CHAPTER III

PLASMA CONDUCTIVITY IN A TRANSVERSE MAGNETIC FIELD

INTRODUCTION

In several communications from this laboratory (Sen & Ghosh 1966, Gupta & Mandal(1967); Sen & Gupta,1969), it has been shown that the measurement of radio frequency conductivity of an ionised gas and its variation with pressure enables us to calculate the varieus parameters of the ionised gas such as the electron density, collision frequency and electron temperature. The measurement in presence of a magnetic field enables us to find the variation of electron temperature with magnetic field. The only effect of magnetic field that has been taken into consideration in the previous papers is the introduction of the concept of equivalent pressure and it has been found that the experimental results can be satisfactorily explained quantitatively for small values of (H/P) only where H is the magnetic field and P the pressure; this is due to the fact that the equivalent pressure concept is valid only for small values of (H/P). The effect of a transverse magnetic field on an ionised gas has been considered by Beckman (1948) and he has shown that the magnetic field

(a) Changes the radial electron density from the axis of the discharge tube and

(b) Increases the axial electric field of the discharge.

The incorporation of these deductions has been quite helpful to explain very satisfactorily the variation of current in a glow discharge (Sen and Gupta, 1971); variation of current, voltage and power in arc plasma (Sen & Das, 1973) and the variation of intensity of emitted radiation in a glow discharge in a transverse magnetic field (Sen, Das & Gupta, 1972). It is thus thought worth while to study the variation of radio frequency conductivity in a variable transverse magnetic field taking into consideration the theoretical conclusions of Beckman. The plan of the present work is to derive the variation from theoretical consideration and predict results for some specific cases. Some experimental results in connection with the theory developed will also be presented.

THEORETICAL CONSIDERATION

In order to derive the expression for the radio frequency conductivity in a transverse magnetic field let us assume that the discharge current is flowing along the x-axis, the radio frequency field used for measurement of of radio frequency conductivity along y-axis and the magnetic field along the z-axis. Then the equations of motion of the electron are

$$m\frac{dv_y}{dt} + m\nu v_y + He v_x = e E_0 e^{J\omega t}$$

$$\frac{dv_y}{dt} + \nu v_y + \omega_B v_x = \frac{eE_0}{m} . e^{J\omega t} \tag{3.1}$$

where ν is the frequency for momentum transfer, ω is the frequency of the measuring radio frequency field and $\omega_B = \frac{eH}{m}$ the electron cyclotron frequency and similarly,

$$\frac{dv_x}{dt} + \nu v_x - \omega_B v_y = 0 \tag{3.2}$$

let $v_x = A e^{J\omega t}$ and $v_y = B e^{J\omega t}$

Then from eqn. (3.2)

$$A = \frac{\omega_B}{\nu + J\omega} . B$$

and from equation (3.1)

$$B = \frac{eE_0(\nu + J\omega)}{m\left[(\nu + J\omega)^2 + \omega_B^2\right]}$$

then $(\sigma_{rf})_H$ which is the real part of radio frequency conductivity in a direction perpendicular to transverse magnetic field is given by

$$\sigma_{(rf)H} = \frac{ne^2\nu\{\nu^2+\omega^2+\omega_B^2\}}{m\{(\omega^2-\nu^2-\omega_B^2)^2+4\omega^2\nu^2\}} \tag{3.3}$$

which is the same expression as deduced previously by Appleton and Boohariwalla(1937) and later by Gilàrdini(1959). If we go on changing the magnetic field keeping the pressure and frequency of the measuring field constant, then maximising we get

$$\frac{[\{\omega^2-\nu^2-\omega_B^2\}^2+4\omega^2\nu^2]2\omega_B+4[(\omega^2+\nu^2+\omega_B^2)(\omega^2-\nu^2-\omega_B^2)]\omega_B}{[\{\omega^2-\nu^2-\omega_B^2\}^2+4\omega^2\nu^2]^2} = 0$$

or

$$\omega_{B\,max}^2 = -[\omega^2+\nu^2]+\sqrt{4\omega^2(\omega^2+\nu^2)}$$

where $\omega_{B\,max} = \frac{e\,H_{max}}{m}$ and H_{max} is the magnetic field at which the radio frequency conductivity becomes a maximum.

SOME SPECIAL CASES

Case I $\omega \gg \nu$, then $\omega_{B\,max} = \omega$

Case II $\omega \ll \nu$, then $\omega_{B\,max} = \sqrt{2\omega\nu - \nu^2}$

which is $+ve$ only if 2ω is $> \nu$

Case III $\omega = \nu$, then $\omega_{B\,max} = 0{\cdot}9\,\omega$ (3.3 a)

Thus it is evident that as $\omega_B = 1{\cdot}76 \times 10^7\,H$, to have some measurable magnetic field at which the radio frequency will become a maximum, the measuremnt should preferably be in the microwave region.

If $\nu = \omega$

$$\frac{(\sigma_{rf})_H}{\sigma_{rf}} = \frac{2\omega^2(2\omega^2 + \omega_B^2)}{\omega_B^4 + 4\omega^4} \qquad (3.4)$$

Equation (3.4) is valid if it is assumed that the electron density n is unaffected by the magnetic field but it has been shown by Beckman(1948) that the effect of the magnetic field is to alter the radial electron density according to the equation

$$\frac{n_H}{n} = \exp\left[\frac{-b_1 E C_1^{1/2} H r}{2\,D_a\,P}\right]$$

where n_H = electron density at a distance r from the axis, H = magnetic field,

b_1 = mobility,

D_a = Diffusion coefficient of electrons

and
$$C_1 = \left(\frac{e}{m} \cdot \frac{L}{v_r} \right)^2$$

where L = mean free path of the electron at a pressure of 1 torr,

v_r = random velocity of the electron.

The equation has been shown by Sen and Gupta (1971) to be equivalent to

$$\frac{n_H}{n} = \exp\left[\frac{-eH}{4\sqrt{2mK}} \sqrt{\frac{k}{T_e}} \right] = \exp\left[-0.0806 \times 10^2 H \sqrt{\frac{k}{T_e}} \right]$$

where k = fraction of energy lost by electron due to either elastic or inelastic collision, T_e = electron temperature. Consequently from eqn. (3.4)

$$\frac{(\sigma_{rf})_H}{\sigma_{rf}} = \left[\exp\left(-.0806 \times 10^2 H \sqrt{\frac{k}{T_e}} \right) \right] \frac{2\omega^2(2\omega^2 + \omega_B^2)}{\omega_B^4 + 4\omega^4}$$

and maximising we get

$$\alpha\omega_B^6 + 2\omega^5 + 2\alpha\omega^2\omega_B^4 + 8\omega^2\omega_B^4 + 4\alpha\omega^4\omega_B^2 - 8\omega^4\omega_B^2 + 8\omega^6\alpha = 0.$$

where $\alpha = \frac{e}{4\sqrt{2mK}}\sqrt{\frac{R}{T_e}}$. Instead of solving this eqn.

we can numerically calculate the value of

$$(\sigma_{rf})_H / \sigma_{rf}$$

for various values of magnetic field and frequency of measurement. To take a typical example in case of hydrogen, the experimental values of T_e and R as provided in the literature is $T_e = 1.16 \times 10^4$ and $R = 2.17 \times 10^{-3}$ for values of(E/P) used here so that $\alpha = .003484$

$$\frac{(\sigma_{rf})_H}{\sigma_{rf}} = \left[\exp\left(-.0034848\right)\right]^{\frac{2\omega^2(2\omega^2+\omega_B^2)}{\omega_B^4+4\omega^4}} \quad (3.5)$$

The ratio $(\sigma_{rf})_H / \sigma_{rf}$ has been calculated numerically both from equations (3.4) and (3.5) for valuesof magnetic field varying from zero to 1000 gauss for frequencies 10 MHz, 100 MHz and 1000 MHz and results are plotted in figures 3.1, 3.2 and 3.3.

It is evident from figures 3.1, 3.2 and 3.3, that the magnetic field at which the radio frequency conductivity becomes a maximum occurs according to the relation $(\omega_B)_{max} = 0.9\omega$ i.e. for 3.1 gauss for frequency 10 MHz, for 31 gauss for 100 MHz and for 310 gauss for 1000 MHz. The ratio $(\sigma_{rf})_H / \sigma_{rf}$ however is the same for all the three frequencies according to eqn. (3.4). The dotted

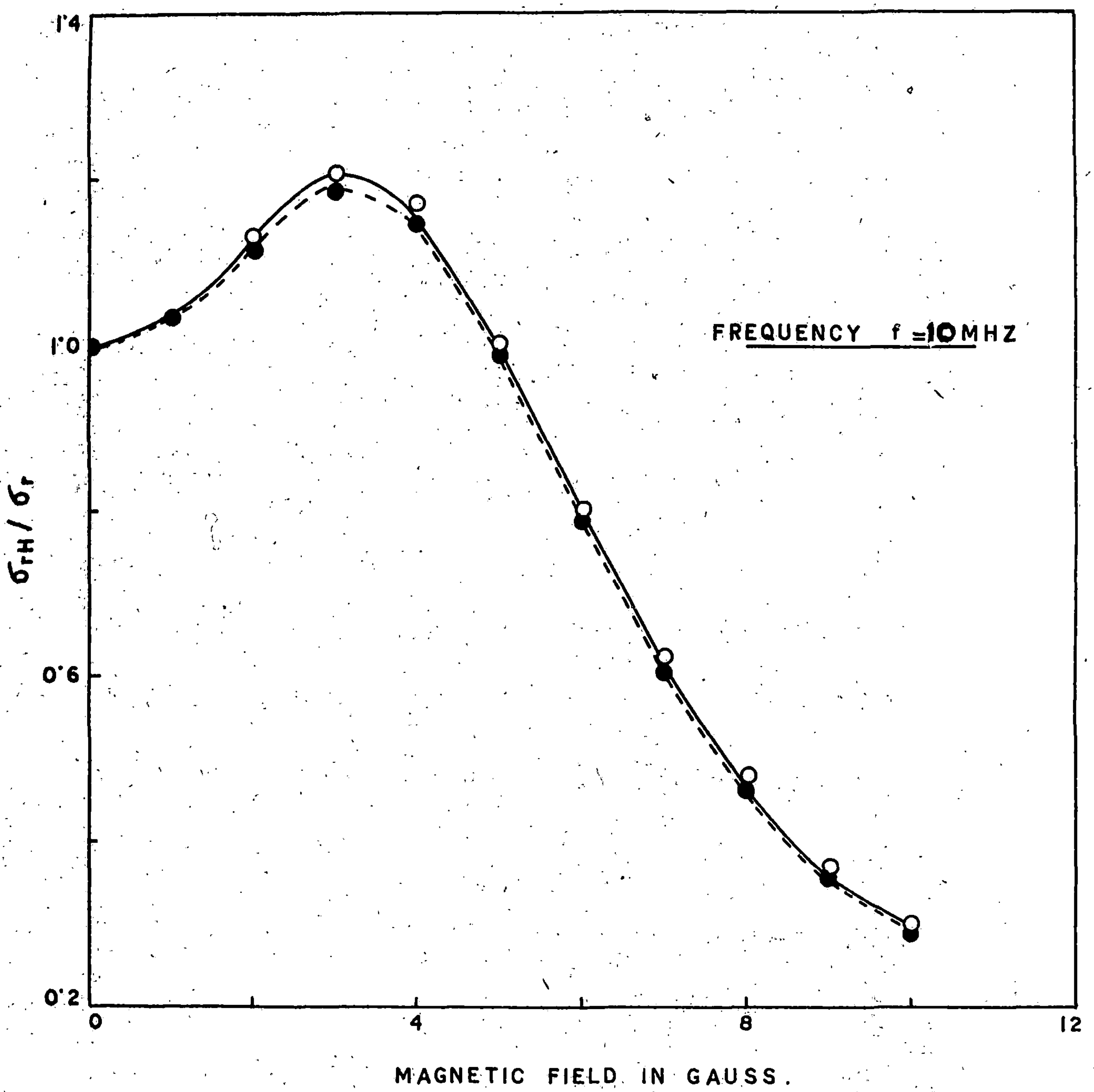

FREQUENCY f = 10 MHZ
σ_{rH} / σ_r
1·4
1·0
0·6
0·2
0
4
8
12
MAGNETIC FIELD IN GAUSS.

Fig - 3.1

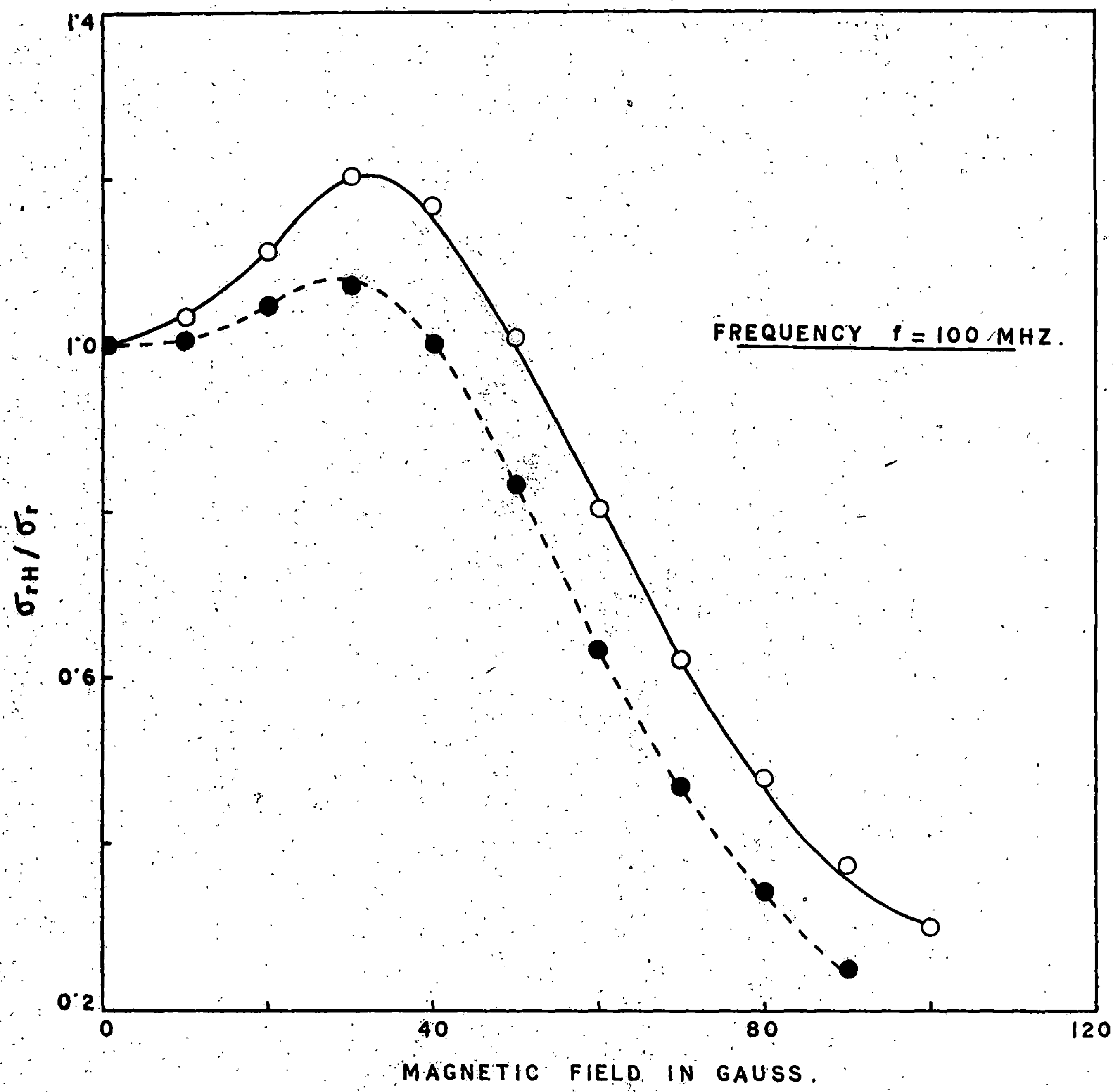
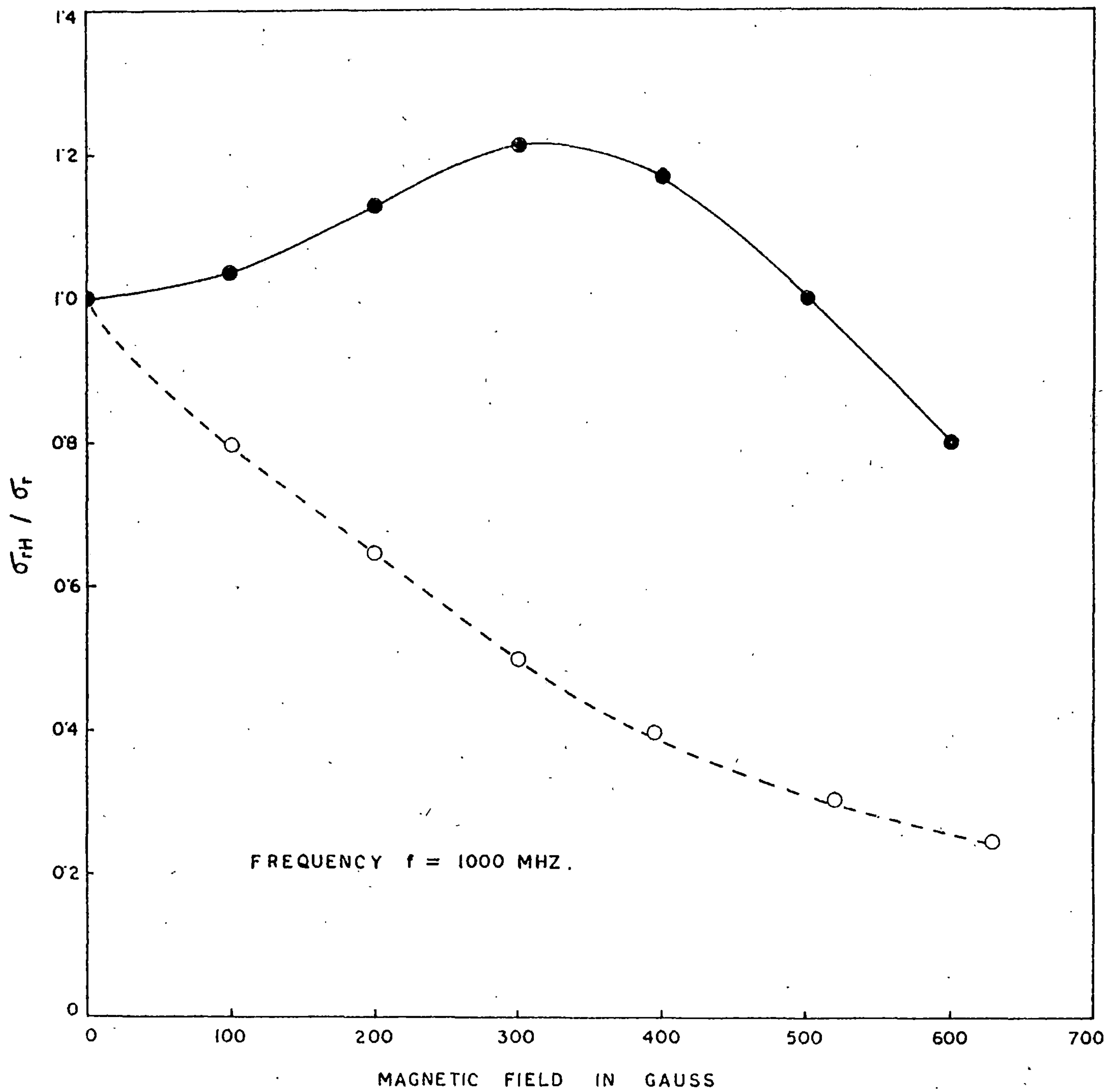

1'4
1'0
0'6
0'2
0
40
80
120
σ_{rH}/σ_r
MAGNETIC FIELD IN GAUSS.
FREQUENCY f = 100 MHZ.

Fig - 3.2



σ_{rH} / σ_r
1·4
1·2
1·0
0·8
0·6
0·4
0·2
0
0
100
200
300
400
500
600
700
FREQUENCY f = 1000 MHZ.
MAGNETIC FIELD IN GAUSS

FIG- 3.3

curves which represent the results as calculated from equation (3.5) show that values are almost identical as calculated from equation (3.4) but the ratio $(\sigma_{rf})_{H max}/\sigma_{rf}$ decreases with the increase of frequency and magnetic field specially for values of magnetic field smaller than 100 gauss. For values of magnetic field greater than 100 gauss, the curve however gradually decreases with the increase of magnetic field without showing a maximum and this is evident due to the fact that the relation

$$\frac{n_H}{n} = \exp\left[\frac{-eH}{4\sqrt{2mK}}\sqrt{\frac{k}{T_e}}\right]$$

as deduced by Sen and Gupta (1971) is valid for small values of (H/P).

It is not necessary to take k into account the variation of axial electric field due to transverse magnetic field as deduced by Beckman because this will not have any effect on the radio frequency conductivity in a direction transverse to both electric and magnetic fields.

From the foregoing analysis it is clear that if the magnetic field is gradually changed then to obtain measurable magnetic field at which the conductivity becomes a maximum, the measuring field must have a frequency in either the ultra high frequency or in the microwave region. Instead, if we go on changing the pressure, then measurements can be made at radio frequency region and a maximum in the prêssure conductivity curve can be obtained. Differentiating equation (3.3) with respect of ν we get

$$\nu^6 + \nu^4(\omega_B^2 + \omega^2) - \nu^2[\omega^4 + \omega_B^4 - 10\omega^2\omega_B^2] - (\omega^2 - \omega_B^2)(\omega^2 + \omega_B^2) = 0$$

As the magnetic field used in the experiment is of the order of order of a few kilogauss, and the measuring field has the frequency of a few magacycles, i.e. $\omega_B \gg \omega$ ~~and~~ we get,

$$(\nu^4 - \omega_B^4)(\nu^2 + \omega_B^2) = 0$$

or

$$\nu = \omega_B$$

$(\sigma_{rf})_H$ will become a maximum when $\nu = \omega_B$. from which the collision frequency at the particular ~~p~~ pressure can be calculated, and

$$\nu_c = \frac{v_r}{\lambda_e} = v_r \frac{P}{L}$$

we get $v_r P_{max}/L = eH/m$

or H/P_{max} = constant,

where P_{max} is the pressure at which the radio frequency conductivity becomes a maximum when the magnetic field H is applied, as

$$(\sigma_{rf})_{H max} = \frac{ne^2}{2m\omega_B} \qquad (3.6)$$

the electron density can be calculated and the maximum value of radio frequency conductivity should decrease with the increase of applied magnetic field. If the variation of radial electron density is considered (Beckman, 1948) then

$$(\sigma_{rf})_H = \frac{n_H e^2}{m} \cdot \frac{\nu}{\nu^2 + \omega_B^2}$$

$$= e^2 n\left[\exp\left(\frac{-eH}{4\sqrt{2mK}}\sqrt{\frac{R}{T_e}}\right)\right] \cdot \frac{\nu}{\nu^2+\omega_B^2}$$

and as before, $(\sigma_{rf})_H$ will become a maximum when $\nu = \omega_B$ and hence

$$(\sigma_{rf})_{H max} = n\left\{\exp\left[\frac{e}{4\sqrt{2mK}}\sqrt{\frac{R}{T_e}}H\right]\right\}\frac{e^2}{2m\omega_B}$$

(3.7)

From which n can be calculated. It is further noted that the deduction of Beckman regarding the variation on the value of radial electron density will have no effect on the value of pressure at which the radio frequency conductivity will become a maximum at a particular value of magnetic field.

EXPERIMENTAL ARRANGEMENT

The present investigation reports results regarding the variation of radio frequency conductivity with pressure in a transverse magnetic field in case of air, hydrogen, oxygen and ammonia. The measuring field has a frequency of 2.45 MHz and the variation of pressure is in the millimeter range and magnetic field employed is 1 kilogauss to 2 kilogauss so that the assumption $\omega << \omega_B$ is justified. The detailed method of measurement is discussed in chapter II and the experimental arrangement is shown in fig. 2.6. Pure and dry air has been used, hydrogen and oxygen have been prepared by the electrolysis of a saturated solution of warm barium hydroxide and dried by potasium hydroxide and phosphoruspentoxide; ammonia was prepared by heating liquid ammonia and dried by dry calcium-oxide. The pressure has been accurately measured by a calibrated Meleod gauge and the magnetic field by a calibrated gauss meter.

RESULTS AND DISCUSSION

The variation of radio frequency conductivity with pressure in transverse magnetic field [1150 G, 1350 G, and 1850] G have been plotted in figures 3.4, 3.5, 3.6 and 3.7 for air, hydrogen, oxygen and ammonia respectively. In each case it is observed that the radio frequency conductivity becomes a maximum at a certain pressure and the pressure at which the conductivity becomes a maximum always shifts towards the higher pressure with the increase of magnetic field and the absolute value ~~x~~of conductivity diminishes with magnetic field for all values of pressure. The experimental results have been entered in table 3.1

It is evident from column VI that the theoretical deduction H/P_{max} = constant for maximum conductivity is well satisfied. From the expression $\nu = \omega_B$ for maximum $(\sigma_{rf})_H$, the value of ν can be obtained and the results are entered in the fifth column in the table 3.1. It is shown that the collision frequency of momentum transfer increases with the increase of magnetic field. Now the electron density n_H in presence of magnetic field has been calculated from the maximum conductivity values $(\sigma_{rf})_{H\,max}$ for values of magnetic field used in the experiment for all the experimental gases such as air, hydrogen, oxygen and ammonia and the results are entered in column VII at table 3.1.

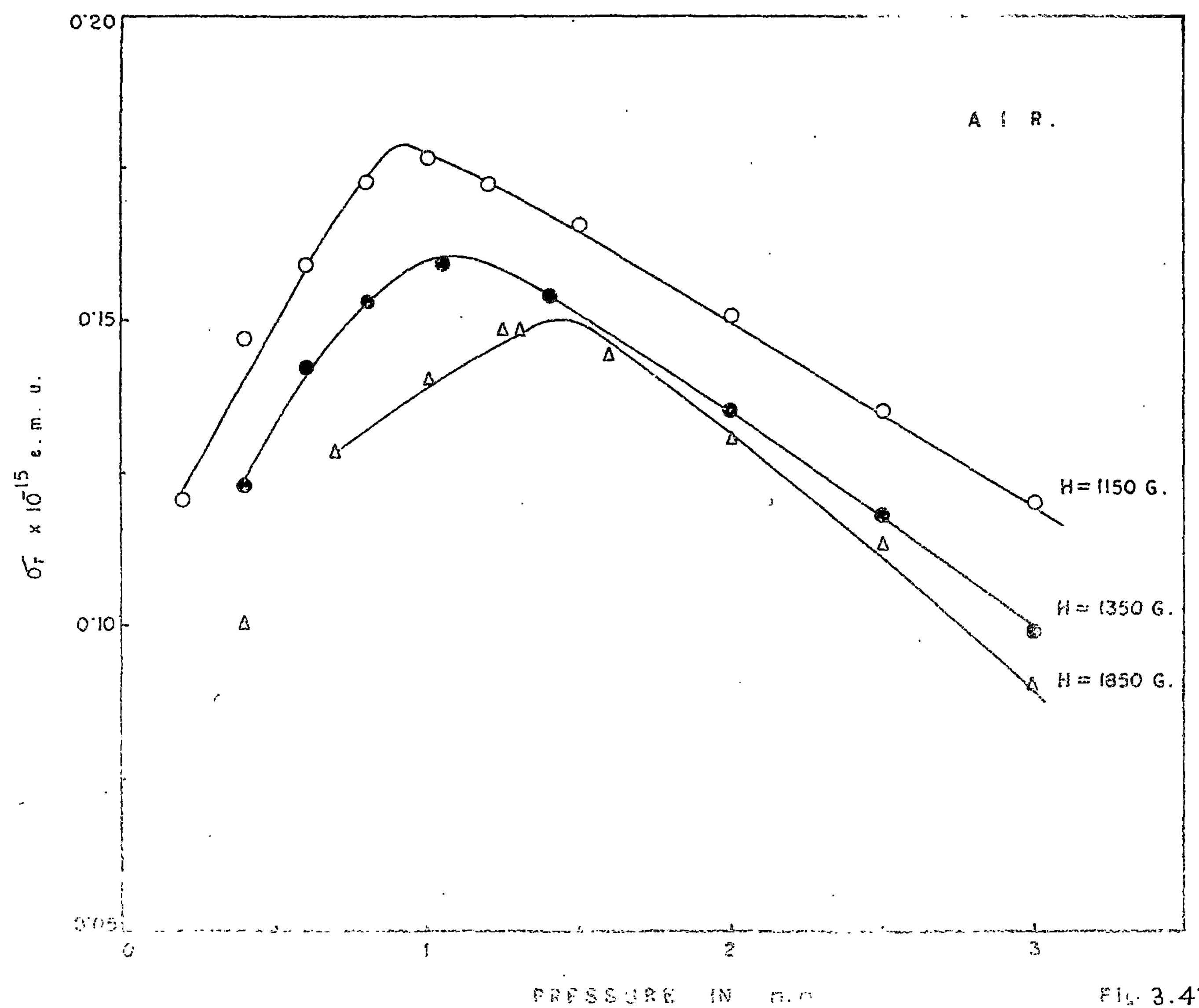

A I R.
H = 1150 G.
H = 1350 G.
H = 1850 G.
0·20
0·15
0·10
0·05
0
1
2
3
$\sigma_T \times 10^{-15}$ e. m. u.
PRESSURE IN m.m

Fig. 3.4

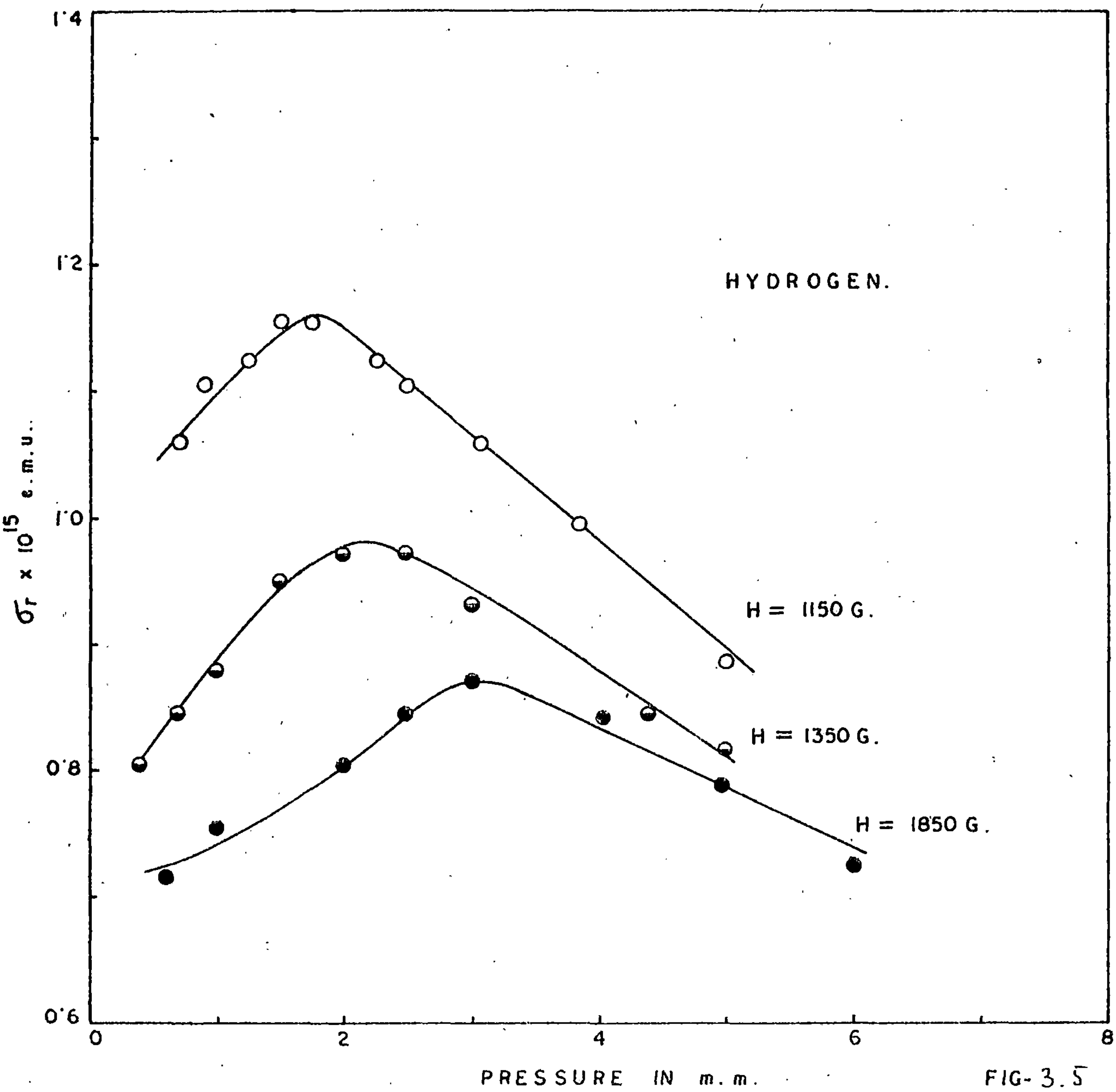
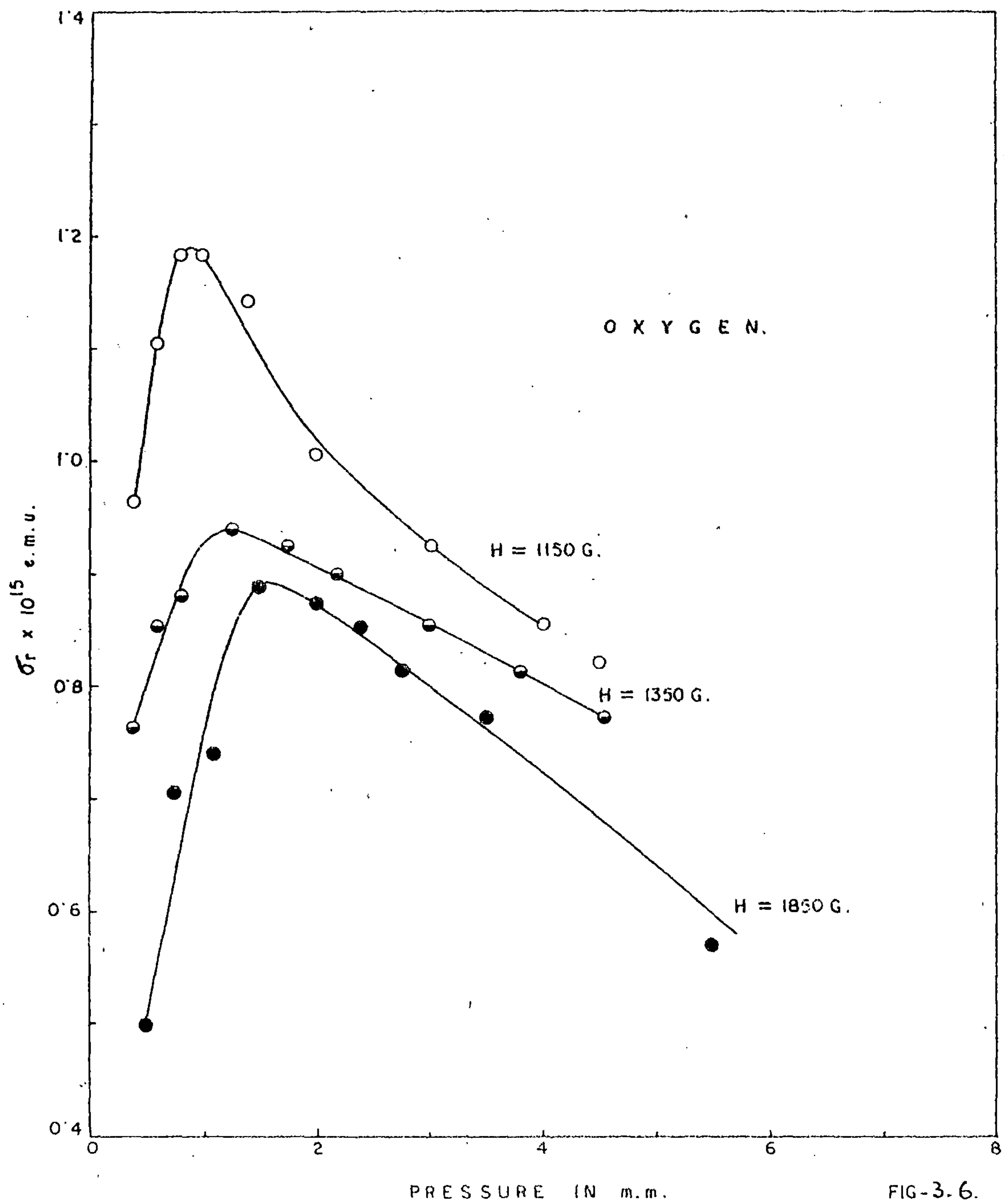
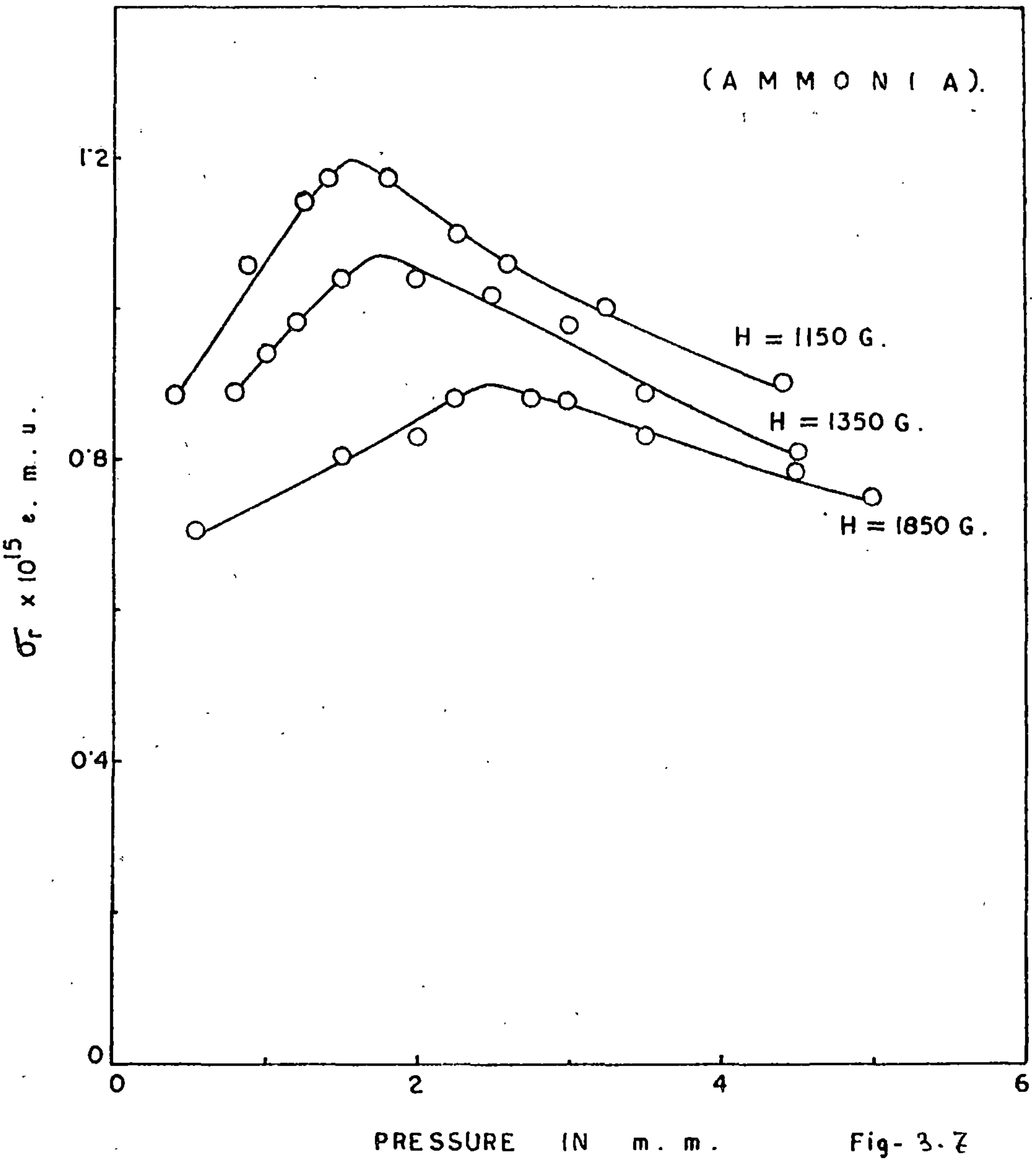

HYDROGEN.
H = 1150 G.
H = 1350 G.
H = 1850 G.
$\sigma_r \times 10^{15}$ e.m.u.
PRESSURE IN m.m.
1·4
1·2
1·0
0·8
0·6
0
2
4
6
8

FIG-3.5



OXYGEN.
H = 1150 G.
H = 1350 G.
H = 1850 G.
$\sigma_r \times 10^{15}$ e.m.u.
PRESSURE IN m.m.

FIG-3.6.



(AMMONIA).
H = 1150 G.
H = 1350 G.
H = 1850 G.
$\sigma_r \times 10^{15}$ e. m. u.
1·2
0·8
0·4
0
0
2
4
6
PRESSURE IN m. m.

Fig- 3.7

TABLE 3.1

Calculated values of electron density (n_H) at different values of magnetic field.

Frequency of measurement = 2.45 MHz.

GAS	Magnetic field in gauss	$(P_H)_{max}$ in m.m.	$(\sigma_{rf})_{H\,max} \times 10^{15}$ e.m.u.	ν	H/P_{max}	n_H
	1150	0.920	0.179	0.3222×10^{10}	1250	0.257×10^{8}
AIR	1350	1.080	0.161	0.379×10^{10}	1256	0.2725×10^{8}
	1850	1.470	0.150	0.5189×10^{10}	1259	0.3472×10^{8}
	1150	1.85	1.16	0.3222×10^{10}	621.6	1.651×10^{8}
HYDROGEN	1350	2.15	0.988	0.3791×10^{10}	626.9	1.674×10^{8}
	1850	3.05	0.87	0.5189×10^{10}	618.0	1.665×10^{8}

(TABLE 3.1 CONTD.)

GAS	Magnetic field in gauss	$(P_H)_{max}$ in m.m.	$(\sigma_{rf})_{H\,max}$ $\times 10^{15}$ e.m.u.	ν	H/P_{max}	η_H
	1150	0.9	1.19	0.3222×10^{10}	1277	1.693×10^8
OXYGEN	1350	1.1	0.95	0.3791×10^{10}	1228	1.628×10^8
	1850	1.5	0.895	0.5184×10^{10}	1236	1.619×10^8
	1150	1.55	1.2	0.3222×10^{10}	742.25	1.727×10^8
AMMONIA	1350	1.80	1.07	0.3791×10^{10}	750.00	1.811×10^8
	1850	2.45	0.9	0.5184×10^{10}	755.10	2.084×10^8

The electron density n_H in presence of magnetic field has been calculated from the maximum conductivity values. Again according to the theory of Beckman[4] (1948).

$$n_H = n \exp(-\alpha H r).$$

where
$$\alpha = \frac{e}{4\sqrt{2mK}}\sqrt{\frac{R}{T_e}}.$$

which shows a gradual radial variation of electron density from the axis to the wall of the tube, which physically means that under the action of the electric and transverse magnetic field most of the electrons will be driven to the walls of the tube. The measuring field which is at right angles to both these fields will thus see less and less number of electrons and consequently the radio frequency conductivity will decrease with the increase of the magnetic field asis also observed experimentally. Beckman's deduction thus provides a physical explanation of the observed results. To calculate the average or mean electron density which is seen by the measuring field let us consider a cylindrical shell of radius lying between r and $r+dr$ and if "a" is the radius of the tube and l the length then $\bar{n}$ the average density when the magnetic field is H is

$$\bar{n} = \frac{1}{\pi a^2 l}\int_0^a n\left[\exp(-\alpha H r)\right] 2\pi r l \, dr$$

$$= \frac{2n}{a^2}\left[\frac{a}{\alpha H}\exp(-\alpha H a) - \frac{\exp(-\alpha H a)}{\alpha^2 H^2} + \frac{1}{\alpha^2 H^2}\right] \qquad (3.8)$$

For a tube of radius 1 cm. such as has been used here the average values of electron density have been calculated for different magnetic fields in case of hydrogen and oxygen from equation (3.8) and results entered in the table 3.2.

TABLE 3.2

Frequency of measurement = 2.45 MHz. Calculated values of average electron density at different values of magnetic fields.

GAS	Magnetic field in gauss.	$(P_H)_{max}$ in mm.	$\bar{n}$
	1150	1.85	1.387×10^9
HYDROGEN	1350	2.15	1.778×10^9
	1850	3.05	4.002×10^9
	1150	0.9	1.423×10^9
OXYGEN	1350	1.1	1.776×10^9
	1850	1.5	4.112×10^9

An alternative way to check the value of n is to calculate the same from the relation $I = nev$ as the discharge current is 10 mA, and $v = \sqrt{\frac{2eV}{m}}$ where V is the discharge voltage which is 600 volts $n \approx 5 \times 10^8$.

It is thus observed that the measurement of radio frequency conductivity of a plasma and its variation with either pressure or magnetic field enables us to calculate the plasma parameters such as collision frequency, collision ~~the~~ cross section and electron density in a plasma with a fair degree of accuracy. One aspect of Beckman's theory namely the radial variaion of electron density in a magneticfield is verifield to a certain extent while the other aspect namely the increase of the axial electric field need not be taken into consideration because measurement of conductivity is made in a direction perpendicular to both the electric and magnetic fields.

It is well known that a magnetic field is used to confine a plasma and hence the study of plasma behaviour in a magnetic field is essential and useful as it will provide us the values of magnetic field and pressure at which the conductivity of the plasma becomes a maximum and hence the heating of the plasma will also be a maximum at that pressure and magnetic field.

REFERENCES

1. Appleton, E.V. & Boohariwalla, D.B., Proc. Phys. Soc. (London) 47 (1937) 1379.
2. Beckman, L. Proc. Phys. Soc. (London) 61 (1948), 515.
3. Gilardini, A., Nuovo Cimento Supplement, Sermov, 10, 13 (1959) 9.
4. Gupta, R. N. & Mandal, S.K., Ind. J. Phys. 41 (1967) 251.
5. Sen, S. N. & Ghosh, A.K., Ind. Jour. Pure & Appl. Phys. 4 (1966) 70.
6. Sen, S. N. & Gupta, R.N., Ind. J. Pure & Appl. Phys. 7 (1969) 462.
7. Sen, S. N. & Gupta, R.N., Jour. Phys. D. Appl. Phys. 4 (1971) 510.
8. Sen, S. N. & Das, R.P., International J. Electronics, 34 (1973) 527.
9. Sen, S. N. & Das, R. P. & Gupta, R.N., J. Phys. D. Appl. Phys. 5 (1972) 1260.