CHAPTER-IV

DATA ANALYSIS AND ERROR ESTIMATION

4.1 Introduction

The data collected in the present experiment are divided into two categories, shower data recorded on a paper tape by means of data handling system (Figure 3.4) and the momentum of muons by means of magnetic spectrograph systems (Figure 3.3). All the data have been collected between November 1984 to May 1985 and November 1985 to February 1986. During this period 17,067 shower data and 1,229 muon data have been registered. All the data have been analysed in two parts (i) The air shower data have been analysed with the help of a computer, 'WIPRO-Z650' of NBU Computer Centre, with specific FORTRAN programing. (ii) Magnetic deflection of muon is measured by means of 'track simulation' method. The maximum detectable momentum (m.d.m) of the spectrograph have been calculated.

An estimate of errors on shower parameters are obtained by analysing a set of artificially generated showers having fluctuated densities. The errors involved in calculating the momentum of muons are also calculated.

4.2 Shower data analysis

A shower is analysed by computer 'WIPRO-Z650' using an iterative procedure. The observed electron densities in different detectors are fitted to Hillas structure function for lateral distribution of electrons in a shower. From the fitting, the best fit values for the shower core position, size and age parameter are obtained. The procedure is stated below.

The density Δ for a particular detector is related to the printed output D, given by

$$\Delta = D/(S.M) \qquad \dots \qquad \dots \qquad (4.1)$$

where S is the area of that detector and M is a constant whose value is obtained by calibration procedure described in Chapter-III.

The air shower parameters are the shower size N_e, core coordinates (X_o , Y_o) and the age s. Assuming a particular lateral distribution function, these parameters can be calculated by knowing only four of the measured densities. But there are certain things which may alter this picture. There could be local fluctuation in densities. Fluctuation also arises due to statistical nature of response of the detectors. There could be random instrumental errors in the measurement of the densities. A statistical fit to the experimental data is therefore necessary to estimate the shower parameters. The most powerful tool for a statistical fit of the experimental data with expected value is χ^2 minimisation. An iterative process of determining the shower parameters is to adjust the value of the shower parameters such that the value of χ^2, defined by

$$\chi^2 = \sum_{i=1}^{n} W_i \left(\Delta_{o,i} - \Delta_{e,i}\right)^2$$

is minimum. Where $\Delta_{o,i}$ is the density observed at the i th density detector, $\Delta_{e,i}$ is the density expected at the i th detector, W_i is the weightage factor attached with each measurement of the predicted densities, which also include the statistical and instrumental errors and n is the number of detectors that have received an usuable density.

In principle it is possible to obtain the best fit values for the four variables X_o, Y_o, s and N_e by solving four simultaneous equations of the form

$$\frac{\partial \chi^2}{\partial \xi_i} = 0$$

where ξ_i are the above four variables.

However, these equations are highly non-linear and cannot be solved easily. Hence an iterative procedure is used for minimizing the χ^2 using the method of steepest descent. For this iterative procedure the initial estimates of these parameters are necessary.

The initial estimate of the core location is made by using the symmetry of the lateral distribution function. The 'centre of gravity' of the density distribution is given by

$$X_o = \sum_{i=1}^{n} \Delta_{o,i} X_i \Big/ \sum_{i=1}^{n} \Delta_{o,i}$$

$$Y_0 = \sum_{i=1}^{n} \Delta_{o,i} Y_i / \sum_{i=1}^{n} \Delta_{o,i}$$

where X_i, Y_i are the coordinates of the i th detector. The showers arriving at the plane of the detectors require some correction for any inclination. It is ignored as the angular distribution of showers is rather steep. An initial estimate of s is obtained from the logarithmic slope of the density distribution around the density weighted centre. Then, given a set of values for X_o, Y_o and s, the corresponding value of N_e is obtained by solving the equation

$$\frac{\partial \chi^2}{\partial N_e} = 0$$

which gives a cubic equation of type

$$N_e^3 + CN_e + t = 0$$

where C and t are the functions of core location and age (s).

With these initial estimates of X_o, Y_o, s and N_e, the values of χ^2 and the various components of $\nabla \chi^2$ are calculated. The weightage factor W_i is simply given by $W_i = 1/\Delta_{e,i}^2$.

The quantity χ^2 can be thought of as to be a surface in 4-dimensional space (X_o, Y_o, s and N_e). The air shower parameters are obtained by locating the minimum of that surface.

With the value of χ^2 and $\nabla\chi^2$ the initial estimates of X_o, Y_o and s are incremented by an amount proportional to the respective component of $\nabla\chi^2$ in the direction of $-\nabla\chi^2$. With these new X_o, Y_o and s the new χ^2 and $\nabla\chi^2$ are calculated. These new values are accepted if the new χ^2 is less than the old. If the new χ^2 is greater than the old χ^2, the new values are rejected and the increments are scaled down by a factor of two and the process is continued until the difference between the successive χ^2 per degree of freedom is less than 0.01 and the current values of X_o, Y_o, s and N_e are taken as the best fit values of the shower parameters. It is found from the analysis of artificial showers that the above condition ensures that the final χ^2 is very close to the minimum. If the above condition is not reached within 200 iterations, the further minimization is abandoned and the current values of the parameters are accepted, since in such cases it has been found that it oscillates between two values very close to the minimum.

A flow chart for χ^2 minimisation procedure for finding the shower parameters is shown in figure 4.1. A regular check on the analysis is kept by plotting the distribution of normalized χ^2 for each set of shower analyzed. A typical distribution of normalized χ^2 is shown in figure 4.2.

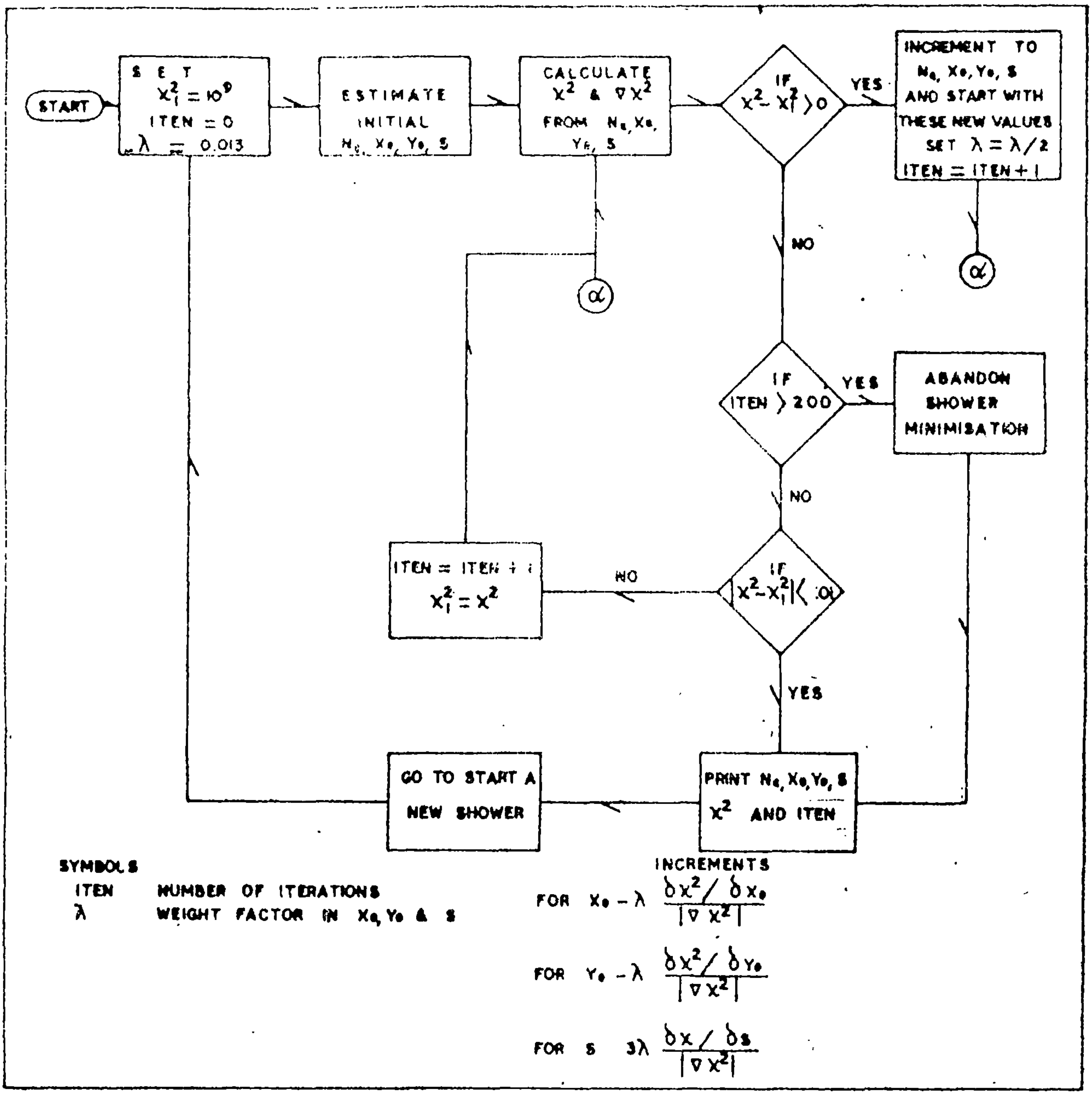


Fig. 4.1 A SIMPLE FLOW CHART FOR χ^2 MINIMISATION PROCEDURE USED FOR THE ANALYSIS OF AIR SHOWER DATA

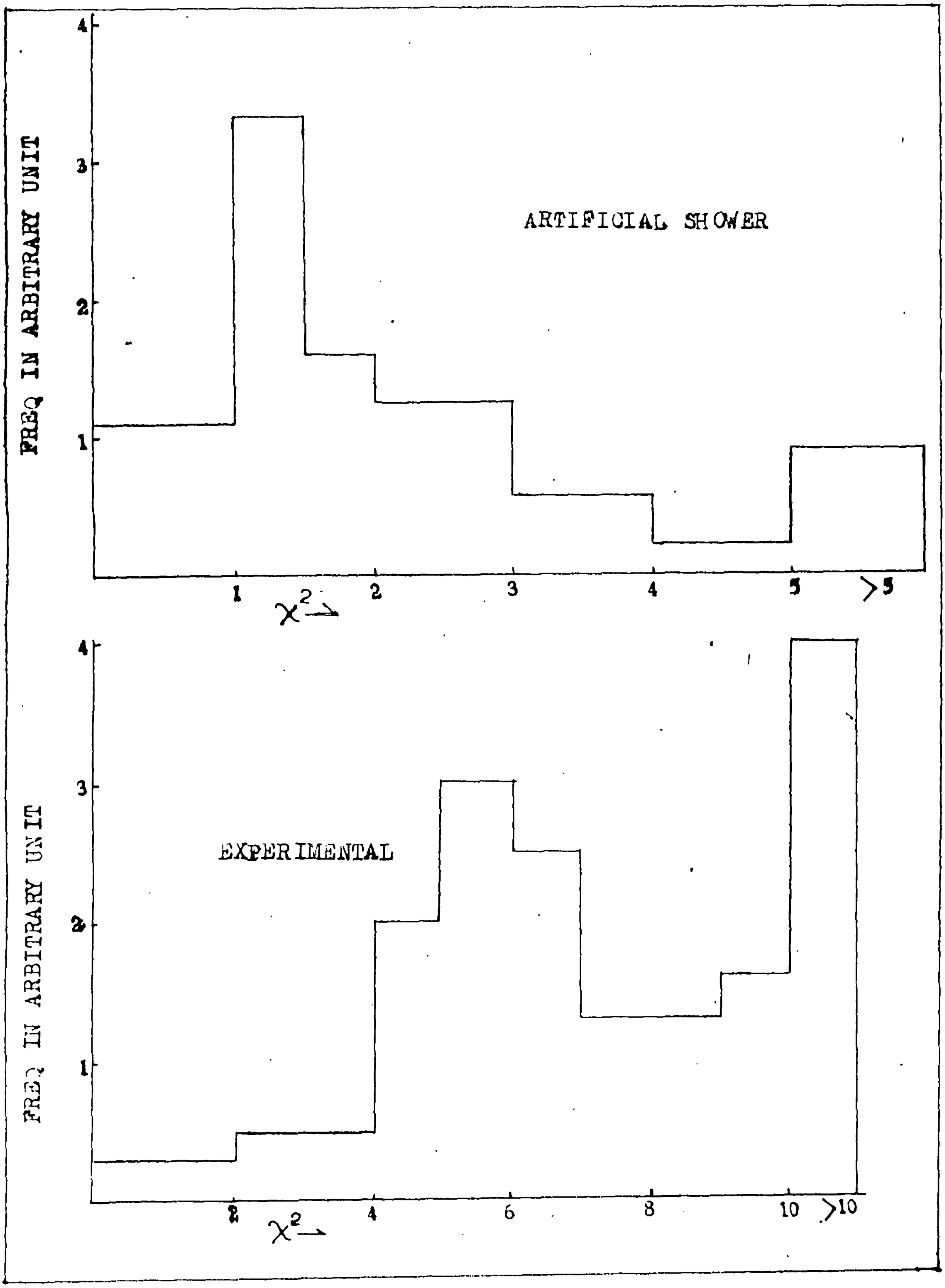


Fig. 4.2 NORMALISED χ^2-DISTRIBUTION FOR OBSERVED AND ARTIFICIAL SHOWERS

4.3 Muon data analysis

The momentum of a muon is determined from the deflection in the magnetic field. The momentum of a particle with charge e, is related to the radius of curvature ρ of its path through a magnetic field B by the equation

$$p = 300\ B\rho\ eV/c \qquad \ldots \qquad \ldots \qquad (4.2)$$

If dl be an element of path normal to the field then

$$\rho = \frac{dl}{d\theta}$$

Hence,

$$\theta = 300 \int \frac{Bdl}{p}$$

Neglecting the energy loss in the material, the momentum p(eV/c) is given by,

$$p = \frac{300 \int Bdl}{\theta} \qquad \ldots \qquad \ldots \qquad (4.3)$$

where B is in gauss, l in cm and Θ is the deflection due to the magnetic field in radians. The angular deflection is calculated from the three or four measured coordinates of the four flash-tube trays (T_1, T_2, T_3, T_4). The coordinates are shown in figure 3.22. The relation between these coordinates and the angular deflection assuming the small angle approximation is given by

$$\Theta = \frac{1}{l_2} \left[(b-a) + (c-d) + (b_o-a_o) + (c_o-d_o) \right] \qquad \ldots \qquad (4.4)$$

for the muon traversing all four trays. If the muon traverse only three trays out of four then equation (4.4) becomes

$$\theta = \frac{1}{l_2}\left[2(b-a) - (c-b) + 2(b_o - a_o) - (c_o - b_o)\right] \qquad \ldots \quad (4.5.a)$$

or, $$\theta = \frac{1}{l_2}\left[(c-b) - 2(d-c) + (c_o - b_o) - 2(d_o - c_o)\right] \qquad \ldots \quad (4.5.b)$$

Therefore, $$p = \frac{1}{\delta}\left[300 \,.\, B \,.\, L \,.\, l_2\right]$$
$$= C/\delta \qquad \ldots \qquad \ldots \quad (4.6)$$

where $\theta = \delta / l_2$

Thus the momentum p is related to the quantity δ, which is directly determined from the coordinates a,b,c,d. For NBU spectrographs B = 16.2 Kgauss, at a current 15 amps, L is the effective magnetic length = 100 cm and thus C = 20.67 GeV/c(t.s.). The coordinates a_o, b_o, c_o, d_o are found from the alignment measurement and a, b, c, d are obtained from the analysis of the photograph for each event. Hence,

$$p(\delta_m \pm \delta_o) = 20.67 \text{ GeV/c (t.s)} \qquad \ldots \quad (4.7)$$

where δ_m is the deflection due to the magnetic field and δ_o is the deflection due to the geometry of the system which depends on the alignment of the spectrograph.

4.4 Maximum detectable momentum of the spectrograph

If the spectrograph is exactly symmetrical about the central line, the trajectory will be as in figure 3.22 i.e., the incident and emergent path should meet at the central line provided that there is no source of error, in measuring δ, and no source of deflection except the magnetic field. But the following three sources of error cause a discrepancy ϵ at the central line of the spectrograph,

(i) track location error due to the finite diameter of the tubes.

(ii) multiple coulomb scattering error.

(iii) loss of energy in traversing the magnet, particularly for low energy particles.

The quantity ϵ determines the accuracy with which measurement of momentum is possible by the spectrograph. For an example, referring to figure 3.22 the

$$\epsilon = X_{ab} - X_{cd} \quad \ldots \quad \ldots \quad (4.8)$$

where $X_{ab} = b - l_2(a-b)/l_1 + b_o - l_2(a_o-b_o)/l_1$

and $X_{cd} = c - l_2(d-c)/l_1 + c_o - l_2(d_o-c_o)/l_1$

Assuming the contribution to ϵ from multiple Coulomb scattering to be small it is possible to derive an expression for m.d.m of the spectrograph taking into account the track location error is the only contribution to ϵ .

Assuming the existance of equal errors σ_i in location of the track in all four trays of flash-tubes the standard error in deflection δ is

$$\sigma_\delta = 2\,\sigma_i$$

Similarly the error in ϵ follows from equation (4.8)

$$\sigma_\epsilon = \sqrt{2}\,\sigma_i \left[(1 + l_2/l_1)^2 + (l_2/l_1)^2 \right]^{\frac{1}{2}}$$

The measured values of l_2 and l_1 are equal to 85 cm

Therefore,

$$\sigma_\epsilon = \sqrt{2}\;\sigma_i\,\sqrt{5}$$

$$= \sqrt{10}\;\sigma_i$$

$$\therefore \quad \sigma_\delta = (2/\sqrt{10})\,\sigma_\epsilon \quad \ldots \quad \ldots \quad (4.9)$$

The m.d.m of the spectrograph is the momentum which corresponds to a deflection equal to the error in the deflection. Hence,

$$p_{m.d.m} = C/\sigma_\delta$$

The m.d.m can also be defined as the momentum which corresponds to a deflection equal to the most probable error on the measured deflection and is given by

$$p_{m.d.m.} = C/0.6745\;\sigma_\delta$$

$$= C/0.4266\;\sigma_\epsilon \ldots \quad \ldots \quad (4.10)$$

The value of $p_{m.d.m}$ for NBU spectrograph is equal to 484($\pm$23)GeV/c for $\sigma_\epsilon = 0.100(\pm 0.005)$t.s. and the lowest value of momentum that can be detectable is 2.5 GeV/c.

4.5 The measurement with zero magnetic field

Without excitation current in the magnets the spectrograph is operated by means of G.M. coincidence (without shower analysing system) for about 15 hours and about 800 particles are collected. The frequency distribution of the deflections is shown in figure 4.3. The distribution is compared with the Gaussian distribution (continuous curve) with the same standard deviation.

The observed deflection is due to (i) an error in location of tracks at each tray and (ii) the multiple Coulomb scattering in the iron. Both the effects cause the deflections positive and negative with equal probability and hence to show the Gaussian distribution with a mean value zero. Sharp Gaussian curve means, the error in alignment of the trays is quite small.

4.6 Momentum measurement technique

(a) Projector method of analysis

To obtain the exact position of the tubes flashed in the trays, first, the films are projected on the vertical boards by a 35 mm film projector. All the boards contain the serial number of the tubes with respect to the fiducial marks. With the

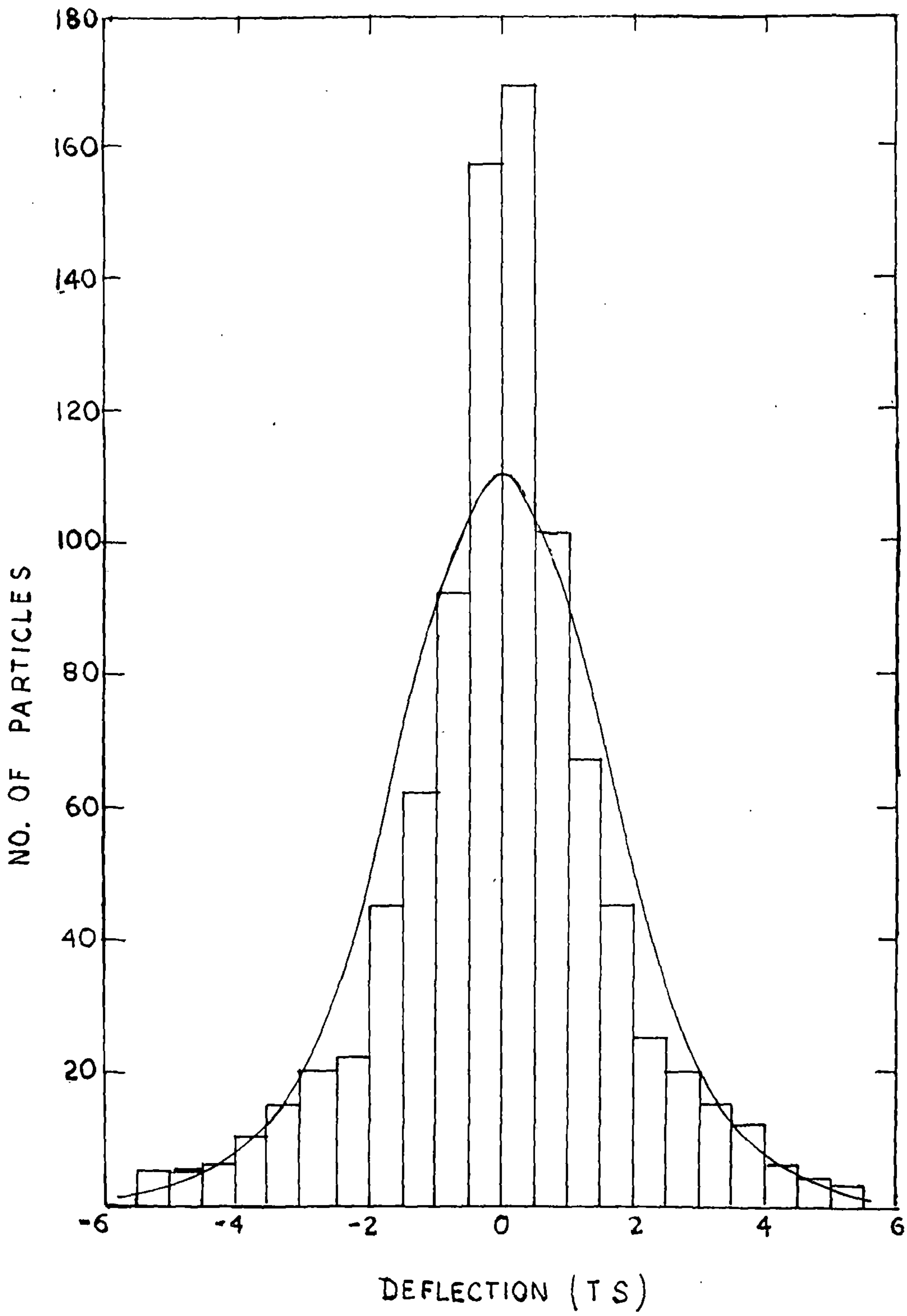


Fig. 4.3 THE ZERO FIELD DISTRIBUTION OF δ

aid of the fiducial marks, images of the flashed tubes are positioned on this reference board and the row number and the column number for each tray are recorded on a data sheet.

(b) Track simulation method of analysis

This technique of analysis was developed by Hyman and Wolfendale[(1)]. This method is quite accurate for high energy particles.

To increase the accuracy in the measurement of a,b,c and d the scale diagram of a portion of flash-tube tray is enlarged by a factor 2.5 in the horizontal direction and by a factor 0.54 in the vertical direction (figure 4.4). A cursor is then adjusted on this scale diagram at an angle previously determined by the projector method of measurement. The mean value of the coordinates of the two extreme limits of the track is then recorded.

From actual distribution of the discrepancy at the centre (ϵ) with the deflection (δ), the standard deviation (σ_ϵ) is calculated to a value equal to 0.100 $\pm$ 0.005 (t.s.). So the m.d.m of the spectrograph by this method of analysis is equal to 484 $\pm$ 23 GeV/c, neglecting the multiple Coloumb scattering and the energy loss in the iron, since these are very small for high energy particles[(2)].

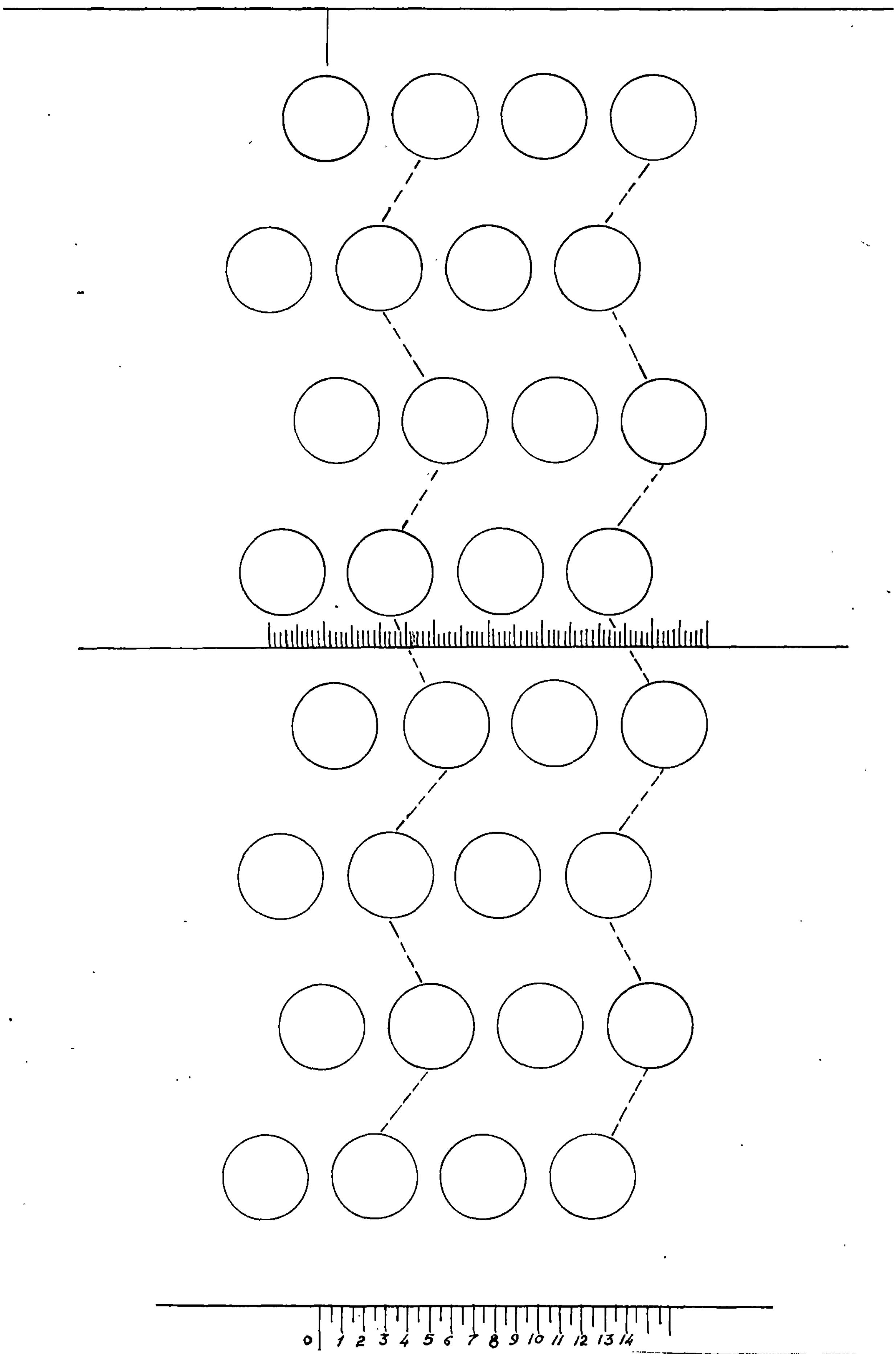
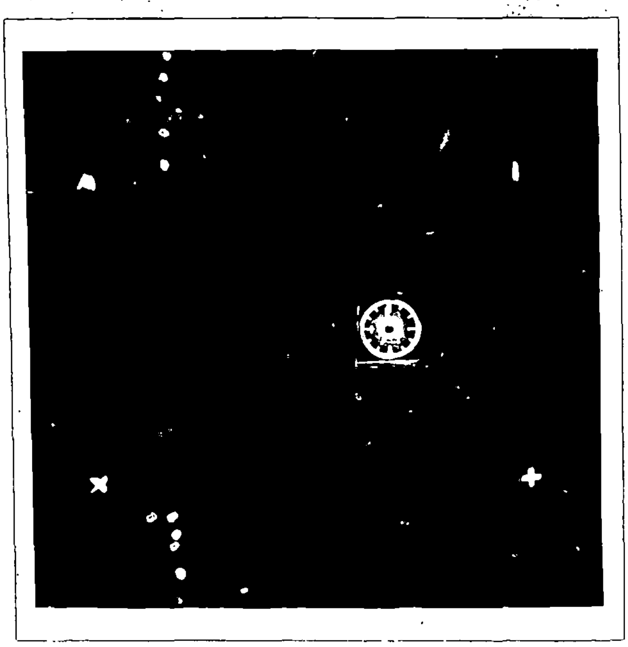
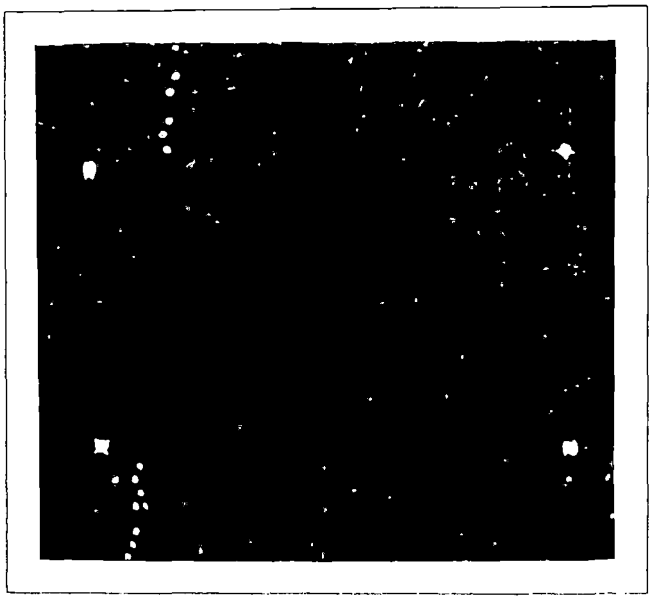


Fig. 4.4 SCALE DIAGRAM OF A PORTION OF FLASH TUBE TRAY

Plate I (a,b) - Plate IV (a,b) : SAMPLE PHOTOGRAPHS OF MUON [ASSOCIATED WITH THE SHOWER] TRAJECTORIES IN THE SPECTROGRAPH UNITS

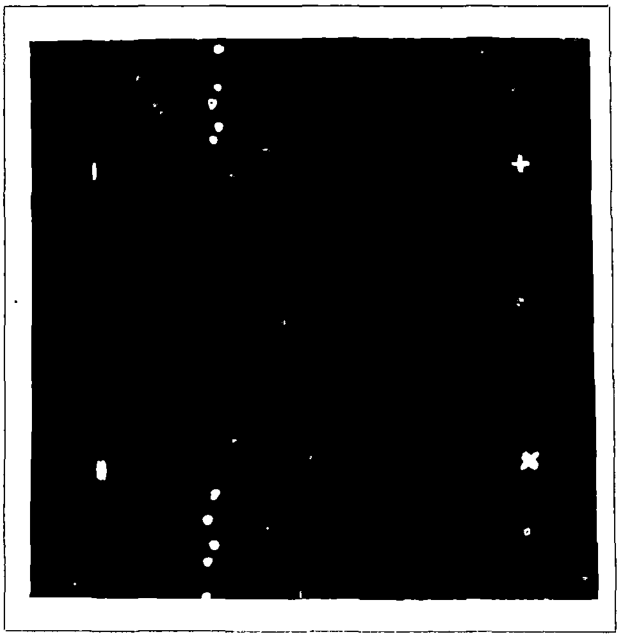


(a) UPPER PART OF THE MAGNET

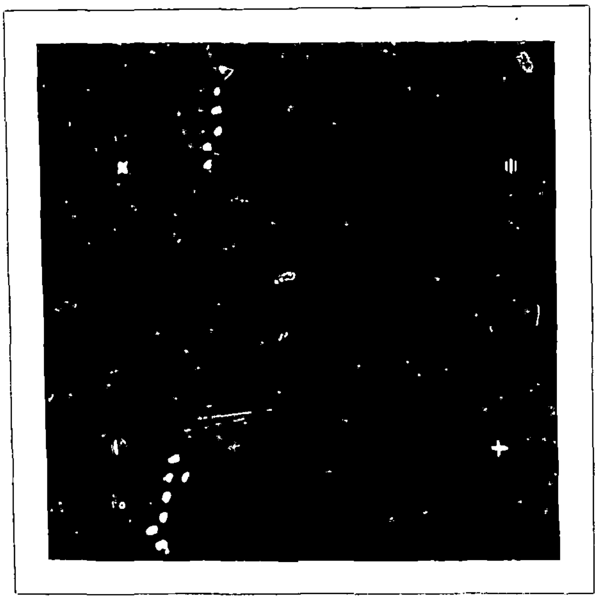


(b) LOWER PART OF THE MAGNET

Plate I(a,b): ONE MUON TRAJECTORY IN ONE LIMB (MAGNET M_1)

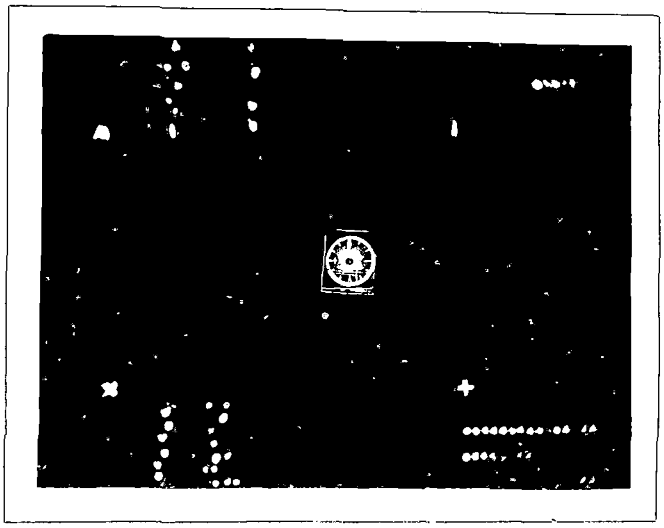


(a) UPPER PART OF THE MAGNET

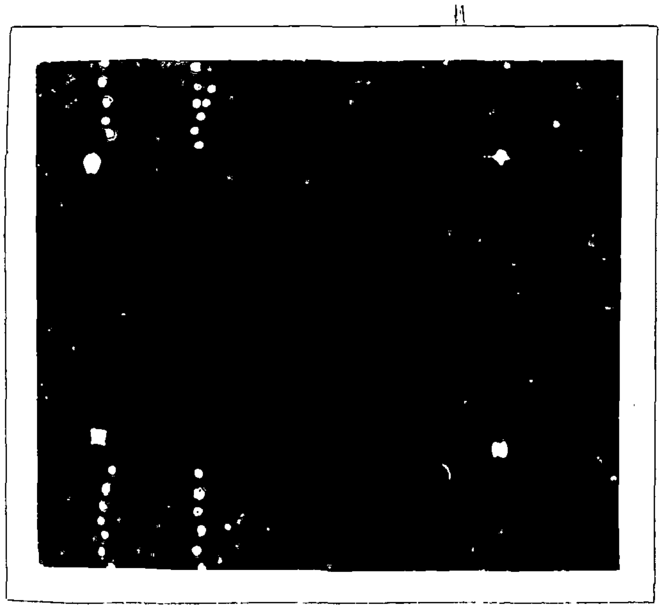


(b) LOWER PART OF THE MAGNET

Plate II(a,b): ONE MUON TRAJECTORY IN ONE LIMB (MAGNET M_2)

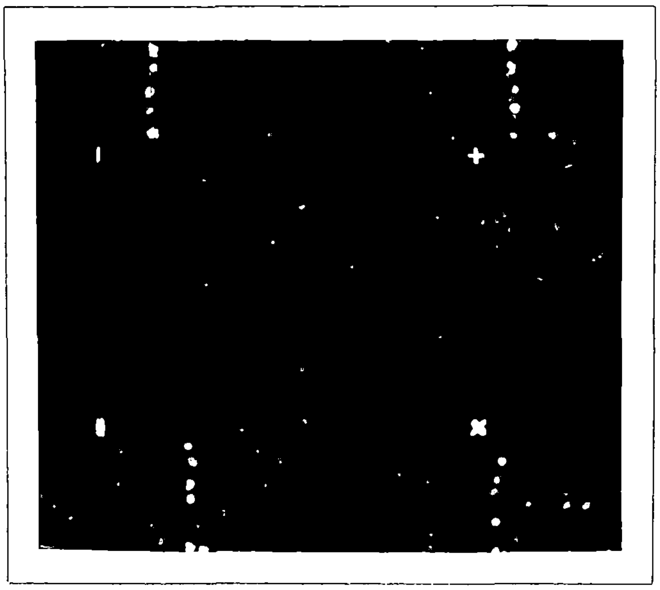


(a) UPPER PART OF THE MAGNET)

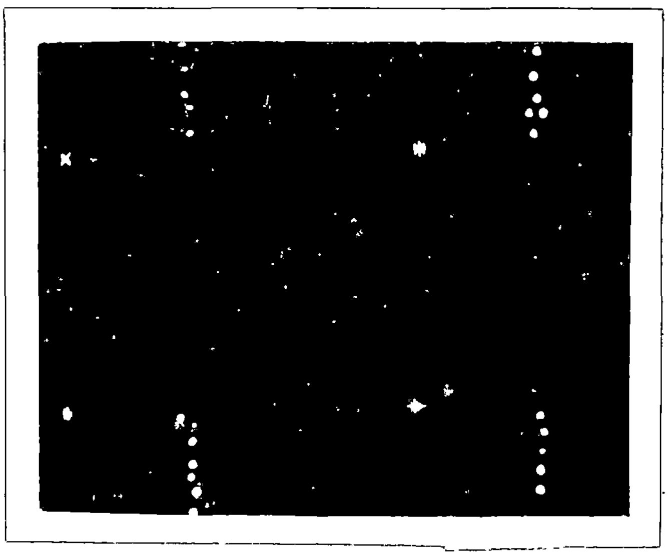


(b) LOWER PART OF THE MAGNET

Plate III(a,b): TWO MUON TRAJECTORIES IN ONE LIMB (MAGNET M_1)



(a) UPPER PART OF THE MAGNET



(b) LOWER PART OF THE MAGNET

Plate IV(a,b): TWO MUON TRAJECTORIES IN TWO LIMBS (MAGNET M_2)

4.7 Estimation of errors on shower parameters

From experimentally observed densities of a particular shower, the shower parameters Xo, Yo, Ne and s are obtained by χ^2 -minimisation procedure. An estimate of the statistical fluctuations and measurement errors on different parameters are obtained in the following manner. A set of artificial showers are simulated by fluctuating the expected densities for a given shower size and age with a fixed core position. The deviations of the fitted parameters from the initial values for these artificial showers give an estimate of the errors.

4.8 Artificial shower analysis

Estimation of errors is done as a function of initial shower size (N_e) and shower age (s). The core position of each shower is selected randomly within an area for an initial value of N_e and s and the expected densities of shower particles in each detector are calculated using a particular lateral distribution function of Hillas et al [3]

$$\Delta(N_e, r, s) = N_e . C(s) \left[r/r_o \right]^{-0.53+1.54(s-1)} \left[1 + r/r_o \right]^{-3.39+0.008(s-1)} \quad \ldots \quad (4.11)$$

The possible errors in densities, due to the statistical fluctuation is given by $\sqrt{a\Delta}$ and the systemetic error in the measurement is estimated to be 25%. Then the total error in Δ is $\left[\Delta(0.0625\Delta + a^{-1}) \right]^{\frac{1}{2}}$, where 'a' is the area of the detector.

The fluctuation is assumed to follow a Gaussian distribution with the total error as a standard deviation. The expected density in each detector is fluctuated with the above error distribution randomly, for all 21 detectors and this set of fluctuated densities is similar to the experimentally observed densities. The distribution of estimated Xo, Yo, Ne, s for initial s of 0.8, 1.0, 1.2 and intial shower sizes of 4×10^4 and 1×10^5 particles are shown in figure 4.5 and 4.6. The arrow mark in each histogram indicates the initial values. The result shows that the error in core location is within $\pm$ 1 m whereas the error in shower age is $\pm$ 0.1 and size error is $\pm$ 0.15 Ne. In the minimisation procedure the threshold is at 1 particle for each detector.

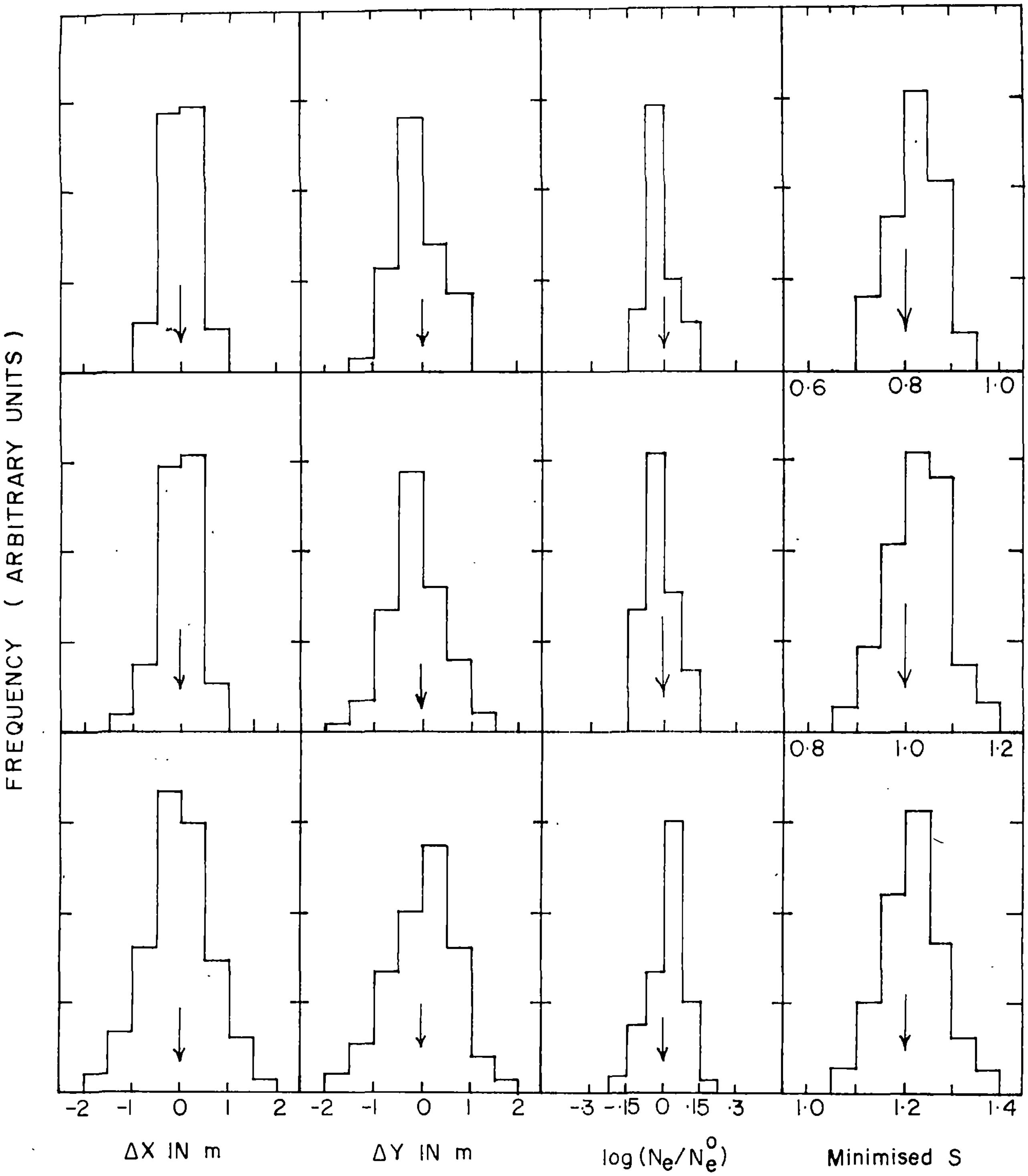


Fig. 4.5 DISTRIBUTION OF ERRORS ON AIR SHOWER PARAMETERS OBTAINED FROM ARTIFICIAL SHOWER ANALYSIS FOR $N_e = 4 \times 10^4$. ARROW INDICATES THE INITIAL VALUES. HISTOGRAMS IN TOP, MIDDLE AND BOTTOM ROWS ARE CORRESPONDING TO THE INITIAL VALUES OF s AS 0.8, 1.0, 1.2 RESPECTIVELY

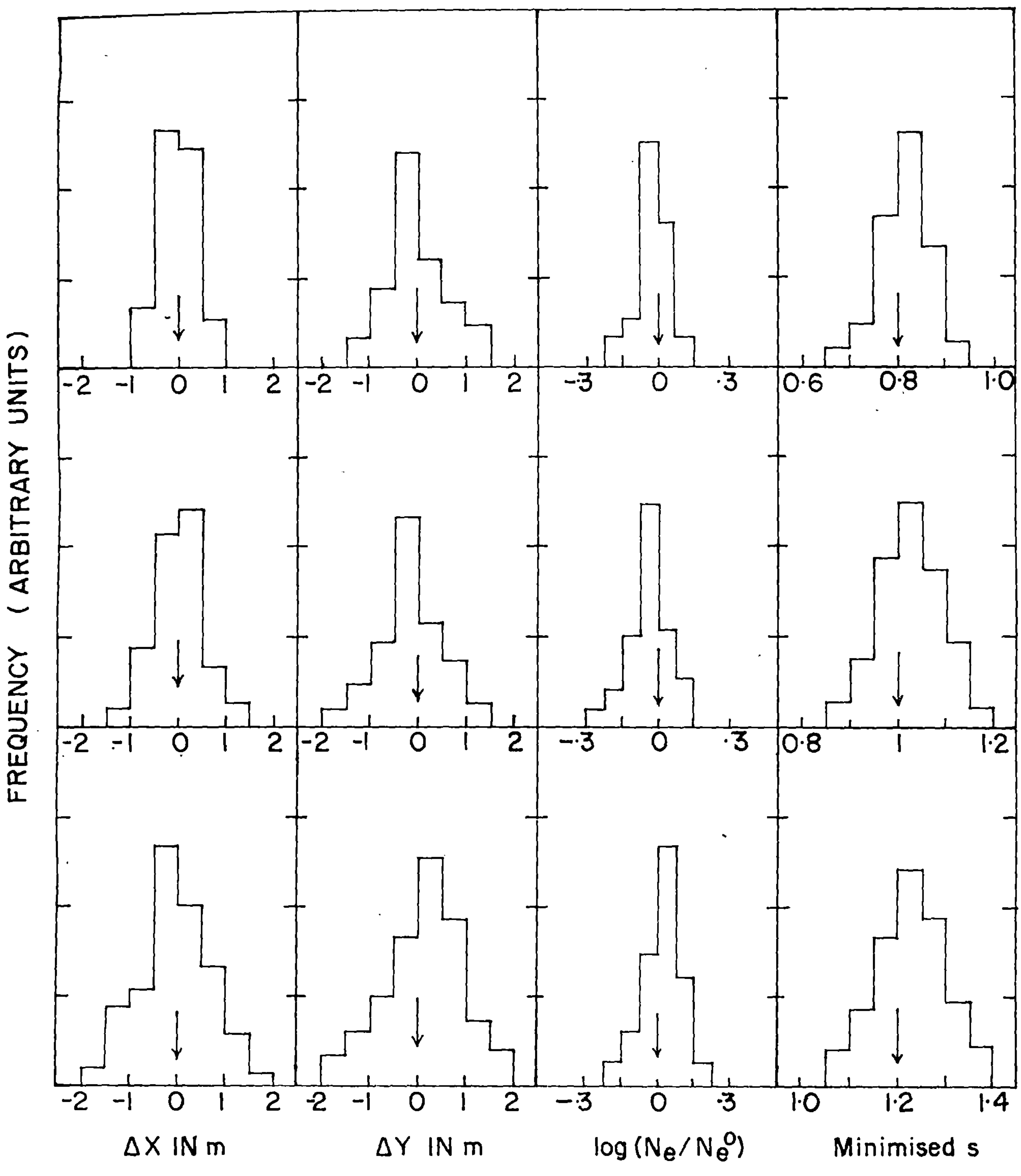


Fig. 4.6 DISTRIBUTION OF ERRORS ON AIR SHOWER PARAMETERS OBTAINED FROM ARTIFICIAL SHOWER ANALYSIS FOR $N_e = 1 \times 10^5$. ARROW INDICATES THE INITIAL VALUES. HISTOGRAMS IN TOP, MIDDLE AND BOTTOM ROWS ARE CORRESPONDING TO THE INITIAL VALUES OF s AS 0.8, 1.0, 1.2 RESPECTIVELY

References

1. Hyman, P.J. and Wolfendale, A.W., Proc.Phys.Soc., 80(1962)710

2. Nandi, B.C. Ph.D. Thesis, University of Burdwan, India, 1970

3. Hillas, A.M and Lapikens, J., Proc.15th ICRC, Plovdiv, 8(1977)460