CHAPTER : IV

MOTION OF VISCOUS FLUID THROUGH A POROUS PIPE.

Part One

Unsteady flow of viscous fluid through a straight long porous Circular tube due to pressure gradient.*

4.1 Introduction:-One dimensional flow problem involving porous media have important application in various disciplines like petroleum engineering , geophysics , agricultural engineering concerned with the movement of oil and gas through reservoir to the hydrologist connected with the study of migration of under ground water etc. The study of blood flow is in pulmonary alveolar sheet where the fluid is found between two permeable layers is of considerable importance in biomechanics. The problem of unsteady flow of viscous incompressible fluid in an annulus of two porous coaxial circular cylinder subject to suction or injection has been studied by Rao (1961) under the presence of a periodic pressure gradient. Singh (1967) has studied the flow of visco-elastic Maxwell fluid in the annulus of the porous concentric circular cylinders under the influence of pressure gradient. Rao(1961) considered the periodic flow through an annulus of two porous concentric cylinders for ordinary viscous fluid. Sexl (1930) considered the motion of viscous

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* PUBLISHED IN THE 'ACTA CIENCIA INDICA' VOL.XIXM,NO.1,045(1993)

liquid through a circular tube under the influence of a periodic pressure gradient. The unsteady flow of viscous fluid through a circular tube has been investigated by Mithal (1960). Das (1977) considered the flow of viscous incompressible fluid through two porous concentric circular cylinder under the influence of pressure gradient which is a function of time alone. Devi Singh (1964) considered the problem of flow of visco-elastic Maxwell fluid through two porous concentric cylinders under the action of exponential pressure gradient. Ramacharyulu and Raja (1984) investigated the run-up flow of a viscous incompressible fluid in a generalised porous medium. Again , Das (1977) considered the problem of unsteady flow of viscous incompressible fluid in the annulus of the two porous concentric cylinders subjected to suction or injection. Singh and Sharma (1992) considered three dimensional magneto-fluid-dynamic of an incompressible fluid over an semiinfinite plate in porous medium with pressure gradient and fluid injection/ suction. Dutta and Arvind kumar Sharma (1993) investigated the run-up flow of an elastico-viscous liquid in a porous medium. Sharma and Harish Kumar (1993) considered unsteady flow of Walters fluid past an infinite porous vertical plate with constant heat flux.

More recently , Agarwal (1994) discussed the magnetohydrodynamic flow of a viscous incompressible fluid along an oscillating porous plate with suction.

In this paper we have studied the unsteady motion of viscous fluid through a straight long porous circular tube due to pressure gradient. The fluid is assumed to be Newtonain and incompressible. The technique of finite Hankel transform has been used to some the problem. The method we have applied put some restriction to injection parameter that it cannot be $\leq -2v$ but in case of suction there is no such restriction.

For large, oscillation when $v >> 1$ the amplitude of the velocity profile gradually diminishes as a suction parameter S increases from O(zero) to onwards which shows that suction tries to anhilate the motion (Table-1) while for injection parameter , the amplitude of the velocity profile increases when $0 > S > -2v$ (Table : 2). But when $S < -2v$ the amplitude of the velocity profile gradually diminishes having a phase lag by π^c (Table:3). Thus $S = -2v$ can be treated as a suction- injection transtion line.

In the present paper pressure gradient is a function of time but is independent of space co-ordinates. The general solution is obtained by using finite Hankel

transform. Velocity distribution for particular types of pressure gradient namely flow under periodic pressure gradient are discussed.

4.2 FORMULATION OF THE PROBLEM :- Applying cylindrical co-ordinate (r,θ,z) with z-axis along the axis of the circular tube. Let us denote by u , v & w the components of velocity along r , θ & z increasing respectively.

Assuming that the motion is symmetrical about the Z-axis.

We have $\frac{\delta}{\delta\theta} = 0$ and the nature of the motion gives v=0.

The Navier-Stokes equations of the motion of viscous incompressible fluid are (in absence of External forces)

$$\frac{\delta u}{\delta t} + u\frac{\delta u}{\delta r} + w\frac{\delta u}{\delta z} = -\frac{1}{\rho}\frac{\delta p}{\delta r} + \nu\left[\frac{\delta^2 u}{\delta r^2} + \frac{1}{r}\frac{\delta u}{\delta r} - u/r^2\right] \quad (1)$$

$$\frac{\delta w}{\delta t} + u\frac{\delta w}{\delta r} + w\frac{\delta w}{\delta z} = -\frac{1}{\rho}\frac{\delta p}{\delta r} + \nu\left[\frac{\delta^2 w}{\delta r^2} + \frac{1}{r}\frac{\delta w}{\delta r} + \frac{\delta^2 w}{\delta z^2}\right] \quad \ldots(2)$$

The equation of continuity is

$$\frac{1}{r}\frac{\delta}{\delta r}(ur) + \frac{\delta w}{\delta z} = 0 \quad \ldots(3)$$

The direction of flow is along the axis of Z.

This show that w is independent of z.

$$\frac{\delta w}{\delta z} = 0 \qquad \ldots(4)$$

By the help of (4) the equation of continuity is

$$\frac{1}{r}\frac{\delta}{\delta r}(ur) = 0$$

Which gives $ur = -s$ (say) $\qquad \ldots(5)$

where $s > 0$ is the suction parameter and $s < 0$ is the injection parameter.

By the help of the equation (5) , equation (1) & (2) reduce to the following forms respectively

$$\frac{1}{\rho}\frac{\delta p}{\delta r} = \frac{S^2}{r^3} \qquad \ldots(6)$$

$$\frac{\delta w}{\delta t} = -\frac{1}{\rho}\frac{\delta p}{\delta z} + \nu\left[\frac{\delta^2 w}{\delta r^2} + \left(1 + \frac{S}{\nu}\right)\frac{1}{r}\frac{\delta w}{\delta r}\right] \qquad \ldots(7)$$

Substituting $\frac{S}{\nu} = 2n \qquad \ldots(8)$

and $-\frac{1}{\rho}\frac{\delta p}{\delta z} = f(t)$

which is a function of t alone , we get $\qquad \ldots(9)$

$$\frac{\delta w}{\delta t} = f(t) + \nu\left[(1 + 2n)\frac{1}{r}\frac{\delta w}{\delta r} + \frac{\delta^2 w}{\delta r^2}\right]$$

Boundary conditions

$$w(r, t) = 0, \quad r = a, \quad t > 0$$

$$u = -\frac{s}{a}, \quad r = a, \quad t > 0 \qquad \ldots(10)$$

Initial condition

$$w(r,0) = 0, \text{when } t = 0, \ 0 \le r \le a \qquad \ldots(11)$$

Writing $w = Fr^{-n}$ where F is a function of r & t, we get

$$\frac{\delta^2 F}{\delta r^2} + \frac{1}{r}\frac{\delta F}{\delta r} - \frac{n^2}{r^2}F = \frac{1}{v}\frac{\delta F}{\delta t} - \frac{r^n}{v} f(t) \qquad \ldots(12)$$

Subject to boundary condition and initial conditions

$$F(r,t) = 0, \quad r = a, \quad t > 0$$

$$F(r,t) = 0, \quad 0 \le r \le a, \quad t = 0 \qquad \ldots(13)$$

4.3 SOLUTION OF THE PROBLEM

We introduce finite Hankel transform defined by

$$W_H = \int_0^a rwJ_n(r\beta_i)\,dr$$

Where β_i is a root of the transcendental equation

$$J_n(a\beta_i) = 0 \qquad \ldots(14)$$

Taking Hankel transform to equation (10) & (11), we get,

$$\frac{dF_H}{dt} + v\beta_i^2 F_H = \frac{a^{n+1}}{\beta_i} J_{n+1}(a\beta_i) f(t)$$

$$- av\beta_i F(a) J'_n(a\beta_i) \qquad \ldots(15)$$

where $n > -1$

Solution of this equation subject to the boundary condition given in equation (13) is

$$F_H = \frac{a^{n+1}}{\beta_i} J_{n+1}(a\beta_i) \int_0^t f(T) e^{-\nu\beta_i^2(T-t)} dT \quad \ldots(16)$$

Applying the inversion formula we get ,

$$F(r,t) = \frac{2}{a^2} \sum_i \frac{a^{n+1}}{\beta_i} \frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)} \int_0^t f(T)\, e^{-\nu\beta_i^2(T-t)} dt \quad \ldots(17)$$

$$W(r,t) = 2\, a^{n-1} r^{-n} \sum_i \frac{J_n(r\beta_i)}{\beta_i J_{n+1}(a\beta_i)} \int_0^t f(T)\, e^{-\nu\beta_i^2(T-t)} dT \quad \ldots(18)$$

where summation extends over the positive roots of the equation

$$J_n(a\beta_i)=0 \quad \ldots(19)$$

4.4 Discussion

We choose $f(T) = e^{ikT}$

Then

$$W(r,t) = 2r^{-n} a^{n-1} \sum_i \frac{J_n(r\beta_i)}{\beta_i J_{n+1}(a\beta_i)} \left[\frac{e^{ikt} - e^{-\nu\beta_i^2 t}}{(ik + \nu\beta_i^2)} \right] \quad \ldots(20)$$

using the result

$$r^{-n}\,\frac{e^{ikt}}{ik}\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_{n+1}\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

$$= 2r^{-n}\, a^{n-1} \sum_i \frac{e^{ikt}}{\beta_i\,(ik + \nu\beta_i^2)}\,\frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)}$$

We get , $W(r,t) = r^{-n}\frac{e^{ikt}}{ik}$

$$\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_n\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

$$- 2r^{-n}\, a^{n-1} \sum_i \frac{J_n(r\beta_i)}{i\,\beta_i J_{n+1}(a\beta_i)}\,\frac{e^{-\nu\beta_i^2 t}}{(ik + \nu\beta_i^2)} \qquad \ldots(21)$$

Real part of the solution

$$\mathrm{Re}\,(w) = \mathrm{Re}\, r^{-n}\frac{e^{ikt}}{ik}\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_{n+1}\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right] \qquad \ldots(22)$$

is a particular solution of the problem of viscous incompressible fluid through a straight porous circular tube due to the influence of periodic pressure gradient.

For high viscous oscillation when $v \gg 1$ we obtain the velocity distribution by expanding Bessel function in equation (22) in power series and neglecting the term of $O(v^{-2})$

we get, $$w(r,t) = \frac{e^{ikt}}{4v\ (n+1)} (a^2 - r^2) \qquad \ldots(23)$$

Real part of $\frac{e^{ikt}}{4v\ (n+1)} (a^2 - r^2)$ is $W_1 = \frac{\cos kt}{4v\ (n+1)} (a^2 - r^2)$

$\ldots(24)$

TABLE-1

Kt->	0°	30°	45°	60°	90°
n=0 w1->	.025	.0216	.0176	.0125	0
n=½ w1->	.016	.014	.012	.008	0
n=1 w1->	.03	.027	.022	.015	0

TABLE-2

Kt->	0°	30°	45°	60°	90°
n= -2 w1->	.03	.027	.022	.015	0
n= -5 w1->	.05	.043	.035	.025	0
n= -9 w1->	.25	.216	.176	.120	0

TABLE -3

Kt->	0°	30°	45°	60°	90°
n= -1.1 w1->	-.25	-.216	-.176	-.125	0
n= -1.5 w1->	-.05	-.043	-.035	-.025	0
n= -2 w1->	-.025	-.0216	-.0176	-.0125	0

When $a = 1$, $r = 0$, $v = 0.10$

Which corresponds to the steady flow of viscous liquid through a porous straight channel due to a periodic pressure gradient. From equation (24) it is revealed that the amplitude of the velocity profile gradually diminishes as suction parameter S increases from 0 (zero) to onwards which shows that suction tries to annihilate the motion (Table-1) while for injection parameter, the amplitude of the velocity profile increases when $0>S>-2v$ (Table-2). But when $S < -2v$ the amplitude of the velocity profile gradually diminishes having a phase lag π^c (Table-3). Thus $S = -2v$ can be treated as a suction-injection transition line.

When there is no suction we obtain the result from (21)

$$W(r,t) = \frac{2}{a} \sum_i \frac{J_0(r\beta_i)}{\beta_i J_1(a\beta_i)} \frac{[e^{ikt} - e^{-v\beta_i^2 t}]}{(ik + v\beta_i^2)} \quad \ldots(25)$$

Then the real part of the solution

$$W_0 = \mathrm{Re}\left[\frac{e^{ikt}}{ik}\left\{1-\frac{J_0\left(\sqrt{\frac{-ik}{\nu}}\,r\right)}{J_0\left(\sqrt{\frac{-ik}{\nu}}\,a\right)}\right\}\right] \qquad \ldots.(26)$$

is a particular solution of the problem of viscous incompressible fluid through a porous straight circular tube due to the influence of pressure gradient which is same as obtained by Grace.

PART — TWO

Unsteady flow of viscous fluid through a long straight porous circular pipe due to time varying pressure gradient with an initial arbitrary velocity distribution.

4.5 Introduction :- The study of flow through porous media is of principal interest due to its importance in petroleum engineering for studying the movement of natural gas , oil and water through the oil reservoirs and to study the underground water in river beds.The flows of viscous fluid through porous medium are of considerable importance for various purposes. Not only because of its significant flow situation and oil extraction but also for the rheometrical aspects, it has been dealt with by a number of researchers who presented solutions of the problems for various models of viscosity characteristics. Many authours like Lapwood (1948) Berman (1958) , Sellers (1955) , Yuan (1956) , Singh (1967) , Sharma et al (1986) , Khamrui (1957) , Verma and Bansal (1967) , Chien Fan (1965) , Mithal (1960) , Bathaiah (1980) , Gersten and Gross (1974) , Reddy and Bathaiah (1982 , 1987),Ahmadi and Manvi (1971) , Yamamto and Iwamura (1976) and Gupta and Babu (1987) made significant contributions in this field. The problem of

Published in the Acta Ciencia Indica Vol. XXM , No. 1 (1994).

unsteady flow of viscous incompressible fluid in an annulus of two porous co-axial circular cylinders subjected to suction or injection has been studied by Rao (1961). Bhattacharya (1980) considered unsteady laminar flow in a channel with porous bed. Pandey (1969) considered slow steady flow of a viscous incompressible fluid between two porous walls at slightly variable distance from each other with distributed suction. Singh (1967) has studied the flow of visco - elastic Maxwell fluid in the annulus of two porous concentric circular tube under the influence pressure gradient. Das (1977) considered the flow of viscous incompressible fluid between two porous concentric circular cylinder with the inner cylinder rotating. Recently , Gupta and Babu (1987) studied the flow of a viscous incompressible fluid through a porous medium near oscillating infinite porous plate in the slip flow regime. Mukesh Gupta and Shalini Sharma (1991) investigated the flow of viscous incompressible and electrically conducting fluid through a porous medium bounded by a oscillating porous infinite plate in slip flow regime under the influence of transverse magnetic field fixed relative to the fluid.

More recently ,Kulshrestha and Singh (1993) considered the MHD flow of viscous incompressible fluid

between two co-axail rotating porous cylinders and discussed the effects of injection , suction parameter and electromagnetric field on velocity distribution.

In this paper we have studied the unsteady motion of viscous fluid through a straight porous channel due to pressure gradient with an initial arbitrary velocity distribution. The fluid assumed to be Newtonian and incompressible. The general solution is obtained by using finite Hankel Transform. Velocity distribution for some particular types of pressure gradients namely , (i) impulsive(ii)periodic are discussed.

4.6 Formulation of Problem :-

Applying cylindrical co - ordinate (r,θ,z) with Z - axis along the axis of the circular tube , let us denote by u,v & w the components of velocity along r,θ,z increasing respectively.

Assuming that the motion is symmetrical about the Z-axis we have $\frac{\delta}{\delta\theta} = 0$ and the nature of motion gives v = 0.

The Navier - Stokes equations of motion of viscous incompressible fluid are (in absence of external forces)

$$\frac{\delta u}{\delta t} + u\frac{\delta u}{\delta r} + w\frac{\delta w}{\delta z} = -\frac{1}{\rho}\frac{\delta p}{\delta r} + v\left[\frac{\delta^2 u}{\delta r^2} + \frac{1}{r}\frac{\delta u}{\delta r} - \frac{u}{r^2}\right]$$

..... (1)

$$\frac{\delta w}{\delta t} + u\frac{\delta w}{\delta r} + w\frac{\delta w}{\delta r} = -\frac{1}{\rho}\frac{\delta p}{\delta r} + \nu\left[\frac{\delta^2 w}{\delta r^2} + \frac{1}{r}\frac{\delta w}{\delta r} + \frac{\delta^2 w}{\delta z^2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

The equation of continuity is

$$\frac{1}{r}\frac{\delta}{\delta r}(ur) + \frac{\delta w}{\delta z} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

The direction of flow is taken along direction of the axis of z.

This shows that w is independent of z.

Therefore, $\frac{\delta w}{\delta z} = 0$ ..... (4)

By the help of (4) the equation of continuity is

$$\frac{1}{r}\frac{\delta}{\delta r}(ur) = 0$$

which gives $ur = -S$ (say) ..... (5)

where $S > 0$ is the suction parameter and $S < 0$ is the injection parameter.

By the help of equation (5), equation (1) & (2) reduce to the following forms respectively.

$$\frac{1}{\rho}\frac{\delta p}{\delta r} = \frac{S^2}{r^3} \quad \ldots\ldots\ldots\ldots (6)$$

$$\frac{\delta w}{\delta t} = -\frac{1}{\rho}\frac{\delta p}{\delta z} + \nu\left[\frac{\delta^2 w}{\delta r^2} + \left(1 + \frac{S}{\nu}\right)\frac{1}{r}\frac{\delta w}{\delta r}\right]$$

..... (7)

Substituting ,

$$\frac{S}{v} = 2n \qquad \ldots\ldots\ldots\ldots (8)$$

$$-\frac{1}{\rho}\frac{\delta p}{\delta z} = f(t) \qquad \ldots\ldots\ldots\ldots (9)$$

Which is a function of t alone , we get

$$\frac{\delta w}{\delta t} = f(t) + v\left[(1+2n)\frac{1}{r}\frac{\delta w}{\delta r} + \frac{\delta^2 w}{\delta r^2}\right]$$

$$\ldots\ldots\ldots\ldots\ldots (10)$$

Writing $W = Fr^{-n}$ where F is a function of r and t,we get

$$\frac{\delta^2 F}{\delta r^2} + \frac{1}{r}\frac{\delta F}{\delta r} - \frac{n^2}{r^2}F = \frac{1}{v}\frac{\delta F}{\delta t} - \frac{1}{v}r^n f(t)$$

$$\ldots\ldots\ldots\ldots\ldots (11)$$

Boundary and initial conditions for the problem are

$$w(r,t) = Fr^{-n} = 0 \qquad r = a\ ,\ t > 0$$

$$u = -u_o = \frac{S}{a} \qquad r = a\ ,\ t > 0 \qquad \ldots\ldots\ldots (12)$$

$$w(r,t) = w_o(r)\ , \qquad 0 \le r \le a\ ,\quad t = 0$$

$$F_o = w_o r^n\ , \qquad t = 0 \qquad \ldots\ldots\ldots (13)$$

4.7 Solution of the Problem :-

We introduce finite Hankel Transform defined by

$$W_H = \int_o^a r\, w\, J_n(r\beta_i)\, dr \qquad \ldots\ldots\ldots (14)$$

Where β_i is a root of the transcedental equation ,

$$J_n(a\beta_i) = 0 \qquad (15)$$

Taking Hankel transform to equations (10) & (11) , we get

$$\frac{1}{\nu}\frac{dF_H}{dt} + \beta_i^2 F_H = -\, a\,\beta_i\, F(a)\, J_n(a\beta_i) + \frac{1}{\nu}\frac{a^{n+1}}{\beta_i}\, f(t)\, J_{n+1}(a\beta_i)$$

Provided , $n > -1$ ..... (16)

$$F_H(0) = \int_0^a r^{n+1}\, W_0(r)\, J_n(r\beta_i)\, dr \qquad \text{.....} \quad (17)$$

With the help of the boundary condition (11) , we get

$$\frac{dF_H}{dt} + \nu\beta_i^2 F_H = \frac{a^{n+1}}{\beta_i}\, \nu J_{n+1}(a\beta_i)\, f(t)$$

Solution of this equation subject to the initial condition (17) is

$$F_H = F_{H(0)}\, e^{-\nu\beta_i^2 t} + \frac{a^{n+1}}{\beta_i} J_{n+1}(a\beta_i) \int_0^t e^{-\nu\beta_i^2 (t-\tau)} f(\tau)\, d\tau \qquad \text{.....} (18)$$

Applying the inversion formula , we get

$$F(r,t) = \frac{2}{a^2}\sum_i F_H(0) e^{-\nu\beta_i^2 t} \left[\frac{J_n(r\beta_i)}{J'_{n+1}(a\beta_i)}\right]^2 + 2a^{n-1}\sum_i \frac{J_{n+1}(a\beta_i)}{\beta_i}\,\frac{J_n(r\beta_i)}{[J'_{n+1}(a\beta_i)]^2} \int_0^t e^{-\nu\beta_i^2 (t-\tau)} f(\tau)\, d\tau$$

$$w(r,t) = \frac{2\, r^{-n}}{a^{1-n}} \sum_i F_{H(o)}\, e^{-\nu \beta_i^2 T} \frac{J_n(r\beta_i)}{J_{n+1}^2(a\beta_i)}$$

$$+ 2\, r^{-n} a^{n-1} \sum_i \frac{J_n(r\beta_i)}{\beta_i J_{n+1}(a\beta_i)} \int_o^t e^{-\beta_i^2 (t-T)} f(T)\, dT \quad \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

4.8 Particular Case and discussion :-

Case : 1

If we take ,

$$W_o(r) = a^2 - r^2$$

$$f(T) = \delta(T)$$

Where $\delta(T)$ is the Dirac Delta function , then ,

$$\therefore w(r,t) = 2\, r^{-n} a^{n-1} \sum_i \left[\frac{4(n+1)}{\beta_i^2} + 1 \right] \frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)} \frac{e^{-\nu \beta_i^2 t}}{\beta_i}$$

$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$ (20)

Taking the roots of the equation

$$J_n(a\beta_i) = 0$$

W (r, t) is tabulated against t.

TABLE : 1

For different values of r (r = 0 ,.25,.5,.75) and a = 1 ,

v = .1 , n = 1.

t	0	1	2	3	4	5	6	7	8
W(0,t)	1.91	.856	.205	.046	.010	.002	.0001	.000	.000
W(.25,t)	1.89	.767	.182	.841	.009	.002	.000	.000	.000
W(.5 ,t)	1.61	.533	.124	.028	.006	.001	.000	.000	.000
W(.75,t)	1.43	.271	.062	.014	.003	.000	.000	.000	.000

TABLE : 2

For different values of r (r=0,.25,.5,.75) & a = 1,v = 1,n=0

t	0	1	2	3	4	5	6	7	8
w(0,t)	2.00	1.57	.85	.48	.27	.15	.08	.05	.03
w(.25,t)	1.94	1.33	.74	.42	.23	.13	.07	.04	.02
w(.5,t)	1.75	1.01	.56	.32	.18	.10	.06	.03	.01
w(.75,t)	1.43	.49	.28	.16	.09	.05	.03	.02	.00

TABLE : 3

For different values of r(r = 0 , .25 ,.5 ,.75) and a = 1 ,

v = 1 , n = − ½

t	0	1	2	3	4	5	6	7	8
w(0,t)	2.03	1.72	1.39	1.09	.85	.67	.52	.40	.31
w(.25,t)	1.95	1.64	1.29	1.01	.79	.61	.48	.37	.29
w(.5,t)	1.76	1.30	1.99	.77	.60	.47	.37	.28	.22
w(.75,t)	1.44	.697	.55	.43	.3358	.26	.2049	.16	.13

From equation (20) it follows that due to the application of impulsive pressure gradient the velocity profile dies out exponentially as t increases and ultimately tends to zero, when $t \to \infty$

Table (1) shows the distribution of velocity when suction parameter is equal to 2v and Table(2) shows the same when the suction parameter is absent. A comparative study of the two tables reveals the fact that suction plays a roll for destroying the motion while Table (2) & (3) shows that injection accelerates that motion.

Case : II

If we take $W_o(r) = a^2 - r^2$

then, $f(T) = e^{ikT}$ .....(21)

$$W(r,t) = 2 r^{-n} a^{n-1} \sum_i \frac{4(n+1)}{\beta_i^3} \frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)}$$

$$- 2 r^{-n} a^{n-1} \sum_i \frac{e^{-\nu\beta_i^2 t}}{\beta_i(ik + \nu\beta_i^2)} \frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)}$$

$$+ 2 r^{-n} a^{n-1} \sum_i \frac{e^{ikt}}{\beta_i(ik + \nu\beta_i^2)} \frac{J_n(r\beta_i)}{J_{n+1}(a\beta_i)}$$

..... (22)

Using the result

$$r^{-n}\frac{e^{ikt}}{ik}\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_n\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

$$= 2\, r^{-n}\, a^{n-1} \sum_i \frac{e^{ikt}}{\beta_i(ik + \nu\beta_i^2)} \frac{J_n(r\,\beta_i)}{J_{n+1}(a\,\beta_i)}$$

We get ,

$$W(r,t) = 2\, r^{-n}\, a^{n-1} \sum_i \frac{4(n+1)}{\beta_i^3} \frac{J_n(r\,\beta_i)}{J_{n+1}(a\,\beta_i)} e^{-\nu\beta_i^2 t}$$

$$- 2\, r^{-n}\, a^{n-1} \sum_i \frac{e^{-\nu\beta_i^2 t}}{\beta_i(ik + \beta_i^2)} \frac{J_n(r\,\beta_i)}{J_{n+1}(a\,\beta_i)}$$

$$+ r^{-n}\frac{e^{ikt}}{ik}\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_n\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

If there is no arbitrary initial velocity distribution , then the real part of the solution.

$$\mathrm{Re}\,(w) = \mathrm{Re}\; r^{-n}\frac{e^{ikt}}{ik}\left[r^n - a^n \frac{J_n\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_n\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

.....(24)

is a particular solution of the problem of viscous incompressible fluid through a straight porous circular tube due to the influence of pressure gradient. In absence of suction parameter s , the above results reduces to the form

$$Re(w) = Re\frac{e^{-ikt}}{ik}\left[1 - \frac{J_o\left(\sqrt{\frac{-ik}{\nu}}\, r\right)}{J_o\left(\sqrt{\frac{-ik}{\nu}}\, a\right)}\right]$$

Which resembles to the result obtained by Grace.

Now we consider the small oscillation of highly viscous fluid. In this case the quantity $\sqrt{\frac{-ik}{\nu}}$ becomes very small.

Then expanding the Bessel's function in equation (24) in power series and neglecting the trems $O(\nu^{-2})$

We get, $$w(r,t) = \frac{e^{ikt}}{ik}\left[1 - \left(1 + \frac{ikr^2}{\{4\nu(n+1)\}}\right)\left(1 - \frac{ika^2}{\{4\nu(n+1)\}}\right)\right]$$

$$= \frac{e^{ikt}}{4\nu(n+1)}\left(a^2 - r^2\right)$$

Which corresponds to the steady flow of viscous liquid through a straight porous channel due to a periodic pressure gradient.

References :—

Ahmadi and Manvi	(1971)	Indian J. Tech. 9, P-441
Agarwal, A.K.	(1994)	The Mathematics Education Vol. xxvii No.4
Berman , A.S	(1958)	J. App1. Physics. Vol-29 (1) P-71.
Bhattacharyya , P	(1965)	Bull. Cal. Math. Soc 57,14
Beavers , G.S. and Joseph, D.D	(1967)	J. Fluid Mech. 30,197
Bathaiah , D	(1980)	Acta Mechanica vol. 35, p-223.
Bhattacharyya , P	(1980)	J. Indian Inst. Sci.62(B) p , 89-90
Bhaskara Reddy , N Bathaiah , D	(1982)	Acta Mechanica vol. 42 , p. 239
Bhaskara Reddy , N and Bathaiah , D	(1987)	Def. Sci, J. Vol32 , No. 4 p.313

Bathaiah , D and K. Sreenivasan	(1993)	Acta ciencia Indica Vol. xix(M) No.4
Chiu, H.H	(1962)	Boundary layer flow with the Suspended particles, princet on Univ. Report.620.
Chien Fan	(1963)	ZAMP, 16,351-360.
Chawla,S,S	(1972)	Int. J.Engg. Sci, 10, 981-987.
Channabassappa, M.N and Ranganna, G	(1976)	Proc. Indian Acad. Sci. Vol. 83A No. 4 Pp 145-155.
Debi Singha	(1964)	Z.A.M.M. 44, 330-3
Das , D.K	(1977)	Ph. D. Thesis , University of North Bengal P. 97
Das , D.K	(1980)	The Mathematics Student. Vol.48,No 4,374-380
Dutt and Arivind Kumar Sharma	(1993)	Indian.J.of theoretical Physics.Vol 41 No. 2 P.111-116

Grace, S.F	(1928)	Phil. Mag(5) P-933
Gersten Kand Gross J.F.	(1974)	ZAMP , 25 , 399
Gupta, A.S and Pop,I	(1975)	Bul. Math. dela soc. sci. Math dela R.S. De Roumanie 19 (67) 291
Gupta, M and Babu,B	(1987)	J. Nat. Acad. Math 5(2) p.106
Gupta M and Sharma S	(1991)	Acta Ciencia Indica Vol xvii(M).No.2
Khamrui, S.R	(1957)	Bull. Cal. Mech.Soc. 47,p1.
Kamal kumar,M.Prasad and P.C Gupta.	(1990)	Acta ciencia Indica.Vol. XIV M, No.3, P.241-244.
Kulshrestha S.P and singha N.P	(1993)	Acta ciencia Indica Vol. XIX (M).No.3
Lapwood,E.R	(1948)	Proc. Camb. Phil.Soc. 44,508
Mithal K.G	(1960)	Bull. Cal.Math.Soc.152(3). 147.
Pandey K.S	(1969)	Bull. Cal. Math .Soc. 61 95-101

Rao, S.P.	(1961)	Appl. Sci. Res.Vol 10 P.298-306
Rao, P.S.	(1969)	Def. Sc. J. 19,135.
Rath,R.S and Parida, D.N	(1982)	Wear, 78. 305-314.
Raja, K.A. and Ramaharyula,P	(1984)	Indian.J. Pure Appl. Maths. 15 (6). 665-670.
Sexl. T	(1930)	Z. Phys. 61, 349.
Sellers, J.R	(1955)	J. Appl. Physics. Vol.26(3) P.489.
Soo, S.L	(1961)	A.I. Ch E. J.7.384.
Singlton, R.E	(1965)	ZAMP. 16 421
Srivastava,A.C and Maiti, M.K.	(1966)	Phys. Fluids. 9,462-465.
Singh, G.S	(1967)	The Mathematics Education.
Sharma H.G. and Kumar Manoj	(1986)	Ind. Journal.Pureand Appl Maths. Vol.17 No 9.P.1154-63