# Introduction

## CHAPTER - I

In this chapter we have dealt with the two problems on surface gravity waves. Here we consider a body of inviscid and in compressible fluid in equilibrium, in a gravitational field and having a plane free surface. We suppose that a small disturbance is applied at some region on the free surface consequently, the free surface will no longer be in equilibrium and hence motion will set up so that waves will be propagated over the free surface of the fluid.

Thus we assume that amplitudes of the waves remain small so that a linearized theory can be used. The velocity of the moving fluid particles is small that the non linear convective term $(\bar{u}\,\nabla\,)\,\bar{u}$, in Eulerian's equation $\frac{\partial \bar{u}}{\partial t} + (\bar{u}\,\nabla)\bar{u} = -\frac{1}{\rho}\,\nabla\,p - g\hat{k}$, can be neglected so that the particle acceleration is described by the linear term $\bar{u}_t$. This will determine under what conditions the nonlinear term is very small in comparison with the linear term.

Here $\bar{u} = (u,v,w)$ is the velocity vector of the fluid motion, P is the pressure, g the acceleration due to gravity, $\hat{k}$ is the unit vector in the positive direction of z.

In case of fluid of infinite depth, both the velocity potential $\phi$ and the free surface elevation $\eta$ satisfy.

$\nabla^2\phi = \phi_{xx} + \phi_{yy} + \phi_{zz} = 0; -\infty < z \leq 0$, with the bottom boundary condition $\phi_z \to 0 \; as \; z \to -\infty$

The concept of variational principle for water waves after Whitham (1965 a, b) [115, 116] and latter on developed by Luke (1967) [73] has been considered.

Two-dimensional cauchy - poison problem for inviscid and also viscid (which has been taken in consideration in latter problems) in compressible fluid of both finite and infinite height with a free horizontal surface has been taken in consideration.

In this regard the works of Miles (1963, 1968),[84, 86] Debnath (1983) [35] are also to be mentioned on the cauchy - poison wave problem in a viscous fluid. We also want to refer to the papers of Rollins and Debnath (1992) [99] and Debnath and Guha (1989) [37] on the Cauchy - poison wave problem in a rotating or stratified fluid.

We have treated $\phi\ (x, z; t)$, $\eta\ (x; t)$ and p(x) as generalized functions in the sense of light hill (1958) [67] so that their Fourier transforms exist with respect to x.

Also the investigation of katsis and Akylas (1987) [53] on the three dimensional waves produced by a moving pressure field acting on the free surface of an inviscid fluid of depth h [Transient wave motion in an inviscid fluid] has been studied.

In consideration of the periodic plane surface waves on deep water, we refer to the Stokes paper (1847), [109] where the Author first established the non linear solutions for periodic plane waves on deep water.

Miche (1944) [81] obtained the maximum wave height in water of finite depth h.

In a fluid of finite depth, the Lagrangian velocity potential components when integrated with respect to time shows that the fluid particles move in an ellipse with the horizontal and vertical semiaxes.

The first paper in this chapter is, ''Note on the surface wave due to prescribed elevation", which has been dealt with the motion of the wave, initiated and maintained by the prescribed surface elevation. Two cases have been considered, i) when the prescribe elevation is a function of exponentially decaying force $\frac{1}{\varepsilon\sqrt{\pi}} \cdot e^{-\frac{r^2}{\varepsilon^2}}$, as the radius vector increases, it alsoceases when r is zero, but exhibits a greater elevation immediately after that. The motion has been studied graphically, it experiences the variation of surface elevation with the variation of g-the acceleration due to gravity, it oscillates with the variation of time, leaving behind the highest elevation initially at t = 0, and diminishes asymptotically as $\frac{r}{\varepsilon} \to 1$

The second problem in this chapter is "Generation of surface waves in a rotating sea by wind stress.

In this problem, the motion of the wave has been studied in a rotating fluid, rotates with the uniform angular velocity $\Omega$. Also there is an impressed periodic force of frequency $\omega$. The motion of the fluid has been studied under the circumstances when $2\Omega > \omega$ and $2\Omega = \omega$, but not in case of $2\Omega < \omega$, which gives an imaginary solution.

From the graphical study it has been observed that the motion degenerates when it experiences the exponentially decaying surface force.

When $2\Omega > \omega$, the surface elevation diminishes parabolically as $\frac{r}{\varepsilon} \to 1$, other wise the surface elevation increases parabolically when $2\Omega = \omega$, while no definite information can be obtained when $2\Omega < \omega$, due to the presence of the imaginary term in the solution.

In this respect, the following references on the surface waves are worthy of mention.

Choudhury, K[26] (1969), Chen, B and Saffman, [23] P.G. (1980a), Chu, V. M. and Mei, C.C. [27] (1971), Glazman, R. E. [42] (1992), Kamenkovich, V. M. [50] (1977), Kinsman, B. [56] (1965), Le Blond, P.H., and Mysak, L. A. [64] (1978).

# CHAPTER II

For the case of long waves, the depth of the fluid is small compared to the wave length and the disturbances effects the motion of the whole of the fluid. Such waves in water are generally produced by disturbing forces such as wind pressure or by such causes as irregularities in the bed of a stream, so that neglecting viscocity, the motion is irrotational.

In this chapter we will assume that the vertical acceleration of the fluid particles may be neglected, or, more precisely, that the pressure at any point (x,y) is sensibly equal to the statical pressure due to the depth,the free surface,

$$p - p_0 = g\rho(y_0 + \eta - y) \quad (1)$$

Where $p_0$ is the uniform external pressure

Hence, $\frac{\partial p}{\partial x} = g\rho \frac{\partial \eta}{\partial x}$ (2)

This is independent of y, so that the horizontal acceleration is the same for all particles in a plane perpendicular to x. It follows that all particles which once lie in such a plane always do so, in other words u = u (x, t) and the free surface elevation $\eta = \eta(x; t)$

The equation of horizontal motion is,

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

for small motions, omitting $u\frac{\partial u}{\partial x}$ weget,

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} = -g\frac{\partial \eta}{\partial x} \qquad (3)$$

Let $\xi = \int u dt$

Then, equation (3) becomes, $\frac{\partial^2 \xi}{\partial t^2} = -g\frac{\partial \eta}{\partial x}$

and $\eta = -h\frac{\partial \xi}{\partial x}$ (cf. Lamb 169, 255)[62]

and subsequently weget, $\frac{\partial^2 \xi}{\partial t^2} = gh\frac{\partial^2 \xi}{\partial x^2}$

Where h is the depth of the fluid.

Introducing the velocity potential $\phi$ we can reach to the two dimensional cauchy - Poisson problem which is governed by the equation.

$$\phi_{xx} + \phi_{yy} = 0; \; -h \leq y \leq 0, \; -\infty < x < \infty; \; t > 0$$

$$\left.\begin{array}{l} \phi_y - \eta_t = 0 \\ \phi_t + g\eta = 0 \end{array}\right\} on \; y = 0, \, t > 0$$

$$\phi_y = 0, \; y = -h \; or \; y \to -\infty$$

Long waves are dispersive in characteristic where the dispersion relation is governed by the relation

$\omega^2 = gk \tanh kh$ [cf. Debnath L.] [36]

Where $\omega$ is the frequency and k is the wave number.

Wave motions are usually called dispersive or, nondispersive, according as the phase velocity of the waves is dependent or, independent of the wave length.

In this respect we would like to refer to some more developments on long waves.

Korteweg - de - vries [60] (kdv) equation for long water waves.

$$\phi_t + c_0 \phi_x + \alpha \phi_{xxx} = 0 \ Where\ c_0^2 = gh,\ \ \omega = c_0 k - \alpha k^3$$

Boussinesq [16] equation for long water waves :

$$\phi_{tt} - c_0^2 \phi_{xx} - \beta^2 \phi_{xxtt} = 0;\ \omega^2 = c_0^2 k^2 \left(1 + \beta^2 k^2\right)^{-1}$$

Benjamin[10], Bona and Mohany (BBM) equation

$$\phi_t + c_0 \phi_x - \alpha\ \phi_{xxt} = 0,\ \omega = c_0 k\ (1 + \alpha k^2)^{-1}$$

Klein - Gordon equation :

$$\phi_{tt} - c^2 \nabla^2 \phi + \beta^2 \phi = 0,\ \omega = \pm\left(c^2 k^2 + \beta^2\right)^{\frac{1}{2}}$$

The first problem of this chapter is "Note on the motion of long waves set up by finite body forces."

Four types of forces have been taken into consideration in this problem.

i) Force decaying exponentially with time.
The motion has been studied graphically. It reveals the fact that the amplitudes and wave length diminishes as the time elapses.

ii) Force acting for a finite time - Heavisides unit function has been utilised for the purpose, it consists of two possibilities, o < t < T and t > T.

Graphical study has been made on these two possibilities, it has been observed that when o < t < T, the surface elevation diminishes parabolically where as it traces the parabollic curve when t > T.

iii) Impulsive force - the Dirac delta function on time has been used for the pur pose and it has been observed graphically that the surface elevation experiences the oscillatory wave motion with the variation of time.

iv) Periodic Pulse - There are two possibilities in this consideration,

$$0 \le t \le \frac{\pi}{\omega} \ and\ t > \frac{\pi}{\omega}$$

The motion has also been studied graphically and it has been observed that when, $0 \le t \le \frac{\pi}{\omega}$ the surface experiences the depression and irregular wave motion, with the variation of time while it experiences the S.H.M. in case of $t > \frac{\pi}{\omega}$

In this respect we would like to quote the works of Miles [83] J.W. (1962), Madson, [75], 0.5., Mei, c.c. and savage, R. P. (1970), Mc. Lean, [80], J.W. (1982b), Mondal [87], C. R. (1986), Amick, C. J. and Toland, J. F. (1981b), Barakat [6], R. and Houston, A (1968), Bryant, P.J. [18, 19] (1973, 1982 b)

The second problem in this chapter is "Long wave in a canal of variable depth."

In this problem, the motion has been excited, in addition to the gravity, by a time dependent oscillatory force of frequency $\omega$. The graphical study has been made from the different points of view i) when x varies alone, ii) When t varies alone iii) when x and t both vary.

It has been observed that when x varies the motion diminishes parabolically, when t varies the motion maintains the S.H.M. and when both x and t vary, the motion initiating from a point of elevation makes S.H.M.

Inrespect to the problem we would like to refer to the works of Benjamin, [11], T. B. Baczar - Karakiewic Z, B., and Prit chand, W. G. (1987), Carrier, [21], G.F. and Gremspan, H. P. (1958) Pramanik, A. K. (1973a). [94]

# CHAPTER - III

When wind blows over the surface of the fluid, it's motion is irregular and eddying. It exerts the tangential as well as normal stress on the surface of the fluid.

When the air moves in the direction in which the wave train moves, but with a greater velocity, there will evidently be an excess of pressure on the rear slopes, as well as a tangential drag on the exposed crests. The aggregate effect of these forces will be a surface drift, and the residual tractions, whether normal or tangential, will be to increase the amplitude of the waves to such a point that the dissipation balances the work done by the surface forces. On the other hand wave trains moving faster than the wind blows, in favour or against the wind, will have their amplitude continually reduced. [cf. Airy]. Also the viscocity has been called into play in this chapter.

In this regard we would like to refer the works of Akylas, T.R. [2,3] (1984 a,b) Cherkesov, L.V. [24] (1962), Crapper, [28], G.D. (1964), Debnath, L. [33] (1969a), Kakutani [48], T and Matsunchi, K. (1975) Miles, J. W. [82] (1957) Pramanik, [95], A.K, (1973b), Light hill, [68] M. J. (1962).

The problems in this chapter is dealt with the waves due to wind stress on the free surface of the viscous fluid. It consists of two problems. The first problem is "Waves due to normal stress applied on the free surface of a viscous in compressible fluid."

The motion has been initiated due to the normal stress on the free surface, it has been considered that $P_{xy}$, the tangential stress is absent and normal stress is $P_{yy} = H(t-a)\cos \omega x \cos nt$, a Heaviside unit function on time which is being played after a certain time say a, it is associated with an oscillatory function of time and oscillatory function on the horizontal space coordinate x.

The graphical study has been made and it has been observed that the surface elevation diminishes continually as time increases, when x remains constant, but oscillates when x increases, t remains constant. It has also been observed that when x and t both vary, the motion of the fluid oscillates on the surface.

Our second problem in this chapter is "Waves due to tangential stress on the free surface of a viscous in compressible fluid." In this problem the motion has been studied in absence of the normal stress but in presence of the tangential stress, $P_{yy} = 0$ and $P_{xy} = WH(t-a) \cos \omega x \cos nt$, the same force but in different direction.

The motion has been observed graphically, the surface elevation experiences the parabollic motion as t increases, when x remains unchanged and experiences the irregular oscillatory motion when x increases, t remains unchanged. But when both x and t vary, as in the case of travelling force, the surface experiences the oscillatory motion, initiating from a point of depression.

**Equation of Motion :-**

If we assume that the motion is confined to the dimensions x, y. Weget the equation of motion, affected by the viscocity on water waves.

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial v}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g + \nu\nabla^2 v$$

with $\frac{\partial u}{\partial t} + \frac{\partial v}{\partial y} = 0$ (*c.f. Lamb* 349, 625)[62]

# CHAPTER - IV

In this chapter we have dealt with the problems, in highly viscous fluid and it consists of two problems. The first problem is "Generation of surface wave in a semi infinite viscous incompressible fluid due to ring load and disc load."

In this problem the motion of the wave has been generated due to ring load $\frac{F_0}{2\pi r}\delta\,(ct - r)$ and a disc load -

$\frac{F_0}{\pi(ct)^2}\,H\,(ct - r)$ on the free surface of the fluid.

The observations have been made for the two cases.

For the case of ring load, it has been observed that the surface elevation varies asymptatically with the variation of time when the radius remains constant, But in case of disc load, the surface elevation diminishes slowly with the variation of the radius, when t remains constant. In this regard, we refer to the works of kochin, [58,59], N. E. (1939) (1940).

Our second problem in this chapter is "Generation of wave motion in a rotatory semi infinite v is cous fluid due to certa in impulsive velocity prescribed on the surface of the fluid."

In this problem, the motion of the wave has been generated and maintained by an impulsive force prescribed on the velocity Component, $V = f(r)\,\delta(t-a)$ where $f(r) = re^{-b^2r^2}$, a geophysic function.

The graphical study has been made and it is observed that the surface elevation varies uniformly with the variation of r, when t remains unchanged and diminishes parabolically with the variation of t when r remains constant.

The study has also been made when both r and t vary, where the surface being elevated initially, diminishes regularly.

The relevant works of Reutov, V. A. [97] (1976), Hooper, A. P.[46] and Grimshaw, R. (1985), Le Blond, [65] P.H. (1987), Liu [69], A. K. and Davis, S. H. (1977) are to be mentioned.

# CHAPTER - V

In this chapter, two problems have been included.

The first problem is "Wave motion of liquid in a rectangular duct due to variable pressure."

In this problem, the study has been made on the two - dim ensional wave motion, initiated due to the disturb-ances, caused by the variable pressure which is a function of x, y ; associated with any function of time.

The particular cases have been considered in this problem. The first one is an impulsive pressure concentrated at the origin, the second one is the impulsive pressure periodic in spatial coordinates and the third one is the pressure, periodic in time, concentrated at the origin on the surface.

The graphical studies have been made and it has been observed that in case I, the wave motion is S.H. with the variation of time where spatial co ordinates remain unchanged, in case II, the motion also exhibits the S.H.M with the variation of time and spatial co ordinates. In Case III, we have considered two possibilities, when $\alpha > \sigma$, Where the motion exhibits the oscillatory motion with diminishing of the amplitudes and lengths of the waves, with the variation of time, when spatial coordinates remain unchanged and in case of $\alpha < \sigma$, the motion also exhibits the same oscillatory motion, but it's phase is different.

Our second Problem in this chapter is, "Note on the two - dimensional wave motion set up by a certain prescribed pressure on the surface of the fluid."

In this problem we have dealt with the unsteady motion of the two dimensional surface gravity wave, proceeding along a straight canal.

The motion is initiated by a pulse of periodic pressure of period $\frac{\pi}{\omega}$.

There are two possibilities in this problem, when $0 \le t \le \frac{\pi}{\omega}$ *and* $t > \frac{\pi}{\omega}$

It has been observed that in first case, the motion experiences the diminishing wave amplitudes and lengths, the motion also exhibits the same characteristic in the second consideration but with different phase; with the variation of time, when other spatial co ordinates remain unchanged.

In these regard, the references may be made on the works of Barnard, B.J.S. [7] Mohany, J.J. and Prit chand, W. G. (1977), Tayler, [113] A. B. and van Den Driess che, P (1974), Novikov, [90], S.P. (1974), Penney[91], W. G., and Price A. T. (1952 b), Pramani K [96] A. K. (1975) Garabedian[40], P. R. (1965) Newell, [89], A. C. (1978)

# CHAPTER - VI

In this chapter, wehave dealt with the problem of "On the motion of fluid on porous media with suction or, injection."

In this context, the study has been made on the motion set up in a semi infinite viscous fluid flow over a porous media, by a certain impulsive velocity, prescribed over a circular area. The motion has been studied graphically and it has been observed that the surface elevation diminishes asymptotically in case of suction where as it increases in case of injection.

In this context, references are made on the works of Bagnold, [5], R. A. (1946), Liu [70], P.L. - F. (1973).