Chapter 1

1.1 Introduction

Einstein's general relativity [1] is widely considered as the standard theory of gravity, at least at the classical level. This is mainly because the theory has an elegant and well-understood structure and it is in good accordance with all the standard experimental tests of gravity till now. However, tests of gravitation conducted till date [2] only probe mainly up to the first order (post-Newtonian) effects of the theory. Since the most exciting predictions of the theory, such as existence of black hole etc., are strong field (strong field is distinguished from the weak gravity through the quantity GM/rc^2, in the strong field regime higher order terms in GM/rc^2 can not be ignored) features, it is important to examine and test the higher order effects of the theory.

Some of the features of general relativity are not even without difficulties. Particularly a major problem of the theory is the occurrence of unavoidable space-time singularities. It is generally suspected that the classical description, provided by the general relativity, breaks down in a domain where the curvature is large. Hence, the question of quantization of gravity arises. But sustained failure of reconciling general relativity with quantum mechanics indicates that the general relativity may need some modifications. Several alternative theories to general relativity have been proposed which are modifications of general relativity in the sense that they have the same post-Newtonian limit as general relativity while are different theories in other regimes. Among them Scalar-tensor theories of gravity [3], in which gravity is mediated by one or several long-range scalar fields together with the usual tensor field, are considered as best motivated class of viable non-Einsteinian theories of gravity till date. They arise naturally as the low energy limit in several modern theoretical attempts to quantizing gravity, such as the Superstring theory [4] or the Kaluza-Klein theory [5].

Since all viable alternative theories coincide with general relativity in the post-Newtonian limit, it is important to study higher order effects in which general relativity may give different predictions than those of alternative theories. At present technology has advanced to the point that the present onboard gravitational experiments or near future experiments are expected to improve the accuracy of the measurement by at least two orders. For instance the Stanford Gyroscope experiment (the Gravity Probe-B mission) is expected to measure the post-Newtonian parameter γ with accuracy of 5×10^{-5} [6] against the current limit of accuracy 3×10^{-3} [2] whereas the Laser Astrometric Test of Relativity mission is expected to test relativistic gravity at the accuracy better than second order in gravitational field strength [7]. Thus there is genuine possibility of measuring small deviations from the predictions of general relativity.

Different authors obtained few theoretical predictions of gravitational theories with accuracy up to second order (or even higher accuracy) in gravitational strength during the last two decades. For instances bending of light [8] and radar echo delay [9] in standard and scalar tensor theories have been estimated with such accuracies. Considering the recent progress in experimental front it is now important to explore other second and higher order physical effects those can be used to test Einstein theory at higher order level and also to discriminate it from the alternative theories.

1.2 Broad outline of the aim and plan of the thesis:

In the present investigation mainly some higher order physical effects of gravity, such as bending of massive particles or strong field lensing for Brans-Dicke scalar tensor theory, have been explored and their observational aspects are examined.

In the week field regime parametrized post-Newtonian (PPN) formalism [10,2] is generally employed to map almost all metric theories of gravity. Hence the study is carried out in the PPN formalism for estimating the second order effects so that the results can be useful not only to test Einstein theory at higher orders but also for discriminating alternative theories from general relativity.

In the present work an emphasis has been given on the study of physical effects in the strong field regime since the theories of gravity can be tested in their full form only in this region. Specifically gravitational lensing in the strong field regime has been studied. Such a study for general relativity [11] has already been made in the literature. The objective of the present investigation is to use the effect to discriminate Einstein theory from alternative theories. It is, however, not possible to parametrize gravitational theories in the strong field regime. Hence a desirable pre-requisite for studying strong field situation is to have knowledge of exact explicit solution(s) of the field equations. A problem with several alternative theories including scalar-tensor theories is that there exist several exact solutions and not all of them are physically relevant. Since only physically acceptable solutions are of astrophysical interest, efforts have been made to examine physical viability of the different solutions. Specifically a study has been made for the best known and simplest scalar tensor theory, the Brans-Dicke theory, and physically viable solution of the theory has been identified. Then strong field lensing effects for the physical solution of the theory are studied and compared with those due to Einstein theory.

Scalar tensor theories can be formulated two conformally related frames, namely the Jordan frame and the Einstein frame [12]. Physical equivalences of the two formulations in these two frames have been debated for long [12]. Here a study has been made to examine whether physical equivalence of the two formulations hold at higher order effects. In another work physically viable traversable wormhole geometry in Brans-Dicke theory has been categorized and usability of this wormhole geometry for interstellar travel has been examined.

The organization of the thesis is as follows. After giving a short review on the tests of gravity at the first PPN order, a brief survey on theoretical predictions of some known effects of gravity at higher orders are discussed in the next chapter. The bending of massive particles due to gravity is estimated with second PPN order accuracy in chapter 3 along with a discussion on experimental feasibility of measuring the effect and its advantage over the deflection of light to test the theory at the second order. In chapters 4-6 possibility of discriminating Einstein theory from the scalar tensor theories through study of gravitational deflection of light at higher order accuracies has been discussed. In chapter 4 the physically viability of solutions of the Brans-Dicke theory, the best-known alternative theory of GR, is studied. The strong field gravitational lensing effects due to Brans-Dicke scalar tensor theory and its comparison with that of the Einstein theory are discussed in chapter 5. Second order gravitational bending of light in the Jordan and the Einstein frame formulations of scalar tensor theories is studied in chapter 6 and physical equivalences of the two formulations are examined in the same chapter. The wormhole geometry in Brans-Dicke scalar tensor theory is examined in chapter 7. Finally the results of this thesis are briefly summarized in chapter 8.

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