CHAPTER 6

ON NAKED BLACK HOLES

6.1. INTRODUCTION

In the context of scalar-tensor theory of gravity, a new effect has recently been discovered by Horowitz and Ross [56,57]. This discovery exposes yet another secret of GR. They assumed that there exist what may be called naked black hole (NBH) solutions in a wide variety of theories involving scalar fields including the low-energy limit of the string theory. These are black holes for which the area of the event horizon is large and all curvature invariants are small near the horizon. Nevertheless, any object which falls in experiences enormous tidal forces outside the horizon. The tidal forces are given by the Riemann tensor in a frame associated with ingoing geodesics. Thus, for these black holes, the geodesic components are much larger than the invariants constructed from them.

The examples considered by Horowitz and Ross [56,57] are charged black holes either at or near extremality. These black holes are called NBH as they violate the spirit of cosmic censorship in that large curvatures are visible outside of the event horizon. The analysis by Horowitz and Ross [56,57] also reveal that the size of the components of the Riemann tensor in the geodesic frame is determined by proper time remaining along the geodesic before the singularity is reached. Also, the large contributions to the Riemann tensor come from the Ricci tensor and hence from the stress-energy of the matter field. In the case of a spherically symmetric field, only the dilaton field in dilaton gravity can have the nearly null stress-energy needed to produce the type of curvature required. It is therefore expected that NBH solutions occur only in theories with scalar fields. The occurrence of large tidal forces outside the horizon may have considerable implications, both for cosmic censorship hypothesis and black hole information puzzle.

The examples of NBH considered in Ref. [56] include Neveu-Schwarz charged black hole solution of the string modified four-dimensional gravity. The dilatonic and $U(1)^2$ black holes follow as a special case. In arriving at the general conditions for NBH, Horowitz and Ross had to invoke the condition of positivity of energy: $G_{oo} = \rho \geq 0$. On this point, a remark is in order. Recently, Kar [82] has shown that both Neveu-Schwarz and its dual solution known as the magnetic black hole violate the WEC given by $T_{\mu\nu}u^{\mu}u^{\nu} \geq 0$ for any timelike vector u^{μ}. For a diagonal $T_{\mu\nu} \equiv [\rho(r), \tau(r), p(r), p(r)]$, the WEC reduces to the inequalities [4] $\rho \geq 0$, $\rho + \tau \geq 0$, $\rho + p \geq 0$. Violation of any of the inequalities must be regarded as a violation of WEC. Therefore, in a static frame, it may so happen that $\rho \geq 0$ but either $\rho + \tau < 0$ or $\rho + p < 0$. It is then possible to go over to a Lorentz boosted frame in which the transformed energy density $\rho' < 0$. Thus the positivity of energy density is no longer a Lorentz covariant statement if WEC is violated. According to Morris and Thorne [1], it is perhaps only a small step to arrive at the conclusion that the static observer also sees a negative energy density. This situation warrants that some caution be excercised in using the condition of positivity of energy in the search for NBH solutions under WEC violating circumstances. The above ambiguity immediately provokes a natural inquiry: Does there exist NBH solutions for which WEC is already satisfied? The aim of this chapter is to address this question and the answer appears to be in the affirmative which obviously places the idea of NBH on firmer grounds.

In Sec. 6.2, we shall rewrite the conditions for NBH in a slightly more convenient way that allows us to discern relevant factors. In Sec. 6.3, we shall consider the Chan-Mann-Horne solution of dilaton-Maxwell gravity in the Einstein frame and show that it does represent NBH. The same solution in the string frame also represents NBH and is dealt with in Sec. 6.4. Finally, some concluding remarks are appended in Sec. 5.5.

6.2. NBH CONDITIONS

Horowitz and Ross consider the generic metric in d dimensions which also include black p-branes (p=d-n-3):

$$ds^2=-\frac{F(r)}{G(r)}\,dt^2+\frac{dr^2}{F(r)}+R^2(r)d\Omega_{n+1}+H^2(r)dy^i dy_i \qquad (6.2.1)$$

where $i=n+3,\cdots,d-1$. We ignore p-branes in this chapter and take d=4, n=1.

As mentioned before, the basic idea is to obtain conditions which ensure that the curvature components in a static frame are vanishingly small at or near the horizon and that these are large in a freely falling frame near the horizon. The nonvanishing curvature components in a static orthonormal basis are R_{0101}, R_{0202}, R_{0303}, R_{1212}, R_{1313} and R_{2323}. It has been shown in Ref. [56] that the difference between the Lorentz boosted frame and the static frame is the same for all curvature components and is proportional to $(R_{0k0k}+R_{1k1k})$, where k=2,3. Hence, it is enough to consider the increment in just one of the curvature components, say, in R_{0202}. In the boosted (^) frame, it becomes,

$$R_{\hat{0}\hat{2}\hat{0}\hat{2}} = -\frac{1}{R}\left[R''(E^2G-F) + \frac{R'}{2}(E^2G'-F')\right] \qquad (6.2.2)$$

where $E^2=(F/G)(1-v^2)^{-1}$ and primes on the right denote derivatives with respect to r. It has been stated in Ref. [56] that the terms proportional to E^2 correspond to the enhancement of the curvature in the geodesic frame over the static frame. This statement may be slightly rephrased in order to find out exactly which piece we need to have enlarged. Note that the conserved energy E^2 can be decomposed as

$$E^2 = (F/G) + v^2(1-v^2)^{-1}(F/G) = E_s^2 + E_{ex}^2. \qquad (6.2.3)$$

The first term represents the value of E^2 in the static frame (E_s^2) and the second term E_{ex}^2 represents the enhancement in E_s^2 due to geodesic motion. Incorporating this, we can also decompose $R_{\hat{0}\hat{2}\hat{0}\hat{2}}$ as follows:

$$R_{\hat{0}\hat{2}\hat{0}\hat{2}} = -\frac{1}{R}\left[\frac{R}{2}\left(E_s^2 G' - F'\right)\right] - \frac{1}{R}\left[R''G + \frac{R'G'}{2}\right]E_{ex}^2$$

$$= R^{(s)}_{0202} + R^{(ex)}_{0202}. \qquad (6.2.4)$$

It is easy to verify that the term $|R^{(s)}_{0202}|$ actually represents the curvature component in the static frame, viz., $R^{(s)}_{0202} = R_{0202}$. Thus, only the term $|R^{(ex)}_{0202}|$ represents overall enhancement in curvature in the Lorentz boosted frame over the static frame. This overall enhancement consists of the product of two factors. The first, which comes from the geodesic motion itself, is in the form of E_{ex}^2. The second is the coefficient of E_{ex}^2 and it is precisely *this* part that needs to be enlarged in order to make $|R^{(ex)}_{0202}|$ as large as we please. Thus, one has the NBH condition (in Planck units)

$$\left|\frac{1}{R}\left(R''G + \frac{R'G'}{2}\right)\right| > 1 \tag{6.2.5}$$

outside the horizon. The other NBH condition can not come from the imposition of positive energy density criterion as we want to investigate solutions for which the positivity requirement is already Lorentz covariantly satisfied, whatever be the values of the free parameters. It has to come directly from the definition of NBH itself: The free parameters in the solution be so choosen as to make the static curvature components small at the horizon. We must emphasize that this rephrasing does not essentially alter the original analysis of Horowitz and Ross but helps us to clearly identify the factors contributing to the enhancement in curvature.

In fact, the expression on the left side of the inequality (6.2.5) is exactly the one that occurs also in the coefficient of E^2 in Eq.(6.2.2), but the difference now is that, in Eq.(6.2.4), E_{ex}^2 is *not* a conserved quantity. The enhanced tidal force in the proper orthonormal frame can be made large simply by making E_{ex}^2 large. Therefore, in order to make the analysis meaningful, we must restrict the class of geodesics, as has actually been done in Ref.[56], to the one for which the conserved energy E^2 is of order unity. In that case, the maximum values of E_s^2 and E_{ex}^2 are also unity. This may be achieved by recalling that $v^2=1-E^{-2}(F/G)$ and considering geodesics that start with small velocity ($v\cong 0$) at some point ($r=r_o$), where (F/G) is of order unity. As the object approaches the horizon, $(F/G)\to 0$ so that $v\to 1$, as expected. With this understanding, let us now investigate a couple of specific examples.

6.3. CHAN-MANN-HORNE (CMH) SOLUTION IN THE EINSTEIN FRAME

This electric solution [99] represents asymptotically non-flat black hole solutions in the dilaton-Maxwell gravity. We shall consider the solution in the Einstein as well as in the string frames. The two frames are related to each other by $g^E_{\mu\nu}=g^S_{\mu\nu}e^{-2\phi}$ where ϕ is the massless dilaton field. The string frame is defined as the one in which the variables $(g^S_{\mu\nu},\phi)$ appear in the action given originally by Brans and Dicke in their scalar-tensor theory [63].

In the Einstein frame, the metric is

$$ds^2 = -\frac{F(r)}{G(r)}\,dt^2 + \frac{dr^2}{F(r)} + R^2(r)d\Omega^2 \qquad (6.3.1)$$

$$F = \left(\frac{1}{4} - \frac{\gamma^2 M}{r^2}\right),\quad G = \frac{\gamma^4}{4r^2},\quad R = r \qquad (6.3.2)$$

while the dilaton and Maxwell fields are

$$\phi(r) = (-1/2)\,\ln\,(2Q^2) + \ln\,r \qquad (6.3.3)$$

$$F_{01} = \frac{Qe^{2\phi}}{r\gamma^2}. \qquad (6.3.4)$$

The dilaton rolls from $-\infty$ to $+\infty$ as r changes its value from 0 to ∞. The horizon radius occurs at $r_h^2 = 4\gamma^2 M$ where $(F/G)=0$, and it is easy to verify that all the WEC inequalities are satisfied for $r \geq r_h$. We are considering geodesics that start at $r = r_o = (\gamma^4 + 4\gamma^2 M)^{1/2}$ where $(F/G)=1$.

The non-vanishing components of the static curvature are

$$R_{0202} = R_{0303} = \frac{1}{4r^2} \qquad (6.3.5)$$

$$R_{1212} = R_{1313} = -\frac{\gamma^2 M}{r^4} \qquad (6.3.6)$$

$$R_{2323} = \frac{3}{4r^2} + \frac{\gamma^2 M}{r^4} . \tag{6.3.7}$$

At the horizon, all curvature components are of the order of r_h^{-2}. Hence, the horizon area will be large and the static curvature will be small if $r_h^2 >> 1/2$.

It may be seen from Eqs.(6.3.5)-(6.3.7) that the Chan-Mann-Horne solution has a remarkable feature: $R_{0k0k}+R_{1k1k}=0$ at the horizon and non-zero outside. This implies that the magnitudes of tidal forces are more outside the horizon than what they are at the horizon. This is already an indication that the solution may serve as a natural candidate for NBH.

The enhancement condition, inequality (6.2.5), gives, near the horizon,

$$\left|\frac{G'}{2r}\right| > 1 \rightarrow r_h^2 < \frac{\gamma^2}{2} . \tag{6.3.8}$$

Thus, finally, the NBH conditions together yield $(1/2) << r_h^2 < \frac{\gamma^2}{2}$. This range automatically implies that $\gamma >> 1$ indicating a large value of r_h or a large static black hole for a given M.

6.4. CMH SOLUTION IN THE STRING FRAME

By performing conformal rescaling via the dilaton field ϕ, the CMH solution in the string frame can be written as [82]

$$ds^2=-(r^2/\gamma^4)(1-Ar^{-1})dt^2+(1-Ar^{-1})^{-1}dr^2+r^2d\Omega^2, \qquad (6.4.1)$$

where $A=2\sqrt{2}\gamma^2 M/Q$. The string coupling e^{ϕ} changes from 0 to ∞ as r goes from 0 to ∞. The functions F,G and R are given by

$$F=1-Ar^{-1},\ G=\gamma^4 r^{-2},\ R=r. \qquad (6.4.2)$$

The horizon occurs at r_h = A and it has been shown in Ref.[82] that all the WEC inequalities are satisfied for $r\geq r_h$. The geodesics start from $r = r_o = (1/2)[A\pm(A^2+4\gamma^4)^{1/2}]$ and the non-vanishing static curvature components turn out to be

$$R_{0101}= -\frac{A}{2r^3} \qquad (6.4.3)$$

$$R_{0202}= R_{0303}= \frac{1}{r^2} - \frac{A}{2r^3} \qquad (6.4.4)$$

$$R_{1212}=R_{1313}= -\frac{A}{2r^3} \qquad (6.4.5)$$

$$R_{2323} = \frac{A}{r^2} . \tag{6.4.6}$$

These components are of the order of r_h^{-2} near the horizon, just as in the previous case and they will be small if $r_h^2 >> 1$. To compute the enhancement in the boosted frame, notice once again that there follows the same remarkable result, viz., $R_{0k0k} + R_{1k1k} = 0$ at $r = r_h$, where $k=2,3$. The inequality (6.2.5) gives $r_h^2 < \gamma^2$. So, finally, one lands up with the range $1 << r_h^2 < \gamma^2$. What happens to the horizon area $4\pi A^2$? It is certainly enlarged provided that the large value of γ^2 is choosen in such a manner that its product with (M/Q) remains large. For an exclusive γ^2 enlargement effect on A, the most reasonable choice of (M/Q) would be of the order of unity which corresponds to a near extremal black hole. The aspect of near extremality is in perfect accordance with the considerations of Ref.[56].

6.5. CONCLUDING REMARKS

We have shown that at least two WEC satisfying NBH solutions exist in the dilaton-Maxwell gravity. The first is the CMH electric solution and the second is the one obtained by conformal rescaling via the massless dilaton field ϕ. Clearly, the NBH character is not destroyed by such rescalings. Both the solutions exhibit a remarkable feature that $R_{0k0k}+R_{1k1k}=0$ at the horizon and non-zero outside. The analysis reveals that, for an infalling object, the tidal forces are vanishingly small at the horizon while these can be enlarged outside the horizon by choosing the parameter γ such that the source is a large static black hole. The fact that we need no longer bother about WEC violating "exotic" materials, lends a more realistic possibility to the idea of NBH.

As a further study, it would be interesting to examine if NBH solutions occur also in Brans-Dicke theory. Generally, the energy momentum tensor in this theory has an indefinite sign, leading to the possibility of WEC violation. However, it is always possible to go over to its conformally rescaled form where the WEC is not violated [62,97]. Work is underway.