INTRODUCTION

The phenomenon of stress wave propagation in elastic solid offers us a rich variety of waves which was developed a century ago. The first mathematician to describe the vibrations of pendulums, the resonance phenomenon and the vibration of strings was Galileo. The first stage of the investigation on wave propagation associated with the names of Navier, Cauchy, Hooke, Poisson, Stokes, Rayleigh, Kelvin, Green, Lame, Clebsch, Ostrogradsky, Christoffel is characterized by development of the extensive theory of elasticity to the problem of wave propagation and vibrating bodies in the elastic meterial.

In the first three decades of the twentieth century the subject lost much of its glamour and interest to the research workers due to the lack of sophisticated instrument, electronic devices like high speed computer and practical methods for observing the passage of stress wave in elastic solids. But in the later part of the century the interest in the study of elastic waves has been growing rapidly because of the application of the theory in Seismology, Geophysics and in Engineering Science. Since that time, the wave propagation has remained an interesting area in seismology because of the need for detailed information on earthquake phenomena, prospecting techniques and the detection of nuclear exploisions. Bullen [1963], Ewing et al [1957], Cagniard [1962], Pilant [1979] and Aki and Richards [1980] have discussed elaborately about the problems involving seismic waves in their respective books.

Here we point out some of the milestones of progress in elastic waves in chronological order:

1678 : Robert Hooke (England) established the stress-strain relation for elastic bodies.

1760 : John Michell (England) recognized that earthquakes originate within the earth and send out elastic waves through earth's interior.

1821 : Louis Navier (France) derived the differential equations of the theory of elasticity.

1828 : Simeo-Denis Poisson (France) predicted theoretically the existence of longitudinal and transverse elastic waves.

1849 : George Gabriel Stokes (England) conceived the first mathematical model of an earthquake source.

1857 : First systematic attempt to apply physical principles to earthquake effects by Robert Mallet (Ireland).

1885 : C. Somigliana (Italy) produced formal solutions to Navier equations for a wide class of sources and boundary conditions.

1885 : Lord Rayleigh (England) predicted the existence of elastic surface waves.

1899 : C. G. Knott (England) derived the general equations for the reflection and refraction of plane seismic waves at plane boundaries.

1903 : A. E. H. Love (England) developed the fundamental theory of point sources in an infinite elastic space.

1904 : Horace Lamb (England) solved the problem on the propagation of tremors over the surface of an elastic solid.

1907 : Vito Voltera (Italy) published the theory of dislocations based on Somigliana's solution.

1940 : Sir Harold Jeffreys (England) and K. E. Bullen (Australia) published travel time tables for seismic waves in the earth.

1949 : Lapwood, E. R. First considered the distribution due to a line source in a semi-infinite elastic medium.

1959 : Ari Ben-Menahem (Isreal) discovered that the energy release in earthquakes take place through a propagating rupture over the causative fault.

During the last three decades there has been a remarkable revival of interest in this subject. Most of the experimental works carried out on the wave propagation of elasticity are concerned with studying propagation in specimens of comparatively simple geometrical shape, the results of this experiment could be compared directly with exact or approximate theoratical predictions. With increasing confidence in the experimental techniques and in the interpretation of observations, it is now possible to study more complicated problems of elastodynamics.

All the elastic bodies may be divided roughly into two catagories :

(I) homogeneous and non-homogeneous

(ii) isotropic and anisotropic

A homogeneous body is the body whose elastic properties are the same at different points and a non-homogeneous body has different elastic properties at different points. If the elastic moduli vary from point to point in a continuous manner, the non homogeneity may also be termed continuous. If, however, the elastic moduli undergo discontinuties in passing from point to point, for example change abruptly, the non-homogeneity is said to be discontinuous or discrete.

An isotropic body, with regard to its elastic properties, is one in which these properties are the same for all directions drawm through a given point. An anisotropic body has, in general, different elastic properties for different directions drawn through a given point. A body may be isotropic or anisotropic and at the same time homogeneous or non-homogeneous depending on its own structure.

However, natural or artificial materials in our surroundings are generally inhomogeneous and anisotropic.

Since 1950 the theory of elasticity for anisotropic bodies has been continually developed and enriched with new investigations of both serious problems as a general nature and individual aspects of these problems. Thus the general theory has been placed on a rigorous scientific basis and a number of laws have been established with the result that this theory, first worked out by B. De Saint Venant and P.V. Bekhterev [1925], has been revived.

Of great importance is the development and construction of many entirely new anisotropic materials which possesses a number of advantages over those previously known (for example, glass-fibre reinforced plastics). Thus, over two or three decades this branch of science has made great progress, both in a theoratical and a purely practical way, i.e. in constructing new anisotropic materials.

Now we recapitulate the fundamental principles of the theory of elasticity and the general equations which will be used in what follows for the construction of solutions to specific problems of the theory of elasticity for anisotropic bodies.

In studying the states of the stress and strain in anisotropic bodies produced by an external load, we make a number of assumptions imposing certain restrictions. The most important of these assumptions reduce to the following :

(I) A body is solid (a continuous medium), the stresses on any plane within the body and on its surface are forces per unit area,. In other words, the couple stresses are neglected, as is done in the classical theory of elasticity.

(II) The relation between the components of strain and the projections of displacement and their first derivatives with respect to the co-ordinate is linear.

(III) The stress-strain relations are linear i.e. the material follows the generalized Hooke's law, the co-efficients in these linear relations may be either constant (homogeneous body) or variable, i.e. functions of position, continuous or discontinuous (in the case of a non-homogeneous body).

(IV) The initial stresses i.e. those existing without any external load, including the thermal stresses are disgarded; specific problems of dynamics are not considered.

Thus, the theory of anisotropic elastic bodies can be studied from the classical linear theory of homogeneous or non-homogeneous elastic bodies.

In the general case of anisotropy each strain component is a linear function of all six components of stress. For a homogeneous body having anisotropy of the most general kind, the equations expressing the generalized Hooke's law for this system are

$$\epsilon_{ij} = a_{ijkl}\,\sigma_{kl} \qquad (i,j,k,l = 1,2,3). \tag{1}$$

The number of all constants a_{ijkl} with four subscripts is 81, but, when grouped, they reduce to 21 (of these 18 constants are independent) elastic constants.

The generalized Hooke's law equations, solved for the stress components, are of the form

$$\sigma_{ij} = A_{ijkl}\,\epsilon_{kl}\ . \tag{2}$$

The notation for the elastic constant "a" and "A" with four subscripts has been used by Malmeister, Tanuzh and Terters (1972) in their book.

However, if the elastic body is isotropic, then all directions in the body are elastically equivalent and the generalized Hooke's law for an isotropic body reduces to

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \gamma(\sigma_y + \sigma_z)\right]$$

$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \gamma(\sigma_z + \sigma_x)\right] \qquad (3)$$

$$\epsilon_z = \frac{1}{E}\left[\sigma_z - \gamma(\sigma_x + \sigma_y)\right]$$

$$\gamma_{yz} = \frac{1}{G}\tau_{yz}, \quad \gamma_{xz} = \frac{1}{G}\tau_{xz}, \quad \gamma_{xy} = \frac{1}{G}\tau_{xy}$$

where E, Young's modulous, γ Poisson ratio and G shear modulous.

In recent years, considering the elastic medium to be either isotropic or anisotropic, problems involving diffraction of elastic waves by crack or by inclusions have attracted considerable attention in view of their application in Seismology and Geophysics. Cracks or inclusions are present in essentially all structural materials either as natural defects or as a result of fabrication processes. Moreover, on many cases the cracks or inclusions are sufficiently small so that their pressence does not significantly reduce the strength of the material. In other cases, however, the imperfections are large enough through fatigue, stress corosion cracking etc., so that they must be taken into account in determining the strength. From the stand point of engineering applications it has been the macroscopic theories based on the notions of continuum solid mechanics and classical thermodynamics which have provided the quantitative working tools for dealing with the fracture of structural materials. In the microscopic continuum approach to fracture it is implicitly assumed that the material contains some macroscopic flaw which may act as fracture nuclei and that the medium is a homogeneous continuum in the sense that the size of the macroscopic flaws is large in comparison with the characteristic microstructural dimension of the material. The problem is then to study the effects of the applied loads, the flaw geometry and the environmental conditions on the fracture process in the solid.

Fracture mechanics is concerned with the analysis of the stability of cracks. A fracture criterion can subsequently be employed to determine the conditions for crack propagation, both stable and unstable, and for crack arrest.

Fracture mechanics problems that have to be treated as dynamic problem may be classified in two types :

(1) Cracked bodies subjected to rapidly varying loads

(2) Bodies containing rapidly propagating cracks.

In both the cases the crack tip is an environment disturbed by wave motions.

Impact and vibration problems fall into the first type of dynamic problems. It is often found that at inhomogeneities in a body the dynamic stresses are higher than the stresses computed from the corresponding problem of static equillibrium in the analysis of this type of problem.

The second type of problem is equally important. There are several kinds of large engineering structures e.g. gas transmission pipelines, ship-hulls, aircraft fuselages and nuclear reactor components, in which rapid crack growth is a definite possibility. The study of earthquake mechanisms is the another area to which the analysis of rapidly propagating cracks is relevant.

Recently, there have been a number of comprehensive articles in the general area of fracture mechanics. Some references are those of Achenbach [1972, 1976], Freund [1975, 1976, 1990] and Kanninen [1978].

Engineering structures requiring protection against the possibility of large scale catastrophic crack propagation are, however, generally constructed of ductile, tough materials. Current progress in this area, and a starting point for the development of a dynamic plastic propagating crack tip analysis have recently been presented by Achenbach and Kanninen.

A problem of central importance in dynamic fracture mechanics is that of predicting the way in which a crack will grow in a deformable solid, given the geometrical configuration of the solid, a characterization of the material, the applied load distribution and suitable initial conditions. In the interpretation of laboratory data on rapid crack propagation, a problem of equal importance is that of determining the values of fracture characterization parameters from measurements of the crack motion and applied load distribution.

In order to determine an equation of motion for a crack tip, two main ingradients are essential. The first of these is a crack propagation criterion which must be stated as a fundamental physical postulate, distinct from the postulates dealing with bulk material behaviour and momentum balance. Generally these later potulates can be satisfied for any motion of the crack tip. It is the role of the fracture criterion to select the motion of the crack tip from the class of all such dynamically admissible motions.

The only geometrical configuration for which exact solutions of the elastodynamic field equations, valid for non-uniform crack motion, have been found is a semi-infinite crack motion in an otherwise unbounded solid or configurations which can be shown to be equivalent to this by linear superposition arguments. The solution for antiplane deformation was presented by Kostrov [1964.

1966] and Achenbach [1970] and in-plane deformation by Freund [1973], Burridge [1976] and Kostrov [1975]. Although these solutions have been of major importance in addressing certain fundamental questions on rapid crack propagation, they have been found to be inadequate for describing some dynamic fracture processes of practical importance.

The shape of the cracks which have been studied uptil now are as follows :

(I) Semi-infinite plane cracks

(II) Finite Griffith cracks

(III) Penny shaped and annular cracks

(IV) Non-planer cracks.

A transient problem in which a semi-infinite crack appears suddenly in a stretched elastic sheet was solved by Maue [1954] and was also discussed by Ang [1958] in his dissertation. Baker [1962] solved the problem of a semi-infinite crack suddenly appearing and growing at a constant velocity in a stretched body. A steady state problem in which a semi-infinite crack extends at constant speed through an elastic sheet was solved by Craggs [1960]. Using the method of matched asymptotic expansion the problem involving diffraction of plane elastic waves by a semi-infinite boundary of finite width was solved by Viswanathan and Sharma [1978] and by Viswanathan, Sharma and Datta [1982].

The diffraction problem of a semi-infinite crack has been solved by the Wiener-Hopf technique (Noble [1958]).

In 1921 Griffith considered the problem of a fracture of a glass containing crack like defects. Griffith's work presented a theory of fracture. Among other workers investigating crack problems are Orowan [1948], Sack [1946], Irwin [1957] etc.. A number of crack problems in the theory of classical elasticity can be found in the literature (e.g. Sneddon and Lowengrub [1969], Sih [1972]}).

Yoffe [1951] considered the inplane problem of propagation of a finite Griffith crack of fixed length at a constant speed in an isotropic elastic solid of infinite extend. Other references treating elastodynamic problems involving a single finite Griffith crack are of Sato [1961], Williams [1957, 1961], Karp and Karal [1962], Ang and Knopoff [1964], Loeber and Sih [1968], Sih and Loeber [1968, 1969, 1970], Willis [1967], Atkinson and Eshebly [1968], Mal [1970a, 1970b, 1972], Hilton and Sih [1971], Chang [1971], Thau and Lu [1971], Sih, Embley and Ravera [1972], Kanninen [1974], Chen [1978], Sih and Chen [1980], Takei, Shindo and Atsumi [1982], Ueda, Shindo and Atsumi [1983], Shindo [1985]. Some other references are of Srivastava, Palaiya and Karaulia [1980a, 1980b], Srivastava, Gupta and Palaiya [1981], Erguven [1987].

The problem of diffraction of finite Griffith crack along the interface of two dissimilar elastic media have been solved by Goldshtein [1966, 1967], Brock and Achenbach [1973a, 1973b, 1974], Atkinson [1974], Matczynski [1974], Brock [1975], Srivastava, Gupta and Palaiya [1978], Neerhoff [1979], Srivastava, Palaiya and Karaulia [1980]. Bostrom [1987] solved the two dimensional scalar problem of scattering of elastic waves under antiplane strain from an interface crack between two elastic half-spaces. The problem of interaction of antiplane shear waves by a Griffith crack at the interface of two bonded dissimilar elastic half-spaces was considered by Srivastava, Palaiya and Karaulia [1980].

The transient stress and displacement fields arround an embedded crack in the shape of a circle were first investigated by Embley and Sih [1971] for extensional impact and by Sih and Embley [1971] for torsional impact. Their method of solution involves isolating the singular portion dynamic stresses in the Laplace transform domain such that the dynamic stress intensity factor can be obtained by direct application of the Laplace inversion theorem. Some other references are Mal [1968, 1970a], Olesiak and Sneddon [1959], Pal and Sridharan [1980a, 1980b], Arwin and Erdogan [1971], Green [1949], Dhawan [1973], Krenk and Schimidt [1982]. Robertson [1967] solved the problem of diffraction of a plane longitudinal wave by penny-shaped crack. Carrier [1946] studied the propagation of waves in orthotropic medium. Achenbach and Bazant [1975] considered the problem of elastodynamic near-tip stress and displacement fields for rapidly propagating cracks in orthotropic materials. Kassir and Tse [1983] also solved the problem of moving a Griffith crack in an orthotropic material.

The study of interfacial cracks between anisotropic media is of great impotance in fracture analysis of composites. For detection and quantitative measurement of interfacial cracks in a fiber reinforced composite solid, one needs to study the crack scattering problem in an anisotropic solid. A typical example of interfacial cracking in anisotropic solids is the delamination of fiber-reinforced composite laminates. The interfacial cracks in composites are introduced by fabrication defects such as incomplete wetting or trapped air bubles between layers, or by debonding of two laminas as a result of high stress concentration at geometric or material discontinuities (c.f. Pipes and Pagano [1970]). Kuo [1984] determined transient stress intensity factors of an interfacial crack between two dissimilar anisotropic half-spaces. To this effect, the mathematical problem has firstly been reduced to three coupled integral equations. Solution of the singular integral equation has been obtained by the use of the Jacobi polynomial and expressions for stress intensity factors at the crack tip of the interfacial crack has been determined. The surface response of a layered half-space, for both isotropic and anisotropic materials, due to a plane SH-wave, incident at an arbitrary angle, has been studied by Karim and Kundu [1988] using the technique of Neerhoff [1979]. The scattering of elastic waves by a circular crack in an anisotropic solid has also been studied by Kundu and Bostrom [1991].

We now discuss a certain type of mixed boundary value problems which are known as contact problem in the theory of elasticity. The contact problem is formulated as a problem about the influence of a rigid body on an elastic body.

Hertz investigated the punch problem in 1882. In this time many researchers followed his work. Chaplygin [1950] collected a number of punch problems worked out during the 19th century. Many authors such as Glagolev [1942], Mushkelishvilli [1953, 1963], Mossakovski [1958], Ufliand [1956], Spence [1968, 1975] investigated punch problems. In the literature (e.g. Gladwell [1980]) a variety of punch problems can be found.

The problem of diffraction of antiplane shear wave by one or more finite rigid strip at the interface has been treated by Palaiya and Majumder [1981], Singh and Dhaliwal [1984], Tait and Moodie [1981], Mandal and Ghosh [1992a, 1992b]. Palaiya amd Majumder [1981] considered the problem of diffraction of antiplane shear wave by a finite rigid strip at the interface of two bonded dissimilar half-spaces. The problem of diffraction of antiplane shear wave by a pair of parallel rigid strips at the interface of two bonded dissimilar elastic media was solved by Mandal and Ghosh [1992a]. De Sarkar [1985a, 1985b] solved the punch problem on a micropolar elastic solid.

Different techniques have been adopted by many authors to solve these type of crack and inclusion problems. From this standpoint, these problems may be divided into two catagories :

(I) one for low frequency oscillation of the source or long wave scattering or transmission; and

(II) the other for high frequency oscillation or short wave scattering or transmission in the medium.

The term long and short are used in comparison to the region of the source of disturbance or the size of the crack or strip etc. inside the medium to the wave length of disturbance.

Here we briefly discuss **some techniques** which are generally used in crack problems in elastodynamics.

INTEGRAL TRANSFORM TECHNIQUE

As the equations of motion in the theory of elasticity are partial differential equations which may be discussed with reference either to Helmholtz equation or to Laplace's equation, the method of integral transform is one of the most effective methods for solving such equations as application of this method to such equations results in the lowering of the dimension of an equation by one. There are several forms of integral transform and the choice of an integral transform depends on the structure of the equation and the geometry of the domain.

The integral transform $\bar{f}(p)$ of a function f(x) defined on an interval (a, ∞) is an expression of the form

$$\bar{f}(p) = \int_a^\infty f(x)\, K(x,p)\, dx \qquad (4)$$

where 'a' is a real number and p is a complex parameter varying over some region D of the complex plane. K(x,p) is called the kernel of the transformation. The transformation (4) becomes particularly useful if it possesses inverse mapping. In that case one can express f(x) in terms of its integral transform by

$$f(x) = \frac{1}{2\pi i} \int_\Gamma \bar{f}(p)\, M(x,p)\, dp \qquad (5)$$

where M(x,p) is a suitable function defined in $a < x < \infty$ and $p \in D$ and is called the kernel of the inverse transform, which is defined for all x in the interval (a, ∞). The complex parameter p is in the region D while Γ is a suitable path of integration in D. After reducing the governing partial differential equation, the reduced problem can be solved for $\bar{f}(p)$. The solution of the original equation can be expressed in terms of the inverse integral, which may then be evaluated. The inversion from the transformed space to the space of actual variables usually involves very complicated integrations. In many cases even the numerical integration can not be performed successfully because of the highly oscillatory character of the integrands (cf Eringen and Suhubi [1975]; Chap.7, Achenbach [1975]; Chap.7). In particular, mixed boundary value problems like the dynamic response of a punch on an elastic half-space and the problem involving the presence of a crack or a strip inside an elastic medium may be reduced to Fredholm integral equation of first kind or to dual integral equations.

NOBLE'S METHOD

Suppose that a mixed boundary value problem is formulated by suitable integral transform so as to be governed by a set of dual integral equations of the form

$$\int_0^\infty x^{-1}\left[1 + K(x)\right] S(x)\, J_\nu(rx)\, dx = f(r); \qquad 0 \le r < a \tag{6}$$

$$\int_0^\infty S(x)\, J_\nu(rx)\, dx = g(r); \qquad r > a \tag{7}$$

where the functions K(x), f(r) and g(r) are known.

According to Noble [1963], when $\nu > -1/2$

$$S(x) = \sqrt{\frac{2x}{\pi}} \left\{ \int_0^a \sqrt{t}\, \Theta(t)\, J_{\nu-\frac{1}{2}}(xt)\, dt + \int_a^\infty t^{\nu+\frac{1}{2}} G(t)\, J_{\nu-\frac{1}{2}}(xt)\, dt \right\} \tag{8}$$

where $\Theta(t)$ satisfies the Fredholm integral equation

$$\Theta(t) + \frac{1}{\pi}\int_0^a M(\varsigma,t)\,\Theta(\varsigma)\,d\varsigma = t^{-\nu}F(t) - H(t); \qquad 0 \le t < a \tag{9}$$

in which

$$M(\varsigma,t) = \pi\sqrt{\varsigma t}\int_0^\infty xK(x)\,J_{\nu-\frac{1}{2}}(\varsigma x)\,J_{\nu-\frac{1}{2}}(tx)\,dx \tag{10}$$

$$F(t) = \frac{d}{dt}\int_0^t f(r)\,r^{\nu+1}(t^2 - r^2)^{-1/2}\,dr \tag{11}$$

$$H(t) = \sqrt{t}\int_0^\infty xK(x)\,J_{\nu-\frac{1}{2}}(xt)\,dx \int_a^\infty \xi^{\nu+\frac{1}{2}}G(\xi)\,J_{\nu-\frac{1}{2}}(x\xi)\,d\xi \tag{12}$$

$$G(\xi) = \int_\xi^\infty g(r)\,r^{-\nu+1}(r^2 - \xi^2)^{-1/2}\,dr \tag{13}$$

The integral equation (9) can be solved for $\Theta(t)$ iteratively for low frequency and consequently $S(x)$ can be determined.

All the axisymmetrical contact problems may be solved by using Hankel transforms and they then reduce to the solution of a number of sets (or pairs) of dual integral equations. To solve these dual integral equations there are various methods one of which is Tranter's method. We discuss briefly the method of Tranter [1968] in solving axisymmetric problems.

TRANTER'S METHOD

The solution of certain physical problems involving axisymmetric geometry can be reduced to the determination of $F(p)$ from so called dual integral equations of the form

$$\int_0^\infty G(p)\,F(p)\,J_\nu(rp)\,dp = f(r), \qquad 0 < r < 1 \tag{14}$$

$$\int_0^\infty p\,F(p)\,J_\nu(rp)\,dp = 0, \qquad 1<r<\infty \tag{15}$$

where G(p) and f(r) are known functions.

A solution F(p) of the above integral equations as a series of Bessel functions can be found by setting

$$F(p) = p^{-k}\sum_{m=0}^{\infty} a_m\, J_{\nu+2m+k}(p) \tag{16}$$

where k is at present an arbitrary parameter, and proceeding as follows :

Substituting from (16) in the second equation of (15) and changing the order of integration and summation, one gets

$$\int_0^\infty p\,F(p)\,J_\nu(rp)\,dp = \sum_{m=0}^{\infty} a_m \int_0^\infty p^{1-k}\,J_\nu(rp)\,J_{\nu+2m+k}(p)\,dp \tag{17}$$

provided $\nu > -1$ and $k > 0$, the formula

$$I(\nu,\mu,\lambda,a,b) = \int_0^\infty \frac{J_\nu(at)\,J_\mu(bt)}{t^\lambda}\,dt \tag{18}$$

$$= \frac{b^\mu\,\Gamma\left(\frac{\nu}{2}+\frac{\mu}{2}-\frac{\lambda}{2}+\frac{1}{2}\right)}{2^\lambda a^{\mu-\lambda+1}\,\Gamma(\mu+1)\,\Gamma\left(\frac{\lambda}{2}+\frac{\nu}{2}-\frac{\mu}{2}+\frac{1}{2}\right)}\; F_1\left(\frac{\nu+\mu-\lambda+1}{2}, \frac{\mu-\lambda-\nu+1}{2}; \mu+1; \frac{b^2}{a^2}\right)$$

shows that all the integrals on the right hand side of (17) vanish when r>1 (because of the factor Γ(-m) in the denominator of the term multiplying the hypergeometric function) and hence the series in (16) automatically satisfies the second of the dual equations (15). The coefficients a_m have now to be chosen so that the series in (16) satisfies the first of the dual equations (15). For this purpose we need the result

$$p^{-k} J_{\nu+2n+k}(p) = \frac{\Gamma(\nu+n+1)}{2^{k-1}\Gamma(\nu+1)\Gamma(n+k)} \int_0^1 r^{\nu+1}(1-r^2)^{k-1} F_n(k+\nu, \nu+1, r^2)\, J_\nu(pr)\, dr \qquad (19)$$

where n is a positive integer or zero and

$$F_n(\alpha, \gamma, x) = F_1(-n, \alpha+n; \gamma : x) \qquad (20)$$

is Jacobi's polynomial.

Substituting from (16) in the first of (15), multiplication by

$r^{\nu+1}(1-r^2)^{k-1} F_n(k+\nu, \nu+1, r^2)$,

integration with respect to r between 0 and 1, interchange of the order of integrations and use of (19) give

$$\sum_{m=0}^{\infty} a_m \int_0^{\infty} G(p)\, p^{-2k} J_{\nu+2m+k}(p)\, J_{\nu+2n+k}(p)\, dp = E(\nu, n, k) \qquad (21)$$

where

$$E(\nu, n, k) = \frac{\Gamma(\nu+n+1)}{2^{k-1}\Gamma(\nu+1)\Gamma(n+k)} \int_0^1 f(r)\, r^{\nu+1}(1-r^2)^{k-1} F_n(k+\nu, \nu+1, r^2)\, dr. \qquad (22)$$

Equation (12) with n=0, 1, 2, 3, ... gives a set of simultaneous equations for the determination of the coefficients a_m. These simultaneous equations can be rewritten in more convenient form by making use of the formula

$$\int_0^{\infty} G(p)\, p^{-1} J_{\nu+2m+k}(p)\, J_{\nu+2n+k}(p)\, dp = \begin{cases} 0, & m \neq n \\ (2\nu+4n+2k)^{-1}, & m=n \end{cases} \qquad (23)$$

this being the form taken by equation

$$\int_0^{\infty} \frac{J_\nu(at)\, J_\mu(at)}{t}\, dt = \frac{\Gamma(\frac{\nu}{2}+\frac{\mu}{2})}{2\Gamma(1+\frac{\nu}{2}-\frac{\mu}{2})\Gamma(1+\frac{\nu}{2}+\frac{\mu}{2})\Gamma(1-\frac{\nu}{2}+\frac{\mu}{2})} = \frac{2}{\pi} \frac{\sin\frac{1}{2}(\mu-\nu)\pi}{\mu^2-\nu^2} \qquad (24)$$

when μ and ν are replaced respectively by $\nu+2n+k$, $\nu+2m+k$ and when 'at' is replaced by p, we find in this way

$$a_n + \sum_{m=0}^{\infty} L_{m,n}\, a_m = (2\nu + 4n + 2k)\, E(\nu, n, k) \qquad (25)$$

where

$$L_{m,n}' = (2\nu+4n+2k) \int_0^{\infty} \left(G(p)\, p^{1-2k} - 1\right) p^{-1} J_{\nu+2m+k}(p)\, J_{\nu+2n+k}(p)\, dp. \qquad (26)$$

The iterative solution of the simultaneous equations (25) is

$$a_n = E_n - E_n' + E_n'' - \ldots\ldots.. \qquad (27)$$

where

$$E_n = (2\nu+4n+2k)\, E(\nu, n, k), \quad E_n' = \sum_{m=0}^{\infty} L_{m,n}\, E_m, \quad E_n'' = \sum_{m=0}^{\infty} L_{m,n}\, E_m' \qquad (28)$$

and so on.

Equations (16), (27), (28), (26) and (23) provide a theoratical solution of the dual integral equations. For a practical solution it is necessary to be able to choose the parameter k so that the expression $\left(G(p)\, p^{1-2k} - 1\right)$, which occurs in the formula (26) for $L_{m,n}$, is fairly small.

Low frequency diffraction due to disc, cone and rigid cylinder have been studied by Asvestas and Kleinman [1970], Senior [1971], Datta [1974], Roy [1982a, 1982b, 1982c], Sleeman [1967], Roy and Sabina [1982], Datta [1970] considered the problem of diffraction of a plane compressional elastic wave by a rigid circular disc.

Singh and Dhaliwal [1984] solved the closed form solutions of dynamic punch problems by integral transform method. The mixed boundary value problem was reduced to a set of dual integral equations with trigonometrical kernals. The solutions were obtained by using Hilbert transform technique (Srivastava and Lowengrub [1968]). We now discuss the Hilbert transform technique as follows :

HILBERT TRANSFORM TECHNIQUE

Using the theorem (Tricomi [1951]) if $p \in L_2(a,b)$, then the equation

$$\frac{1}{\pi}\int_a^b \frac{h(x)}{x-y}\,dx = p(y); \quad y \in (a,b) \tag{29}$$

has the solution

$$h(x) = -\frac{1}{\pi}\sqrt{\frac{x-a}{b-x}}\int_a^b \sqrt{\frac{b-y}{y-a}}\frac{p(y)}{y-x}\,dy + \frac{c}{\sqrt{(x-a)(b-y)}} \tag{30}$$

where c is an arbitrary constant and the first term belongs to the class $L_2(a,b)$. Using the above theorem, Srivastava and Lowengrub [1968] found that the solution of the integral equation

$$\frac{1}{\pi}\int_a^b \frac{2th(t^2)}{t^2-y^2}\,dt = p(y); \quad y \in (a,b) \tag{31}$$

(Provided that p satisfies the conditions of the above theorem) is given by

$$h(t^2) = -\frac{1}{\pi}\sqrt{\frac{t^2-a^2}{b^2-t^2}}\int_a^b \sqrt{\frac{b^2-y^2}{y^2-a^2}}\frac{2yp(y)}{y^2-t^2}\,dy + \frac{c}{\sqrt{(t^2-a^2)(b^2-t^2)}} \tag{32}$$

where c is an arbitrary constant.

Using Hilbert transform technique problems involving pair of cracks or strips can easily be solved. Using Hilbert transform technique Sarkar, Mandal and Ghosh [1994] solved the elastodynamic problem involving two co-planar Griffith cracks in a orthotropic medium. Using the same technique Das and Ghosh [1992] treated four co-planar Griffith crack problems in an infinite elastic medium. Itou [1978] solved the problem of dynamic stress concentration around two co-planar Griffith cracks in an infinite elastic medium using Schmidt method. Itou [1980a, 1980b] also considered two different problems involving two finite cracks using the same technique. The problem of determining the transient stress distribution in an infinite elastic medium weakened by two coplanar Griffith cracks has been reduced by Itou to the following equation

$$\sum_{n=1}^{\infty} c_n(s)\left[-\frac{4c_1^3}{k^2s^2b}\int_0^{\infty} g(s,\xi)\sin\left(\frac{a+b}{2}\xi-\frac{n\pi}{2}\right)J_n\left(\frac{b-a}{2}\xi\right)\cos(\xi x)\,d\xi\right] = -pc_L(bs), \quad a<x<b \tag{33}$$

with

$$g(s,\xi) = \frac{\left[\xi^2 + k^2 s^2/(2c_L^2)\right]^2 - \xi^2 \gamma_1 \gamma_2}{\xi \gamma_1} \tag{34}$$

where locations of the cracks are $a \le |x| \le b$, $|y| < \infty$, $z = 0$, $c_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}$, $c_T = \sqrt{\frac{\mu}{\rho}}$, $k = c_L / c_T$ and $c_n(s)$ are unknown coefficients.

To determine the coefficients $c_n(s)$ by Schmidt's method [1958] equation (33) can be rewritten as

$$\sum_{n=1}^{\infty} c_n(s) E_n(s,x) = -u(s,x), \qquad a<|x|<b \tag{35}$$

where $E_n(s,x)$ and $u(s,x)$ are known functions and the coefficients $c_n(s)$ are unknown.

A set of functions $P_n(s,x)$ which satisfy the orthogonality conditions

$$\int_0^b P_m(s,x) P_n(s,x)\, dx = N_n \delta_{mn}, \qquad N_n = \int_0^b P_n^2(s,x)\, dx \tag{36}$$

can be constructed from the function, $E_n(s,x)$, such that

$$P_n(s,x) = \sum_{n=1}^{\infty} \frac{M_{in}}{M_{nn}} E_i(s,x) \tag{37}$$

where M_{in} is the cofactor of the element d_{in} of D_n, which is defined as

$$D_n = \begin{vmatrix} d_{11} & d_{12} & \dots & \dots & d_1 n \\ d_{21} & d_{22} & \dots & \dots & d_2 n \\ & & \vdots & & \\ d_{n1} & \dots & \dots & \dots & d_{nn} \end{vmatrix} \tag{38}$$

$$d_{in} = \int_a^b E_i(s,x) E_n(s,x)\, dx.$$

Using equations (35) and (36) one can obtain

$$c_n(s) = \sum_{j=n}^{\infty} q_j \frac{M_{nj}}{M_{jj}} \tag{39}$$

with

$$q_j = -\frac{1}{N_j}\int_a^b u(s,x)\,P_j(s,x)\,dx \; . \qquad (40)$$

KELLER'S GEOMETRICAL METHOD

Keller's theory of geometrical diffraction applied to elastodynamics states that the two conical surfaces of diffracted rays are generated when an incident ray strikes an edge. The surface of the inner cone consists of rays of longitudinal motion, while the surface of the outer cone is composed of rays of transverse motion. The half-angles of the cones are related by Snell's law. Fig.1 shows the cones generated by an incident longitudinal ray. For this case the diffracted longitudinal rays make the same angle ϕ_L with the tangent to the edge as the incident ray, and the diffracted rays of transverse motion are under an angle ϕ_T with the edge, where $C_L \cos\phi_T = C_T \cos\phi_L$.

For a straight diffracting edge, and an incident longitudinal ray, the diffracted displacement fields are related quantitatively to the incident field by

$$\vec{u}_d^{\,L} = e^{i\omega s_1/c_L}\left[S_1(1+S_1/R_L)\right]^{-1/2} D_L\, \hat{i}_L^{\,d} A\, e^{i\omega(S_0/C_L - t)} \qquad (41)$$

$$\vec{u}_d^{\,T} = e^{i\omega s_2/c_T}\left[S_2(1+S_2/R_d)\right]^{-1/2} D_T\, \hat{i}_T^{\,d} A\, e^{i\omega(S_0/C_L - t)} \qquad (42)$$

Here $A\exp[i\omega(S_0 / C_L - t)]$ defines the amplitude and the phase of the incident field at the point of diffraction, and D_L and D_T are diffraction coefficients which relate the diffracted field to the incident field. Also S_1 and S_2 are the smaller of the principal radii of curvature of the diffracted wave front, or equivalently the distances along the diffracted rays from the points of diffraction to the observation point. The unit vectors $\hat{i}_L^{\,d}$ and $\hat{i}_T^{\,d}$ relate the directions of displacement of the diffracted field to the direction of displacement of the incident field. For a straight diffracting edge

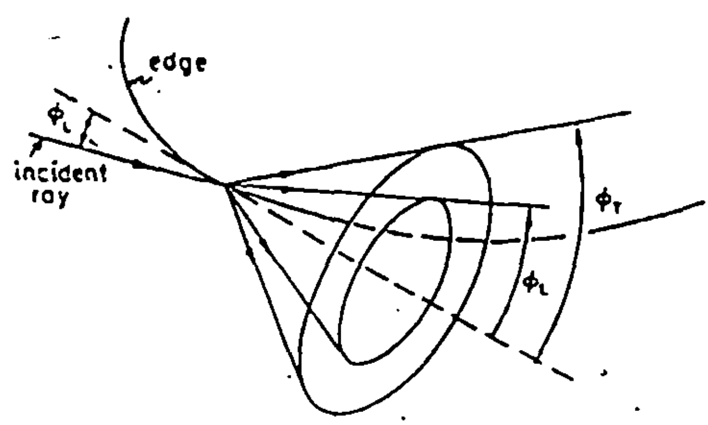

Fig.1. Cones of diffracted longitudinal and trandverse rays for an incident longitudinal ray.

R_L is the radius of curvature at the point of diffraction of the curve formed by the intersection of the incident wave front and the plane which contains the incident ray and the edge, and

$$R_d = R_L \frac{\sin\phi_T}{\sin\phi_L} \frac{\tan\phi_T}{\tan\phi_L}. \tag{43}$$

In case of high frequency oscillation Wiener-Hopf technique (Noble, [1958]) and Keller's [1958] geometrical theory are found to be most suitable. We now briefly discuss this methods.

THE WIENER-HOPF TECHNIQUE

Let a function $\phi(z)$ analytic in the interval $y_- < \mathrm{Im}\, z < y_+$ be defined in the plane of a complex variable z. It is required to express $\phi(z)$ in the form

$$\phi(z) = \phi_+(z)\,\phi_-(z) \tag{44}$$

where $\phi_+(z)$ and $\phi_-(z)$ are functions analytic in the half-plane $\mathrm{Im} z > y_-$ and the half-plane $\mathrm{Im} z < y_+$ respectively. The problem is called factorization problem. In a more general case, it is required to define two functions $\phi_+(z)$ and $\phi_-(z)$ which are analytic in the same half-planes respectively and which satisfy the following relation in the interval

$$A(z)\,\phi_+(z) + B(z)\,\phi_-(z) + C(z) = 0 \tag{45}$$

where A(z), B(z) and C(z) are given analytic functions in the interval. It is obvious that if C(z)=0, we obtain the representation (44) after the corresponding changes in the notation.

Let us assume that the function $\phi(z)$ which is to be factorised does not have any zeros in the interval $y_- < \mathrm{Im}\, z < y_+$ and tend to infinity as $x \to \infty$. In this case, neither of the functions $\phi_+(z)$ and $\phi_-(z)$ will have any zero, and we can take the logarithm of both sides of the relation (44)

$$\log\phi(z) = \log\phi_+(z) + \log\phi_-(z). \tag{46}$$

The function $F(z) = \log\phi(z)$ satisfies the condition

$$|F(x+iy)| < C|x|^{-p}; \qquad (p>0 \text{ for } x\to\infty) \tag{47}$$

and hence the relation (46) can always be solved with the help of the transformation

$$F(z) = F_+(z) + F_-(z)\,. \tag{48}$$

Finally, we get

$$\phi(z) = e^{F_+(z)} . e^{F_-(z)} = \phi_+(z)\,\phi_-(z)\,. \tag{49}$$

If the function $\phi(z)$ has zeros in the intervals we must consider a new function

$$\phi_1(z) = \frac{(z^2+b^2)^{N/2}\,\phi(z)}{\prod_{i=1}^{N_1}(z-z_i)^{\alpha_i}} \tag{50}$$

where z_i and α_i are the zeros, their multiplicity in the interval $N_1 \le N$, where N is the total number of zeros, $b > (y_+, y_-)$. The factor in the numerator of (50) ensures that the properties of auxiliary functions are conserved at infinity.

Let us now consider the relation (45) and carry out its factorization into L_+ and $1/L_-$ for the same interval of the ratio A/B. The relation (45) can be represented in the form

$$L_+(z)\,\phi_+(z) + L_-(z)\,\phi_-(z) + L_-(z)\,C(z)/B(z) = 0. \tag{51}$$

The expression $L_-(z)\,C(z)/B(z)$ Can be represented in the following form in accordance with (48)

$$E_+(z) + E_-(z)$$

where $\phi_+(z)$ and $\phi_-(z)$ are functions analytic in the half-plane $y > y_-$ and the half-plane $y < y_+$ respectively. Taking this into account, we get

$$L_+(z)\,\phi_+(z) + E_+(z) = -L_-(z)\,\phi_-(z) - E_-(z). \tag{52}$$

It follows from the generalized Liouville's theorem that the left as well as right hand side of (52) represents the same polynomial $P_n(z)$ of nth degree.

Wiener-Hopf technique and different other techniques for solving partial differential equation arising in solid mechanics have been elaborately discussed by Duffy [1994] in his book.

BOUNDARY ELEMENT METHOD

The formulation of general wave diffraction problem by a crack in the half-space with the aid of the frequency-domain direct Boundary Element Method (BEM) has been done by Niwa and Hirose [1986] and has also been elaborately decribed in the book Boundary Element Method in Elastodynamics [1988].

Consider the crack $\sum$ with two faces Σ^- and Σ^+ in the half-space V with a free surface S subjected to an oblique harmonic wave that creates a free displacement field $\bar{u}_i^f$.

For this type of geometry one can write the two integral equations

$$-\int_{S+\Gamma} \bar{G}_{ik}\bar{t}_k^s ds + \int_{S+\Gamma} \bar{F}_{ik}\bar{u}_k^s ds = \begin{cases} \bar{u}_i^s(x), & x \in V \\ 0, & x \in V^c \end{cases} \quad (53)$$

$$+\int_{\Gamma} \bar{G}_{ik}\bar{t}_k^f ds - \int_{\Gamma} \bar{F}_{ik}\bar{u}_k^f ds = \begin{cases} \bar{u}_i^f(x), & x \in \Omega \\ 0, & x \in \Omega^c \end{cases} \quad (54)$$

where Γ represents the surface of a flat cavity Ω that in the limit becomes the crack Σ, superscripts s and f stand for scattered and free fields, respectively and superscript c denotes the complementary domain and G_{ij} is the Green function in infinite medium due to the body force

$$b_j = \delta(\vec{x} - \vec{\xi})\,\delta(t)\,e_j \quad (55)$$

where δ denotes Dirac delta function and e is a constant unit vector and F_{ij} is the corresponding stress component given by

$$F_{ij} = \left\{\lambda G_{mk,m}\delta_{ij} + \mu(G_{jk,j} + G_{jk,i})\right\} n_j ; \tag{56}$$

n_j being the outward pointing normal vector.

Combination of equations (53) and (54) results in

$$\int_S \bar{F}_{ik}\bar{u}_k^s ds + \int_\Gamma \bar{F}_{ik}\bar{u}_k ds = \begin{cases} \bar{u}_i^s(x), & x\in V \\ -\bar{u}_i^f(x), & x\in\Omega \\ 0, & x\in V^c\cap\Omega^c \end{cases} \tag{57}$$

where the traction free boundary condition on Γ has been taken into account as well as the fact that $\bar{u}_i = \bar{u}_i^f + \bar{u}_i^s$. For crack problems (57) is unsuitable because it leads to an undetermined integral equation. Thus equation (57) is differentiated according to the constitutive law to give

$$\int_S \bar{W}_{ijk}\bar{u}_k^s ds + \int_\Gamma \bar{W}_{ijk}\bar{u}_k ds = \begin{cases} \bar{t}_i^s(x), & x\in V \\ -\bar{t}_i^f(x), & x\in\Omega \\ 0, & x\in V^c\cap\Omega^c \end{cases} \tag{58}$$

where

$$\bar{W}_{ijk} = \lambda \bar{F}_{kj,j} n_j + \mu\left(\bar{F}_{ki,j} + \bar{F}_{kj,i}\right) n_j \tag{59}$$

In the limit as the flat cavity approaches a crack, the above equation becomes

$$\int_S \bar{W}_{ijk}\bar{u}_k^s ds + \int_\Sigma \bar{W}_{ijk}\left[\bar{u}_k\right] ds = \begin{cases} \bar{t}_i^s(x), & x\in V \\ 0, & x\in V^c \end{cases} \tag{60}$$

where the third brackets denote the discontinuity across Σ, i.e.

$$\left[\bar{u}_k\right] = \bar{u}_k^+ - \bar{u}_k^- . \tag{61}$$

Taking the limit of equation (60) as the field point x approaches the boundary S or Σ, one obtains the traction boundary integral equations :

$$\int_S \bar{W}_{ijk} \bar{u}^s_k ds + \int_\Sigma \bar{W}_{ijk} \left[\bar{u}_k\right] ds = 0 ; \qquad x \text{ on } S \tag{62}$$

$$\int_S \bar{W}_{ijk} \bar{u}^s_k ds + \int_\Sigma \bar{W}_{ijk} \left[\bar{u}_k\right] ds = -\bar{t}^f_i ; \qquad x \text{ on } \Sigma \tag{63}$$

where the two parallel line segments on the integral sign signify a finite part integral in the sense of Kutt [1975a-c]. Equations (62) and (63) constitute a system of equations to be solved for the unknowns $\left[\bar{u}_k\right]$ on Σ and $\bar{u}^s_k$ on S following standard direct BEM procedures. The singularities in (62) and (63) are of order r^{-3} and as such stronger than then r^{-2} singularities of (57). They are computed by using the rigid body motion concept and by introducing an auxiliary surface close to the discretized portion of S.

Another special BEM is a direct method which starts with a reciprocity relation that finally yields for the scattered displacement field the integral representation

$$\bar{u}^s_k = \int_\Sigma \bar{F}_{ik} \left[\bar{u}_i\right] ds \, . \tag{64}$$

Equation (64) is then substituted into Hooke's law to derive expressions for the scattered stress tensors in terms of $\left[\bar{u}_i\right]$. Finally, use of the boundary conditions produces a set of singular integral equations for the $\left[\bar{u}_i\right]$. These equations are solved numerically either by employing special quadrature rules or by expanding known and unknown functions in terms of suitable basis functions and then determining the expansion co-efficients of of the $\left[\bar{u}_i\right]$. This method has been succeessfully used by Achenbach et al [1983, 1984], Vander Hijden and Neerhoff [1984a-b] and McMaken [1984] for the solution of wave diffraction problems in the infinite or semi-infinite half-plane containing a crack or in the infinite space containing circular cracks.

The thesis presented here consists of some boundary value problems in elastodynamics involving wave propagation due to some finite source or cracks. The work has been presented in four chapters.

The **first chapter** deals with high frequency diffraction of elastic wave by Griffith cracks which are either stationary or moving in character.

In the **second chapter**, inclusion problem in elastodynamic in infinitely long elastic strip is considered.

The **third chapter** deals with the scattering of elastic waves by vertical crack.

Finally in the **last chapter**, elastodynamic Green's function due to ring source has been treated.

The summary of the thesis is presented here chapterwise .

In the **first problem of chapter-1**, the high frequency elastodynamic problem involving the excitation of an interface crack of finite width lying between two dissimilar anisotropic elastic half planes has been analysed. The crack surface is excited by a pair of time harmonic antiplane line sources situated at the middle of the cracked surface. The problem has first been reduced to one with the interface crack lying between two dissimilar isotropic elastic half planes by a transformation of relevant co-ordinates and parameters. The problem has then been formulated as an extended Wiener-Hpof equation (cf. Noble. 1958) and the asymptotic solution for high frequency has been derived. The expression for the stress intensity factor at the crack tips has been derived and the numerical results for different pairs of materials have been presented graphically.

In the **second problem of this chapter**, the transient elastodynamic problem involving the scattering of elastic waves by a Griffith crack of finite width lying at the interface of two dissimilar anisotropic half planes has been analysed. The crack faces are subjected to a pair of suddenly applied antiplane line loads situated at the middle of the cracked surface. The problem has first been reduced to one with the interface crack of finite width lying between two dissimilar isotropic elastic half planes by a transformation of relevant co-ordinates and parameters. Spatial and time transforms are the applied to the governing differential equations and to the boundary conditions which yield generalized Wiener-Hopf type equations. The integral equations arising are solved by the standard interation technique. Physically each successive order of iteration correspond to successive scattered or rescattered wave from one crack tip to the other. Finally, expression for the resulting mode III stress intensity factor are determined as a function of time for both symmetric and antisymmetric loadings. Each crack tip stress intensity factor has been plotted versus time for four pairs of different types of materials.

The **third problem** deals with the scattering of horizontally polarised shear wave by a Griffith crack moving with uniform velocity along a bimaterial interface has been investigated. Using Fourier transform technique, the mixed boundary value problem has been reduced to the solution of a pair of dual integral equations. These equations are further reduced to a pair of coupled Fredholm integral equation of the second kind. The singular character of the dynamic stress near the crack tip has been examined and the expression for dynamic stress intensity factor has been derived. The dynamic stress intensity factors for several values of wave number, angle of incidence, crack speed and material constants have been depicted by means of graphs.

In **chapter-2**, we have considered the problem of diffraction of normally incident SH-wave by two co-planar finite rigid strips placed symmetrically in an infinitely long isotropic elastic strip perpendicular to the lateral suface of the elastic strip. The mixed boundary value problem gives rise to the determination of the solution of the triple integral equations which finally have been reduced to the solution of a Fredholm integral equation of second kind. The equation has been solved numerically for low frequency range. Finally the elastodynamic stress intensity factors are obtained. The variations of the stress intensity factors at the tips of the rigid strips with frequency have been depicted by means of graphs.

The problem of **chapter-3** deals with the scattering of elastic wave by an edge crack. A time harmonic plane longitudinal wave is incident on a half-space containing a vertical edge crack. Both the incident field as well as the scattered field have been decomposed into symmetric and antisymmetric fields with respect to the plane of the crack so that the problem is reduced to the boundary value problem for a 90^0 wedge. In both the symmetric and antisymmetric problem, incident body waves are at first diffracted by the edge of the crack. For high frequency solution, the diffracted body wave being insignificant after a few wave lengths, the significant part of the diffracted wave is the Rayleigh wave which is reflected back from the corner of the wedge giving rise to diffracted Rayleigh wave from the crack tip. This process of reflection of surface wave from the corner of the wedge and subsequent diffraction by the crack tip continues. Considering the contribution from the incident body waves and all the reflected Rayleigh waves, stress intensity factors have been determined and their dependence on the frequency and on the angle of incidence has been depicted by means of graphs.

In the **last chapter** (chapter-4), the elastodynamic Green's functions for time harmonic radial and axial ring sources and for time harmonic torsional ring source as well in a transversely isotropic medium has been derived. The axis of material symmetry and the axis of ring source coincides. Fourier and Hankel transforms have been used to derive the solution in integral form.

Finally, stationary phase method has been used to evaluate the displacement at large distances away from the ring source. For Graphite-epoxy composite material, the displacements have been depicted by means of graphs.

With this much of introduction, we now present the thesis chapterwise. References given in the thesis do not include all the previous workers in this line. But attempt has been made to include most of them.