Chapter - VI

DIFFERENTIAL ENERGY SPECTRUM OF VERTICAL SINGLE MUONS

6.1 Introduction :

The interrelation of the characteristics of the primary CR particles, their interactions with the atmospheric matter and the secondary components has been a subject of great interest for a quite long time. Several theoretical analyses[1-5] have been made on this subject. But the conclusion inferred from these studies mutually disagree on several points. In this chapter the sea-level differential energy spectrum of muons has been derived based on the primary nucleon spectrum[6] and on a new formula representing all single-pion and kaon invariant cross-sections of inclusive proton-proton (p-p) interactions in the energy range (12 GeV-1500 GeV) of the incident proton in the laboratory system.

6.2 Method of Calculation :

The experimentally determined cross-sections[7-9] for single-charged-pion and kaon production in the inclusive process $p + p \rightarrow i$ + anything, for incident-proton laboratory energies in the range (12 GeV - 1500 GeV) have been used and have been fitted to the phenomenological form for the invariant cross-section

$$E\frac{d^3\sigma}{dp^3} = A_1\left(1+\frac{1}{E_p}\right)^{-n} \exp\left(-\frac{1}{E_p}\right) \exp\left\{-\left(B_1 x + C_1 p_t\right)\right\} \quad \ldots.(6.1)$$

where E_p in unit of $2m_p$ (rest mass of the p-p system) is the incident-proton laboratory energy, P_t is the transverse momentum of the particle i, $x=E/E_p$ (for $x \geqslant 0.02$), E being the laboratory energy of the particle i. The values of the parameters of the best fit to the whole set of the cross-section data are given in table below :-

Particle (i)	Parameters			
	n	A_1 $\left(mb/(GeV^2/c^3)\right)$	B_1	C_1 $(GeV/c)^{-1}$
π^+	1.2	102.3	4.13	5.02
π^-	4.8	112	5.43	5.16
K^+	5.8	11.9	4.26	4.14
K^-	9.8	12.9	6.82	4.81

Equation (6.1) gives a good fit (χ^2/degree of freedom in the range 0.85 to 1.5 for the incident proton energy range 12 GeV - 1500 GeV). The P_t range is from 0.1 - 0.8 GeV/c over X =0.05 to 0.8.

6.2.1 Calculation of the Production Spectra of CR Mesons :

The production spectra of CR mesons produced due to the interactions of protons and other nuclear components of the primary beam with the air nuclei can be calculated by applying equation (1) [ignoring the effects of target in nucleon-nucleus collision and of nucleus-nucleus collision]. Now for the nucleon-nucleon process at a given incident energy and x value the number distribution of the produced particles i in the collision is given by

$$x\frac{d\sigma}{dx} = E\frac{d\sigma}{dE} = \pi\int E\frac{d^3\sigma}{dp^3}\,dp_t^2 \qquad .....(6.2)$$

Assuming the spectrum of the incident nucleons to be a power law $E_p^{-\gamma} dE_p$ the production spectrum of secondary mesons i of energy E is obtained from above as

$$F_i(E)dE = I(E)\frac{1}{\sigma_{inel}}\left[\int x^{\gamma-1}\frac{d\sigma}{dx}\,dx\right]dE$$

$$\text{or,}\quad F_i(E)dE = I(E)f_i\,dE \qquad \text{....(6.3)}$$

σ_{inel} being the total inelastic cross-section of p-p interaction; taken to be constant at 35 mb (from accelerator experiments[10]. It is further assumed that nucleons in the incident nuclei behave as if they were free nucleons in interacting with the air nuclei. Hence the spectrum of the primary total nucleon beam can be expressed in terms of the number of nucleons in unit interval of energy E_p per nucleon by

$$I(E_p)dE_p = I_o E_p^{-\gamma}\,dE_p \qquad \text{..................(6.4)}$$

Using equation (6.4) the production spectra of CR pions and kaons have been calculated.

6.2.2 Relationship between the Primary Nucleon Spectrum and Sea-level Muon Spectrum :

In deriving a simplified relationship between the incident-primary-nucleon spectrum and sea-level muon spectrum in the vertical direction let us consider only generation of muons from decay of first generation of mesons i (pions and kaons).

Now neglecting the energy loss term the spectra of mesons from pion-muon and kaon-muon decay (branching ratio =0.63) at any vertical depth X are given by the solution of the diffusion equation

$$\frac{\partial N_i(E,X)}{\partial X} = - N_i(E,X)\left[\frac{1}{\lambda_i} + \frac{h_i}{X}\right] + \frac{F_i(E)}{\lambda} \exp\left(- X/\lambda_a\right) \qquad \ldots.(6.5)$$

where $N_i(E,X)$ is the spectrum of mesons i at energy E, $F_i(E)$ is the production spectrum [as given by equation (6.3)], λ_i is the interaction mean free path of mesons i [λ_π=120 gcm^{-2}, λ_k =150 gcm^{-2}], $h_i = \frac{H m_i c}{\tau_i E_i}$ [H being the scale height]. The solution of equation (6.5) is used to obtain the sea-level muon spectrum due to the decay of mesons i in the form

$$N_\mu(E_\mu, X)\, dE_\mu = \left[\int_{E_\mu}^{r_i^2 E_\mu} \int_X \frac{1}{\lambda}\left(\frac{1}{1 - 1/r_i^2}\right) \exp\left(-X/\lambda_a\right) dX \right.$$
$$\left. F_i(E)\, \Lambda(g, h_i)\, \frac{dE_i}{E_i}\right] dE_\mu \qquad \ldots.(6.6)$$

where $r_i = m_i/m_\mu$, $g = \frac{X}{L'}$, $\frac{1}{L'} = \left(1/\lambda - 1/\lambda_a\right)$

$$\Lambda(g, h_i) = h_i \int_0^1 t^h \exp\left[g(1-t)\right] dt$$

t=1 at X_o = 1030 gcm^{-2} [i.e. sea level]

Equation (6.6) on integration over X and then over E_i yields (at sea-level)

$$N_\mu(E_\mu, X)\, dE_\mu = \frac{1}{\alpha + \gamma}\left[\frac{1 - 1/r_i^{2(\alpha+\gamma)}}{1 - 1/r_i^2}\right].$$
$$M(h_\mu)\, F(E_\mu)\, dE_\mu \qquad \ldots.(6.7)$$

where $M(h_\mu >> 1) \simeq \frac{h_\mu}{h_\mu + 1}\ \frac{\lambda_a}{\lambda}$; $h_\mu = \frac{H m_i c}{\tau_i E_\mu}$

$$M(h_\mu << 1) \simeq \frac{h_\mu \lambda_a}{\lambda_a - \lambda} \ln(\lambda_a/\lambda)$$

$$F(E_\mu) = I(E_\mu) f_i$$

and $\alpha = \left[1 + \frac{h_i \lambda \ln(\lambda_a/\lambda)}{\lambda_a - \lambda} \right]^{-1}$.

6.2.3 Improved Calculation of Sea-level Muon Spectrum :

The generation of mesons from secondary pion collisions with air nuclei is taken into account by folding it with the production spectrum of the secondary mesons from primary nucleon-nucleus collisions. The production spectrum of mesons i at a depth X' in the atmosphere and local Zenith angle $\theta^*(X')$ is taken as -

$$N_i(E', X', \theta^*(X'))dE'dX' = \frac{F_i(E')}{\lambda} \sec\theta^*(X')dX' \cdot \exp\left[-\int_0^{X'} \frac{\sec\theta^*(X'')}{\lambda'} dX''\right] dE' \quad \ldots.(6.8)$$

The differential spectrum of mesons at X is

$$N_i(E, X, \theta^*(X))dE = \int_0^X \left(\frac{F_i(E')}{\lambda} \sec\theta^*(X') \exp\left[-\int_0^{X'} \frac{\sec\theta^*(X'')}{\lambda'} dX''\right] dE \cdot \left(\exp\left[-\int_{X'}^{X} \left[\frac{1}{\lambda_i} + \frac{H m_i c}{\tau_i E_i'' \rho(X'')} \right] \sec\theta^*(X'') dX'' \right] \right) \cdot \sec\theta^*(X')dX' \right.$$

$$\ldots.(6.9)$$

where the expression in the second parentheses is the probability that mesons i produced at X' can reach at depth X, $\rho(X'')$ is the density of air at depth X", λ' is the attenuation mean free path of all hadrons generating mesons with a constant value of 120 gcm^{-2}. Hence at the observational level of depth X_o

and Zenith angle θ the differential muon spectrum is

$$N_\mu(E,X_0,\theta) = \int_{E_\mu}^{r_i^2 E_\mu} \int_X \frac{b_i \sec\theta^*(X)}{\rho(X)} \cdot \left(\frac{1}{1-r_i^{-2}}\right) \cdot W(E,X,X_0,\theta)\, N_i(E,X,\theta(X)) \frac{dE_i}{E_i^2}\, dX \qquad \ldots(6.10)$$

where $b_i = \frac{Hm_i C}{\tau_i}$ and $W(E,X,X_0,\theta)$ is the survival probability which means that a muon produced with energy E_μ at depth X is characterised with a probability $W(E,X,X_0,\theta)$ to survive down with energy E to a depth X_0 along the direction of Zenith angle θ given by -

$$W(E,X_0,X,\theta) = \exp\left[-\frac{m_\mu c^2}{\tau_\mu c}\int_X^{X_0} \frac{\sec\theta^*(X')}{E_\mu}\frac{dX'}{\rho(X')}\right] \qquad \ldots(6.11)$$

where θ*(X), the local Zenith angle at depth X along the straight path of the muon, arriving at depth X_0 with Zenith angle θ, is related to θ by

$$\sin\theta^*(X) = \frac{R_z}{R_E + h(X)} \sin\theta \qquad \ldots\ldots\ldots\ldots(6.12)$$

where $R_z = R_E + Z_0$, R_E = radius of the earth

Z_0 = altitude of the observational place,

$\rho(X)$ and $h(X)$ are the air density and the altitude of the atmospheric depth X.

6.3 Results:

In figure 6.1 the survival probability has been shown as a function of muon energy. In this calculation it has been

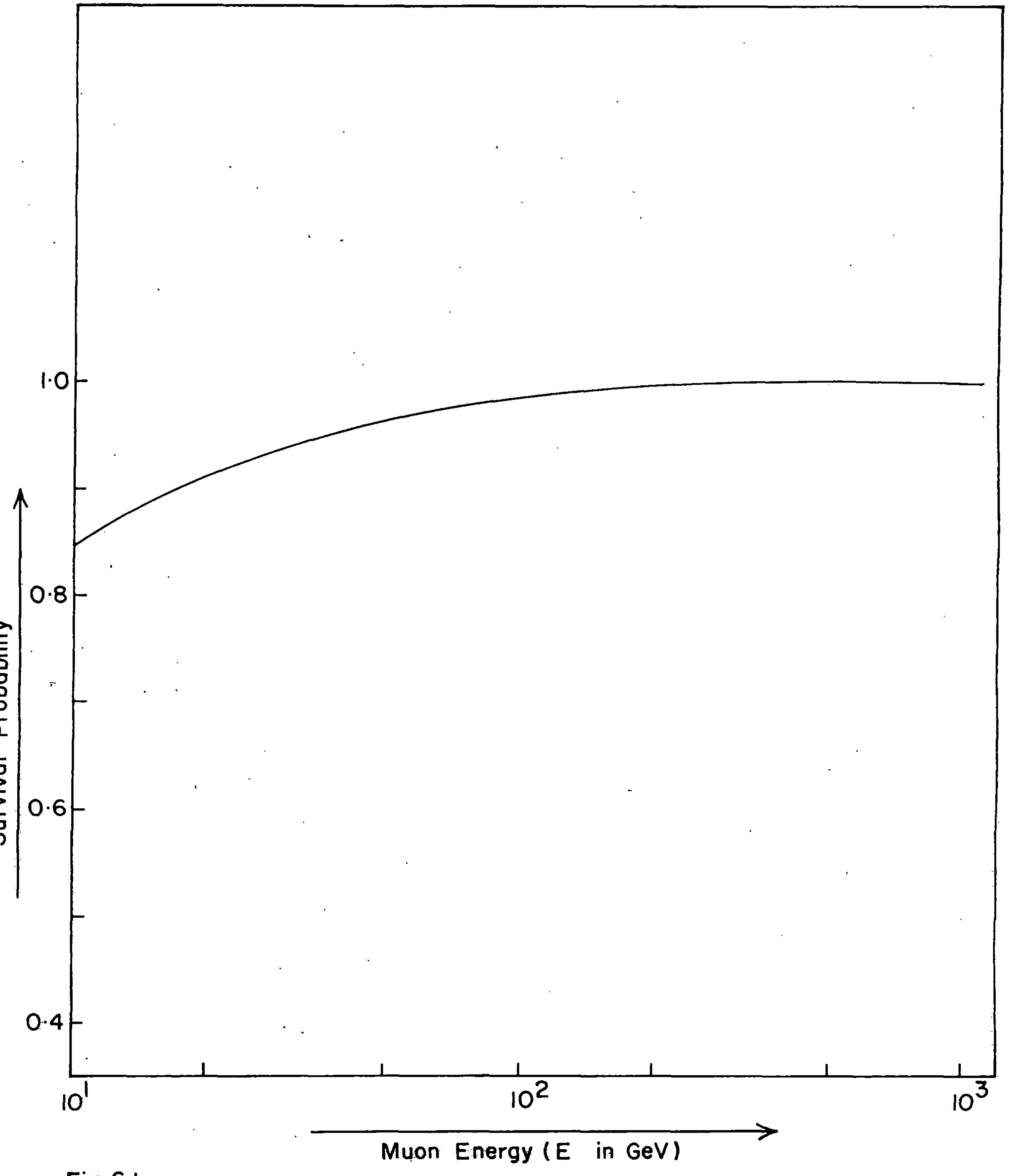

Fig. 6·1

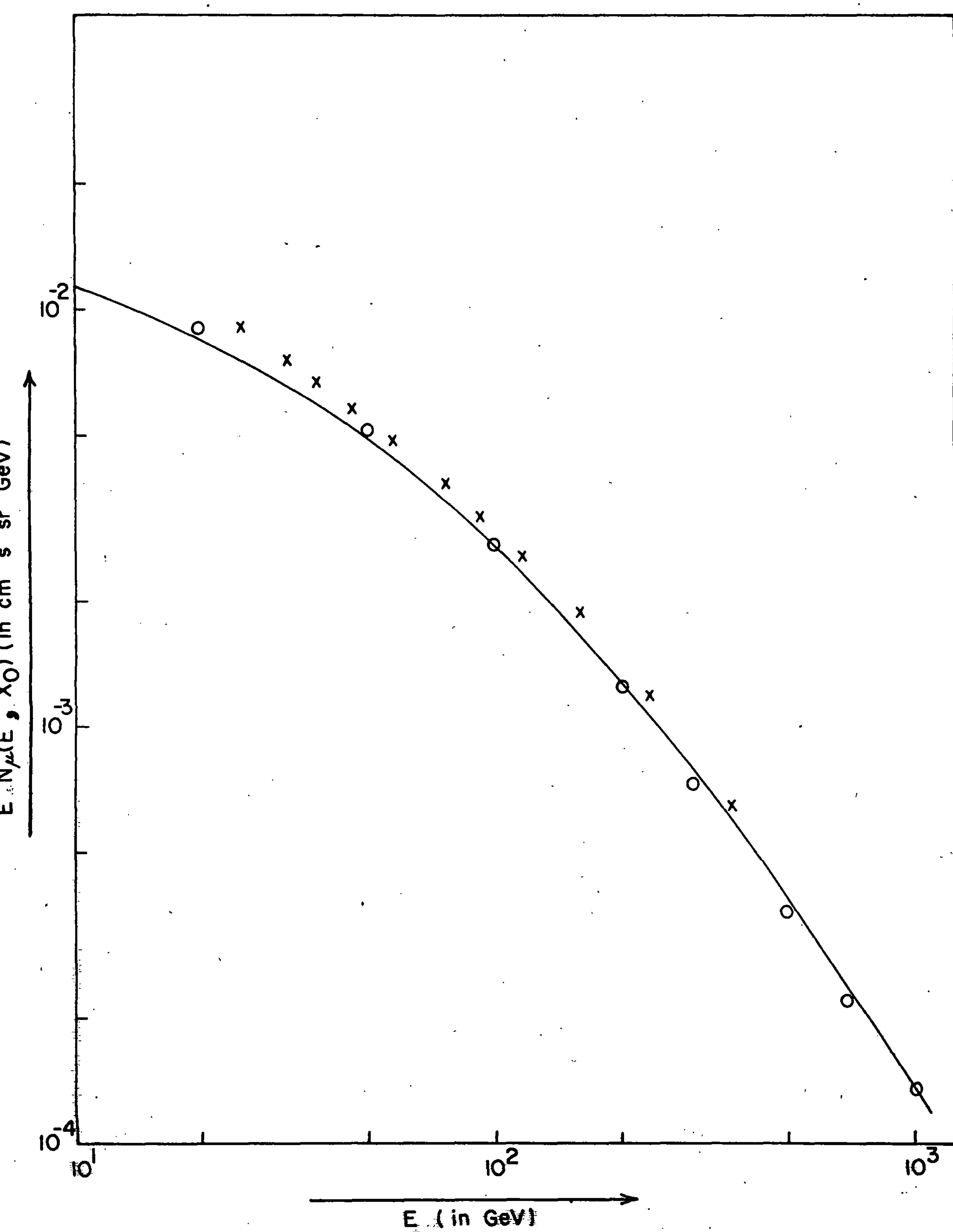

Fig. 6·2

assumed that the muons are produced at an atmospheric depth of 200 gcm^{-2} where the thermal gradient of the atmospheric temperature is low.

In calculating the differential energy spectrum of vertical muons the primary nucleon spectrum of Pal and Bhattacharyya[6] viz. $I(E)dE = 2.36E^{-2.7}dE\ (cm^2 SSr\ GeV/n)^{-1}$ has been taken. The resulting spectrum has been shown in fig. 6.2 along with the experimental data of Nandi[11] and Ayre et al[12]. The agreement is found to be excellent.

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Figure Caption

Fig. 6.1 - Survival probability of vertical single muons as a function of energy.

Fig. 6.2 - Differential energy spectrum of vertical single muons in the form of

$$E^2 N\ (E, X_o) \text{ vs } E,\ X_o = 1030 \text{ gcm}^{-2} \text{ (sea-level)}$$

O indicates the data of Nandi[11]

X " " " " Ayre et al[12].