# SUMMARY

In the introductory chapter, we have attempted to provide basic materials in order to make the thesis self-contained as far as possible. The focus has entirely been on classical wormholes. Since each chapter ends with a summary of its own, we shall only report here the salient features of our results.

For the first time, we have found the correct range of $\omega$ in the Jordan frame viz., $-3/2<\omega<-4/3$ for static, spherically symmetric wormholes in the BD theory. This range is consistent with the pN value of $C(\omega)$ and allows one to go over to the conformally rescaled Einstein frame. In this frame, wormhole solutions are not permitted at all since all the energy conditions are satisfied. Magnano and Sokołowski [62] have argued that only the Einstein frame variables are physical due to the fact the solutions are stable. Our results indicate that everything is not lost in the Jordan frame : There *does* exist a narrow interval of $\omega$ for which stable wormhole solutions are available also in the Jordan frame.

Recent investigations by Vollick [8] and Anchordoqui *et al* [25] do buttress our conclusion above. Ref. [8] shows that

interaction between gravity and matter fields gives rise to negative energies which maintain a classical wormhole. This is certainly the case in the Jordan frame where $\rho<0$. Ref. [25] shows that the presence of extra matter fields do not destabilize a static wormhole. This result is quite consistent with the fact that the range $-3/2<\omega<-4/3$ leaves the option to go over to a stable Einstein frame version open. Ref. [25] also contains a number of differing viewpoints about the physicality of Jordan or Einstein frame variables. At least in the context of wormhole physics, we now see that there is a common ground of agreement symbolized in the range of $\omega$ derived above.

An exceedingly important branch of physics is the string theory, which, in the low-energy limit gives rise to a scalar-tensor theory of 4-dimensional gravity with additional electromagnetic fields. We have studied charged Neveu-Schwarz and Turyshev solutions and shown that they represent wormholes with exactly opposite features. This result gives us an idea as to how the wormhole solutions in string theory would behave. In two consecutive Chapters (4,5), we have studied the "no-hair" theorems which find relevance in any type of scalar-tensor theory. First, we have focused on a problem relating to the limit processes involved and the need to remain aware of it in order to avoid meaningless conclusions. Secondly, we have proposed a modified version of Saa's "no-hair" conjecture which we believe to be more exact.

We next go over to a re-examination of the status of no-hair theorem in the case of BD solutions. The cue has been provided by Bekenstein's novel no-hair theorem stated in Chapter 5. Its inapplicability in the conformal scalar field can allow such hairs to exist around black holes. However, conformal solutions are distinct from BD solutions with $\omega$=constant. Nonetheless, these two types of solutions show remarkable similarity in that both violate Bekenstein's theorem and consequently, both allow scalar hairs to exist provided the right range of values for the parameters are chosen. This result is interesting in its own right and it poses a more basic question : For $(C+1)/\lambda = 2$, can we regard the BD Class I solution as representing a WEC violating black hole? If we insist that the finiteness of curvature has to be Lorentz invariant, then the answer is in the negative. If we relax Lorentz invariance, then the answer is positive.

The final Chapter (Chapter 6) deals with a very recent proposal of naked black holes (NBH) by Horowitz and Ross

[56,57]. We have presented two examples from string-inspired scalar-tensor theory in the form of Chan-Mann-Horne solutions. The interesting point is that this solution satisfies WEC. Its importance lies in the fact that in the string modified gravity, appropriate singularity theorems for black holes are not yet available. One may adopt a conservative stance and adhere to the old singularity theorems and attempt to find out as many WEC satisfying NBH as possible.

The contents of this thesis reveal that scalar-tensor theories of gravity have more secrets in their fold than have so far been discovered. These theories are not only useful for the interpretation of many physical phenomena but also extremely interesting from the theoretical point of view.