CHAPTER –VI

STUDY OF COINTEGRATION

6.1 Introduction

In economic analysis it is imperative to examine the existence of long-run equilibrium relationship among macroeconomic variables concerned. Engle and Granger (1987) established that long run relationship among variables could exist if and only if the variables are '*cointegrated*'. As a matter of fact, '*cointegration*' is the study concerning the existence of long run equilibrium relationship among variables. The study allows the researcher to examine the existence of an equilibrium relationship among two or more time series, each of which is individually non-stationary. According to Engle and Granger, the variables will be *cointegrated* when the linear combination of non-stationary variables is stationary. *Cointegration* provides the elimination of the cost of differencing by rationalizing terms in levels. However the non- stationary variables involved must be *'integrable'* of the same order.

In our study both budget deficit (BD_t) and trade deficit (TD_t) define random walk processes. These variables attain stationarity upon first differencing indicating that these are I(1) variables. It, therefore, becomes pertinent on our part to examine if these variables were cointegrated. The present Chapter is devoted to address this issue.

6.2 Engle- Granger Cointegration Test: The Model

According to Engle and Granger (1987) the variables will be cointegrated when the linear combination of non-stationary variables is stationary. In this study, BD_t and TD_t are found to be integrated of order one i.e. I (1). The linear combination of the two variables are:

$$TD_t = \alpha + \beta BD_t + u_t \qquad (6.1)$$

$$\text{Or, } u_t = TD_t - \alpha - \beta BD_t \qquad (6.2)$$

$$\text{Again } BD_t = \gamma + \delta TD_t + v_t \qquad (6.3)$$

$$\text{Or, } v_t = BD_t - \gamma - \delta TD_t \qquad (6.4)$$

Now if any of u_t and v_t were stationary, i.e. I (0), then BD_t and TD_t are cointegrated at level i.e. CI (1,0). Consequently, the *Cointegration* between BD_t and TD_t may be studied through the estimation of the equations (6.1) or equation (6.3).

6.3 Results of Estimation of the Equation (6.1)

Results of estimation of the equation (6.1) are given below.

$$\hat{TD}_t = -677.2999 + 3.774828\ BD_t$$

S.E: (214.4711) (0.892902)

t- stat. : [-3.1580] [4.2276]

prob.: (0.0044) (0.0003)

$R^2 = 0.4373$

Adj. $R^2 = 0.4128$

Durbin-Watson stat. = 0.4196

Akaike info Criterion =15.89072

Schwarz Criterion = 15.98823

6.4 Test of Stationarity of Residuals

Stationarity of the corresponding residuals $\hat{u}_t$ has been examined through the *ADF* and *PP* tests. Results of the tests are given by Tables 6.1 and 6.2

Table -6.1

Augmented Dickey Fuller Unit Root Test on Residuals ($\hat{\mu}_t$)

Null Hypothesis: *The residual has a unit root*

Exogenous: Constant **Lag length** ;-(Automatic based on SIC, Max Lag =8)

Variable	ADF test statistic	Prob* value	Lag length	Test critical Values 1%	5%	10%
$\hat{\mu}_t$	-1.545442	0.4940	0	-3.737853	-2.991878	-2.635542
Exogenous : None						
$\hat{\mu}_t$	-1.599763	0.1017	0	-2.664853	-1.955681	-1.608793

* Mac Kinon (1996) One- sided P-values

Table - 6.2

Phillips-Perron Unit Root Test on Residuals ($\hat{\mu}_t$)

Null Hypothesis: *The residual has a unit root*

Exogenous: Constant **Bandwidth** ;-(Newey-West using Bartlett kernel)

Variable	P-P test statistic	Prob* value	Band Width**	Test critical Values 1%	5%	10%

$\hat{\mu}_t$	-1.533575	0.4999	1	-3.737853	-2.991878	-2.635542
Exogenous : None						
$\hat{\mu}_t$	-1.601040	0.1015	1	-2.664853	-1.955681	-1.608793

* Mac Kinon(1996) One- sided P-values ** Newey-West using Bartlett kernel

6.5 Findings From the Tables 6.1- 6.2

It is observed from the tables 6.1- 6.2 that,

(i) level since the ADF test statistics (absolute value) falls short of the corresponding initial values at 10 % level of significance.

(ii) *the null hypothesis of unit root* in $\hat{u}_t$ has been accepted even at 10 % level, since the PP test statistics (absolute value) is lower than the corresponding critical value at 10 % level of significance.

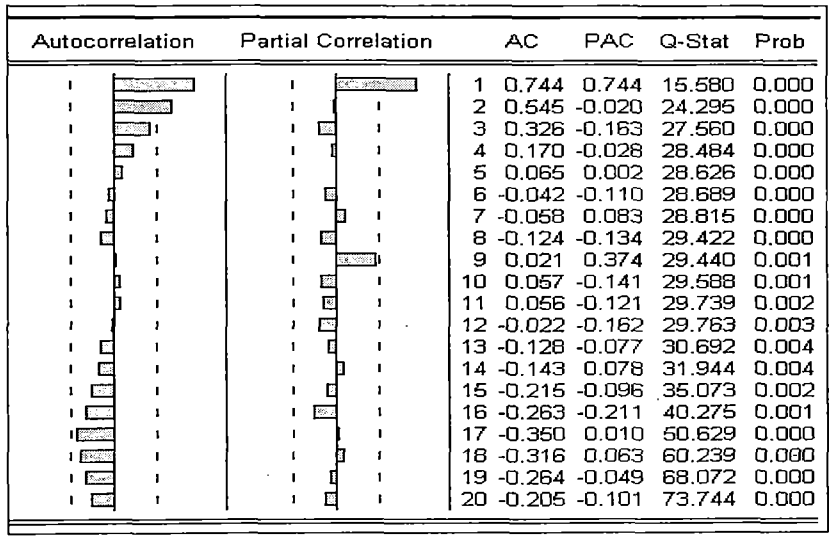
It, therefore, follows that both the ADF test and Phillips–Perron tests testify for 'non-stationarity' of the residuals $\hat{u}_t$. Consequently, BD_t and TD_t are not Cointegrated at level.

6.6 Stationarity of residuals $\hat{u}_t$: Correlogram Study.

The stationarity of residuals $\hat{u}_t$ has been examined through the plots of *Autocorrelation Function* (ACF) and *Partial Autocorrelation Function* (PACF) across different lags. Figure 6.1 presents the Correlogram of the residuals $\hat{u}_t$

Figure 6.1

Correlogram of the $\hat{u}_t$ (at level)



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.744	0.744	15.580	0.000
		2	0.545	-0.020	24.295	0.000
		3	0.326	-0.163	27.560	0.000
		4	0.170	-0.028	28.484	0.000
		5	0.065	0.002	28.626	0.000
		6	-0.042	-0.110	28.689	0.000
		7	-0.058	0.083	28.815	0.000
		8	-0.124	-0.134	29.422	0.000
		9	0.021	0.374	29.440	0.001
		10	0.057	-0.141	29.588	0.001
		11	0.056	-0.121	29.739	0.002
		12	-0.022	-0.162	29.763	0.003
		13	-0.128	-0.077	30.692	0.004
		14	-0.143	0.078	31.944	0.004
		15	-0.215	-0.096	35.073	0.002
		16	-0.263	-0.211	40.275	0.001
		17	-0.350	0.010	50.629	0.000
		18	-0.316	0.063	60.239	0.000
		19	-0.264	-0.049	68.072	0.000
		20	-0.205	-0.101	73.744	0.000

6.7 Findings From the Correlogram

The Correlogram of the residual is given by the Fig 6.1. The plots of the *ACF* and *PACF* of the residual show that

(i) there is declining step spike pattern in the *ACF* before it crosses the base, and

(ii) there exists a significant spike at lag in the *PACF*.

These findings indicate non-stationarity of the residuals $\hat{u}_t$*.Consequently,* BD_t *and* TD_t *series at level are not cointegrated and there exists no long-run relationship between the variables concerned at level.*

6.8 Cointegration Between BD_t and TD_t: The CRDW Method:

An alternative method of testing *Cointegration* among variables is the CRDW Test. The CRDW Test has been performed to examine if BD_t and TD_t were *cointegrated.* The results of the CRDW test along with critical values have been presented in the Table 6.3.

Table-6.3

Results of Durbin –Watson Test Results of Co-integrating Regression (CRDW)

Null Hypothesis	D-W statistic	Critical Values		
		1%	5%	10%
No Co-integration between TD_t & BD_t	0.4196	0.511	0.386	0.322

6.9 Findings From the CRDW Test

The Table 6.3 show that

(i) DW statistics = 0.4196 is below the corresponding critical value at 1% level of significance. Consequently, the '*null hypothesis of the no Cointegration*' between BD_t and TD_t (at level) has been accepted at 1% level.

(ii) BD_t and TD_t (at levels) are, therefore, not cointegrated (at 1% level) and there did exist no long run relationship between BD_t and TD_t (at level).

These findings from the CRDW Test also corroborate the findings from the *ADF* Test, *PP* Test and *Correlogram Analysis.*

6.10 Cointegration Between BD_t and TD_t : Johansen Cointegration Test.

The *Johansen Method of Cointegration* has been performed for further confirmation of our findings of '*No Cointegration*' between BD_t and TD_t at levels. The results of the Test (based on the Johansen Method) have been presented through the Tables (6.3) and (6.4).

Table- 6.4
Johansen Cointegration Test

Unrestricted Cointegration Rank Test **Lags interval** (in 1st differences): 1to 2
Series: BD_t TD_t **Trend assumption**: Linear deterministic trend

Null Hypothesis	Alternative Hypothesis		Eigen Value	Trace Statistic	5% Critical Value	1% Critical Value	Max-eigen statistic	5% Critical Value	1% Critical Value
	λ_{max} tests	Trace tests							
r = 0	r =1	r ≥ 1	0.329967	8.826838	15.41	20.04	8.809421	14.07	18.63
r ≤ 1	r =2	r ≥ 2	0.000791	0.017418	3.76	6.65	0.017418	3.76	6.65

*(**) denotes rejection of the hypothesis at the 5 %(1%) level
Trace test indicates no Cointegration at both 5% and 1% level
Max-eigen value test indicates no Cointegration at both 5% and 1% levels.

Table- 6.5
Johansen Cointegration Test

Unrestricted Cointegration Rank Test **Lags interval** (in 1st differences): 1to 2
Series: BD_t TD_t **Trend assumption**: No deterministic trend (restricted constant)

Null Hypothesis	Alternative Hypothesis		Eigen Value	Trace Statistic	5% Critical Value	1% Critical Value	Max-eigen statistic	5% Critical Value	1% Critical Value
	λ_{max} tests	Trace tests							
r = 0	r =1	r ≥ 1	0.344433	14.88376	19.96	24.60	9.289608	15.67	20.20
r ≤ 1	r =2	r ≥ 2	0.224525	5.594152	9.24	12.97	5.594152	9.24	12.97

*(**) denotes rejection of the hypothesis at the 5 %(1%) level
Trace test indicates no Cointegration at both 5% and 1% level
Max-eigen value test indicates no Cointegration at both 5% and 1% levels.

6.11 Findings From the Table (6.4)

Table (6.4) shows that

(i) trace statistics λ_{trace} = 8.826838 for the *maintained regression equation* with '*linear deterministic trend*' is lower than the critical value at 5% level of significance. Consequently, the '*null hypothesis of no cointegration*' be r = 0 has been accepted at 5% level.

(ii) λ_{max} = 8.809421 for the *maintained regression equation* with '*No Deterministic Trend*' falls short of the critical value at 5% level. Consequently, *the null hypothesis of 'no-cointegration'* (r = 0) has been accepted at 5% level.

These findings testify for the fact that BD_t and TD_t are *not cointegrated* at level and there exists no long-run relationship between BD_t and TD_t when the estimated maintained regression equations contain *linear deterministic trend.*

6.12 Findings From The Table 6.5

The Table 6.5 presents the results of the *Johansen Test* where the *maintained regression equation* contains no '*deterministic trend*'. The Table 6.5 shows that

(i) trace statistic λ_{trace} = 14.88376 for r = 0 is lower than the critical value even at 5% level.

(ii) λ_{max} = 9.28908 for r = 0 falls short of the critical value even at 5% level.

Consequently the *'null hypothesis of no cointegration'* between BD_t and TD_t (at level) i.e. r = 0 is being accepted even at 5 % level.

6.13 Overview of Findings in Sections 6.11 and 6.12

The findings in Sections 6.11 and 6.12 indicate that *there exists no cointegration between BD_t and TD_t at level.* This confirms the findings on cointegration between BD_t and TD_t through the Engle-Granger and *CRDW* methods.

6.14 Engle-Granger Test of Cointegration Between BD_t and TD_t (with First Differenced Data sets): The Model

The *cointegration* between BD_t and TD_t has been enquired into with first differenced data sets. The relevant estimable model is as follows

$$\Delta TD_t = \alpha + \beta\, \Delta BD_t + v_t \qquad (6.5)$$

$$v_t = \Delta TD_t - \alpha - \beta\, \Delta BD_t \qquad (6.6)$$

where,

ΔTD_t = First Difference of Trade Deficit (DTD_t)

ΔBD_t = First Difference of Budget Deficit (DBD_t), and

$$v_t \sim \text{iid } N(0, \sigma^2_u)$$

6.15 Results of Estimation of the Equation (6.5):

The corresponding estimated equation is given by equation (6.7) below

$$\Delta TD_t = -97.812 + 0.577117\ \Delta BD_t \qquad (6.7)$$

S.E : (48.359) (0.4434)

t- stat. : [-2.0225] [1.3015]

prob.: (0.0554) (0.2066)

$R^2 = 0.071$

Adj. $R^2 = 0.029$

Durbin-Watson stat. = 2.348

Akaike Info Criterion =13.8384

Schwarz Criterion = 13.9365

6.16 Stationarity of the Residuals ($\hat{v}_t$).

Stationarity of the residual ($\hat{v}_t$) of the estimated equation (6.7) has been examined through the *ADF* and *Philips –Perron* Tests. Results of such tests are given below by the Tables (6.6) – (6.7).

Table -6.6

Augmented Dickey Fuller Unit Root Test on Residuals ($\hat{v}_t$)

Null Hypothesis: The residual has a unit root

Exogenous: Constant **Lag length**: (Automatic based on SIC, Max Lag =8)

Variable	ADF test statistic	Prob* value	Lag length	Test critical Values 1%	5%	10%
$\hat{v}_t$	-4.691328	0.0014	2	-3.788030	-3.012363	-2.646119
Exogenous : None						
$\hat{v}_t$	-4.841086	0.0000	2	-2.679735	-1.958088	-1.607830

* Mac Kinon (1996) One- sided P-values

Table - 6.7

Phillips-Perron Unit Root Test on Residuals

Null Hypothesis: The residual has a unit root

Exogenous: Constant **Bandwidth** ;-(Newey-West using Bartlett kernel)

Variable	P-P test statistic	Prob* value	Band Width**	Test critical Values 1%	5%	10%
$\hat{v}_t$	-6.318631	0.0000	3	-3.752946	-2.998064	-2.638752

Exogenous : None						
$\hat{v}_t$	-6.547354	0.0000	3	-2.669359	-1.956406	-1.608495

* Mac Kinon(1996) One- sided P-values ** Newey-West using Bartlett kernel

6.17 Findings From the Table (6.6) – (6.7)

The Tables (6.6) – (6.7) show that

(i) absolute values of *ADF Test statistics* exceed the (absolute) critical values at 1% level, when the maintained regression equation is estimated with or with out an exogenous constant (intercept) term. Consequently, the '*null hypothesis of unit root*' in $\hat{v}_t$ is rejected at 1% level and $\hat{v}_t$ is found to be stationary.

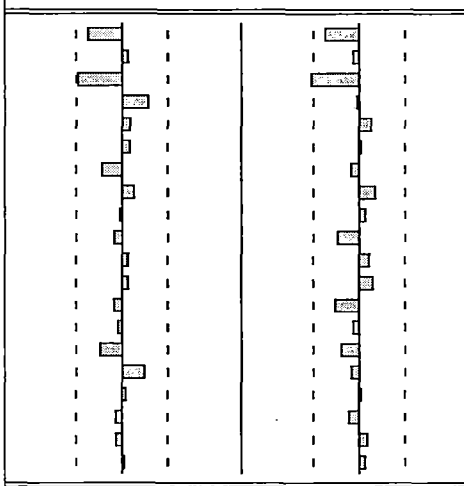
(ii) absolute values of *PP-Test* statistics also exceed the (absolute) critical values at 1% level when the maintained regression equation is estimated with or without an intercept term. As a result, the null hypothesis of unit root in $\hat{v}_t$ is found to be stationary.

Consequently ΔBD_t and ΔTD_t appear to be cointegrated signifying the existence of long-run equilibrium relationship between first differenced series for BD_t and TD_t.

6.18 Stationarity of $\hat{v}_t$ Through the Study of Correlogram:

The stationarity of $\hat{v}_t$ has further been examined through the study of its correlogram. The Figure 6.2 presents the correlogram of $\hat{v}_t$

Fig.- 6.2
Correlogram of Residual (Vt)



	AC	PAC	Q-Stat	Prob
1	-0.294	-0.294	2.3485	0.125
2	0.044	-0.047	2.4024	0.301
3	-0.377	-0.414	6.6292	0.085
4	0.224	-0.019	8.1919	0.085
5	0.062	0.110	8.3192	0.139
6	0.076	0.008	8.5176	0.203
7	-0.170	-0.057	9.5801	0.214
8	0.100	0.137	9.9736	0.267
9	-0.015	0.049	9.9829	0.352
10	-0.060	-0.181	10.143	0.428
11	0.048	0.094	10.254	0.508
12	0.053	0.125	10.401	0.581
13	-0.078	-0.200	10.745	0.632
14	-0.036	-0.050	10.825	0.700
15	-0.181	-0.161	13.103	0.594
16	0.187	-0.064	15.839	0.464
17	0.028	0.009	15.907	0.530
18	-0.043	-0.086	16.102	0.585
19	-0.045	0.062	16.357	0.633
20	0.024	0.055	16.445	0.689

6.19. Findings From the Correlogram of the Residuals ($\hat{v}_t$)

The plots of the *ACF* and *PACF* of the residual ($\hat{v}_t$) show that

(i) the residual ($\hat{v}_t$) dataset display no significant spike in the *ACF* at the first lag.

(ii) the *PACF* is free from any significant spike at the first lag.

These findings confirm stationarity of dataset for ($\hat{v}_t$) .Consequently ΔTD_t and ΔBD_t are Cointegrated.

6.20 Cointegration Between ΔBD_t and ΔTD_t through the Johansen Method.

Results of the corresponding tests are being presented through the Tables – (6.8).

Table- 6.8

Johansen Cointegration Test of ΔBD_t and ΔTD_t

Trend assumption: Linear deterministic trend				
Series: DBD DTD				
Lags interval (in first differences): 1 to 2 Unrestricted Cointegration Rank Test				
Hypothesized No. of CE(s)	**Eigen value**	**Trace Statistic**	**5 % Critical Value**	**1 % Critical Value**
None **	0.574533	24.45897	15.41	20.04
At most 1 *	0.266659	6.513032	3.76	6.65
Hypothesized No. of CE(s)	**Eigen value**	**Max-Eigen Statistic**	**5 % Critical Value**	**1 % Critical Value**
None *	0.574533	17.94594	14.07	18.63
At most 1 *	0.266659	6.513032	3.76	6.65

6.21 Findings From the Table 6.8

(A) It is observed from the Table 6.8 that

(i) for the *null hypothesis of r =0* against r =1, λ_{trace}=24.45 exceeds the corresponding critical values even at 1% level. Consequently the '*null hypothesis of no cointegration*' between ΔBD_t and ΔTD_t has been rejected at 1% level.

(ii) for *the null hypothesis of r=0 against r=1*, λ_{max}=17.9459 exceeds the corresponding critical value at 5% level. Consequently, '*the null hypothesis of 'no cointegration'* between ΔBD_t and ΔTD_t is being rejected at 5% level.

All these observations indicate that ΔBD_t and ΔTD_t are cointegrated at 5% level.

(B) It is further observed from the Table 6.8 that

(i) for *the null hypothesis of* $r \leq 1$ *against* $r=2$ (i.e. there exists not more than one *cointegrating* equation), λ_{trace} = 6.513032 falls short of the corresponding critical value at 1% level and λ_{trace} exceeds the corresponding critical value at 5% level.

(ii) for the *null hypothesis of* $r \leq 1$ *against* $r=2$, λ_{max} = 6.513032 falls short of the corresponding critical value at 1% level though it exceeds the corresponding critical value at 5% level.

All these observations testify for the fact that

(i) ΔBD_t and ΔTD_t *are cointegrated at 5% level*

(ii) there exists only one cointegrating equation at 5% level of significance.

6.22 Summary of the Findings in Chapter VI

Findings in Section (6.2) through Section (6.22) may be summarized as follows.

(i) study of co-integration between budget deficit (BD_t) *and Trade Deficit* (TD_t) *is possible since* $BD_t \sim I(1)$ *and* $TD_t \sim I(1)$.

(ii) BD_t *and* TD_t *series at levels are not cointegrated. Consequently, long-run equilibrium relation between Budget Deficit* (BD_t) *and Trade Deficit* (TD_t) *can not be expected to exist.*

(iii) ΔBD_t *and* ΔTD_t *series are cointegrated. Consequently, a long-run equilibrium relationship exists between first differenced series for Budget deficits* (BD_t) *and Trade Deficit* (TD_t).

(iv) the Cointegration relation between budget deficit and trade deficit is CI(1,1) in the economy of Maldives.
