

** CHAPTER I*

INTRODUCTION

** This is mostly a review of earlier works.*

I.1. Introduction - Background and Objective :

During the last two decades, the developments in particle physics have mostly centred around gauge symmetries of particle interactions. It is now generally believed that Quantum Chromodynamics (QCD), the gauge theory of colour $SU(3)$, describes the strong interactions of particles while the Salam-Weinberg-Glashow $SU(2)_L \times U(1)$ gauge theory describes their weak and electromagnetic interactions. A further step in the direction of finding a complete and fundamental law for particle interaction will be to try to unify these forces. An intensive study of QCD and the interquark interaction, in particular, may provide useful information in our quest for a successful theory of unification. The recently discovered quarkonium states present objects of great interest in this connection.

The quarkonium spectroscopy provides a field where the concepts of QCD could be tested experimentally. Deep-Inelastic Lepton scattering processes have already provided some confirmation of the perturbative QCD. The quarkonium spectroscopy has the potential of becoming another field, within the range of our experimental capabilities, where QCD may be tested qualitatively at all distance scales. The results obtained so far tend to confirm the expectation that QCD is the most promising candidate for a dynamical theory of strong interactions. It is a non-abelian local gauge theory which describes the interaction of quarks with

massless coloured spin 1 gauge bosons, the gluons. Unlike in QED, the effective coupling constant in QCD becomes feeble at large momentum transfers (at short-distances), so that quarks no longer interact strongly at small interquark separations. This concept of asymptotic freedom¹ is consistent with the deep-inelastic scattering results.² In QCD, the three colours of a quark are assumed to transform as a fundamental representation whereas the gluons transform as an adjoint representation of $SU(3)_c$. An important constraint on the theory is that elementary coloured quarks and gluons are not directly observable in an experiment. Thus the observed bound states are supposed to be singlets in the colour space.

In QCD calculations, the results of a perturbation theory are expected to be accurate for large momentum transfers. Since the effective quark gluon coupling becomes small at high energies, the one-gluon-exchange term dominates the potential and the interaction potential is roughly of the coulomb type $V(r) \sim 1/r$. However, perturbative QCD does not give any explanation of the quark confinement. The general expectation is that at a large separation, quarks feel an increasingly strong restoring force which is responsible for their confinement. The Confining hypothesis now seems to be an essential ingredient for building models of quarkonia. The interaction is supposed to guarantee that all hadronic states observable in nature are colour singlets. The large distance part of the potential is, however, not yet determined accurately. Its behavior may be as given by the

string³ and flux-tube⁴ models. Lattice gauge theories⁵ provide some useful guidance here. It is not yet clear whether the rate of growth of the potential is logarithmic⁶ or some power^{7,8} of distance. A popular belief is that the potential rises linearly with the inter-quark distance, i.e. $V(r) \sim kr$ where k is the string constant, a behavior also expected from Wilson's area law. While the choice of the long distance potential gets some guidance from our theoretical bias, practically not much is known about the nature of the potential in the intermediate range. This is where the heavy quarkonium spectroscopy proves to be useful. The spectroscopy carries information about the shape of the potential at all distances, including the cross-over region, i.e. in the gap between the one-gluon-exchange potential and the confining potential.

It may be recalled that the existence of five flavours of quarks (u,d,c,s,b) has so far been found to be consistent with the experimental results. However, on theoretical grounds, a sixth massive quark (top quark) is essential for the standard three generation model.⁹ Although the top quark is yet to be discovered, the search is on, particularly for the toponium states, which would have properties similar to ψ and Y systems. All these states should be described fairly accurately by the Schrödinger equation with a flavour independent non-relativistic potential $V(r)$.¹⁰ Since the mass of the quark is assumed to be large compared to its binding energy, the relativistic effects should give only small corrections.

To test the concepts outlined above, one has to calculate the energy levels, leptonic decay widths as well as E1 transition rates for heavy quarkonia by considering the spin-independent potential as the sum of the short and long-range potentials. We have undertaken a similar study in chapter II. As a first step, we have considered a non-relativistic potential consisting of a short-range 2-loop QCD potential matched to a confining potential at a large distance. Apart from the $c\bar{c}$ and $b\bar{b}$ states, we have also studied the postulated $t\bar{t}$ states using the same flavour independent potential. We repeated the calculations including the relativistic corrections. We have noted that the standard Breit-Fermi type of spin-dependent potentials cannot satisfactorily explain the fine-hyperfine splittings of heavy quarkonium states. The recent observations of 1P_1 states of the $b\bar{b}$ and $c\bar{c}$ families have sharply focussed on the limitations of the standard potentials. In chapter III, we have tried to fit the experimental results by considering an additional contribution to the fine-hyperfine interaction potentials staying within the framework of Breit-Fermi form. In chapter IV, we have made use of the general properties of Schrödinger equation to predict some results for the wave-function at the origin pertaining to 3S_1 and 1S_0 states of $Q\bar{Q}$ systems. In the last chapter, using a similar technique, some useful results for the two gluino bound states have also been obtained by making use of supersymmetry (SUSY).

The aim of this thesis was to study the properties of some newly discovered mesons which are identified as bound states of a

heavy quark and its antiparticle so as to obtain some constraints on QCD. Considerable phenomenological studies have already been made in this field but this rapidly developing area of studies make it necessary that our theoretical results be continuously compared with new experimental results. The phenomenology of heavy quarkonia provides tests for the entire model based on QCD, the standard electroweak theory and other theoretical ideas of particle interactions. Using SUSY as an additional input, we could study the two gluino bound states within the same formalism. Our results on $\tilde{g}\tilde{g}$ bound states will be useful in the confirmation of both QCD and SUSY and in establishing the interplay between the two most appealing theoretical ingredients of high energy physics.

II.2. Heavy Quarkonia - Experimental Results :

Considerable experimental results are now available for a detailed study of quarkonium spectroscopy. The spectroscopic notation for a $Q\bar{Q}$ level is denoted by $n^{2S+1}L_J$ where the symbols have their usual meaning. The two spin-half quarks can form spin-singlet and spin-triplet states. The state with $L=0$ is split into 3S_1 and 1S_0 states. For $L \geq 1$, one expects four states $^3L_{L-1}$, 3L_L , $^3L_{L+1}$ and 1L_1 which become non-degenerate due to fine-hyperfine interactions. Some states have already been discovered.

Electron-positron colliders are well-suited for the study of heavy quarkonia. The triplet S-states are produced directly in

e^+e^- annihilations whereas other states are produced via electromagnetic and hadronic transitions. The present experimental information about the S-states, P-states and their radial excitations for $b\bar{b}$ and $c\bar{c}$ systems are presented below.

Four $b\bar{b}$ resonance, Y ,¹¹ Y' ,^{12,13} Y'' ,¹² and Y''' .¹⁴ have been observed in e^+e^- annihilation. But no singlet S-state (η_b) and $b\bar{b}({}^3D_1)$ state have so far been seen. Four experiments, i.e. CUSB,¹⁵ CLEO,¹⁶ ARGUS¹⁷ and Crystal Ball^{18,19} detectors have observed the 1^3P_J states from the radiative decays of $Y(2S)$. Each group confirmed the existence of χ^1 and χ^2 states. But some discrepancy in the mass of χ^0 state has been reported. However, the mass value reported by ARGUS is possibly more reliable because of their accurate measurement of photon energy. The existence of 2P bound states χ_b^J have been reported by CUSB group²⁰ while the spin-singlet P-state in bottomonium, h_b , has been reported by CLEO Collaboration.²¹

The charmonium states ψ and ψ' have been observed in $p\bar{p}$ annihilation.²² Subsequently, further states, viz. η_c , χ_c^1 and χ_c^2 states were also observed in $p\bar{p}$ annihilation by the R704 Collaboration.²³ Gaiser *et al.*²⁴ have measured the masses of χ_c^J states by studying the process $\psi(2S) \rightarrow \gamma\chi$. The η_c state has been detected²⁴ in the inclusive photon spectra of both ψ and ψ' . In addition, another paracharmonium state η'_c has been discovered by the Crystal Ball group.²⁵ The $\psi({}^3D_1)$ state²⁶ has also been observed in the e^+e^- cross-section. The R704 Collaboration²⁷ has reported some preliminary results about the

spin-singlet state h_c which lies below the ψ' level. A review of particle properties by Particle Data Groups²⁸ gives us the experimental information for different states of the quarkonium systems. The masses of all the confirmed levels have been shown in Table 3.3 and Table 3.4. So far there is no signature of a toponium and the general belief is that the mass of the top quark, if it exists, should be much higher⁷⁴ than 30-50 GeV, as anticipated by UA1 Collaboration.²⁹ The aspects of the quarkonium spectroscopy which are usually studied are discussed briefly in the following :

1) Masses of quarks and quarkonia :

The mass spectrum of a quarkonium family is given by

$$M_n(Q\bar{Q}) = 2M_Q + E_n(M_Q, V) \quad (1.1)$$

where M_Q is the quark mass and E_n is the energy eigenvalue of the non-relativistic Schrödinger equation with the chosen $Q\bar{Q}$ potential $V(r)$. The input mass in this calculation is the mass assigned to the relevant quark flavour. Since free quarks have not yet been seen, it is difficult to measure its mass in the usual way. The quark masses are known indirectly from measurements on hadrons and different measurements may, in principle, lead to different masses. Two different types of masses are usually associated with the quark of a given flavour, the current mass and the constituent mass. The processes involving large momentum transfers are associated with the current mass. For dealing with the static

properties of hadrons, constituent mass is more suitable. It may be noted that inside the hadron, quarks are associated with gluon field. An alternative definition of the constituent quark mass is that it minimizes the effects of gluons and sea quarks in a calculation of the static properties. However, the constituent quark masses are usually treated as free parameters in a potential model calculation of the quarkonium.

ii) Leptonic Width :

The leptonic decay mode of a vector meson is due to $Q\bar{Q}$ annihilation via a virtual photon. The non-relativistic formula for the decay of a quark-antiquark bound state into a lepton pair³⁰ is

$$\Gamma_{ee} = \frac{4 e_Q^2 \alpha^2}{M_{Q\bar{Q}}^2} |\phi(0)|^2 \quad (1.2)$$

Thus the annihilation amplitude is proportional to $\phi(0)$, the value of the wave-function of the $Q\bar{Q}$ state at zero separation. The formula should, however, be corrected for vacuum polarization effect, which has been treated by a number of authors.³¹ In most cases the correction may be included by following the suggestion of Poggio and Schnitzer, viz. by replacing $\phi(0)$ by $\phi(1/M_Q)$, where M_Q is the quark mass. Physically this expresses the simple fact that the annihilation may take place once the particles are within the de-Broglie wavelengths of each other. The measured leptonic

widths of the heavy quarkonia provide useful constraints on the wave-function and hence on the $Q\bar{Q}$ potential.

iii) E1 Transitions :

The transitions of heavier quarkonia to lighter ones take place through either strong or electromagnetic interaction. Some of the electromagnetic transitions which have been observed are $1^3P_J \rightarrow 1^3S_1$, $2^3P_J \rightarrow 1^3S_1$, $2^3S_1 \rightarrow 1^3P_J$, $2^3P_J \rightarrow 2^3S_1$, $3^3S_1 \rightarrow 1^3P_J$ and $3^3S_1 \rightarrow 2^3P_J$. These are all E1 transitions. Some M1 transitions have also been observed but this will not be treated here. The quarkonium wave-function which is approximately determined by the potential model calculation helps in the evaluation of the relevant decay width, Γ_{E1} . The E1 transition rates of the quarkonia are given by

$$\Gamma(n^3P_J \rightarrow \gamma + n'^3S_1) = \frac{4}{9} \alpha e_Q^2 \omega^3 \left(\int_0^\infty R_{n0} R_{n'1} r^3 dr \right)^2, \quad (1.3)$$

$$\Gamma(n^3S_1 \rightarrow \gamma + n'^3P_J) = \frac{4}{3} \frac{2J+1}{9} \alpha e_Q^2 \omega^3 \left(\int_0^\infty R_{n0} R_{n'1} r^3 dr \right)^2 \quad (1.4)$$

where e_Q is the electric charge of the quark, ω represents the photon energy and R , the radial wave-function. This transition occurs between states with opposite parity and the partial width is proportional to the square of an overlap integral involving the

1P and 1S wave-functions. Thus one can measure the overlap between the two wave-functions from the transition rate.

It is now well-known that the electric dipole transition $\psi' \rightarrow \chi$, is relatively suppressed. A number of factors seem to contribute to this effect. The dipole matrix element is influenced by the relativistic wave-function distortions. The quantity $|\langle \chi | r | \psi' \rangle|^2$ is particularly sensitive to the relativistic corrections. This is because the integral is found to be the sum of the two contributions of opposite sign due to the presence of a node in the 2S wave-function. A shift in the wave-function caused by the relativistic corrections reduce the value of the matrix element. The coupled channel effects³² for ψ' which also reduce the overlap between its wave-function and that of the P-wave χ state should also be considered. It is important to note that relativistic corrections in the Y system are much smaller than that in the ψ system.

I.3. Heavy Quarkonia - Theoretical aspects :

For heavy quarkonia, a non-relativistic treatment based on a Schrödinger equation with a static potential should be a very good approximation, so far as the spin-averaged properties are concerned. A number of authors have discussed relativistic spin-dependent calculations making use of this non-relativistic potential. The spin-dependent effects have been obtained by inserting the one-gluon-exchange interaction into the relativistic

wave-equation, such as Bethe-Salpeter³³ (BS) equation with a suitable kernel. The Breit-Fermi reduction^{34,35} of the BS equation leads to the well-known spin-dependent interaction terms. However, the reduction involves an approximation of the BS equation which essentially reduces it to a non-relativistic Schrödinger equation with correction terms including both spin-dependent and spin-independent terms. One of the many well-known difficulties in the treatment of the BS equation is related to the relative time coordinate. A number of authors^{36,37} have solved the BS equation for quarkonium in the 'instantaneous' approximation so that in the centre of mass frame, the relative time coordinate vanishes. This reduces the BS equation to a Salpeter equation.³⁸ A QCD oriented BS equation has been discussed by Mittal and Mitra³⁹ in the 'null-plane' approximation in order to obtain the mass-spectra of heavy as well as light quarkonia. Jacobs *et al.*⁴⁰ also have considered an 'instantaneous' approximation to solve the BS equation in momentum space with vector and scalar kernels but the results seem to be quite complicated. It may be pointed out that the non-relativistic Schrödinger equation with the usual Breit-Fermi correction terms can be derived from the Breit equation itself.

Eichten and Feinberg⁴¹ have described the spin-dependent forces in heavy quark systems by taking into account the non-abelian nature of QCD and using a gauge invariant formalism. They start from the wilson loop where a $1/M$ expansion of the quark propagator is inserted. They relate the spin-dependent parts of

the potential to correlation functions of electric and magnetic field strengths. They assume that the colour-magnetic field interactions vanish at a large distance. Assuming that the long-range part of the potential transforms like a Lorentz scalar, it is argued that the confinement is associated with a purely electric term while the hyperfine potential is associated with the magnetic one. Gromes⁴² also has claimed that the confining potential must be a scalar. Gromes has obtained an important relation connecting V_1 , V_2 and V , $V = V_2 - V_1$, which follows from the Lorentz invariance of the theory where V_1 and V_2 represent the spin-orbit potentials and V gives the non-relativistic potential. Representing the magnetic field by a space-like loop integral and applying the area law, Gromes arrives at the conclusion that the sign of the spin-orbit term is opposite to that obtained by Eichten and Feinberg⁴¹ and in an earlier paper by Gromes⁴³ and is identical with that of Buchmüller.⁴⁴

Using a modified Richardson potential instead of linear plus coulomb form, Moxhay and Rosner⁴⁵ have included the relativistic corrections to the spin-independent potential. Following the approach of EF, they calculated the energies, leptonic widths and dipole transition rates of ψ and Y systems. Some consequences of relativistic effects have been discussed by McClary and Byers⁴⁶ choosing a scalar potential for the confining potential. The sign of the spin-orbit contribution arising from a scalar potential is the same as proposed by Buchmüller⁴⁴ and Gromes.⁴² Pantaleone et al.⁴⁷ also adopt an approach based on the

work of EF to calculate the spin-dependent corrections to the static QCD potential upto α_s^2 term. They extend the formalism for both equal and unequal masses to obtain spin-dependent terms which contain logarithmic as well as inverse powers of the heavy quark mass.

On the otherhand, Gupta, Radford and Repko^{48,49} (GRR) have given a potential which incorporates higher order perturbative corrections to spin-dependent and spin-independent terms of the $Q\bar{Q}$ potential. They have calculated the fine-hyperfine splittings by a perturbative approach. They consider a renormalization scale parameter μ which is fixed by minimizing the effect of higher order terms which occur in a renormalization group improvement of the potential. Their study reveals that the confining potential may be regarded as the effect of a scalar exchange. Using GRR scheme, Igi and Ono⁵⁰ have investigated the properties of $c\bar{c}$ and $b\bar{b}$ states for various values of $\Lambda_{\overline{MS}}$ and observed that the fine-hyperfine interaction is not only sensitive to the choice of the $Q\bar{Q}$ potential but also very sensitive to the value of $\Lambda_{\overline{MS}}$. Fulcher⁵¹ has developed an abbreviated form of GRR scheme which is based on the numerical solution of the Schrödinger equation. He has considered the α_s^2 correction to the static potential but neglected these corrections for the spin-dependent part. He has included a coulomb term in the confinement potential by introducing a join radius, where the running coupling constant stops running. All these modifications yielded better results for the Y system than what one gets in the original GRR potential.

In the BRR renormalization scheme, the perturbative potential not only depends on the renormalization scheme, but also on the renormalization scale parameter. The method generally involves a long calculation. On the otherhand, in the EF approach, the quantity $\Lambda_{\overline{MS}}$ is usually chosen as one of the free parameters. In our calculation, we have followed the EF approach.

I.4. Supersymmetry and bound states :

The supersymmetric theories have been found to offer a very exciting field of research in the context of the unification schemes of the elementary particle interactions. The idea of supersymmetry (SUSY) was initially introduced to solve the gauge hierarchy problem in GUTS. It has also an aesthetic appeal. It is, therefore, not surprising that these theories have opened up a very attractive field of research.

In supersymmetric theories, elementary bosons have supersymmetric fermion partners and vice-versa. The supersymmetric partners have the same mass when SUSY is unbroken. The fermionic and bosonic masses and couplings get related to each other when SUSY is imposed on a theory, thereby reducing the number of free parameters in the theory.

If SUSY is broken, the energy of the lowest lying state need not be exactly zero. One expects SUSY to be approximate in nature, so that the superpartners will be split. One of the important features of a supersymmetric theory is that it predicts

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the existence of associated particles differing by one-half unit of spin. Among the SUSY particles, the gluino, the SUSY partner of the gauge boson gluon, seems to offer a wide range of interactions accessible by the present experimental facilities. Each superpartner carries a good quantum number, R-parity,⁵² which plays the role of a flavour-conserving quantum number. In almost all models, the gluino is expected to be the lightest sparticle, apart from the photino, the spin 1/2 partner of photon. Photino is expected to be a very light particle and its detection is very much difficult due to its relatively weak interaction. In most models, supersymmetric particles may be detected by their decay products. The gluino decays predominantly via the process $\tilde{g} \rightarrow \tilde{00} \tilde{\gamma}$ where photino is assumed to be long-lived. The photino carries off energy and momentum which are not detected.

Like gluon, gluino is a colour octet. Due to this nature and its large coupling strength, the cross-section for production of a gluino pair is twenty times larger than those of the quark pair of equal mass. Since the lifetime is believed to be relatively long, it can possibly be easily identified. The light gluino search can be carried out in Beam dump experiments⁵³ with beam energy ~ 1 TeV. At a hadron collider, gluino pair production occur via gluon-gluon scattering.⁵⁴ The hadron collider is a suitable place for the search of a heavy gluino. The UA1 Collaboration⁵⁵ have analysed the E_T data and have concluded that the mass limit of gluino is greater than 53 GeV. Explicit search for gluino of a mass upto 150-200 GeV at the Fermilab Tevatron have been suggested

by different authors.⁵⁶ Barnett et al.⁵⁷ surveyed the decay modes and signatures of gluinos in minimal supersymmetric model. The important features of these decays and their phenomenological implication have also been discussed.

The gluino, being a self-conjugate majorana fermion, may bind with quarks, gluons and other gluinos and form colour singlet bound states. Like ordinary fermion-antifermion states, the parity of gluinonium ($\tilde{g}\tilde{g}$) state is given by $P = -(-1)^L$ and C-parity positive. The binding energy and wave-function at the origin of $\tilde{g}\tilde{g}$ state are large. Since the gluino mass is heavy, the gluinonium state can be described by the non-relativistic potential model as in the $Q\bar{Q}$ system although some differences occur in the case of gluinos. The differences arise mainly because gluinos are majorana fermions and also because they belong to a colour octet. The short-range part of the potential may be represented as $V_{\tilde{g}\tilde{g}}(r) = \frac{9}{4} V_{Q\bar{Q}}(r)$ where $9/4$ comes from the ratio of quadratic casimir operators for the adjoint and fundamental representations. This is valid only for the short-range part of the potential. Some authors,^{58,59} however, assumed that this relation holds for the entire $\tilde{g}\tilde{g}$ potential. Among other gluino states, the glueballino ($\tilde{g}g$) states may be formed by replacing one gluino in a gluinonium state by one gluon. These states should be experimentally identified more easily than other glue ball states. Mitra and Ono⁶⁰ have studied the properties of the glueballino and hybrino ($\tilde{g}Q\bar{Q}$) states by using a BS model. The discovery of any of these states will be extremely exciting.

I.5. Summary of the work done :

The following gives the chapterwise summary of the work reported in the thesis :

a) In chapter II, the properties of the heavy quarkonia have been studied by the non-relativistic potential model. The potential chosen is the vacuum polarization corrected 2-loop QCD potential supplemented by a confining potential. The energy levels, leptonic decay widths with and without Poggio-Schnitzer correction and E1 transition rates of ψ and Y families are calculated and are found to be in good agreement with the experimental results. The toponium spectroscopy is studied for the expected range of top quark mass (30-70 GeV). Considering the experimental branching ratio, the total width and the hadronic decay width of the χ_b^J states are also determined. Potential model calculations seem to be fairly successful in describing the non-relativistic properties of heavy quarkonia.

b) In chapter III, we discuss the spin-dependent interactions in $Q\bar{Q}$ systems. We note that the newly discovered 1P_1 states of $b\bar{b}$ and $c\bar{c}$ systems, in particular, pose serious problems to the standard models. We present an extensive analysis to show that the standard Breit-Fermi type of interactions cannot fit the experimental results accurately. A modification of the formalism is suggested. A model calculation with a realistic potential is carried out. The potential consists of a 2-loop QCD short-range potential and a scalar long-range potential with an additional term, as suggested

by Lüscher and also supported by lattice-gauge results. The energy level splittings are calculated with this potential. The possibility of including the spin-dependent contribution of a pseudo-scalar exchange is also considered. It is seen that although the modified formalism just about accommodates the trend of data for 1P_1 states, the Breit-Fermi interactions in general lead to severe constraints on the parameters of the potential when one tries to fit the entire range of quarkonia data. The choice of the Breit-Fermi form for the spin-dependent interactions do not allow enough freedom to fit the recent data on 1P_1 levels.

c) In chapter IV, we note that apart from the inadequacy of Breit-Fermi type of interactions for describing the fine-hyperfine splittings of heavy quarkonia, the presence of highly singular terms like $\delta^3(r)$ and $1/r^3$ terms also tend to make the perturbative calculations unreliable. Exact results or bounds, even if weak, will be very useful in this context. Using Martin's techniques, some weak inequalities for the values of the wave-function at the origin for the triplet and singlet S-states have been derived for a large class of $Q\bar{Q}$ potentials, including the recently proposed Gupta's potential. The inequalities could be used to predict bounds for the decay widths of the η_b states. This inequality will be useful in quarkonium spectroscopy.

d) In chapter V, the effect of the long-range confining potential on the two gluino bound states has been studied in a particular potential model. A power law potential has been chosen as the long-range part of the potential. The asymptotically free nature

of QCD suggests that the short-range behavior is due to one-gluon-exchange. From supersymmetric considerations, we may expect that the short-distance part of the $\tilde{g}\tilde{g}$ potential, $V_{\tilde{g}\tilde{g}}$ is $9/4$ times the $Q\bar{Q}$ potential, $V_{Q\bar{Q}}$. However, the relation between long-range parts of $V_{\tilde{g}\tilde{g}}$ and $V_{Q\bar{Q}}$ is completely unknown. We have chosen a long-range $V_{\tilde{g}\tilde{g}}$ potential which is β -times the long-range potential for $Q\bar{Q}$ where β lies in the interval $0.5 \leq \beta \leq 3$. We have applied Martin's techniques to derive some useful inequalities for the value of the wave-function at the origin, $\psi_{\tilde{g}\tilde{g}}(0)$, for a general class of potentials. The results are useful for estimating various decay widths and production cross-sections.

To summarize, our calculations seem to confirm that the phenomenology of the heavy quarkonia are in general consistent with the basic concepts of QCD, although some details are yet to be understood. With the new accelerators becoming operational in near future, a more definite conclusion may hopefully be drawn.