EPILOGUE

The approach developed in the thesis is mathematically simpler and physically intuitive as it employs the familiar language of optics/mechanics. Indeed, eq.(2.2.7) - with the Evans-Rosenquist parameter A defined by eq.(1.1.10) - covers a lot of physics: It applies to geometrical optics of isotropic inhomogeneous media, to classical mechanics in velocity-independent potentials (for which we put $n = 1$ and $A \rightarrow t$), as well as to the orbits of light and particles in metrics representing a fairly wide class of physical situations. The expression of particle mechanics and geometrical optics in the same mathematical language is an aspect of the optical-mechanical analogy. There are several different formulations of this analogy, but eq.(2.1.6) (with its generalization, eq.(2.2.8)) is the simplest.

The Newtonian forms, eqs.(2.2.7) and (2.2.8), for the geodesic equations of motion offer some practical advantages for calculation. In particular, they facilitate the writing down of exact general relativistic expressions simply by analogy to classical formulas. Thus, they constitute one more tool for the relativist's tool kit. But the most interesting consequence of *extending the optical-mechanical analogy to general relativity is* that one simple equation of motion, eq.(2.2.7), now summarizes three fields of study: classical geometrical optics, classical particle mechanics and geodesic motion of both massive and massless particles in general relativity. Of course, our treatment is restricted to isotropic fields and media. Nevertheless, this *unified* approach, based on the use of optical action, possesses considerable flexibility and scope. A single variational principle (2.1.16) governs all three domains.

When the equations governing separate domains of physics can be cast into similar forms, two possibilities arise. The similarity might be due to mere contrivance on the part of the physicist. In this case, the mathematical analogy can be exploited to transfer methods and results from the more to the less familiar field, but *the underlying physics of the two fields could be quite different.* The second possibility is that the mathematical analogy results from a deeper connection between the two apparently disparate fields. In the case of the optical-mechanical analogy, this is certainly the situation.

The variational principle (2.1.16) puts a new twist on a conclusion mentioned before, in Sec.3.1: Maupertuis' principle amounts to a special case of Fermat's principle - the special case involving the matter waves of quantum mechanics. In the case of the propagation of light and the motion of particles in a gravitational field, both Fermat's principle and Maupertuis' principle may be regarded as special, or limiting cases of eq.(2.1.16), which, in turn, follows from the geodesic property of the trajectories. There is *nothing* of waves involved in eq.(2.1.16), which follows entirely from considerations of the behavior of particles in general relativity. How can it be the case that (1) Maupertuis' principle is a special case of Fermat's in the context of wave mechanics and that (2) both Maupertuis' principle and Fermat's principle are special cases of a variational principle deduced from general relativity?

The optical-mechanical analogy, as developed by Hamilton, can now be seen as providing a crucial hint about the wave-like nature of material particles, a hint that lay unnoticed until de Bròglie took it up a century later. A great, and so far unrealized, goal of late-twentieth-century physics is the unification of gravity with other forces of nature. The program of unification has been profoundly successful, beginning from the nineteenth century unification of optics, radiant heat, electricity and magnetism. But one could still maintain the possibility that gravity has nothing to do with other forces. The program of unification with it might represent only a unjustifiable act of faith by physicists in the unity of nature. The fact that Maupertuis' principle can simultaneously be the geometrical-optics limit of wave mechanics and a special case of the geodesic equation in general relativity seems to argue strongly in favor of an underlying unity of quantum mechanics and gravitation theory.