Chapter 2

2.1 Introduction

Einstein's General Theory of Relativity is widely recognized as the standard theory of gravitation. The gravitational phenomenon as described by Einstein is radically different from the image painted by the Newton's universal law of gravitation since it is the consequence of geometric space-time distortions. General Relativity and Newton's gravitational theory, however, make essentially identical predictions as long as the strength of the gravitational field is weak. Nonetheless there are few crucial weak field predictions where the two theories diverge and thus can be tested with careful experiments. Einstein himself proposed three tests - precession of the perihelion, gravitational bending of light and gravitational red shift. However, now it has been clear that the gravitational red shift is a test of the Einstein equivalence principle (or more correctly as a test of local position invariance principle) rather than that of general relativity. On the other hand Shapiro in the year 1964 proposed another crucial observational test of general relativity through measurement of relativistic time delay that was confirmed experimentally later. As mentioned earlier parameterized post-Newtonian (PPN) formalism [1] is generally employed to map almost all metric theories of gravity in the week field regime. After introducing PPN formalism, we briefly discuss those standard tests of relativity below.

2.2 The PPN formalism:

In the week field and slow motion limit, the space-time metric predicted by most of gravity theory has the same structure, which can be expressed as an expansion about the Minkowski metric in terms of dimensionless small gravitational potential

$$U(x,t) = \int \frac{\rho(x',t)}{|x-x'|} d^3x' \tag{2.1}$$

so that for static case the metric coefficients take the form

$$g_{oo} = -1 + 2U - 2\beta U^2 + 4\psi - \varsigma\phi \tag{2.2}$$

$$g_{ij} = \delta_{ij}(1 + 2\gamma U) \tag{2.3}$$

where

$$\psi = \int \frac{\rho(x',t)\eta(x',t)}{|x-x'|} d^3x' \tag{2.4}$$

$$\phi(x,t) = \int \frac{\rho(x',t)\left[(\vec{x}-\vec{x}').\vec{v}(x',t)\right]}{|x-x'|^3} d^3x' \tag{2.5}$$

$$\eta = \beta_1 \vec{v}^2 + \beta_2 U + \frac{1}{2}\beta_3 \Pi + \frac{3}{2}\beta_4 P/\rho \tag{2.6}$$

(the Newtonian potential for the Sun produce the first non-vanishing contribution, which is of the order of ε^2, by definition of the parameter ε. Accordingly the potentials ψ and ϕ are of the order of ε^4) P is the pressure, $\Pi = \frac{\rho_o - \rho}{\rho}$, **v** represents the velocity of matter in the solar system and the constants $\gamma, \beta, \beta_1, \beta_2, \beta_3, \beta_4$ and ζ are the PPN parameters.

Actually ten PPN parameters completely characterize the weak-field behavior of a wide class of metric theories of gravity including general relativity when rotating part is also included. Out of these parameters, namely γ and β have fundamental importance. The PN parameter γ quantifies how much curvature produced by a unit mass, β described non-linearity in superposition law. For GR as well as for the scalar tensor theories γ and β are the only non-vanishing PPN parameters.

Hence the PPN metric

$$ds^2 = -B(r)dt^2 + A(r)\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\varphi^2\right) \qquad (2.7)$$

can be described in most cases by the metric coefficients

$$\mathrm{B(r)} = 1 - 2\frac{m}{r} + 2\beta\frac{m^2}{r^2} \qquad (2.8)$$

and

$$\mathrm{A(r)} = 1 + 2\gamma\frac{m}{r} \qquad (2.9)$$

(we have taken G = C = 1).

2.3 The classical tests of general relativity:

The solar system experiments allow one to map out weak field gravity at the first post-Newtonian level.

2.3.1. Precession of the perihelion

The orientation of planets orbit is found to precess in space over time, which is commonly called the "precession of the perihelion". Relative to the local inertial frame of the solar system the precision of Mercury's orbit is observed as ~572′ seconds of arc per century (a second of arc is 1/3600 of an angular degree) out of which 529″ can be accounted for in the framework of Newton's theory by the action of other planets. But Newton's law of gravitation cannot account the remaining 43 seconds of arc per century in the observed precession.

For the PPN metric as given by the Eqs.(2.7)-(2.9), the relativistic expression for precession, is given by

$$\Delta\varphi = 2\pi \frac{mG}{L}(2+2\gamma-\beta) \text{ radian per revolution.}$$

where $L=a(1-e^2)$ is the semi-latus rectum of the orbit with the semi major axis a and eccentricity e, and m is the total mass of the two body system. For GR i.e. for γ and β equal to 1, $\Delta\varphi = 43$ arcsec per century.

For the Brans-Dicke theory the above expression becomes

$$\Delta\varphi = 6\pi \frac{MG}{L}\left(\frac{3\omega+4}{3\omega+6}\right)$$

2.3.2. Bending of light

The theory of general relativity predicts the deflection of light in the presence of a mass distribution. If a photon approaching the sun from very great distances the deflection of the orbit from a straight line is

$$\Delta\varphi = 2\left|\varphi(r_0) - \varphi_\infty\right| - \pi \qquad (2.10)$$

If this is positive, then the angle φ changes by more than $180°$, that is the trajectory is bent toward the sun ; if $\Delta\varphi$ is negative then the trajectory is bent away from the sun. For the metric given by Eqs.(2.7)-(2.9)

$$\varphi(r) - \varphi_\infty = \int_r^\infty A^{1/2}(r)\left[\left(\frac{r}{r_0}\right)^2\left(\frac{B(r_0)}{B(r)}\right) - 1\right]^{-1/2} \frac{dr}{r} \qquad (2.11)$$

To the first order in $\frac{MG}{r_0}$, the deflection angle thus becomes

$$\Delta\varphi = \frac{4MG}{r_0}\left(\frac{1+\gamma}{2}\right) \tag{2.12}$$

For a light ray deflected by the sun we must use $M = M_0 = 1.97 \times 10^{33} g$, that is, $MG = M_0 G = 1.475 km$, and the minimum value of r_0 is $R_0 = 6.95 \times 10^5 km$, so

$$\Delta\varphi = \left(\frac{R_0}{r_0}\right)\theta_0 \text{ where } \theta_0 = \frac{4MG}{R_0}\left(\frac{1+\gamma}{2}\right) = 1.75''\left(\frac{1+\gamma}{2}\right)$$

General relativity gives $\gamma = 1$ so it predicts a deflection toward the sun $\theta_0 = 1.75''$ which is confirmed by the observations with a high accuracy.

For Brans-Dicke theory the expression for θ_0 is $\frac{4MG}{R_0}\left(\frac{2\omega+3}{2\omega+4}\right)$.

2.3.3. Gravitational time delay

The time required for light to move from r_0 to r in a gravitational field described by the Eq.(2.7) is

$$t(r, r_0) \cong \sqrt{r^2 - r_0^2} + (1+\gamma)MG \ln\left(\frac{r + \sqrt{r^2 - r_0^2}}{r_0}\right) + MG\left(\frac{r - r_0}{r + r_0}\right)^{1/2} \tag{2.13}$$

The leading term $\sqrt{r^2 - r_0^2}$ is just the time required for light to travel in straight lines (Euclidian) at unit velocity. The other terms gives a general-relativistic delay in the time. Several high precision measurements were made using radar ranging to targets passing through superior conjunction. The targets employed included planets, such as Mercury. All those experiments support the predictions of GR to high accuracy.

2.4 A quest for higher order effects:

The first order predictions of general relativity have been extensively tested in the solar system and the theory is found to agree with all data gathered to date with great success. Experiments like the microwave ranging to the Viking Lander on Mars [2], Lunar laser ranging [3], the astrometric observations of quasars with very long based Interferometry (VLBI) [4] yielded accuracy nearly 1% in the tests of general relativity. The recent experiments with the Cassini spacecraft improved the accuracy of the first order test of Einstein theory to $\sim 0.2\%$ [5].

With the advancement of technology now plans are being developed to study second and higher order effects. In fact, already experiments are launched to detect gravitational waves, which is practically a feature of strong field regime.

2.4.1 The second order effects:

Plans are being developed to launch orbiting observatories of microarcseconds angular resolution. For the purpose optical interferometry technique will be employed. Since at the limb of the Sun $GM/r \sim 2 \times 10^{-6}$, one might expect that second order corrections would be order of few μ arcsecs. Thus, possibility has developed to access post post-Newtonian corrections experimentally in near future and hence a clear understanding on such corrections is necessary.

It appears that only gravitational deflection of light at second order accuracy could be measurable in near future. Other effects such as gravitational delay and precession of the perihelion are comparatively difficult to detect with such high accuracy. The theoretical development of computing gravitational deflection of light commenced in the early eighties [6]. The deflection angle for the PPN metric (Eqs.(2.7)-(2.9)) to the second order accuracy is given by

$$\alpha = \frac{2(1+\gamma)M}{r_0} + \frac{M^2}{r_0{}^2}\left[\{2(1+\gamma) - \beta + 3/4\,\delta\}\pi - 2(1+\gamma)^2\right] \qquad (2.14)$$

Where δ is a post PPN parameter. In GR the second order effect contributes a deflection of $\sim 3.5 \times 10^{-6}$ arc sec.

When the rotation and quadrupole moment of the Sun are taken into account non-trivial contributions of μarcsecs order would also appear. Details of those contributions and feasibility of accessing the post PPN parameter δ through measurement of deflection angle at second order accuracy has been discusses in the next chapter.

2.4.2. Strong field lensing:

Gravitational lensing by a massive compact lens appears a potential tool for studying and testing gravity theories in the strong field regime. Theoretical investigations [7-10] suggest that while propagating close to massive compact object (e.g. a black hole) light rays take several turns around the lens before reaching the observer and as a result apart from primary and secondary images a set of infinite images on both side of the optic axis will be produced which are termed as relativistic images. Though in such situation the primary and secondary lensed images carry important information on various orders of post-post-Newtonian effects [11] but these relativistic images are the main signature of the strong field lensing. However, unless the source is almost perfectly aligned with the lens and the observer, relativistic images will be very faint as a result of high demagnification. With the indication that the radio-source Sgr A* in the galactic centre hosts a supermassive object (black hole) of 3.6×10^6 solar masses [12] a possibility has developed of studying lensing phenomena in the strong gravity regime.

Lensing theory in the strong field regime has been developed in stages by several researchers. The occurrence of relativistic images was brought forward by Darwin [13] and Atkinson [14] in their pioneering works in the field. The lens equation in the strong regime was mainly developed by Fritelli and Newman [15], Virbhadra and Ellis [10], Bozza et al [16] and Perlick [17]. After a detailed numerical study of strong field lensing produced by a Schwarzschild black hole, Virbhadra and Ellis [10] first explored observational consequences of the phenomena when the lens is the massive black hole of the galactic centre. Noting the possibility that detection of relativistic images may not be impossible in future and hence they could be used to test strong field gravity, extensive study of relativistic images started to take place. Bozza et al [16] developed an analytical technique of obtaining deflection angle in the strong field situtation and showed that the deflection angle diverges logarithmically as light rays approach the photon sphere of a Schwarzschild black hole. Such a study was extended by Eiroa et al. [18] for lensing due to the Reissner-Nordstrom (RN) spacetime. Later Bozza [19] extended the method of analytical lensing for general class of static spherically symmetric metrics and demonstrated that the logarithmic divergence of deflection angle at photon sphere is a common feature for such space-times. Exploiting the Bozza's method, strong field lensing has been carried out to several interesting cases, such as lensing due to the charged black hole of heterotic string theory [20], black holes from braneworlds [21], Einstein-Born-Infeld black holes [22], wormholes, monopole [17] etc. Very recently Bozza et al [23] have studied strong field lensing due to the Kerr black hole for equatorial observers. On the other hand Keeton and Peters have introduced a technique for computing corrections to lensing observables beyond the weak deflection limit [24]. Recently Amore and his collaborators [25] introduced a new formalism for estimating deflection angle of light. Their approach allows one to convert the integral for the angle into a geometrical convergent series, whose terms can be calculated analytically.

An interesting consequence of strong field gravitational deflection is the retro lensing [26] which occurs when the source is in between the observer and the lens or the observer is in between the source and the lens in contrast to the case of standard lensing where lens is situated in between the source the observer. The phenomenon is almost same to the standard lensing except the fact that relativistic images are formed in this case for deflection angles closer to odd multiples of π rather than even multiples. Holtz and Wheeler [26] studied retro lensing due to a Schwarzschild black hole in the Galactic buldge with the Sun as a source. Eiroa and Torres [27] considered the analytical retro lensing due to a general spherically symmetric static lens. Without remaining confined to the highly aligned case of source, lens and observer geometry, Bozza and Mancini [28] explored retro lensing due to the massive black hole of Galactic centre with the nearby (

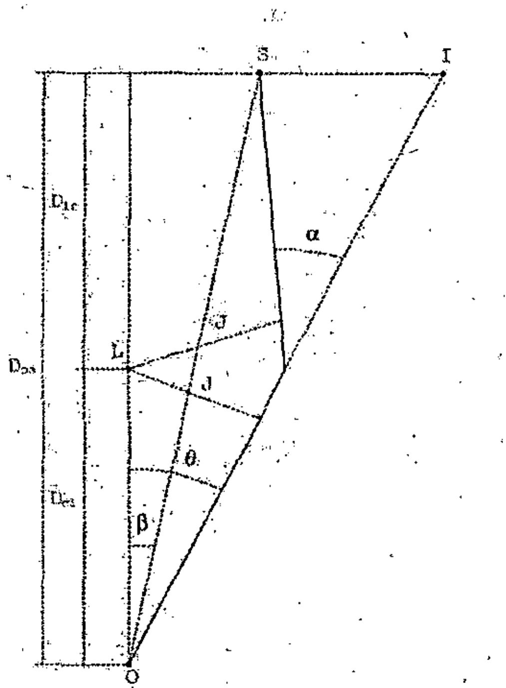

FIG. 1. Lens diagram. The observer (O), the lens (L), the source (S), and the image (I) positions are shown. D_{ol}, D_{os}, and D_{ls} are, respectively, the observer-lens, the observer-source, and the lens-source distances. α is the deflection angle and J is the impact parameter.

to lens) bright star S2 as source. The time delay between different relativistic images was estimated by Bozza and Mancini [29] which was later applied by several authors to some interesting cases [30].

An analytical approach of estimating deflection angle at strong gravity regime is given in the chapter 5 following the technique developed by Bozza [19]. The analytical expression of the deflection angle close to the divergence can be written as

$$\alpha(\theta) = -u \log\left(\frac{\theta\, \mathrm{D_{OL}}}{\mathrm{b_{ps}}} - 1\right) + v + O\left(b - b(x_{ps})\right) \tag{2.15}$$

The parameters u, v, b_{ps} can be estimated for a theory analytically/numerically from the expressions as given in the chapter 5. For GR these parameters take the following value

u = 1, v = -0.4002 and $b_{ps} = \frac{3\sqrt{3}}{2}$ [19].

Several other efforts are going on to detect higher order effects including strong gravity effects. This includes detection of gravitational waves, binary pulsars measurements, exploration of the spacetime near black holes and neutron stars through accreting matter . Comprehensive reviews on these features are given in [31].

References:

[1] C.M.Will, " The Confrontation between General Relativity and Experiment", *Living Reviews.* S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1972) C.W.

Misner, K.S. Thorne, and J.A. Wheeler, Gravitation (Freeman, San Francisco, 1973).

[2] R. D. Reasenberg, et al. *Astrophys. J. Lett.*, **234**, L219-L221 (1979). I. I. Shapiro, C. C. Counselman, III, & R. W.King, *Phys. Rev. Lett.*, **36**, 555 (1976). I. I. Shapiro, et al., *Journal of Geophysical Research.*, **82**, 4329-4334 (1977).

[3] K. Nordtvedt, Jr., *Phys. Rev.*, **170**, 1168 (1968). K.Nordtvedt, Jr., *Phys. Rev.* D **43**, 10 (1991). K.Nordtvedt, Jr., *Class. Quantum Grav.* **15**, 3363 (1998). K.Nordtvedt, *CQG* **16**, A101 (1999). K.Nordtvedt, Jr., arXiv:gr-qc/0301024. J. G. Williams, Newhall, X X, & J. O. Dickey, *Phys. Rev.* D **53**, 6730 (1996). J. G. Williams, J. D. Anderson, D. H. Boggs, E.L. Lau, & J. O. Dickey, *AAS Meeting*, Pasadena, CA, June 3-7, 2001, BAAS 33, 836 (2001).

[4] D. S. Robertson, W. E. Carter & W. H. Dillinger, *Nature*, **349**, 768 (1991).D. E. Lebach, B. E. Corey, I. I. Shapiro, M. I. Ratner, J. C. Webber, A. E. E. Rogers, J. L. Davis, & T. A. Herring, *Phys. Rev. Lett.*, **75**, 1439 (1995).

[5] B. Bertotti *et al, Nature* **425**, 374 (2003)

[6] G.W. Richter and R.A. Matzner, *Phys. Rev.* D 26, 1219 (1982). E. Fischbach and B.S. Freeman, *Phys. Rev.* D 22, 2950 (1982). R. Epstein and I.I. Shapiro, *Phys. Rev.* D 22, 2947 (1980).

[7] C. Darwin, *Proc. Roy. Soc.* (London) A **249**, 180 (1959)

[8] R.D. Atkinson, *Astron. J* **70**, 517 (1965)

[9] J.P. Luminet, *Astron. Astrophys.* **75**, 228 (1979); H.C. Ohanian, *Am. J. Phys.* **55**, 428 (1987);R. J. Nemiroff, it *Am. J. Phys.* **61**, 619(1993)

[10] K.S. Virbhadra and G.F.R.Ellis, *Phys. Rev.* D **62**, 084003(2000)

[11] C.R.Keeton, A.O.Petters *Phys.Rev.*D **72,** 104006 (2005).

[12] F. Eisenhaur et al, *Astrophys.* J **628**, 246 (2005); D. Richstone et al, *Nature* **395,** A14 (1998); J. Magorrian et al, *AJ* **115** 2285 (1998).

[13] C. Darwin, *Proc. Roy. Soc.* (London) A **249**, 180 (1959)

[14] R.D. Atkinson, *Astron. J* **70**, 517 (1965).

[15] S. Frittelli, and E.T. Newman, *Phys. Rev.* D **59**, 124001(1999); S. Frittelli, T.P. Kling and E.T. Newman, *Phys. Rev.* D **61**, 064021(2000)

[16] V. Bozza, S. Capozziello, G. Iovane and G. Scarpetta, *Gen. Rel. Grav.* **33,** 1535 (2001).

[17] V. Perlick, *Phys. Rev.* D **69**, 064017 (2004).

[18] E.F. Eiroa, G.E. Romero and D.F. Torres, *Phys. Rev.* D. **66,** 024010 (2002)

[19] V. Bozza, *Phys.Rev.* D **66**, 103001 (2002)

[20] A. bhadra, *Phys. Rev.* D.**67** 103009 (2003)

[21] R. Whisker *Phys. Rev.* D **71**, 064004(2005); E.F. Eiroa *Phys. Rev.* D **71,** 083010 (2005).

[22] E.F. Eiroa gr-qc/0511065

[23] V. Bozza, F.De Luca, G. Scarpetta, M. Sereno *Phys. Rev.* D **72,** 083003 (2005)

[24] C.R.Keeton and A.O.Petters, *Phys. Rev.* D **72**, 104006 (2005)

[25] P. Amore and S. Arceo, *Phy. Rev.* D **73,** 083004 (2006); P. Amore, S. Arceo, and F. M. Fernandez, *Phy. Rev.* D **74,** 083004 (2006).

[26] D.E. Holtz and J.A. Wheeler, *Astrophys. J.* **578**, 330 (2002).

[27] E.F. Eiroa and D.F. Torres *Phys. Rev.* D **69**, 063004 (2004).

[28] V. Bozza, L.Mancini *Astrophys. J.* **611**, 1045 (2004).

[29] V. Bozza, L.Mancini *Gen. Rel. Grav.* **36**,435 (2004).

[30] E.A. Larranaga Rubio gr-qc/0309108.

[31] C.M.Will, " The Confrontation between General Relativity and Experiment", *Living Reviews.*