CHAPTER 1

GENERAL INTRODUCTION

The topic of wormhole physics occupies a distinguished place in the frontiers of theoretical physics today. Wormholes are topology changes that connect two distant asymptotically flat regions. Over the last few years, considerable amount of investigation has gone into this area of theoretical physics, following especially the seminal works of Morris, Thorne and Yurtsever [1,2]. There have been efforts that concentrated on the exciting possibility of constructing time machines or of rapid interstellar travel. The standard methods use solutions of Einstein's field equations which are shown to give rise to "exotic" stress-energy distribution at least in some region of spacetime. The exotic materials violate the energy conditions and it may be that fundamental laws of physics forbid the existence of such materials on macroscopic scales. However, with the developments of quantum field theory, the situation has changed. An early example has been provided by Zel'dovich [3] in which a quantum field leads macroscopically to an exotic equation of state. Casimir effect or squeezed vacuum states

[4-6] also lead to a stress-energy tensor that violates energy conditions. Such violations however need not always contradict the positive mass theorem [7]. Quite recently, Vollick [8] has demonstrated a remarkable result that negative energy densities may occur even in the classical regime via interaction of classical fields with gravity. Therefore, all in all, there is good reason to look for wormhole solutions even when the stress-energy is provided by classical fields.

The primary aim of this thesis is to examine the possibility of static, spherically symmetric wormhole solutions in the scalar-tensor theories such as the Brans-Dicke (BD) theory and the string modified four- dimensional gravity. As a related study, we shall also attempt to focus on the "no-hair" theorem attendent upon such theories.

In order to make the contents of the thesis self-contained as far as possible, we shall systematically present relevant basic materials in different sections. In Sec. 1.1, we describe what is known as a Morris-Thorne wormhole. Sec. 1.2 provides a survey of recent works including those that focus on the applications of wormhole physics. In Sec. 1.3, we display energy conditions relevant to wormhole scenarios. This will be

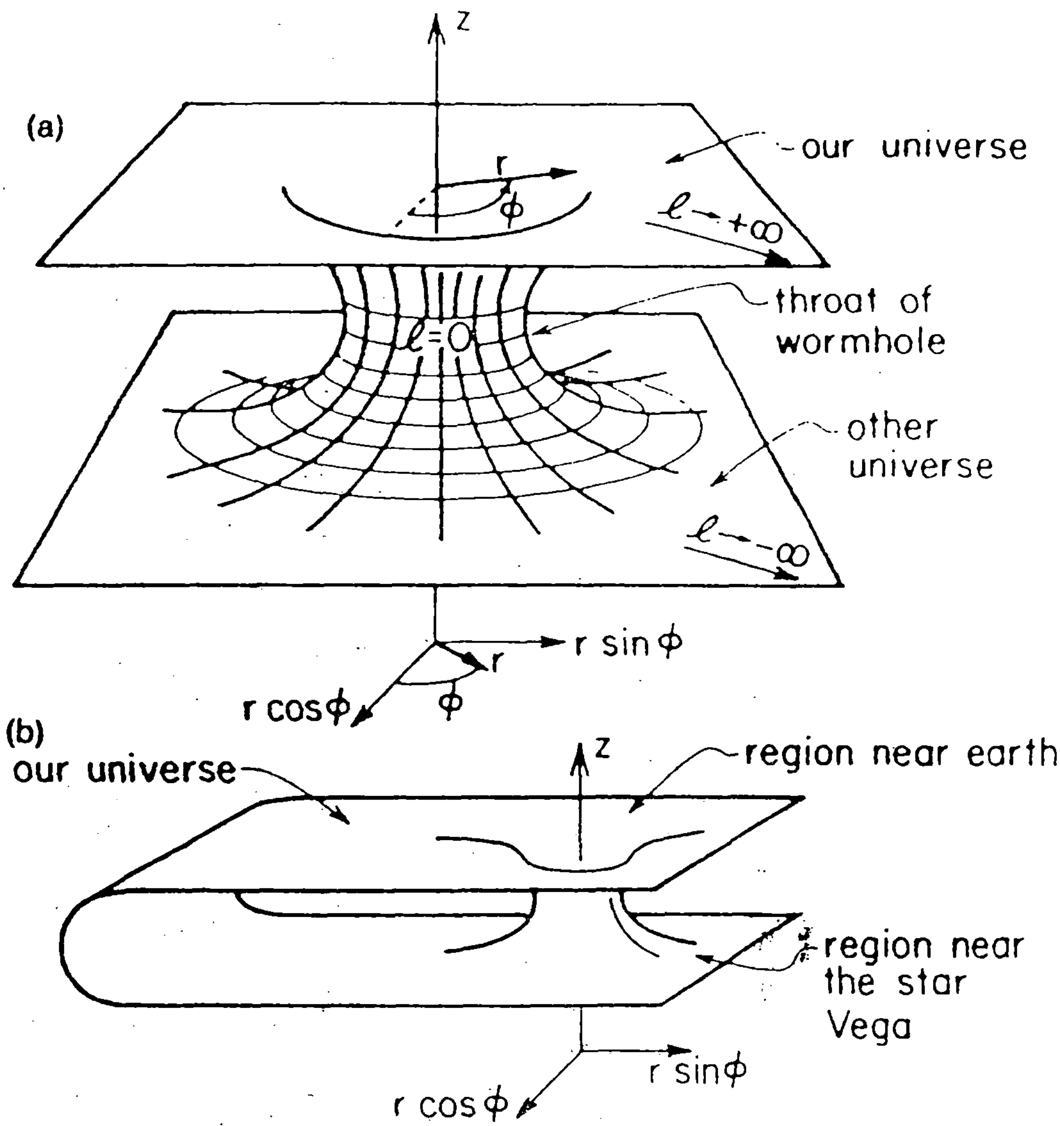

Fig. 1. (a) Embedding diagram for a wormhole that connects two different universes. (b) Embedding diagram for a wormhole that connects two distant regions of our own universe. Each diagram depicts the geometry of an equatorial ($\theta = \pi/2$) slice through space at a specific moment of time ($t = \text{const}$)

followed by a detailed statement in Sec. 1.4 of the objectives pursued in this thesis.

1.1. MORRIS-THORNE TRAVERSABLE WORMHOLE

Wormhole geometries can be embedded into the spacetime that follow as exact solutions of Einstein's equations. The embedding diagram connects two different universes or two distant regions of our own universe (Fig. 1). Only the topologies differ and Einstein field equations do not constrain the topology of a solution. Depending on the physical conditions, it is possible that a traveler may pass from one region to another through what is termed as the "throat" of the wormhole. These are called traversable wormhole. Below we shall quote its essential properties from Ref. [1] :

(i) Basic Wormhole Criteria

(1) The metric should be both spherically symmetric and static (time independent). The requirement is imposed only to simplify the calculations, and one should keep in mind that the wormhole might be unstable to spherical or nonspherical perturbations.

(2) The solution must everywhere obey the Einstein field equations. We assume the correctness of general relativity theory.

(3) To be a wormhole the solution must have a throat that connects two asymptotically flat regions of spacetime.

(4) There should be no horizon, since a horizon, if present, would prevent two-way travel through the wormhole.

(ii) Usability Criteria

(1) The tidal gravitational forces experienced by a traveler must be bearably small.

(2) A traveler must be able to cross through the wormhole in a finite and reasonably small proper time (e.g., less than a year) as measured not only by herself/himself, but also by observers who remained behind or who await her/him outside the wormhole.

(iii) Physical Criteria

The matter and fields that generate the wormhole's spacetime curvature must have a physically reasonable stress-energy tensor. It turns out that the form of the stress-energy tensor is strongly constrained by the preceding six properties. That constrained form in fact violates what we usually mean by "physically reasonable".

(2) The solution should be perturbatively stable (especially as a spaceship passes through). Enforcing this requirement would involve a time-dependent and nonspherical analysis.

(3) It should be possible to assemble the wormhole. For instance, the assembly should require both much less than the mass of the universe and much less than the age of the universe. Although not enough is known to permit a quantitative analysis, present knowledge of quantum gravity suggests that assembly *might* be possible.

(a) Form of the metric

The form of the metric is taken in the Morris-Thorne canonical form given by

$$ds^2=-e^{2\Phi}c^2dt^2+dr^2/(1-b/r)+r^2(d\theta^2+\sin^2\theta\ d\varphi^2). \quad (1.1.1)$$

Here, $\Phi=\Phi(r)$ and $b=b(r)$ are two arbitrary functions of radius only and c is the speed of light in vacuum. The function b(r) determines the spatial shape of the wormhole, so we shall call it the "shape function" and $\Phi(r)$ determines the gravitational redshift, so we shall call it the "redshift function". Notice that the radial coordinate r has special geometric significance: $2\pi r$ is the circumference of a circle centered on the wormhole's throat, and thus r is equal to the embedding-space radial coordinate. As a result, r is nonmonotonic: It decreases from $+\infty$ to a minimum value, b_o, as one moves through the lower universe of Fig. 1 toward the wormhole and into the throat; then it increases from b_o back to $+\infty$ as one moves out of the throat and into the upper universe.

We define an orthonormal static coordinate system for which the basis vectors are

$$e_{\hat{t}} = e^{-\Phi}e_t, \quad e_{\hat{r}} = (1 - b/r)^{1/2}e_r, \quad (1.1.2)$$

$$e_{\hat{\theta}} = r^{-1} e_{\theta}, \qquad e_{\hat{\varphi}} = (r \sin \theta)^{-1} e_{\varphi} \tag{1.1.3}$$

In this basis the metric coefficients take on their standard, special relativity forms,

$$g_{\hat{\alpha}\hat{\beta}} = e_{\hat{\alpha}} \cdot e_{\hat{\beta}} = \eta_{\hat{\alpha}\hat{\beta}} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1.1.4}$$

The nonzero components of the Einstein tensor in this proper reference frame are

$$G_{\hat{t}\hat{t}} = \frac{b'}{r^2}, \tag{1.1.5}$$

$$G_{\hat{r}\hat{r}} = -\frac{b}{r^3} + \frac{2(1 - b/r)\Phi'}{r}, \tag{1.1.6}$$

$$G_{\hat{\theta}\hat{\theta}} = G_{\hat{\varphi}\hat{\varphi}} = \left(1 - \frac{b}{r}\right)\left(\Phi'' - \frac{b'r - b}{2r(r - b)}\Phi' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r - b)}\right). \tag{1.1.7}$$

The stress-energy tensor $T_{\hat{\mu}\hat{\nu}}$ must have the same algebraic structure as the $G_{\hat{\mu}\hat{\nu}}$ and we take them as

$$T_{\hat{t}\hat{t}} = \rho(r)c^2, \qquad T_{\hat{r}\hat{r}} = -\tau(r), \qquad \text{and} \qquad T_{\hat{\theta}\hat{\theta}} = T_{\hat{\varphi}\hat{\varphi}} = p(r). \tag{1.1.8}$$

where $\rho(r)$ is the total density of mass-energy that they measure (in units of g/cm^3), $\tau(r)$ is the tension per unit area that they measure in the radial direction (i.e., it is the negative of the radial pressure and has units dyn/cm^2) and $p(r)$ is the pressure (in dyn/cm^2) that they measure in lateral directions (directions orthogonal to radial).

We can rewrite Eqs. (1.1.5)-(1.1.7) in a form

$$\rho = \frac{b'}{8\pi G c^{-2} r^2}, \tag{1.1.9}$$

$$\tau = \frac{b/r - 2(r - b)\Phi'}{8\pi G c^{-4} r^2} \tag{1.1.10}$$

$$p = \frac{r}{2}\left[\left(\rho c^2 - \tau\right)\Phi' - \tau'\right] - \tau. \tag{1.1.11}$$

Clearly, the choices of $b(r)$ and $\Phi(r)$ determine the stress-energy tensor required to construct the Morris-Thorne wormhole.

(b) Embedding

Let us consider the line element for an equatorial slice,

$\theta=\pi/2$, and t=constant from Eq. (1.1.1). It is given by

$$ds^2 = (1 - b/r)^{-1}dr^2 + r^2 d\varphi^2. \qquad (1.1.12)$$

We shall remove this slice and embed it in Euclidean space for

which the metric has the form

$$ds^2 = dz^2 + dr^2 + r^2 d\varphi^2. \qquad (1.1.13)$$

The embedded surface will be axially symmetric described by a single function z=z(r). On that surface the line element will be

$$ds^2 = \left[1 + \left(\frac{dz}{dr}\right)\right]dr^2 + r^2 d\varphi^2. \qquad (1.1.14)$$

Identifying Eq. (1.1.14) with Eq. (1.1.12), we find that the embedded surface z=z(r) satisfies

$$\frac{dz}{dr} = \pm\left(\frac{r}{b(r)} - 1\right)^{-1/2}. \qquad (1.1.15)$$

(c) Wormhole throat

The minimum radius of the wormhole, called its throat, occurs at a radial value $r=r_o$ for which $b(r_o)=r_o$. Also, the proper radial distance is defined by

$$l(r) = \pm \int_{b_o}^{r} \frac{dr}{[1 - b(r)/r]^{1/2}} \qquad (1.1.16)$$

In order that l(r) is well behaved everywhere, we require that

$$(1 - b/r) \geq 0 \text{ throughout spacetime.} \qquad (1.1.17)$$

Far from the throat in both radial directions space must become asymptotically flat; i.e., $dz/dr = \pm(r/b - 1)^{-1/2}$ must approach zero as $l \to \pm \infty$; i.e., $b/r \to 0$, as $l \to \pm \infty$. Note that Eqs. (1.1.15) and (1.1.16) imply that for the embedded wormhole

$$\frac{dz}{dl} = \pm\left(\frac{b}{r}\right)^{1/2} \text{ and } \frac{dr}{dl} = \pm\left(1 - \frac{b}{r}\right)^{1/2}. \qquad (1.1.18)$$

<u>(d) Absence of a horizon and tidal forces</u>

A wormhole will be traversable if it does not possesses any horizon. This implies that $\Phi(r)$ must be everywhere finite. The tidal gravitational forces as measured by a static observer depend on the values of the curvature tensor in the proper reference frame at that point. The equation is given by the well known equation for geodesic deviation. For the metric (1.1.1) the nonvanishing curvature components are given by $R_{\hat{0}\hat{1}\hat{0}\hat{1}}$, $R_{\hat{0}\hat{2}\hat{0}\hat{2}}$, $R_{\hat{0}\hat{3}\hat{0}\hat{3}}$, $R_{\hat{1}\hat{2}\hat{1}\hat{2}}$, $R_{\hat{1}\hat{3}\hat{1}\hat{3}}$ and $R_{\hat{2}\hat{3}\hat{2}\hat{3}}$. In a Lorentz boosted frame (*) the nonvanishing curvature will be

$$R_{0^*1^*0^*1^*}=R_{0101}, \quad R_{0^*k1^*k^*}=\cosh\alpha\ \sinh\alpha(R_{0k0k}+R_{1k1k}), \qquad (1.1.19)$$

$$R_{0^*k0^*k^*}=R_{0k0k}+\sinh^2\alpha(R_{0k0k}+R_{1k1k}), \qquad (1.1.20)$$

$$R_{1^*k1^*k^*}=R_{1k1k}+\sinh^2\alpha(R_{0k0k}+R_{1k1k}), \qquad (1.1.21)$$

$$\cosh\alpha=(1-v^2)^{-1/2}, \quad \sinh\alpha=v(1-v^2)^{-1/2}, \qquad (1.1.22)$$

and R_{k1k1} for k=2,3. For a wormhole to be traversable the tidal acceleration must be finite both in the static and Lorentz boosted frame.

(e) Flaring out condition

The requirement that the wormhole be connectible to asymptotically flat spacetime entails at the throat that the embedding surface flare outward. This requires that the embedding function r(z) must satisfy $d^2r/dz^2>0$ at or near the throat, r=b. That is

$$\frac{d^2r}{dz^2} = \frac{b - b'r}{2b^2} > 0. \qquad (1.1.23)$$

We can define a dimensionless function,

$$\zeta \equiv \frac{\tau - \rho c^2}{|\rho c^2|} = \frac{b/r - b' - 2(r - b)\Phi'}{|b'|}, \qquad (1.1.24)$$

and rewrite it in the form

$$\zeta = \frac{2b^2}{r|b'|}\left(\frac{d^2r}{dz^2}\right) - 2(r - b)\frac{\Phi'}{|b'|}. \qquad (1.1.25)$$

The finiteness of b' and the fact that $(r-b)\Phi'\to 0$ at the throat enables us to rewrite the flaring out condition as

$$\zeta_0 = \frac{\tau_0 - \rho_0 c^2}{|\rho_0 c^2|} > 0 \qquad (1.1.26)$$

at or near the throat, $r=b=b_0$.

Matter field satisfying $\tau>\rho c^2$ is called "exotic". One of the reason is that the signal speed $|\tau/\rho|^{1/2}$ in the exotic material exceeds the vacuum speed of light. An equivalent condition of $\tau > \rho c^2 > 0$ can be obtained by going over to a Lorentz boosted frame. It then follows that it is sufficient to have $\rho c^2 < 0$ for matter to be exotic.

So far we have reproduced the basic outline of the Morris-Thorne wormhole as developed in Ref. [1]. Whenever necessary other aspects will be provided at appropriate places.

1.2. ENERGY CONDITIONS

Energy conditions are supposed to provide constraints on classical or quantum wormholes threaded by classical or quantum fields. There exist several pointwise or global energy conditions [4]. For example, the weak energy condition (WEC) states that $T_{\hat{\mu}\hat{\nu}}\xi^{\mu}\xi^{\nu} \geq 0$ for any timelike vector ξ_{ν}. Written out in full, this implies

$$\rho \geq 0, \quad \rho + \tau \geq 0, \quad \rho + p \geq 0. \qquad (1.2.1)$$

For quantum fields, Ford and Roman [9] have proposed, on the basis of certain assumptions, an inequality that constrains the magnitude of negative energy density at the throat of a traversable wormhole. A fundamental assumption for quantum wormholes is that the stress-energy of the spacetime is a renormalized expectation value of the energy momentum operator in some quantum state, say, $|\psi>$. In the literature [10,11], one actually considers field equations of semiclassical gravity in the form $G_{\mu\nu}=8\pi<\psi|T_{\mu\nu}|\psi>$. However, some doubts have been raised, notably by Unruh [12], as to whether field equations in this form could be an exact description of gravity [13]. On the other hand, quantized source fields obey well defined uncertainty relations and it is expected that uncertainty in the source would induce uncertainty in the gravidynamic variables and in the light cone structure of spacetime [14,15]. If the source is taken as $<T_{\mu\nu}>$, such fluctuations would not occur. Despite these questions, it must be emphasized that field equations in the above form provide a very good approximation in many physical situations, especially in the description of early universe [16].

There also exist classical fields playing the role of "exotic matter" that violates the WEC, at least at the throat of the wormhole. Examples are provided by the stress-energy tensors occurring in theories where the action contains: $R+R^2$ terms [17], an antisymmetric 3-form axion field coupled to scalar fields [18], minimally coupled fields with a self-interacting potential [19]. Other theories include string inspired 4-dimensional gravity coupled nonminimally to a scalar field [20], Zee's induced gravity [21] and the BD scalar-tensor theory [22]. Most of the analyses dealt with dynamic rather than static wormholes. Unfortunately, investigations aimed at finding out static wormhole solutions in the alternative theories of gravity such as BD scalar-tensor theory have been relatively few [23,24]. On the other hand, it is well known that in the limit $|\omega|\to\infty$, BD theory goes over to Einstein's general relativity (GR). Hence, it is desirable that a thorough investigation is carried out with regard to the possible existence of wormhole solutions in the theories of gravity coupled to scalar and vector fields.

It has recently been shown by Vollick [8] that an interaction stress-energy tensor between matter fields does not satisfy WEC although the interacting matter fields individually do. It is this interaction energy that maintains a wormhole with a scalar field. It has also been shown recently by Anchordoqui, Bergliaffa and Torres [25] that the wormhole throat in the BD theory in the presence of matter need not be necessarily threaded with exotic matter.

1.3. APPLICATIONS OF WORMHOLE PHYSICS

There have been attempts to interpret and predict new physical phenomena in terms of wormhole scenarios. Cramer, Forward, Morris, Visser, Benford and Landis [26] attempted to interpret the phenomenon of gravitational lensing in terms of Visser wormholes [4] which are born out of cut-and-paste surgery. Take two copies of Schwarzschild spacetime, and remove from each manifold the four-dimensional regions described by

$$\Omega_{1,2} \equiv (r_{1,2} \le a \mid a > 2M) \qquad (1.3.1)$$

where *a* is a constant. The resulting manifolds have boundaries given by the timelike hypersurfaces

$$\partial\Omega_{1,2} \equiv (r_{1,2} = a \mid a > 2M). \qquad (1.3.2)$$

Now *identify* these two timelike hypersurfaces (i.e., $\partial\Omega_1 \equiv \partial\Omega_2$). The resulting manifold *M* is geodesically complete and possesses two asymptotically flat regions connected by a traversable Lorentzian wormhole. The throat of the wormhole is at $\partial\Omega$.

Because *M* is piecewise Schwarzschild, the Einstein tensor is zero everywhere *except* at the throat, where it is formally singular: the Einstein tensor is a Dirac distribution on the manifold. Using the field equations, the surface stress-energy tensor can be calculated in terms of the jump in the second fundamental form across $\partial\Omega$. This prescription is known as the "thin-shell formalism" [4].

Visser [4] suggested a particular thin-shell wormhole configuration, a flat-space wormhole that is framed by "struts" of an exotic material, a variant of the cosmic string solutions of Einstein's equations [27,28]. To satisfy the Einstein field equations the cosmic string framing Visser wormholes must have a *negative* string tension [29] of $-1/4G$ and therefore a negative mass density. However, for the total mass of the wormhole system, the negative mass density of the struts should be combined with the effective positive mass density of the wormhole's gravitational field. The overall object could, depending on the details of the model, have positive, zero, or negative net external mass. Note that, in hypothesizing the existence of such a wormhole, one has to abandon the averaged null energy condition [1,2]. Indeed, in a generic curved spacetime, it is possible to prove that the averaged null energy condition fails [30]. Therefore, the hypotheses underlying the positive mass theorem no longer apply and there is nothing, in principle, to prevent the occurrence of negative total mass [31].

If a particle with positive electric charge passes through such a wormhole, its lines of force, threading through the wormhole aperture, give the entrance mouth an effective positive charge (flux lines radiating outward) and give the exit mouth an effective negative charge (flux lines converging inward). Similarly, when a massive object passes through the wormhole, the same back-reaction mechanism might cause the entrance mouth to gain mass and the exit mouth to lose mass [32]. Now let us consider a stable Visser wormhole with near-zero mass residing in the mass-energy-rich environment of the early universe. The expected density fluctuations of the early universe suggest that the separated wormhole mouths will reside in regions of differing mass density, leading to a mass flow between the regions they connect. As this mass passes through the wormhole, the entrance mouth will gain mass while the exit mouth will lose mass by the same amount. Soon, if the mass flow continues, the exit wormhole mouth will acquire a net negative mass.

This will lead to a gravitational instability, since the positive mass mouth will attract more mass through its aperture while the negative mass mouth will gravitationally repel nearby mass [33]. Thus the positive-negative imbalance will be fed by gravity and will continue to grow. If this process proceeds without interruption, the exit wormhole mouth might develop a stellar-scale negative mass. Visser wormholes, therefore, provide at least one motivation for seriously considering the possible existence of naturally occurring astronomical objects of sizable negative mass.

Negative mass objects, while repelling all nearby mass (positive or negative), would themselves be attracted [33] by the mass of a nearby galaxy and might form part of a galactic halo. They would have unusual gravitational properties that could produce detectable gravitational lensing effects. These lensing effects, for the same absolute mass, are of the same magnitude as those recently detected for massive cosmic halo objects (MACHO's) [34,35]. Cramer *et al* [26] examined in detail the lensing effects of gravitationally negative anomalous compact halo objects (GNACHO's) and they recommend that MACHO search data be analyzed for GNACHO's.

Another interesting application of wormhole topology lies in the field of cosmology. For example, Hochberg and Kephart [36] attempted to solve cosmological problem in terms of dynamic wormhole "handles" which connect a Friedmann universe onto itself. The present observation of isotropicity and uniformity in temperature of the cosmic microwave background radiation (CMBR) can not be explained by the standard big bang model. The reason is that the CMBR is received from regions which were not in causal contact at the time of last scattering. This is commonly known as the horizon problem and the generally accepted resolution invoke an inflationary period in the early universe. On the other hand, as we saw in Sec. 1.1, wormhole solutions require that the WEC be violated in the vicinity of the wormhole throat. It is also known that WEC violating quantum fields occur in the gravitationally squeezed state [5,6]. Therefore, the best place to explore wormhole physics is in the quantum or semiclassical regime during the planck epoch in the early universe. On these grounds, Hochberg and Kephart [36] suggest another resolution which envisages that the universe was populated with traversable wormholes connecting otherwise causally disconnected regions of the early universe leading to a rapid thermalization which has persisted in the observed CMBR today.

Most recently González-Díaz [37] has proposed the idea of spacetime tunnels which are wormholes and "ringholes" existing naturally in the universe. Two-sphere tunnel topology give rise to wormholes and two-torus topologies give rise to what are called ringholes. Embedding them in the Friedmann universe leads to many possible observable effects including lensing, frequency shift of emitting sources, discontinuous change of background temperature, broadening and intensity enhancement of spectral lines, as well as an unexpected increase of luminosity of any object at the tunnel's throat.

Cavaglià, de Alfaro and de Felice [38] have suggested that gravitational Euclidean wormholes (+,+,+,+) obtained through coupling of gravity to electromagnetism can be interpreted as a tunneling to a hyperbolic baby universe. This Euclidean instanton can also be interpreted as a static wormhole that joins two asymptotically flat spaces of a Reissner-Nordström type solution. Euclidean wormholes in the early universe lead to consequences such as quantum decoherence [39-41] and the vanishing of cosmological constant [42,43].

On the theoretical side some notable works are worth mentioning. Coule and Maeda [18] have discussed wormhole solutions in a theory with an axion field coupled to scalar fields. Earlier works involving the 3-form axion field carried out by Giddings and Strominger [44]. Lee [45] obtained wormhole solutions for a spontaneously broken complex field. Brown, Bergess, Kshirsagar, Whiting and York [46] used an Euclideanised scalar field and Hosoya and Ogura [47] used SU(2) Yang-Mills theory to obtain wormhole solution. Hochberg, Popov and Sushkov [10] obtained Lorentzian wormhole solutions of semiclassical Euclidean field equations coupled to a quantum scalar field. Schein and Aichelburg [11] constructed static axisymmetric traversable wormhole solutions of two charged shells kept in equilibrium by their electromagnetic repulsion. Kar and Sahdev [48] have obtained evolving Lorentzian wormholes for which the time spent over which the WEC satisfied can be made as large as one wishes. Evolving wormholes were also considered by Roman [49], Wang and Letelier [50]. Barceló, Garay, González-Díaz and Marugán [51] have constructed a quantum theory for asymptotically anti-de Sitter wormholes. Dynamic Euclidean wormhole solutions have been obtained as a solution of the Wheeler-DeWitt equation in the context of BD theory have been obtained by Xiao, Carr and Liu [52]. Kar [53] has obtained exact solutions of the string equations of motion in a specific Lorentzian wormhole background. Hochberg and

Visser [54] have investigated into the geometric structure of generic static traversable wormhole throat. They have also derived generalized theorems regarding violations of the energy conditions which apply to a wide-ranging class of wormholes. Balbinot, Barrabès and Fabbri [55] have discussed the traversability and stability of wormholes consisting of closed Friedmann-like regions connected by Reissner-Nordström black holes.

1.4. DETAILED OBJECTIVES

As already mentioned before, our primary concern in this thesis would be to examine the possibility of static, spherically symmetric wormhole solutions in the scalar-tensor theories of gravitation. For the vacuum BD theory, we consider both the original (Jordan) frame and the conformally rescaled (Einstein frame) frame. Our aim is to find out the exact range of the coupling parameter ω for which such wormholes are possible. We shall also investigate the question of traversability. These are accomplished in Chapter 2. A further realm of investigation is provided by the now well known string theory which, in the low-energy limit, provides a four-dimensional theory of gravity characterized by an action containing couplings among gravity, dilaton and Maxwell fields. We shall be particularly interested in the wormhole solutions when the kinetic term in the action has a negative sign. These stringy wormholes are investigated in Chapter 3.

In a theory where gravity couples to scalar fields, a pertinent inquiry is whether the no-hair theorem is satisfied. In the context of BD theory, we shall examine, in Chapter 4, the no-hair theorem and focus on the limiting process involved. Carrying the analysis further, we shall also take a cue from Bekenstein's no-hair theorem and compare the scalar dressing of black holes in the BD and conformally coupled scalar field theory. This is carried out in Chapter 5.

Very recently, Horowitz and Ross [56,57] have proposed the idea of naked black holes (NBH). We shall present, in Chapter 6, two NBH solutions coming from dilaton-Maxwell gravity which satisfy WEC.

For convenience, the chapters are divided into subsections as displayed in the CONTENTS . Each chapter is provided with an introduction and concluded with a summary and/or remarks. All the references are appended in the end. A general summary of the contents is also annexed.