CHAPTER - 2

INCLUSION PROBLEM

[4]SHEAR WAVE INTERACTION WITH A PAIR OF RIGID STRIPS EMBEDDED IN AN INFINITELY LONG ELASTIC STRIP

1. INTRODUCTION

In recent years great interest has been developed in studying elastic wave interaction with singularities in the form of cracks or inclusion located in an elastic medium, in view of their application in engineering fracture mechanics and geophysics. Most of the attempts have been based on the assumption that the crack or the inclusion is situated sufficiently far from the neighbouring boundaries. Mathematically, this type of problem reduces to the study of the elastic field due to the presence of cracks or inclusions in an infinite elastic medium. A detailed reference of work done on the determination of the dynamic stress field around a crack or an inclusion in an infinite elastic solid has been given by Sih [1997]. However in the presence of finite boundaries, the problem becomes complicated since they involve additional geometric parameters, describing the dimension of the solids. Papers involving a crack or a rigid strip in an infinitely long elastic strip are very few. The problem of an infinite elastic strip containing an arbitrary number of Griffith cracks of unequal size, located parallel to its surfaces and opened by an arbitrary internal pressure, has been treated by Adam [1980]. Finite crack perpendicular to the surface of the infinitely long elastic strip has been studied by Chen [1978] for impact load, and by Srivastava et al [1981] for normally incident waves. Recently Shindo et al [1986] considered the problem of impact response of a finite crack in an orthotropic strip. Itou [1980] also studied the response of a central crack in a finite strip under inplane compression impact.

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But these solutions were limited to the problems involving a single crack or a finite rigid strip embedded in an elastic strip because of severe mathematical complexity involved in finding solutions for two or more cracks or inclusions. Recently Srivastava et al [1983] considered the problem of interaction of shear waves with two co-planar Griffith cracks situated in an infinitely long elastic strip. Tai and Li [1987] also derived the elastodynamic response of a finite strip with two co-planar cracks under impact loading. The solution of the mixed boundary value problem was expressed in terms of two Cauchy-type singular integral equations which were solved numerically, following a collocation scheme due to Erdogan and Gupta [1972]. A numerical Laplace transform inversion technique described by Miller and Guy [1966] are then used to obtain the solution.

In our paper, we have considered the diffraction of normally incident SH-wave by two co-planar finite rigid strips situated in an infinitely long isotropic elastic strip perpendicular to the lateral surface. The mixed boundary value problem gives rise to the determination of the solution of triple integral equations which finally have been reduced to the solution of a Fredholm integral equation of second kind. The equation has been solved numerically for low frequency range. Finally the elastodynamic stress intensity factors are obtained. The variations of the stress intensity factors at the tips of the rigid strips with variable frequency have been depicted by means of graphs.

2. FORMULATION OF THE PROBLEM

Consider an infinite long homogeneous isotropic elastic strip of width 2H containing two coplanar rigid strips embedded in it. Consider a rectangular Cartesian co-ordinate system (X,Y,Z) with origin at the centre of the elastic strip, such that the rigid strips occupy the region $-b \le X \le -a$; $a \le X \le b$, $|Y| < \infty$, $Z=0$. A time-harmonic antiplane shear wave is assumed to be incident normally on

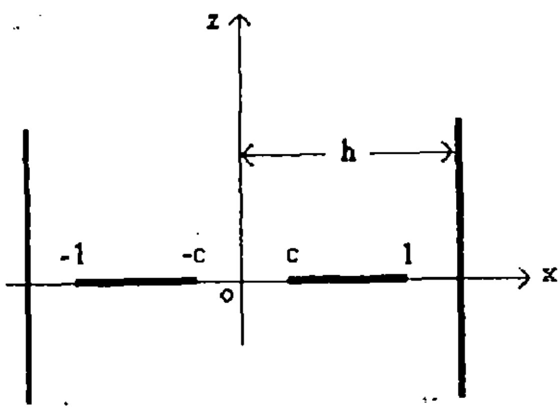

Fig.1. Geometry of the problem.

the rigid strips.

Since the non-vanishing component of displacement is only the component V, all stress components except σ_{YZ} and σ_{XY} vanish identically. Thus the problem is to find the stress distribution near the edge of strips subject to the following boundary conditions :

$$V(X,0+) = V(X,0-) = -V_0 e^{-i\omega t}; \qquad a \le |X| \le b,$$

$$\sigma_{YZ}(X,0+) = \sigma_{YZ}(X,0-) = 0; \qquad |X|>b, \quad |X|<a,$$

and $\sigma_{XY}(\pm h, Z) = 0.$ (2.1)

The displacement V satisfies the wave equation

$$\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} = \frac{1}{C_2^2}\frac{\partial^2 V}{\partial t^2}, \tag{2.2}$$

C_2 being shear wave velocity. It is convenient to normalize all lengths with respect to b so that

$$\frac{X}{b} = x, \quad \frac{Y}{b} = y, \quad \frac{Z}{b} = z, \quad \frac{V}{b} = v, \quad \frac{V_0}{b} = v_0, \quad \frac{a}{b} = c, \quad \frac{H}{b} = h.$$

Therefore the strips are defined by $-1 \le x \le -c$, $c \le x \le 1$, $|y|<\infty$. $z=0$ (Fig.1). Supressing the time factor $e^{-i\omega t}$, the boundary conditions reduce to

$$v(x,0+) = v(x,0-) = -v_0; \qquad c \le |x| \le 1,$$

$$\sigma_{yz}(x,0+) = \sigma_{yz}(x,0-) = 0; \qquad |x|>1, \quad |x|<c, \tag{2.3}$$

and $\sigma_{xy}(\pm h, z) = 0.$

The scattered field v subject to the above boundary conditions should be a solution of the equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + k_2^2 v = 0 \tag{2.4}$$

where $k_2^2 = \dfrac{\omega^2 b^2}{c_2^2}$.

The solution of equation (2.4) can be taken as

$$v(x,z) = \int_0^\infty A(\xi)e^{-\beta|z|}\cos(\xi x)\,d\xi + \int_0^\infty B(\zeta)\cosh(\beta_1 x)\cos(\zeta z)\,d\zeta \tag{2.5}$$

so that

$$\sigma_{yz}(x,z) = \mu\left[-\mathrm{sgn}(z)\int_0^\infty \beta A(\xi)e^{-\beta|z|}\cos(\xi x)\,d\xi - \int_0^\infty \zeta B(\zeta)\cosh(\beta_1 x)\sin(\zeta z)\,d\zeta\right], \tag{2.6}$$

where

$$\beta = \begin{cases} \sqrt{\xi^2-k_2^2}; & \xi>k_2, \\ -i\sqrt{k_2^2-\xi^2}; & \xi<k_2, \end{cases} \quad \text{and} \quad \beta_1 = \begin{cases} \sqrt{\zeta^2-k_2^2}; & \zeta>k_2, \\ -i\sqrt{k_2^2-\zeta^2}; & \zeta<k_2, \end{cases}$$

so that

$$\beta_1 = -i\sqrt{(k_2^2-\zeta^2)} = -i\beta_1' \quad \text{where} \quad \zeta<k_2.$$

3. DERIVATION OF INTEGRAL EQUATION

The condition of vanishing of σ_{yz} at z=0 outside the strips yields

$$\int_0^\infty \beta A(\xi)\cos(\xi x)\,d\xi = 0; \qquad |x|<c, \quad |x|>1. \tag{3.1}$$

Again the boundary condition $v(x,0) = -v_0$ at $c \le |x| \le 1$ gives

$$\int_0^\infty A(\xi)\cos(\xi x)\,d\xi + \int_0^\infty B(\zeta)\cosh(\beta_1 x)\cos(\zeta z)\,d\zeta = -v_0; \qquad c\le|x|\le 1. \tag{3.2}$$

Using the boundary condition $\sigma_{xy}(\pm h, z) = 0$ one obtains

$$\int_0^\infty \beta_1 B(\zeta)\sinh(\beta_1 h)\cos(\zeta z)\,d\zeta = \int_0^\infty \xi A(\xi)e^{-\beta|z|}\sin(\xi h)\,d\xi$$

which after Fourier cosine inversion yields

$$\beta_1 B(\zeta)\sinh(\beta_1 h) = \frac{2}{\pi}\int_0^\infty \frac{\xi\beta}{\beta^2+\zeta^2}A(\xi)\sin(\xi h)\,d\xi. \tag{3.3}$$

Eliminating $B(\zeta)$ from equations (3.2) and (3.3) one obtains

$$\int_0^\infty A(\xi)\cos(\xi x)\,d\xi = -v_0 - \frac{2}{\pi}\int_0^\infty \frac{\cosh(\beta_1 x)}{\beta_1\sinh(\beta_1 h)}\,d\zeta\int_0^\infty \frac{\xi\beta}{\beta^2+\zeta^2}A(\xi)\sin(\xi h)\,d\xi; \qquad c\le|x|\le 1. \tag{3.4}$$

Replacing $\beta A(\xi)$ by $C(\xi)$, equations (3.1) and (3.4) become

$$\int_0^\infty C(\xi)\cos(\xi x)\,d\xi = 0; \qquad |x|<c, \quad |x|>1 \tag{3.5}$$

and

$$\int_0^\infty \xi^{-1}[1+H(\xi)]C(\xi)\cos(\xi x)\,d\xi = -v_0 - \frac{2}{\pi}\int_0^\infty \frac{\cosh(\beta_1 x)}{\beta_1 \sinh(\beta_1 h)}d\zeta \int_0^\infty \frac{\xi C(\xi)}{\beta^2+\zeta^2}\sin(\xi h)\,d\xi; \quad c \le |x| \le 1, \quad (3.6)$$

where

$$H(\xi) = \left\{\frac{\xi}{\beta} - 1\right\} \to 0 \quad \text{as} \quad |\xi| \to \infty. \quad (3.7)$$

In order to solve the integral equations (3.5) and (3.6) we set

$$C(\xi) = \int_c^1 \frac{h(t^2)}{t}\{1 - \cos(\xi t)\}\,dt \quad (3.8)$$

where the unknown function $h(t^2)$ is to be determined.

Substituting $C(\xi)$ from (3.8) in equation (3.5) we note that

$$\int_0^\infty C(\xi)\cos(\xi x)\,dx = \pi \int_c^1 \frac{h(t^2)}{t}\left[\delta(x) - \frac{1}{2}\delta(x+t) - \frac{1}{2}\delta(|x-t|)\right]dt$$

so that equation (3.5) is automatically satisfied.

Again, the substitution of the value of $C(\xi)$ from (3.8) in equation (3.6) yields

$$\frac{1}{2}\int_c^1 \frac{h(t^2)}{t}\log\left|\frac{x^2-t^2}{x^2}\right|dt = -v_0 - \int_c^1 \frac{h(t^2)}{t}\,dt$$

$$\times\left[\int_{k_2}^\infty \frac{\cosh(\beta_1 x)e^{-h\beta_1}}{\beta_1 \sinh(\beta_1 h)}\{1-\cosh(t\beta_1)\}d\zeta - \int_0^{k_2} \frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\beta_1' \sin(\beta_1' h)}\{1-\cos(\beta_1' t)\}d\zeta\right]$$

$$- \int_c^1 \frac{h(t^2)}{t}\,dt \int_0^\infty \xi^{-1}H(\xi)\cos(\xi x)\{1-\cos(\xi t)\}\,d\xi, \qquad c \le |x| \le 1 \quad (3.9)$$

where the result

$$\int_0^\infty \frac{\cos(\xi x)\{1-\cos(\xi t)\}}{\xi}\,d\xi = \log\left|\frac{x^2-t^2}{x^2}\right|$$

has been used.

Differentiating both sides of equation (3.9) with respect to x and next multiplying by $(-2x/\pi)$,

one obtains

$$\frac{2}{\pi}\int_c^1 \frac{t h(t^2)}{t^2-x^2}\,dt = \frac{2x}{\pi}\int_c^1 \frac{h(t^2)}{t}\,dt\left[\int_{k_2}^\infty \frac{\sinh(\beta_1 x)e^{-h\beta_1}}{\sinh(\beta_1 h)}\{1-\cosh(t\beta_1)\}d\zeta\right.$$

$$+\int_0^{k_2} \frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\sin(\beta_1' h)}\left\{1-\cos(\beta_1' t)\right\}d\zeta - \int_0^{\infty} H(\xi)\sin(\xi x)\{1-\cos(\xi t)\}d\xi \Bigg]; \quad c \le |x| \le 1. \quad (3.10)$$

It is known that using Hilbert transform technique, the solution of the integral equation (Srivastava and Lowengrue [1968])

$$\frac{2}{\pi}\int_a^b \frac{t h(t^2)}{t^2-y^2}dt = R(y), \quad a<y<b$$

can be obtained in the form

$$h(t^2) = -\frac{2}{\pi}\sqrt{\frac{t^2-a^2}{b^2-t^2}}\int_a^b \sqrt{\frac{b^2-y^2}{y^2-a^2}}\frac{yR(y)}{y^2-t^2}dy + \frac{D}{\sqrt{(t^2-a^2)(b^2-t^2)}} \quad (3.11)$$

with condition that R must be an even function of y so as to make integral convergent. D is an arbitrary constant.

Following (3.11), the solution of equation (3.10) is given by

$$h(u^2) + \int_c^1 \frac{h(t^2)}{t}\left\{K_1(u^2,t^2) + K_2(u^2,t^2)\right\}dt = \frac{D}{\sqrt{(u^2-c^2)(1-u^2)}} \quad (3.12)$$

where

$$K_1(u^2,t^2) = -\frac{4}{\pi^2}\sqrt{\frac{u^2-c^2}{1-u^2}}\int_c^1 \sqrt{\frac{1-x^2}{x^2-c^2}}\frac{x^2 dx}{x^2-u^2}$$

$$\times\left[\int_{k_2}^{\infty} \frac{\cosh(\beta_1 x)e^{-h\beta_1}}{\sinh(\beta_1 h)}\left\{1-\cos(t\beta_1)\right\}d\zeta + \int_0^{k_2} \frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\sin(\beta_1' h)}\left\{1-\cos(\beta_1' t)\right\}d\zeta\right] \quad (3.13)$$

and

$$K_2(u^2,t^2) = +\frac{4}{\pi^2}\sqrt{\frac{u^2-c^2}{1-u^2}}\int_c^1 \sqrt{\frac{1-x^2}{x^2-c^2}}\frac{x^2 dx}{x^2-u^2}\int_0^{\infty} H(\xi)\sin(\xi x)\{1-\cos(\xi t)\}d\xi. \quad (3.14)$$

In order to determine the arbitrary constant D, equation (3.9) is multiplied by $\frac{x}{\sqrt{(x^2-c^2)(1-x^2)}}$ and integrated from c to 1 with respect to x, and using the result

$$\int_c^1 \frac{x\log|1-t^2/x^2|}{\sqrt{(x^2-c^2)(1-x^2)}}dx = \frac{\pi}{2}\log\left|\frac{1-c}{1+c}\right|$$

we finally obtain

$$\int_c^1 \frac{h(u^2)}{u}du = -\frac{2v_0}{\log\left|\frac{1-c}{1+c}\right|} - \frac{4}{\pi\log\left|\frac{1-c}{1+c}\right|}\int_c^1 \frac{h(t^2)}{t}dt$$

$$\times \int_c^1 \frac{x}{\sqrt{(x^2-c^2)(1-x^2)}}\left[A_1(x,t^2)+A_2(x,t^2)\right]dx \tag{3.15}$$

where

$$A_1(x,t^2) = \int_{k_2}^{\infty} \frac{\cosh(\beta_1 x)e^{-h\beta_1}}{\beta_1 \sinh(\beta_1 h)}\{1-\cosh(t\beta_1)\}d\zeta$$

$$-\int_0^{k_2} \frac{\cos(\beta_1' x)\cos(\beta_1' h)}{\beta_1' \sin(\beta_1' h)}\left\{1-\cos(\beta_1' t)\right\}d\zeta, \tag{3.16}$$

$$A_2(x,t^2) = \int_0^{\infty} \xi^{-1}H(\xi)\cos(\xi x)\{1-\cos(\xi t)\}d\zeta$$

$$= \frac{1}{2}\log\left|\frac{x^2-t^2}{x^2}\right| - \frac{\pi i}{2}H_0^{(1)}(xk_2) + \frac{\pi i}{4}H_0^{(1)}\left\{(x+t)k_2\right\} + \frac{\pi i}{4}H_0^{(1)}\left\{|x-t|k_2\right\}. \tag{3.17}$$

Again, by substituting $h(u^2)$ from equation (3.12) in the left-hand side of equation (3.15) and simplifying, one obtains

$$D = -\frac{2v_0 c}{\pi \log\left|\frac{1-c}{1+c}\right|} - \frac{8c}{\pi^2 \log\left|\frac{1-c}{1+c}\right|}\int_c^1 \frac{h(t^2)}{t}dt$$

$$\times \int_c^1 \frac{x}{\sqrt{(x^2-c^2)(1-x^2)}}\left[A_1(x,t^2)+A_2(x,t^2)\right]dx$$

$$+ \frac{2c}{\pi}\int_c^1 \frac{h(t^2)}{t}dt \int_c^1 \frac{1}{u}\left\{K_1(u^2,t^2)+K_2(u^2,t^2)\right\}du. \tag{3.18}$$

Eliminating D from equations (3.12) and (3.18) and simplifying on obtains

$$\sqrt{(u^2-c^2)(1-u^2)}\,h(u^2) + \int_c^1 \frac{h(t^2)}{t} + \left[K_a(u^2,t^2)+K_b(u^2,t^2)\right]dt$$

$$= -\frac{4v_0 c}{\pi \log\left|\frac{1-c}{1+c}\right|} \tag{3.19}$$

where

$$K_a(u^2,t^2) = -\frac{4}{\pi^2}(u^2-c^2)\int_c^1 \sqrt{\frac{1-x^2}{x^2-c^2}}\frac{x^2}{x^2-u^2}dx\left[\frac{\partial}{\partial x}\left\{A_1(x,t^2)+A_2(x,t^2)\right\}\right], \tag{3.20}$$

$$K_b(u^2,t^2) = \frac{8c}{\pi^2 \log\left|\frac{1-c}{1+c}\right|} \int_c^1 \frac{x}{\sqrt{(x^2-c^2)(1-x^2)}}\left[A_1(x,t^2)+A_2(x,t^2)\right]dx, \tag{3.21}$$

$$K_c(u^2,t^2) = -\frac{4c^2}{\pi^2}\int_c^1 \sqrt{\frac{1-x^2}{x^2-c^2}}\left[\frac{\partial}{\partial x}\left\{A_1(x,t^2)+A_2(x,t^2)\right\}\right]dx. \tag{3.22}$$

Next for further simplification we put

$$\sqrt{(u^2-c^2)(1-u^2)}\; h(u^2) = H(u^2)$$

and make the substitution

$$u^2 = c^2\cos^2\phi + \sin^2\phi \quad \text{and} \quad t^2 = c^2\cos^2\theta + \sin^2\theta$$

in equation (3.19) which then reduces to the form

$$G(\phi) + \int_0^{\pi/2} \frac{G(\theta)}{c^2\cos^2\theta + \sin^2\theta}\left[K_a'(\phi,\theta) + K_b'(\phi,\theta) + K_c'(\phi,\theta)\right]d\theta$$

$$= -\frac{4v_0 c}{\pi \log\left|\frac{1-c}{1+c}\right|} \tag{3.23}$$

where

$$G(\phi) = H(c^2\cos^2\phi + \sin^2\phi), \tag{3.24}$$

$$G(\theta) = H(c^2\cos^2\theta + \sin^2\theta), \tag{3.25}$$

$$K_a'(\phi,\theta) = K_a(c^2\cos^2\phi + \sin^2\phi,\, c^2\cos^2\theta + \sin^2\theta), \tag{3.26}$$

$$K_b'(\phi,\theta) = K_b(c^2\cos^2\phi + \sin^2\phi,\, c^2\cos^2\theta + \sin^2\theta), \tag{3.27}$$

$$K_c'(\phi,\theta) = K_c(c^2\cos^2\phi + \sin^2\phi,\, c^2\cos^2\theta + \sin^2\theta). \tag{3.28}$$

4. STRESS INTENSITY FACTOR

From equation (2.6) for $z \to 0$, $c \le |x| < 1$, one obtains

$$\sigma_{yz}(x,0\pm) = \mp \mu \int_0^\infty \beta A(\xi)\cos(\xi x)\,d\xi.$$

It is useful to determine the difference of the stress components on the lower and upper surfaces of the strips. Weput

$$\Delta\sigma_{yz}(x,0) = \sigma_{yz}(x,0+) - \sigma_{yz}(x,0-);$$

then

$$\Delta\sigma_{yz}(x,0) = -2\mu \int_0^\infty C(\xi)\cos(\xi x)\,d\xi, \qquad c<|x|<1.$$

Substituting the value of $C(\xi)$ and next changing the order of integration and integrating, one obtains

$$\Delta\sigma_{yz}(x,0) = \frac{\mu\pi h(x^2)}{x}. \tag{4.1}$$

Since

$$h(x^2) = \sqrt{(x^2-c^2)(1-x^2)}\,H(x^2)$$

and $\quad x^2 = c^2\cos^2\phi + \sin^2\phi,$

and hence equation (4.1) becomes

$$\Delta\sigma_{yz}(x,0) = \frac{\mu\pi G(\phi)}{x\sqrt{(x^2-c^2)(1-x^2)}}. \tag{4.2}$$

So the stress intensity factors N_c and N_1 at the two tips of the strip can be expressed as

$$N_c = \underset{x\to c+}{Lt}\left[\frac{\Delta\sigma_{yz}(x,0)}{\mu\pi}\sqrt{(x-c)}\right] \tag{4.3}$$

and

$$N_1 = \underset{x\to 1-}{Lt}\left[\frac{\Delta\sigma_{yz}(x,0)}{\mu\pi}\sqrt{(1-x)}\right]. \tag{4.4}$$

With the aid of equation (4.2) one obtains

$$N_c = \frac{G(0)}{c\sqrt{2c(1-c^2)}} \Rightarrow G(0) = c\sqrt{2c(1-c^2)}\,N_c \tag{4.5}$$

and

$$N_1 = \frac{G(\pi/2)}{c\sqrt{2(1-c^2)}} \Rightarrow G(\pi/2) = \sqrt{2(1-c^2)}\,N_1. \tag{4.6}$$

Making c tend to zero, the two strips merge into one and in that case

$$N_1 = \frac{1}{\sqrt{2}}G(\pi/2).$$

5. RESULTS AND DISCUSSIONS

The numerical calculations have been carried out for the determination of stress intensity factors for different values of the dimensionless frequency k_2 within the range 0.1 to 0.8. The integrals $A_1(x,t^2)$ and $A_2(x,t^2)$ given by (3.16) and (3.17), respectively, appearing in the kernel of integral equation (3.23) have been evaluated using the Gauss quadrature formula. Following Fox and Goodwin [1953], the solution of integral equation (3.23) has been obtained by converting it into a system of linear algebraic equations. Substituting these values of $[G(\phi)]$ in equations (4.3) and (4.4), the stress intensity factors N_c and N_1 at the inner and outer tips, respectively, of the rigid strips have been found to be related with $G(0)$ and $G(\pi/2)$ through the relations (4.5) and (4.6). The amplitudes $|G(0)|$ and $|G(\pi/2)|$ have been plotted against k_2 with different values of h for c=0.2, 0.4, 0.6; the values chosen for k_2 range from 0.1 to 0.8, at step of 0.05.

From the graphs it can be concluded that for fixed values of h, the stress intensity factor near the inner tip of the rigid strip decreases with the increase in the values of frequency within the range 0.1 to 0.8 (Figs. 2, 4 and 6), and for fixed values of h the stress intensity factor near the outer tip of the rigid strip at first decreases, attains a minimum and then it gradually increases with the increase in the values of frequency within the range 0.1 to 0.8 (Figs. 3, 5 and 7) for different values of c (c=0.2, 0.4 and 0.6).

It is interesting to note that for different values of k_2 within the range 0.1 to 0.8, the stress intensity factor of the inner tip of the strips, for a given value of k_2, increases with the increase in the values of h, whereas the stress intensity factor at the outer tip of strips, within the given range of values of k_2, decreases with the increase in the values of h for small values of k_2 but shows the reverse character for higher values of k_2 for any given value of the parameter c.

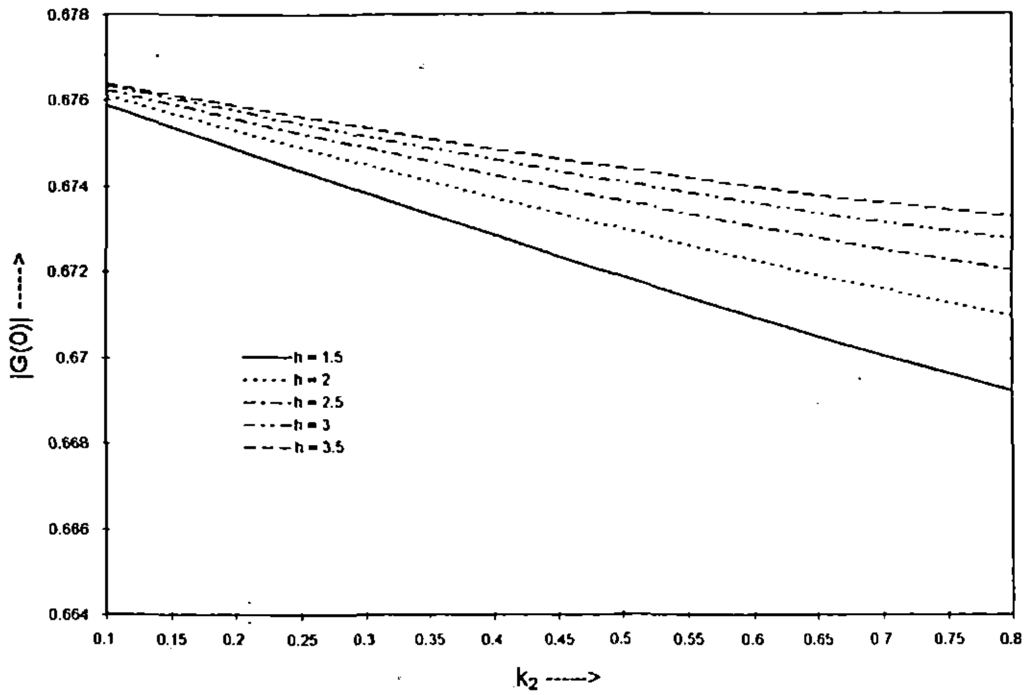

Fig.2. Amplitude of |G(0)| plotted against dimensionless frequency k_2 for c=0.2.

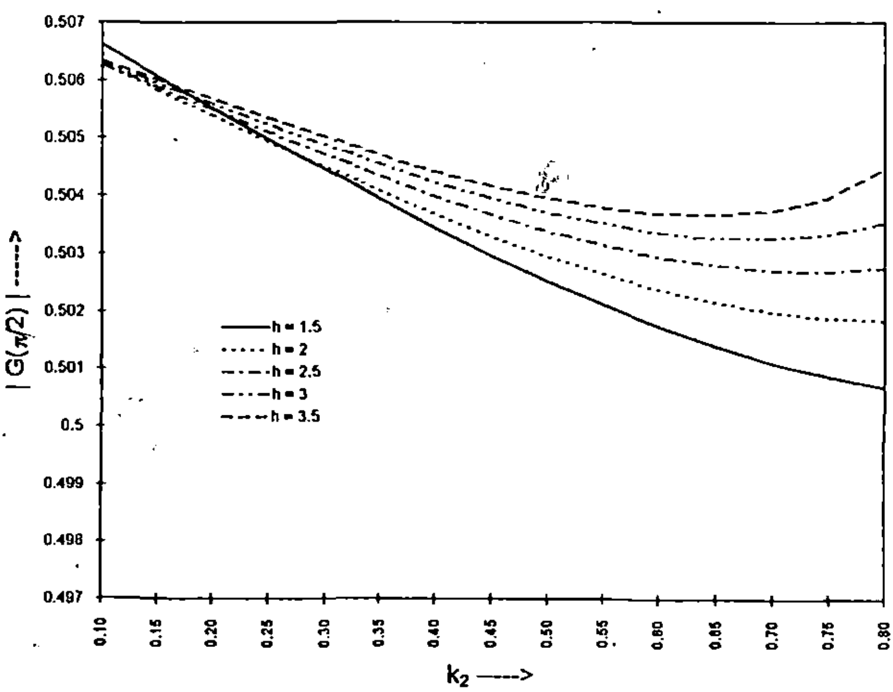

Fig.3. Amplitude of |G(π/2)| plotted against dimensionless frequency k_2 for c=0.2.

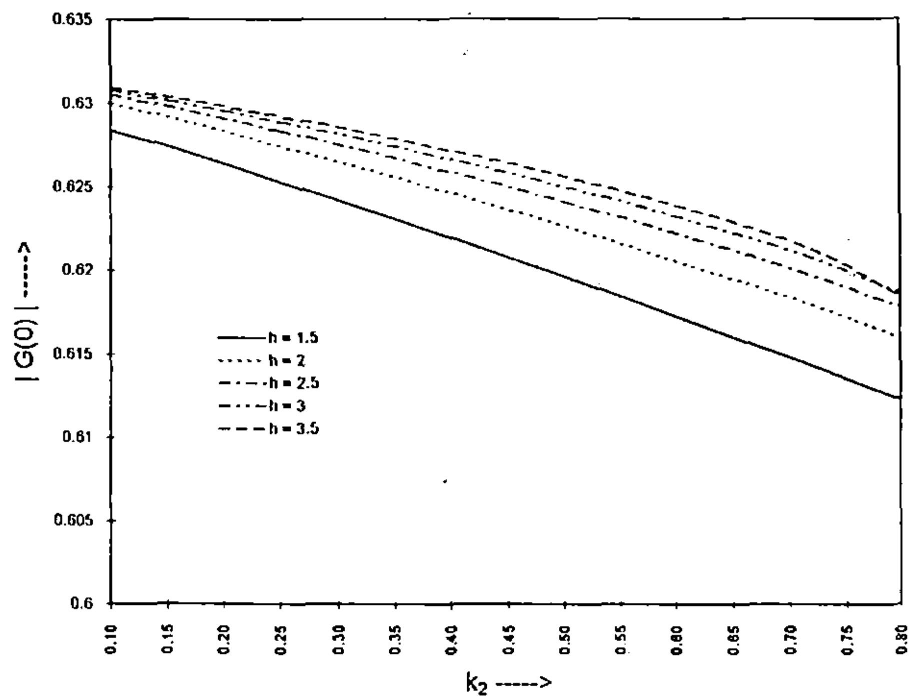

Fig.4. Amplitude of |G(0)| plotted against dimensionless frequency k_2 for c=0.4.

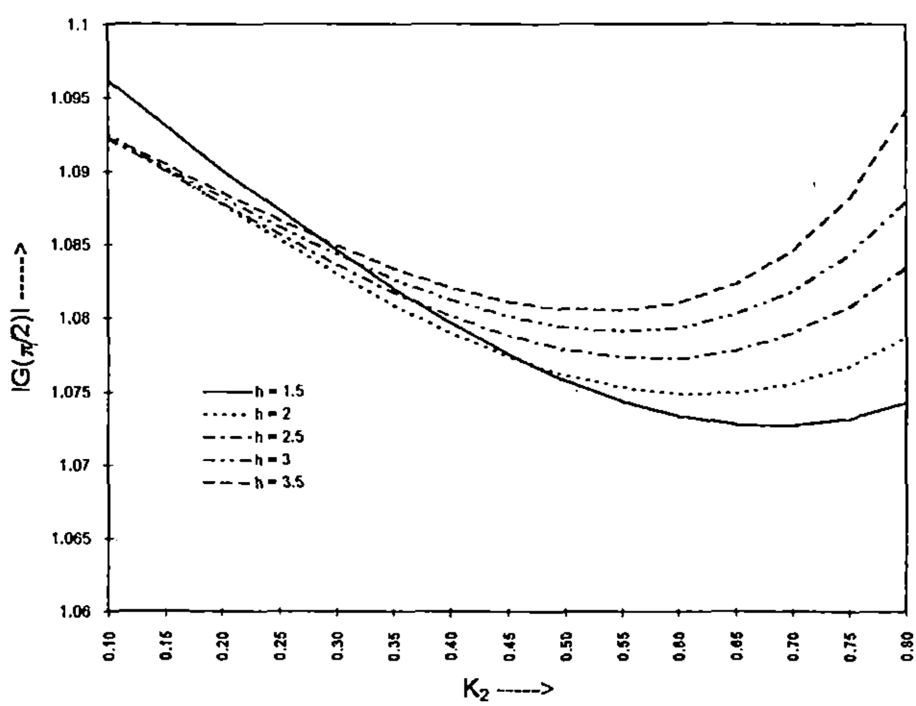

Fig.5. Amplitude of $|G(\pi/2)|$ plotted against dimensionless frequency k_2 for c=0.4.

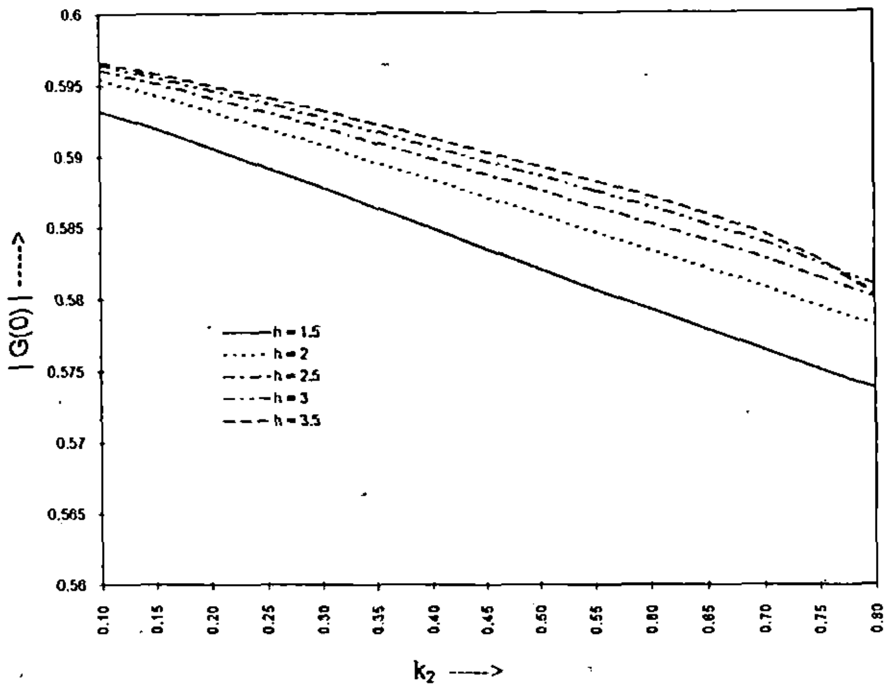

Fig.6. Amplitude of |G(0)| plotted against dimensionless frequency k_2 for c=0.6.

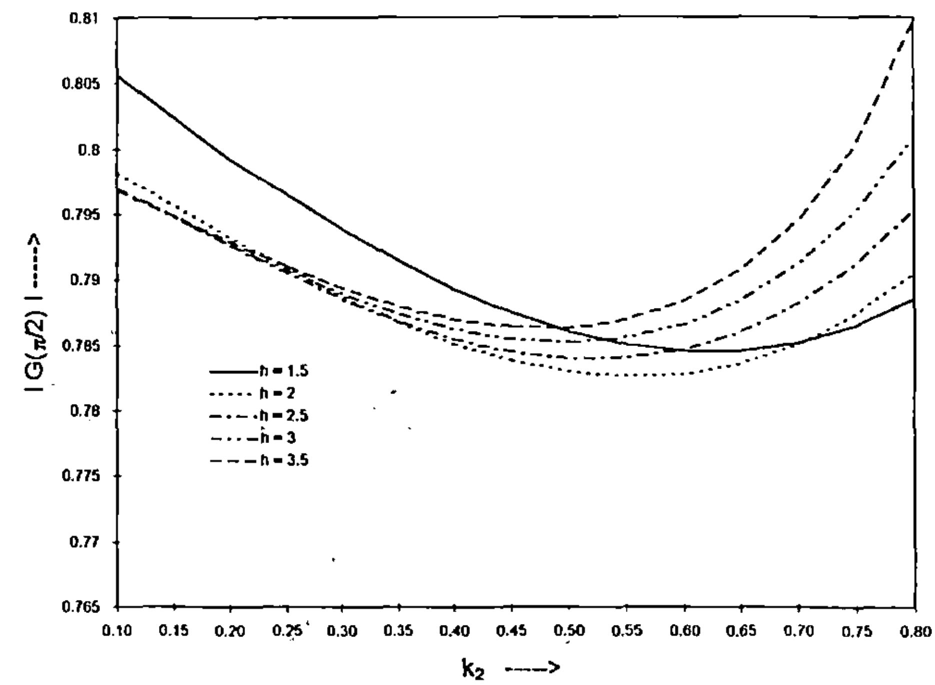

Fig.7. Amplitude of |G(π/2)| plotted against dimensionless frequency k_2 for c=0.6.