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An Analysis of Cosmic Ray Air Showers for the Determination of Shower Age

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Abstract

A sample of 8651 air showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$ has been analysed to determine the distribution of the measured age in terms of (i) the number of showers in a specified size range, and (ii) the radial distances in individual showers. It is shown that the radial age distribution in an individual shower leads to an average shower age approximately the same as the prediction of the electron–photon cascade theory. The other results include a study of the variation of (i) shower age, as measured by the χ^2-minimisation technique, with shower size of vertically incident showers, and (ii) the measured electron density at any point with its radial distance from the shower axis, as a function of the age of a large shower group with very small spread in size. A comparison of similar measurements with relevant theory is also included.

1. Introduction

The development in the longitudinal direction of electron–photon cascades in cosmic ray extensive air showers is described by a parameter called the shower age s. The cascade grows to a maximum ($s = 1$) and then rapidly decays. In the lateral direction from the axis of the shower, the electron density distribution in the shower is measured in terms of the radial age $S(r)$ as one of the parameters. In most earlier experiments (Idenden 1990; Hara *et al.* 1981, 1983; Abdullah *et al.* 1981, 1983), the shower age determined by the standard least-squares fitting technique differs from the theoretical value at all atmospheric depths. This was taken to be an indication that a shower must be described by two age parameters, one for its longitudinal development and the other for its lateral development (Hara *et al.* 1983; Sasaki 1971; Capdevielle and Gawin 1982, 1985; Dai *et al.* 1990). This aspect of extensive air showers has been under investigation in recent years at various centres (Idenden 1990; Dai *et al.* 1990; Cheung and MacKeown 1987; Samorski and Stamm 1983). In the present work a critical experimental examination is made of the techniques used to determine the shower age from new measurements on smaller air showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$. An analysis of shower age has also been made to show its dependence on various shower parameters.

2. Experiment

The air shower array at the North Bengal University campus has been developed in stages since 1980 (Basak *et al.* 1984). At present it consists of 21 electron-

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density-sampling plastic scintillation detectors, eight fast timing detectors and two magnet spectrographs. The total area covered by the array is 1176 m^2. The shower size threshold for the array is $N_e = 10^{4 \cdot 2}$. The radial electron density distribution and muon density distribution are measured simultaneously over a radial distance from the array centre to about 30 m and the muon energy in the range 2·5–220 GeV.

To determine the size of a shower, the electron densities at radial distance intervals of 8 m ($\sim r_0/10$, where r_0 is the Molière radius in air at sea level) were measured by a cluster of 21 scintillation detectors installed at sea level. The dynamic range of the detectors is 1–250 particles/detector and each detector was operated at a threshold of one particle. The shower direction was determined by measuring relative arrival times, while shower size N_e, age parameters s and core location (x_0, y_0) determination was carried out by fitting the radial electron density data of a shower event to an interpolating lateral structure function as given by Hillas and Lapikens (1977):

$$f(r) = c(s)\,(r/r_1)^{a_1+a_2(s-1)}(1+r/r_1)^{b_1+b_2(s-1)}, \qquad (1)$$

where $c(s)$ is the normalisation constant and $a_1 = -0\cdot53$, $a_2 = 1\cdot54$, $b_1 = -3\cdot39$, $b_2 = 0$ and $r_1 = 24\ m$.

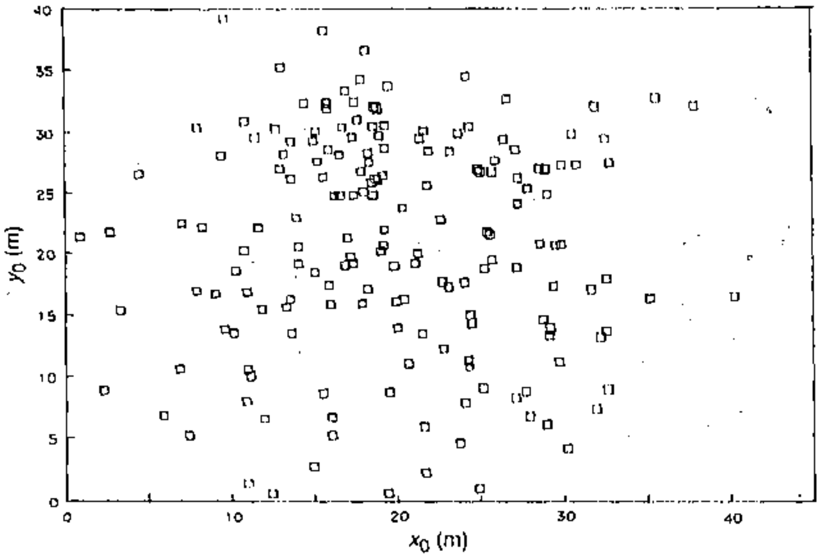

Fig. 1. Distribution of shower core location (x_0, y_0) for a group of 178 showers of $\overline{N}_e = 1 \cdot 2\times 10^5$ with $\bar{s} = 1$.

The computed results on the shower core location (x_0, y_0) form a random distribution, as shown in Fig. 1 for a group of 178 showers of size $\overline{N}_e = 1 \cdot 2\times10^5$ with age $\bar{s} = 1$.

(2a) Determination of Radial Age Parameter

Using the Hillas–Lapikens (HL) structure function (1) and assuming that the normalisation constants do not vary much at two neighbouring radial points r_i and r_j measured from the core position (e.g. Fig. 1), we obtain for the radial age parameter, at radial location $r_i - r_j$,

$$S_{ij}(r) = \ln(F_{ij}\, X_{ij}^{2\cdot 07}\, Y_{ij}^{3\cdot 39})/1\cdot 54 \ln X_{ij}\,, \tag{2}$$

where $F_{ij} = f(r_i)/f(r_j)$, $X_{ij} = r_i/r_j$, $Y_{ij} = (1+x_i)/(1+x_j)$, with $x = r/r_1$. Substitution of the measured electron densities at the radial points r_i and r_j in the above formula gives $S_{ij}(r)$. With the Nishimura–Kamata-Greisen (NKG) function (Greisen 1960), the expression for $S_{ij}(r)$ under the same conditions as in (2) is

$$S_{ij}(r) = \ln(F_{ij}\, X_{ij}^{2}\, Y_{ij}^{4\cdot 5})/\ln(X_{ij}\, Y_{ij})\,. \tag{3}$$

Some representative results are shown in Table 1.

Table 1. Radial variation of shower age $S_{ij}(r)$ for three shower sizes

		(a) Shower size $N_e = 5\cdot 3\times 10^4$			
Radial distance interval (m)	2·5-5	5·5–8·5	8·5–12·5	12·5–17·5	17·5–22·5
HL	$1\cdot 481^{+0\cdot 075}_{-0\cdot 059}$	$1\cdot 535^{+0\cdot 026}_{-0\cdot 024}$	$1\cdot 592^{+0\cdot 034}_{-0\cdot 032}$	$1\cdot 464^{+0\cdot 015}_{-0\cdot 014}$	
NKG		$1\cdot 759^{+0\cdot 038}_{-0\cdot 034}$	$1\cdot 687^{+0\cdot 047}_{-0\cdot 044}$	$1\cdot 384^{+0\cdot 019}_{-0\cdot 018}$	$1\cdot 761^{+0\cdot 006}_{-0\cdot 007}$
		(b) Shower size $N_e = 1\cdot 2\times 10^5$			
Radial distance interval (m)	12·5–17·5	17·5–22·5	22·5–27·5	27·5–32·5	32·5–37·5
HL	$1\cdot 177^{+0\cdot 009}_{-0\cdot 008}$	$1\cdot 779^{+0\cdot 025}_{-0\cdot 023}$	$1\cdot 666^{+0\cdot 007}_{-0\cdot 007}$	$1\cdot 550^{+0\cdot 015}_{-0\cdot 014}$	$1\cdot 738^{+0\cdot 113}_{-0\cdot 123}$
NKG	$1\cdot 002^{+0\cdot 012}_{-0\cdot 011}$	$1\cdot 699^{+0\cdot 032}_{-0\cdot 030}$	$1\cdot 491^{+0\cdot 009}_{-0\cdot 009}$	$1\cdot 312^{+0\cdot 019}_{-0\cdot 017}$	$1\cdot 513^{+0\cdot 133}_{-0\cdot 144}$
		(c) Shower size $N_e = 1\cdot 2\times 10^6$			
Radial distance interval (m)	22·5–27·5	27·5–32·5	32·5–37·5	37·5–45	45–55
HL	$1\cdot 189^{+0\cdot 049}_{-0\cdot 069}$	$1\cdot 971^{+0\cdot 009}_{-0\cdot 163}$	$1\cdot 688^{+0\cdot 121}_{-0\cdot 096}$	$1\cdot 877^{+0\cdot 031}_{-0\cdot 112}$	$1\cdot 775^{+0\cdot 058}_{-0\cdot 052}$
NKG	$1\cdot 271^{+0\cdot 061}_{-0\cdot 085}$	$1\cdot 820^{+0\cdot 085}_{-0\cdot 197}$	$1\cdot 455^{+0\cdot 142}_{-0\cdot 114}$	$1\cdot 654^{+0\cdot 151}_{-0\cdot 128}$	$1\cdot 526^{+0\cdot 065}_{-0\cdot 058}$

The average age parameter of a shower at a particular size is given by

$$\bar{S} = \sum_{ij} \frac{2w_{ij}}{r_j^2 - r_i^2} \int_{r_i}^{r_j} S_{ij}(r)\, r\, \mathrm{d}r\,, \tag{4}$$

where (r_i, r_j) is the radial distance interval within which the radial age parameters are measured experimentally, and w_{ij} is the statistical weight factor of that particular radial distance bin (i, j).

According to electron–photon cascade theory, the shower age is

$$S(\text{theor.}) = 3t/[t + 2\ln(E_0/\epsilon_0) + 2\ln z], \tag{5}$$

where $\epsilon_0 = 0{\cdot}0842$ GeV is the critical energy of an electron in air, t is the air depth in radiation lengths, E_0 is the primary energy and $z = r/r_0$. The average values at different shower sizes are found in the following way:

$$\bar{S}(\text{theor.}) = \frac{6t}{z_2^2 - z_1^2} \int_{z_1}^{z_2} z\, dz/[t + 2\ln(E_0/\epsilon_0) + 2\ln z]. \tag{6}$$

The average radial ages $\bar{S}$(HL) and $\bar{S}$(NKG), determined from (4) using equations (2) and (3) for the HL and NKG lateral structure functions, are compared with the theoretical average values in Table 2.

Table 2. Comparison of average radial ages with the theoretical average for three shower sizes

N_e	$5{\cdot}3\times10^4$	$1{\cdot}2\times10^5$	$1{\cdot}2\times10^6$
$\bar{S}$(HL)	1·494	1·493	1·733
$\bar{S}$(NKG)	1·703	1·324	1·476
$\bar{S}$(theor.)	1·517	1·434	1·325

(2b) Measurement of Age Parameter from Electron Density Data

The χ^2-minimisation technique using the gradient search method has been used to fit the measured electron densities of individual showers to the chosen interpolation function. The distribution of measured shower age for a sample of 8651 showers in the size range $10^{4\cdot3}$–$10^{6\cdot2}$ is shown in Fig. 2. Experimentally measured shower ages are compared with those of the Moscow group (quoted in Capdevielle and Gawin 1985) in Table 3.

3. Effect of Age Parameter on Lateral Structure

The radial electron densities ρ, measured at various radial points in a group of 893 showers in the size intervals $(5{\cdot}6{\cdot}5)\times10^4$, $(1\text{–}1{\cdot}5)\times10^5$ and $(1\text{–}1{\cdot}5)\times10^6$, and with the age distribution shown in Fig. 2, are presented in Figs 3, 4 and 5. The fixed size showers of ages s differing by $\sim0{\cdot}1$ are distinguishable only in the data at small core distances, as is made evident in these radial electron density distributions.

The observed showers belonging to the age distribution in Fig. 2 are shown in Fig. 6 as a distribution of shower size $\bar{N}_e$ in shower age $\bar{s}$. The error bars represent the standard errors in the mean s. Standard deviations are shown in the same figure. The plot shows that the age of the electron cascade in a shower observed in a vertical direction at sea level decreases with an increase in shower size. The theoretical calculations on this feature given by Capdevielle and Gawin (1982) and the experimental results of the Akeno group (Hara *et al.* 1981) are shown in the same figure.

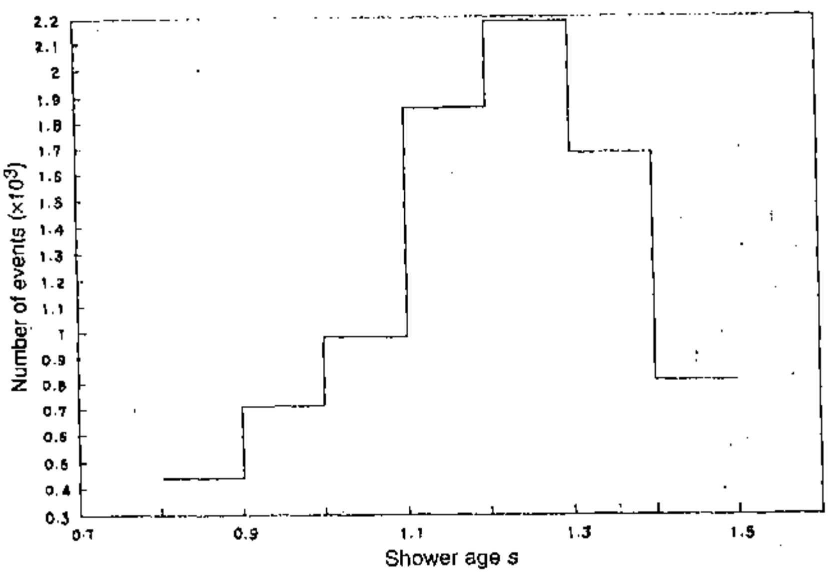

Fig. 2. Distribution of shower age measured by χ^2 minimisation (total 8651 showers).

Table 3. Comparison of present experimental shower ages with those of the Moscow group (quoted in Capdevielle and Gawin 1985)

N_e	$5\cdot3\times10^4$	$1\cdot2\times10^5$	$1\cdot2\times10^6$
Present	1·19	1·10	1·00
Moscow	1·126	1·068	0·924

4. Discussion

The age parameter of cosmic ray extensive air showers has been the subject of further study in recent years. It has been used (Idenden 1990; Cheung and MacKeown 1987; Samorski and Stamm 1983) to distinguish between ultra-high-energy photon-initiated showers and charged cosmic ray particle-initiated showers. Some workers (Hara *et al.* 1983; Sasaki 1971; Capdevielle and Gawin 1982, 1985; Dai *et al.* 1990) have used the radial age parameter in the shower analysis, in addition to the longitudinal age, to describe the longitudinal development of the shower in the atmosphere. In the present work, it has been shown that the average of the radial shower age at different radial distances over the whole shower disk is almost identical to the theoretical average value of the shower age, as given by electron–photon cascade theory. The age value of a particular shower group determined by the χ^2-minimisation technique is dependent on the shower-detecting area and the detector spacing. A comparison is shown in Table 3 for the present work and the Moscow experiment, which had nearly the same detecting area as in the present experiment.

The shower age measured by the χ^2-minimisation technique has been used as a parameter to show the measured radial electron density distributions in Figs 3, 4 and 5 for three shower size groups, each with different age values.

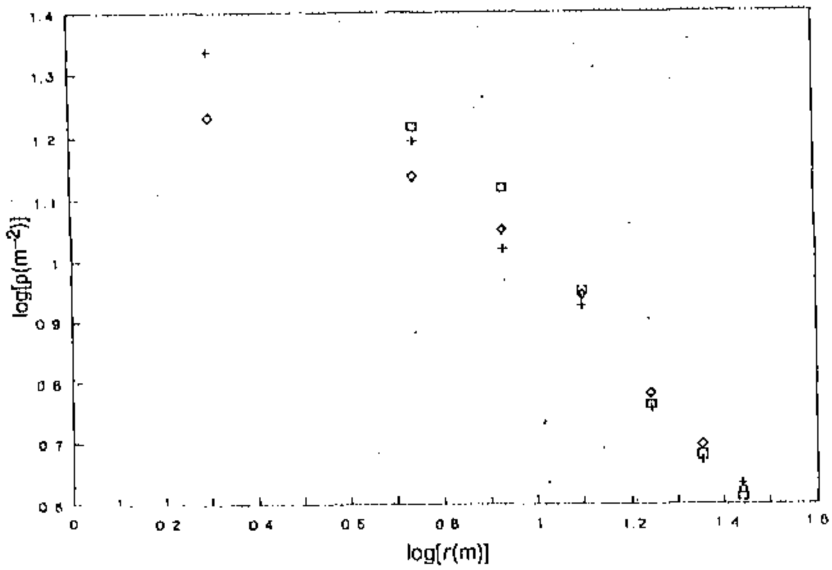

Fig. 3. Radial density distribution of the electron component for N_e in the range (5–5·5)×10^4: □ for $s = 0·99$; + for $s = 1·11$; ◇ for $s = 1·19$.

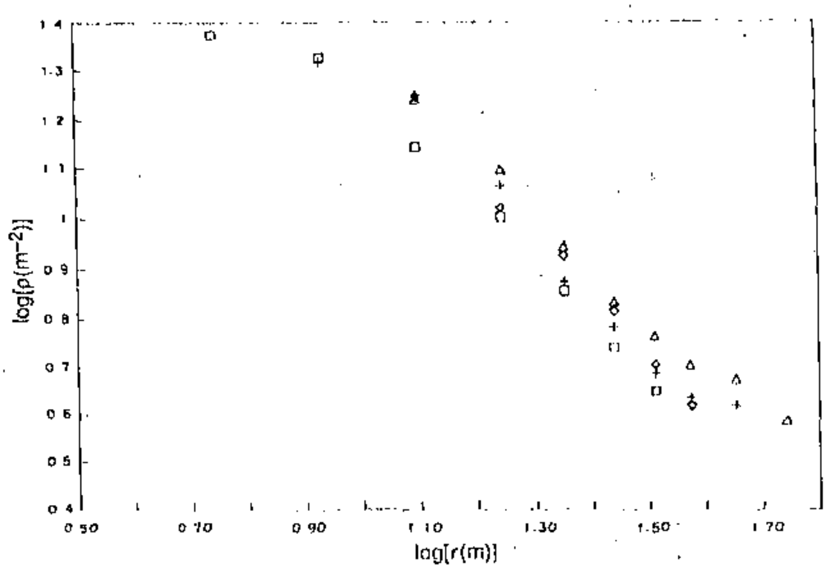

Fig. 4. Radial density distribution of the electron component for N_e in the range (1–1·5)×10^5: □ for $s = 0·89$; + for $s = 1·00$; ◇ for $s = 1·10$; and △ for $s = 1·20$.

These results are in agreement with expectation (Hillas and Lapikens 1977). A reconfirmation of the earlier results on the variation of the shower age measured by the minimisation technique with shower size published by the Akeno group (Hara et al. 1981) and Clay et al. (1981) is also given in the present work for

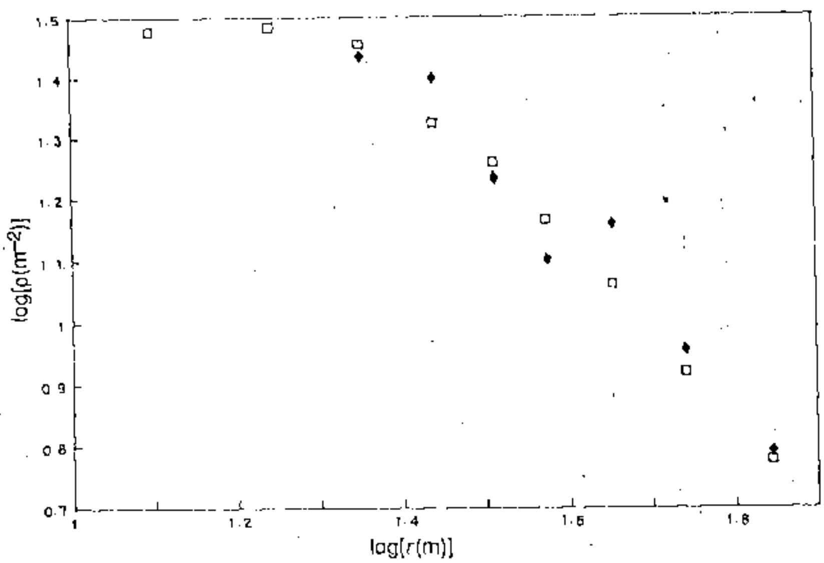

Fig. 5. Radial density distribution of the electron component for N_e in the range $(1-1\cdot5)\times10^6$: □ for $s=1\cdot00$; ◆ for $s=1\cdot07$.

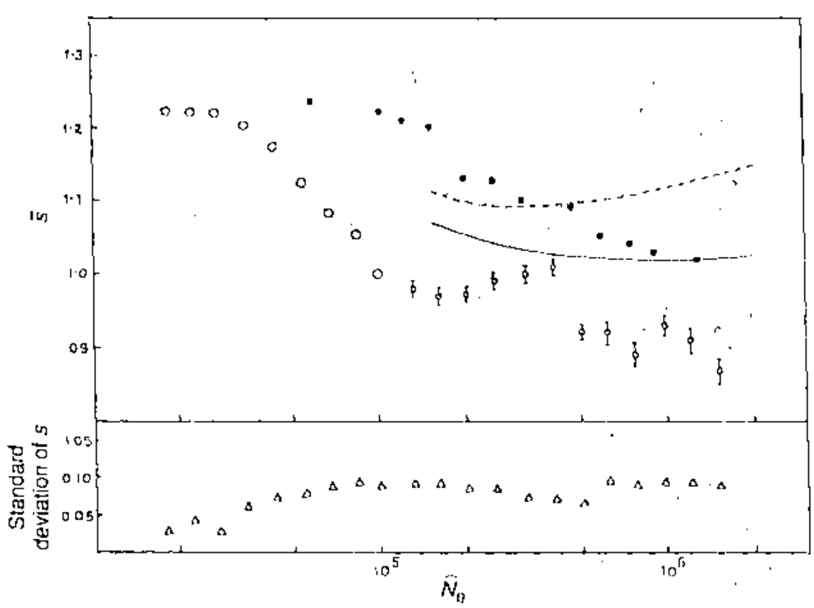

Fig. 6. Distribution of shower size $\bar{N}_e$ with shower age $\bar{s}$: ○, present experiment (sea level); ●, Akeno experiment (900 g cm^{-2}, Hara *et al.* 1981); simulation results (solid curve, scale breaking model; dashed curve, high multiplicity model) of Capdevielle and Gawin (1982). Triangles represent the standard deviation at each point.

vertically incident showers. As can be seen from Fig. 6, a shower of given size developing in the vertical direction over one attenuation length ($\bar{\Lambda} = 112\ \mathrm{g\,cm^{-2}}$ for $N_e \geq 5\times10^5$, Sasaki 1971) increases in age by $\sim$0·07. This result is in good agreement with that measured (0·06/100 $\mathrm{g\,cm^{-2}}$, by Clay *et al.* (1981) and that from the measurements of Hara *et al.* (1981) in the shower size range 10^5–10^6. A similar trend was obtained by Capdevielle and Gawin (1982) for two models, as shown also in Fig. 6. The present results on the variation of shower age with radial distance and with shower size are in accordance with the predictions of the electron–photon cascade theory.

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A New Lateral Distribution Function for Electrons in Extensive Air Showers (EAS) Detected near Sea Level.

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Summary. — A detailed analysis of Extensive Air Showers in the size range 10^4–10^6 particles detected near sea level has yielded a new distribution function for the radial distribution of EAS electrons. The goodness-of-fit criteria applied to the present and already existing similar distribution functions confirm that the present function is appropriate in EAS at radial distances beyond 20 m from the shower axis.

PACS 94.40.My – Cascade studies (*e.g.*, extensive air showers).

1. – Introduction.

There has been a number of recent studies on the lateral structure of Cosmic-Ray Extensive Air Showers (EAS) with a view to distinguishing between Primary-Cosmic-Ray (PCR) protons or nuclei-initiated EAS and ultra-high-energy cosmic gamma-ray photon-initiated EAS. In both kinds of EAS a photon-electron cascade develops together with a nucleon cascade longitudinally from the atmospheric depths to which the initiating particles penetrate to make their first nuclear collisions. A photon-electron cascade in EAS with radial symmetry is described laterally at a distance r from the EAS axis by expressing the shower particle density $\Delta(r)$ by

$$\Delta(r) = \frac{N}{r_0^2} f(r/r_0, s), \tag{1}$$

with the shower particle density defined as

$$\Delta(r) = \frac{\Delta N}{(r_0^2)\, 2\pi(r/r_0)\, d(r/r_0)},$$

where N is the total number of particles (size) in EAS; $f(r/r_0, s)$ the lateral structure function of the EAS; s the age of the electron-photon cascade in the EAS and r_0 the unit of distance chosen for measuring the radial distance of any point in EAS from the EAS axis.

An exact form of the function $f(r/r_0, s)$ is necessary to determine the EAS parameters (shower axis location coordinates (x_0, y_0), shower size N, and shower age s) from a number of measured densities $\Delta(r)$ at various radial distances r from the EAS axis.

A critical analysis of several forms of $f(r/r_0, s)$ used in EAS work in the last four decades was given by Basak *et al.* [1]. The form of $f(r/r_0, s)$ referred to as NKG distribution function was first introduced by Greisen [2] to represent the theoretical results of Nishimura and Kamata [3]. The various forms of $f(r/r_0, s)$ in use [4] are the NKG form and the different modifications [5-13] of the NKG form to take care of the discrepancies with measurement of $\Delta(r)$ *vs.* r observed over a wide range of r.

The purpose of the present paper is to determine, from the experimentally observed lateral particle density distribution, the radial ranges in which three extensively used forms of $f(r/r_0, s)$ are valid on the basis of rigorous «goodness of fit» criterion. A new form for $f(r/r_0, s)$ has also been derived from such analysis of the observed sea level EAS data on $\Delta(r)$ in individual EAS.

2. – EAS data collection and method of analysis.

A closely packed well-defined EAS array operating near sea level at the North Bengal University (26°45′ N) has 35 unshielded scintillation detectors to measure shower particle density $\Delta(r)$ in individual EAS. Eight (8) of these detectors are used to measure relative time delays between their output pulses to determine the arrival directions of the detected EAS. The angular accuracy in direction measurement is within 2° and the error in the EAS axis location is about 1 m.

The shower parameters are determined by fitting a chosen function $f(r/r_0, s)$ to the observed radial distribution of the densities $\Delta(r)$ by minimizing with respect to each of the shower parameters simultaneously the entity defined as

$$\chi^2(x_0, y_0, N, s) = \sum_{i=1}^{n} W_i(\Delta_i^o - \Delta_i^e)^2 . \tag{2}$$

Here Δ_i^o, Δ_i^e are the observed and expected particle densities at the i-th detector in the EAS array and the weight factor W_i of the i-th density data point is the inverse of the variance of the i-th point density Δ_i^o. If the fitting function $f(r/r_0, s)$ is chosen appropriately to predict densities $\Delta_i^e(r)$, a good fit of $f(r/r_0, s)$ to the observed densities $\Delta_i^o(r)$ can be obtained by minimizing χ^2 and hence the constants and parameters in the fitting function can be determined. The number of data points in an EAS is denoted by n.

The method of searching for the minimum value of $\chi^2(x_0, y_0, N, s)$ with respect to each of the EAS parameters simultaneously is the gradient search method in the direction of steepest descent. For fitting the observed density $\Delta^o(r)$ *vs.* r distribution in individual EAS, three forms of the function $f(r/r_0, s)$ have been tried. These are: NKG function $f_{\rm NKG}(r/r_0, s)$ [2], Hillas function $f_{\rm H}(r/r_0, s)$ [14] and Capdevielle function $f_{\rm C}(r/r_0, s)$ [15]. The steepest-descent iterative process of minimizing χ^2 was done by the gradient search method and when the minimum of the χ^2-hypersurface was attained the gradient was reduced to one-half of its former value. This procedure was repeated until the χ^2-value between two successive steps was close enough to a preassigned value.

3. – Results.

Results of a sample of some five thousand recorded EAS events with more than 50% detectors registering particle densities have been analysed shower-size-wise by the standard χ^2 minimization procedure discussed above in sect. **2**. The mean of the minimum of the χ^2-values represents the goodness of fit of the observed density distribution of particles in EAS of given size to the fitting function chosen to describe the data.

3'1. *Least-square fitting to the observed density* $\Delta^{e}(r)$ *data using* NKG *function* $f_{NKG}(r/r_0, s)$ *for* $\Delta^{e}(r)$. – The shower parameters N and s determined on the basis of $f_{NKG}(r/r_0, s)$ (eq. (3)) are given in table I.

$$\Delta^{e}(r) = \frac{N}{r_0^2} f_{NKG}(r/r_0, s) = \frac{N}{r_0^2}\left[C(s)\left(\frac{r}{r_0}\right)^{s-2}\left(1+\frac{r}{r_0}\right)^{s-4.5}\right], \tag{3}$$

where the photon-electron cascade parameter s is a measure of the development of EAS down to the depth of observation in the atmosphere. Theoretically this shower age parameter s is a function of depth t (measured in radiation unit), the energy of the initiating particle and the radial range r of an EAS. $C(s)$ is the normalization constant to be determined by the fitting procedure, $r_0 = 79$ m (Moliere unit of displacement at sea level).

The observed probability distribution P_χ corresponding to ν degrees of freedom for the reduced chi square $\chi_\nu^2 (= \chi^2/\nu)$ for a given shower size over the whole radial range is shown in fig. 1 and for the range 0–20 m in fig. 2.

3'2. *Least-square fitting to the density* $\Delta^{e}(r)$ *data using Hillas function* $f_H(r/r_0, s)$. – The observed probability distribution for the reduced χ_ν^2 for the same shower size is given in fig. 3 and fig. 4 when $f_H(r/r_0, s)$ (derived from Monte Carlo simulation data) given below (eq. (4)) was used for fitting the EAS density data

$$\Delta^{e}(r) = \frac{N}{r_0^2} f_H(r/r_0, s) = \frac{N}{r_0^2}\left[C(s)\left(\frac{r}{r_0}\right)^{a_1+a_2(s-1)}\left(1+\frac{r}{r_0}\right)^{b_1+b_2(s-1)}\right], \tag{4}$$

where the constants r_0, a_1, a_2, b_1, b_2 are the fitting parameters.

TABLE I.

Range of estimated shower size N (No. of particles)	Radial range of density $\Delta^{e}(r)$ (measurement in metres)	Range of best-fitting values of s
$(1-5)\cdot 10^4$	0–120	0.90–1.35
$(5-9)\cdot 10^4$	0–120	0.90–1.35
$(1-5)\cdot 10^5$	0–120	0.90–1.35

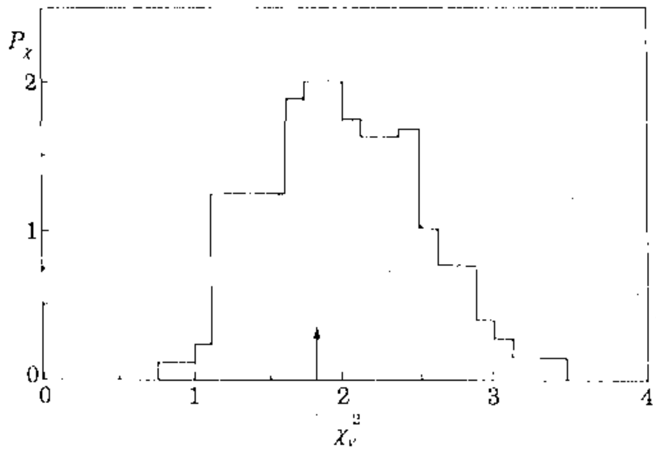

Fig. 1. – The observed probability distribution P_χ for the reduced chi square χ_ν^2 using the NKG function in the radial range 0–120 m.

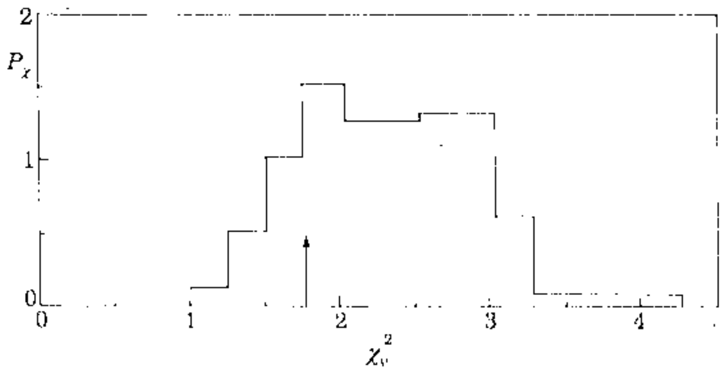

Fig. 2. – Same as in fig. 1, but for the radial range 0–20 m.

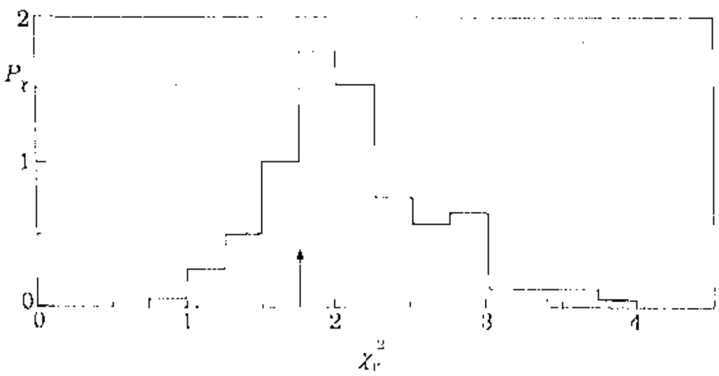

Fig. 3. – Same as in fig. 1, but using Hillas function.

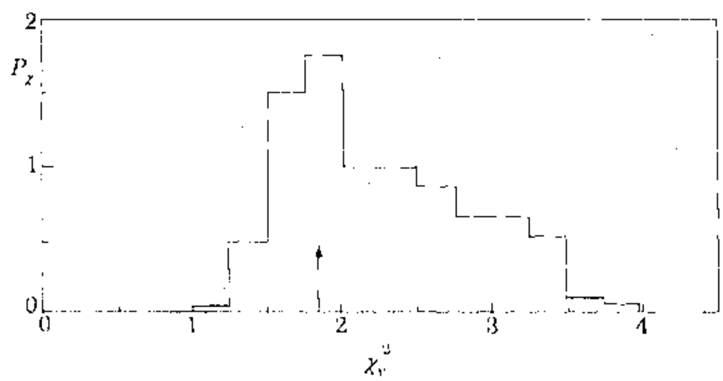

Fig. 4. – Same as in fig. 1, but using Hillas function in the radial range 0–20 m.

3·3. *Least-square fitting to the density* $\Delta^e(r)$ *data using* $f_C(r/r_0, s)$. – Capdevielle *et al.* [15] assumed that the shower age parameter in the fitting function should be the «effective age» for radial development of shower and defined it as

$$s(r) = \alpha \log \beta(r/r_0) + s_l\,, \qquad \text{for} \qquad r \leqslant 150\ \text{m} \tag{5}$$

where s_l is the longitudinal age parameter at the level of observation and α, β, s_l are constants at sea level for a given shower size.

The observed probability distribution for the reduced χ_ν^2 for the same shower size using the $f_C(r/r_0, s)$ (eq. (6)) for fitting the density data is given in fig. 5 and fig. 6.

$$\Delta^e(r) = \frac{N}{r_0^2} f_C(r/r_0, s) = \frac{N}{r_0^2}\left[C(s)\left(\frac{r}{r_0}\right)^{s(r)-2}\left(1+\frac{r}{r_0}\right)^{s(r)-4.5}\right], \tag{6}$$

where $r_0 = 79$ m (Moliere unit of displacement at sea level). The summary of χ_ν^2 results from the distribution in fig. 2 to 6 and similar such other distributions (not shown) are given in table II and III.

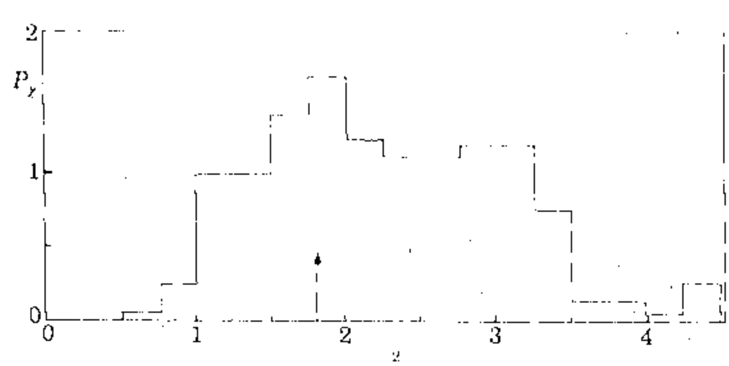

Fig. 5. – Same as in fig. 1, but using Capdevielle function.

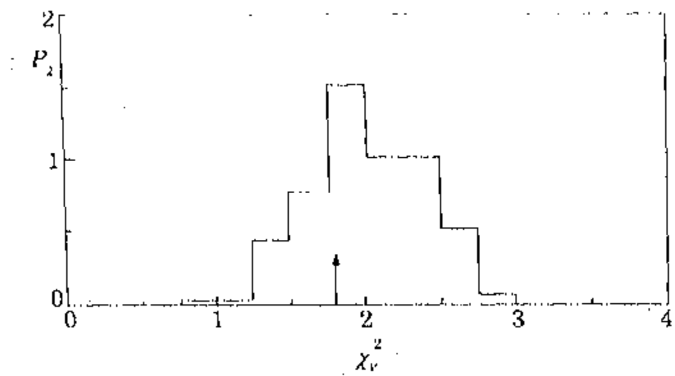

Fig. 6. – Same as in fig. 1, but using Capdevielle function in the radial range 0–20 m.

TABLE II. – *Mean values of the reduced* χ_ν^2 *from fig.* 1, 3 *and* 5 *for the distribution functions* (*for* EAS *radial range* 0–120 m).

$f_{NKG}(r/r_0, s)$	$f_H(r/r_0, s)$	$f_C(r/r_0, s)$	Proposed $f^{\prime}(r/r_0, s)$
1.81	1.77	1.80	1.72

TABLE III. – *Mean values of the reduced* χ_ν^2 *in different radial ranges in* EAS *for distribution functions.*

EAS radial ranges in metres	$f_{NKG}(r/r_0, s)$	$f_H(r/r_0, s)$	$f_C(r/r_0, s)$	Proposed function (eq. (7))
0–20	1.77	1.82	1.80	1.83
20–80	1.79	1.77	1.76	1.77
80–120	1.83	1.55	1.82	1.45

4. – Proposed radial distribution function.

The present shower data have also been analysed by using a new distribution function $f^{\prime}(r/r_0, s)$ (proposed) (eq. (7)) which incorporates two features:

1) the dependence of radial shower age on radial distance and

2) the unit of distance r_0 is taken as the parameter of the fitting function instead of choosing for it a constant value of 79 m (Moliere unit of displacement at sea level).

$$(7)\quad \Delta^{e}(r) = \frac{N}{r_0^2} f^{\prime}(r/r_0, s) = \frac{N}{r_0^2}\left[C(s)\left(\frac{r}{r_0}\right)^{0.58+1.54(s(r)-1)}\left(1+\frac{r}{r_0}\right)^{-3.39+0.01(s(r)-1)}\right],$$

where $s(r) = \alpha \ln \beta(r/r_0) + s_1$, for $r \leqslant 150$ m. Here r_0, α, β and s_1 are the fitting parameters.

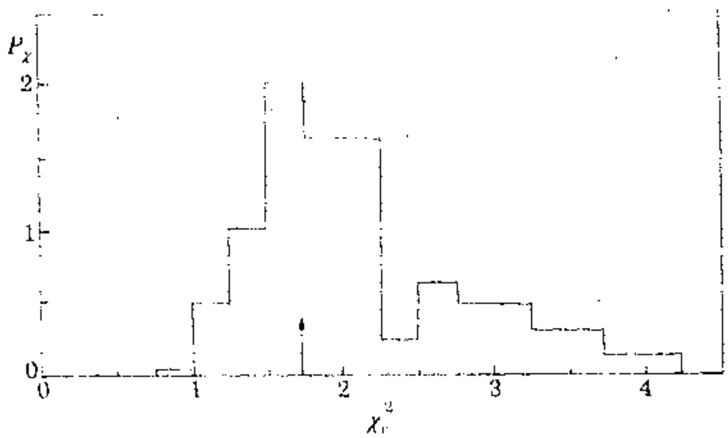

Fig. 7. – Same as in fig. 1, but using the proposed function.

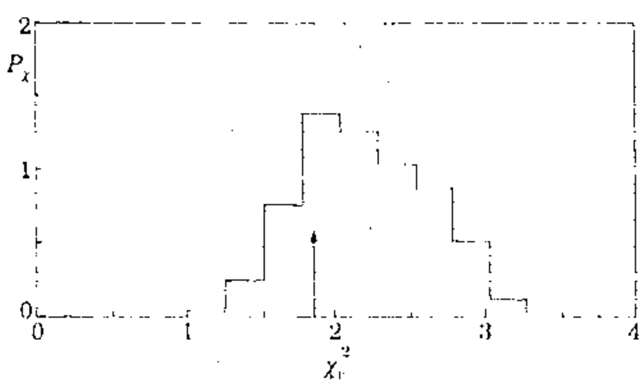

Fig. 8. – Same as in fig. 1, but using the proposed function in the radial range 0–20 *m*.

Some results for the probability distribution for χ^2_ν using eq. (7) are shown in fig. 7 and fig. 8. The mean values of χ^2_ν for the same shower size in the whole region and in three different radial ranges 0–20 m, 20–80 m, 80–120 m are given in table II and III, respectively.

5. – Discussion and conclusion.

It is necessary to touch upon a few points in connection with the present air shower measurements and analysis. The transition effect arising from multiplication or absorption (absorption is predominant over multiplication) of shower particles in the finite thickness of a plastic scintillator in a density detector of the EAS array was taken into account by correcting the observed density in the manner discussed previously by Basak *et al.* [1, 16] and Asakimori *et al.* [17, 18]. The shape of the average lateral distribution function and the value of the local shower age parameter *s* measured [1, 16-18] by using thin plastic scintillators is not much dependent on the transition effect near cores of showers in the size range $\sim 10^6$ particles.

The cores of EAS striking the detecting points within the well-defined periphery of the EAS array were located by fitting the measured particle densities registered at the struck detectors to eq. (3) for interpolation of the measured density readings. It has been checked that shower cores thus located are insensitive to the interpolation function chosen. With a close-packed (small detector spacing) well-defined detector array as in the present experiment, the uncertainty in the shower core location from the measured density readings is expected to be minimum compared to what is expected from a detector arrangement with large spacings.

The weighting factor W_i in eq. (2) is the inverse of the variance σ_i^2 (which describes the uncertainty of the i-th data point evaluated by assuming Poisson distribution). Consequently the fits obtained with different lateral distribution function (l.d.fs) will not depend on the detector spacing of an array with well-defined perimeter, the weighting factor W_i and the shower core location procedure.

To obtain a fit of the measured density data of a recorded EAS event to a fitting function (l.d.f) with several parameters, the gradient search method of least squares was used for determining simultaneously the optimum values of the parameters which give a minimum to the function χ^2 (eq. (2)) defined with that l.d.f. This

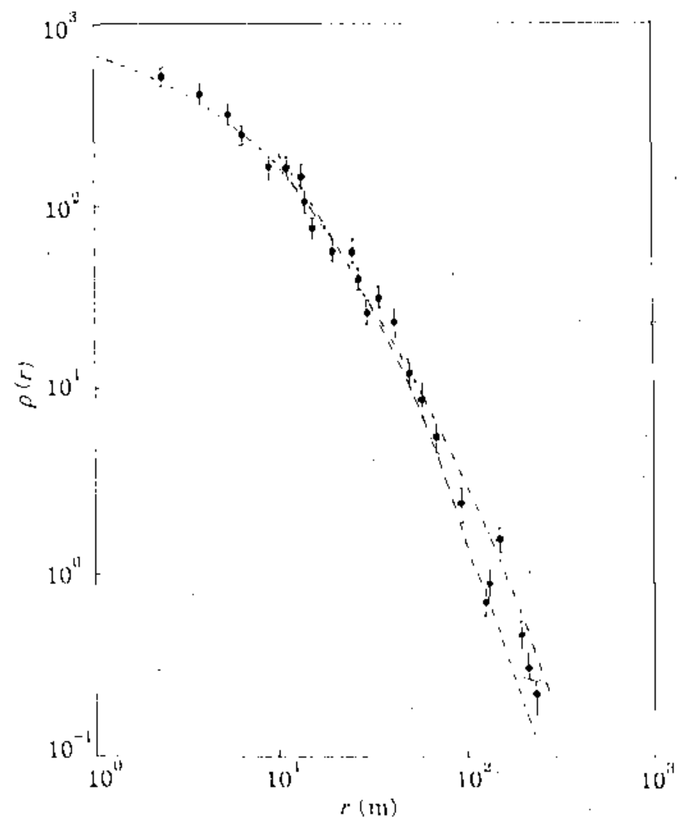

Fig. 9. - Observed lateral electron density distribution along with the theoretical distributions using Hillas (dashed line), Capdevielle (dot-dashed line) and the proposed function (dotted line) in the radial range 0–120 m. $N = 6 \cdot 10^6$, $s = 1.25$.

procedure is expected to obtain a best fit for the multiparameter l.d.f to a large number of density readings in a registered EAS event from a large sample of EAS of fixed size N. The probability distribution for χ^2 as well as the χ^2-values found in the present work by the gradient search method are larger than the expected values due to large intrinsic fluctuations from shower to shower and random sample of density data of an individual EAS with small values of «sample standard deviation σ_i». The error in attaining χ^2-minima by the gradient search with steepest-descent iterative procedure adopted in the present work may contribute to the size of χ^2-near minima.

The results of the fit with different l.d.fs are shown in fig. 9 to indicate the extent to which the present experimental data could discriminate them.

It is seen from table II that $f_H(r/r_0, s)$ among the three distributions (f_{NKG}, f_H, f_C) considered above is the best fit to the observed density distribution in the whole 0–120 m radial range of EAS. The results of the reduced χ^2_ν test (table III) for the three successive smaller radial ranges in an EAS of the same size as that used in table II show a varying degree of goodness of fit of the observed data to the same fitting function. Whereas $f_{NKG}(r/r_0, s)$ represents a good fit to 0–20 m radial range, in the 20–80 m radial range $f_C(r/r_0, s)$ shows a slight improvement over $f_H(r/r_0, s)$ which is best in the 80–120 m range. None of these three functional forms can give a reasonably good fit to the observed data in the whole range 0–120 m.

The choice of $s(r)$ variation with r and the choice of r_0 as an adjustable parameter of the fitting function in place of Moliere unit of displacement at sea level are adjusted in the radial range 20–80 m and 80–120 m. In the 80–120 m range the function under eq. (7) gives the better fit to the observed data ($\chi^2_\nu = 1.45$) than the fits obtained by other functions (eq. (1) to (3)). The result of the χ^2_ν-test (table II) made over the whole shower range 0–120 m shows that the proposed function (eq. (7)) is better than any other fitting functions considered above.

The measured age parameter s (table I) of an EAS of given size is the mean of the s-values obtained at the χ^2 minima using eq. (3) or eq. (4) as the fitting function. The relation between the values of s and s_l values obtained by the analysis using eq. (7) as the fitting function is

$$s_l = s + s_e\,, \qquad \text{for} \qquad r \leq 120\ \text{m}\,, \tag{8}$$

with $s_e = 0.15$–0.3.

This experimental relation has been obtained from the analysis over the radial range 0–120 m in muon-rich normal EAS initiated presumably by PCR protons or nuclei. However this form is applicable to recorded EAS events of similar shower size having arrival directions from specific stellar point sources of ultra-high-energy gamma-ray photons. From the measured s-values of such EAS events, one can determine s_l values from relation (8). This determination together with a determination of muon size and hadron size simultaneously in such EAS may unambiguously identify such events as ultra-high-energy gamma-ray photon-initiated events. This relation gives a comparison of the performance of the proposed function (eq. (7)) with the NKG function or Hillas function in terms of the measured longitudinal age parameter s_l and the measured s which represents the shower age parameter of a given shower size that one obtains from a best fit of the data with $f_{NKG}(r/r_0, s)$ or $f_H(r/r_0, s)$.

Recently the lateral distributions of particles in simulated EAS have also been studied by using cosmic atomic nuclei and cosmic gamma-ray photons [14, 19, 20]. In the simulation work of Mikocki *et al.*, the lateral distribution of particles in EAS of size range 10^5–10^6 particles at sea level was studied using NKG formula (eq. (3)) as the fitting function. Their χ_ν^2 test for the simulation over the radial range 0–100 m yielded, at the minimum value of $\chi_\nu^2 (= 6)$ and the fitting parameters, $r_0 = 41$ m and $\bar{s} = 1.29$–1.35. These results show that $f_{NKG}(r/r_0, s)$ is not an appropriate fitting function for the observed $\Delta(r)$ distribution over a wide radial range in EAS.

In conclusion it may be stated that the proposed function for radial particle density distribution in EAS can be applied to analyse small- and medium-size EAS of the radial range extending to 120 m at least. It can be also used to derive the longitudinal age parameter s_l from the measured shower age parameters $\bar{s}$.

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Studies on the Lateral Distribution of the Soft Component in the EAS

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ABSTRACT

In the NKG formula two defects have been identified. Firstly it is found that the age parameter increases with distance & secondly Moliere radius is somewhat less than what was expected. Different modifications were proposed by various authors by considering one of the above defects. This paper takes into consideration both the defects simultaneously & claims to obtain better results.

1. INTRODUCTION

In the studies of Cosmic Rays, the most used lateral distribution function for the soft component of the EAS, is the NKG formula which is given by

$$\rho(r)=c(s)(N/r_m^2)(r/r_m)^{(s-2)}(1+r/r_m)^{(s-4.5)} \quad (1)$$

Here the Moliere radius r_m =80 m. However by using the Monte Carlo method Allan et al (1975) first noted that the widths of the showers are very much less than that given by NKG formula. The results of Messel & Crawford (1970) were quite wrong. Hillas et al (1977) carried on Monte Carlo calculations by using several new factors which were not used in earlier calculations. They found that the spread of the shower is narrower & the lateral distribution of electrons fit the following formula.

$$\rho(r)=c(s)N/r_h^2(r/r_h)^{a+a(s-1)}(1+r/r_h)^{b+b(s-1)} \quad (2)$$

Here the radius of the disc r_h=24m i.e., nearly 1/4 of Moliere radius. Such effects were also observed by Lagutin et al (1979). Hillas (1981) cited papers to claim that his simulation results agree well with the calculations of the Lagutin.

Some other authors, like Linsley J et al (1962), have taken into consideration the effect of the Zenith angle. Another line of thinking arose by Miyake et al (1968) and Kristiansen (1971). They proposed that a single age parameter is insufficient to describe the lateral distribution of electrons.

Linsley (1973), Aguirre (1973) & Porter (1973) have been noticed that the age parameter increases with distance and it was confirmed by Kristiansen et al (1975) and Kawaguchi (1975). Capdevielle et al (1977) have considered this point in detail. By simulation and from the experimental results of the Tien-Shan experiment he concluded that the age parameter varies with the distance according to the formula

$$s(r)=\alpha\ln\beta(r/r_o)+s_t \qquad 150m>r>15m \qquad (3)$$

and the lateral distribution function becomes

$$\rho(r)=c(s)N/r_m^2(r/r_m)^{s(r)-2}(1+r/r_m)^{s(r)-4.5} \qquad (4)$$

2. PROPOSED LATERAL DISTRIBUTION FUNCTION

It is found that NKG function still remain as basis of all other distribution functions. Others only modified it. In the present work all the three curves viz. the NKG, the Hillas and the Capdevielle are fitted with the shower data available in the NBU Cosmic Ray Research Centre. It is found that Hillas function fitted better than NKG at a large distance from the core of the axis of the shower where as the Capdevielle distribution fitted better for a smaller distance. So there are some discrepancies in the NKG function. It can be argued that if the discrepancy is due to the effect of the Moliere radius then the Hillas distribution would be appropriate. On the other hand if the variation of the age parameter with distance is responsible for the discrepancy then the Capdevielle function would be appropriate. Moreover the Hillas distribution makes the curve steeper in the regions far away from the axis and the Capdevielle distribution makes it steeper near the axis. Hence there would be no ambiguity in finding out the factors responsible for the departure from the NKG function.

Hence a new formula is proposed which includes both the above features and is given by

$$\rho(r)=c(s)N/r_o^2(r/r_o)^{a_1+a_2(s(r)-1)}(1+r/r_o)^{b_1+b_2(s(r)-1)}$$

and $s(r)=\alpha\ln\beta(r/r_o)+s_t$

where $a_1=-0.5, a_2=1.54, b_1=-3.39, b_2=0$ and $r_o=30$ m & N, r, α, β & s_t are the parameter to be fitted

3 RESULTS

The lateral distribution of electrons for different distribution functions and the variation of age parameter with distance, obtained from the data available in the NBU Cosmic Ray Research Centre are displayed in Fig 1 and Fig 2 respectively.

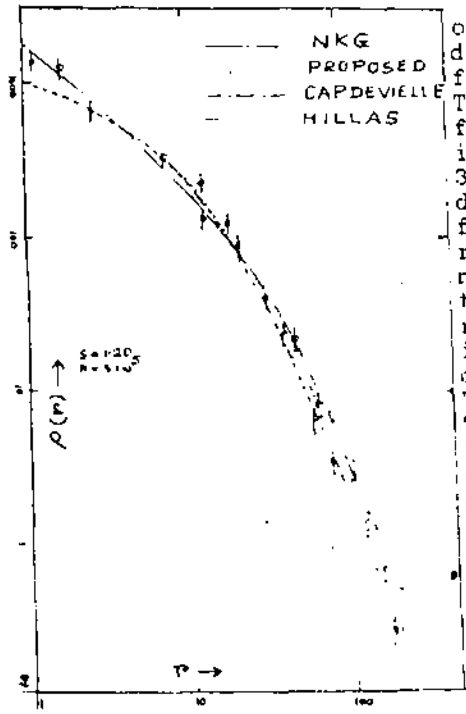

Fig.1: Lateral distribution of electrons.

The average values of χ^2-distribution for different distribution functions are given in Table 1 and the same for the Proposed function is displayed in Fig 3. To find out which distribution function fits better in which region, the whole distance is devided into three segments and the reduced χ^2 distribution is also calculated for different distributions. The results is given in Table 2. Thus it is found that NKG function, Capdevielle function and the Proposed function fits better than other functions in the (0-20)m, (20-80)m and (80-120)m range respectively. In the (20-120)m region the Proposed function fits better than other functions. However in the (0-20)m region the Proposed function does not fit well.

TABLE 1

Function	NKG	HILLAS	CAP	PROPOSED
χ^2	1.81	1.77	1.80	1.72

TABLE 2
Values of chi-square

DISTANCE	(0-20)m	(20-80)m	(80-120)m
NKG	1.77	1.79	1.83
HILLAS	1.82	1.77	1.55
CAP	1.80	1.76	1.82
PROPOSED	1.83	1.77	1.45

CONCLUTION

It can be easily seen that the effect of Moliere radius is felt in the (80-120)m region and the reduction is well justfied. In the (20-80)m

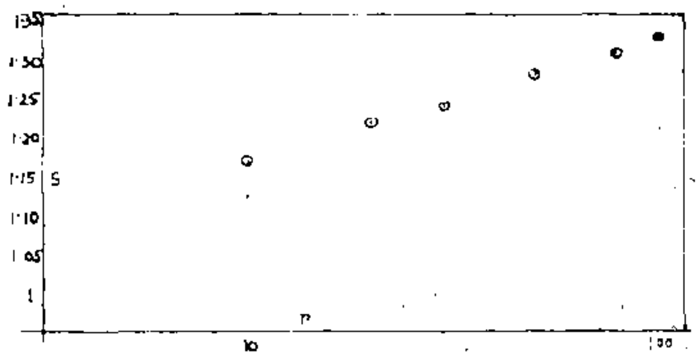

Fig.2 : Variation of Age parameter.

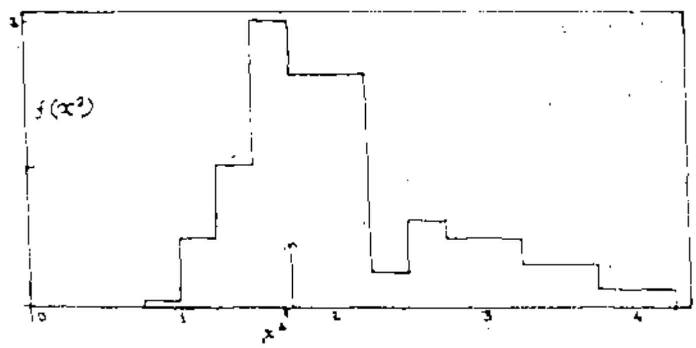

Fig.3 : χ^2 -distribution.

region the shower age parameters variation is most prominent. The proposed form of shower age variation is not applicable in the (0-20)m region and hence the failure of the distribution function in this region takes place.

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Measurement of the Charge Ratio of High Energy Muons in Cosmic Ray Extensive Air Shower(EAS).

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ABSTRACT

New results of measurements on the charge ratio of high energy muons in cosmic ray extensive air shower (EAS) are presented. A comparison with some similar results is given for drawing a conclusion.

1. INTRODUCTION

Measurement of muons in cosmic ray EAS studies provide information not only of the primary composition but also on the composition of hadrons in the nuclear active cascade in an EAS. At an average primary cosmic ray energy per nucleon of about ten thousand GeV producing an air shower of size about $\sim 10^4$ particles at sea level, the no of high energy muons, say above 50 GeV, arise from the decay of hadrons(eg, pions, kaons & hyperons) of a number of successive generations forming the nuclear active cascade. Measuring the charge ratio of these muons as a function of their energy one can infer the composition and the relative contribution of pion, kaon and hyperon decay to muons. The first measurement of the charge ratio of air shower muons at energy around 10 GeV is that of Bennett (review article by K.Greisen, Ann.Revs.Nuclear Science 10, 1960). The result for the charge ratio of this measurement is nearly 1.0 indicating that at low energy charge symmetric pion production is predominent.

At higher energies of hadrons, the hadron-nucleus collisions in the nuclear cascade, the kaon and hyperon production is expected to contribute through decay process to muons.

In the present work, measurement on high energy muons in EAS of size between $\sim 10^4$ and $\sim 10^6$ particles, have been carried out by using two solid iron magnets of MDM 500 GeV/c. Low and high energy muons in wide energy range have been recorded simultaneously and the momentum as well as the sign of the charge on each muon determined.

2. EXPERIMENT

Data collection & analysis : An array of 21 closely packed plastic scintillation detectors with spacing of about 8 m at a site near sea level detects incident air shower of size range 10^4 to 10^6 particles over an estimated core distance range of about ~40 m. The shower trajectory and the size of a recorded shower have been determined from the measured time delays and the distribution of radial electron densities. Two shielded magnetic spectrographs installed near the centre of the array determined the trajectories through the spectrographs of the muons in an incident shower. The trajectories of both low and high energy muons in the range 2.5 to 500 GeV have been recorded simultaneously.

3. RESULTS

some two thousand six hundred muons in the recorded air showers in the size range $\sim 10^4$ to $\sim 10^6$ particles have been so far recorded. An analysis has been made to obtain the radial distribution, the energy spectrum at certain radial distances and the charge ratio of muons. Some representative examples of muon radial distribution and energy spectrum are shown in fig.1 & 2. The present results on the charge ratio with momentum are shown graphically in fig.3. It is found that the muon charge ratio has a value close to 1 upto the muon energy of about 20 GeV. Above this energy, the charge ratio increases to about 1.25 around the muon energy of about 450 GeV.

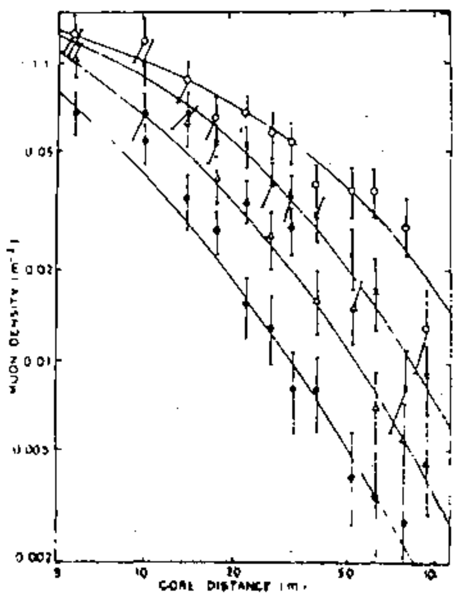

fig. 1: Radial distribution of muon at different muon threshold energies : 2.5 GeV. (O), 11.3 GeV. (X), 25.5 GeV. (Δ), 53.7 GeV. (●) for shower of size Ne ~ 1.5 X 10^4

4. DISCUSSION

In fig.4 a comparison is given of the present results with similar results of few earlier measurements on the muon charge ratio. The most of the data points above

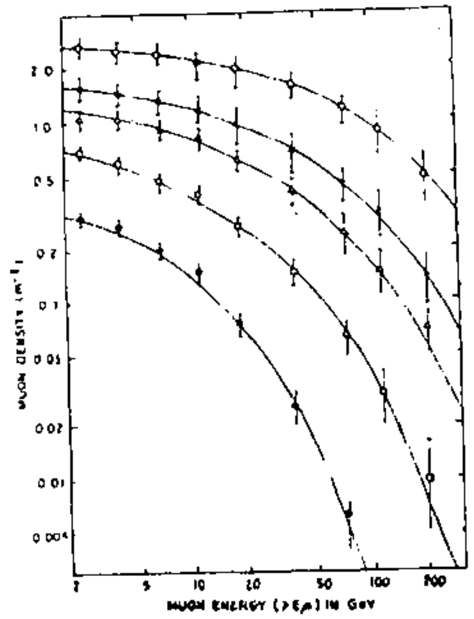

Fig.2 Muon energy spectrum at different radial distances ;05.7 m (O) , 13.6 m (X) , 21.2 m (▲) , 42.4 m (□) , 89.5 m (●) ,

about muon energy of 25 GeV, except those of Hawkes et al, show a trend for the muon charge ratio rising with increasing energy. It means that at such high energies the hadron-nucleus collisions in the hadronic cascade of EAS produce charge symmetric

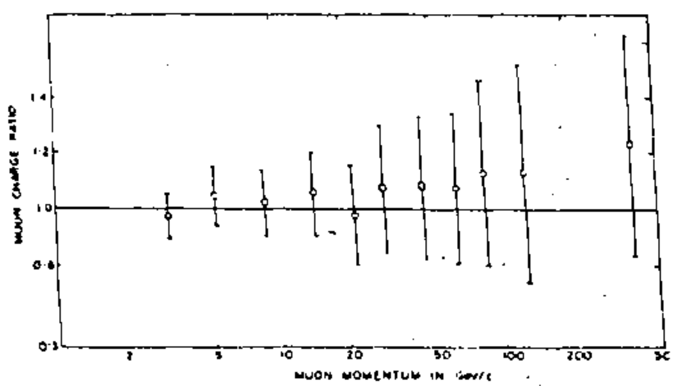

Fig.3: Dependence of charge ratio on muon momentum for the shower size range ~ 10^4 to 10^6 particles.

pions as well as heavier mesons and hyperons which generate through decay excess of positive muons. At such high energies the positive charge excess is similar to that for single muons (unassociated) measured recently by Basini et al (1991).

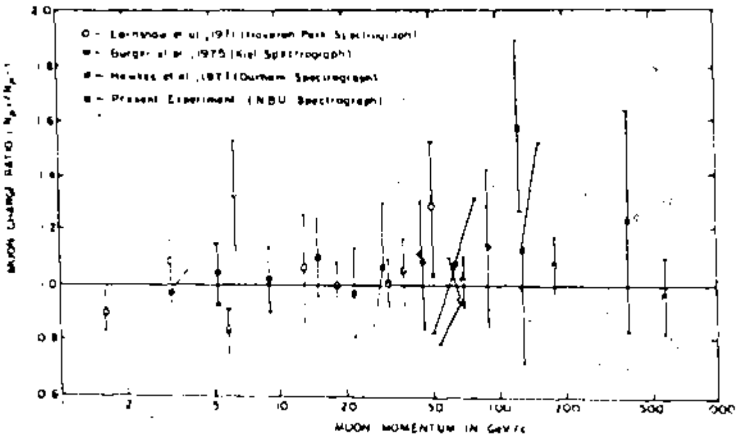

Fig.4: A comparison of the present result, showing the dependence of muon charge ratio on momentum, with the other results.

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A Study on the Cosmic Ray EAS Age Parameter

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ABSTRACT

An analysis of 8651 air showers in the size range $10^{4.3}-10^{6.2}$ particles shows that (i)for a particular shower the average of the radial shower ages over the whole shower disk is nearly identical with that of the theoretical average value,(ii)the age of a shower of given size developing in the vertical direction increases by~0.07 over 1 attenuation length.

1. INTRODUCTION

In most of the earlier experiments (Abdullah et al.1981,1983; Hara et al.1981,1983;Idenden.1990) the air shower age (s) is determined by standard least square fitting of electron density informations at various radial points of extensive air shower(EAS) front. The age(s) values measured by this method differ from the theoretical values at all atmospheric depths. Some workers(Sasaki.1971; Capdevielle & Gawin.1982,1985; Hara et al.1983; Dai et al.1990) suggested that a shower has to be described by two age parameters, one for its longitudinal development and the other for its lateral development. The present work is concerned with an analysis of shower age in the size range $10^{4.3}-10^{6.2}$ particles.

2. EXPERIMENT

The EAS array consists of 21 electron density sampling plastic scintillation detectors at radial distance intervals of about 8m, 8 fast timing detectors and 2 magnet spectrographs covering an area of 1176 m^2. The radial electron density distributions and muon density distributions are measured simultaneously over a radial distance from the array centre upto about 30 m and muon energy in the range 2.5-220 GeV. EAS direction has been determined by measuring relative arrival times and shower size(Ne) , age(s) and core location(x_0,y_0) determination was carried out by chi square minimization of the radial electron density data using gradient search method of an EAS event to an interpolating lateral structure function as given by Hillas & Lapikens(1977).

3. DATA ANALYSIS & RESULTS

From Hillas-Lapikens (HL) structure function we obtain for the radial age, $S_{ij}(r)$ at radial location r_i-r_j the following formula,

$$S_{ij}(r)=\ln(F_{ij}X_{ij}^{2.07}Y_{ij}^{3.39})/1.54\ \ln X_{ij} \quad (1)$$

With NKG function (Greisen, 1960) the formula becomes,

$$S_{ij}(r)=\ln(F_{ij}X_{ij}^{2}Y_{ij}^{4.5})/\ln(X_{ij}Y_{ij}), \quad (2)$$

where $F_{ij}(r)=f(r_i)/f(r_j)$, $X_{ij}=r_i/r_j$,

$Y_{ij}=(1+x_i)/(1+x_j)$ with $x=r/r_0$.

The average of the radial ages over the whole shower disk from HL[S(HL)] and from NKG[S(NKG)] lateral structure functions are compared with the theoretical average values[S(theo.)] for different shower sizes(Ne) in table 1 below :

TABLE 1.

Ne	$5.3.10^4$	$1.2.10^5$	$1.2.10^6$
S(HL)	1.512	1.612	1.77
S(NKG)	1.665	1.422	1.503
S(theo)	1.517	1.434	1.325

The distribution of shower size(Ne) in the shower age(s) is shown in fig.1. The plot shows that the age(s) of the electron cascade in a shower observed in vertical direction at sea level has structure with a decreasing behaviour with increase in size.

4. DISCUSSION

The age parameter of cosmic ray EAS has been a subject of further study because it has been used (Samorski & Stamm, 1983; Cheung & MacKeown, 1987; Idenden, 1990) to distinguish between ultrahigh energy photon initiated EAS and charged cosmic ray particle initiated showers. Some workers (Sasaki, 1971; Capdevielle & Gawin, 1982, 1985; Hara et al., 1983) have used in the shower analysis the radial age parameter in addition to longitudinal age to describe longitudinal development of the shower in the atmosphere. In the present work it has been shown that the average value of the radial shower ages over the whole shower disk is nearly identical with the theoretical average value of the shower age as given by the electron-photon cascade theory.

The variation of the measured shower age (s) by chi-square minimization technique with Ne for vertically incident showers is shown in fig.1. The present results are compared with Akeno group (Hara et al.1981) in the same plot. As can be seen from fig.1 a shower of given size developing in the vertical direction over one attenuation length ($\bar{\Lambda}$ = 112 gm. cm.$^{-2}$ for Ne$\geq$ 5.10^5, Sasaki.1971) increases in age by $\sim$ 0.07. This result is in good agreement with that measured (0.06/100 gm.cm.$^{-2}$) by Clay et al. (1981) and with the measurements of Hara et al. (1981) in the Ne range 10^5-10^6 particles. A trend of such variation is obtained by Capdevielle & Gawin (1982) on two models is also included in the same figure.

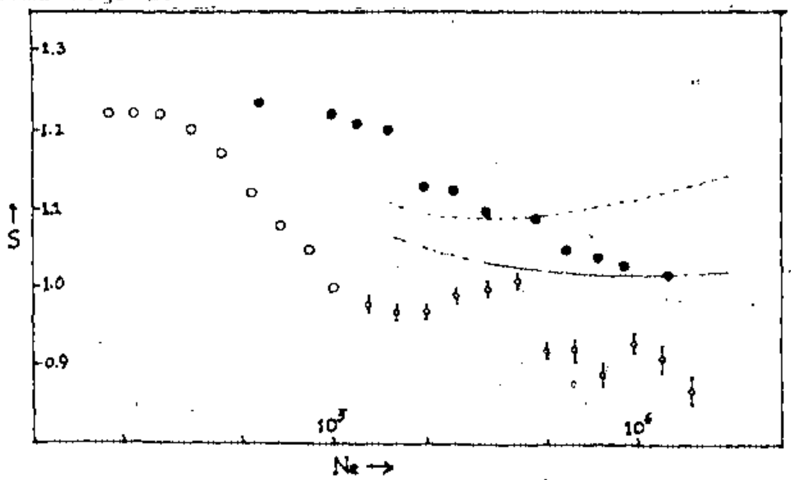

Fig.1:Distribution of Ne in s : O present expt. (sea level); ● Akeno expt.(900 gm.cm.$^{-2}$,Hara et al. (1981); simulation results of Capdevielle & Gawin (1982):—— scale breaking model,----high multiplicity model.

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