CHAPTER- XII

SPECTRAL ANALYSIS FOR CONFIRMATION OF THE NATURE OF GRANGER CAUSALITY

12.1 Introduction:

The spectral representation of a stationary time series $\{X_t\}$ essentially decomposes $\{X_t\}$ into a sum of sinusoidal components with uncorrelated random coefficients. In conjunction with this decomposition, there is a corresponding decomposition into the sinusoids of the autocovariance function of $\{X_t\}$. The spectral decomposition is thus an analogue for stationary processes of the more familiar '*Fourier Representation*' of deterministic functions. The analysis of stationary processes by means of their spectral representation is often referred to as the '*Frequency Domain Analysis*' of time series or '*Spectral Analysis*'. It is essential to '*Time Domain*' analysis based on the '*Autocovariance Functions*', but provides an alternative way of viewing the process.

The '*Spectral Analysis*' has been a very useful tool for studying econometric issues like *trad e cycle* separation, *the analysis of co-movements of series*, inspecting *cyclical phenomenon* and highlighting '*lead-lag*' *relations* among series. It also provides a rigorous and versatile way to define formally and quantitatively each series components and by means of filtering. It provides a reliable extraction method. In particular, '*Cross Spectral Analysis*' allows a detailed study of the correlation among series.

In Chapter V through Chapter XI we have undertaken '*time domain*' analysis of the series like budget deficit and trade deficit in the economy of Maldives. The '*time domain*' analysis is based on '*Auto-correlation Function*' and '*Correlogram*' enabled us to examine, '*Stationarity*', '*Integrability*', '*Cointegration*', and '*Granger Causality*' between the variables concerned. We seek to reexamine all these aspects of the concerned macroeconomic variables through the '*Spectral Analysis*' in this Chapter.

12.2 Spectral Estimation: Methodology

12.2.1 Fourier Transformation

Given a function h (t) of real variable t, the Fourier Transformation of h (t) can be defined as

$$H(w) = \int_{-\infty}^{+\infty} h(t)e^{iwt}\,dt \qquad (12.1)$$

provided the integral exists for real w.

A sufficient condition for H(w) to exist is

$$\int_{-\infty}^{+\infty} h(t)dt < \infty$$

If (11.1) is regarded as an integral equation for h(t) given H(w), then a simple inversion formula exists of the form

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} H(w) e^{iwt} dw$$

and h(t) is called the *Fourier Transform* of H(w)

In time series the discrete form of the *Fourier Transform* is used when h(t) is only defined for integral values of t.

Then we have $$H(w) = \sum_{-\infty}^{+\infty} h(t) e^{-iwt} , \qquad -\pi \leq w \leq \pi$$

It is the *Fourier Transform* of h (t), here H (w) is defined only in the interval $[-\pi, \pi]$.The *Inverse Fourier Transform* is given by

$$h(t) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} H(w) e^{iwt} dw \qquad (12.2)$$

12.2.2 Periodogram and Auto-Spectrum

Let us consider a finite series U(j) of length T = N Δt

where N = Number of data

Δt = the sampling periodicity

v_k = the frequency = $\dfrac{k}{N\Delta t}$

tj = the time = j Δt

The Discrete Fourier Transform (DFT) U(k) of U(j) and its inverse (IDFT) for finite series are

$$U(k) = \frac{1}{N} \sum_{j=0}^{N-1} u(j) e^{-i2\pi jk/N} \qquad (12.3)$$

$$U(j) = \frac{1}{N} \sum_{j=0}^{[n-1]/2} u(j) e^{-i2\pi jk/N} \qquad (12.4)$$

where, $k \in [-(N/2), (\frac{N-1}{2})]$ and j = 0,....,N-1

Equation (12.3) can only be an approximation of the corresponding real quantity, since it provides only for finite set of discrete frequencies. The quantity

$P_u(k) = |U(K)|^2$ is the power Spectrum and its estimators is '*Schuster's periodograms*'

$$P_u(K) = \Delta t \sum_{j=-[N-1]0}^{[N-1]} \gamma_{un}(J) e^{-12\pi jk/N}$$

$$= \sum_{J=-[N-1]0}^{[N-1]} \gamma_{un}(J) Cos \frac{2\pi Jk}{N} \qquad (12.5)$$

where, $\gamma_{un}(-J) = \gamma_{un}(J) \quad N^{-1} \sum_{J=-(N-j)}^{N-j} [\{u(J) - u\}\{u(J+J) - u\}]$ (12.6)

and $\gamma_{un}(J)$ is '*Standard Sample Estimation*' at lag J of the '*Auto Co-variance Function*'.

The technique of '*Windowing*' is applied for building a '*Spectral Estimation*' which has a smaller variance than $P_u(k)$. The result of '*Windowing*' is the '*Smoothed Spectrum.*'

$$\hat{S}_u(k) = \Delta t \sum_{J=-(N-1)}^{N-1} w(J)\gamma_{un}(J) Cos \frac{2\pi jk}{N} \qquad (12.7)$$

Here the *Convolution of the Periodogram* $P_u(k)$ with *Fourier Transformation* of $W_M(J)$ is the '*Spectral Window*' $W_M(K)$ of width $M' = M^{-1}$

Thus the '*Smoothed Spectrum*' is nothing but the *Periodogram* seen through a window opened on a covariant interval around k.

12.2.3 Cross Spectrum

Cross Spectrum is obtained by substituting the *Cross-Covariance Function* in equation (12.6) for the *Autocovaiance Function*. Thus we have two time series $u_1(J)$ and $u_2(J)$ and their *Cross Covariance Function* $\gamma_{22}(J) = \gamma 22(-J)$, the *Cross Spectrum* is

$$\hat{S}_u(k) = \Delta t \sum_{J=-(N-1)}^{N-1} w(J)\gamma_{12}(Je) e^{-i2\pi jk/N}$$

$$= \hat{Q}_{12}(k) - i\hat{Q}(k) \qquad (12.8)$$

The real part $\hat{Q}_{12}(k)$ is the '*Cospectrum*' and the imaginary $\hat{Q}_{12}(k)$ the '*Quadrature Spectrum*'.

Here the '*Coherence Spectrum*' is

$$K_{12}(k) = \frac{|\hat{S}_{12}(k)|}{\sqrt{\hat{S}_1(k)}} \qquad (12.9)$$

And the '*Phase Spectrum*' is

$$\hat{Q}_1(k) = \arctan\left(-\frac{\hat{Q}_1(k)}{\hat{Q}_1(k)}\right) \qquad (12.10)$$

Again the '*Gain Spectrum*' is

$$\hat{G}_{12}(k) = \frac{\left|\hat{S}_{12}(k)\right|}{\hat{S}_1(k)} \qquad (12.11)$$

12.3 Features of Spectral Estimation

We have estimated *'Periodograms by Frequency', 'Periodograms by Period', 'Spectral Densities by Frequency', 'Spectral Densities by Periods', 'Cospectral Densities by Frequency'; 'Cospectral Densities by Period', 'Coherence Spectrum', 'Gain Spectrum', 'Phase Spectrum'* for the Analysis. Appropriate care has been undertaken with respect to

a. Aliasing
b. Filtering
c. Tapering
d. Window Closing, and
e. Fast Fourier Transformation (FFT)

12.4 Univariate Periodograms : Nature and Significance

The '*Univariate Periodograms by frequency*' for trade deficit (DTD_t) and budget deficit (DBD_t) are given by the figures 12.1-12.2. Corresponding '*Univariate Periodograms by Periods*' are being presented through Figures 12.3-12.4

Figure 12.1

Periodograms of DTD_t by Frequency

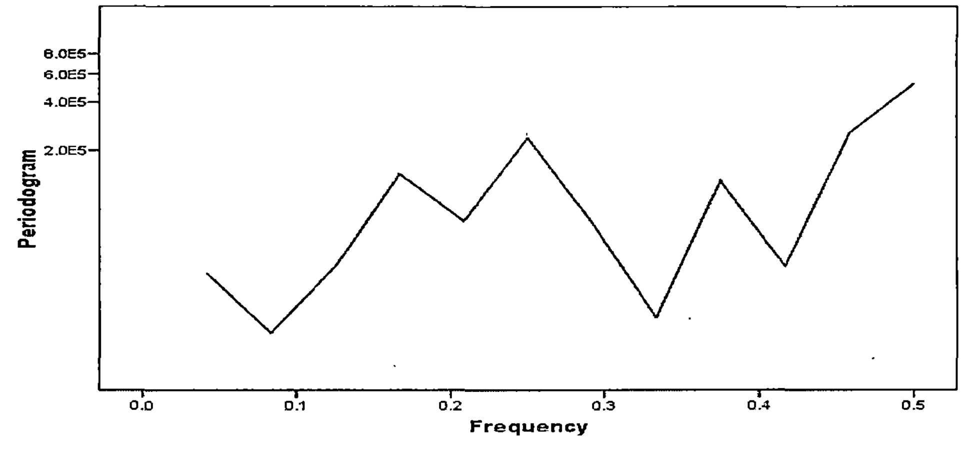


Figure 12.2

Periodograms of DBD$_t$ by Frequency

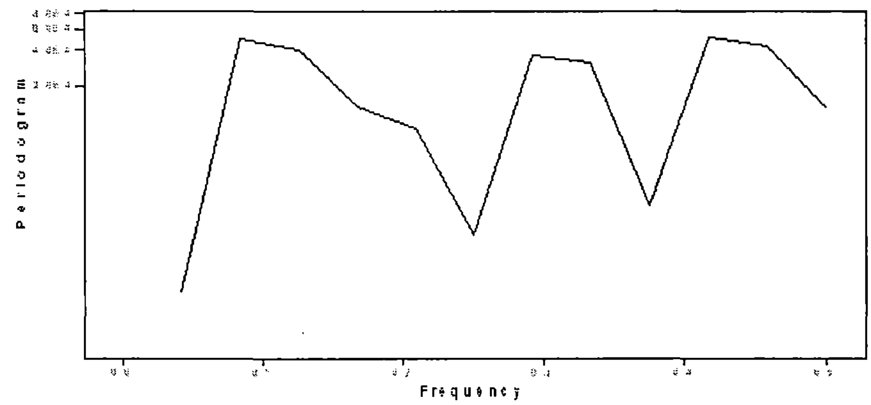


Figure 12.3

Periodograms of DTD$_t$ by Period

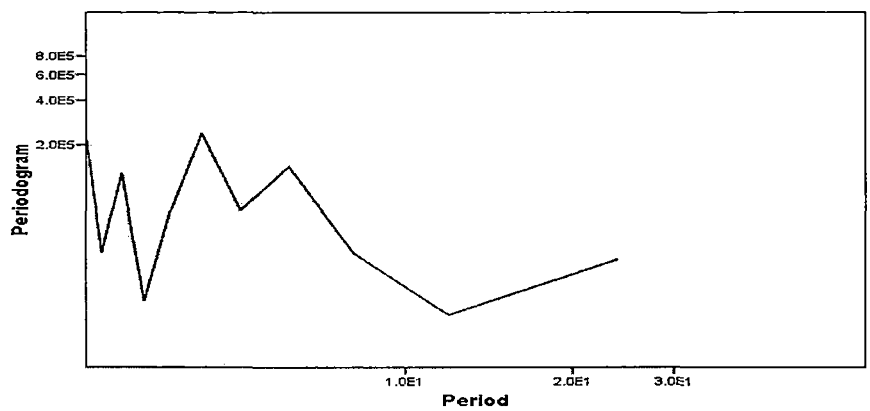
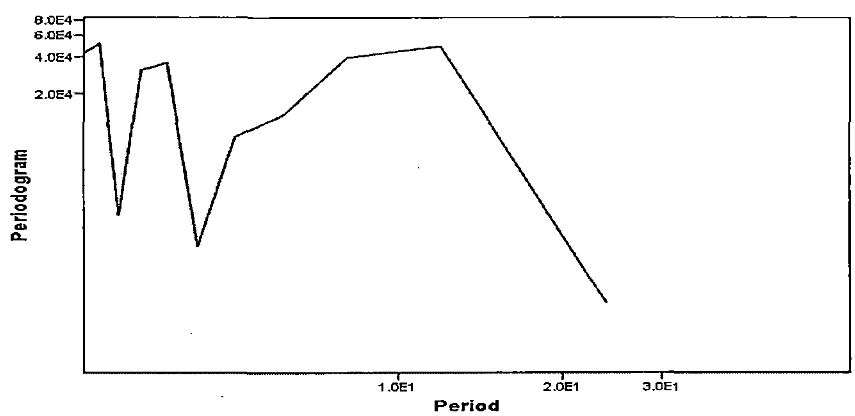


Figure 12.4

Periodograms of DBD$_t$ by Period



12.4.1 'Periodogram by frequency' for Trade Deficit (DTD_t) Series.

The Figure 12.1 indicates that

(i) the *Periodogram* of trade deficit (DTD_t) is not a horizontal straight line and it has several peaks at different frequencies. Consequently, trade deficit series exhibited periodicity in it.

(ii) the existence of several peaks indicates that variances were decomposed without '*uniformity*' over different frequencies. Consequently, the trade deficit series (DTD_t) were not a *'White noise'* one.

(iii) several upswings and downswings in the series across different frequencies testify for the stationarity of the series concerned.

(iv) several upswings and downswings also testify for the presence of AR(p) stochastic structure for the trade deficit series (DTD_t).

12.4.2 'Periodogram By Period' for Trade Deficit (DTD_t) Series

The Figure 12.3 presents the 'Periodogram' of trade deficit series (DTD_t) by periods. It shows

(i) the concentration of ups and downs in the early periods (1-5periods) indicating an AR(p) stochastic structure of the trade deficit series(DTD_t).

(ii) the existence of a prominent and predominant peak at period 3 (or close to it) hints at the possibility of AR(3) structure for the trade deficit series.

This is supported by the finding in our study with the VAR Model in Chapter VIII. The estimated equation (8.2) in the table 8.2 shows AR (3) structure for the trade deficit series.

12.4.3 'Periodogram by Frequency' of Budget Deficit (DBD_t) Series

The Figure 12.2 presents the '*Periodogram of Budget Deficit*' by frequency. It shows that

(i) the '*Periodogram of Budget deficit*' is not a horizontal straight line. Consequently, the budget deficit series displays periodicities.

(ii) the existence of ups and downs indicates that variances were decomposed without '*Uniformity*' over different frequencies.

(iii) the absence of any sharp peak in the '*Periodogram*' and the existence of some plateaus over some frequencies. These hint at the '*White Noise*' property of the series concerned.

12.4.4 'Periodogram by Period' for Budget Deficit (DBD_t) Series

The Figure 12.4 presents the '*Periodogram'* of budget deficit series by period. It shows

(i) the absence of any predominant peak at any period.

(ii) also the existence of *plateau* over some periods. This also testifies for the absence of any AR (p) stochastic structure for the budget deficit (DBD_t) series.

'White Noise' property of the DBD_t series seems to be plausible in view of the insignificant estimated auto-regressive coefficients for DBD_t in equation (8.4) presented in the Table (8.3) for the VAR model. Moreover, the *Correlogram* for the DBD_t series (BD_t series at first difference) as given by the Figure 5.3 in Chapter V also testifies for the absence of any AR (p) structure for the series concerned. The BD_t series is ARIMA (0.1, 0) by nature.

12.5 Spectral Density Representations: Nature and Significance.

Figures (12.5)-(12.6) present the '*Spectral Density by Frequency'* for DTD_t and DBD_t respectively. These figures also indicate that each **'Spectral Density' is free from any noticeable 'Periodicity'. Absence of sharp peaks in the 'Spectral Densities' at regular intervals of frequencies indicates that the Auto-Spectra of the variables (DTD_t and DBD_t) do not exhibit any cyclical behavior.**

Figure 12.5

Spectral Density of DTD_t by Frequency.

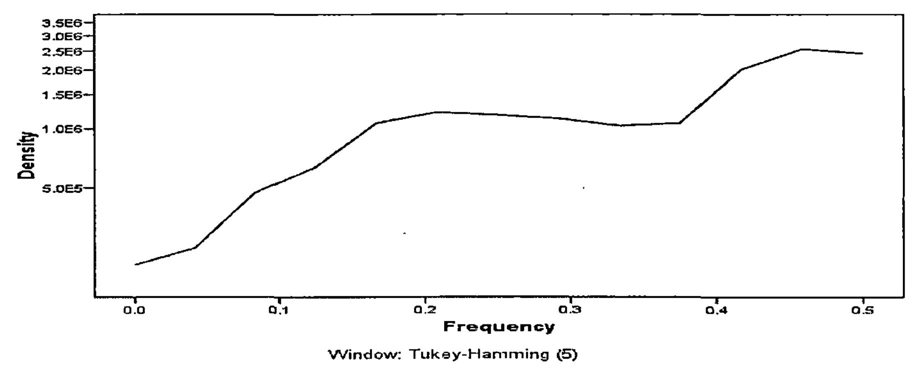


Figure 12.6

Spectral Density of DBD_t by Frequency.

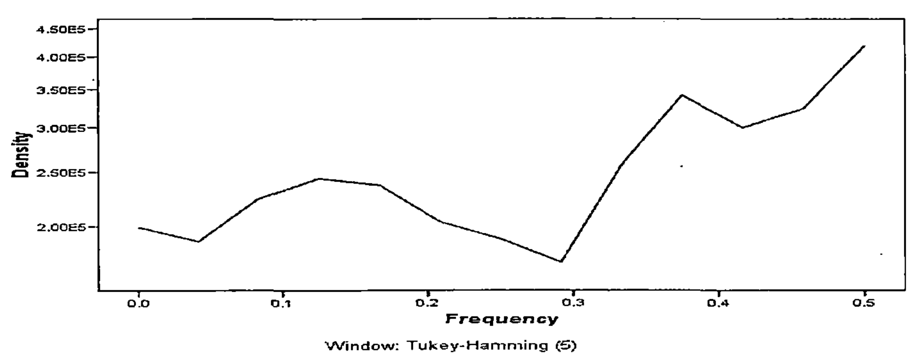


Figure 12.7

Spectral Density of DTD_t by Period

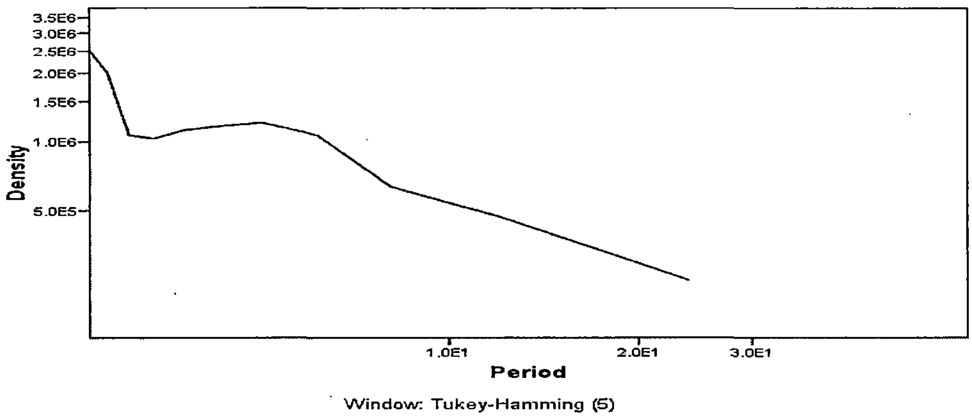
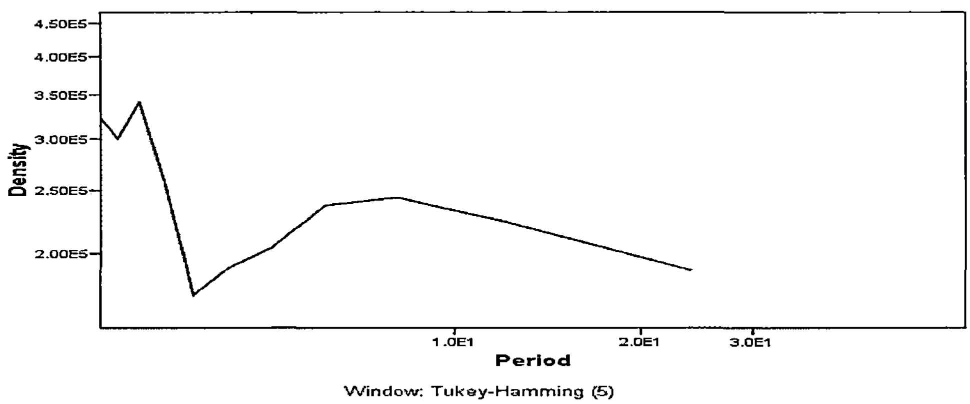


Figure 12.8

Spectral Density of DBD_t by Period



The Corresponding '*Spectral Densities' (Auto-Spectra*) by period for trade deficit and budget deficit series are given by the Figures 12.7 and 12.8. These figure exhibit no '*Periodicity*' in the '*Spectral Densities'* concerned. Cyclical behaviors are also absent in the series.

12.6 The 'Cospectral Densities by Frequency' for Trade Deficit (DTD_t) and Budget Deficit (BD_t)

The '*Cospectral Density by Frequency'* for DTD_t and DBD_t is given by the Figure 12.9 below.

Figure 12.9

Cospectral Density of DBD_t and DTD_t by Frequency

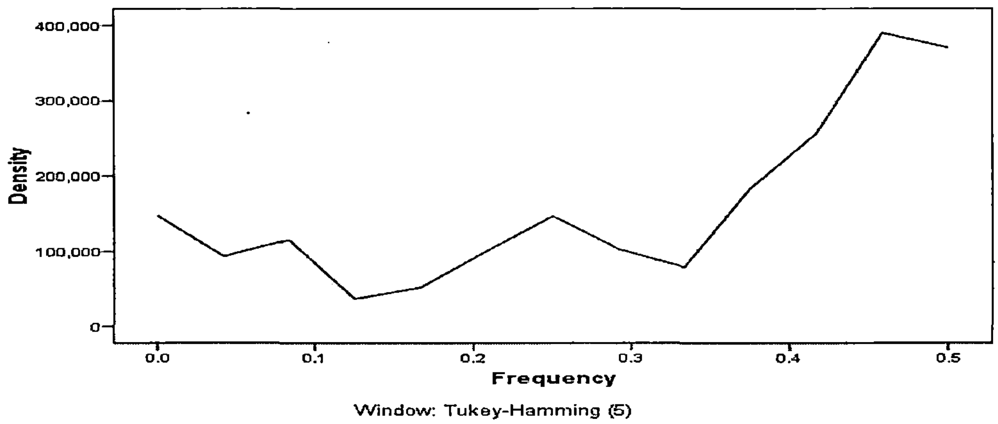
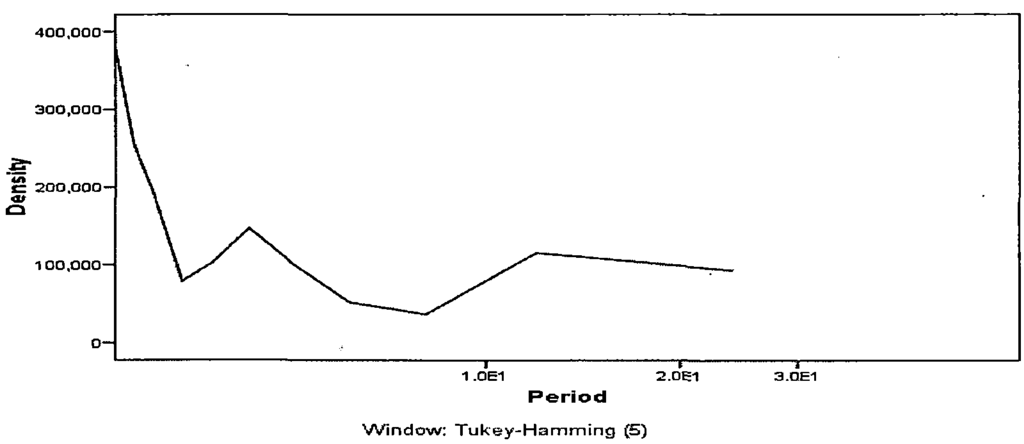


Figure 12.10

Cospectral Density of DBD_t and DTD_t by period



The Figure 12.9 shows that the '*Cospectral Density*'

(i) is not a horizontal straight line

(ii) is marked by the presence of several ups and downs

(iii) contains no sharp peak at any frequency.

The '*Cospectral Density*' by periods for these variables, as given by the Figure 12.10, also exhibit the features stated above.

All these observations indicate that

(i) there are co-movements of DTD_t and DBD_t over the period of study.

(ii) the co-movements is marked by the absence of periodicity.

These findings confirm that **DTD_t and DBD_t are '*Cointegrated*' and the long-run relationship between these variables is 'stable'.**

12.7 'Coherency Spectrum' for DTD_t and DBD_t

The '*Coherency Spectrum*' by frequency for DTD_t and DBD_t is being presented through Figure 12.11 while the Figure 12.12 presents the corresponding '*Coherency Spectrum*' by period.

Figure 12.11

Coherencies of DTD_t and DBD_t by Frequency

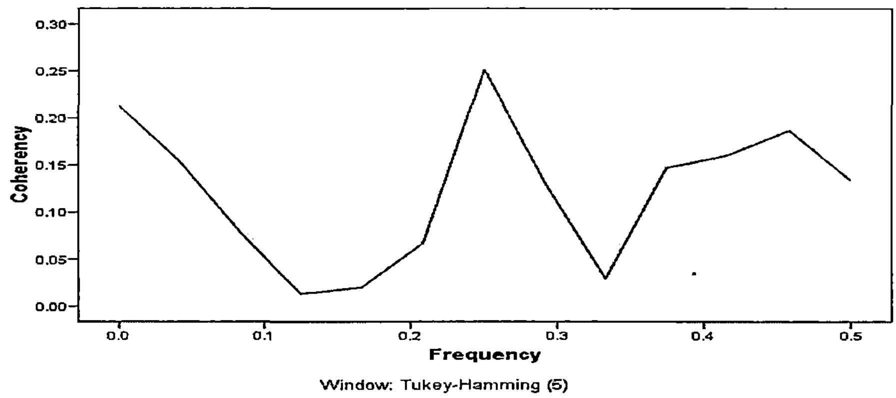
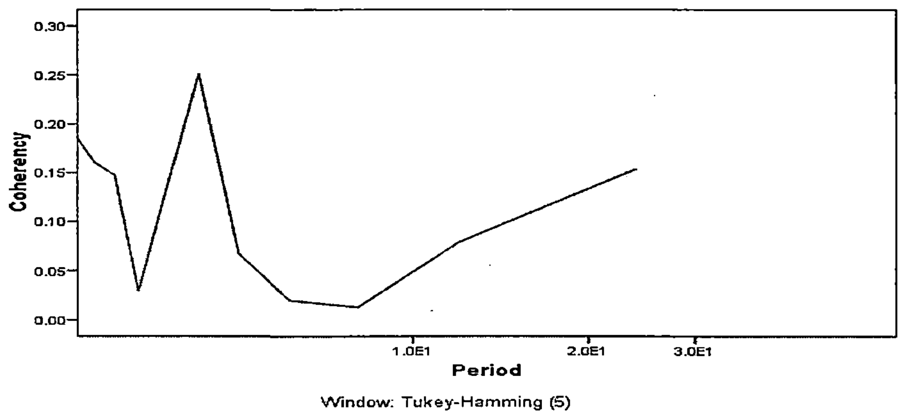


Figure 12.12

Coherencies of DTD_t and DBD_t by Period



The Figure 12.11 shows that

(i) the '*Coherency Spectrum*' of the variables DTD_t and DBD_t is only 0.25 at frequency 0.275 (approx)

(ii) the '*Coherency*' is lower than 0.25 at all other frequencies.

The '*Coherency Spectrum*' in Figure 12.12 correspondingly shows that

(i) the '*Coherency*' is as high as 0.25 at period 3, and

(ii) the '*Coherency*' is lower than 0.25 at all other periods.

All these observations confirm that

(i) **there does not exist high intensity of co-movements of the variables concerned.**

(ii) **there does exist a 'stable' long-run relationship between these series**

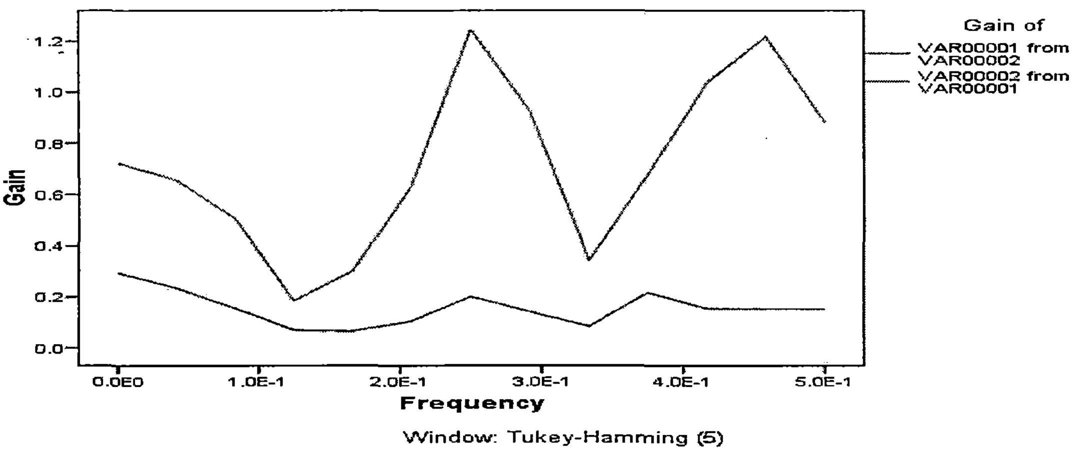
(iii) **no significant 'Periodicity' exists in such relationship over the period of study or over the corresponding frequencies.**

12.8 'Gain Spectrum' For DTD_t and DBD_t

The '*Gain Spectrum*' for DTD_t and DBD_t by frequency is given by the Figure 12.13

Figure 12.13

Gain of DTD_t and DBD_t by Frequency



The '*Gain Spectrum*' in the Figure 12.13 shows that

(i) the '*Gain*' of trade deficit' (DTD_t) from budget deficit (DBD_t) lies over the '*Gains*' of Budget Deficit (DBD_t) from trade deficit across all the frequencies.

(ii) the '*Gain*' of trade deficit (DTD_t) from budget deficit (DBD_t) is about 1.2 at frequency 0.25 (approx).

All these observations indicate that

(i) the contribution of budget deficit (DBD_t) variations to those in trade deficit (DTD_t) was higher than the contribution of variations in trade deficit (DTD_t) to those in budget deficit (DBD_t) at all frequency levels.

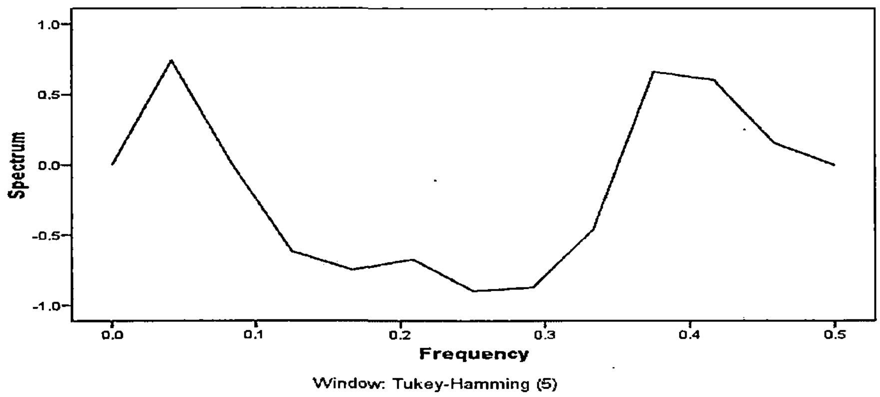
(ii) the highest 'Gain' of trade deficit (DTD_t) is insignificant. So the possibility of '*Unidirectional Causality*' running budget deficit to trade deficit is not appreciable. Consequently, these two variables appear to be independent of each other. This finding confirms '*time domain*' findings of '*absence of Granger Causality*' between budget deficit and trade deficit in Chapters VII through Chapter XI.

12.9 'Phase Spectrum' for DTD_t and DBD_t

The '*Phase Spectrum*' for DTD_t and DBD_t is being presented through the Figure 12.14

Figure 12.14

Phase Spectrum of DTD_t and DBD_t by Frequency



The '*Phase Spectrum*' shows that

(i) the '*phase difference*'[$Q_{12}(k)$] is negative over the frequency range (0.1 -0.350)

(ii) the '*phase difference*' [$Q_{12}(k)$] is positive over the frequency range (0.0-0.1) and (0.350-5)

(iii) the frequency range for the positive '*phase difference*' almost equals the frequency range for the **negative** '*phase difference*'.

All these observations indicate that both the budget deficit and trade deficit (DTD_t) possess almost equal dominance as the '*Lead*' variable over the admissible ranges of frequencies. These findings testify for **the absence of '*Causal relation*' between these variables. Budget Deficit (DBD_t) and Trade Deficit (DTD_t) appear to be '*independent*' of each other.** This finding of '*independence*' is in conformity with our findings in Chapters VIII and XI.

12.10 Overview of Findings in the Spectral Analysis

The overall findings in the 'Spectral Analysis' are as follows:

(i) 'Univariate Periodograms' for DTD_t and DBD_t confirm that DTD_t and DBD_t series are 'Stationary', i.e. $DTD_t \sim I(0)$ and $DBD_t \sim I(0)$. Consequently, $TD_t \sim I(1)$ and $BD_t \sim I(1)$.

(ii) 'Auto-Spectra' confirm absence of periodicity in DTD_t and DBD_t series across different frequencies and the incidence of statistically significant auto-regressive structure for DTD_t.

(iii) *'Auto-Spectrum' for* DBD_t *confirms 'White Noise' property i.e. ARIMA (0, 1, 0) stochastic structure for the series.*

(iv) *The 'Cospectrum' for trade deficit* (DTD_t) *and budget deficit* (DBD_t) *confirms that the series are 'Cointegrated' and the long run relationship between these series is 'stable'.*

(v) *The 'Coherence Spectrum' for* DTD_t *and* DBD_t *series confirms that there exists no strong 'Coherence' in their co-movements over the period of study.*

(vi) *The 'Gain Spectrum' for the series confirms the absence of 'Granger Causality' between these variables.*

(vii) *The 'Phase Spectrum' for the variables further confirm that neither of the variables 'Granger Causes' the other in the economy of Maldives over the period of study.*

(viii) **The 'Ricardian Equivalence Hypothesis' is operationally valid in the economy of Maldives over the period of study.**