CHAPTER II

A BRIEF SURVEY OF RECENT EXPERIMENTS AND MONTE CARLO SIMULATIONS OF EAS

The Cosmic Rays Extensive Air Shower is a product of multiple generation of particles initiated by the interaction of primary cosmic rays (PCR) incident at the top of the atmosphere. The development of any extensive air shower (EAS) depends on the nature of the PCR initiating the shower and also on the characteristics of the particle interactions at high energies. So, the studies of different components of the EAS have been very useful to investigate the nuclear mass composition, the energy spectrum of the PCR and the mechanism of the particle interactions at high energies. Over the years a number of experiments designed to study one or more than one component of EAS have been performed and are still being performed at different places of the world. However, the study of EAS is an indirect way of observing the interaction of PCR with the upper atmosphere as the target and sampling the resulting cascade of particles deep in the atmosphere. In general, many interactions intervene between the initial collision and the observed cascade particles. So, detailed reconstruction of EAS events with extensive modeling and computer simulation is required to interpret the experimental observations. Different investigators have used different models, proposed for describing the characteristic properties of particle interaction at high energies, to simulate EAS events.

A brief survey of recent experiments investigating the electron and muon components of EAS and their results about these two components are presented here in the first two sections of this chapter. And, a brief account of various models used by different investigators for Monte Carlo simulations of EAS, especially those models whose calculated results were used for a comparative analysis of the measurements obtained in the present experiment, is presented in the last section.

2.1 DISTRIBUTION OF ELECTRONS IN EAS

One of the most important observational features of EAS is the distribution of electrons. The fundamental parameters of the EAS - the shower size (Ne), the shower age (s) and the core position (X_0, Y_0) are estimated by fitting the electron densities directly measured at different points of the shower front to an assumed analytical function, called the structure function which is supposed to describe the distribution of electrons in an EAS.

According to the cascade theory for pure electromagnetic shower, the density (ρ_e) of electrons at a distance 'r' from the shower core is written as,

$$\rho_e\,(Ne, s, r) = \frac{Ne}{r_0^2}\; f\,(s, \frac{r}{r_0})\; m^{-2}$$

where Ne is the shower size, s is the age parameter, r_0 is the Moliere unit (a characteristic unit of length in scattering theory) and f (s, r/r_0) is the lateral structure function of the electrons.

Different forms of the function $f(s,r/r_0)$ have been proposed and used by different investigators. One of the most popular functions which has been used and tested by many investigators of EAS is the NKG (Nishimura-Kamata-Griesen) function (f_{NKG}), first introduced by Greisen [1] to represent the theoretical results of Nishimura and Kamata [2]. The semi empirical formula proposed by Greisen to represent the distribution of electrons in the EAS can be written as;

$$\rho_e\,(Ne,s,r) = \frac{Ne}{r_0^2}\, f_{NKG}\,(\frac{r}{r_0}, s) = c(s)\, Ne\, (\frac{r}{r_0})^{s-2}\, (1+\frac{r}{r_0})^{s-4.5}\; m^{-2}$$

where c(s) is the normalising constant and its expression used in actual evaluation is,

$$c(s) = 0.443\, s^2\, (1.9\text{-}s) \qquad \text{for } s < 1.6$$
$$= 0.366\, s^2\, (2.07 - s)^{5/4} \quad \text{for } s < 1.8$$

Many investigators have used this form of the structure function (f_{NKG}) to approximate their experimental measurements. However, many authors [3-6] have reported problems in using it and have shown that the lateral distribution of electrons in an EAS cannot be well represented by this function with a single age parameter. Tonwar [7] has pointed out the same type of indications on the basis of his analysis of different experimental measurements over a large range of distances from the core of the showers having sizes of 10^4 to 10^8 particles observed both at mountain altitude and sea level. Similarly, Capdevielle and Gawin [8], with their three dimensional simulation of EAS at different altitudes, have investigated in detail the various difficulties of fitting experimental data with NKG function such as shower size under-estimation, variation of age parameter with distances etc.

On the other hand, investigators of Kobe University [9] and Lodz EAS array [10] have reported of observing their experimental measurements of electron densities being well represented by NKG function with single age parameter except for smaller distances i.e. the vicinity of the core. The investigators of Hong Kong University EAS array [11] have reported that their experimental data when fitted with NKG function resulted in the age parameter s=1.35 for core distances <30m and s=0.8 for longer core distances. Similarly, recent measurements obtained with Utah-Michigan-Chicago (UMC) array, as reported by J.C.Van der Velde et al. [12], were observed to be in fairly good agreement with that calculated with NKG function in the primary energy region of $E_0 \sim 5 \times 10^{13}$ to 3×10^{15} eV when the age parameter was chosen as a function of shower size and zenith angle of shower arrival direction.

Besides this problem of age parameter the other problems related with NKG function is that the experimental distribution of electrons is found to be steeper than that predicted by NKG function. Many authors [13-14] have pointed out that to fit their measurements to the NKG function approximately half the Moliere unit ($r_o/2$) must be used.

Hillas and Lapikens [13] with their extensive simulation of electron-photon cascade in the atmosphere taking account of various energy dependence of electron and photon interactions down to an energy of less than 1 MeV have proposed another form of structure function $f_H(r/r_o,s)$, with which the distribution formula can be written as,

$$\rho_e\,(Ne, s, r) = \frac{Ne}{r_o^2}\, f_H\left[\frac{r}{r_o}, s\right] = \frac{Ne}{r_o^2}\, c(s) \left[\frac{r}{r_o}\right]^{a_1+a_2(s-1)} \left[1+\frac{r}{r_o}\right]^{b_1+b_2(s-1)}$$

where c(s) is normalising constant and the parameters obtained by them for the best fit of experimental data were as r_o= 24m, a_1= -0.53, a_2 = 1.54, b_1= - 3.39 and b_2 = 0 for the shower age in the range s=0.6 to s=1.5.

Recently, Bhattacharya et al.[15] have reported of analysing the data collected with NBU air shower array by testing their goodness of fit to different structure functions. Fitting the electron densities measured in the radial distance range of 0-120m to three structure functions proposed by Greisen[1], Hillas & Lapikens[13] and Capdevielle[16], the above authors have reported of observing the mean values of reduced χ^2 as, respectively, 1.81, 1.80 and 1.77 for these three structure functions indicating slightly better agreement of the experimental measurements with the distribution given by the function proposed by Hillas & Lapikens.

Among the recent measurements with Giant Air Shower arrays investigating air showers with large sizes, the investigators of giant air shower array of Akeno (AGASA) that covers an area of 100 Km^2 and has been in operation since Feb.'90

have reported [17] of using another structure function proposed by Linsley [18]. The densities of electrons in the showers initiated by primary CR with energies $E_o > 10^{18}$ eV as measured by the above array were observed to be well represented by the equation proposed by Linsley [18] which is of the form,

$$\rho_e = Ne\, Ce\, \left(\frac{r}{r_o}\right)^{-\alpha} \left(1 + \frac{r}{r_o}\right)^{-(\eta-\alpha)}$$

Taking r_o and α to be respectively 91.6m and 1.2 for the distance range 300 to 1000m, the value of η, considered to be a function of the zenith angle (θ), was determined by fitting as (3.96±0.03) - (1.08±0.14) (sec θ - 1).

Updating their results for larger distances the investigators of AGASA have reported [19] of observing the lateral distribution of electrons becoming steeper beyond 1 km. And, to account for this observation they have introduced an additional term in the structure function and have represented the distribution by the equation written as,

$$\rho_e(r) = C \left(\frac{r}{r_o}\right)^{-1.2} \left(1 + \frac{r}{r_o}\right)^{-(\eta - 1.2)} \left[\left(1 + \left(\frac{r}{1Km}\right)^2\right]^{-\delta}$$

The value of new parameter δ was determined by fitting the measured densities as equal to 0.6 ± 0.1.

2.2 Distribution of Muons in EAS :

The other component of EAS, which has been investigated by many investigators, is the muon component. It carries different information regarding the mass composition of primary cosmic rays and their interaction with the air nuclei. In any air shower, muons spread out to wider area than electrons. The size of an air shower depends strongly on the total energy of the primary while the total number of muons in the shower depends upon the energy per nucleon of the primary initiating the shower. Different features regarding the distribution of muons in the EAS such as, the radial distribution of muons having different energies, the energy spectrum at different radial distances from the core and the variation of total number of muons with the shower size etc. are studied in different EAS experiments.

In the past many experiments at different places e.g. Moscow State University (MSU) in Russia, Kiel in Germany, University of Durham in U.K., Akeno in Japan, TIFR in India, to name only a few, have been performed to study the muon component and different authors [20-32] have reported their results.

Compiling the works of different investigators Greisen [1] had proposed an empirical equation for representing the distribution of muons in an EAS as a function of muon threshold energies (E_μ) and shower size (Ne) as,

$$\rho_\mu(>E_\mu, Ne, r) = \frac{14.4\, r^{-0.75}}{(1+\frac{r}{320})^{2.5}} \left[\frac{Ne}{10^6}\right]^{0.75} \left[\frac{51}{E_\mu + 50}\right] \left[\frac{3}{E_\mu + 2}\right]^{0.14\, r^{0.37}}$$

where E_μ is in GeV, r in meters and ρ_μ is the density of muons m^{-2} with energy $\geq E_\mu$ at a distance r from the shower core.

And, the total number of muons N_μ in the shower as a function of threshold energy and shower size is expressed as,

$$N_\mu(>E_\mu, Ne) = 1.7 \text{ X } 10^5 \left[\frac{2}{E_\mu + 2}\right]^{1.37} \left[\frac{Ne}{10^6}\right]^{0.75}$$

Many workers have used this form of muon distribution popularly known as Greisen function, both in its unchanged and in slightly modified forms.

Among the recent experiments, the investigators of Utah-Michigan-Chicago (UMC) [12,33 & 34] and those of EAS-TOP[35] experiments have reported of observing their experimentally measured muon distribution in EAS being approximated well by the Greisen function.

UMC (Utah-Michigan-Chicago) experiment was performed with Utah-Michigan array and the Chicago air shower array (CASA) located at the site of Fly's Eye installation at Dugway in Utah which has an atmospheric depth of 870g/cm^2. The array and its operation has been described by Sinclair [33]. The investigators of this experiment have reported [12 & 34] that the experimentally measured muon densities out to a distance of 300 m from the core of EAS initiated by the primaries with energy E_0 = 5 X 10^{13} to 3 X 10^{15}eV, fit extremely well with the Greisen function. It has further been observed that the data at larger zenith angles (~45°) also fit well to the same distribution function.

Similar observations have been reported by the investigators of EAS-TOP experiment [35] that is being performed at the height of 2005m a.s.l. in Campo Imperatore on the top of the underground Gran Sasso laboratory in Central Italy. Their measurements of density of muons with energy $E_\mu \geq 2$ GeV in the near vertical EAS of sizes $10^{4.5}$ to 10^6 particles were observed to be well represented by the Greisen formula written in simplified form as,

$$\rho_\mu(r) = K r^{-0.75} (1 + r/r_0)^{-2.5}$$

The value of the parameter r_0 obtained by fitting the experimental measurements is reported to be equal to 662m.

In another experiment that was performed at Chacaltya Cosmic Ray observatory situated at the height of 5200m a.s.l. in Bolivia, Martinic and Aliaga [36] have reported of using a Greisen type distribution function for fitting the experimental measurements. In this experiment, the measured densities of muons in the EAS corresponding to primaries with energy 2 X10^{14} to 6 X 10^{15} eV were fitted to the equation written as,

$$\rho_\mu(r) = K\left(\frac{Ne}{10^5}\right)^{\gamma} r^{-\alpha} \left(1 + \frac{r}{r_0}\right)^{-\beta}$$

Here, the parameter r_0, considering it to be a function of shower size (Ne) and the zenith angle (θ) of the arriving showers, was expressed as,

$$r_0 = 2(A + B \log Ne)$$

where, $A = -(22.15 + 125.47 \operatorname{Sec}\theta)$m.

$B = (5.54 + 25.78 \operatorname{Sec}\theta)$m.

And, taking the value of γ to be 1, the values of other exponents α and β estimated by fitting were, reported to be respectively, 0.44±0.01 and 2.47±0.12.

Furthermore, this Greisen type of distribution function had also been used by the investigators studying air showers of large sizes, the so called giant air showers (GAS). For instance, as reported by Dyakonov et al.[37], the densities of muons with energy $E_\mu \geq 1$GeV associated with EAS of sizes 10^7 to 10^9 measured with Yakutsk array at an atmospheric depth of 1020 g/cm^2, were fitted to the equation-

$$\rho_\mu(r) = K\left(\frac{E_0}{10^{18}}\right)^{0.87} r^{-0.75} \left(1 + \frac{r}{280}\right)^{0.75-b}$$

Taking the coefficient K as,

$$K = 910 \exp[(1 - \operatorname{Sec}\theta)\, 1020/440]$$

the exponent 'b', considered to be a function of zenith angle (θ) of the showers and primary energy E_0 was expressed as,

$$b(\theta, E_0) = b_0 - b_1 (1 - \cos\theta) + b_2 \log(E_0/10^{18})$$

And, the values of the parameter b_0, b_1 & b_2 were found to be

$b_0 = 3.26 \pm 0.03$, $b_1 = 2.28 \pm 0.03$, $b_2 = 0.09 \pm 0.01$

Comparing these measurements of Yakutsk array with those measured by Michigan muon array operating in coincidence with Fly's eye experiment, Cassiday et al.[38] have reported of observing excellent consistency between the two. The authors, using the same muon distribution equation with same values of the parameters obtained by the investigators of Yakutsk array but taking due considerations of the differences in atmospheric depth between Yakutsk ($1020 g/cm^2$) and Fly's eye experimental sites ($870 g/cm^2$), have observed the mean ratio between the two measurements at a distance of 1000m as,

$$< \log (\rho_\mu^{FE} (1000) / \rho_\mu^{Yakutsk} (1000))> \leq 0.017 \pm 0.025$$

implying an agreement within 4%.

Similarly, the measurements of Akeno Giant Air Shower Array (AGASA), too, have been reported of being well represented by Greisen type of distribution. Presenting the results of measurements obtained with 1 Km^2 array (A1) and 20 Km^2 array (A20), a part of the AGASA, during a considerably long period of time up to '90 starting from '81 and '84 respectively for the above two arrays, Hayashida et al.[17] have reported of fitting the measured densities of muons with energy $E_\mu \geq 1GeV$ in EAS initiated by primaries of energies in the range $10^{16.5}$ to $10^{18.5}$ eV to the equation written as,

$$\rho_\mu = N_\mu \frac{C\mu}{r_0^2} (\frac{r}{r_0})^{-0.75} (1+\frac{r}{r_0})^\beta$$

C_μ being the normalization constant, the exponent β obtained by fitting the data for near vertical showers was found to be equal to 2.58±0.02. The parameter r_o, considered to depend upon the zenith angle of the shower was observed to be approximated by the equation,

$$\log r_0 = (0.5 \pm 0.04) (\mathrm{Sec}\,\theta - 1) +(2.43 \pm 0.05)$$

The report further adds, the average total number of muons N_μ, also determined by fitting the measured ρ_μ, was expressed as function of shower size and zenith angle θ, as

$$N_\mu =(2.8 \pm 0.2) \mathrm{X}\, 10^{5+a} (Ne /10^7)^b$$

And, a & b were found to be,

$$a = (13 \pm 0.1) (\mathrm{Sec}\,\theta -1) \quad \text{and}$$

$$b = (0.79 \pm 0.02) + (0.09 \pm 0.05) (\mathrm{Sec}\,\theta - 1).$$

However, the analysis of their subsequent measurements obtained with $100Km^2$ array (A100), which is reported to have become operational from Feb.'90 and is able to measure muon densities at larger distances from the core of showers initiated by higher primary energies, have shown significant deviation of the measured muon densities at distances above 800m from the shower core than that was expected from the previously used Greisen type distribution. To take account of this observation, the authors, Chiba et al. [39] have reported of introducing a new term in the distribution formula used previously as,

$$\rho_\mu = N_\mu \frac{C_\mu}{r_0^2} \left(\frac{r}{r_0}\right)^{-0.75} \left(1+\frac{r}{r_0}\right)^{\beta} \left[\left(1+\frac{r}{800}\right)^{3}\right]^{-\delta}$$

and, the value for the exponent δ of the additional term, determined by fitting the measured data up to a distance of 2000m, was found to be equal to 0.6.

Apart from the Greisen's function the other form of function that was proposed to represent the distribution of muons in an EAS was that of Khrenov and Linsley [40]. On the basis of their study of experimental data collected with magnetic spectrometer of Moscow State University (MSU) they have proposed an empirical equation to represent the density of muons with threshold energies from 5GeV to $6X10^3$ GeV in the air showers of sizes ranging from $3X10^4$ to 10^6 particles as,

$$\rho_\mu(>E_\mu, Ne, r) = \frac{5.10^3}{(E_\mu + 250)^{1.4}} r^{-0.55} \eta^{0.1} \psi^{0.07} \exp\left(-\eta^{0.62} \frac{r}{80}\right) \psi^{0.78}$$

$$N_\mu(>E_\mu, Ne) = 2.10^5 (E_\mu + 250)^{-0.65} \eta^{-1.25} \psi^{0.78}$$

where,

$$\eta = \frac{(E_\mu + 2)}{12} \quad \text{and} \quad \psi = \frac{Ne}{2.10^5}$$

Many investigators of MSU [41-43] studying the distribution of muons with different threshold energies have reported of observing the measured densities of muons to be in good agreement with that calculated with the above equation.

Atraskevich et al.[41], reporting the results of their study of energy spectra and lateral distribution of muons with threshold energies between 10-500 GeV in EAS of size $Ne=10^6$ measured with MSU array incorporating the underground muon magnetic spectrometer installed at a depth of 40m.w.e., have observed the measured energy spectra of muons at different distances up to 100m from the shower core being well represented by Khrenov & Linsley's equation. Similarly, Bazhutov et al.[42] have reported of fitting the measured densities of muons having energies more than 50 to 500 GeV in EAS having different sizes between $1.3X10^4$ to $4X10^6$particles with the same type of equation written in simplified form as,

$$\rho_\mu = K\left(\frac{Ne}{106}\right)^{\alpha} r^{-n} \exp\left[-\frac{r}{r_o(Ne)}\right]$$

where K and n were taken to be of the form,

$$K = \frac{1.3X10^4}{(E_\mu+250)^{1.4}} \quad \text{and} \quad n = 0.55\left[\frac{E_\mu+2}{12}\right]^{0.1}$$

And, the parameters α and r_o, determined by fitting the data for different values of threshold energies E_μ of muons, were found to be,

E_μ(GeV)	50	100	200	500
α	0.78±0.04	0.77±0.05	0.76 ± 0.06	0.77±0.10

$$r_o = 80\left[\frac{E_\mu+2}{12}\right]^{-0.62}\left(\frac{Ne}{10^6}\right)^{0.05} \quad \text{for} \quad 50 \le E_\mu \le 200\text{GeV}$$

$$= 10\pm3 \qquad \text{for} \quad E_\mu = 500 \text{ GeV}$$

They have further indicated in their report that the lateral distributions of muons show weak dependence on the size of the EAS. And, the variation of total number of muons with shower size, when expressed as $N_\mu \sim Ne^\alpha$, resulted in the exponent $\alpha = 0.78$ for the muons having energies $E_\mu \ge 10$ to 500GeV in the EAS of sizes $6X10^4$ to $3X10^6$ particles.

Among the other experiments on muon component of EAS the results of Haverah Park experiment is worth mentioning. Several reports of this experiment [44-49], have described their study of the dependence of the muon content of EAS on energy of the primary CR. Representing the primary energy E_o by different shower size parameters such as S_{50}, which is the response of unshielded scintillation detectors at 50m from the shower axis, or ρ_c , the response of 9 m^2 wide and 120 cm deep water Cerenkov detector placed close to muon detector, they have analysed the variation of muon content of EAS with primary energy E_o for a large region of energy between 10^{15} to10^{19} eV. Assuming the dependence of muon density ρ_μ on ρ_c of the form $\rho_\mu \sim \rho_c^\alpha$ and finding the best fit value of α from the experimental data, they have observed the value of α remaining constant (~0.94) for primary energy region 10^{16} to 10^{19} eV indicating a smooth decrease of muon content of EAS implying no change or very slow change in the primary composition . But, for lower energy region 10^{15} to 5×10^{15} eV they have reported of observing muon content of EAS decreasing more rapidly with primary energy than at higher energies ($>10^{16}$ eV). They have pointed out that α changes from 0.94 at $E_0 > 3 \times 10^{15}$ eV to ~0.75 at $E_0 < 3 \times 10^{15}$ eV and have suggested the possibility of change in the primary mass at this energy region.

2.3 Monte Carlo Simulations of EAS using Different Models

The correct interpretation of different observational features of EAS requires a detailed knowledge of composition of primary cosmic ray (PCR) and the characteristics of particle interaction at high energies. The experimental constraints arising due to the low flux of PCR have limited the direct measurements of its composition only up to energies of about a few hundred GeV per nucleon [50,51]. So, at higher energies the study has to be made with the help of EAS phenomenon. Any attempt to deduce the primary composition from these indirect ways has so far been rather indecisive because of the dependence of the conclusions on the particle interaction model considered. So, a key feature, in any such attempts for determining primary composition or the interpretation of EAS phenomenology as a whole, is a model for particle interactions at high energies.

A model is proposed by taking information from accelerator experiments about the behavior of some parameters which characterizes the interaction of particles and predicting their behavior at higher energies. Any such model proposed to represent the characteristics of high-energy particle interactions has to describe equally well the results of the accelerator experiments and at the same time, predict different energy behavior at higher energy region.

2.3.1 The Scaling Model

Great progress has been made over the past three decades in extending the energy regime in which studies of particle interactions could be made in laboratories. But, it has not been an easy proposition as far as predicting the interaction characteristics in the cosmic rays energy region on the basis of such studies is concerned. In the early seventies, with the proposition of Feynman's Scaling hypothesis [52] and its moderate successes in the then accelerator energy region, it seemed a way out was found. In this model an asymptotic condition is assumed to be reached by $\sim10^{11}$eV such that most of the interaction energy is taken off by a small, constant number of secondaries and hence gives a very definite and straightforward way of extrapolating to higher energies.

The predictions of scaling hypothesis were applied by many investigators in calculating the production and propagation of secondary cosmic rays in the atmosphere. However, the different characteristics of EAS calculated on the basis of simple scaling hypothesis were seen to be in clear contradiction with experimental observations if the primary cosmic rays were considered to have a composition that does not change much from that measured at lower energies [53-58]. The extension of such scaling to EAS energies was seen to require an improbable enrichment of the primary radiation with heavy nuclei. But, the indications of experiments investigating primary composition from EAS studies [59,60] suggested the composition of the primary flux at energies $\sim10^{14}$-10^{16} eV to be proton dominant as at lower energies where direct measurements exist. So, the questions regarding the applicability of the scaling hypothesis in the cosmic ray energy region was raised and evidences against it accumulated, more often, in the case of cosmic ray experiments. However, due to the indirect ways of EAS investigations they were taken to be more suggestive than decisive.

Different suggestions, apart from the enrichment of PCR by heavy components to have the scaling predictions valid in the high energy region, such as scaling coupled with rising interaction cross section with energy [61] and the use of radial scaling variable X_R (defined as E/E_0, where E & E_0 are, respectively, the energies of secondary and primary particles) in place of X_F, the Feynman scaling variable (defined as p_l / p_{max}, where p_l & p_{max} are, respectively, the longitudinal component of the momentum of secondary particles and the maximum available momentum) were suggested [62,63]. Ouldridge and Hillas [61] had shown that the predictions of scaling was not inconsistent with the observations even if normal mixed composition of primary same as that directly measured at lower energies were taken provided that inelastic collision cross sections were assumed to rise with increasing energy. Their assumption of cross section rise with energy E was represented as,

$$\sigma = \sigma_0[1+0.039\{\log_{10}(E/100\text{GeV})\}^{1.8}] \qquad \text{for } E > 100\text{GeV}.$$

The other argument given against the scaling hypothesis was that of its prediction of slow degradation of the particle energy, as a result of which the theoretical calculations based on it showed that the air shower maximum occurred too low in the atmosphere contradicting the experimental observations. So, the secondary multiplicites were thought to rise faster than the logarithmic growth as required by scaling model and hence the models like CKP, fire-ball and Landau models, which assume rapid growth of multiplicity with energy were advocated [64-66]. Greider [64], using different variations of fire-ball or the two component cluster models, had calculated different characteristic features of EAS and comparing these theoretically calculated results with those observed in experiments had reported that models which incorporate stronger energy dependence of the multiplicity produce, in general, better results. In his calculations, taking the dependence of average total multiplicity $\langle n \rangle$ on energy of the form $\langle n \rangle \sim s^{\delta}$, where s stands for center of mass energy squared, δ was assumed to lie between 0.25 to 0.5.

2.3.2 THE SCALE BREAKING MODELS

In the beginning of the eighties, when the experimental results of CERN SPS$p\bar{p}$ collider experiments began to appear, the evidences of scaling violation in the energy region of TeV became stronger. The reports of UA1 and UA5 collaboration experiments [67-68], showing the pseudorapidity (η) distribution of charged particles around the central region of $\eta<5$ measured at CERN over an energy range from $\sqrt{s}$ = 53 to 900 GeV, gave clear indications of violation of Feynman scaling in the so called central region (small X_F). This observation was also supported by CDF collaboration experiments performed with FNAL Tevatron. Their reports [69] also showed that pseudorapidity density of charged produced particles, in terms of $(1/\sigma_{inel})\, d\sigma/d\eta$, increases with the incident energy in the central region of the collision, indicating the violation of the scaling in the central region.

In addition to these quite strong evidences against the scaling violation in the central region, UA5 collaboration results also threw light in the behavior of multiplicity of secondary particles with the rise of incident energy. Alner et al. [70], reporting the multiplicity distribution for non single-diffractive events produced at CM energies $\sqrt{s}$=200 and 900 GeV measured in UA5 collaboration experiments, had indicated that the energy variation of the average charged multiplicity could be represented by a quadratic in ln s or a $s^{0.13}$ dependence.

As a result of these firm indications provided by accelerator experiments, whose energy regime by then had elevated from $\sim 10^{12}$eV (ISR) to $\sim 10^{14}$ (S $p\bar{p}$ S), of scaling violation and the secondary multiplicity not rising as fast as advocated by the earlier 'high multiplicity models', in the mid of eighties a number of so called 'scale breaking models' of particle interaction were proposed. These models had different characteristic features describing the particle interactions at high energies. Some of them violated the scaling very strongly like the one used by Wdowczyk and Wolfendale [71-73] but some of them featured only mild violation of scaling as in the case of 'independent particle emission' model used by Yodh et al.[74]. Similarly, the 'multi-cluster phenomenological model' used by Capdevielle et al. [75,76] violated scaling in both central and fragmentation region but the model (W00) used by Mikocki et al. [77-79] violated scaling only in central region but retained it in the fragmentation region. Again, the model used by Cheung and MacKeown [80] had typical features of strongly violating scaling only for leading nucleon interactions but retained scaling (in radial variable X_R) for all other interactions. Wrotniak and Yodh [81] reporting the results of their extensive Monte Carlo (MC) simulation of EAS with a number of different models have examined the sensitivity of different air shower observables to the changes in interaction models. The models used in their simulation vary from the one with constant interaction cross-section and scale invariant production distribution to a model with rapidly rising cross-section and large violation of scaling in both central and fragmentation region.

The important characteristic features of some of these models, which were utilized for simulating air shower events for studying the muon and electron components by different investigators are summarized here.

Yodh et al. [74], preferring to use radial scaling variable X_R, first introduced by Yen [62], in place of Feyman's scaling variable X_F have used 'independent-particle-emission' model for the simulation of air shower events. In this model, putting an energy dependent term in the X_R distribution, a mild scaling violation was incorporated. Secondary momenta in the center-of-mass (c.m.) system were chosen from the probability distributions obtained from scaled invariant single-particle inclusive cross-sections. Kaons (both charged and neutral) and baryon productions as well as prominent decay modes of pions and kaons were taken into consideration. A large transverse momentum component in the transverse momentum distribution was also included in the simulation. The hadron-air nucleus inelastic interaction cross-section was assumed to increase logarithmically with energy. For nucleus-nucleus interactions superposition model, which considers the interaction of a nucleus having mass number A and energy E_0 to be interaction of A nucleons with energy E_0/A, was used. Analysing the experimental measurements, representing various features of high energy muons in EAS observed in different experiments-Tien Shan($E_\mu \geq 5$GeV), KGF($E_\mu \geq 220$GeV) and MSU($E_\mu \geq 10$GeV), on the basis of the information obtained by simulating the EAS events by using the above model, the authors have indicated evidences of heavy-nuclei dominant composition of primary CR at energies of the order of 10^{15} eV.

Wdowczyk and Wolfendale [82], analyzing the accelerator data representing inclusive spectra of the secondary pions produced in hadron-hadron collisions at different energies E_0 (laboratory system) observed in FNAL ($E_0 \sim 10^{10}$eV) and ISR ($E_0 \sim 10^{12}$eV) experiments, had pointed out indications of scaling violation in these energy region. To estimate the extent of this violation they had proposed of introducing a scale-breaking factor $(s/s_0)^\alpha$ into the expression for the energy spectrum of the secondary particles as,

$$\frac{1}{\sigma_t}\frac{d\sigma}{dp_l\,dp_t} = \frac{1}{E}\left(\frac{s}{s_0}\right)^\alpha f\left[\frac{p_l}{p_{max}}\left(\frac{s}{s_0}\right)^\alpha, p_t\right]$$

where σ_t is the total cross-section for $p\bar{p}$ collision in which a secondary pion has momentum components p_l (longitudinal) , p_t (transverse) and energy E. The parameter α was supposed to represent the degree of scaling violation between the energies $\sqrt{s_0}$ and $\sqrt{s}$ in center of mass system and $\alpha = 0$ corresponded to strict scaling. The value of α, initially estimated by them from the analysis of the experimental data at $E_0 \leq 10^{12}$eV, was found to be equal to 0.13.

Again, in their subsequent analysis [71,72] of data from CERN $Sp\bar{p}S$ collider experiments in the higher energy range $E_0 \sim 2.10^{14}$eV ($\sqrt{s} = 540$ GeV), taking note of indications showing reduction of inelasticity (K) for the production of the charged pions at higher energies, they have introduced a new inelasticity reduction factor $K(s,s_0)$ in the above scale breaking formula as,

$$\frac{1}{\sigma_t}\frac{d\sigma}{dp_l\,dp_t} = \frac{K(s,s_0)}{E}\left(\frac{s}{s_0}\right)^\alpha f\left[\frac{p_l}{p_{max}}\left(\frac{s}{s_0}\right)^\alpha, p_t\right]$$

where $K(s,s_0) = \dfrac{K_{\pi}\pm(s)}{K_{\pi}\pm(s_0)}$

Using this new expression for the energy distribution of secondaries, the authors have reported of observing the values of $K(s,s_0) = 0.6$ and α between 0.18 to 0.20 were best suited to reproduce the experimental measurements obtained at energy $\sqrt{s} = 540$ GeV. They have further pointed out that this increase of scale breaking parameter α from 0.13 to 0.18 at higher energy of $E_0 \sim 2.10^{14}$eV was a clear evidence for continued and in fact enhanced scaling violation with the increase of E_0.

Continuing their investigation in still higher energies, the cosmic rays energy region, they have calculated different EAS characteristics by simulating air shower events using their scale breaking formula. In their calculations [72,73], both the parameters α and $K(s,s_0)$ were considered to change with energy. The values of α for different energies were taken to be $\alpha = 0.173, 0.21, 0.248$, and 0.285 at $E_0 = 10^{14}, 10^{15}, 10^{16}$, and 10^{17} eV respectively. And, $K(s,s_0)$ was taken as,

$$K(s,s_0) = \frac{K_{\pi}\pm(s)}{K_{\pi}\pm(s_0 = 387\text{GeV}^2)} = \left(\frac{s}{s_0}\right)^{-0.061}$$

and $$K_{\pi^0}(s,s_0) = \left(\frac{s}{s_0}\right)^{-0.042}$$

Comparing their theoretically calculated results with those measured in various experiments investigating the different components of EAS and gamma-hadron families at high altitude cosmic ray stations of Kanbala (520g/cm^2), Pamir (594g/cm^2) and Fuji (650g/cm^2), they have shown considerable consistency between the two. They have further pointed out that the results calculated by assuming normal composition of primary CR above $\sim 10^{16}$ eV were in better agreement with those observed in experiments.

The model of particle interaction used by Cheung and MacKeown [80] for their calculations of different EAS characteristics incorporate the scaling violation upto the extent as estimated by Wdowczyk and Wolfendale [71,72] for proton-nucleon interactions at energies greater than 2.10^{13}eV. But, scaling in radial variables were used for the distributions of secondaries in all other hadron interactions. The average charge multiplicity in proton-nucleon collisions was obtained directly from the particle distribution and the resulting energy dependence was seen to be faster than expected from a quadratic in ln s but less than that given by $E^{1/4}$ law. Fluctuations in multiplicity following KNO scaling were incorporated in the model. Five types of secondaries $\pi^{\pm}$, π^{o}, K and B (proton-antiproton and other baryons) were considered. The generation probabilities of these secondaries were assumed to be function of nucleon energy. A gamma distribution was taken for the transverse momenta of secondaries, with a mean $\langle p_t \rangle$, which shows the non-scaling behavior with particle density in rapidity space.

The hadron-air nucleus interaction cross-section ($\sigma_{inel}{}^{hA}$) above 100 GeV was assumed to rise as the squared logarithm of the particle energy and the interaction length λ was taken to be of the form.

$$\lambda(E) = \frac{\lambda(0)}{[a + b \ln^2(E/GeV)]}$$

where a= 0.926, b= 0.00352 and $\lambda(0)$ the interaction length below 100 GeV were- $\lambda_{\pi\text{-air}} = \lambda_{K\text{-air}} = 120 g/cm^2$ and $\lambda_{p\text{-air}} = 90\ g/cm^2$.

The elasticity was taken to be $\langle\eta\rangle_{p\text{-air}} = \langle\eta\rangle_{p\text{-carbon}} = 0.36$ and the multiplicity $\langle n\rangle_{p\text{-air}} = 1.58\langle n\rangle_{p\text{-}p}$.

For heavier incident nuclei partial fragmentation model was used, in which primary nucleus breaks up into fragment nuclei, α-particles and nucleons in the first interaction. A fraction of nucleons released from fragmenting nuclei interact with air target to produce secondaries.

Using the model with features described above, the authors had carried out a three dimensional simulation of proton and iron-initiated EAS events with primary energies in the range 10^{15}-10^{17} eV. Comparing their calculated results of muon fluctuations, muon-electron size correlation and equi-intensity curves of EAS development with those observed in experiments, they have reported of observing overall consistency between the measured and the calculated data when the Fe fraction in PCR was taken to be non-zero but not exceeding 25% in energy region of 10^{15}-10^{17} eV.

Mikocki et al., in their extensive simulation of EAS [77-79] generated by gamma and proton primaries had used a model of particle interaction code named as W00 and incorporated in SHOWERSIM [83]. SHOWERSIM is a modular software system used for the Monte Carlo simulation of cascades initiated by ultra-high energy cosmic rays in the atmosphere and has several default models of particle interactions. The one used by the above authors had features which show scaling below 1 TeV but at higher energies it was mildly violated in the fragmentation region and strongly violated in the central region. The rapidity plateau at 150 TeV (S $p\bar{p}$S collider beam) was found to be 2.3 times higher than at 1.5 TeV (ISR energy range).

In this model the inelasticity distribution was assumed to be independent of energy, on average 0.5 for nucleons and 0.66 for mesons. The multiplicity was increased considerably with energy, e.g. there were 5.84 charged pions per interaction at 1.5 TeV and 12.78 at 150 TeV.

The secondaries considered were only pions and kaons. The fractions of π^o and kaons were assumed to increase with interaction energy. For $E_0 \leq 1$TeV there were 30% of π^o, 10% of kaons and 60% of $\pi^{\pm}$. The fraction of π^o and kaons increased respectively by 4.2% and 0.6% per decade of energy.

The transverse momentum of produced particles was assumed to be energy independent and was sampled according to the distribution function,

$$\frac{\delta N}{\delta p_t} \propto \frac{p_t}{\left(1+\frac{p_t}{w}\right)^4}$$

the value of w was so chosen that the mean value for pions and kaons were 325 MeV/c and 371 MeV/c respectively. The hadron-air interaction cross section was taken to be constant below 0.1 TeV and equal to 280 mb. At higher energies, it was allowed to rise according to the Gaisser parameterization as,

$$\sigma(\mathrm{mb}) = A+B \log^{1.8}(E/0.1)$$

where, E is incident energy in TeV, A=280 mb and B=2.5mb for nucleons and corresponding value of A and B for pions and kaons were 1.41 and 1.55 times lower. The results calculated with this model, representing the shower size fluctuations, lateral distributions of electrons and muons ($E_\mu \geq$ 1, 3 and 10 GeV) in showers initiated by γ-rays and protons are reported in [79].

2.3.3 The Quark Gluon String (QGS) Model

Apart from these so called 'scale breaking' models other type of models like Dual Parton Model (DPM)[84] or Quark Gluon String (QGS) model [85,86], which describe the hadron interaction at high energies with quark-gluon (parton) picture of the hadron structure have also been proposed and used by many authors. The early indications of success of these models in describing the data from accelerator experiments quite satisfactorily and their subsequent developments by inclusion of hadron-nucleus interactions have encouraged many investigators to use these models at cosmic ray energy region for the simulation of EAS events in the atmosphere. Among the different models proposed in recent years, QGS model is seen to be one of the most widely used models for the analysis of different cosmic ray experiments being performed both at sea level and in mountain altitude investigating various EAS components and gamma-family events. A brief survey of simulations performed by different authors using QGS model of particle interactions and results of their comparison with the experimental measurements is presented here.

In QGS model, the multiple hadron production at the high-energy interaction of particles is considered to be the result of creation and breaking of quark-gluon strings, created by gluon exchange and stretching between the two colliding hadrons. Kaidalov and Termartirosyan [85,86] had observed that this model of quark-gluon string together with the theory of super critical pomeron (a pomeron whose intercept of Regge trajectory at the zero momentum transfer $\alpha_P(0) > 1$) can describe the high energy multiple production data obtained from SPS collider experiments. Taking the form of intercept $\alpha_P(0) = 1+\Delta$, the value of parameter Δ initially estimated from the accelerator data was found to be between 0.07 to 0.09. Encouraged by this initial observation, which was further strengthened by subsequent studies of the same authors [87,88], the QGS model was further improved to include hadron-nucleus interactions [89-92]. The interaction cross-section and the inclusive spectra of secondary particles have been calculated (with Δ=0.14 for higher energies) in terms of functions describing the distribution of quarks in colliding hadrons and fragmentation of quarks and diquarks into secondary hadrons. These calculations of inclusive spectra of secondaries with QGS model show violation of scaling both in the central and fragmentation region. Furthermore, as pointed out by Shabelskii [91], this violation gets stronger in the case of hadron-nucleus collisions. The author has pointed out that the secondary spectra from nuclear targets vary with increasing energy much more rapidly than in the case of hadron-nucleon collisions.

Utilizing these information about the QGS model of particle interaction at high energies, Kalmykov and his co-workers [93-95] have calculated different EAS characteristics. And, comparing them with those measured in different experiments, they have reported of observing results of their calculation with normal composition of the primary, as suggested by Simon et al.[96], to be in better agreement with the experimental observations than those calculated with the scaling model. They have also indicated that increasing the value of parameter Δ improves the agreement further. Similarly, Fomin et al.[97], analysing the experimental measurements of different EAS components and gamma family events, have pointed out that inclusion of various other features of high energy interactions such as the productions of other particles in addition to pions, the jet production and nucleus-nucleus interaction characteristics in the model could improve the agreement between the experimental measurements and those calculated theoretically with the model.

So, observing the results of previous studies, which indicated that QGS model could serve as the basic model of particle interaction in the ultrahigh energy region of cosmic rays, Vashkevich et al. [98] have performed an extensive analysis of the measurements obtained in MSU experiments investigating high energy muons ($E_\mu \geq$ 50-500 GeV) in EAS of size Ne~5.10^4 - 3.10^6 on the basis of calculated results obtained by using QGS model. The EAS events were simulated using the characteristics of hadron-nucleus interactions calculated by other authors [90,91,94,95]. In their calculation, the productions of both pions and kaons, whose decay contribute to the number of muons in EAS, have been taken into consideration. The authors, investigating the influence of interaction parameters in the calculated characteristics of EAS, have reported of observing varying sensitivity of different features of muon component to the change in parameters Δ and K (the inelasticity coefficient). The total number of muons N_μ in an EAS and its variation with the size of EAS were seen to be the most sensitive feature in comparison to the lateral distributions and the energy spectra of muons.

In further effort of improving the calculations of EAS characteristics based on QGS model, Kalmykov et al.[99,100] have investigated the effect of charm production and inclusion of jet generation in the base model. Their study shows that the average value of variation in the number of particles at cascade maximum due to charm production is less than 1% if the calculation is performed with QGS model. The variation was seen to increase with the consideration of upper limit of charm particle inclusive spectra, as in the case of Recombination of Quark-Parton (RQP) model [101], but still its value was observed to be less than 10%. The authors [100], have reported of observing better agreement of the calculated results with experimental measurements when the primary composition changing with energy according to diffusion model was considered, which otherwise was seen to deviate considerably if the permanent non-changing composition is assumed. However, they have further observed that the agreement could still be fairly good, even with the permanent composition of primary, if the calculation is performed with inclusion of jet generation in the bounds of QGS model. Representing the variation of total number of muons N_μ with the size of the air shower Ne as $N_\mu \sim Ne^{\alpha}$, the values of exponent 'α' calculated by them with the assumption of different primary compositions are shown in the Table-2.1 along with those observed in different experiments.

Table-2.1: The values of exponent calculated with QGS model and different primary

			Diffusion model	Permanent composition	
Expt.	$\geq E_\mu$ (GeV)	α_{expt}	α_{QGS}	α_{QGS}	$\alpha_{QGS(Jet)}$
MSU	10	0.78±0.01	0.78	0.73	0.76
Akeno	1	0.83±0.01	0.82	0.80	0.82
Tien Shan	5	0.80±0.01	0.80	0.77	0.80
Chacaltya	0.6	0.87±0.03	0.87	0.89	0.90

In recent years, the same groups of investigators [102-104] have further improved the traditional QGS model by including new features of high-energy interactions observed in different experiments. They have pointed out that the fluctuations in EAS generated by primary nuclei exceed the predictions of superposition model, which was used earlier for calculations of nucleus-nucleus interactions. Hence, the configuration of nucleus-nucleus interactions was simulated directly according to Glauber approach. The diffractive dissociation of nucleons was included in the model. Fragmentation of the spectator part of the projectile nucleus was taken care by Percolation-evaporation model [105]. In hadron-nucleus interactions the fluctuations associated with the number of interacting nucleons of target nucleus, the number of pomeron strings and the process of string fragmentation have been taken into account. Similarly, taking note of the indications from modern collider data about the increasing influence of the so-called semi-hard processes on high-energy hadron interactions, this mechanism has been included in the model.

However, the authors have pointed out that the inclusion of all these new features in the model only enhances the fluctuations but does not seem to change the average characteristics of EAS significantly. Only at higher energies of $E_0 \rangle 10^{17}$eV the inclusion of semi-hard processes was seen to produce some changes in the calculated characteristics of EAS compared to the previous calculations. Furthermore, to investigate the composition of primary CR, they have compared various experimental measurements representing the fluctuation of muon number in EAS of different sizes and variation of EAS maximum depth X_{max} with primary energy E_0 with those calculated theoretically on the basis of this updated QGS model. And, have reported [102,103] of observing the calculations performed by considering the primaries, whose composition changes with energy according to Peter-Zatsepin diffusion model [106,107] from normal before the knee of the primary spectrum to the one enriched by the heavy nuclei with approximate abundance at E_0~10^{17}eV as p : α : M : H : Fe = 0.14 : 0.14 : 0.25 : 0.24 : 0.23, agree well with the experimental measurements. They have further ascertained that at higher energies if the diffusion process is assumed to stop at some critical energy $E_{crit}(Z) = 3.10^{17}Z$ eV for primary nucleus of charge Z then at 10^{19}eV it would again lead to the normal composition with p : α : M : H : Fe = 0.32 : 0.23 : 0.21 : 0.14 : 0.10. And, the authors have reported of observing the measurements of X_{max} obtained in Yakutsk and Fly's Eye EAS experiments being well represented by the calculated results obtained with such composition of primary CR in the energy region of 10^{15} to 10^{19}eV.

Many other authors, too, have reported about their analysis of experimental data collected by various experimental groups on the basis of calculated results obtained with the QGS model. Slavatinsky [108] and Borisov et al.[109], analysing the data obtained in the experiment 'PAMIR' on gamma-hadron families by X-ray emulsion, carbon and lead chambers, have reported of observing the experimental measurements in good agreement with the results calculated with QGS model with increasing total inelasticity $\langle K_{tot} \rangle$. Similarly, Dunaevsky et al. [110,111] have reported the results of simulation of gamma-families accompanied by EAS based on the same model for the experiment HADRON performed at Tien-Shan installation consisting of emulsion chambers with X-ray films and EAS array. These authors, investigating the correlation between the calculated gamma-family energy characteristics and the total number of electrons in EAS, have reported that experimental data from 'HADRON' experiment do not contradict the calculated results of energy distributions in families simulated with QGS model.

On the other hand, Efimov et al. [112] studying the attenuation lengths of shower size (Ne) and the muon size (N_μ) in EAS initiated by primaries with energy $E_0 > 10^{17}$ eV from the experimental data measured by joint registration of charged particles, muons ($E_\mu \geq 1$ GeV) and the EAS Cerenkov light at Yakutsk array, have pointed out that calculations obtained with the QGS model only qualitatively agree with experimental measurements. They have indicated that the increase of the muon portion up to 20% or stronger growth of multiplicites of secondary particles with energy could improve the agreement between the two.

Capdevielle et al. [113] have reported the results of their comparative analysis of the data from PAMIR and Tien Shan experiments on gamma-hadron families on the basis of the results calculated theoretically using various interaction models featuring different behavior of total inelasticity $\langle K_{tot} \rangle$. Among the different Monte Carlo generators of EAS used by the authors for calculating $h^{14}N$ and h-Pb interactions (h= p, n, π, K, Λ) in the energy range of 10^2 to 10^6 TeV, the MQ generator based on QGS model shows an increase of K_{tot} with energy. The other one, MCP generator [114] based on the semi empirical model proposed by Capdevielle [115] shows decrease of K_{tot} from 0.53 at 100GeV to 0.43 at 10^4 TeV. And, in generator MSF[116] the K_{tot} remains constant about 0.56. The authors have reported of observing the experimental measurements to be closer to the calculations based on QGS model with K_{tot} increasing up to 0.84 at 10^4TeV.

So, the QGS model of particle interactions at high energies is seen to be one of the most widely used and considerably successful in predicting and interpreting the various features of particle interactions both at accelerator and cosmic ray energies. It has been utilised by many investigators for simulating the EAS and gamma-family events to analyse the experimental measurements obtained at both sea level and mountain altitude experiments.

A comparative study of the various features of muons with energy $E_\mu \geq 2.5$ - 100 GeV in EAS of sizes 10^4-10^6 particles measured in the present experiment with some of those calculated and reported by different authors using QGS model is presented in the section 4.6.3 of Chapter IV of this thesis.

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