Chapter - I

INTRODUCTION

1.1 INTRODUCTORY REMARKS

The contents of this thesis are arranged in four chapters. *Chapter - I* is of review nature and deals with the introduction to the thesis and a brief discussion on the basic concepts of Newtonian fluid, non-Newtonian fluid, Micropolar fluid, dusty fluid flows, rotating fluid flows, flow through porous medium, free convection and mass transfer flow and unsteady convective dipersion. Attempts are also made to give a brief survey of previous results so that the work presented in this thesis could be seen in its proper perspective.

Chapter - II is devoted to the study of unsteady flow of Newtonian and non-Newtonian fluids with the suspension of dust particles under different geometrical circumstances. These problems are becoming increasingly important due to their varied applications in the field of science and technology. This chapter is divided into two parts. In part one of this chapter we analysed the effect of dust particles on MHD rotational flow of viscous fluid in porous medium. The second part of this chapter deals with the study of two-dimensional unsteady free convective flow of an incompressible dusty micropolar fluid past a semi infinite vertical porous plate embedded in a porous medium in presence of an applied pressure gradient. This study of dusty micropolar fluids is of practical importance, particularly for the flow through packed beds, sedimentation, environmental pollution and centrifugal separation of particles. Considering blood as a micropolar fluid, this study also gives some insight into the flow of the blood through veins and arteries.

Chapter - III is devoted to the study of free convection flow and mass transfer with or without rotation under different geometrical conditions. This chapter is divided into three parts. In the first part an attempt is made to study the unsteady free convection and mass transfer flow of viscous fluid through porous medium bounded by an infinite porous plate in a rotating system when there is an oscillating free stream velocity and oscillating temperature distribution. The second part of this chapter deals with the study of free convection and mass transfer flow of an electrically conducting rotating viscous fluid in porous medium past a vertical infinite porous plate with constant suction under the assumption of rigid body rotation. In part three of this chapter we consider MHD free convection flow and mass transfer of viscous incompressible electrically conducting fluid through an inclined open rectangular channel with a bed of varying permeability. Natural convection in porous medium has received considerable attention due to its numerous applications in geophysics and energy related engineering problems. Such type of application includes natural circulation in isothermal reserviors, acquifers porous insulation heat storage bed, grain storage, extraction of geothermal energy and thermal insulation design. Keeping this in view, we have studied above mentioned three problems which may shed some light in this field of research.

The last chapter is devoted to the study of dispersion of matter through Newtonian and non-Newtonian fluids following generalised dispersion model proposed by Gill and Sankarasubramanian [1] under different laminar conditions. This chapter is divided into three parts. In the first part of this chapter we studied the dispersion of solute in combined free and forced convective flow between two parallel plate channel, when there is a uniform axial temperature variation. The second part of this chapter presents an exact analysis of unsteady dispersion of solute in an MHD flow through a vertical channel. Special attention is paid to the study of the role of radiation on the time dependent dispersion coefficients. The third, i.e. last part of this chapter deals with the study of dispersion of passive impurity in blood stream flowing between two parallel plates treating blood as a micropolar fluid. The effects of non-Newtonian behaviour of fluids on the dispersion coefficients are studied.

Before we discuss various problems we present below a general introduction on Newtonian/non-Newtonian fluid, micropolar fluid, dusty fluid flows, rotating fluid flows, flow through porous medium, free convective flow and mass transfer which are directly related to the concerned problems of this thesis.

1.2 NEWTONIAN FLUIDS

For a laminar flow where the fluid particles move in straight, parallel lines, the Newton's viscosity law states that for certain fluids, called Newtonian fluids, the shear stress on an interface tangent to the direction of flow is proportional to the distance rate of change of velocity, wherein the differentiation is taken in a direction normal to the interface. Mathematically, this is stated as

$$\tau \propto \frac{\partial v}{\partial n} \qquad .....(1.1)$$

Figure- 1.1 may further explain this relationship.

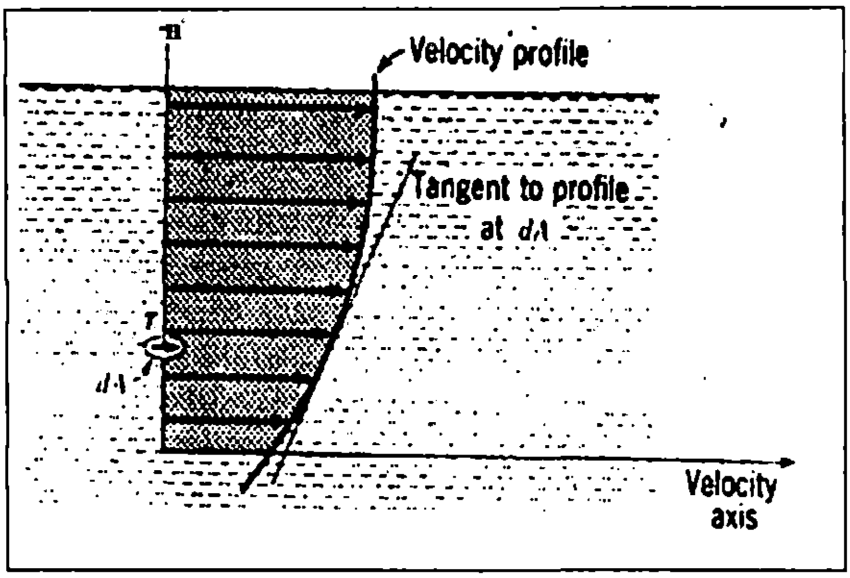

Figure- 1.1

An infinitesimal area in the flow is chosen parallel to the horizontal velocity axis. The normal **n** to the area is drawn. The fluid velocities at points along the normal are plotted, thus forming a velocity profile. The slope of the profile toward the n-axis at the position corresponding to the area element is the value $\partial v / \partial n$ which is related to the shear stress τ shown on the interface. Inserting the coefficient of proportionality into Newton's viscosity law leads to the result

$$\tau = \mu \frac{\partial v}{\partial n} \qquad .....(1.2)$$

where μ is called the coefficient of viscosity. This theory has been extensively investigated during the last century.

1.3 NON-NEWTONIAN FLUIDS

The relationship (1.1) explains reasonably well most of the phenomena like drag, lift, skin-friction, separation etc. occurring in the flows of fluids. However, it fails to explain the occurrence of Weissenberg effect [2], Merrington effect [3] and Poynting effect [4]. In order to explain these effects, the stress-rate of strain relation should be generalised so that the constitutive equation becoms non-linear. The fluids under this category are termed as non-Newtonian fluids. Non-Newtonian fluids form an extremely wide class of different materials, whose only common features are fluidity and a failure to obey equation (1.1). The behaviour of such type of fluids is shown in the stress, shear-rate curve (1.2).

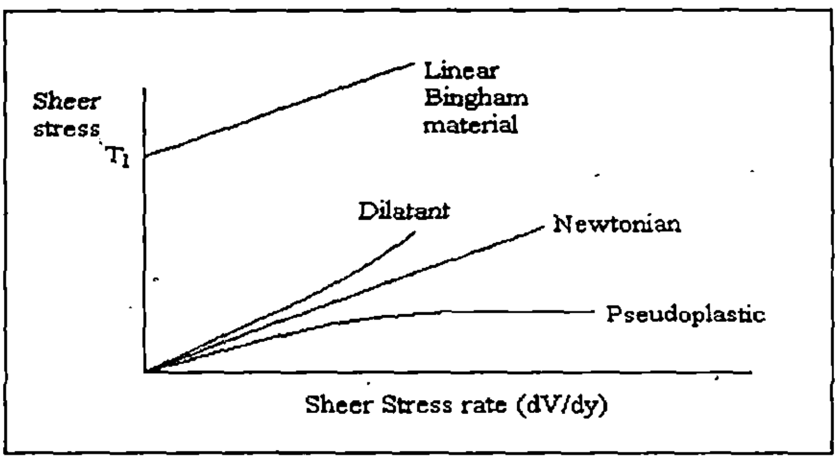

Figure- 1.2

The exceptions to Newton's viscous law are not of rare occurrence, in fact the so-called non-Newtonian fluids are to be found close at hand everywhere. The fluids like blood, honey, condensed milk, mud flows, solutions or suspensions (slurries), liquid lubricants, printing inks, starch, rasin pastes, plastics, high polymers, salad dressings, butter, whipped cream and daughs, egg white, paints, certain varieties of oil and many other materials of industrial importance fall under the category of non-Newtonian fluids [5].

1.3.1 MICROPOLAR FLUIDS

Micropolar fluids are fluids with microstructure belonging to a class of fluids with asymmetrical stress tensor. Physically, they represent fluids consisting of randomly oriented particles suspended in a viscous medium [6, 7, 8]. Eringen [9] introduced the concept of micropolar fluid in an attempt to explain the behaviour of a certain fluid containing polymeric additives and naturally occurring fluids such as animal blood and is expected to provide a mathematical model for non-Newtonian behaviour observed in these fluids. In this model, the fluid is assumed to have a microstructure displaying spin inertia and to have the capacity of sustaining stress and body moments. Willson [10] studied boundary layer flows of a micropolar fluid with special reference to the flow near a stagnation point. Ahmadi [11] studied the flow characteristics of the boundary layer flow of a micropolar fluid over a semi-infinite plate. Gorla [12] studied the heat transfer characteristics of the micropolar fluid flow in the vicinity of a stagnation point. The forced heat transfer characteristics of the micropolar boundary layer flow over a flate plate have been investigated by Gorla [13]. Gorla [14] studied the effects of buoyancy force on forced cenvection on a continuously moving vertical cylinder. Later Gorla [15] analysed the combined free and forced convection in micropolar boundary layer flow on a vertical flat plate. Hassanien [16] studied combined force and free convection in boundary layer flow of a micropolar fluid over a horizontal plate. Helmy [17] studied unsteady two-dimensional laminar free convection flow of an incompressible viscous electrically conducting micropolar fluid through a porous medium. Recently Helmy [18] studied the unsteady free convection flow of a micropolar fluid cast into a matrix from using the state spaee and he solved the problem with the help of Laplace transform technique. More recently Hsu and How [19] analysed effect of natural convection of micropolar fluids over a uniformly heated horizontal plate.

1.4 ROTATING FLUID FLOWS

The stimulus for scientific research on fluid system in rotating environments is originated from geophysical and fluid engineering applications. Many aspects of the motion of terrestrial and planetary atmospheres are influenced by the effects of rotation. The broad subjects of oceanography, meteorology, atmospheric science all contain some important and essential aspects of rotating flows. Rotating flow theory is utilised in determining the viscosity of fluids and in the construction of turbines and other centrifugal machines. Also the study of flows through porous media in a rotating system is of considerable interest in many scientific and engineering applications, viz., to the petroleum engineers concerned with the movement of oil and gas through reservoir; to the hydrologist in his study of migration of underground water etc.. The complete literature pertaining to rotating fluids is enormous and an excellent review can be found in the monograph by Greenspan [20]. Rotation in a fluid system produces two effects, viz., the coriolis and the centrifugal forces, on the fluid particles. The balance between the coriolis forces and the pressure gradient with correction for the viscous action at the boundaries emerges as the backbone of the entire theory of rotating flows. In considering flows in rotating environment we come across situations where the entire fluid is in a solid body rotation or only the solid boundaries are rotating. In the later case it is preferable to use an inertial co-ordinate system fixed in space. On the other hand the flow behaviour in the former case can be described in a co-ordinate system which rotates with the fluid, and in this frame of reference the fluid is at rest.

In a steadily rotating system, a balance is struck between coriolis and frictional forces in a thin layer over horizontal boundaries. This layer, called the Ekman layer, was first noticed by Ekman [21] and plays a very fundamental role in the rotating fluid flows. The study of flows through porous media has been motivated by its immense importance and continuing interest in many engineering and technological fields. Many researchers have paid their attention towards the flow in rotating system. Gupta [22] obtained an exact solution of the three-dimensional Navier Stokes steady state equations for the flow past a plate with uniform suction/injection (blowing) in a rotating system. Debnath and Mukerjee [23] studied the motion of an incompressible, homogeneous, viscous fluid bounded by porous plate with uniform suction/injection. Puri [24] discussed the fluctuating flow of a viscous fluid on a porous plate in a rotating medium. Kishore et al. [25] obtained a solution describing the hydrodynamic boundary layer flow in a rotating system with uniform suction/injection. Kim [26] studied the unsteady two dimensional laminar flow of a viscous incompressible electrically polar fluid via a porous medium past a semi-infinite vertical porous moving plate in the presence of transverse magnetic field. Recently Jat and Jhankel [27] analysed three dimensional unsteady flow of an incompressible viscous fluid in presence of transverse magnetic field through a porous medium past an oscillating plate in a rotating system. In this thesis we consider certain problems in rotating system due to their varied applications in the field of technology.

1.5 DUSTY FLUID FLOWS

Fluid flows with particulate suspensions, when the suspended matter may consist of solid particles, liquid droplets, gas bubbles or combinations of these, are commonly termed as dusty gas flows or dusty fluid flows. They are also referred to as two-phase flows, since they involve a composite of two-phase or two materials with different distinguishable properties - one phase being the fluid medium which is a continuous phase and the other phase being the particulate suspensions which are scattered throughout the fluid medium and hence known as the dispersive phase or discrete phase or simply particulate phase.

The flows of fluids with suspended material particles abound in nature, classical examples being pollution of air and contamination of water. The earth's atmosphere is a predominantly gaseous envelope of air surrounding the earth and it contains solid particles and liquid droplets. Besides it is also being constantly polluted by a number of dust particles like carbon, sulpher, and many other toxic elements which arise as inevitable consequence and natural by-products of industrialization.

Dusty gas flows assume importance in such engineering problems as fluidization (flow through packed beds), sedimentation, powder technology, aerosol filtration, gas purification, flows in rocket tubes where small carbon or metalic fuel particles are present. Further, problems concerned with atmospheric fallout, rain errosion of guided missiles and aircrafticing are some of the areas where the dynamics of dusty gases play a prominent role.

A knowledge of dusty fluid flows is useful to some extent in understanding the rheology of blood flows through capillaries, where red blood cells can be regarded as rigid particles embedded in the plasma. Another biological situation where the study of two-phase flows assumes importance is the phenomena of particle deposition in the respiratory tract.

The formulation of the fundamental equations of the dusty fluid flows is guided in a reasonably simple manner under certain basic assumptions : (i) the fluid is an incompressible viscous fluid; (ii) dust particles are spherical and undeformable, all having equal radius and mass; (iii) the density of the dust particles is high compared with the fluid density and the mass fraction of the dust particles is not extremely high so that the volume occupied by the dust particles is negligible; (iv) the volume-fraction of the dust particles is so small that the interaction between individual dust particles may be neglected and so the fluid phase contributes the entire pressure; (v) dust particles form a cloud of pseudo-fluids with negligible viscosity; (vi) the thermal and gravitational forces are neglected; (vii) the radius of the dust particles being small, the only force acting due to interaction between dust and fluid particles is the drag force given by Stoke's law, where it is also supposed that the Reynold's number of the relative motion of the dust and fluid is small compared to unity; (viii) the distortion of the flow field around the dust particles is neglected.

Under these assumptions, following Saffman's formulation [28], the equations of continuity, equations of motion of viscous fluid and the dust particles are

$$u_{i,i} = 0 \qquad .....(1.3)$$

$$\frac{\partial N}{\partial t} + (N v_i)_{,i} = 0 \qquad .....(1.4)$$

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j u_{i,j}\right) = p_{ij,j} + KN\,(v_i - u_i) \qquad .....(1.5)$$

$$m\left(\frac{\partial v_i}{\partial t} + v_j v_{i,j}\right) = K\,(u_i - v_i) \qquad .....(1.6)$$

where u_i, v_i are the local velocity vectors of liquid and dust particles respectively, p_{ij} the stress tensor defined by the equation

$$p_{ij} = (\lambda \Delta - p)\delta_{ij} + 2\mu e_{ij} \qquad .....(1.7)$$

in which the rate of strain tensor e_{ij} defined in terms of the velocity field u_i by

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \qquad .....(1.8)$$

p the pressure, Δ the dilatation, λ, μ are coefficients of the medium, ρ the density, K the Stoke's resistance coefficient (for spherical particles of radius d, it is $6\pi\mu d$), *N the number density of dust particle and m the mass of dust particle.*

Based upon the theoretical model proposed by Saffman [28], numerous authors investigated a number of dusty gas flow problems under several physical and geometrical circumstances. Kazakevich and Krapivin [29] and Sproull [30] *performed experimental works on dusty fluid and observed that the aerodynamical* resistance of the dusty gas is less than that of a clean gas. Soo [31, 32, 33, 34] made pioneering works in the two-phase problems including the problems of dusty fluid flows. Marble [35], Pai [36], Jain [37] and Goddard [38] reviewed the subject of dynamics of dusty fluids. Rudinger and Chang [39] and Rudinger [40] considered *the effects of volume-fraction of dust particles on the flow of dusty gases and* also analysed unsteady two-phase flow. Wallis [41] studied one-dimensional two phase flow. Davidson [42] gave some features on the flow of fluid-solid mixtures. Torobin and Gauvin [43, 44], Crooke [45], Hinch [46] and Purcell [47] put forward some noble ideas on the fundamental aspects of the flow of dusty fluids.Later Mandal et al. [48] have studied rotational motion of dusty viscous fluid contained in the semi-infinite circular cylinder due to an initially applied impulse on the surface. Recently Varshney and Varshney [49] discussed the problem of Mandal et al. [48] with porous medium. Kannan and Ramamurthy [50] discussed the flow of a dusty viscous fluid through a circular cylinder with volume fraction of dust particles being taken into account. More recently Rashmi Kumari [51] extended the analysis of Varshney and Varshney [49] to the flow of dusty viscous fluid in circular cylinder in presence of magnetic field.

In *chapter-II* of the present thesis we consider two problems on unsteady flow of dusty Newtonian and non-Newtonian fluid. In part one, the effect of dust particles on MHD rotational flow of viscous fluid in porous medium is studied. Problems dealing with the influence of dust particles on viscous flow in a rotating system have importance on the pollution problems as well as on the motion of aerosol over the rotating earth.

Among many fluid mechanical problems which are of direct technological interest, one of the most important classes is that involvimg the motion of particles in polymer solutions or melts and other non-Newtonian fluids. A considerable amount of both theoretical and experimental work in this line of study exists in literature. Leal [52] examined the slow motion of slender rod like axi-symmetric particles on a second order fluid while Goddard [53, 54] discussed the stress field of slender and rod like particles respectively flowing in a non-Newtonian fluid. Later Leal [55] presented a report on the studies of motion of small particles in non-Newtonian fluids. In a series of papers [56, 57, 58, 59, 60] authors discussed the dynamics of dusty non-Newtonian fluid through pipes and channels. Mandal et al. [61] considered the unsteady flow of dusty visco-elastic liquid between two oscillating plates. Later Mandal et al. [62] studied unsteady flow of dusty elastico-viscous liquid in the Ekman layer and also [63] studied unsteady axi-symmetric rotational flow of dusty elastico-viscous liquid. Recently Singh et al. [64] studied free convection and mass transfer in dusty flow of a visco-elastic liquid between two vertical heated, porous parallel plates in presence of magnetic field acting perpendicular to the flow. Considering the importance of study of dusty non-Newtonian fluid, in part two, we have considered two dimensional free convective flow of dusty micropolar fluid past a semi-infinite porous plate embedded in a porous medium in the presence of an applied pressure gradient. The free stream velocity follows an exponentially decreasing and increasing small perturbation law. The porous surface absorbs the micropolar fluid with a time varying suction velocity which has a small amplitude. The effects of material parameters on the velocity and temperature field of fluid and dust particles across the boundary layer are investigated. Numerical results of velocity and temperature distributions of micropolar fluid and dust particles are carried out and the results of velocity distribution of micropolar fluids are compared with the corresponding flow problem for Newtonian fluid. The effects of dust on velocity and temperature distribution are studied. This investigation may have applications on the problems of transport of solid particles suspended in non-Newtonian fluids through pipes and channels polluted oil extraction, polymer extrusion, paint spraying and boundary layer growth of non-Newtonian fluids with suspended dust particles.

1.6 FLOW THROUGH POROUS MEDIUM

In recent years the flows of fluid through porous medium have attracted the attention of a number of scholars because of their possible application in many branches of science and technology. In fact a porous material containing the fluid is a non homogeneous medium but it may be possible to treat it as a homogeneous one, for the sake of analysis, by taking its dynamical properties to be equal to the local average of original non homogeneous continuum. Thus a complicated problem of the flow through a porous medium get reduced to the flow problem of homogeneous fluid with some additional resistance. Flows of fluid through porous medium are of principal interest because these are quite prevalent in nature. Such flows are important in the field of agricultural engineering to study the underground water resources, seepage of water in river beds, in petroleum technology to study the movement of natural gas, oil and water through the oil reservoirs, in chemical engineering for filtration and purification process. In view of the geophysical application of the flows through porous medium, a series of investigations have been made by Raptis et al. [65, 66, 67] into the steady flow past a vertical wall.

Studies associated with flows through porous medium have been based on the Darcy's empirical equation

$$\vec{q}\,' = -\frac{const}{\mu} \cdot \vec{\nabla} p \qquad \text{.....(1.9)}$$

where $\vec{q}'$ is the mean filter velocity, μ is the viscosity of the fluid and $\vec{\nabla}p$ is the pressure gradient. Later Muskat [68] has shown that the constant in equation (1.9) must depend on the permeability of the porous medium and showed that

$$\vec{q}' = -\frac{K'}{\mu}\vec{\nabla}p \qquad \text{...(1.10)}$$

where K' is the permeability of the porous medium. Following Yamamoto and Iwamura [69], we regard the porous medium as an assemblage of small identical spherical particles fixed in space and the equation (1.10) for incompressible fluid and unsteady flow, takes the form

$$\frac{\partial \vec{q}'}{\partial t} + \left(\vec{q}'.\vec{\nabla}\right)\vec{q}' = -\frac{1}{\rho}\vec{\nabla}p - \frac{\nu}{K'}\vec{q}' + \nu\nabla^2\vec{q}' - g \qquad \text{...(1.11)}$$

where ν is the kinematic viscosity, t is the time and g is the acceleration of gravity.

1.7 FREE CONVECTION AND MASS TRANSFER FLOW THROUGH POROUS MEDIUM

Fluid flow due to density differences in the force field is generally called free convection. Such external forces are gravity forces, and the density difference, a very simple case, is the result of the temperature drop between the solid surface and the fluid. Free convection flow is not of rare occurrence in nature. In fact trade winds are due to convection currents set up in the atmosphere due to unequal heating. Also land and sea breezes arise in a similar manner. Studies on free convection have growing importance on the problem of unsteady free convection flow past an infinite vertical plate as one of the fundamental problem in heat transfer owing to its practical applications. Free convection effects on the Stoke's problem for an infinite vertical plate was investigated by Soundalgekar [70]. This problem is better known as Stokes problem for the vertical plate. Pop and Soundalgekar [71] investigated the free convection flow past an accelerated vertical infinite plate. The effect of magnetic field on free convective flow of electrically conducting fluids past a semi-infinite flat plate is analyzed by Sacheti et al. [72]. Later Satter and Alam [73] studied MHD free convective flow with Hall current in a porous medium for electrolytic solution. Tak and Gehlot [74, 75] studied the effects of suction on skin-friction and heat transfer in the free convection boundary layer flow along a porous vertical semi-infinite plate in presence of transverse magnetic field with or without frictional heat. Soundgalgekar et al. [76] obtained an exact solution of the transient free convection flow past an infinite vertical plate in presence of periodic heat flux. Recently Sahoo et al. [77] studied the unsteady hydromagnetic free convective flow of viscous incompressible and electrically conducting fluids past an infinite vertical porous plate in presence of constant suction and heat absorbing sinks.

However, in nature, along with the free convection currents caused by the temperature differences, the flow is also affected by chemical composition differences and gradieants or by material or phase constitutions. This can be seen in our everyday life in the atmospheric flow which is driven appreciably by both temperature and H_2O concentration differences. In water also the density is considerably affected by the temperature differences and by the concentration of dissolved materials or by suspended particulate matter. The flow caused by density difference which in turn is caused by concentration difference is known as the mass transfer flow. When a mixture of gases or liquids is contained such that there exists a concentration gradient of one or more of the constituents across the system, there will be a mass transfer on a microscopic level as the result of diffusion from a region of high concentration to regions of low concentration. There is also a mass transfer associated with convection in which mass is transported from one place to another in the flow system. This type of mass transfer occurs on a macroscopic level. Due to applications in various technological problems and in agricultural science, effects of mass transfer on the unsteady free convective flow past an infinite porous plate with constant or variable suction were studied by Soundalgekar [78], Soundalgekar and Wavre [79, 80], Soundalgekar [81] and Raptis et al. [82].

Research on fluid flow through porous media finds great application in geothermy, geophysics and technology. Yamamoto and Iwamura [69] considered the flow with convective acceleration through a porous medium assuming the porous medium as an assemblage of small identical spherical particles fixed in space. Raptis et al. [82, 83] studied the steady free convective flow and mass transfer of a viscous fluid through a porous medium bounded by a vertical infinite porous surface with constant suction by using generalized Darcy's law. Raptis et al. [84, 85] studied the influence of the free convective flow on the steady flow of the viscous fluid through the porous medium when there is a constant heat flux. Raptis [86] analysed the influence of free convecton on the unsteady flow of a viscous fluid through a porous medium considering the fluctuation of the surface temperature in time about a constant non-zero mean value. In a subsequent paper [87], under the same geometrical and physical considerations, he studied the influences of both free convective flow and mass transfer through a porous medium. Raptis [88] studied the flow through porous medium bounded by a plate in the presence of free convection and mass transfer flow and free stream velocity, taking into consideration the general form of Darcy's law. Raptis and Perdikis [89] also analysed the steady free convective and mass transfer flow when a viscous and incompressible fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite porous plate. Recently Acharya et al. [90] have analysed free convection and mass transfer in steady flow through porous medium with constant suction in the presence of magnetic field. Samman et al. [91] studied transient free convection flow of a viscous dissipative fluid with mass transfer past a semi-infinite vertical plate. More recently Singh et al. [92] studied hydromagnetic heat and mass transfer in MHD flow of an incompressible electrically conducting viscous fluid past an infinite vertical porous plate embedded with porous medium of time-dependent permeability under oscillatory suction velocity normal to the palte.

Later Mahato and Maiti [93] gave an exact analysis of unsteady free convective flow and mass transfer during the motion of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous surface in a rotating system. Hiremath and Patil [94] studied the effect of free convection flow on the oscillatory flow of polar fluid in a porous medium bounded by vertical plane surface of constant temperature. Siddheshwar et al. [95, 96] discuss the effect of interphase mass transfer on unsteady convective diffusion in a Hagen-Poiseuille flow of a power law fluid. Very recently Vaszi et al. [97] studied two-dimensional conjugate free convection flow over an inclined flat plate in a semi-infinite porous medium under boundary layer approximation. Kim [98] studied the unsteady convective flow of micropolar fluids past a vertical porous plate embedded in a porous medium. Gupta and Johari [99] studied the effect of magnetic field on the three dimensional flow of an incompressible viscous fluid past a porous plate. Singh et al. [100] also presented a theoretical analysis of couette flow of a viscous incompressible fluid through a porous medium between two infinite horizontal parallel porous flat plate. Very recently Jat and Jhankal [101] analysed the effects of transverse applied magnetic field on the three-dimensional free convective flow of an incompressible viscous fluid through a porous medium.

In *chapter-III* of the thesis we consider three problems on the study of free convective flow and mass transfer with or without rotation under different geometrical conditions. In the first part an attempt is made to study the unsteady free convective flow and mass transfer during the motion of viscous fluid through a porous medium bounded by an infinite porous plate in a rotating system when there is an oscillating free stream velocity and oscillating temperature distribution. The second part of this chapter deals with the study of free convective flow through porous medium with mass transfer of an electrically conducting rotating viscous fluid past a vertical infinite porous plate with constant suction under the assumption of rigid body rotation.

It is known that MHD flows have received considerable attention because of their practical applications. On the other hand the significance of suction/injection for the boundary layer control in the field of aerodynamics and space science is well recognized. A theoretical analysis of steady free convective and mass transfer flow was presented by Raptis and Kafousias [85] when a viscous incompressible and electrically conducting fluid flows through a porous medium occupying a semi-infinite region of the space bounded by an infinite vertical porous plate. A magnetic field of uniform strength was applied perpendicular to the plate and constant heat flux at the plate was assumed. In part three of this chapter we consider MHD free convective flow and mass transfer of viscous incompressible electrically conducting fluid through an inclined open rectangular channel with a bed of varying permeability.

1.8 UNSTEADY CONVECTIVE DIFFUSION

The problems of dispersion with interphase mass transfer have possible applications on the dispersion of tracers in blood streams, in the extracorporeal treatment of blood, in the treatment of effluents during their conveyance through tubes, in crude oil conveyance, and in chromatography and mass transfer problems in polymer solution. When a solute is released in a solvent which flows steadily under laminar conditions through a circular tube, it spreads out longitudinally about a plane moving at the mean speed of the flow under the combined effect of lateral molecular diffusion and longitudinal convection and longitudinal molecular diffusion. Extensive investigations were made both theoretically and experimentally by many authors to analyze the dispersion of soluble matter in laminar flow.

Taylor [102] considered the unsteady convective diffusion problem for steady flow in a cylindrical tube. Some restrictions in this model of Taylor [102] was overcome partially by Aris [103] using a statistical approach. These two models are restricted to the study of dispersion of passive solute at large time and small time respectively. The studies of Gill and Sankarasubramanian [1, 104, 105] and Barton [106] on dispersion have all-time validity. In these models the systems considered involved no interphase mass transfer at the boundaries. Bagchi and Maiti [107] discussed the dispersion of solute initially distributed in a fully developed laminar flow in a porous-walled parallel plate channel in presence of suction velocity which may have some bearing in the dialysis of blood in artificial kidneys. Mandal et al. [108] investigated the dispersion of solute in an incompressible electrically conducting viscous fluid in a porous-walled parallel plate channel permitted by transverse magnetic field. Hazra, Gupta and Niyogi [109] presented an exact analysis of the dispersion of a solute in an incompressible viscous fluid flowing slowly in a parallel plate channel under the influence of a periodic pressure gradient. Sankarasubramanian and Gill [110] considered the system which involves interphase mass transfer. They studied the dispersion of solute in a laminar flow through a tube with first order, irreversible, heterogeneous chemical-reaction. But in all the above mentioned studies the solvent is Newtonian. In a variety of applications involving fluids with suspended particles the assumption of a non-reactive solute and Newtonian description for solvent may be too restrictive. Fan and Hwang [111], Fan and Wang [112], Ghoshal [113], Gupta and Gupta [114], Dutta et al. [115], Shah and Cox [116], Prenosil and Jarvis [117], Rao [118], Shukla et al. [119], Soundalgekar [120], Rudraiah et al. [121], Chandra and Agarwal [122], Philip and Chandra [123], Soundalgekar and Haldavnekar [124], and Agarwal et al. [125] are some of the papers on dispersion. These studies are based on the Taylor approach in non-Newtonian fluid flows, with/without, homogeneous/haterogeneous chemical reaction. Sankarasubramanian and Gill [110] studied all time dispersion of non reactive solute in tube flows. Subramanian and Gill [126] studied dispersion of solute in a power-law fluid flow.

Later Krishnamurthy and Subramanian [127] formulated convective diffusion theory for predictive modelling of field-flow fractionation columns used for the separation of colloidal mixture. Jayaraj and Subramanian [128] used the truncated versions of generalised dispersion theory to study the relaxation phenomena in field-flow-fractionation. Annapurna and Gupta [129] and Gupta [130] analysed the unsteady magnetohydrodynamic convective diffusion in electrically conducting fluid flowing in a parallel plate channel. Subsequently Annapurna and Gupta [131] studied the dispersion of matter in flow of a Bingham plastic in a tube using the generalised dispersion model. Later Mukherjee and Maiti [132] studied the dispersion of solute in blood stream flowing through a tube treating blood as a casson fluid model. Siddheshwar et al. [95, 96] studied the effect of interphase mass transfer on it. Siddheshwar and Manjunath [133] have recently studied dispersion of solute in a plane poiseuille flow of a micropolar fluid. Recently Siddheshwar and Markande [134] considered unsteady convective diffusion of solute in a micropolar fluid flow through a cylindrical tube.

In the last chapter, we use the truncated versions of generalised dispersion theory proposed by Gill and Sankarasubramanian [1] to study the problem of dispersion of soluble matter in laminar flow. This chapter is divided into three parts. The first part of this chapter contains a brief discussion on the dispersion of solute in combined free and forced convective laminar flow between two parallel plate channel when there is a uniform axial temperature variation. The dispersion model of Gill and Sankarasubramanian [1] is used in the analysis. The explicit expression of dispersion coefficient in the flow valid for all time is obtained. It is observed that the value of dispersion coefficients for large time remain same for both heating and cooling. The time dependent dispersion coefficients first increases with time and then attains an asymptotic value.

The second part presents an exact analysis of unsteady dispersion of solute in an MHD flow through a vertical channel. The effects of radiation on dispersion coefficients are studied. For different values of magnetic parameter (**M**) and Rayleigh number (**Ra**), the dispersion coefficients $\mathbf{K}_2^*(\infty)$ and $\mathbf{K}_3(\infty)$ decrease with increase in radiation parameter (**F**). Also the dispersion coefficient $\mathbf{K}_2^*(\infty)$ increases slowly for smaller values of $(\mathbf{Ra})^{1/4}$ and then increases rapidly with increase in $(\mathbf{Ra})^{1/4}$. It is aslo seen that dispersion coefficient $[\mathbf{K}^2(\tau)\text{-}\mathbf{Pe}^{-2}]$ increases with increase in time (τ) and then levels off to the asymptotic value $\mathbf{K}_2^*(\infty)$. This paper may have some application in the study of hydromagnetic convection with heat ransfer in MHD generators and shock tubes, where radiation effect should not be neglected.

In the last part of this chapter we studied the dispersion of passive impurity in blood streams flowing between two parallel plates in the presence of transverse magnetic field treating blood as magnetohydrodynamic micropolar fluid. The effects of non-Newtonian behaviour of fluid on the dispersion coefficients are investigated. This study finds application in cooling turbine blades and MHD generator with neutral fluid seedling in the form of rigid micro inclusions.

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