* *CHAPTER V*

TWO GLUINO BOUND STATES

* A part of the contents of this chapter has already been published in Ref.(97).

V.1. Introduction :

Supersymmetric QCD gives a simple method of studying the two gluino ($\tilde{g}\tilde{g}$) bound states. The production and decay rates of $\tilde{g}\tilde{g}$ states have already been studied by a number of authors.[58] The gluinos are expected to have a long life and can, therefore, form bound states which may be detected. The study of $\tilde{g}\tilde{g}$ states is expected to provide a simple method of detecting a sparticle. However, there are considerable uncertainties in the theoretical predictions for the $\tilde{g}\tilde{g}$ states, which stem from the uncertainties in the gluino mass $\mu_{\tilde{g}}$ as well as in the two gluino potential, $V_{\tilde{g}\tilde{g}}$. The speculations about the gluino mass have been centred around 5 GeV (light gluino) or 60 GeV (heavy gluino), with the experimental bias shifting towards a heavier gluino.[101] For these gluino masses, non-relativistic bound state model is adequate to predict wave-functions and binding energies of $\tilde{g}\tilde{g}$ states. The potential for the $\tilde{g}\tilde{g}$ system is only partially known. A simple minded application of supersymmetric QCD to the gluino sector suggests that the short-distance part of the $\tilde{g}\tilde{g}$ potential is related to the short-distance part of the quark-antiquark potential by the colour factor 9/4. There is, however, no such relation for the long-range part of the potentials. Some authors[58,59] have assumed the same proportionality constant 9/4 between the two potentials, even for the non-perturbative long-range part, just because it makes the total $V_{\tilde{g}\tilde{g}}$ potential proportional to the $Q\bar{Q}$ potential, $V_{Q\bar{Q}}$. Since there is a quark in each of the mass ranges presently considered for the gluino, it is indeed convenient to study the $\tilde{g}\tilde{g}$ states by comparing with the quarkonium states, which are rather well-studied. In the hadron colliders, $\tilde{g}\tilde{g}$ states may be produced via the $gg \to \tilde{g}\tilde{g}$ and $Q\bar{Q} \to \tilde{g}\tilde{g}$ subprocesses. Goldman and Haber[102] discussed the method of detection of the gluinonium states which may be produced in hadron collider and in quarkonium decay. A convenient method of detecting the $\tilde{g}\tilde{g}$ states will be to look for a radiative decay, $(t\bar{t}) \to \gamma + \tilde{g}\tilde{g}(^1S_0)$, if permitted kinematically. To estimate this and other decay widths and production crosssection, one needs the particle density at the origin, $|\psi_{\tilde{g}\tilde{g}}(0)|^2$ of the $\tilde{g}\tilde{g}$ states. The uncertainty in the contribution of the long-range potential makes any theoretical prediction for $|\psi_{\tilde{g}\tilde{g}}(0)|^2$ unreliable. We shall show by considering a particular potential model that there is a significant dependence of the theoretical results on the long-range potential. Given this situation, it seems reasonable to look for model independent results or bounds. The observation that heavy quarkonia can be described by a Schrödinger equation with a non-relativistic $Q\bar{Q}$ potential has led to considerable activities in the study of general properties of the Schrödinger equation with confining potentials. The scaling properties of the Schrödinger equation for power-law potentials have been found[6] to be very useful in deriving results of this nature. Rigorous results on level ordering for more general potentials have been obtained by Grosse and Martin.[98] The value of the S-states wave-function at the origin $\psi(0)$ has a special significance in quarkonium spectroscopy as this quantity occurs in the expressions for various decay widths of the state. Martin has obtained rigorous results for the relative magnitudes of the wave-function at the origin for 1S and 2S states for the usual quarkonium potentials. Martin[103] has, in particular, shown that for convex (concave) $Q\bar{Q}$ potentials, $|\psi_{2S}(0)| > |\psi_{1S}(0)|$ $\left(|\psi_{2S}(0)| < |\psi_{1S}(0)|\right)$ which was found useful in comparing the leptonic decay widths of the quarkonia. Some of the techniques used by Martin are now familiar and we shall use them to derive some inequalities relevant for the study of $\tilde{g}\tilde{g}$ states. The purpose of this chapter is to point out that the results can be generalised further to obtain some useful information about the two gluino bound states.

The presentation in the remaining part of this chapter is as follows. In section V.2, the two gluino bound state potential is briefly reviewed. In section V.3, we consider a class of potentials to study the effect of the long-range part of the potential on the spectroscopy of the two gluino system, using some general properties of the Schrödinger equation. The results may provide bounds on decay widths. The last section gives our conclusions.

V.2. Two gluino bound state :

Gluinos are a self-conjugate majorana spinor, transforming as an octet of colour SU(3) and are supersymmetric partners of gluons. It would be exactly massless at the tree level if supersymmetry were an unbroken symmetry. The gluino-gluino bound states are known to follow the general decomposition rule

$$8 \times 8 = 1 + 8_S + 8_A + 10 + 10^* + 27 .$$

We, however, need consider only the singlet sector. Starting from an octet QCD action, Zuk *et al.*[58] followed the method of Brink *et al.*[104] to extend the original QCD lagrangian to the supersymmetric sector. The action is given by

$$S = \int d^4x \left[-\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a + \frac{i}{2} \bar{\tilde{g}}^a \gamma . D \tilde{g}_a \right]$$

which describes the interaction between the SU(3) gauge vector boson and its superpartner gluino fields. D_μ is a covariant derivative, $D_\mu = \partial_\mu + ig\, T^a . A^a_\mu$ and $F^a_{\mu\nu}$ (a = 1,2,------8) are the Yang-Mills fields constructed from the adjoint representation of gluon field. T^a are colour matrices. The gluino (majorana) field $\tilde{g}_a$ satisfies the condition $\tilde{g}^a = c\bar{\tilde{g}}^{aT}$ which is necessary to halve the number of fermionic degrees of freedom to match the number of bosonic degrees of freedom. The majorana constraint is SU(3) gauge invariant. The interaction lagrangian term of gluino and gluon is obtained as

$$L_{int} = -g\, J^\mu_a\, A^a_\mu$$

where J^{μ}_{a} is a gluino current, written as $J^{\mu}_{a}(x) = \frac{1}{2} \bar{\tilde{g}}(x)\gamma^{\mu}T_{a}\tilde{g}(x)$ and g is the strong QCD coupling constant. The matrix elements of the gluino current is

$$\langle \tilde{g}' |: J^{a}_{\mu}(0) :| \tilde{g} \rangle = T^{a}\bar{u}(k's')\gamma_{\mu}u(ks)$$

where k and s denote the momentum and spin of the free gluino states. We now consider $\tilde{g}\tilde{g}$ scattering at the one-gluon-exchange level. The relevant diagram gives the matrix element

$$M_{fi} = g^{2} \left[\frac{1}{8} \mathrm{Tr} \sum_{a} (T^{a})^{2} \right] \frac{1}{q^{2}} \bar{u}(k_2's_2')\gamma^{\mu}u(k_2s_2)\bar{u}(k_1's_1')\gamma_{\mu}u(k_1s_1) .$$

Comparing this matrix element with that for e^-e^- scattering with one-photon-exchange, we see that

$$V_{\tilde{g}\tilde{g}} = - C_2(G)g^2 \, V_{e^-e^-} |_{e^2=1} .$$

Considering a similar graph for $Q\bar{Q} \to Q\bar{Q}$, we note the scaling relation

$$V_{\tilde{g}\tilde{g}} = C_2(G)/C_2(R) \, V_{Q\bar{Q}} = 9/4 \, V_{Q\bar{Q}} . \quad (5.1)$$

The above relation is valid for large Q^2, in the region of perturbative QCD. But at small Q^2, we have no knowledge of QCD potential and the scaling relation need not hold for the long distance part of the potential. In the next section, we show the dependence of the theoretical results on the undetermined long-range part of the potential.

V.3. Long-range potential and results :

To study the effect of the long-range part of the potential, we consider a simple parametrization by assuming that

$$V_{Q\bar{Q}} = V_S(r) + V_L(r) \quad , \tag{5.2}$$

$$V_{\tilde{g}\tilde{g}} = \alpha\, V_S(r) + \beta\, V_L(r) \quad , \tag{5.3}$$

where α, β are real parameters. $V_S(r)$ is the short-range potential, which is attractive and dominant near $r = 0$. The long-range part, $V_L(r)$ becomes positive and divergent as $r \to \infty$. We first consider a particular non-relativistic potential $V_{Q\bar{Q}} = V_S + V_L$, with

$$V_S(r) = f(r)\, V^{(2)}_{QCD}(r) \quad , \qquad V_L(r) = \big(1 - f(r)\big)\, V_M(r) \tag{5.4}$$

$$\text{and} \qquad f(r) = \frac{\left(1 + e^{-a/s} \right)}{\left(1 + e^{(r - a)/s} \right)}$$

where V_S is estimated from the two-loop QCD calculations, given by the Eq. (2.3). We choose $\Lambda_{\overline{MS}} = 0.200$ GeV and $N_f = 4$. In (5.4), V_M is a Martin-type power-law potential given by

$$V_M(r) = -7.392 + 8.080 r^{0.1} \tag{5.5}$$

with r in fm. The choice is motivated so that

$$V_{QCD}(r=a) = V_M(r=a) \quad . \tag{5.6}$$

We have chosen the parameters in the potentials V_S and V_L so that the condition (5.6) is satisfied. Because of this condition, the calculated spin-averaged results are not very sensitive to the choice of the values of a and s.[66] We have chosen a = 0.0723 fm and s = 0.01 fm in the following calculations. The singularity of $V^{(2)}_{QCD}(r)$ at $r = \Lambda^{-1}_{\overline{MS}}$ should be ignored in calculating $V_S(r)$. One may truncate $V_S(r)$ for $r > r_o$, where one chooses $r_o < \Lambda^{-1}_{\overline{MS}}$ so that $(r_o-a)/s \gg 1$. The results are then insensitive to the value of r_o chosen.

We now assume $V_{\tilde{g}\tilde{g}} = 9/4\ V_S + \beta\ V_L$ and calculate the binding energies and the values of $|\psi_{\tilde{g}\tilde{g}}(0)|^2$ of the $\tilde{g}\tilde{g}$ system for a range of β values ($0.5 \le \beta \le 3$)and with different gluino mass. Our results are shown in Figs.5.1-5.4. We do not expect a large deviation for these results for any other acceptable potential model. We note, in particular, that $|\psi(0)|^2$ for both light and heavy gluinos show a significant dependence on β. It is obvious that the long-range part needs more attention.

The decay rates of gluinonium are given by expressions similar to those of the corresponding quarkonia, apart from the group factors

$$\frac{1}{2}\ \frac{8}{3}\ |\ c_2(G)\ /\ c_2(R)\ |^2 = 27/4 \quad .$$

Thus the decay width of two gluino bound state is given by

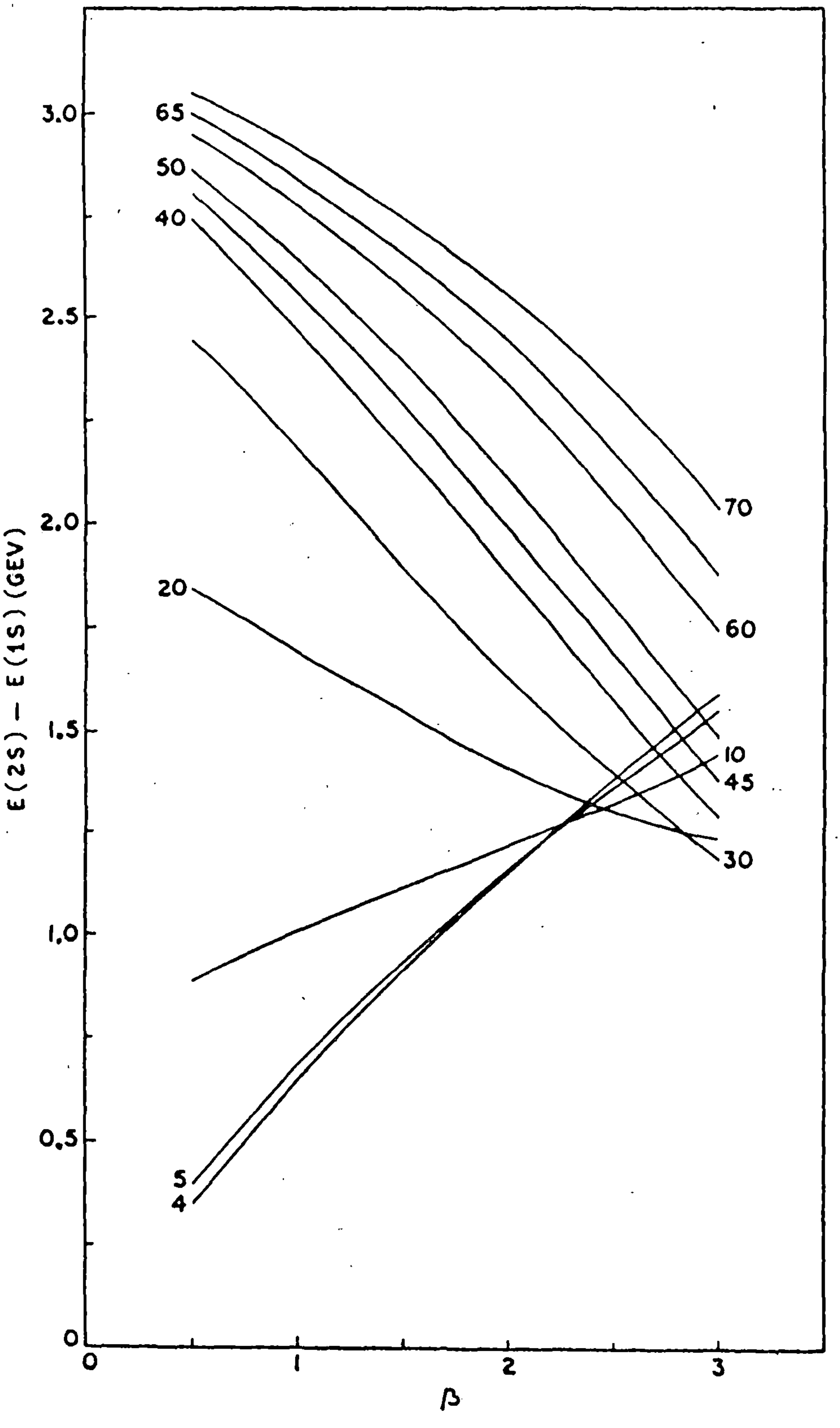

Fig.5.1. The energy level separation, E(2S) − E(1S), of the $\tilde{g}\tilde{g}$ system for $0.5 \leq \beta \leq 3$. The numbers indicate the gluino mass $\mu_{\tilde{g}}$ in GeV.

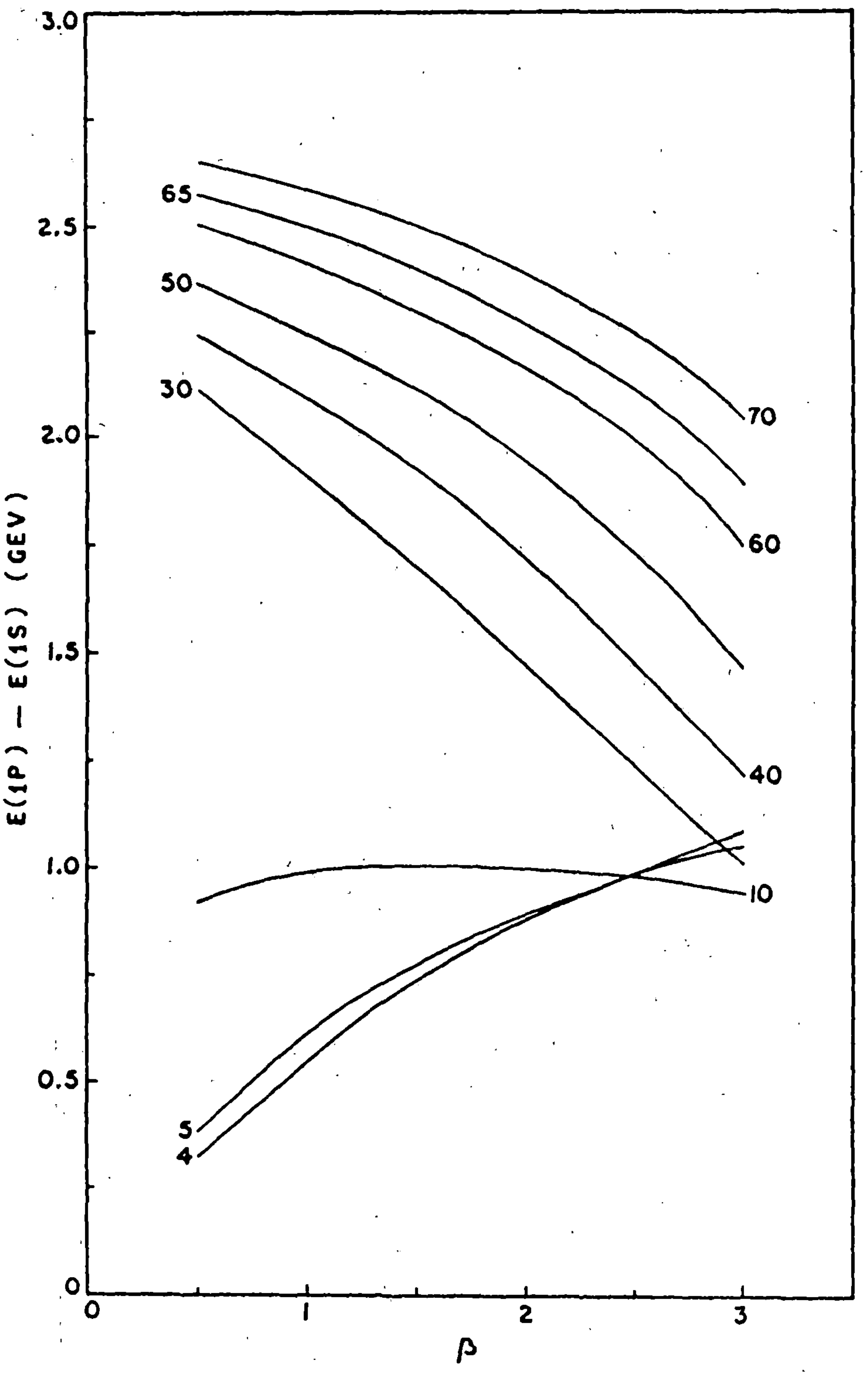

Fig.5.2. The level separation E(1P) − E(1S) of the $\tilde{g}\tilde{g}$ system for $0.5 \leq \beta \leq 3$. The numbers indicate $\mu_{\tilde{g}}$ in GeV.

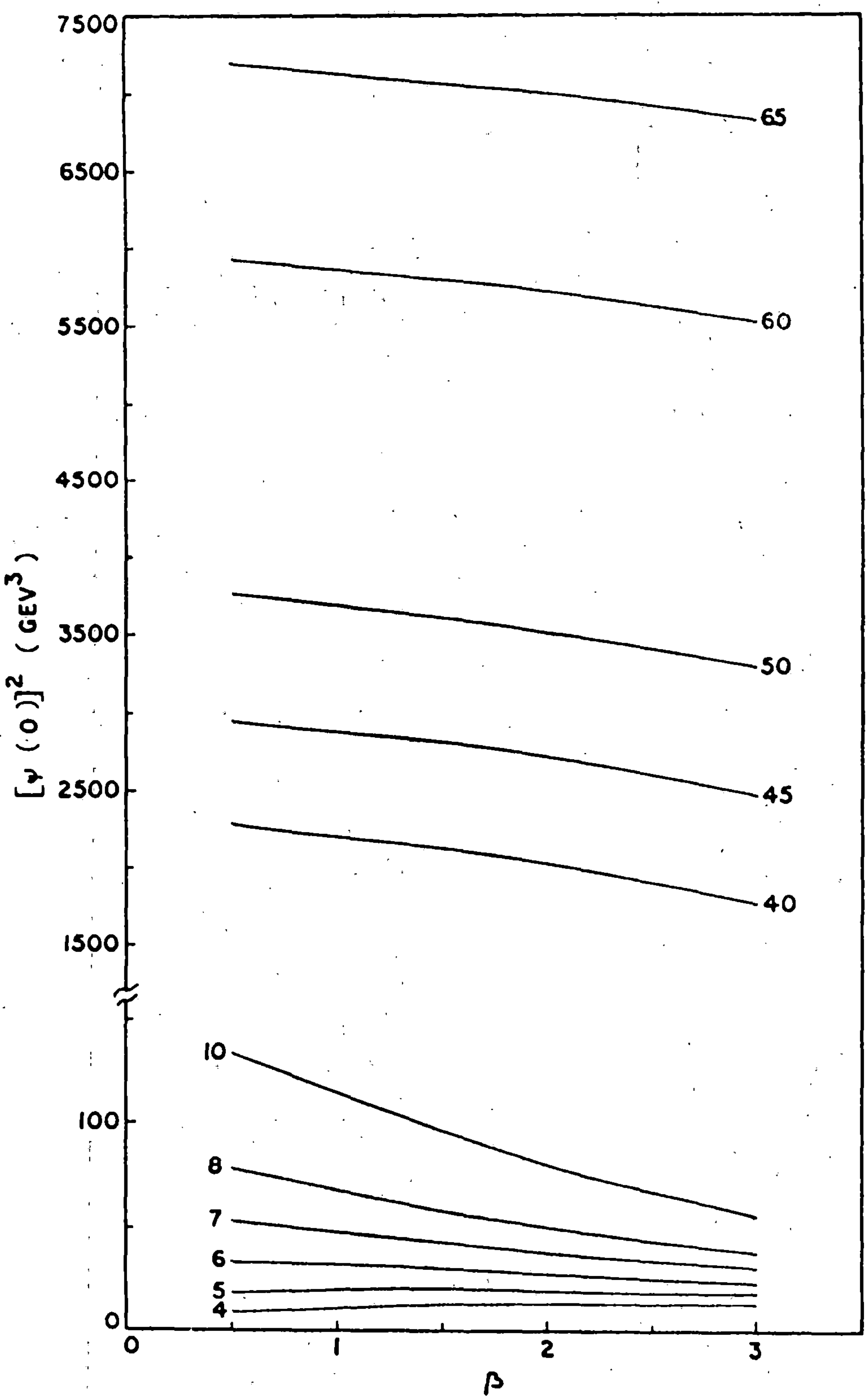

Fig.5.3. The values of $|\psi(o)|^2$ for the $\tilde{g}\tilde{g}$ 1S state. The numbers indicate $\mu_{\tilde{g}}$ in GeV.

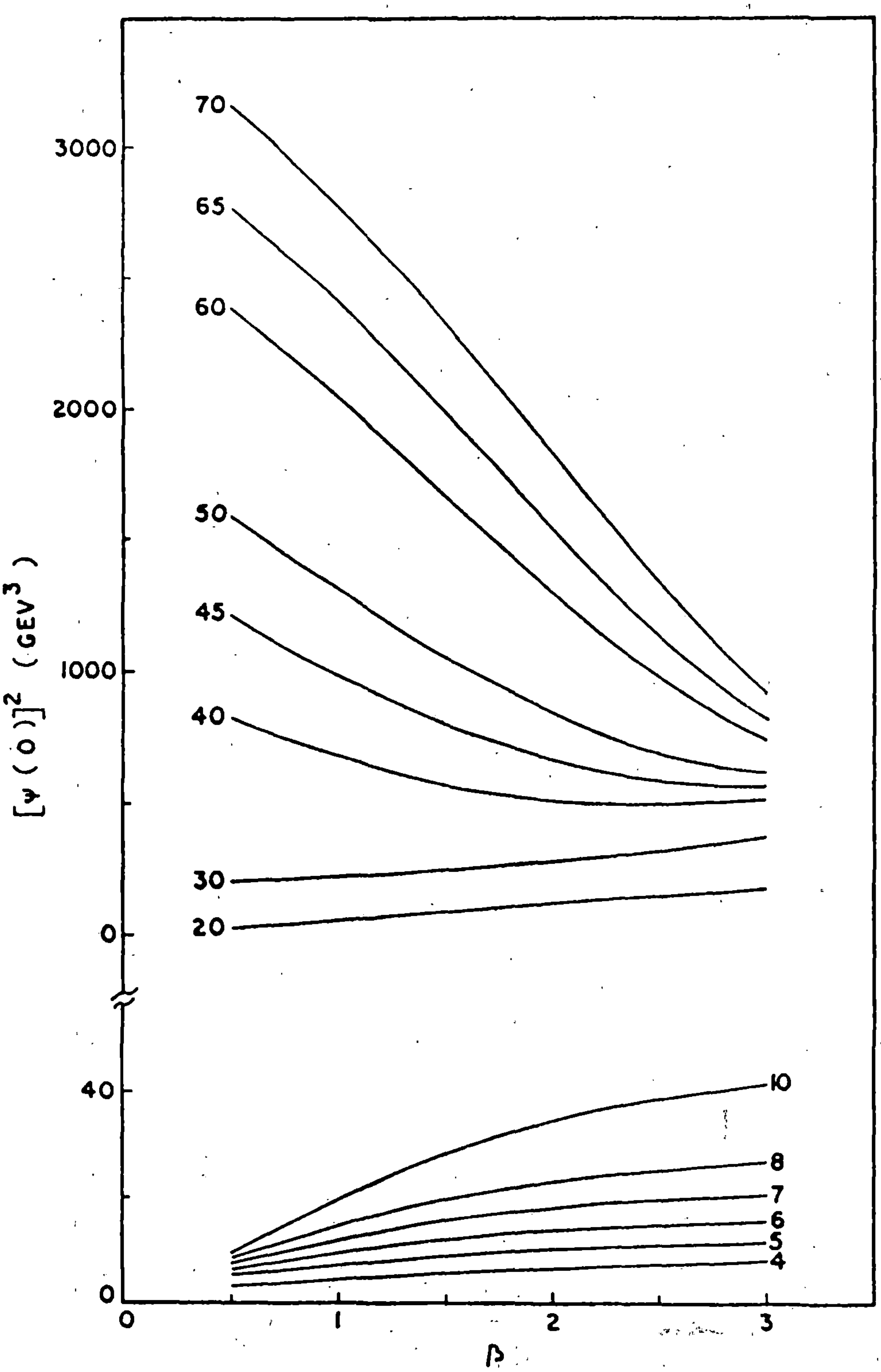

Fig.5.4. The values of $|\psi(0)|^2$ for the $\tilde{g}\tilde{g}$ 2S state. The numbers indicate $\mu_{\tilde{g}}$ in GeV.

$$\Gamma\left(\tilde{g}\tilde{g}(0^{-+}) \to gg\right) = \frac{27}{4}\,\frac{8}{3}\,\alpha_s^2\,|\psi_{\tilde{g}\tilde{g}}(0)|^2/M^2 \,. \tag{5.7}$$

The standard formula for $\tilde{g}\tilde{g}$ production cross-section in hadron collisions is

$$\sigma = (2J+1)\,\pi^2 \frac{\Gamma(\tilde{g}\tilde{g} \to gg)}{8M^3} \int_\tau^1 \frac{dx}{x}\,\tau\, f_g(x)\, f_g(\tau/x) \tag{5.8}$$

where $\tau = M^2/S$, M, the gluinonium mass and $3(1-x)^5/x$ is assumed to give the distribution function for gluons. We have considered only the cross-section for pseudoscalar production. We have shown in Fig.5.5, the cross-section versus $\mu_{\tilde{g}}$ for two different beam energies ($\sqrt{S}$) considering two different values of β. An increase in β decreases the cross-section for a given gluino mass. Our calculated results agree with those of Kühn and Ono[58] but differ slightly from the results of Kane and Leveille[105] at a higher beam energy.

A study of the general features of the potentials (5.2) and (5.3) provides some information which may be obtained in the form of inequalities. The proof of the inequality makes use of the techniques developed by Martin. We assume that both the short-range and the long-range parts of the potential are monotonically increasing. The assumption is true for almost all the potential models considered for the $Q\bar{Q}$ systems. Thus V_S and V_L in (5.2) and (5.3) satisfy

$$\frac{dV_S}{dr} > 0 \;, \quad \frac{dV_L}{dr} > 0 \tag{5.9}$$

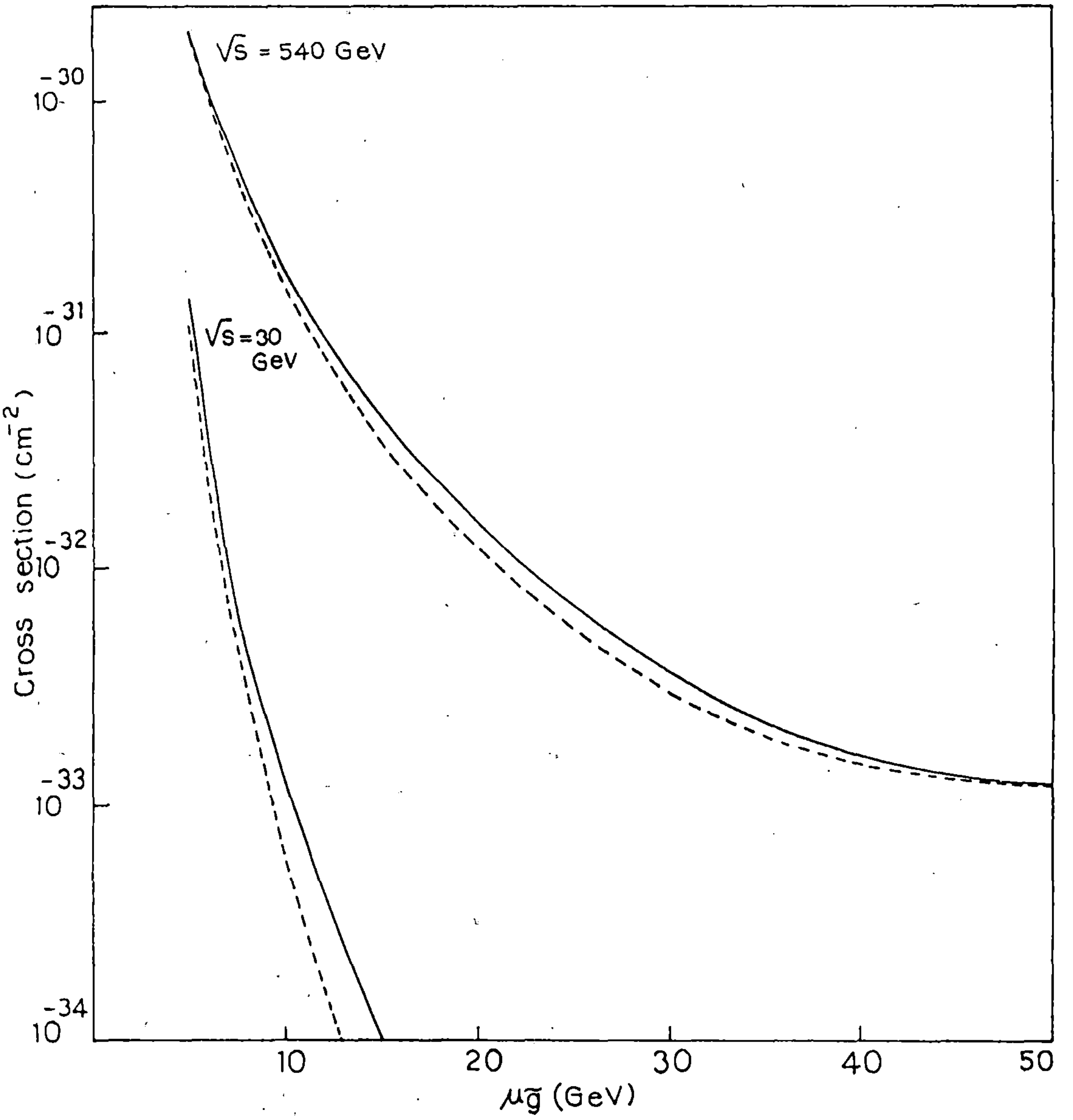

Fig.5.5. The values of cross-section(σ) for pseudoscalar production VS. $\mu_{\tilde{g}}$. The solid line corresponds to β=2.25 and the broken line to β=3 .

and $$\frac{d^2 V_{\tilde{g}\tilde{g}}}{dr^2} < 0 \qquad \text{(concave downwards)} \tag{5.10}$$

but are otherwise arbitrary. We can derive the following results:

(a) If $\mu_{\tilde{g}}/\mu_Q > 1/\alpha$ and $\beta > \mu_Q/\mu_{\tilde{g}}$,

we have for the 1S states

$$|\psi_{\tilde{g}\tilde{g}}(0)|^2 > \mathrm{Min}(\ \alpha,\beta\ )(\ \mu_{\tilde{g}}/\mu_Q)\ |\psi_{Q\bar{Q}}(0)|^2 , \tag{5.11}$$

where Min(α,β) is the smaller of the two parameters.

(b) If $\mu_{\tilde{g}}/\mu_Q < 1/\alpha$ and $\beta < \mu_Q/\mu_{\tilde{g}}$,

$$|\psi_{\tilde{g}\tilde{g}}(0)|^2 < |\psi_{Q\bar{Q}}(0)|^2 . \tag{5.12}$$

Let $\psi_{\tilde{g}\tilde{g}} = v(r)/r$ and $\psi_{Q\bar{Q}}(r) = u(r)/r$ be the 1S radial solutions for the $\tilde{g}\tilde{g}$ and $Q\bar{Q}$ systems respectively. Both u abd v are chosen positive. To prove the inequality (5.11), we follow the steps outlined below:

(i) For $r \to \infty$, we can make use of the large r behavior of the radial equations to show that $v^2 < u^2$ for $r \to \infty$, if $\beta > \mu_Q/\mu_{\tilde{g}}$.
(ii) It can be shown that $v^2 - u^2$ has a unique zero. Consider the Wronskian

$$\begin{aligned} I(r) &= (v'u - u'v)(r) \\ &= \frac{1}{\hbar^2}\int_0^r uv\ [\ (\alpha\ \mu_{\tilde{g}} - \mu_Q)V_S + (\beta\ \mu_{\tilde{g}} - \mu_Q)V_L \\ &\qquad - (E'\ \mu_{\tilde{g}} - E\ \mu_Q)\ ]\ dr \ . \end{aligned} \tag{5.13}$$

Since $I(\infty) = 0$, we may also write the following representation

$$I(r) = -\frac{1}{\hbar^2}\int_r^\infty uv\left[(\alpha\,\mu_{\tilde g} - \mu_Q)V_S + (\beta\,\mu_{\tilde g} - \mu_Q)V_L\right.$$

$$\left. - (E'\,\mu_{\tilde g} - E\,\mu_Q)\right] dr \quad . \qquad (5.14)$$

From (5.13), it follows that for $r \sim 0$, the integrand is $\sim (\alpha\,\mu_{\tilde g} - \mu_Q)V_S$ and negative and hence $I(r) < 0$. For $r \to \infty$, one finds that the integrand is $\sim (\beta\,\mu_{\tilde g} - \mu_Q)V_L$ and hence from (5.14), it follows that I(r) is also negative at large r. Since both V_S and V_L are monotonic , u and v are nonzero and positive. It has, therefore, only one zero, say at $r = r_o$. For $r < r_o$, we use the relation (5.13) and for $r > r_o$, we use the relation (5.14) to show that $I(r) < 0$ for all r. We can now show that v − u vanishes only once, say at $r = r_1$. The relation (5.13) gives

$$u(r_1)\left[\,v'(r_1) - u'(r_1)\,\right] < 0 \quad . \qquad (5.15)$$

Thus $(v' - u')_{r=r_1} < 0$, since $u > 0$. The uniqueness of the zero of $v - u$ and hence of $(v^2 - u^2)$ follows from above.

(iii) Since $v^2 - u^2 < 0$ for $r \to \infty$ and since $v^2 - u^2$ has a unique zero, we conclude that $v^2 - u^2 > 0$, for $r \sim 0$.

(iv) In the last step, we make use of the well-known relations like

$$u'^2(0) = \frac{\mu_Q}{\hbar^2}\int u^2\,\frac{dV}{dr}\,dr \quad . \qquad (5.16)$$

We consider, wih $\beta \le \alpha$,

$$|\psi_{\tilde{g}\tilde{g}}(0)|^2 - \beta \frac{\mu_{\tilde{g}}}{\mu_Q} |\psi_{Q\bar{Q}}(0)|^2$$

$$= v'^2(0) - \beta \frac{\mu_{\tilde{g}}}{\mu_Q} u'^2(0)$$

$$= \frac{\mu_{\tilde{g}}}{\hbar^2} \int dr \ (v^2 - u^2)\frac{dV_{\tilde{g}\tilde{g}}}{dr} + \frac{\mu_{\tilde{g}}}{\hbar^2} \int dr \ u^2(\alpha - \beta)\frac{dV_S}{dr}$$

$$\ge \frac{\mu_{\tilde{g}}}{\hbar^2} \int dr \ (v^2 - u^2) \left[\frac{dV_{\tilde{g}\tilde{g}}}{dr} - \frac{dV_{\tilde{g}\tilde{g}}}{dr}\Big|_{r=r_1} \right]$$

$$> 0 ,$$

since

$$\frac{d^2V_{\tilde{g}\tilde{g}}}{dr^2} < 0 \quad . \tag{5.17}$$

For $\beta > \alpha$, we can show that

$$|\psi_{\tilde{g}\tilde{g}}(0)|^2 - \alpha \ (\mu_{\tilde{g}}/\mu_Q) \ |\psi_{Q\bar{Q}}(0)|^2$$

is positive definite. This completes the proof of the inequality (5.11). For $\mu_{\tilde{g}}/\mu_Q < 1/\alpha$, $\beta < \mu_Q/\mu_{\tilde{g}}$, we may follow the same steps to derive an upper bound for $|\psi_{\tilde{g}\tilde{g}}(0)|^2$, *viz.* the inequality (5.12). Our results are shown explicitly in Fig.5.6 for $\alpha = 9/4$. We cannot make any prediction for the striped regions. We note the following:

(i) For quarkonium states, an useful inequality for two $Q\bar{Q}$ states

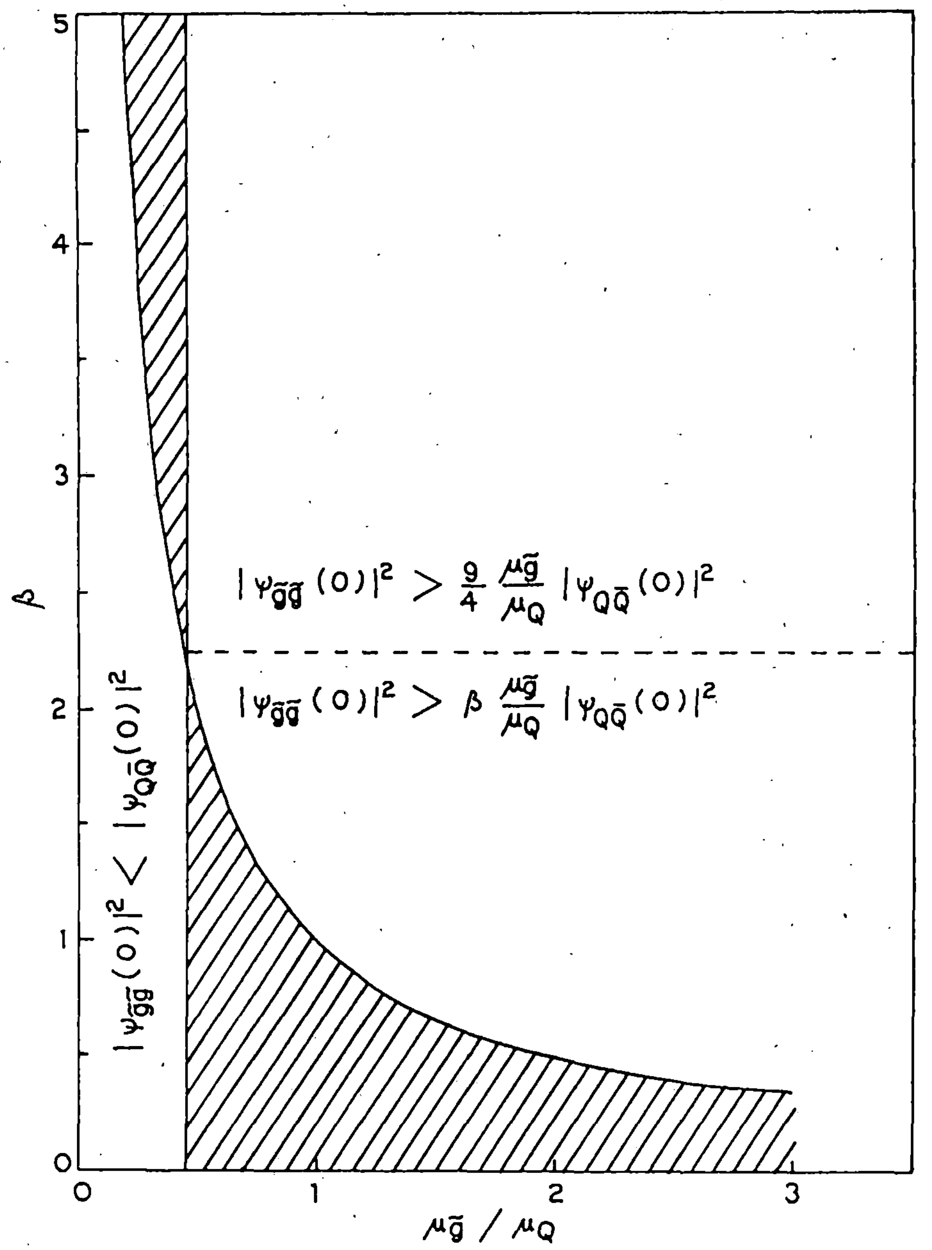

Fig.5.6. The inequalities (5.11) and (5.12) shown explicitly. There is no prediction for the striped region.

with masses μ_Q and $\mu_{Q'}$ was considered by Rosner *et al.*[100].

$$\psi^2_{Q'\bar{Q}'}(0) \geq \frac{\mu_{Q'}}{\mu_Q} \psi^2_{Q\bar{Q}}(0) , \qquad \mu_{Q'} > \mu_Q . \qquad (5.18)$$

The inequality has been shown to be valid for power-law potentials and also for a potential satisfying conditions like (5.9) and (5.10) in the WKB approximation. Our results, with $\alpha = \beta = 1$ provide a rigorous proof of this inequality for a large class of potentials.

(ii) For $\mu_{\tilde{g}} \simeq \frac{4}{9} \mu_t$ we have the equality $\psi_{\tilde{g}\tilde{g}}(0) = \psi_{t\bar{t}}(0)$ for $\beta = 9/4$. For a smaller β, $\psi_{t\bar{t}}(0) > \psi_{\tilde{g}\tilde{g}}(0)$, while for a larger β, the situation is reversed. The variation of $\psi(0)$ with β is expected to be smooth, as has also been seen in our model calculations.

(iii) The results may be useful for estimating bounds on decay widths. As an example, consider the two gluon decay width (5.7). Suppose $\beta \sim 9/4$ and μ_t = 45 GeV. If $\mu_{\tilde{g}}$ = 20 GeV,

$$\Gamma(\tilde{g}\tilde{g}(0^{-+}) \rightarrow gg) \simeq 190 \text{ MeV}, \qquad (5.19)$$

where we have used the result $|\psi_{t\bar{t}}(0)|^2_{1S} = 350 \text{ GeV}^3$, as can be seen from calculations with the potential (5.4). This may be compared with the two gluon width of the 1S toponium state for $\mu_t \sim$ 45 GeV which is about 5 MeV. The latter estimate is in general agreement with the results of Nanopoulos *et al.*,[59] who consider a particular $Q\bar{Q}$ potential. Combining the inequalities of Fig.5.6 with the results (5.19), we can now derive some crude bounds on the decay widths. The knowledge of $\mu_{\tilde{g}}$ (or some strict constraints on $\mu_{\tilde{g}}$) will help in making the predictions sharper. If $\mu_{\tilde{g}}/\mu_t \gtrsim 1$, the inequalities become valid for larger ranges of β.

(iv) For the class of potentials considered, Martin has proved that $\psi_{2S}(0) > \psi_{1S}(0)$. Thus we can obtain a weak inequality if we replace $\psi_{\tilde{g}\tilde{g}}(0)$ of 1S state by $\psi_{\tilde{g}\tilde{g}}(0)$ for 2S state in the relation (5.11).

V.4. Conclusions:

We have shown that the long-range part of the $\tilde{g}\tilde{g}$ potential may affect significantly the theoretical prediction for $|\psi_{\tilde{g}\tilde{g}}(0)|^2$. We have also studied the dependence of the binding energy difference and the value of the wave-function at the origin on the gluino mass for various values of β ($0.5 \le \beta \le 3$). Although, the long-range part of the $\tilde{g}\tilde{g}$ potential is not known, it is interesting to note that by using Martin's technique, one can derive useful inequalities relevant for the study of $\tilde{g}\tilde{g}$ states. The inequality holds for a range of values of β and $\mu_{\tilde{g}}/\mu_Q$ as shown in Fig.5.6. In fact, in the absence of any definite information about either $\mu_{\tilde{g}}$ or β, the theoretical predictions will perhaps be no better than the predictions of the inequalities shown in Fig.5.6.