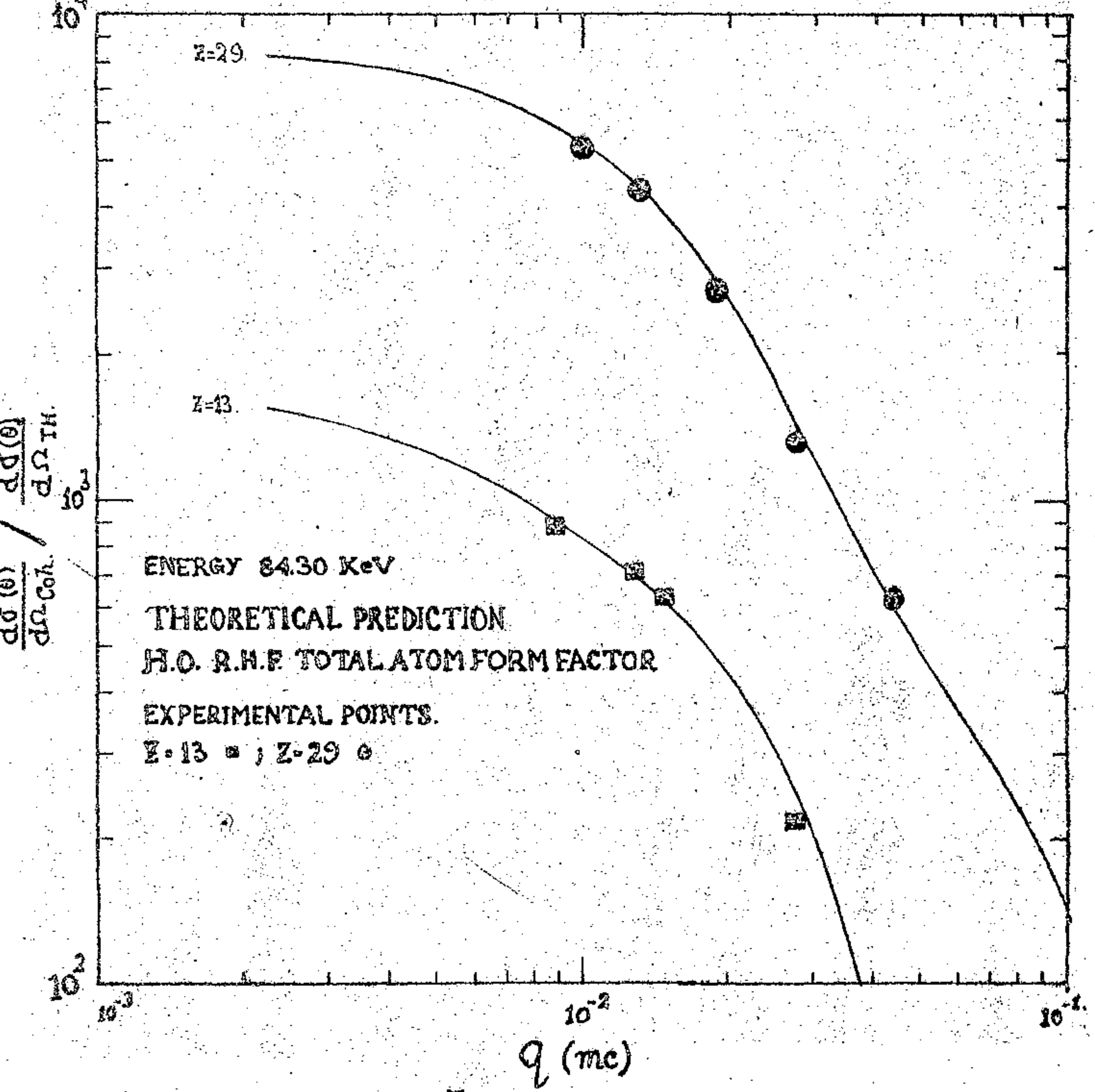

10^4
10^3
10^2
$\frac{d\sigma(\theta)}{d\Omega_{coh.}} \Big/ \frac{d\sigma(\theta)}{d\Omega_{TH.}}$
Z=29.
Z=13.
ENERGY 84.30 KeV
THEORETICAL PREDICTION
H.O. R.H.F. TOTAL ATOM FORM FACTOR
EXPERIMENTAL POINTS.
Z=13 ■ ; Z=29 ●
10^{-3}
10^{-2}
10^{-1}
q (mc)

Fig. 13.

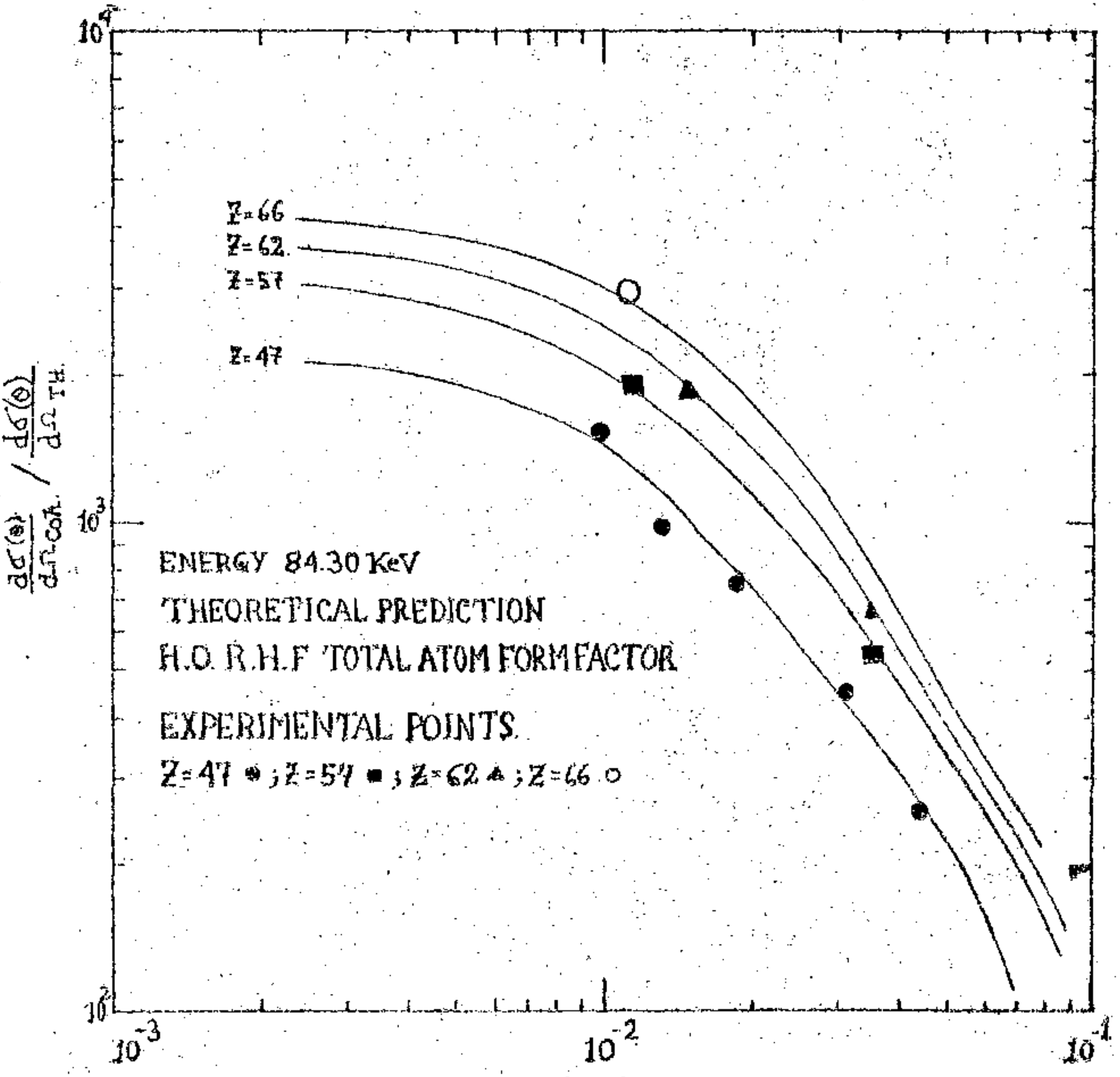

10^4
10^3
10^2
10^{-3}
10^{-2}
10^{-1}
$\frac{d\sigma(\theta)}{d\Omega_{coh.}} / \frac{d\sigma(\theta)}{d\Omega_{TH}}$
$q\,(mc)$
Z=66
Z=62
Z=57
Z=47
ENERGY 84.30 KeV
THEORETICAL PREDICTION
H.O. R.H.F TOTAL ATOM FORM FACTOR
EXPERIMENTAL POINTS
Z=47 ● ; Z=57 ■ ; Z=62 ▲ ; Z=66 ○

Fig. 14

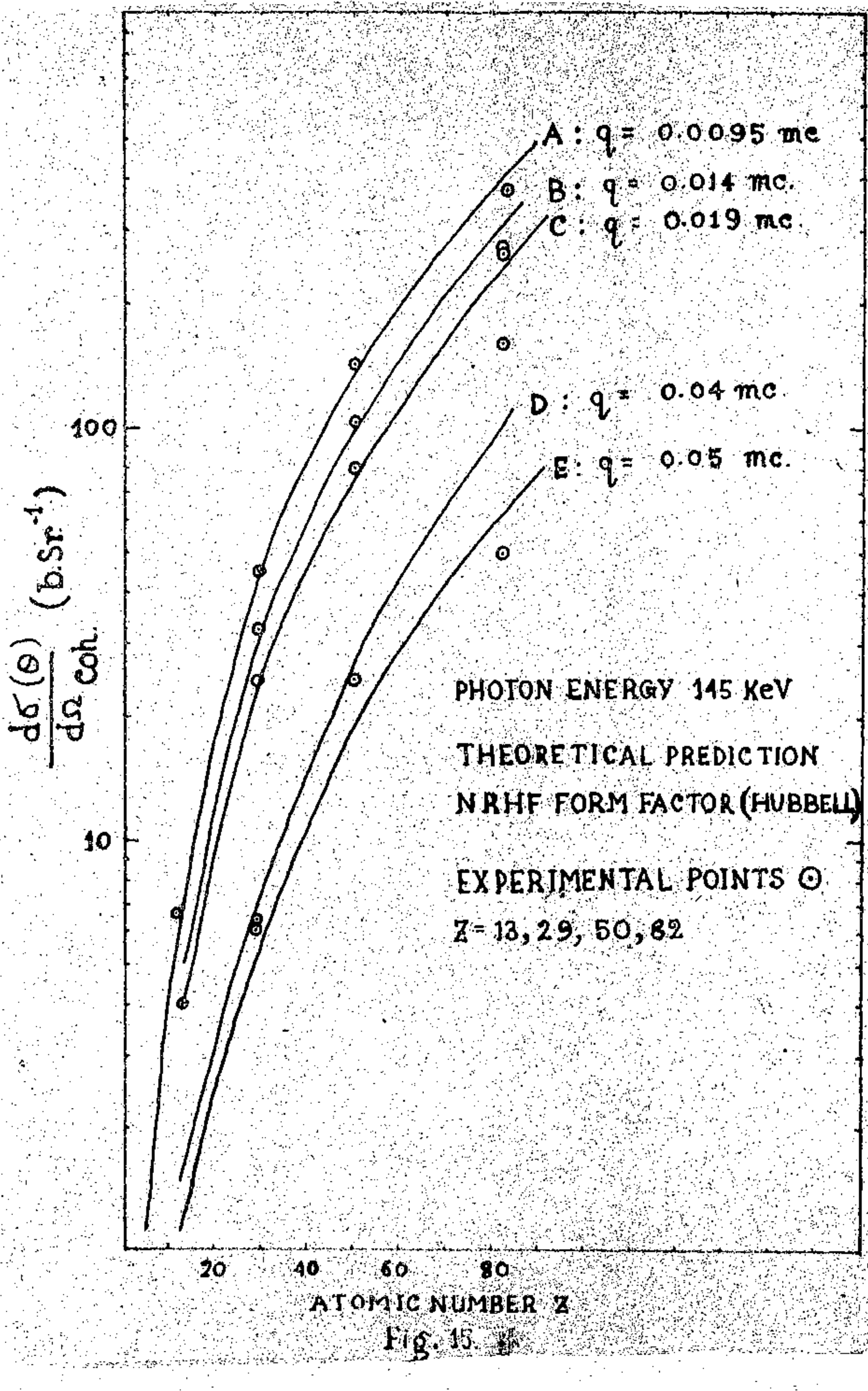

A : q = 0.0095 mc
B : q = 0.014 mc.
C : q = 0.019 mc.
D : q = 0.04 mc
E : q = 0.05 mc.
PHOTON ENERGY 145 KeV
THEORETICAL PREDICTION
NRHF FORM FACTOR (HUBBELL)
EXPERIMENTAL POINTS ⊙
Z = 13, 29, 50, 82
$\frac{d\sigma(\theta)}{d\Omega}_{coh.}$ (b.Sr^{-1})
100
10
20
40
60
80
ATOMIC NUMBER Z

Fig. 15.

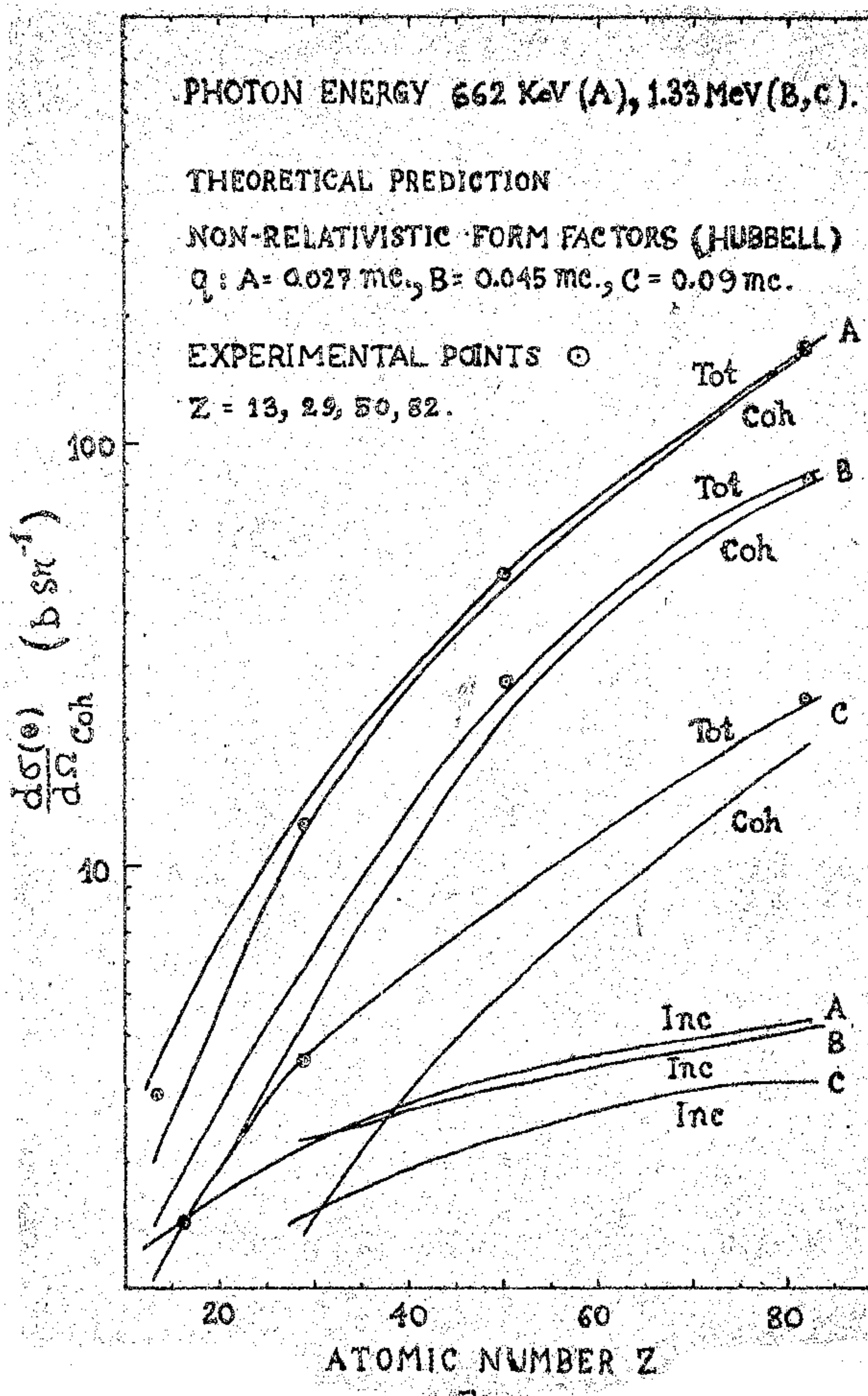

PHOTON ENERGY 662 KeV (A), 1.33 MeV (B, C).
THEORETICAL PREDICTION
NON-RELATIVISTIC FORM FACTORS (HUBBELL)
q : A = 0.027 mc., B = 0.045 mc., C = 0.09 mc.
EXPERIMENTAL POINTS ⊙
Z = 13, 29, 50, 82.
$\frac{d\sigma(\theta)}{d\Omega}_{Coh}$ ($b\,sr^{-1}$)
100
10
20
40
60
80
ATOMIC NUMBER Z
Tot
Coh
A
Tot
Coh
B
Tot
Coh
C
Inc
A
B
Inc
C
Inc

Fig.16

factor approximations appear to be adequate in the low momentum transfer considered.

5.2 100 KeV – 1 MeV

In the intermediate energy range from 100 KeV to 1 MeV, Kissel (Ref. 6. Thesis page 77 and Fig. 5.16) has compared experimental results of Anand and Sood[66] and Schumacher et al[59] with S-matrix calculation of Johnson and Cheng and their own calculation and have found reasonable agreements at 279, 412 and 662 KeV.

We shall compare different theories on Rayleigh scattering in the energy range covering a wide range of momentum transfer with other experimental results together with our own data in the momentum transfer range for small angle scattering. For this purpose we include our data (26 data points) for Z = 82, the most commonly used scatterer in this energy range for six different photon energies: (0.145, 0.280, 0.662, 1.115, 1.17 and 1.33 MeV) and these data are displayed in Fig. 16. together with additional data for Pb (Z = 82) from the following high precision measurements, 1) Schumacher et al, photon energies 59.54 KeV (seven data points), 412 KeV (eight data points), 662 KeV (seven data points), 889 KeV (nine data points), 1.12 MeV (nine data points) and 2.75 MeV (eight data points);

(ii) Hardie et al, photon energy 1.33 MeV (ten data points); (iii) Kahane et al, photon energy 6.84 MeV (one data point), (iv) P.P.Kane et al (two data points). The data points referred are compared with theoretical results including different predictions of Rayleigh scattering amplitudes in Fig. 17. It is seen from the display that form factor theory is sufficient even for high Z atoms over the q -range below 0.5 mc. The RHF form factor theory is appropriate to scattering of photons with energies greater than the K-shell binding energy of the heavy scatterer atom, whereas for photon energies less than the K-shell binding energy, NRHF (non-relativistic Hartree Fock) form factor predictions, consistent with new theoretical predictions by Kissel and Pratt are found to show agreement at 5% level below q = 0.2 mc and within 15% above q = 0.2 mc.

In the q-range above 0.5 mc we observe q-distribution for each photon energy over the range 0.406 - 2.75 MeV. The Florescu - Gavrila high energy approximation approaches the low - q end of the observed q-distribution, while form factor theory approaches the high q end of the distribution at each of the photon energy mentioned. In the intermediate q-range the distribution $\frac{d\sigma(\theta)}{d\Omega_{coh}} / d\sigma(\theta)_T$ is in excellent agreement with the predictions of the energy dependence of the Johnson and Cheng (consistent with Kissel &

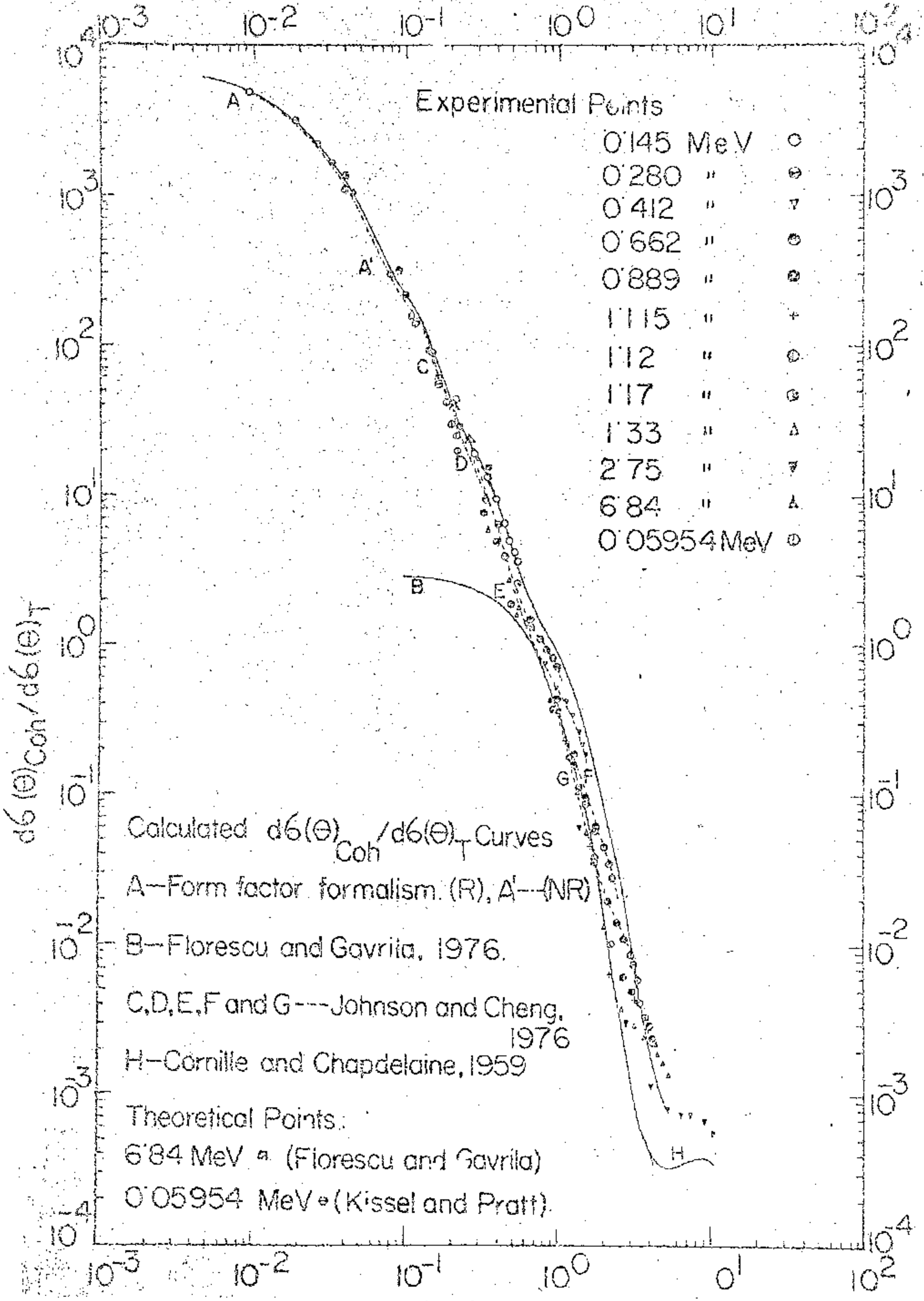

Experimental Points
0·145 MeV
0·280 "
0·412 "
0·662 "
0·889 "
1·115 "
1·12 "
1·17 "
1·33 "
2·75 "
6·84 "
0·05954 MeV
Calculated $d\sigma(\Theta)_{Coh}/d\sigma(\Theta)_T$ Curves
A—Form factor formalism. (R), A'—(NR)
B—Florescu and Gavrila, 1976.
C,D,E,F and G—Johnson and Cheng, 1976
H—Cornille and Chapdelaine, 1959
Theoretical Points.
6·84 MeV (Florescu and Gavrila)
0·05954 MeV (Kissel and Pratt)
A
A'
B
C
D
E
F
G
H
$d\sigma(\Theta)_{Coh}/d\sigma(\Theta)_T$
q (mc)
10^{-3}
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Fig. 17.

Pratt calculations at those energies) calculations.

5.3 1 MeV - 3 MeV

And finally we consider comparison of experimental data with theories for photon energy in the range 1-3 MeV. The main interest in this photon energy range for high Z target atoms is the contribution of real or dispersive part of the Delbruck amplitude. For this purpose two sets of theoretical curves, one including Rayleigh, nuclear Thomson, nuclear resonance (if significant) and imaginary part of Delbruck amplitudes and the other set including also the real part of the Delbruck scattering amplitudes are drawn in Figs. 18,19. In these figures both theoretical and experimental values of differential coherent scattering cross sections expressed in units of Thomson cross section are again plotted as a function of momentum transfer q in mc unit in the foregoing manner. The theoretical curves are drawn for each of the following photon energy, 1.1205, 1.333, 1.70, 2.09 and 2.754 MeV for Z = 82 and 1.3325, 1.70, 2.09 and 2.754 MeV for Z = 92 . The following observations could be made from these displays for momentum transfer varying from 1.0 to 10.0 mc. For Pb (Z = 82) it is seen from Fig. 18 that

(i) for photon energies of 1.1205 and 1.3325 MeV, experimental data agree excellently with theoretical cross sections including real part of Delbruck amplitudes as shown by solid curves at all momentum transfers from 1.34 - 5.03 mc excepting at q = 3.17 mc for 1.1205 MeV photons and at q = 3.32 mc for 1.3325 MeV where the experimental data approaches theoretical cross section excluding the real or dispersive part of Delbruck amplitude as shown by the broken curves.

(ii) for photon energy 1.70 MeV, there is sufficient agreement between experimental data and theoretical cross sections at momentum transfer of 4.05 mc to 6.42 mc. The experimental data appear to be slightly less than the theoretical cross sections including real part of Delbruck amplitudes.

(iii) for photon energy of 2.09 MeV the experimental data are slightly higher than the theoretical cross sections at momentum transfer of 4.09, 5.70 and 7.90 mc.

(iv) for 2.754 MeV photons, there is complete disagreement at all momentum transfer ranging from 2.78 mc to 10.41 mc. The disagreement appears to be narrowing down with increasing momentum transfer starting from 7.62 mc. The disagreement

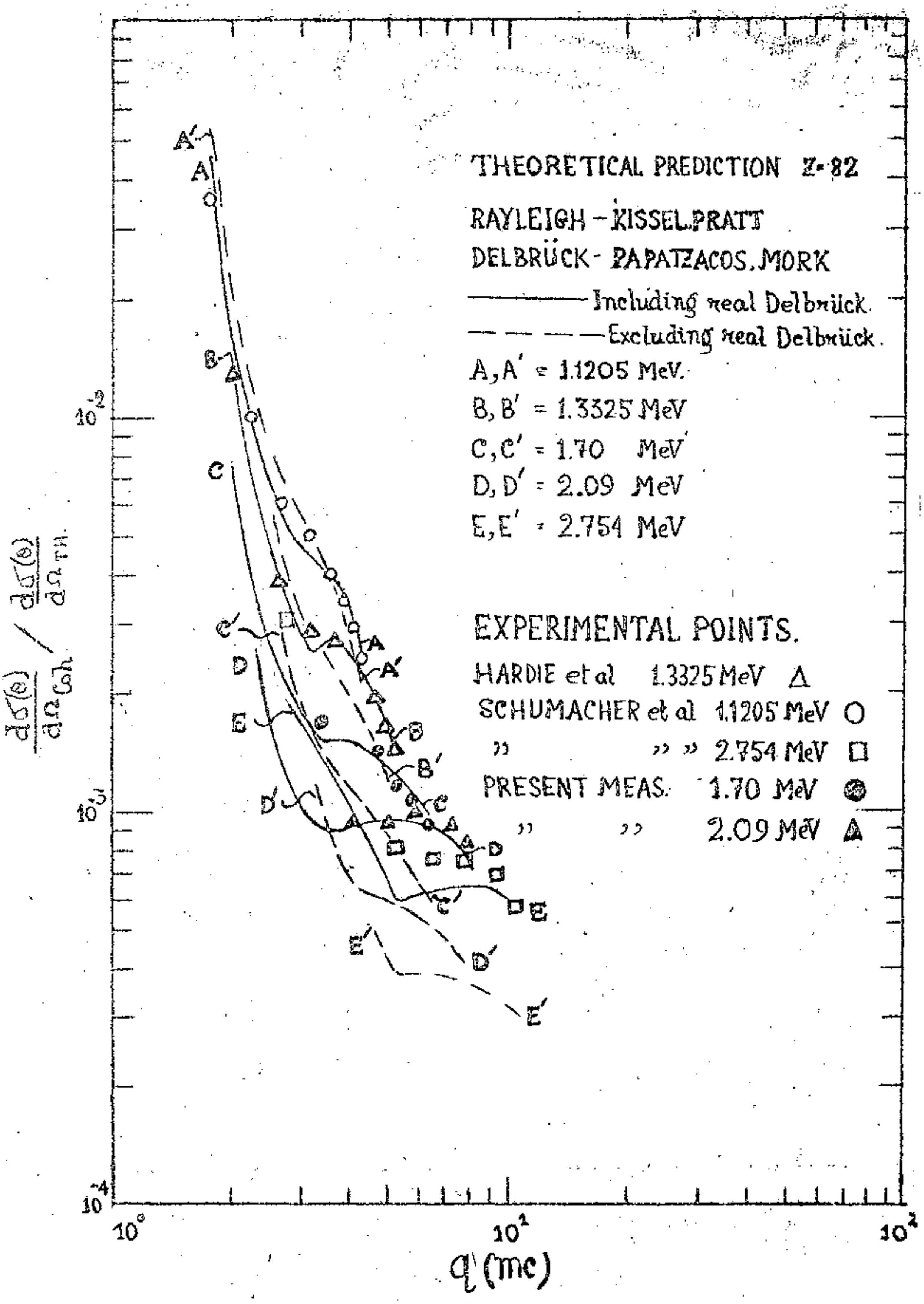

THEORETICAL PREDICTION Z=82
RAYLEIGH - KISSEL.PRATT
DELBRÜCK - PAPATZACOS.MORK
Including real Delbrück.
Excluding real Delbrück.
A, A' = 1.1205 MeV.
B, B' = 1.3325 MeV
C, C' = 1.70 MeV
D, D' = 2.09 MeV
E, E' = 2.754 MeV
EXPERIMENTAL POINTS.
HARDIE et al 1.3325 MeV △
SCHUMACHER et al 1.1205 MeV ○
" " " 2.754 MeV □
PRESENT MEAS. 1.70 MeV ●
" " 2.09 MeV ▲
A'
A
B
C
C'
D
E
D'
A'
B'
C'
D'
E'
D
E
10^{-2}
10^{-3}
10^{-4}
10^{0}
10^{1}
10^{2}
$\frac{d\sigma(\theta)}{d\Omega_{Coh}} / \frac{d\sigma(\theta)}{d\Omega_{TH}}$
q (mc)

Fig. 18.

is maximum at 2.78 mc being 64.69% between experimental data and theoretical cross section including real part of Delbruck amplitude.

For U (Z = 92) the following observations are made from Fig. 19.

(i) for 1.3325 MeV photon energy, there is good agreement at all momentum transfer excepting at q = 1.54 and 3.17 mc, the disagreement being 4.95% and 6.06% respectively between experimental data and theoretical cross sections including real part of Delbruck amplitudes.

(ii) for 1.70 MeV photon energy, the experimental data are slightly higher than the theoretical cross sections at momentum transfer of 3.32, 4.70 and 5.76 mc. The disagreement being maximum at 3.32 mc (6.58%).

(iii) for 2.09 MeV photon energy, the experimental cross sections are higher at momentum transfer of 4.09, 5.73 and 7.08 mc, the disagreement being 11%, 11.5% and 10.2% respectively with theoretical cross sections including real part of Delbruck amplitudes.

(iv) for 2.754 MeV photon energy, there is again complete disagreement at all momentum transfer the maximum disagreement is 77% at 2.78 mc.

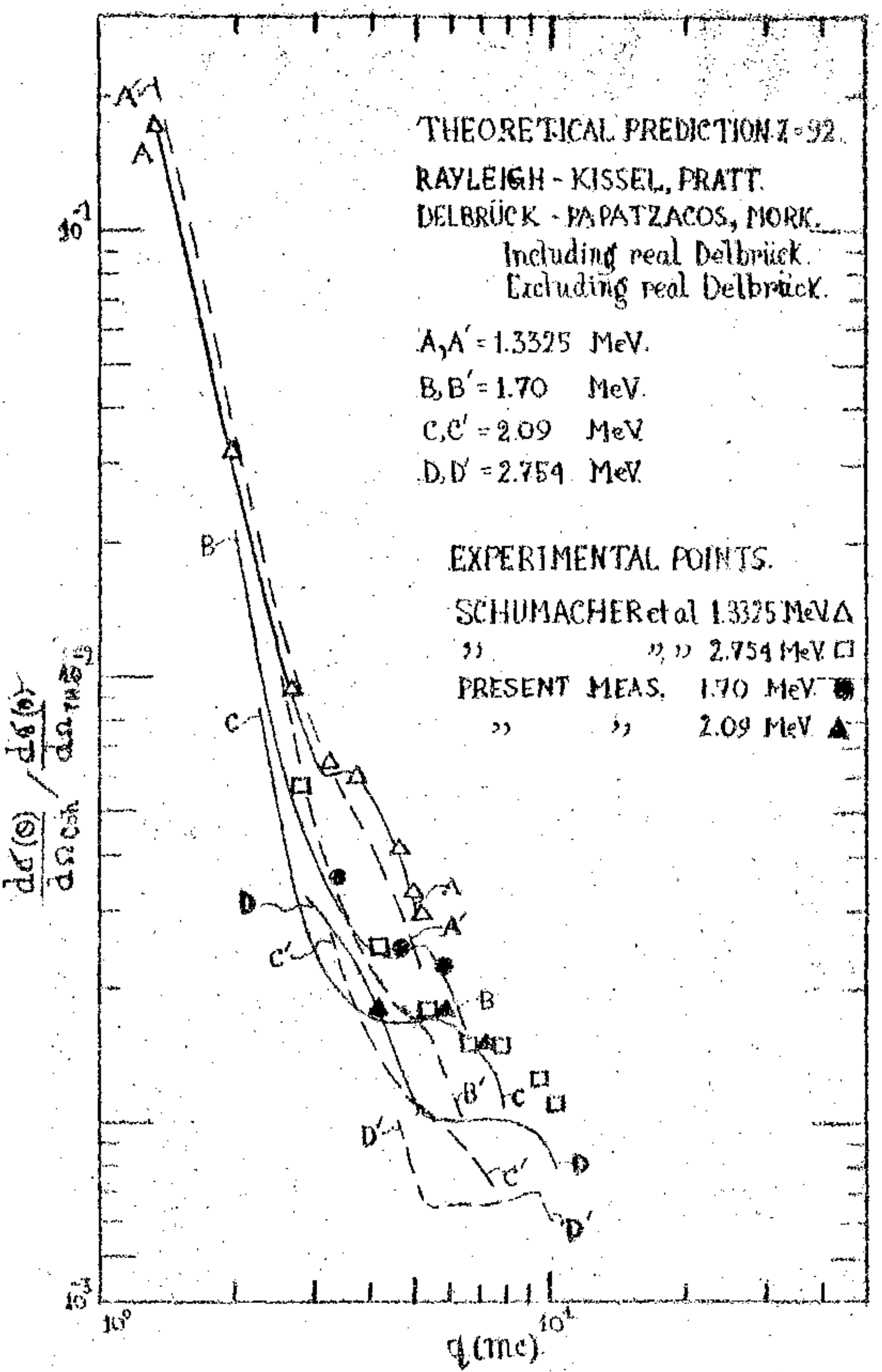

THEORETICAL PREDICTION Z=92.
RAYLEIGH - KISSEL, PRATT.
DELBRÜCK - PAPATZACOS, MORK.
Including real Delbrück.
Excluding real Delbrück.
A, A′ = 1.3325 MeV.
B, B′ = 1.70 MeV.
C, C′ = 2.09 MeV
D, D′ = 2.754 MeV
EXPERIMENTAL POINTS.
SCHUMACHER et al 1.3325 MeV △
" " " 2.754 MeV □
PRESENT MEAS. 1.70 MeV ●
" " 2.09 MeV ▲
A′
A
B
C
D
C′
D′
B′
10^{-1}
10^{-2}
10^{-3}
10^{0}
10^{1}
$\frac{d\sigma(\theta)}{d\Omega_{coh}} / \frac{d\sigma(\theta)}{d\Omega_{TH}}$
q (mc)

Fig. 19

An interesting feature of the displays is that for both Pb(Z = 82) and U(Z = 92) there is considerable discrepancy at momentum transfer around 3 mc. It appears from the displays that existence of real or dispersive part of Delbruck amplitudes is beyond doubt and calculation of Delbruck amplitudes by Papatzacos and Mork are adequate to explain elastic scattering of photons together with Rayleigh scattering amplitudes of Kissel and Pratt for photon energies near threshold and slightly above. The disagreement at photon energies above 2 MeV may be attributed to the non inclusion of higher order terms in the calculation of Delbruck amplitudes from Born approximation, or in the approach in estimating nuclear resonance scattering amplitudes.

As mentioned in Chapter we have also made comparisons of different theoretical predictions on Rayleigh scattering to see the relation of each calculation with others. We make the following observation.

It appears from tables 1-4 and plots 1-3 in the comparison of theoretical cross sections of Lead (Z = 82) using different predictions of Rayleigh scattering amplitudes that both ordinary and modified form factors approach closely to Kissel Pratt calculation at low momentum transfer. Modified form factors appear to be more closer. The agreement between RHF total atom form factors of Hubbell and Øverbø and Kissel Pratt calculation is good at photon energies of 145 KeV, though for momentum transfer below .05 mc. R.H.F. total atom form factors give much higher cross sections. These may be due to non inclusion of higher shell amplitudes in the Kissel Pratt calculation. The modified form factors give much lower values of cross sections at large scattering angles. The agreement between Kissel Pratt K-shell calculation agrees well with Florescu Gavrila K-shell calculation in the intermediate energy range for low momentum transfer. In table 8 we have compared K-shell calculations of Kissel and Pratt with Brown and Meyers and Florescu and Gavrila and RHF total atom calculations for Mercury atoms (Z = 80) and find that for momentum transfer above 1.32 mc there is complete disagreement.

For medium Z atoms we have compared theoretical cross sections for Tin (Z = 50) with available predictions of Rayleigh amplitudes at photon energy of 511 KeV. Florescu Gavrila K-shell amplitudes appear to be in good agreement with Kissel Pratt K-shell calculations at all momentum transfer. The ordinary and modified form factors (K-shell) also agree well with Kissel Pratt calculation of K-shell calculations upto $q = 0.5$ mc. Higher shell calculations of Kissel and Pratt, Johnson and Lin are not available in this case.

In tables 7,8 and 9 we have compared results for low Z (Al, Z = 13) atom with available theories. RHF form factors are in excellent agreement with Kissel Pratt calculations at the photon energies considered. The disagreement at low momentum transfer again may be due to non-inclusion of higher shell amplitudes in the Kissel Pratt calculations.

Finally, we can draw the following conclusions after a careful study made in the foregoing manner.

(i) Form Factor Approximations (both NRHF and RHF) are dependable for Rayleigh scattering at low momentum transfer.

(ii) For photon energies near K-absorption edges of target atoms dispersion corrections are important. There is also need to extend these corrections to higher angles.

(iii) Calculation of Rayleigh scattering amplitudes based on S-matrix formalism provide very useful theoretical data for a wide range of scattering variables. These theoretical data will be completed if (a) higher shell amplitudes are also calculated in the exact manner for low photon energies (below 100 MeV). (b) Exact calculations of L and possibly M-shell amplitudes are also to be calculated in the exact manner for photon energies above 1 MeV.

(iv) Delbruck amplitudes calculated and based on Born Approximation are adequate for photon energies near threshold. The theory fails for photon energies above 2 MeV.