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High frequency scattering due to a pair of time-harmonic antiplane forces on the faces of a finite interface crack between dissimilar anisotropic materials

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Abstract – The high-frequency elastodynamic problem involving the excitation of an interface crack of finite width lying between two dissimilar anisotropic elastic half-planes has been analyzed. The crack surface is excited by a pair of time-harmonic antiplane line sources situated at the middle of the cracked surface. The problem has first been reduced to one with the interface crack lying between two dissimilar isotropic elastic half-planes by a transformation of relevant co-ordinates and parameters. The problem has then been formulated as an extended Wiener–Hopf equation (cf. Noble, 1958) and the asymptotic solution for high-frequency has been derived. The expression for the stress intensity factor at the crack tips has been derived and the numerical results for different pairs of materials have been presented graphically. © 1999 Éditions scientifiques et médicales Elsevier SAS

crack in anisotropic media / high frequency scattering / Wiener–Hopf technique

1. Introduction

The extensive use of composite materials in modern technology has created interest among the scientists for carrying on considerable research work in the modeling, testing and analysis of laminated media. The laminated composites which behave as anisotropic material may be weakened by interface flaws which can lead to serious degradation in load carrying capacity.

Neerhoff (1979), therefore, studied the diffraction of Love waves by a crack of finite width at the interface of a layered half-space. Kuo (1992) carried out numerical and analytical studies of transient response of an interfacial crack between two dissimilar orthotropic half-spaces. Kuo and Cheng (1991) studied the elastodynamic responses due to antiplane point impact loading on the faces of a semi-infinite crack lying at the interface of two dissimilar anisotropic elastic materials. The problem of diffraction of normally incident antiplane shear wave by a crack of finite width situated at the interface of two bonded dissimilar isotropic elastic half-spaces has been studied by Pal and Ghosh (1990).

In the present paper we are interested in finding the high-frequency solution of the diffraction of elastic waves by a Griffith crack of finite width excited by a pair of time-harmonic concentrated antiplane line loads situated at the centres of the cracked surfaces. The materials are assumed to possess certain material symmetry and the crack plane is assumed to coincide with one of the planes of material symmetry, so that the inplane and the antiplane motion are not coupled.

The high-frequency solution of the diffraction of elastic waves by a crack of finite size is interesting in view of the fact that the transient solution close to the wave front can be represented by an integral of the high-frequency component of the solution. The analysis of the paper is first based on the observation of several researchers, e.g., Achenbach and Kuo (1986), Ma and Hou (1989), Markenscoff and Ni (1984) that antiplane shear deformation in an anisotropic solid can be deduced from the corresponding deformations of an isotropic

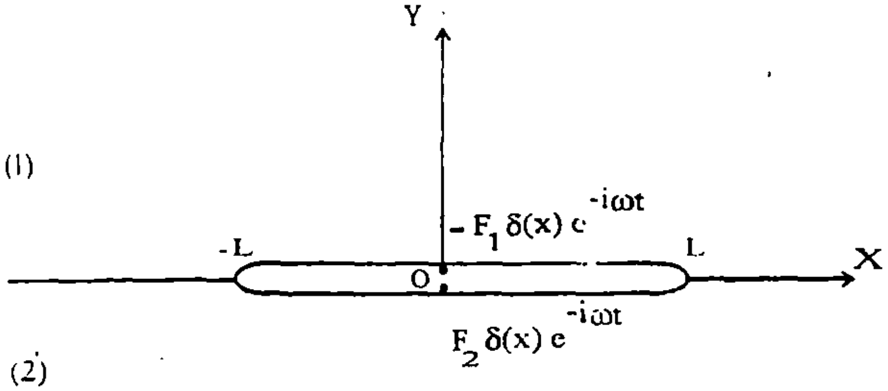

Figure 1. Geometry of the problem.

solid by a transformation of relevant co-ordinates and parameters. Based on this observation, analysis of the interface crack by line loads between two bonded dissimilar anisotropic elastic solids can first be converted to that of a crack between two dissimilar isotropic elastic materials. Later, following the method of Chang (1971), the problem has been formulated as an extended Wiener–Hopf equation. The Wiener–Hopf equation in brief can be found in the book of Achenbach (1973). The asymptotic solutions for high-frequencies or for wave lengths short compared to the width of the crack have been derived. Expression for the dynamic stress intensity factor near the crack tips has been obtained and the results have been illustrated for different pairs of materials.

2. Formulation of the problem

Let (X, Y, Z) be rectangular Cartesian co-ordinates. The X-axis is taken along the interface, Y-axis vertically upwards into the medium and Z-axis is perpendicular to the plane of the paper. Let an open crack of finite width $2L$ be located at the interface of two bonded dissimilar anisotropic semi-infinite elastic solids lying parallel to X-axis. The anisotropic half-planes are characterized by the elastic moduli $(C_{ik})_j$; $(i, k = 4, 5)$ and mass density $\bar{\rho}_j$. The subscript j $(j = 1, 2)$ corresponds to the upper and lower semi-infinite media respectively.

A pair of concentrated time-harmonic antiplane shear forces in the Z-direction of magnitudes F_1 and F_2 act on the crack faces $Y = 0+$ and $Y = 0-$ respectively at $X = 0$ as shown in *figure 1*. Thus the crack boundary conditions are

$$\sigma_{YZ}(X, Y, t) = \begin{cases} -F_1\delta(X)\,\mathrm{e}^{-i\omega t}; & |X| < L,\ Y = 0+, \\ F_2\delta(X)\,\mathrm{e}^{-i\omega t}; & |X| < L,\ Y = 0-, \end{cases} \tag{1}$$

$$\sigma_{YZ}^{(1)}(X, Y, t) = \sigma_{YZ}^{(2)}(X, Y, t) \quad \text{at } Y = 0,\ |X| > L \tag{2}$$

and

$$W_1(X, Y, t) = W_2(X, Y, t) \quad \text{at } Y = 0,\ |X| > L, \tag{3}$$

where ω is the circular frequency. Two dimensional antiplane wave motions of homogeneous anisotropic linearly elastic solids are governed by

$$(C_{55})_j\frac{\partial^2 W_j}{\partial X^2} + 2(C_{45})_j\frac{\partial^2 W_j}{\partial X\partial Y} + (C_{44})_j\frac{\partial^2 W_j}{\partial Y^2} = \bar{\rho}_j\frac{\partial^2 W_j}{\partial t^2} \quad (j = 1, 2), \tag{4}$$

where $W_j(X, Y, t)$ are the out-of-plane displacements.

The XY-plane has been assumed to coincide with one of the planes of material symmetry such that inplane and anti-plane motions are not coupled.

The relevant stress components are

$$\sigma_{XZ}^{(j)} = (C_{55})_j \frac{\partial W_j}{\partial X} + (C_{45})_j \frac{\partial W_j}{\partial Y}, \tag{5}$$

$$\sigma_{YZ}^{(j)} = (C_{45})_j \frac{\partial W_j}{\partial X} + (C_{44})_j \frac{\partial W_j}{\partial Y}. \tag{6}$$

Following Achenbach and Kuo (1986) and Kuo and Cheng (1991) we introduce a co-ordinate transformation

$$\left.\begin{aligned} x &= X - \frac{(C_{45})_j}{(C_{44})_j} Y, \\ y &= \frac{\mu_j}{(C_{44})_j} Y, \\ z &= Z, \end{aligned}\right\} \quad (j = 1, 2) \tag{7}$$

where

$$\mu_j = \left[(C_{44})_j (C_{55})_j - (C_{45})_j^2\right]^{1/2} \quad (j = 1, 2). \tag{8}$$

Equation (7) and the chain rule of differentiation reduced (4) to the standard wave equation

$$\frac{\partial^2 W_j}{\partial x^2} + \frac{\partial^2 W_j}{\partial y^2} = s_j^2 \frac{\partial^2 W_j}{\partial t^2}, \tag{9}$$

where

$$s_j^2 = \frac{\rho_j}{\mu_j} \quad \text{and} \quad \rho_j = \frac{\bar{\rho}_j (C_{44})_j}{\mu_j}, \tag{10}$$

s_j is the slowness of shear waves. Without loss of generality we assume that

$$s_1 < s_2. \tag{11}$$

Assume

$$W_j(x, y, t) = w_j(x, y)\,\mathrm{e}^{-i\omega t}, \quad j = 1, 2, \tag{12}$$

so that $w_j(x, y)$ satisfy the following Helmholtz equations

$$\nabla^2 w_j(x, y) + k_j^2 w_j(x, y) = 0, \quad j = 1, 2, \tag{13}$$

with

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad \text{and} \quad k_j = \omega s_j, \; j = 1, 2.$$

It follows from Eq. (11) that $k_2 > k_1$.

It is easily verified from (4), (5) and (6) that the relevant displacement and the stress component in a physical anisotropic solid are related to those in the corresponding isotropic solid by

$$W_j(X, Y, t) = W_j(x, y, t), \tag{14}$$

$$\sigma^{(j)}_{XZ}(X, Y, t) = \frac{\mu_j}{(C_{44})_j}\sigma^{(j)}_{xz}(x, y, t) + \frac{(C_{45})_j}{(C_{44})_j}\sigma^{(j)}_{yz}(x, y, t), \tag{15}$$

$$\sigma^{(j)}_{YZ}(X, Y, t) = \sigma^{(j)}_{yz}(x, y, t). \tag{16}$$

Further writing

$$\sigma^{(j)}_{yz}(x, y, t) = \tau^{(j)}_{yz}(x, y)\,\mathrm{e}^{-i\omega t}, \quad j = 1, 2, \tag{17}$$

under the changed co-ordinate system the boundary conditions (1), (2) and (3) reduce to

$$\tau^{(1)}_{yz}(x, y) = \mu_1\frac{\partial w_1}{\partial y} = -F_1\delta(x); \quad |x| < L,\ y = 0+, \tag{18}$$

$$\tau^{(2)}_{yz}(x, y) = \mu_2\frac{\partial w_2}{\partial y} = F_2\delta(x); \quad |x| < L,\ y = 0- \tag{19}$$

and

$$\mu_1\frac{\partial w_1}{\partial y} = \mu_2\frac{\partial w_2}{\partial y}, \quad |x| > L,\ y = 0, \tag{20}$$

$$w_1(x, 0+) = w_2(x, 0-), \quad |x| > L. \tag{21}$$

To obtain the solutions to the wave equations (13), introduce the Fourier transform defined by

$$\bar{w}(\alpha, y) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} w(x, y)\,\mathrm{e}^{i\alpha x}\,\mathrm{d}x. \tag{22}$$

The transformed wave equations are

$$\frac{\mathrm{d}^2\bar{w}_1}{\mathrm{d}y^2} - (\alpha^2 - k_1^2)\bar{w}_1(\alpha, y) = 0, \quad y \geqslant 0, \tag{23}$$

$$\frac{\mathrm{d}^2\bar{w}_2}{\mathrm{d}y^2} - (\alpha^2 - k_2^2)\bar{w}_2(\alpha, y) = 0, \quad y \leqslant 0. \tag{24}$$

The solutions of (23) and (24) which are bounded as $y \to \infty$ are

$$\bar{w}_1(\alpha, y) = A_1(\alpha)\,\mathrm{e}^{-\gamma_1 y}; \quad y \geqslant 0, \tag{25}$$

$$\bar{w}_2(\alpha, y) = A_2(\alpha)\,\mathrm{e}^{\gamma_2 y}; \quad y < 0, \tag{26}$$

where

$$\gamma_j = \begin{cases} (\alpha^2 - k_j^2)^{1/2}; & |\alpha| > k_j, \\ -i(k_j^2 - \alpha^2)^{1/2}; & |\alpha| < k_j. \end{cases} \tag{27}$$

Introduce, for a complex α

$$G_+(\alpha) = \frac{1}{\sqrt{2\pi}} \int_L^\infty \tau_{yz}^{(1)}(x,0)\, e^{i\alpha(x-L)}\, dx, \tag{28}$$

$$G_-(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-L} \tau_{yz}^{(1)}(x,0)\, e^{i\alpha(x+L)}\, dx, \tag{29}$$

$$G_j(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-L}^{L} \tau_{yz}^{(j)}(x,0)\, e^{i\alpha x}\, dx. \tag{30}$$

The transformed stress at the interface $y=0$ can therefore be written as

$$\bar{\tau}_{yz}^{(j)}(\alpha,0) = G_+(\alpha)\, e^{i\alpha L} + G_-(\alpha)\, e^{-i\alpha L} + G_j(\alpha). \tag{31}$$

Using the boundary conditions (18) and (19), we get

$$G_j(\alpha) = (-1)^j \frac{F_j}{\sqrt{2\pi}} \quad (j = 1, 2). \tag{32}$$

Now

$$\bar{\tau}_{yz}^{(1)}(\alpha, 0+) = \mu_1 \frac{\partial \bar{w}_1(\alpha, 0+)}{\partial y} = -\mu_1 \gamma_1 A_1(\alpha), \tag{33}$$

$$\bar{\tau}_{yz}^{(2)}(\alpha, 0-) = \mu_2 \frac{\partial \bar{w}_2(\alpha, 0-)}{\partial y} = \mu_2 \gamma_2 A_2(\alpha). \tag{34}$$

Using (33) and (34), Eq. (31) can be written in the form

$$(-1)^j \mu_j \gamma_j A_j(\alpha) = G_+(\alpha)\, e^{i\alpha L} + G_-(\alpha)\, e^{-i\alpha L} + (-1)^j \frac{F_j}{\sqrt{2\pi}}.$$

Therefore,

$$A_j(\alpha) = \frac{(-1)^j}{\mu_j \gamma_j} \left[G_+(\alpha)\, e^{i\alpha L} + G_-(\alpha)\, e^{-i\alpha L} + (-1)^j \frac{F_j}{\sqrt{2\pi}} \right]. \tag{35}$$

Now

$$\bar{w}_1(\alpha, 0+) - \bar{w}_2(\alpha, 0-) = \frac{1}{\sqrt{2\pi}} \int_{-L}^{L} \{ w_1(x, 0+) - w_2(x, 0-) \}\, e^{i\alpha x}\, dx = B(\alpha), \quad \text{say}$$

which is the measure of the displacement discontinuity across the crack surface. Therefore

$$B(\alpha) = A_1(\alpha) - A_2(\alpha). \tag{36}$$

Substituting the values of $A_j(\alpha)$ from (35) in Eq. (36) one finds an extended Wiener–Hopf equation namely

$$G_+(\alpha)\, e^{i\alpha L} + G_-(\alpha)\, e^{-i\alpha L} + B(\alpha) K(\alpha) = \frac{K(\alpha)}{\sqrt{2\pi}} \left\{ \frac{F_1}{\mu_1 \gamma_1} - \frac{F_2}{\mu_2 \gamma_2} \right\}, \tag{37}$$

where

$$K(\alpha) = \frac{\mu_1 \mu_2 \gamma_1 \gamma_2}{\mu_1 \gamma_1 + \mu_2 \gamma_2} = \frac{\mu_1 \mu_2 (\alpha^2 - k_1^2)^{1/2}}{\mu_1 + \mu_2} R(\alpha) \tag{38}$$

and

$$R(\alpha) = \frac{(\mu_1 + \mu_2)(\alpha^2 - k_2^2)^{1/2}}{\mu_1(\alpha^2 - k_1^2)^{1/2} + \mu_2(\alpha^2 - k_2^2)^{1/2}}. \tag{39}$$

In order to obtain the high-frequency solution of the Wiener–Hopf equation given by (37) one assumes that the branch points $\alpha = k_1$ and $\alpha = k_2$ of $K(\alpha)$ possess a small imaginary part. Therefore k_1 and k_2 are replaced by $k_1 + ik_1'$ and $k_2 + ik_2'$ respectively where k_1' and k_2' are infinitesimally small positive quantities which would ultimately be made to tend to zero.

Now $K(\alpha) = K_+(\alpha)K_-(\alpha)$ where $K_+(\alpha)$ is analytic in the upper half-plane $\operatorname{Im}\alpha > -k_2'$ whereas $K_-(\alpha)$ is analytic in the lower half-plane $\operatorname{Im}\alpha < k_2'$ are given by (cf. Pal and Ghosh, 1990; Wickham, 1980)

$$K_\pm(\alpha) = \left(\frac{\mu_2(\alpha \pm k_1)}{1+m}\right)^{1/2} \exp\left[\frac{1}{\pi}\int_1^\gamma \frac{\tan^{-1}\left\{\frac{(t^2-1)^{1/2}}{m(\gamma^2-t^2)^{1/2}}\right\}}{t \pm \frac{\alpha}{k_1}}\,\mathrm{d}t\right],$$

where $m = \frac{\mu_2}{\mu_1}$ and $\gamma = \frac{k_2}{k_1}$.

Since $\tau_{yz}(x, 0)$ decreases exponentially as $x \to \pm\infty$, $G_+(\alpha)$ and $G_-(\alpha)$ have the common region of regularity as $K_+(\alpha)$ and $K_-(\alpha)$. It may be noted that $B(\alpha)$ is analytic in the whole of α-plane.

Now (37) can easily be expressed as two integral equations relating $G_+(\alpha)$, $G_-(\alpha)$ and $B(\alpha)$ as follows:

$$\begin{aligned}
&\frac{G_\pm(\alpha)}{K_\pm(\alpha)} + \frac{1}{2\pi i}\int_{C_\pm} \frac{G_\mp(s)\,\mathrm{e}^{\mp 2isL}}{(s-\alpha)K_\pm(s)}\,\mathrm{d}s \\
&\quad - \frac{1}{2\pi i}\int_{C_\pm} \frac{\mathrm{e}^{\mp isL}K_\mp(s)}{\sqrt{2\pi}(s-\alpha)}\left\{\frac{F_1}{\mu_1\gamma_1(s)} - \frac{F_2}{\mu_2\gamma_2(s)}\right\}\mathrm{d}s \\
&= -B(\alpha)K_\mp(\alpha)\,\mathrm{e}^{\mp i\alpha L} - \frac{1}{2\pi i}\int_{C_\mp} \frac{G_\mp(s)\,\mathrm{e}^{\mp 2isL}}{(s-\alpha)K_\pm(s)}\,\mathrm{d}s \\
&\quad + \frac{1}{2\pi i}\int_{C_\mp} \frac{\mathrm{e}^{\mp isL}K_\mp(s)}{\sqrt{2\pi}(s-\alpha)}\left\{\frac{F_1}{\mu_1\gamma_1(s)} - \frac{F_2}{\mu_2\gamma_2(s)}\right\}\mathrm{d}s,
\end{aligned} \tag{40}$$

where C_+ and C_- are the straight contours situated within the common region of regularity of $G_+(s), G_-(s)$, $K_+(s)$ and $K_-(s)$ as shown in *figure 2*.

In the first equation of (40) (i.e. the equation involving upper subscripts), the left-hand side is analytic in the upperhalf plane whereas the right-hand side is analytic in the lowerhalf plane and both of them are equal in the

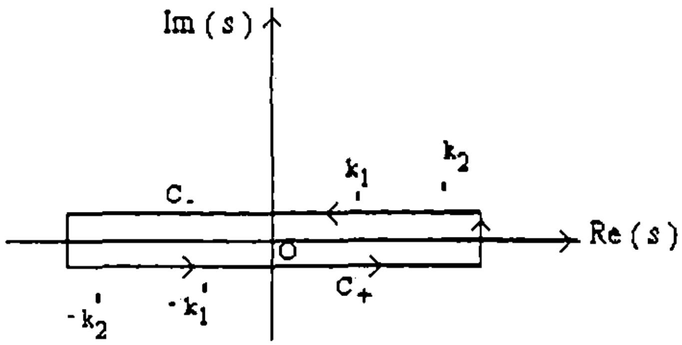

Figure 2. Path of integration in the complex s-plane.

common region of analyticity of these two functions. So by analytic continuation, both sides of the equation are analytic in the whole of the s-plane. Now, since

$$\tau_{yz} = O(|x \mp L|^{-1/2}) \quad \text{as } x \to \pm L$$

so

$$G_+(\alpha) = O(\alpha^{-1/2}) \quad \text{as } |\alpha| \to \infty, \ \operatorname{Im}\alpha > 0$$

and also

$$K_\pm(\alpha) = O(\alpha^{1/2}) \quad \text{as } |\alpha| \to \infty, \ \operatorname{Im}\alpha \gtrless 0.$$

So it follows that

$$\frac{G_+(\alpha)}{K_+(\alpha)} = O(\alpha^{-1}) \quad \text{as } |\alpha| \to \infty, \ \operatorname{Im}\alpha > 0.$$

Presumably one has $w_1(x, 0+) - w_2(x, 0-) = O(|x \mp L|^{1/2})$ as $x \to \pm L$.

Then it follows by standard Abelian asymptotics (cf. Noble, 1958; p. 36) that

$$B(\alpha) = e^{i\alpha L} O(\alpha^{-3/2}) + e^{-i\alpha L} O(\alpha^{-3/2}) \quad \text{as } |\alpha| \to \infty.$$

Consequently one has

$$B(\alpha) K_-(\alpha) e^{-i\alpha L} = O(\alpha^{-1}) \quad \text{as } |\alpha| \to \infty, \ \operatorname{Im}\alpha < 0.$$

Thus both sides of the first equation of (40) are $O(\alpha^{-1})$ as $|\alpha| \to \infty$ in the respective half-planes.

Therefore by Liouville's Theorem, both sides of the first equation of (40) are equal to zero. The second equation of (40) (i.e. the equation involving lower subscripts) can be treated similarly. Therefore from (40) one obtains the system of integral equations given by

$$\frac{G_\pm(\alpha)}{K_\pm(\alpha)} + \frac{1}{2\pi i}\int_{C_\pm} \frac{G_\mp(s)\, e^{\mp 2isL}}{(s-\alpha)K_\pm(s)}\,ds - \frac{1}{2\pi i}\int_{C_\pm} \frac{e^{\mp isL} K_\mp(s)}{\sqrt{2\pi}(s-\alpha)}\left\{\frac{F_1}{\mu_1\gamma_1(s)} - \frac{F_2}{\mu_2\gamma_2(s)}\right\} ds = 0. \tag{41}$$

Since $\tau_{yz}^{(1)}(x, 0)$ is an even function of x, so from (28) and (29) it can be shown that $G_+(-\alpha) = G_-(\alpha)$ and that $K_+(-\alpha) = iK_-(\alpha)$ (cf. Pal and Ghosh, 1990). Using these results and replacing α by $-\alpha$ and s by $-s$ in the first equation of (41) it can easily be shown that both the equations in (41) are identical. So $G_+(\alpha)$ and $G_-(\alpha)$ are to be determined from any one of the integral equations in (41).

3. High frequency solution of the integral equation

To solve the second integral equation of (41) in the case when the normalized wave number $k_1 L \gg 1$, the integration along the path C_- in (41) is replaced by the integration along the contours L_{k_1} and L_{k_2} around the branch cuts through the branch points k_1 and k_2 of the function $K_-(s)$ as shown in *figure 3*. Thus the second equation in (41) takes the form

$$\begin{aligned} G_-(\alpha) = &-\frac{K_-(\alpha)}{2\pi i}\int_{L_{k_1}+L_{k_2}} \frac{G_+(s)\, e^{2isL}}{(s-\alpha)K_-(s)}\,ds \\ &+ \frac{K_-(\alpha)}{2\pi i}\int_{L_{k_1}+L_{k_2}} \frac{e^{isL} K_+(s)}{\sqrt{2\pi}(s-\alpha)}\left\{\frac{F_1}{\mu_1\gamma_1(s)} - \frac{F_2}{\mu_2\gamma_2(s)}\right\} ds. \end{aligned} \tag{42}$$

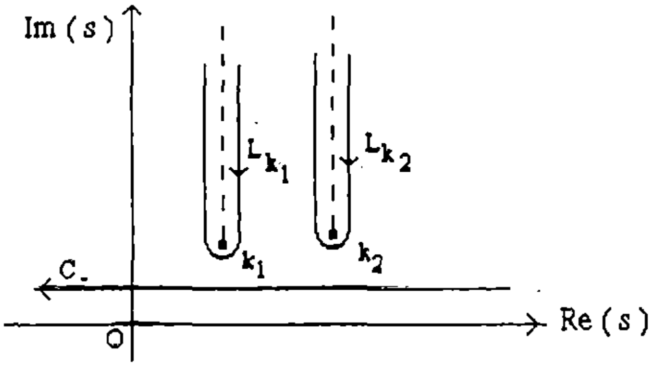

Figure 3. Paths of integration C_-, L_{k_1}, L_{k_2}.

For $k_1 L \gg 1$, it can be shown that

$$\int_{L_{k_j}} \frac{G_+(s)\,\mathrm{e}^{2isL}}{(s-\alpha)K_-(s)}\,\mathrm{d}s \approx -\frac{1}{\mu_j}\sqrt{\frac{\pi}{k_j L}}\frac{G_+(k_j)K_+(k_j)}{(k_j-\alpha)}\,\mathrm{e}^{i\pi/4}\,\mathrm{e}^{2ik_jL}, \quad j=1,2, \tag{43}$$

and

$$\int_{L_{k_1}+L_{k_2}} \frac{K_+(s)}{\sqrt{2\pi}(s-\alpha)}\left\{\frac{F_1}{\mu_1(s^2-k_1^2)^{1/2}} - \frac{F_2}{\mu_2(s^2-k_2^2)^{1/2}}\right\}\mathrm{e}^{isL}\,\mathrm{d}s \approx \sum_{j=1}^{2}(-1)^j\frac{F_j}{\mu_j}\frac{K_+(k_j)}{(k_j-\alpha)}\frac{\mathrm{e}^{i(k_jL+\pi/4)}}{(k_jL)^{1/2}}. \tag{44}$$

Using the results of (43) and (44) and also the relations $G_+(-\alpha) = G_-(\alpha)$ and $K_-(\alpha) = -iK_+(-\alpha)$ one obtains from (42)

$$F_+(-\alpha) + \sum_{j=1}^{2}\frac{A(k_j)\,\mathrm{e}^{2ik_jL}}{\mu_j(k_j-\alpha)(k_jL)^{1/2}}F_+(k_j) = -C(-\alpha), \tag{45}$$

where

$$F_+(\xi) = \frac{G_+(\xi)}{K_-(-\xi)} = \frac{G_-(-\xi)}{K_-(-\xi)}, \tag{46}$$

$$A(\xi) = \frac{[K_+(\xi)]^2\,\mathrm{e}^{i\pi/4}}{2\sqrt{\pi}}, \tag{47}$$

$$C(\xi) = \frac{1}{2\pi i}\sum_{j=1}^{2}(-1)^{j+1}\frac{F_j}{\mu_j}\frac{K_+(k_j)}{(k_j+\xi)}\frac{\mathrm{e}^{i(k_jL+\pi/4)}}{(k_jL)^{1/2}}. \tag{48}$$

Substituting $\alpha = -k_1$ and $\alpha = -k_2$ in (45) one obtains respectively the equations

$$\left[1 + M_1(k_1)\,\mathrm{e}^{2ik_1L}\right]F_+(k_1) + \frac{\mu_1}{\mu_2}M_1(k_2)\,\mathrm{e}^{2ik_2L}F_+(k_2) = -C(k_1) \tag{49}$$

and

$$\frac{\mu_2}{\mu_1}M_2(k_1)\,\mathrm{e}^{2ik_1L}F_+(k_1) + \left[1 + M_2(k_2)\,\mathrm{e}^{2ik_2L}\right]F_+(k_2) = -C(k_2), \tag{50}$$

where

$$M_j(\xi) = \frac{A(\xi)}{\mu_j(k_j + \xi)\sqrt{\xi L}}. \tag{51}$$

Now solution of (49) and (50) gives

$$F_+(k_m) = \left[\frac{\mu_m}{\mu_n} M_m(k_n) C(k_n)\, e^{2ik_n L} - C(k_m)\{1 + M_n(k_n)\, e^{2ik_n L}\}\right] P(k_1, k_2)$$
$$(\text{for } m = 1,\ n = 2 \text{ and for } m = 2,\ n = 1), \tag{52}$$

where

$$P(k_1, k_2) = \left[1 + M_1(k_1)\, e^{2ik_1 L} + M_2(k_2)\, e^{2ik_2 L} + \left(\frac{k_1 - k_2}{k_1 + k_2}\right)^2 M_1(k_1) M_2(k_2)\, e^{2i(k_1+k_2)L}\right]^{-1}. \tag{53}$$

For high-frequency, expanding $P(k_1, k_2)$ up to $O(k_j L)^{-1}$ and neglecting the terms involving $(k_j L)^{-2}$ and the higher order terms in $F_+(k_1)$ and $F_+(k_2)$ in (52) respectively, one obtains from Eqs (45) and (46)

$$G_-(\alpha) = \frac{K_-(\alpha)}{2\pi i} \sum_{j=1}^{2} (-1)^j L(k_j) F_j \left\{ \frac{1}{(k_j - \alpha)\mu_j} - \sum_{m=1}^{2} \frac{M_j(k_m)}{\mu_m (k_m - \alpha)}\, e^{2ik_m L} \right.$$
$$\left. + \sum_{m=1}^{2} \sum_{n=1}^{2} \frac{M_j(k_m) M_m(k_n)}{\mu_n (k_n - \alpha)}\, e^{2i(k_m+k_n)L} \right\}, \tag{54}$$

where

$$L(\xi) = \frac{K_+(\xi)\, e^{i(\xi L + \pi/4)}}{\sqrt{\xi L}}. \tag{55}$$

Replacing α by $-\alpha$ and using the relation $K_-(-\alpha) = -i K_+(\alpha)$ and $G_-(-\alpha) = G_+(\alpha)$ one obtains,

$$G_+(\alpha) = -\frac{K_+(\alpha)}{2\pi} \sum_{j=1}^{2} (-1)^j L(k_j) F_j \left\{ \frac{1}{(k_j + \alpha)\mu_j} - \sum_{m=1}^{2} \frac{M_j(k_m)}{\mu_m (k_m + \alpha)}\, e^{2ik_m L} \right.$$
$$\left. + \sum_{m=1}^{2} \sum_{n=1}^{2} \frac{M_j(k_m) M_m(k_n)}{\mu_n (k_n + \alpha)}\, e^{2i(k_m+k_n)L} \right\}. \tag{56}$$

4. Stress intensity factor near the crack tips

For $\alpha \to +\infty$ along the real axis,

$$K_\pm(\alpha) \sim \alpha^{1/2} \sqrt{\frac{\mu_1 \mu_2}{\mu_1 + \mu_2}}. \tag{57}$$

From (53) and (56) one obtains,

$$G_+(\alpha) \sim S\alpha^{-1/2} \quad \text{and} \quad G_-(\alpha) \sim -i S\alpha^{-1/2}, \tag{58}$$

where

$$S=-\frac{1}{2\pi}\sqrt{\frac{\mu_1\mu_2}{\mu_1+\mu_2}}\sum_{j=1}^{2}(-1)^j L(k_j)F_j\left\{\frac{1}{\mu_j}-\sum_{m=1}^{2}\frac{1}{\mu_m}M_j(k_m)\,e^{2ik_mL}\right.$$
$$\left.+\sum_{m=1}^{2}\sum_{n=1}^{2}\frac{1}{\mu_n}M_j(k_m)M_m(k_n)\,e^{2i(k_m+k_n)L}\right\}. \tag{59}$$

Using (57) and (58), Eq. (37) yields

$$B(\alpha)=\frac{S}{\alpha\sqrt{\alpha}}\{i\,e^{-i\alpha L}-e^{i\alpha L}\}\left(\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right)+\frac{1}{\sqrt{2\pi}}\left\{\frac{F_1}{\mu_1}-\frac{F_2}{\mu_2}\right\}\frac{1}{\alpha}. \tag{60}$$

From Eqs (18), (19) and (20) one obtains,

$$\tau_{yz}^{(1)}(x,0+)-\tau_{yz}^{(2)}(x,0-)=-(F_1+F_2)\delta(x).$$

Taking Fourier transformation on both sides, we obtain

$$\bar{\tau}_{yz}^{(1)}(\alpha,0+)-\bar{\tau}_{yz}^{(2)}(\alpha,0-)=-\frac{(F_1+F_2)}{\sqrt{2\pi}}$$

or

$$\mu_1\gamma_1A_1(\alpha)+\mu_2\gamma_2A_2(\alpha)=\frac{(F_1+F_2)}{\sqrt{2\pi}}. \tag{61}$$

From Eqs (60), (61) and (36) one obtains when $\alpha\to+\infty$ along the real axis,

$$A_j(\alpha)=\frac{(-1)^{j+1}S}{\mu_j\alpha\sqrt{\alpha}}[i\,e^{-i\alpha L}-e^{i\alpha L}]+\frac{1}{\sqrt{2\pi}}\frac{1}{\alpha}\frac{F_j}{\mu_j};\quad j=1,2. \tag{62}$$

Now

$$\tau_{yz}^{(j)}(x,y)=\mu_j\frac{\partial w_j(x,y)}{\partial y}=\mu_j\frac{\partial}{\partial y}\left[\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}A_j(\alpha)\,e^{-\gamma_j|y|-i\alpha x}\,d\alpha\right]$$
$$=(-1)^j\frac{\mu_j}{\sqrt{2\pi}}\int_0^{\infty}\gamma_jA_j(\alpha)\,e^{-\gamma_j|y|}[e^{-i\alpha x}+e^{i\alpha x}]\,d\alpha \tag{63}$$

as by Eq. (35) $A_j(\alpha)$ is an even function of α.

Substituting the values of $A_j(\alpha)$ as $|\alpha|\to\infty$ we can write the stress in the vicinity of the crack tip as

$$\tau_{yz}^{(j)}(x,y)\approx\frac{S}{\sqrt{2\pi}}\int_0^{\infty}\frac{e^{-\alpha|y|}}{\sqrt{\alpha}}[e^{i\alpha(x+L)}-i\,e^{i\alpha(x-L)}-i\,e^{-i\alpha(x+L)}+e^{-i\alpha(x-L)}]\,d\alpha$$
$$+(-1)^j\frac{F_j}{\pi}\int_0^{\infty}e^{-\alpha|y|}\cos\alpha x\,d\alpha$$
$$=\frac{S(1-i)}{\sqrt{2\pi}}\int_0^{\infty}\frac{e^{-\alpha|y|}}{\sqrt{\alpha}}[\cos\alpha(x+L)-\sin\alpha(x+L)+\cos\alpha(x-L)+\sin\alpha(x-L)]\,d\alpha$$
$$+(-1)^j\frac{F_j}{\pi}\int_0^{\infty}e^{-\alpha|y|}\cos\alpha x\,d\alpha$$
$$=S(1-i)\left[\frac{1}{\sqrt{r_1}}\cos\frac{\theta_1}{2}+\frac{1}{\sqrt{r_2}}\cos\frac{\theta_2}{2}\right]+(-1)^j\frac{F_j}{\pi}\frac{|y|}{x^2+y^2}, \tag{64}$$

where

$$(x-L)+iy=r_1\,e^{i\theta_1}, \qquad -(x+L)+iy=r_2\,e^{i\theta_2}, \qquad \pi \leqslant \theta_{1,2} \leqslant \pi. \tag{65}$$

It is to be noted that the final term in Eq. (64) which can be reduced to $-\frac{F_j}{\pi}\frac{y}{x^2+y^2}$ describes the behaviour of the stress near the source. Therefore at the interface ($y=0$) we obtain

$$\tau_{yz} \approx \frac{S(1-i)}{\sqrt{x-L}} \quad \text{as } x \to L+0, \tag{66}$$

$$\tau_{yz} \approx \frac{S(1-i)}{\sqrt{-(x+L)}} \quad \text{as } x \to -L-0. \tag{67}$$

Now the dimensionless stress intensity factor is defined by,

$$K=\left|\frac{S(1-i)}{F_1\sqrt{k_1}}\right|, \tag{68}$$

where S is given by (59).

5. Results and discussions

Since from Eqs (7) and (16) we note that for $Y=0$, $x=X$ and $y=0$ and that $\sigma_{YZ}^{(j)}(X,0,t)=\sigma_{yz}^{(j)}(x,0,t)$, therefore, the elastodynamic mode-III stress intensity factor of the interface crack in an anisotropic bimaterials is *the same as that of an interface crack of the corresponding isotropic bimaterial given by (68).*

Numerical calculations have been carried out for both the cases of antisymmetric ($F_1=-F_2=F$) and symmetric ($F_1=F_2=F$) antiplane loadings. For numerical evaluation of the stress intensity factors, the three material pairs (Nayfen, 1995), given in *table 1*, have been considered.

The absolute values of the complex stress intensity factors defined by (68) have been plotted against $k_1 L$ in *figures 4–6*, for symmetric as well as for antisymmetric loadings for values of dimensionless frequency $k_1 L$ varying from 1.01 to 10.

It is interesting to note that in the case of symmetric loading, the stress intensity factor first increases with increasing $k_1 L$, attains a maximum and then with further increase of $k_1 L$, decreases gradually with oscillatory

Table I. Engineering elastic constants of different materials.

Medium Name	$\hat{\rho}$ (kg m^{-3})	C_{44} (GPa)	C_{55} (GPa)	C_{45} (GPa)
Type of material pair: I				
1. Carbon-epoxy	1.57×10^3	3.98	6.4	0
2. Graphite-epoxy	1.60×10^3	6.55	2.6	0
Type of material pair: II				
1. Isotropic Chromium	7.20×10^3	115.2	115.2	0
2. Isotropic Steel	7.90×10^3	81.91	81.91	0
Type of material pair: III				
1. Graphite	1.79×10^3	5.52	28.3	0
2. Carbon-epoxy	1.57×10^3	3.98	6.4	0

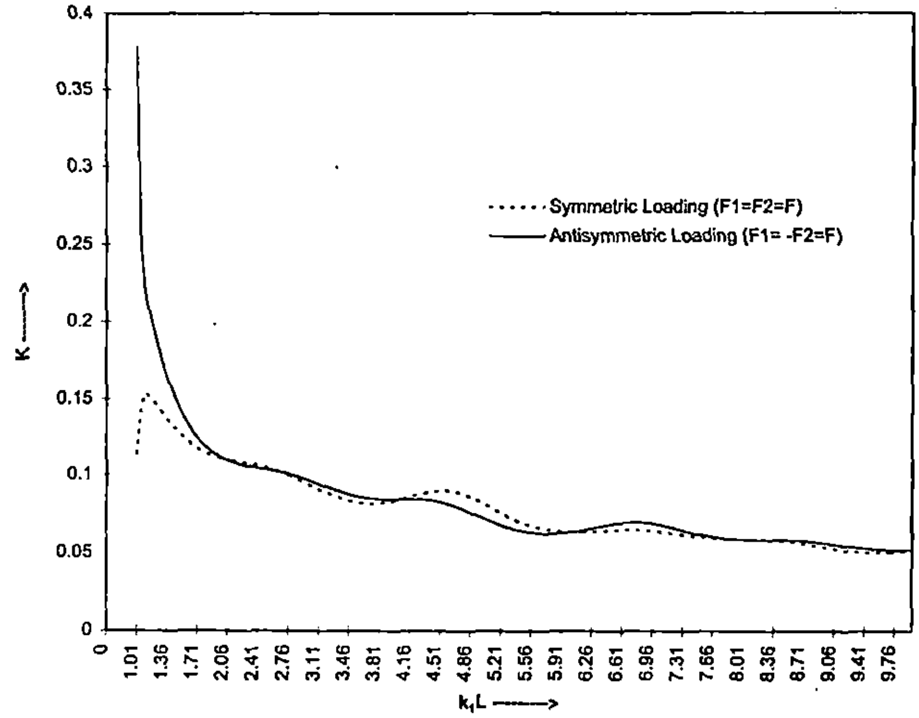

Figure 4. Stress intensity factor K versus dimensionless frequency $k_1 L$ for Type-I material pair.

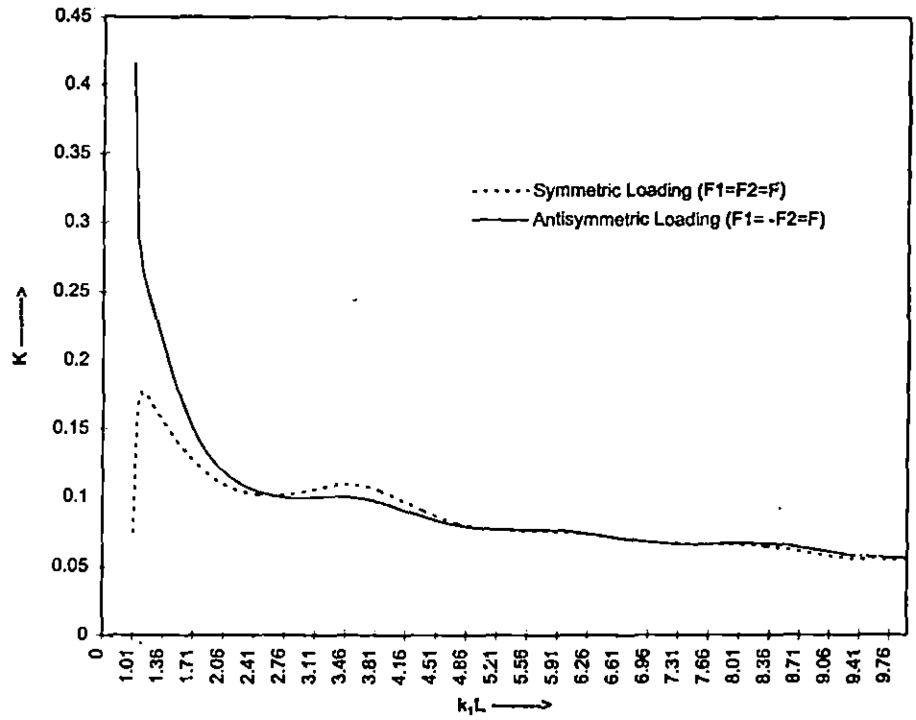

Figure 5. Stress intensity factor K versus dimensionless frequency $k_1 L$ for Type-II material pair.

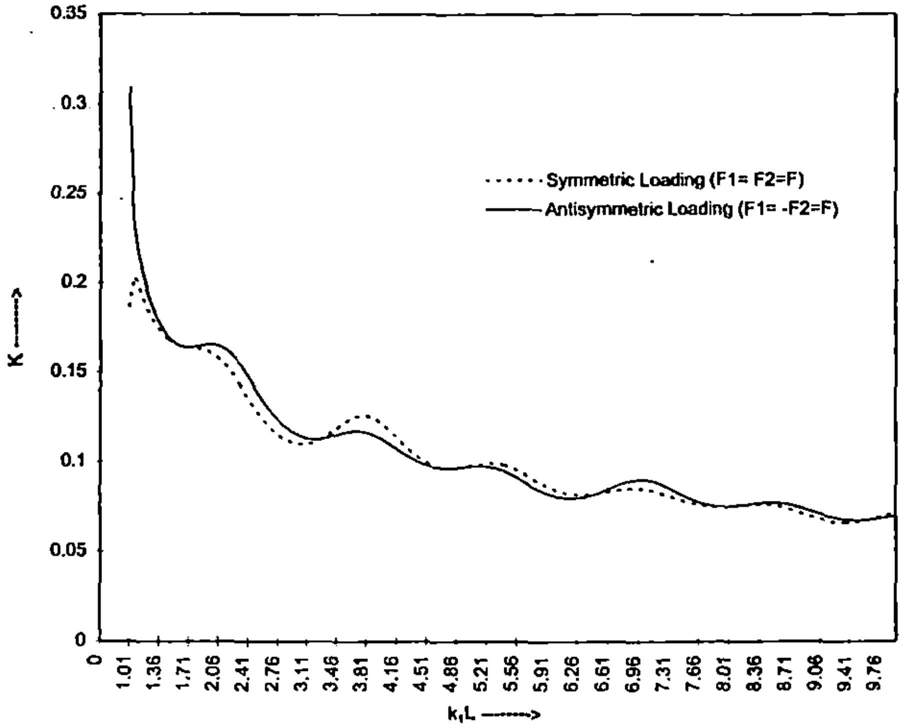

Figure 6. Stress intensity factor K versus dimensionless frequency k_1L for Type-III material pair.

behavior. On the other hand in the case of antisymmetric loading, stress intensity factor at first decreases sharply but with the increase of k_1L, it shows almost the same behaviours as the case for symmetric loading. The general oscillatory feature for the curves in *figures 4–6* are due to the effect of interaction between the waves generated by the two tips of the crack.

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Transient response due to a pair of antiplane point impact loading on the faces of a finite griffith crack at the bimaterial interface of anisotropic solids

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Abstract

The transient elastodynamic problem involving the scattering of elastic waves by a Griffith crack of finite width lying at the interface of two dissimilar anisotropic half planes has been analysed. The crack faces are subjected to a pair of suddenly applied antiplane line loads situated at the middle of the cracked surface. The problem has first been reduced to one with the interface crack of finite width lying between two dissimilar isotropic elastic half planes by a transformation of relevant coordinates and parameters. Spatial and time transforms are then applied to the governing differential equations and boundary conditions which yield generalized Wiener–Hopf type equations. The integral equations arising are solved by the standard iteration technique. Physically each successive order of iteration corresponds to successive scattered or rescattered wave from one crack tip to the other. Finally, expressions for the resulting mode III stress intensity factors are determined as a function of time for both symmetric and antisymmetric loadings. Each crack tip stress intensity factor has been plotted versus time for four pairs of different types of materials. © 1998 Elsevier Science Ltd. All rights reserved.

1. Introduction

The problem of a crack in an elastic material under the action of impulsive loading has been a subject of considerable interest recently. Sih et al. [1] have considered the problem for an infinite isotropic material and Kassir and Bandyopadhyay [2] studied infinite orthotropic

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material. Stephen and Hwel [3] also investigated the problem of diffraction of transient SH-waves by a crack of finite width and a rigid ribbon, also of finite width.

However in present years the extensive use of composite materials in the modern technology has created interest among scientists for carrying on considerable research work in the modeling, testing and analysis of laminated media. The laminated composites which behave as anisotropic material may be weakened by interface flaws which lead to serious degradation in load carrying capacity.

Kuo [4] carried out numerical and analytical studies of transient response of an interfacial semi-infinite crack between two dissimilar orthotropic half spaces. The problem of diffraction of transient horizontal shear waves by a finite crack located at the interface of two bonded dissimilar elastic half spaces has been treated by Takei et al. [5].

Neerhoff [6] studied the diffraction of Love waves by a crack of finite width at the interface of a layered half space. Kuo and Cheng [7] considered the elastodynamic response due to antiplane point impact loadings on the faces of an interface semi-infinite crack along dissimilar anisotropic materials.

In our present paper, we are interested in the antiplane transient elastodynamic responses and stress intensity factors of a Griffith crack of finite width lying along the interface of two dissimilar anisotropic elastic materials. The crack is subjected to a pair of suddenly applied antiplane concentrated line loading situated at the middle of the cracked surface. The materials are assumed to possess certain material symmetry and the crack plane is assumed to coincide with one of the planes of material symmetry, so that the inplane and the antiplane motion are not coupled.

The analysis of the paper is first based on the observation of several researches, e.g. Markenscoff and Ni [8], Achenbach and Kuo [9], that antiplane shear deformation in an anisotropic solid can be deduced from the corresponding deformations of an isotropic solid by a transformation of relevant co-ordinates and parameters. Based on this observation, analysis of the interface crack by transient line loads between two bonded dissimilar anisotropic elastic materials has first been converted to the corresponding problem between two dissimilar isotropic elastic solids. Later following Thau and Lu [3], spatial and time transforms are applied to the governing differential equations and generalized Wiener–Hopf type equations are obtained. The integral equation arising are solved by the standard iteration procedure. Physically, each successive order of iteration corresponds to successive scattered or rescattered wave from one crack tip to other.

Finally results are presented for the stress intensity factor near the crack tips. Each crack tip stress intensity factor is plotted versus time for a pair of different type of anisotropic materials.

2. Formulation of the problem

Consider antiplane deformation of a Griffith crack of finite width $2L$ lying between dissimilar anisotropic half planes which are characterized by the elastic moduli $(c_{ik})_j$; $i, k = 4, 5$ and mass densities $\hat{\rho}_j$. The subscripts j $(j = 1, 2)$ refers to the upper and lower media respectively. Let (X, Y, Z) be Cartesian co-ordinates. The X-axis is taken along the interface,

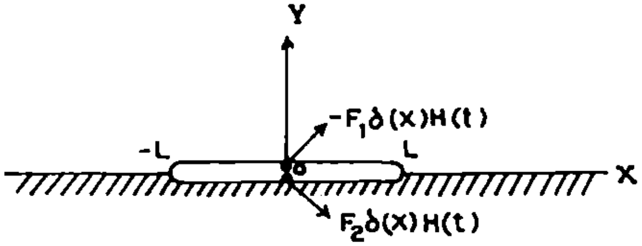

Fig. 1. Geometry of the problem.

Y-axis vertically upwards into the medium and Z-axis is perpendicular to the plane of the paper (Fig. 1).

For time $t < 0$, the elastic solids are at rest. For time $t \geq 0$, a pair of concentrated antiplane shear forces in the Z-direction of magnitudes F_1 and F_2 act on the crack faces $Y = 0+$ and $Y = 0-$ respectively at $X = 0$. Thus the crack face boundary conditions are

$$\hat{\sigma}_{YZ}(X, Y, t) = \begin{cases} -F_1\delta(X)H(t); |X| < L, Y = 0+ \\ F_2\delta(X)H(t); |X| < L, Y = 0- \end{cases}, \tag{1}$$

where $H(\,)$ and $\delta(\,)$ are the Heaviside step and Dirac delta functions respectively. Ahead of the crack tips, the interface boundary conditions which corresponds to the continuity of the displacement and traction along the welded part of the interface along $|X| > L$, $Y = 0$ are

$$\hat{\sigma}^{(1)}_{YZ}(X, 0, t) = \hat{\sigma}^{(2)}_{YZ}(X, 0, t) \tag{2}$$

$$\hat{W}_1(X, 0, t) = \hat{W}_2(X, 0, t). \tag{3}$$

Two dimensional antiplane wave motions of homogeneous anisotropic linearly elastic solids are governed by [10]

$$(C_{55})_j \frac{\partial^2 \hat{W}_j}{\partial X^2} + 2(C_{45})_j \frac{\partial^2 \hat{W}_j}{\partial X \partial Y} + (C_{44})_j \frac{\partial^2 \hat{W}_j}{\partial Y^2} = \hat{\rho}_j \frac{\partial^2 \hat{W}_j}{\partial t^2}; \quad (j = 1, 2), \tag{4}$$

where $\hat{W}_j(X, Y, t)$ is the out-of-plane displacement.

The crack plane has been assumed to coincide with one of the plane of material symmetry such that inplane and outplane motions are not coupled.

The relevant stress components are

$$\hat{\sigma}^{(j)}_{XZ}(X, Y, t) = (C_{55})_j \frac{\partial \hat{W}_j}{\partial X} + (C_{45})_j \frac{\partial \hat{W}_j}{\partial Y} \tag{5}$$

$$\hat{\sigma}^{(j)}_{YZ}(X, Y, t) = (C_{45})_j \frac{\partial \hat{W}_j}{\partial X} + (C_{44})_j \frac{\partial \hat{W}_j}{\partial Y}. \tag{6}$$

Following Achenbach and Kuo [9] and Ma [11], we introduce a co-ordinate transformation which has also been used by Kuo and Cheng [7]

$$\left.\begin{aligned} x &= X - \frac{(C_{45})_j}{(C_{44})_j} Y \\ y &= \frac{\mu_j}{(C_{44})_j} Y \\ z &= Z \end{aligned}\right\} (j = 1, 2), \tag{7}$$

where

$$\mu_j = [(C_{44})_j (C_{55})_j - (C_{45})_j^2]^{1/2}, \quad (j = 1, 2). \tag{8}$$

Transformation given by Eq. (7) reduce Eq. (4) to the standard wave equation

$$\frac{\partial^2 \hat{W}_j}{\partial x^2} + \frac{\partial^2 \hat{W}_j}{\partial y^2} = s_j^2 \frac{\partial^2 \hat{W}_j}{\partial t^2}, \tag{9}$$

where

$$s_j^2 = \frac{\rho_j}{\mu_j} \text{ and } \rho_j = \frac{\hat{\rho}_j (C_{44})_j}{\mu_j} \tag{10}$$

s_j is the slowness of shear waves. Without any loss of generality we assume that

$$s_1 < s_2 \tag{11}$$

It is easily verified from Eqs. (4)–(6) that the relevant displacement and the stress component in the physical anisotropic solid are related to those in the corresponding isotropic solid by

$$\hat{W}_j(X, Y, t) = w_j(x, y, t), \tag{12}$$

$$\hat{\sigma}_{XZ}^{(j)}(X, Y, t) = \frac{\mu_j}{(C_{44})_j} \sigma_{xz}^{(j)}(x, y, t) + \frac{(C_{45})_j}{(C_{44})_j} \sigma_{yz}^{(j)}(x, y, t), \tag{13}$$

$$\hat{\sigma}_{YZ}^{(j)}(X, Y, t) = \sigma_{yz}^{(j)}(x, y, t). \tag{14}$$

From Eqs. (9) and (12), the antiplane wave motions of the corresponding isotropic bimaterial in the transformed co-ordinate are governed by the standard wave equation

$$\frac{\partial^2 w_j}{\partial x^2} + \frac{\partial^2 w_j}{\partial y^2} = s_j^2 \frac{\partial^2 w_j}{\partial t^2}; \quad (j = 1, 2) \tag{15}$$

and the relevant stress component is

$$\sigma_{yz}^{(j)}(x, y, t) = \mu_j \frac{\partial w_j}{\partial y}. \tag{16}$$

Under the changed co-ordinate system the boundary conditions Eqs. (1)–(3) reduce to

$$\sigma_{yz}(x, y, t) = \begin{cases} -F_1\delta(x)H(t); |x|<L, y=0+ \\ F_2\delta(x)H(t); |x|<L, y=0- \end{cases} \tag{17}$$

$$\sigma_{yz}^{(1)}(x, y, t) = \sigma_{yz}^{(2)}(x, y, t); \quad |x| > L, \quad y=0, \tag{18}$$

$$w_1(x, y, t) = w_2(x, y, t); \quad |x| > L, \quad y=0. \tag{19}$$

Hence

$$\mu_1 \frac{\partial w_1}{\partial y} = -F_1\delta(x)H(t); \quad |x|<L, \quad y=0+ \tag{20}$$

$$\mu_2 \frac{\partial w_2}{\partial y} = F_2\delta(x)H(t); \quad |x|<L, \quad y=0- \tag{21}$$

and

$$\mu_1 \frac{\partial w_1}{\partial y} = \mu_2 \frac{\partial w_2}{\partial y}; \quad |x| > L, \quad y=0 \tag{22}$$

$$w_1(x, 0, t) = w_2(x, 0, t); \quad |x| > L, \quad y=0. \tag{23}$$

We begin the analysis by introducing unknown functions w_j and $\partial w_j/\partial y$ along the x-axis over the intervals where the functions are not specified by Eqs. (22) and (23).

Assume that

$$w_j(x, 0, t) = g_j(x, 0, t); \quad -L<x<L \tag{24}$$

and

$$\mu_j \frac{\partial w_j}{\partial y} = \begin{cases} \phi(x+L, t); y=0 x+L<0 \\ \phi(x-L, t); y=0 x-L>0. \end{cases} \tag{25}$$

Now we introduce Laplace and Fourier transforms defined as

$$F(x, y, p) = \int_0^\infty f(x, y, t)\mathrm{e}^{-pt}\mathrm{d}t, \qquad \bar{F}(\zeta, y, p) = \int_{-\infty}^\infty F(x, y, p)\mathrm{e}^{-\mathrm{i}\zeta x}\mathrm{d}x \tag{26}$$

so that their inverse transforms are

$$f(x, y, t) = \frac{1}{2\pi i}\int_{BR} F(x, y, p)e^{pt}dp, \qquad F(x, y, p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \bar{F}(\zeta, y, p)e^{i\zeta x}d\zeta. \tag{27}$$

Taking Laplace transform with respect to t of both sides of Eq. (24) (for $|x| < L$)

$$W_j(x, 0, p) = \int_0^{\infty} w_j(x, 0, t)e^{-pt}dt = \int_0^{\infty} g_j(x, 0, t)e^{-pt}dt = G_j(x, 0, p); \quad |x|<L. \tag{28}$$

Next taking Laplace and Fourier transform on wave Eq. (15) one obtains

$$\frac{\partial^2 \bar{w}_1}{\partial y^2} - (\zeta^2 + k_1^2)\bar{w}_1 = 0; \quad (y > 0), \tag{29}$$

$$\frac{\partial^2 \bar{w}_2}{\partial y^2} - (\zeta^2 + k_2^2)\bar{w}_2 = 0; \quad (y<0), \tag{30}$$

where

$$k_j^2 = s_j^2 p^2; \quad (j = 1, 2). \tag{31}$$

The solutions of the Eqs. (29) and (30) which are bounded as $|y| \to \infty$ are

$$\bar{w}_1(\zeta, y, p) = A_1(\zeta)e^{-\gamma_1 y}; \quad y > 0, \tag{32}$$

$$\bar{w}_2(\zeta, y, p) = A_2(\zeta)e^{\gamma_2 y}; \quad y<0, \tag{33}$$

where

$$\gamma_j = (\zeta^2 + k_j^2)^{1/2}; \quad (j = 1, 2). \tag{34}$$

The transformed stress at the interface $y = 0$ can be written as

$$\mu_j \frac{\partial \bar{W}_j(\zeta, 0, p)}{\partial y} = e^{i\zeta L}\bar{\Phi}_+(\zeta, p) + \frac{1}{p}c_j F_j + e^{-i\zeta L}\bar{\Phi}_-(\zeta,p), \qquad [j = 1, 2, c_j = (-1)^j], \tag{35}$$

where

$$\bar{\Phi}_+(\zeta, p) = \int_{-\infty}^{-L} e^{-i\zeta x}\left[\int_0^{\infty} \phi(x + L, t)e^{-pt}dt\right]dx$$

and

$$\bar{\Phi}_-(\zeta, p) = \int_L^{\infty} e^{-i\zeta x}\left[\int_0^{\infty} \phi(x - L, t)e^{-pt}dt\right]dx.$$

$\bar{\Phi}_+$ and $\bar{\Phi}_-$ are analytic in the complex half plane $\mathrm{Im}(\zeta) > -k_1$ and $\mathrm{Im}(\zeta) < k_1$ respectively. So from Eqs. (32) and (33) one obtains

$$\mu_1 \frac{\partial \bar{W}_1(\zeta, 0, p)}{\partial y} = -\mu_1\gamma_1 A_1(\zeta), \qquad \mu_2 \frac{\partial \bar{W}_2(\zeta, 0, p)}{\partial y} = \mu_2\gamma_2 A_2(\zeta) \tag{36}$$

Eq. (35) with aid of Eq. (36) yields

$$(-1)^j \mu_j\gamma_j A_j(\zeta) = \mathrm{e}^{\mathrm{i}\zeta L}\bar{\Phi}_+(\zeta, p) + \mathrm{e}^{-\mathrm{i}\zeta L}\bar{\Phi}_-(\zeta, p) + (-1)^j \frac{F_j}{p}; \quad (j = 1, 2). \tag{37}$$

Taking aid of Eqs. (19), (28), (32), and (33) one obtains

$$\bar{W}_1(\zeta, 0, p) - \bar{W}_2(\zeta, 0, p) = \int_{-\infty}^{\infty} [W_1(x, 0, p) - W_2(x, 0, p)]\mathrm{e}^{-\mathrm{i}\zeta x}\mathrm{d}x$$

$$= \int_{-L}^{L} [G_1(x, 0, p) - G_2(x, 0, p)]\mathrm{e}^{-\mathrm{i}\zeta x}\mathrm{d}x = B(\zeta) \text{ (say)}$$

so that

$$A_1(\zeta) - A_2(\zeta) = B(\zeta). \tag{38}$$

By the help of Eqs. (37) and (38) one finds an extended Wiener–Hopf equation namely

$$K(\zeta)B(\zeta) = -\bar{\Phi}_+(\zeta, p)\mathrm{e}^{\mathrm{i}\zeta L} + \bar{\Phi}_-(\zeta, p)\mathrm{e}^{-\mathrm{i}\zeta L} + \frac{K(\zeta)}{p}\left[\frac{F_1}{\mu_1\gamma_1} - \frac{F_2}{\mu_2\gamma_2}\right], \tag{39}$$

where

$$K(\zeta) = \frac{\mu_1\mu_2\gamma_1\gamma_2}{\mu_1\gamma_1 + \mu_2\gamma_2} = \frac{\mu_1\mu_2(\zeta^2 + k_1^2)^{1/2}}{\mu_1 + \mu_2} R^1(\zeta), \tag{40}$$

$$= \frac{\mu_1\mu_2(\zeta^2 + k_2^2)^{1/2}}{(\mu_1 + \mu_2)} R^2(\zeta) \tag{41}$$

so that

$$R^1(\zeta) = \frac{(\mu_1 + \mu_2)(\zeta^2 + k_2^2)^{1/2}}{\mu_1(\zeta^2 + k_1^2)^{1/2} + \mu_2(\zeta^2 + k_2^2)^{1/2}}, \tag{42}$$

$$R^2(\zeta) = \frac{(\mu_1 + \mu_2)(\zeta^2 + k_1^2)^{1/2}}{\mu_1(\zeta^2 + k_1^2)^{1/2} + \mu_2(\zeta^2 + k_2^2)^{1/2}}. \tag{43}$$

The solution of Eq. (39) along with two transform inversions completes the problem. Here we shall concentrate on finding and then inverting $\bar{\Phi}_+$ and $\bar{\Phi}_-$ since $\phi(x + L, t)$ and $\phi(x - L, t)$ from Eq. (25) are equal to the shear stresses directly ahead of the crack tips. Hence they are required for the determination of dynamic stress intensity factors at the crack tips.

In order to solve Eq. (39), the function $K(\zeta)$ is at first made single valued by drawing branch cuts along the η-axis (recall $\zeta = \xi + i\eta$) from $\eta = k_1$ to ∞ and from $\eta = -k_1$ to $-\infty$. It is then broken up into the product of two functions which are analytic in the overlapping regions $\mathrm{Im}(\zeta) > -k_1$ and $\mathrm{Im}(\zeta) < k_1$ so that

$$K(\zeta) = K_+(\zeta)K_-(\zeta). \tag{44}$$

Next we divide Eq. (39) by $K_+(\zeta)$ and change ζ to ζ' in it and redivide it by $2\pi i e^{i\zeta' L}(\zeta' - \zeta)$ which yields

$$\frac{e^{-i\zeta' L}K_-(\zeta')B(\zeta')}{2\pi i(\zeta' - \zeta)} = -\frac{\bar{\Phi}_+(\zeta')}{2\pi i K_+(\zeta')(\zeta' - \zeta)} + \frac{\bar{\Phi}_-(\zeta')e^{-2i\zeta' L}}{2\pi i K_+(\zeta')(\zeta' - \zeta)} + \frac{K_-(\zeta')e^{-i\zeta' L}}{2\pi i(\zeta' - \zeta)p} \times \left[\frac{F_1}{\mu_1(\zeta'^2 + k_1^2)^{1/2}} - \frac{F_2}{\mu_2(\zeta'^2 + k_2^2)^{1/2}}\right] \tag{45}$$

Now with $\zeta' = \xi' + i\eta'$, take a line L_1 in the ζ'-plane lying in the strip $-k_1 < \eta' < k_1$; choose ζ to be a point lying above L_1 (i.e. $\eta > \eta'$) and integrate Eq. (45) along L_1 from $-\infty < \xi' < \infty$

$$\int_{L_1} \frac{e^{-i\zeta' L}K_-(\zeta')B(\zeta')}{2\pi i(\zeta' - \zeta)} d\zeta' = -\int_{L_1} \frac{\bar{\Phi}_+(\zeta')}{2\pi i K_+(\zeta')(\zeta' - \zeta)} d\zeta' + \int_{L_1} \frac{\bar{\Phi}_-(\zeta')e^{-2i\zeta' L}}{2\pi i K_+(\zeta')(\zeta' - \zeta)} d\zeta' + \int_{L_1} \frac{K_-(\zeta')e^{-i\zeta' L}}{2\pi i(\zeta' - \zeta)p}\left[\frac{F_1}{\mu_1(\zeta'^2 + k_1^2)^{1/2}} - \frac{F_2}{\mu_2(\zeta'^2 + k_2^2)^{1/2}}\right] d\zeta'. \tag{46}$$

Since $B(\zeta')$ is analytic in the entire plane and $K_-(\zeta')e^{-i\zeta' L}$ is analytic in the lower half plane, so considering semicircular contour in the lower half plane the first integral is found to be equal to zero.

Again while evaluating the second integral, a semicircular contour in the upper half plane is considered. Consequently the second integral is found to yield the value $\bar{\Phi}_+(\zeta)/K_+(\zeta)$.

Next for the last two integrals the integration path is deformed to the path round the branch cut through the branch points $\zeta = -ik_1$ and $-ik_2$ as shown in Fig. 2 so that finally Eq. (46) takes the form

$$\begin{aligned}
\bar{\Phi}_+(\zeta) = {} & \frac{iK_+(\zeta)}{\pi\mu_1}\int_0^\infty \frac{\bar{\Phi}_-[-ik_1(1+\lambda)]K_-[-ik_1(1+\lambda)]e^{-2Lk_1(1+\lambda)}}{[ik_1(1+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}} d\lambda \\
& + \frac{iK_+(\zeta)}{\pi\mu_2}\int_0^\infty \frac{\bar{\Phi}_-[-ik_2(1+\lambda)]K_-[-ik_2(1+\lambda)]e^{-2Lk_2(1+\lambda)}}{[ik_2(1+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}} d\lambda \\
& + \frac{iF_1K_+(\zeta)}{\pi\mu_1 p}\int_0^\infty \frac{K_-[-ik_1(1+\lambda)]e^{-Lk_1(1+\lambda)}}{[ik_1(1+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}} d\lambda \\
& - \frac{iF_2K_+(\zeta)}{\pi\mu_2 p}\int_0^\infty \frac{K_-[-ik_2(1+\lambda)]e^{-Lk_2(1+\lambda)}}{[ik_2(i+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}} d\lambda.
\end{aligned} \tag{47}$$

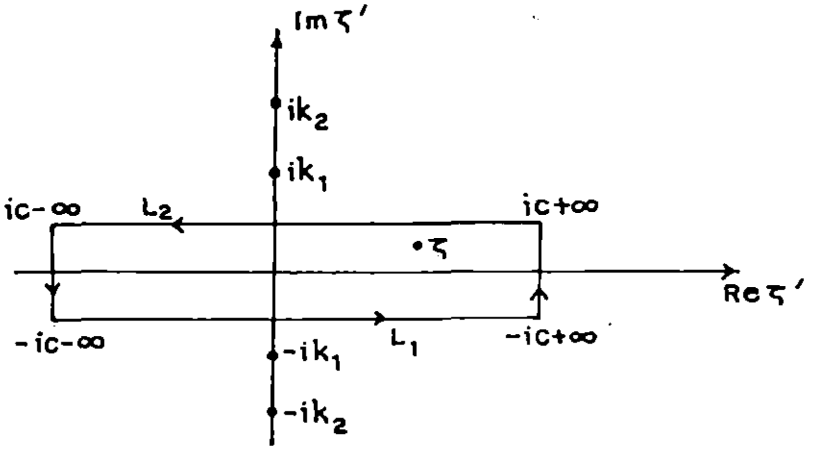

Fig. 2. Path of integration.

Similarly we can derive an equation for $\bar{\Phi}_-(\zeta)$ by dividing Eq. (39) by $2\pi i e^{-2\zeta' L} K_-(\zeta')(\zeta' - \zeta)$ after first changing ζ to ζ' and then choosing a line of integration L_2 in the strip $-k_1 < \eta' < k_1$. The point ζ is taken below L_2 and the result analogous to Eq. (47), then becomes;

$$\begin{aligned}\bar{\Phi}_-(\zeta) = &\frac{iK_-(\zeta)}{\pi\mu_1}\int_0^\infty \frac{\bar{\Phi}_+[ik_1(1+\lambda)]K_+[ik_1(1+\lambda)]e^{-2Lk_1(1+\lambda)}}{[ik_1(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda \\ &+\frac{iK_-(\zeta)}{\pi\mu_2}\int_0^\infty \frac{\bar{\Phi}_+[ik_2(1+\lambda)]K_+[ik_2(1+\lambda)]e^{-2Lk_2(1+\lambda)}}{[ik_2(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda \\ &-\frac{iF_1K_-(\zeta)}{\pi\mu_1 p}\int_0^\infty \frac{K_+[ik_1(1+\lambda)]e^{-Lk_1(1+\lambda)}}{[ik_1(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda \\ &+\frac{iF_2K_-(\zeta)}{\pi\mu_2 p}\int_0^\infty \frac{K_+[ik_2(1+\lambda)]e^{-Lk_2(1+\lambda)}}{[ik_2(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda. \end{aligned} \tag{48}$$

The integral equations have been solved by the standard iteration method and it may be noted that each successive order of iteration is a solution of the problem for successively increasing units of time starting from $t = 0$. Since each unit of time here corresponds exactly to the time required for an SH-wave to traverse the crack width, we can interpret physically each order of iteration in terms of the successive scatterings of waves from one crack to other and back again. Now we consider the zeroth order solutions of Eqs. (47) and (48) as

$$\begin{aligned}\bar{\Phi}_+^{(0)}(\zeta) = &\frac{iF_1K_+(\zeta)e^{-Lk_1}}{\pi\mu_1 p}\int_0^\infty \frac{K_-[-ik_1(1+\lambda)]e^{-Lk_1\lambda}}{[ik_1(1+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda \\ &-\frac{iF_2K_+(\zeta)e^{-Lk_2}}{\pi\mu_2 p}\int_0^\infty \frac{K_-[ik_2(1+\lambda)]e^{-Lk_2\lambda}}{[ik_2(1+\lambda)+\zeta][\lambda(\lambda+2)]^{1/2}}d\lambda \end{aligned} \tag{49}$$

and

$$\bar{\Phi}_-^{(0)}(\zeta) = -\frac{iF_1 K_-(\zeta)e^{-Lk_1}}{\pi\mu_1 p}\int_0^\infty \frac{K_+[ik_1(1+\lambda)]e^{-Lk_1\lambda}}{[ik_1(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}\,d\lambda$$

$$+\frac{iF_2 K_-(\zeta)e^{-Lk_2}}{\pi\mu_2 p}\int_0^\infty \frac{K_+[ik_2(1+\lambda)]e^{-Lk_2\lambda}}{[ik_2(1+\lambda)-\zeta][\lambda(\lambda+2)]^{1/2}}\,d\lambda. \tag{50}$$

Due to the presence of exponentially decaying terms in the integrands the main contribution to the integrals would be from small values of λ. So approximately evaluating the integrals we obtain finally

$$\bar{\Phi}_+^{(0)}(\zeta) = \frac{F_1 K_+(\zeta)K_+(ik_1)e^{-Lk_1}}{\mu_1 p(\zeta+ik_1)(2\pi Lk_1)^{1/2}} - \frac{F_2 K_+(\zeta)K_+(ik_2)e^{-Lk_2}}{\mu_2 p(\zeta+ik_2)(2\pi Lk_2)^{1/2}}, \tag{51a}$$

$$\bar{\Phi}_-^{(0)}(\zeta) = \frac{iF_1 K_-(\zeta)K_+(ik_1)e^{-Lk_1}}{\mu_1 p(\zeta-ik_1)(2\pi Lk_1)^{1/2}} - \frac{iF_2 K_-(\zeta)K_+(ik_2)e^{-Lk_2}}{\mu_2 p(\zeta-ik_2)(2\pi Lk_2)^{1/2}}. \tag{51b}$$

The expressions for $\bar{\Phi}_+^{(0)}(\zeta)$ and $\bar{\Phi}_-^{(0)}(\zeta)$ may be recognised as the solutions corresponding to the separate problems of diffraction of semi-infinite cracks $y = 0$, $x > -L$ and $y = 0$, $x < L$ respectively because until the scattered wave emanating from a given crack tip reaches the opposite crack tip, the semi-infinite crack solution must apply.

The waves originating from concentrated line sources at $x = 0$, $y = 0+$ and $x = 0$, $y = 0-$ arrive at the crack edges at $t = s_1L$ and $t = s_2L$ respectively.

The waves arriving at one edge at time $t = s_1L$ and $t = s_2L$ respectively through the upper and lower media reach the opposite edge at times $t = 3s_1L$, $s_2L + 2s_1L$ through the upper medium and at time $t = s_1L + 2s_2L$, $3s_2L$ through the lower medium. So the first order solution $\bar{\Phi}_+^{(1)}(\zeta)$ and $\bar{\Phi}_-^{(1)}(\zeta)$ which we obtain by substituting Eqs. (51a)–(b) into the integral Eqs. (47) and (48) gives the effect of these waves and it is valid until $t = 5s_1L$ when the second scattered wave from the opposite edge first arrives. So the first order iteration becomes

$$\bar{\Phi}_+^{(1)}(\zeta) = \sum_{r=1}^{2}\frac{K_+(\zeta)\bar{\Phi}_-^{(0)}(-ik_r)K_+(ik_r)e^{-2Lk_r}}{2\mu_r(\zeta+ik_r)(\pi Lk_r)^{1/2}} - \sum_{r=1}^{2}\frac{(-1)^r F_r K_+(\zeta)K_+(ik_r)e^{-Lk_r}}{\mu_r p(\zeta+ik_r)(2\pi Lk_r)^{1/2}} \tag{52a}$$

and

$$\bar{\Phi}_-^{(1)}(\zeta) = -i\sum_{r=1}^{2}\frac{K_-(\zeta)\bar{\Phi}_+^{(0)}(ik_r)K_+(ik_r)e^{-2Lk_r}}{2\mu_r(\zeta-ik_r)(\pi Lk_r)^{1/2}} - i\sum_{r=1}^{2}\frac{(-1)^r F_r K_-(\zeta)K_+(ik_r)e^{-Lk_r}}{\mu_r p(\zeta-ik_r)(2\pi Lk_r)^{1/2}}. \tag{52b}$$

For stress intensity factor since we are interested in the singular part of the stress near the crack tips, so making $|\zeta| \to \infty$ and noting that $R_+^{1,2}(\zeta)$ tends to unity as $|\zeta| \to \infty$ we obtain

$$\bar{\Phi}_+^{(1)}(\zeta) = \left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)^{1/2}\left[\sum_{r=1}^{2}\frac{\bar{\Phi}_-^{(0)}(-ik_r)K_+(ik_r)e^{-2Lk_r}}{2\mu_r(\zeta+ik_r)^{1/2}(\pi Lk_r)^{1/2}} - \sum_{r=1}^{2}\frac{(-1)^r F_r K_+(ik_r)e^{-Lk_r}}{\mu_r p(\zeta+ik_r)^{1/2}(2\pi Lk_r)^{1/2}}\right] \tag{53a}$$

and

$$\bar{\Phi}_{-}^{(1)}(\zeta)=\left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)^{1/2}\mathrm{i}\left[-\sum_{r=1}^{2}\frac{\bar{\Phi}_{+}^{(0)}(\mathrm{i}k_r)K_+(\mathrm{i}k_r)\mathrm{e}^{-2Lk_r}}{2\mu_r(\zeta-\mathrm{i}k_r)^{1/2}(\pi Lk_r)^{1/2}}-\sum_{r=1}^{2}\frac{(-1)^r F_r K_+(\mathrm{i}k_r)\mathrm{e}^{-Lk_r}}{\mu_r p(\zeta-\mathrm{i}k_r)^{1/2}(2\pi Lk_r)^{1/2}}\right]. \tag{53b}$$

Taking inverse Fourier transform we obtain,

$$\begin{aligned}\Phi_{\pm}(x\pm L)=&\pm\left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)^2\frac{1}{2\pi(L|x\pm L|)^{1/2}}\left[-\frac{F_1}{\mu_1^2}\frac{[R_+^1(\mathrm{i}k_1)]^3}{(\pi s_1 L)^{1/2}}\frac{\mathrm{e}^{s_1p(\pm x-2L)}}{p^{3/2}}\right.\\&+\frac{F_2}{\mu_1\mu_2}\frac{R_+^1(\mathrm{i}k_1)R_+^1(\mathrm{i}k_2)R_+^2(\mathrm{i}k_1)}{(\pi s_2 L)^{1/2}}\frac{\mathrm{e}^{p(\pm s_1x-s_1L-s_2L)}}{p^{3/2}}\\&-\frac{F_1}{\mu_1\mu_2}\frac{R_+^2(\mathrm{i}k_2)R_+^2(\mathrm{i}k_1)R_+^1(\mathrm{i}k_2)}{(\pi s_1 L)^{1/2}}\frac{\mathrm{e}^{p(\pm s_2x-s_1L-s_2L)}}{p^{3/2}}\\&\left.+\frac{F_2}{\mu_2^2}\frac{[R_+^2(\mathrm{i}k_2)]^3}{(\pi s_2 L)^{1/2}}\frac{\mathrm{e}^{s_2p(\pm x-2L)}}{p^{3/2}}\right]\pm\frac{1}{\pi}\left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)\frac{1}{(L|x\pm L|)^{1/2}}\\&\times\sum_{r=1}^{2}\frac{(-1)^{r+1}F_rR_+^1(\mathrm{i}k_r)\mathrm{e}^{\pm s_rxp}}{\mu_r p}\quad\text{as } x\longrightarrow\mp(L+0).\end{aligned} \tag{54}$$

Next from Eq. (54) the normalized stress intensity factors $K_{\pm L}(t)$ where subscripts $-L$, $+L$ refer to the corresponding values at the crack tips at $x=-L$ and $x=L$ respectively have been derived.

Noting that $R_+^{1,2}(\mathrm{i}k_1)$ and $R_+^{1,2}(\mathrm{i}k_2)$ are independent of p and using shifting theorem, the inverse Laplace transform finally gives the normalised dynamic stress intensity factors as

$$\begin{aligned}|K_{\mp L}(t)|=&\left|\frac{1}{F_1}Lt_{x\to\mp L}\frac{\phi(x\pm L)}{(L|x\pm L|)^{1/2}}\right|=\left|-\frac{1}{\pi(1+m)}\left[mR_+^1(\mathrm{i}k_1)H(\tau-1)-\frac{F_2}{F_1}R_+^2(\mathrm{i}k_2)H(\tau-\gamma)\right]\right.\\&+\frac{1}{\pi^2(1+m)^2}\left[m^2[R_+^1(\mathrm{i}k_1)]^3\sqrt{\tau-3}\,H(\tau-3)\right.\\&-\frac{F_2}{F_1}mR_+^1(\mathrm{i}k_1)R_+^1(\mathrm{i}k_2)R_+^2(\mathrm{i}k_1)\sqrt{\frac{\tau}{\gamma}-\frac{2}{\gamma}-1}\,H(\tau-2-\gamma)\\&+mR_+^2(\mathrm{i}k_2)R_+^2(\mathrm{i}k_1)R_+^1(\mathrm{i}k_2)\sqrt{\tau-2\gamma-1}\,H(\tau-2\gamma-1)\\&\left.\left.-\frac{F_2}{F_1}[R_+^2(\mathrm{i}k_2)]^3\sqrt{\frac{\tau}{\gamma}-3}H(\tau-3\gamma)\right]\right|,\quad 1<\tau<5,\end{aligned} \tag{55}$$

where

$$m = \frac{\mu_2}{\mu_1}, \quad \gamma = \frac{s_2}{s_1} \text{ and } \tau = \frac{t}{s_1 L}$$

It may be noted that stress intensity factors at the both edges $|K_{+L}(t)|$ and $|K_{-L}(t)|$ are the same which is also obvious from the symmetry of the problem.

3. Results and discussions

From Eqs. (7) and (14) it is to be noted that for $Y = 0$, $x = X$ and $y = 0$ and that $\hat{\sigma}^{(j)}_{YZ}(X, 0, t) = \sigma^{j}_{yz}(x, 0, t)$.

Therefore, elastodynamic mode III stress intensity factors at the crack tips of the interface crack in an anisotropic bimaterial are the same as that of the interface crack of the corresponding isotropic bimaterial given by Eq. (55).

While carrying out numerical calculations both the cases of symmetric ($F_1 = F_2 = F$) and antisymmetric ($F_1 = -F_2 = F$) loading have been treated. For numerical evaluation of stress intensity factors at the tips of the cracks of finite width situated at the interface, the four material pairs [12], given in Table 1, have been considered.

The absolute value of the stress intensity factors defined by Eq. (55) has been plotted against $\tau(=t/(s_1 L))$ for different material pairs in Figs. 3–6 for both the symmetric and antisymmetric loading for values of τ varying from 1.0 to 5.0.

It is to be noted that in the case of antisymmetric loading, stress intensity factor increases in two steps, the first step corresponds to the first arrival of the wave at the crack tip moving along the upper face of the crack from the source and the second jump occurring because of the arrival of the wave at the crack tip due to wave moving along the lower face of the crack.

Table 1
Engineering elastic constants of different materials

Medium Name	$\hat{\rho}$ (kg m^{-3})	C_{44} (GPa)	C_{55} (GPa)	C_{45} (GPa)
Type of material pair II				
(1) Carbon-epoxy	1.57×10^3	3.98	6.4	0
(2) Graphite-epoxy	1.6×10^3	6.55	2.6	0
Type of material pair II				
(1) Isotropic chromium	7.2×10^3	115.2	115.2	0
(2) Isotropic steel	7.9×10^3	81.91	81.91	0
Type of material pair III				
(1) Isotropic aluminium	2.7×10^3	26.45	26.45	0
(2) Carbon-epoxy	1.57×10^3	3.98	6.4	0
Type of material pair IV				
(1) Copper coated stainless steel	8×10^3	91	135	0
(2) Isotropic aluminium	2.7×10^3	26.45	26.45	0

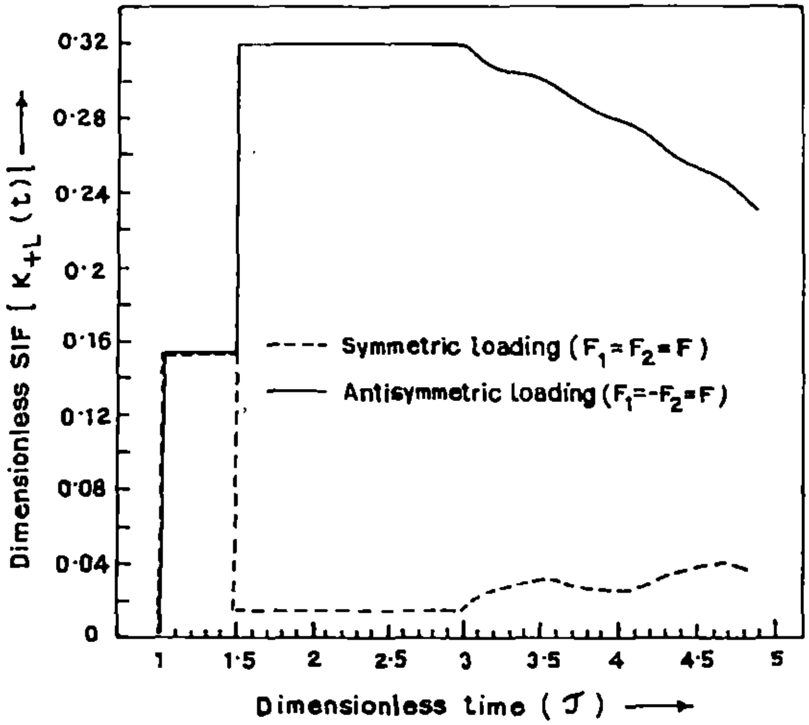

Fig. 3. Stress intensity factor versus dimensionless time for type I material pair.

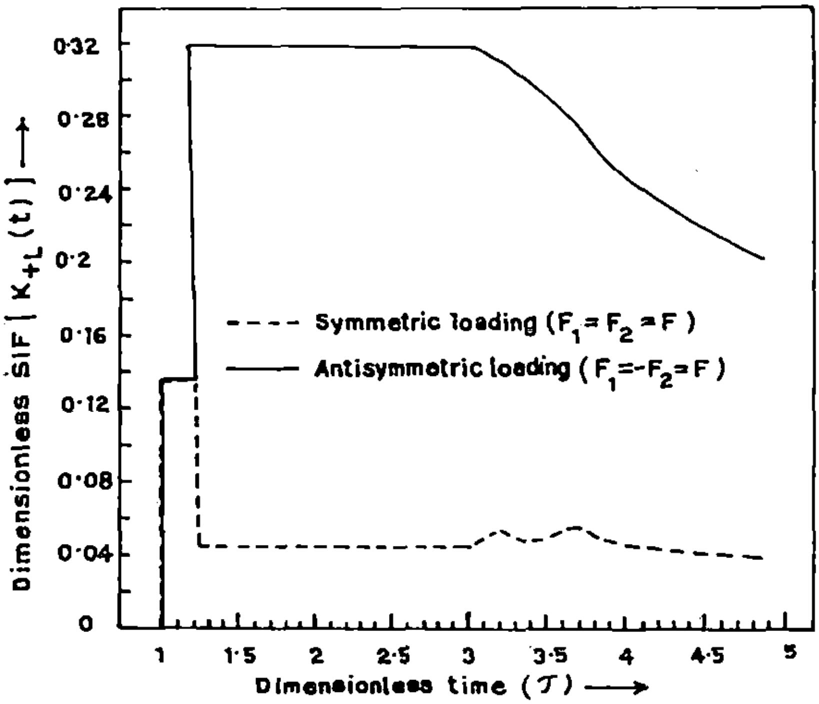

Fig. 4. Stress intensity factor versus dimensionless time for type II material pair.

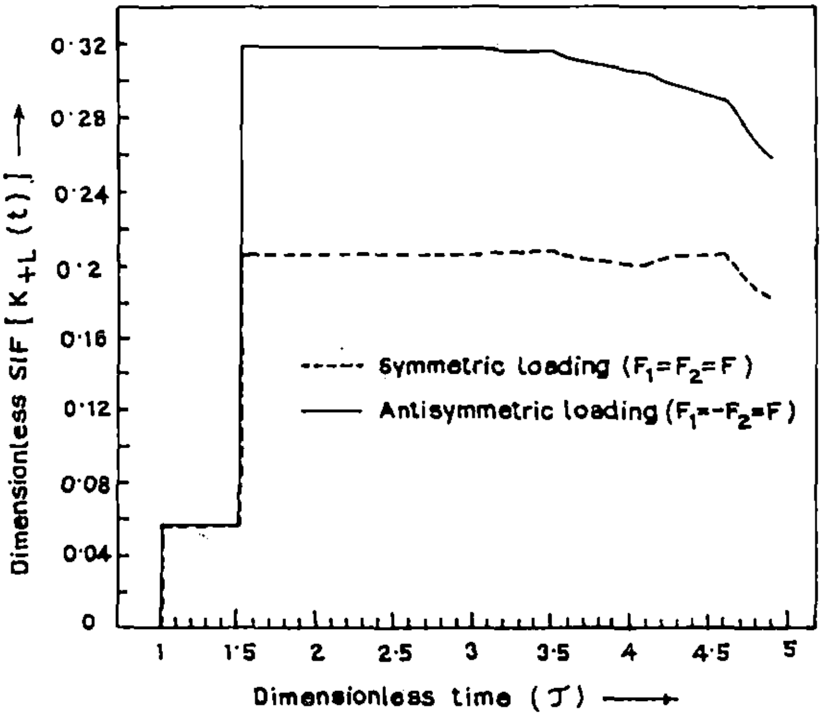

Fig. 5. Stress intensity factor versus dimensionless time for type III material pair.

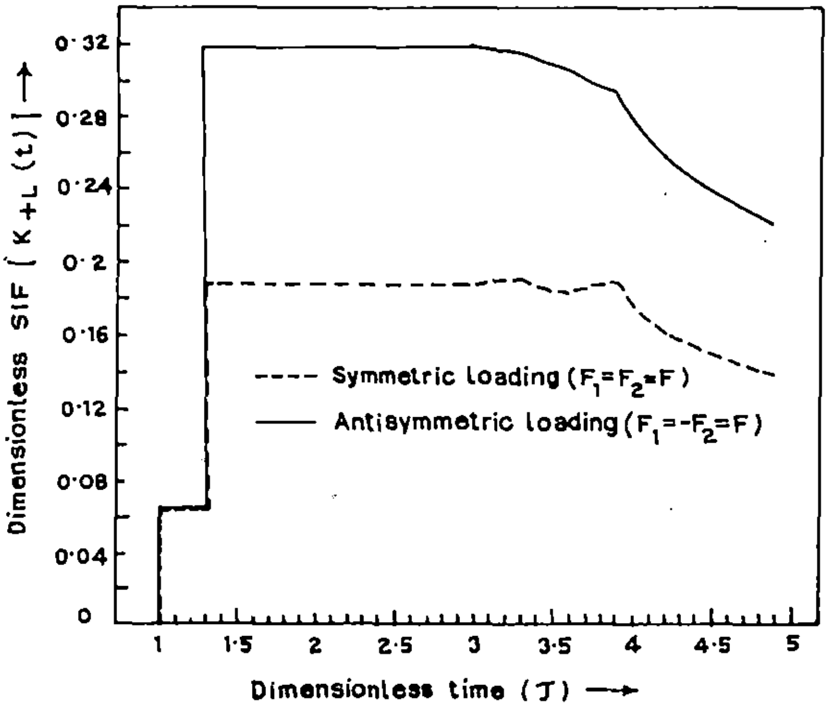

Fig. 6. Stress intensity factor versus dimensionless time for type IV material pair.

It is interesting to note that after the arrival of the first scattered wave from the opposite edge of the crack, the stress intensity factor gradually decreases in the case of antisymmetric loading.

However in the case of symmetric loading stress intensity factor at first increases when the wave moving from the source along the upper face of the crack surface reaches the crack tip but as soon as the wave from the source moving along the lower face of the crack reaches the crack tip, suddenly it decreases for type I and Type II material pairs and increases for type III and type IV material pairs until the scattered wave from the opposite crack tip arrives when the stress intensity factor shows tendency of increasing but with slow oscillations.

Acknowledgements

We take the opportunity of thanking the referee for suggesting some improvement of the paper.

Appendix A

From Eq. (40) we obtain

$$R^1(\zeta) = \frac{(\mu_1+\mu_2)(\zeta^2+k_2^2)^{1/2}}{\mu_1(\zeta^2+k_1^2)^{1/2}+\mu_2(\zeta^2+k_2^2)^{1/2}} \tag{A.1}$$

$$R^1(\zeta) = R_+^1(\zeta)R_-^1(\zeta) = \frac{1}{m/(1+m)+(1/(1+m))((\zeta^2+k_1^2)/(\zeta^2+k_2^2))^{1/2}},$$

where

$$m = \frac{\mu_2}{\mu_1}$$

Taking logarithm on both sides, one obtains

$$\log R^1(\zeta) = \log R_+^1(\zeta) + \log R_-^1(\zeta) = -\log\left[\frac{m}{1+m}+\frac{1}{1+m}\left(\frac{\zeta^2+k_1^2}{\zeta^2+k_2^2}\right)^{1/2}\right].$$

So

$$\log R_+^1(\zeta) = \frac{1}{2\pi i}\int_{-ic-\infty}^{-ic+\infty}\frac{\log R^1(z)}{z-\zeta}dz.$$

Replacing z by $-z$ and using $R^1(-z) = R^1(z)$

$$\log R^1_+(\zeta) = -\frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty}\frac{\log R^1(z)}{z+\zeta}dz = \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty}\frac{\log\left[\frac{m}{1+m}\left\{1+\frac{1}{m}\left(\frac{z^2+k_1^2}{z^2+k_2^2}\right)^{1/2}\right\}\right]}{(z+\zeta)}dz$$

$$= \frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty}\frac{\log\left[1+\frac{1}{m}\left(\frac{z^2+k_1^2}{z^2+k_2^2}\right)^{1/2}\right]}{(z+\zeta)}dz = \frac{1}{2\pi i}\int_{k_1}^{k_2}\frac{\log\left[1+i\left(\frac{u^2-k_1^2}{m^2(k_2^2-u^2)}\right)^{1/2}\right]}{(u-i\zeta)}du-$$

$$-\frac{1}{2\pi i}\int_{k_1}^{k_2}\frac{\log\left[1-i\left(\frac{u^2-k_1^2}{m^2(k_2^2-u^2)}\right)^{1/2}\right]}{(u-i\zeta)}du$$

which yields,

$$R^1_+(\zeta) = \exp\left[\frac{1}{\pi}\int_{k_1}^{k_2}\frac{\tan^{-1}[(z^2-k_1^2)/m^2(k_2^2-z^2)]^{1/2}}{(z-i\zeta)}dz\right]. \tag{A.2}$$

Similarly,

$$R^1_-(\zeta) = \exp\left[\frac{1}{\pi}\int_{k_1}^{k_2}\frac{\tan^{-1}\left[(z^2-k_1^2)/m^2(k_2^2-z^2)\right]^{1/2}}{z+i\zeta}dz\right]. \tag{A.3}$$

Similarly, it can be shown that

$$R^2_\pm(\zeta) = \exp\left[-\frac{1}{\pi}\int_{k_1}^{k_2}\frac{\tan^{-1}[m^2(k_2^2-z^2)/(z^2-k_1^2)]^{1/2}}{z\mp i\zeta}dz\right]. \tag{A.4}$$

where $R^2(\zeta)$ is given by Eq. (43).

Using Eq. (40) or Eq. (41) it can be shown that

$$K_\pm(\zeta) = \left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)^{1/2}(\zeta\pm ik_1)^{1/2}R^1_\pm(\zeta) \tag{A.5}$$

$$= \left(\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right)^{1/2}(\zeta\pm ik_2)^{1/2}R^2_\pm(\zeta) \tag{A.6}$$

From either Eq. (A.5) or Eq. (A.6) it can be easily shown that

$$K_-(-\zeta) = -\mathrm{i}K_+(\zeta).$$

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SHEAR WAVE INTERACTION WITH A PAIR OF RIGID STRIPS EMBEDDED IN AN INFINITELY LONG ELASTIC STRIP

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In this paper, the problem of diffraction of normally incident SH wave by two co-planar finite rigid strips placed symmetrically in an infinitely long isotropic elastic strip perpendicular to the lateral surface of the elastic strip has been treated. The mixed boundary value problem gives rise to the determination of the solution of triple integral equations which finally have been reduced to the solution of a Fredholm integral equation of second kind. The equation has been solved numerically for low frequency range. Finally the elastodynamic stress intensity factors are obtained. The variations of the stress intensity factors at the tips of the rigid strips with frequency have been depicted by means of graphs.

1. Introduction

In recent years great interest has been developed in studying elastic wave interaction with singularities in the form of cracks or inclusion located in an elastic medium, in view of their application in engineering fracture mechanics and geophysics. Most of the attempts have been based on the assumption that the crack or the inclusion is situated sufficiently far from the neighbouring boundaries. Mathematically, this type of problem reduces to the study of the elastic field due to the presence of cracks or inclusions in an infinite elastic medium. A detailed reference of work done on the determination of the dynamic stress field around a crack or an inclusion in an infinite elastic solid has been given by SIH [1]. However in the presence of finite boundaries, the problem becomes complicated since they involve additional geometric parameters, describing the dimension of the solids. Papers involving a crack or a rigid strip in an infinitely long elastic strip are very few. The problem of an infinite elastic strip containing an arbitrary number of Griffith cracks of unequal size, located parallel to its surfaces and opened by an arbitrary internal pressure, has been treated by ADAM [2]. Finite crack perpendicular to the surface of the infinitely long elastic strip has been studied by CHEN [3] for impact load, and by SRIVASTAVA *et al.* [4] for normally incident waves. Recently SHINDO *et al.* [5]

considered the problem of impact response of a finite crack in an orthtropic strip. ITOU [6] also studied the response of a central crack in a finite strip under inplane compression impact.

But these solutions were limited to the problems involving a single crack or a finite rigid strip embedded in an elastic strip because of severe mathematical complexity involved in finding solutions for two or more cracks or inclusions. Recently SRIVASTAVA *et al.* [7] considered the problem of interaction of shear waves with two co-planar Griffith cracks situated in an infinitely long elastic strip. TAI and LI [8] also derived the elastodynamic response of a finite strip with two co-planar cracks under impact loading. The solution of the mixed boundary value problem was expressed in terms of two Cauchy-type singular integral equations which were solved numerically, following a collocation scheme due to ERDOGAN and GUPTA [9]. A numerical Laplace transform inversion technique described by MILLER and GUY [10] are then used to obtain the solution.

In our paper, we have considered the diffraction of normally incident SH wave by two co-planar finite rigid strips situated in an infinitely long isotropic elastic strip perpendicular to the lateral surface. The mixed boundary value problem gives rise to the determination of the solution of triple integral equations which finally have been reduced to the solution of a Fredholm integral equation of second kind. The equation has been solved numerically for low frequency range. Finally the elastodynamic stress intensity factors are obtained. The variations of the stress intensity factors at the tips of the rigid strips with variable frequency have been depicted by means of graphs.

2. Formulation of the Problem

Consider an infinitely long homogeneous isotropic elastic strip of width 2H containing two coplanar rigid strips embedded in it. Consider a rectangular Cartesian coordinate system (X, Y, Z) with origin at the centre of the elastic strip, such that the rigid strips occupy the region $-b \leqslant X \leqslant -a$; $a \leqslant X \leqslant b$, $|Y| < \infty$, $Z = 0$. A time-harmonic antiplane shear wave is assumed to be incident normally on the rigid strips.

Since the non-vanishing component of displacement is only the component V, all stress components except σ_{YZ} and σ_{XY} vanish identically. Thus the problem is to find the stress distribution near the edges of strips subject to the following boundary conditions:

$$\begin{aligned} V(X,0+) &= V(X,0-) = -V_0 e^{-i\omega t}; \qquad a \leqslant |X| \leqslant b, \\ \sigma_{YZ}(X,0+) &= \sigma_{YZ}(X,0-) = 0; \qquad |X| > b, \qquad |X| < a, \end{aligned} \tag{2.1}$$

and

$$\sigma_{XY}(\pm h, Z) = 0.$$

The displacement V satisfies the wave equation

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} = \frac{1}{C_2^2}\frac{\partial^2 V}{\partial t^2}, \tag{2.2}$$

C_2 being shear wave velocity. It is convenient to normalize all lengths with respect to b so that

$$\frac{X}{b} = x, \quad \frac{Y}{b} = y, \quad \frac{Z}{b} = z, \quad \frac{V}{b} = v, \quad \frac{V_0}{b} = v_0, \quad \frac{a}{b} = c, \quad \frac{H}{b} = h.$$

Therefore the strips are defined by $-1 \leqslant x \leqslant -c$, $c \leqslant x \leqslant 1$, $|y| < \infty$, $z = 0$ (Fig. 1). Suppressing the time factor $e^{-i\omega t}$, the boundary conditions reduce to

$$\begin{aligned} v(x,0+) &= v(x,0-) = -v_0; \qquad c \leqslant |x| \leqslant 1, \\ \sigma_{yz}(x,0+) &= \sigma_{yz}(x,0-) = 0; \qquad |x| > 1, \qquad |x| < c, \end{aligned} \tag{2.3}$$

and $\sigma_{xy}(\pm h, z) = 0$.

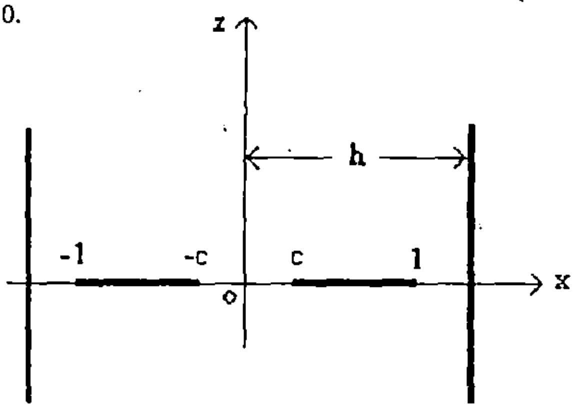

FIG. 1. Geometry of the problem.

The scattered field v subject to the above boundary conditions should be a solution of the equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + k_2^2 v = 0 \tag{2.4}$$

where $k_2^2 = \dfrac{\omega^2 b^2}{c_2^2}$.

The solution of Eq. (2.4) can be taken as

$$v(x,z) = \int_0^\infty A(\xi) e^{-\beta|z|} \cos(\xi x) d\xi + \int_0^\infty B(\zeta) \cos h(\beta_1 x) \cos(\zeta z) d\zeta \tag{2.5}$$

so that

$$\sigma_{yz}(x,z) = \mu \left[-\operatorname{sgn}(z) \int_0^\infty \beta A(\xi) e^{-\beta|z|} \cos(\xi x) d\xi - \int_0^\infty \zeta B(\zeta) \cosh(\beta_1 x) \sin(\zeta z) d\zeta \right], \tag{2.6}$$

where

$$\beta = \begin{cases} (\xi^2 - k_2^2)^{1/2}; & \xi > k_2, \\ -i(k_2^2 - \xi^2)^{1/2}; & \xi < k_2, \end{cases}$$

and

$$\beta_1 = \begin{cases} (\zeta^2 - k_2^2)^{1/2}; & \zeta > k_2, \\ -i(k_2^2 - \zeta^2)^{1/2}; & \zeta < k_2, \end{cases}$$

so that $\beta_1 = -i(k_2^2 - \zeta^2)^{1/2} = -i\beta_1'$ where $\zeta < k_2$.

3. Derivation of Integral Equation

The condition of vanishing of σ_{yz} at $z = 0$ outside the strips yields

$$\int_0^\infty \beta A(\xi)\cos(\xi x)d\xi = 0; \qquad |x| < c, \qquad |x| > 1. \tag{3.1}$$

Again the boundary condition $v(x,0) = -v_0$ at $c \leqslant |x| \leqslant 1$ gives

$$\int_0^\infty A(\xi)\cos(\xi x)d\xi + \int_0^\infty B(\zeta)\cosh(\beta_1 x)d\zeta = -v_0; \qquad c \leqslant |x| \leqslant 1. \tag{3.2}$$

Using the boundary condition $\sigma_{xy}(\pm h, z) = 0$ one obtains

$$\int_0^\infty \beta_1 B(\zeta)\sinh(\beta_1 h)\cos(\zeta z)d\zeta = \int_0^\infty \xi A(\xi)e^{-\beta|z|}\sin(\xi h)d\xi$$

which after Fourier cosine inversion yields

$$\beta_1 B(\zeta)\sinh(\beta_1 h) = \frac{2}{\pi}\int_0^\infty \frac{\xi\beta}{\beta^2 + \zeta^2}A(\xi)\sin(\xi h)d\xi. \tag{3.3}$$

Eliminating $B(\zeta))$ from equations (3.2) and (3.3) one obtains

$$\int_0^\infty A(\xi)\cos(\xi x)d\xi = -v_0 - \frac{2}{\pi}\int_0^\infty \frac{\cosh(\beta_1 x)}{\beta_1\sinh(\beta_1 h)}d\zeta \times \int_0^\infty \frac{\xi\beta}{\beta^2+\zeta^2}A(\xi)\sin(\xi h)d\xi; \qquad c \leqslant |x| \leqslant 1. \tag{3.4}$$

Replacing $\beta A(\xi)$ by $C(\xi)$, Eqs. (3.1) and (3.4) become

$$\int_0^\infty C(\xi)\cos(\xi x)d\xi = 0; \qquad |x| < c; \quad |x| > 1 \tag{3.5}$$

and

$$\int_0^\infty \xi^{-1}\left[1+H(\xi)\right]C(\xi)\cos(\xi x)d\xi = -v_0 - \frac{2}{\pi}\int_0^\infty \frac{\cosh(\beta_1 x)}{\beta_1\sinh(\beta_1 h)}d\zeta \times \int_0^\infty \frac{\xi C(\xi)}{\beta^2+\zeta^2}\sin(\xi h)d\xi; \qquad c\leqslant |x|\leqslant 1, \tag{3.6}$$

where

$$H(\xi) = \left\{\frac{\xi}{\beta}-1\right\}\to 0 \quad \text{as} \quad |\xi|\to\infty. \tag{3.7}$$

In order to solve the integral Eqs. (3.5) and (3.6) we set

$$C(\xi) = \int_c^1 \frac{h(t^2)}{t}\left\{1-\cos(\xi t)\right\}dt \tag{3.8}$$

where the unknown function $h(t^2)$ is to be determined.

Substituting $C(\xi)$ from (3.8) in equation (3.5) we note that

$$\int_0^\infty C(\xi)\cos(\xi x)dx = \pi\int_c^1 \frac{h(t^2)}{t}\left[\delta(x)-\frac{1}{2}\delta(x+t)-\frac{1}{2}\delta(|x-t|)\right]dt$$

so that Eq. (3.5) is automatically satisfied.

Again, the substitution of the value of $C(\xi)$ from (3.8) in equation (3.6) yields

$$\frac{1}{2}\int_c^1 \frac{h(t^2)}{t}\log\left|\frac{x^2-t^2}{x^2}\right|dt = -v_0 - \int_c^1 \frac{h(t^2)}{t}dt \tag{3.9}$$

$$\times\left[\int_{k_2}^\infty \frac{\cosh(\beta_1 x)e^{-h\beta_1}}{\beta_1\sinh(\beta_1 h)}\left\{1-\cosh(t\beta_1)\right\}d\zeta - \int_0^{k_2}\frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\beta_1'\sin(\beta_1' h)}\left\{1-\cos(\beta_1' t)\right\}d\zeta\right]$$

$$-\int_c^1 \frac{h(t^2)}{t}dt\int_0^\infty \xi^{-1}H(\xi)\cos(\xi x)\left\{1-\cos(\xi t)\right\}d\xi, \qquad c\leqslant|x|\leqslant 1$$

where the result

$$\int_0^\infty \frac{\cos(\xi x)\{1-\cos(\xi t)\}}{\xi}d\xi = \log\left|\frac{x^2-t^2}{x^2}\right|$$

has been used.

Differentiating both sides of Eq. (3.9) with respect to x and next multiplying by $(-2x/\pi)$, one obtains

$$\frac{2}{\pi}\int_c^1 \frac{th(t^2)}{(t^2-x^2)}dt = \frac{2x}{\pi}\int_c^1 \frac{h(t^2)}{t}dt \left[\int_{k_2}^{\infty} \frac{\sinh(\beta_1 x)e^{-h\beta_1}}{\sinh(\beta_1 h)}\{1-\cosh(t\beta_1)\}\,d\zeta \right. \tag{3.10}$$

$$+\int_0^{k_2} \frac{\sin(\beta_1' x)\cos(\beta_1' h)}{\sin(\beta_1' h)}\{1-\cos(\beta_1' t)\}\,d\zeta$$

$$\left. -\int_0^{\infty} H(\xi)\sin(\xi x)\{1-\cos(\xi t)\}\,d\xi \right]; \qquad c \leqslant |x| \leqslant 1.$$

It is known that using Hilbert transform technique, the solution of the integral equation (Srivastava and Lowengrue [11])

$$\frac{2}{\pi}\int_a^b \frac{th(t^2)}{(t^2-y^2)}dt = R(y), \qquad a<y<b$$

can be obtained in the form

$$h(t^2) = -\frac{2}{\pi}\left(\frac{t^2-a^2}{b^2-t^2}\right)^{1/2}\int_a^b \left(\frac{b^2-y^2}{y^2-a^2}\right)^{1/2}\frac{yR(y)}{y^2-t^2}dy \tag{3.11}$$

$$+\frac{D}{(t^2-a^2)^{1/2}(b^2-t^2)^{1/2}}$$

with condition that R must be an even function of y so as to make integral convergent. D is an arbitrary constant.

Following (3.11), the solution of Eq. (3.10) is given by

$$h(u^2) + \int_c^1 \frac{h(t^2)}{t}\{K_1(u^2,t^2)+K_2(u^2,t^2)\}\,dt = \frac{D}{(u^2-c^2)^{1/2}(1-u^2)^{1/2}} \tag{3.12}$$

where

$$K_1(u^2,t^2) = -\frac{4}{\pi^2}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2}\int_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2}\frac{x^2dx}{x^2-u^2} \tag{3.13}$$

$$\times\left[\int_{k_2}^{\infty} \frac{\sinh(\beta_1 x)e^{-h\beta_1}}{\sinh(\beta_1 h)}\{1-\cos(t\beta_1)\}\,d\zeta + \int_0^{k_2} \frac{\sin(\beta_1' x)\cos(\beta_1' h)}{\sin(\beta_1' h)}\{1-\cos(\beta_1' t)\}\,d\zeta\right]$$

and

$$(3.14)\qquad K_2(u^2,t^2) = +\frac{4}{\pi^2}\left(\frac{u^2-c^2}{1-u^2}\right)^{1/2}\int_c^1\left(\frac{1-x^2}{x^2-c^2}\right)^{1/2}\frac{x^2dx}{x^2-u^2}$$
$$\times\int_0^\infty H(\xi)\sin(\xi x)\left\{1-\cos(\xi t)\right\}d\xi.$$

In order to determine the arbitrary constant D, Eq. (3.9) is multiplied by $\dfrac{x}{(x^2-c^2)^{1/2}(1-x^2)^{1/2}}$ and integrated from c to 1 with respect to x, and using the result

$$\int_c^1 \frac{x\log|1-t^2/x^2|}{(x^2-c^2)^{1/2}(1-x^2)^{1/2}}dx = \frac{\pi}{2}\log\left|\frac{1-c}{1+c}\right|$$

we finally obtain

$$(3.15)\qquad \int_c^1\frac{h(u^2)}{u}du = -\frac{2v_0}{\log\left|\dfrac{1-c}{1+c}\right|} - \frac{4}{\pi\log\left|\dfrac{1-c}{1+c}\right|}\int_c^1\frac{h(t^2)}{t}dt$$
$$\times\int_c^1\frac{x}{(x^2-c^2)^{1/2}(1-x^2)^{1/2}}\left[A_1(x,t^2)+A_2(x,t^2)\right]dx$$

where

$$(3.16)\qquad A_1(x,t^2) = \int_{k_2}^\infty \frac{\cos h(\beta_1 x)e^{-h\beta_1}}{\beta_1\sinh(\beta_1 h)}\left\{1-\cosh(t\beta_1)\right\}d\zeta$$
$$-\int_0^{k_2}\frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\beta_1'\sin(\beta_1' h)}\left\{1-\cos(\beta_1' t)\right\}d\zeta,$$

$$(3.17)\qquad A_2(x,t^2) = \int_0^\infty \xi^{-1}H(\xi)\cos(\xi x)\left\{1-\cos(\xi t)\right\}d\xi$$

$$= \frac{1}{2}\log\left|\frac{x^2-t^2}{x^2}\right| - \frac{\pi i}{2}H_0^{(1)}(xk_2) + \frac{\pi i}{4}H_0^{(1)}\left\{(x+t)k_2\right\} + \frac{\pi i}{4}H_0^{(1)}\left\{|x-t|k_2\right\}.$$

Again, by substituting $h(u^2)$ from Eq. (3.12) in the left-hand side of equation (3.15) and simplifying, one obtains

(3.18)
$$D = -\frac{4v_0 c}{\pi \log\left|\frac{1-c}{1+c}\right|} - \frac{8c}{\pi^2 \log\left|\frac{1-c}{1+c}\right|} \int\limits_c^1 \frac{h(t^2)}{t} dt$$
$$\times \int\limits_c^1 \frac{x}{(x^2-c^2)^{1/2}(1-x^2)^{1/2}} \left[A_1(x,t^2) + A_2(x,t^2)\right] dx$$
$$+ \frac{2c}{\pi} \int\limits_c^1 \frac{h(t^2)}{t} dt \int\limits_c^1 \frac{1}{u} \left\{K_1(u^2,t^2) + K_2(u^2,t^2)\right\} du.$$

Eliminating D from Eqs. (3.12) and (3.18) and simplifying one obtains

(3.19)
$$\left[(u^2-c^2)(1-u^2)\right]^{1/2} h(u^2) + \int\limits_c^1 \frac{h(t^2)}{t}$$
$$+ \left[K_a(u^2,t^2) + K_b(u^2,t^2) + K_c(u^2,t^2)\right] dt = -\frac{4v_0 c}{\pi \log\left|\frac{1-c}{1+c}\right|}$$

where

(3.20)
$$K_a(u^2,t^2) = -\frac{4}{\pi^2}(u^2-c^2) \int\limits_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2} \frac{x^2}{x^2-u^2} dx$$
$$\times \left[\frac{\partial}{\partial x}\left\{A_1(x,t^2) + A_2(x,t^2)\right\}\right],$$

(3.21)
$$K_b(u^2,t^2) = \frac{8c}{\pi^2 \log\left|\frac{1-c}{1+c}\right|} \int\limits_c^1 \frac{x\,dx}{(x^2-c^2)^{1/2}(1-x^2)^{1/2}}$$
$$\times \left\{A_1(x,t^2) + A_2(x,t^2)\right\},$$

(3.22)
$$K_c(u^2,t^2) = -\frac{4c^2}{\pi^2} \int\limits_c^1 \left(\frac{1-x^2}{x^2-c^2}\right)^{1/2}$$
$$\times \left[\frac{\partial}{\partial x}\left\{A_1(x,t^2) + A_2(x,t^2)\right\}\right] dx.$$

Next for further simplification we put

$$\left[(u^2-c^2)(1-u^2)\right]^{1/2} h(u^2) = H(u^2)$$

and make the substitution

$$u^2 = c^2\cos^2\phi + \sin^2\phi \qquad \text{and} \qquad t^2 = c^2\cos^2\theta + \sin^2\theta$$

in Eq. (3.19) which then reduces to the form

$$G(\phi) + \int_0^{\pi/2} \frac{G(\theta)}{c^2\cos^2\theta + \sin^2\theta}\left[K_a'(\phi,\theta) + K_b'(\phi,\theta) + K_c'(\phi,\theta)\right] d\theta = -\frac{4v_0 c}{\pi \log\left|\dfrac{1-c}{1+c}\right|} \tag{3.23}$$

where

$$G(\phi) = H(c^2\cos^2\phi + \sin^2\phi), \tag{3.24}$$

$$G(\theta) = H(c^2\cos^2\theta + \sin^2\theta), \tag{3.25}$$

$$K_a'(\phi,\theta) = K_a(c^2\cos^2\phi + \sin^2\phi, c^2\cos^2\theta + \sin^2\theta), \tag{3.26}$$

$$K_b'(\phi,\theta) = K_b(c^2\cos^2\phi + \sin^2\phi, c^2\cos^2\theta + \sin^2\theta), \tag{3.27}$$

$$K_c'(\phi,\theta) = K_c(c^2\cos^2\phi + \sin^2\phi, c^2\cos^2\theta + \sin^2\theta). \tag{3.28}$$

4. Stress Intensity Factor

From equation (2.6) for $z \to 0$, $c \leqslant |x| < 1$, one obtains

$$\sigma_{yz}(x, 0\pm) = \mp\mu \int_0^\infty \beta A(\xi)\cos(\xi x) d\xi.$$

It is useful to determine the difference of the stress components on the lower and upper surfaces of the strips. We put

$$\Delta\sigma_{yz}(x,0) = \sigma_{yz}(x,0+) - \sigma_{yz}(x,0-);$$

then

$$\Delta\sigma_{yz}(x,0) = -2\mu \int_0^\infty C(\xi)\cos(\xi x) d\xi, \qquad c < |x| < 1.$$

Substituting the value of $C(\xi)$ and next changing the order of integration and integrating, one obtains

$$\Delta\sigma_{yz}(x,0) = \frac{\mu\pi h(x^2)}{x}. \tag{4.1}$$

Since

$$h(x^2) = \left[(x^2-c^2)(1-x^2)\right]^{-1/2} H(x^2)$$

and

$$x^2 = c^2\cos^2\phi + \sin^2\phi,$$

and hence Eq. (4.1) becomes

$$\Delta\sigma_{yz}(x,0) = \frac{\mu\pi G(\phi)}{x\left[(x^2-c^2)(1-x^2)\right]^{1/2}}. \tag{4.2}$$

So the stress intensity factors N_c and N_1 at the two tips of the strip can be expressed as

$$N_c = \underset{x\to c+}{Lt}\left[\frac{\Delta\sigma_{yz}(x,0)}{\mu\pi}(x-c)^{1/2}\right] \tag{4.3}$$

and

$$N_1 = \underset{x\to 1-}{Lt}\left[\frac{\Delta\sigma_{yz}(x,0)}{\mu\pi}(1-x)^{1/2}\right]. \tag{4.4}$$

With the aid of Eq. (4.2) one obtains

$$N_c = \frac{G(0)}{c\sqrt{2c(1-c^2)}} \Rightarrow G(0) = c\sqrt{2c(1-c^2)}N_c \tag{4.5}$$

and

$$N_1 = \frac{G(\pi/2)}{\sqrt{2(1-c^2)}} \Rightarrow G(\pi/2) = \sqrt{2(1-c^2)}N_1. \tag{4.6}$$

Making c tend to zero, the two strips merge into one and in that case

$$N_1 = \frac{1}{\sqrt{2}}G(\pi/2).$$

5. Results and Discussions

The numerical calculations have been carried out for the determination of stress intensity factors for different values of the dimensionless frequency k_2 within the range 0.1 to 0.8. The integrals $A_1(x,t^2)$ and $A_2(x,t^2)$ given by (3.16) and (3.17), respectively, appearing in the kernel of integral Eq. (3.23) have been evaluated using the Gauss quadrature formula. Following Fox and Goodwin [12], the solution of integral equation (3.23) has been obtained by converting it into a system of linear algebraic equations. Substituting these values of $[G(\phi)]$ in equations (4.3) and (4.4), the stress intensity factors N_c and N_1 at the inner and outer tips, respectively, of the rigid strips have been found to be related with $G(0)$ and $G(\pi/2)$ through the relations (4.5) and (4.6). The

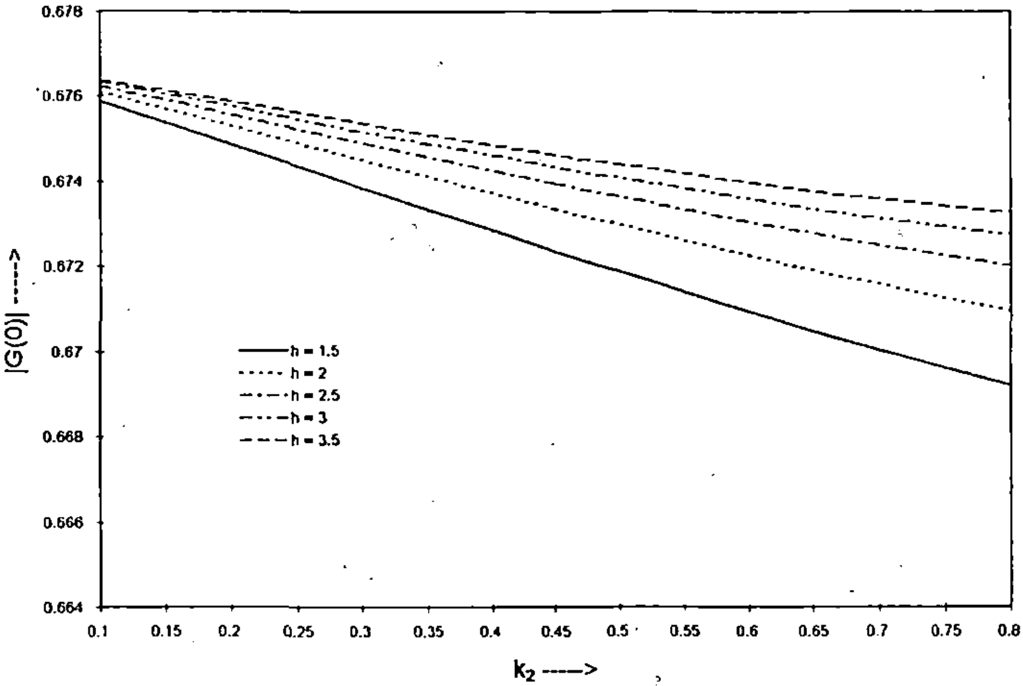

FIG. 2. Amplitude of $|G(0)|$ plotted against dimensionless frequency k_2 for $c = 0.2$.

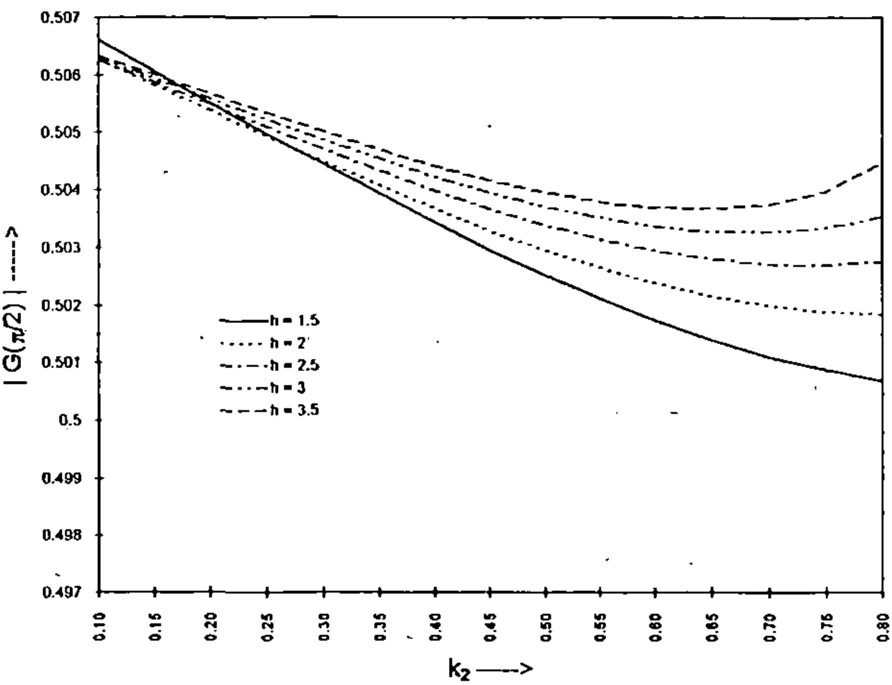

FIG. 3. Amplitude of $|G(\pi/2)|$ plotted against dimensionless frequency k_2 for $c = 0.2$.

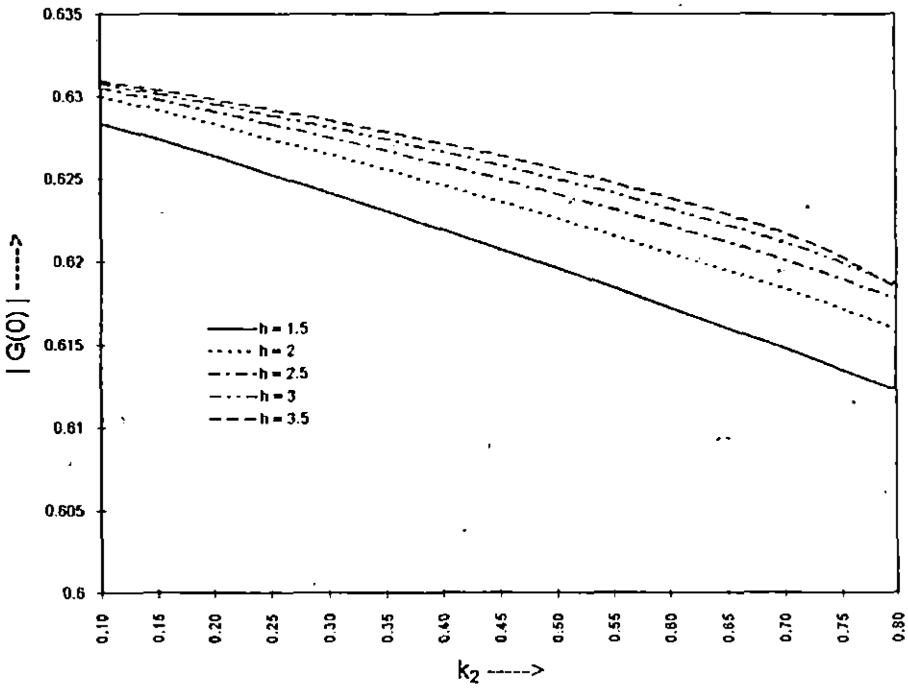

FIG. 4. Amplitude of $|G(0)|$ plotted against dimensionless frequency k_2 for $c = 0.4$.

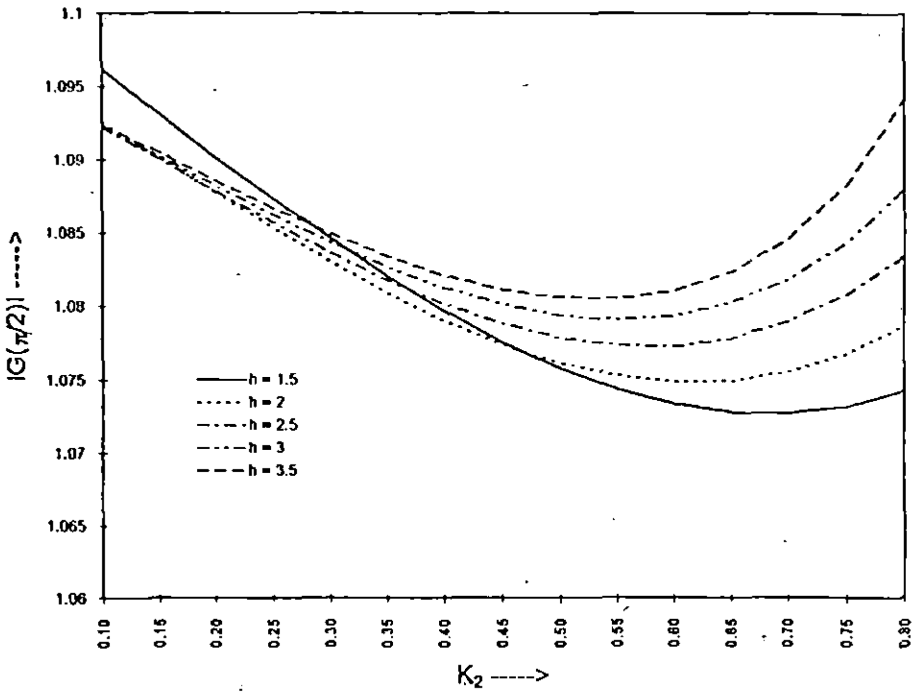

FIG. 5. Amplitude of $|G(\pi/2)|$ plotted against dimensionless frequency k_2 for $c = 0.4$.

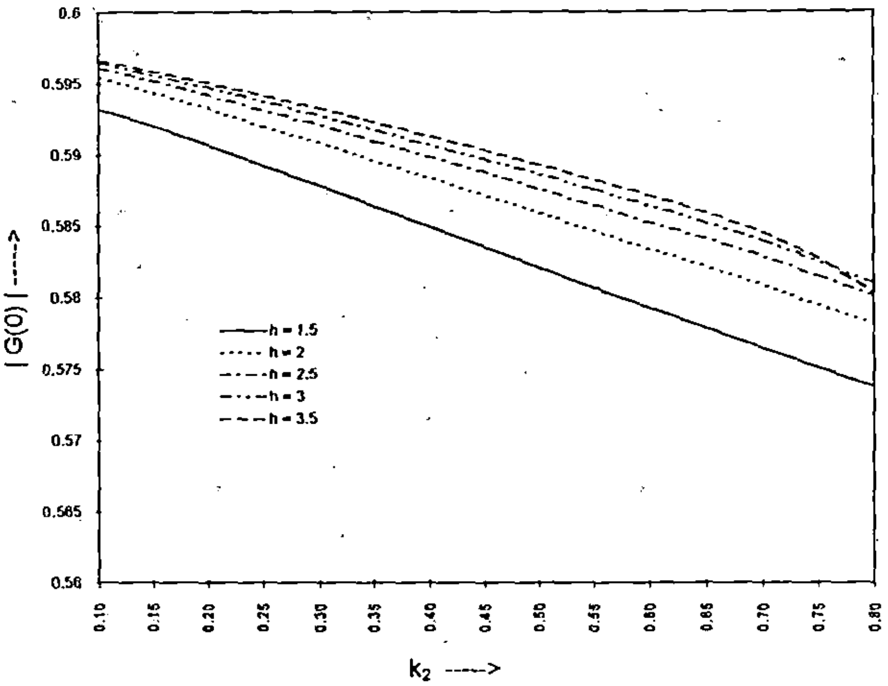

FIG. 6. Amplitude of $|G(0)|$ plotted against dimensionless frequency k_2 for $c = 0.6$.

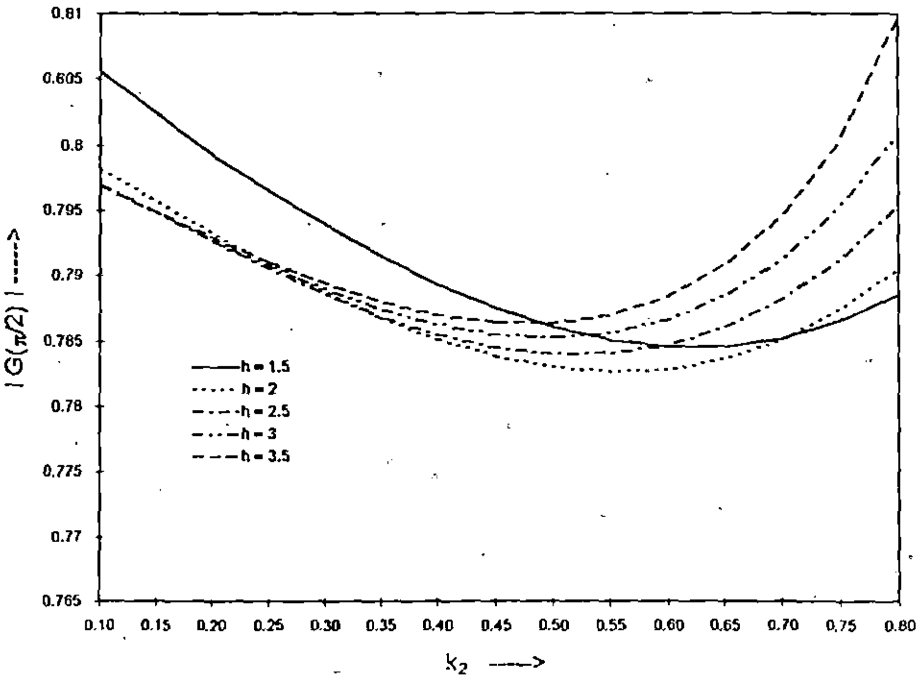

FIG. 7. Amplitude of $|G(\pi/2)|$ plotted against dimensionless frequency k_2 for $c = 0.6$.

amplitudes $|G(0)|$ and $|G(\pi/2)|$ have been plotted against k_2 with different values of h for $c = 0.2, 0.4, 0.6$; the values chosen for k_2 range from 0.1 to 0.8, at step of 0.05.

From the graphs it can be concluded that for fixed values of h, the stress intensity factor near the inner tip of the rigid strip decreases with the increase in the values of frequency within the range 0.1 to 0.8 (Figs. 2, 4 and 6), and for fixed values of h the stress intensity factor near the outer tip of the rigid strip at first decreases, attains a minimum and then it gradually increases with the increase in the values of frequency within the range 0.1 to 0.8 (Figs. 3, 5 and 7) for different values of c ($c = 0.2$, 0.4 and 0.6).

It is interesting to note that for different values of k_2 within the range 0.1 to 0.8, the stress intensity factor of the inner tip of the strips, for a given value of k_2, increases with the increase in the values of h, whereas the stress intensity factor at the outer tip of the strips, within the given range of values of k_2, decreases with the increase in the values of h for small values of k_2 but shows the reverse character for higher values of k_2 for any given value of the parameter c.

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