CHAPTER IV B

RADIO FREQUENCY CONDUCTIVITY OF AN IONISED GAS IN A TRANSVERSED MAGNETIC FIELD

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Measurement of plasma current and the capacitative current in a radiofrequency gas discharge

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By introducing a variable choke in parallel with the discharge tube and noting the resonant current in the main circuit as well as in the parallel circuit the plasma current as well as the capacitative current can be separated and measured. A mathematical formulation of the theory of measurement has been presented.

When a discharge occurs in a cylindrical glass tube, fitted with internal or external plane parallel electrodes and excited by a radio frequency source then in addition to the two components of discharge current whose phase difference with the applied radio frequency field depends upon the ratio of applied frequency and collision frequency of electrons with neutral atoms, there will be a current due to capacitative effect of the electrodes fitted in the discharge tube. If the discharge current is measured by a radio frequency meter then the two currents one due to flow through the plasma and the other flowing due to capacitative effect of the discharge tube (vacuum displacement current) cannot be separated from one another.

Francis and von engel[1] measured the actual current flowing through the discharge where the capacitative current flowing across the external electrodes is balanced by a bridge, the bridge becoming unbalanced when current flows through the gas. By means of a similar procedure Penfield and Warder[2] developed and tested an electronic circuit to measure the current in a high voltage radio frequency plasma. Clark *et al.*[3] determined the complex impedance of a radio-frequency discharge excited in hydrogen by a similar technique. It is however, observed in practice that it is difficult to balance the bridge accurately and a lot of adjustment and screening is necessary throughout the measurements at different ranges of applied voltages. An alternative method of separating and measuring the plasma current and the capacitative current is suggested here.

We consider the circuit as shown in Fig. 1. R is the ohmic resistance of the plasma and C is the capacity of the discharge tube with two electrodes; in parallel with the discharge tube there is a variable inductance L in series with a radio frequency milliammeter M_2 whereas M_1 is the radio frequency milliammeter which indicates the main current in the discharge. R_L is the ohmic resistance of the inductance L. Hence, the total current

$$I = I_R + I_{CP} + I_C + I_L \qquad \ldots (1)$$

where I_R is the resistive part of the current through the plasma of resistance R, I_{CP} is the capacitative current through the plasma due to capacity of the plasma of capacitance C_P, I_C through the capacitance C and I_L through the inductance L.

$$I = \frac{V}{R} + J\omega C_P V + \frac{V}{R_L + J\omega L} + J\omega CV$$

$$= \frac{V}{R} + \frac{VR_L}{R_L^2 + \omega^2 L^2} + J\omega V(C + C_P) - \frac{J\omega VL}{R_L^2 + \omega^2 L^2}$$

where V is the radio frequency voltage applied to excite the discharge.

At resonance

$$I_0 = \frac{V}{R} + \frac{VR_L}{R_L^2 + \omega^2 L^2} \qquad \ldots (2)$$

For resonance

$$C + C_P = \frac{L}{R_L^2 + \omega^2 L^2} \qquad \ldots (3)$$

now $C_P = C\varepsilon$, where ε is the dielectric constant of the plasma and $\varepsilon = [1 - \omega_P^2/(\omega^2 + \nu_C^2)]$ where ω_P is the electron plasma frequency, $\omega_P = (4\pi ne^2/m)^{1/2}$ and ν_C is the collision frequency of electrons with neutral molecules. For low density plasma, ε is almost equal to unity and $C + C_P = 2C$.

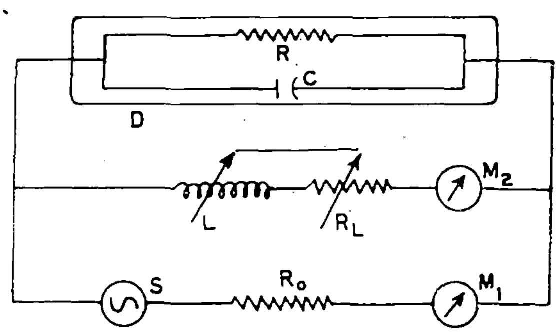

Fig. 1—Circuit for measurement of plasma and capacitative currents

So from Eq. (3) $C=\frac{L}{2[R_L^2+\omega^2L^2]}$ ... (4)

and $I_0=\frac{V}{R}+\frac{VR_L}{R_L^2+\omega^2L^2}=\frac{V}{R}+\frac{2VR_LC}{L}$ ... (5)

In the case of glow discharge tubes $C=A/(4\pi d)$, where A is the area of the electrodes and d is the distance between them. If A/d is not much different from unity, then

$$C=\frac{A}{4\pi d\times 9\times 10^{11}}\,\mathrm{F}\approx 10^{-12}\mathrm{F}.$$ From Eq. (4)

$$\omega^2L^2-\frac{L}{2C}+R_L^2=0$$

$$L=\frac{\frac{1}{2C}\pm\sqrt{\frac{1}{4C^2}-4\omega^2L^2}}{2\omega^2} \quad \text{(at resonance)}$$

So there are two values of L for resonance namely L_1 and L_2 and

$$L_1+L_2=\frac{1}{2\omega^2C}=\frac{1}{2\times 10^{12}\times 10^{-12}}\approx 0.5 \text{ if } \omega=10^6$$

and $L_1L_2=\frac{R_L^2}{\omega^2}$

If we design the coil L so that R_L is of the order of a few ohms then $L_1L_2\simeq 10^{-12}$ and as $L_1+L_2\simeq 0.5$ then L_1 is of the order of one henry and L_2 is of the order of 10^{-12}H. If we use the higher value of L for resonance then the total current at resonance I_0 from Eq. (5) is:

$$I_0=\frac{V}{R}+\frac{2VR_LC}{L}$$

If it is assumed that the radio frequency voltage is of the order 10^3 V

$$I_0=\frac{V}{R}+\frac{2\times 10^3\times 10^{-12}}{1}$$

but as the total discharge current is of the order of a few milliamperes then I_0 the current at resonance is

$I_0=V/R$ ... (6)

and $I_C=J\omega CV\approx 10^6\times 10^{-12}\times 10^3\approx 10^{-3}$ A

So the current through the condenser will also be of the order of a few milliamperes.

$$I_C=J\omega CV=-\frac{J\omega VL}{R_L^2+\omega^2L^2} \quad \text{... (7)}$$

which can approximately be written as

$$I_C=\frac{R_LV}{R_L^2+\omega^2L^2}-\frac{J\omega VL}{R_L^2+\omega^2L^2} \quad \text{... (8)}$$

As $R_L\approx 1\,\Omega$ and $L\approx 1$ H, the contribution of the first term is insignificant compared to the second term

$$I_C=\frac{V(R_L-J\omega L)}{R_L^2+\omega^2L^2}=\frac{V}{R_L+J\omega L}=I_L \quad \text{... (9)}$$

Hence noting the current at resonance in meter M_1 we can get I_R and noting the current in meter M_2 we can get I_C.

So by inserting a variable choke in parallel to the discharge tube and attaining resonance by changing L we can directly measure the current through the plasma and the capacitative current through the discharge tube separately. It is thus evident that as the capacitative current is of the same order as the current through the plasma, its contribution to the main current should be taken into consideration in calculating the radio frequency conductivity of the ionised gas.

In the paper by Francis and von Engel[1] no data were provided for capacitative current; so actual comparison cannot be made. We are taking measurements and the results will be reported in our future communication.

The capacitative effect of plasma has been taken into consideration and as it is a low density plasma, the dielectric constant of the plasma is almost equal to unity.

This method is valid for applied radiofrequency of the order of few megacycles, voltage of the order of 10^3V and radio frequency current of the order of a few milliamperes.

References

1 Francis G & von Engel A, *Phil Trans Roy Soc A (London)*, 246 (1953) 143.
2 Penfield A S & Warder Jr R C, *Rev Sci Instrum (USA)*, 38 (1967) 1533.
3 Clark J L, Earl R G & New J, *Proceedings of the international conference on gas discharge, London*, (1970) 172.

CHAPTER - IV

PART B

RADIO FREQUENCY CONDUCTIVITY OF AN IONISED GAS IN A TRANSVERSE MAGNETIC FIELD

INTRODUCTION

The real part of radio frequency conductivity σ_r of ionised gases (air and nitrogen) was measured by Sen and Ghosh (1966) at various pressures (5 to 300μ) and also at values of discharge current (10, 20 and 30 m.a). It was observed that σ_r increases with pressure, becomes a maximum at a pressure of 30μ in case of air and at a pressure of 84 μ in case of nitrogen. From theoretical analysis of the results the values of n (electron density) v_r the random velocity and T_e the electron temperature were obtained. The nature of variation of n and T_e were explained. A radio frequency probe was used for measurement, it was assumed that the ionised gas acts like a lossy dielectric and a mathematical analysis was presented for calculation of σ_r by representing the ionised column as an equivalent circuit of capacitance and a lossy resistance.

Gupta and Mandal (1967) also measured the real part of radio frequency conductivity in the case of air and carbon dioxide by the same method as adopted by Sen and Ghosh (1966) over a pressure range of a few microns to 0.3 torr but in presence of some fixed values of transverse magnetic field

(H = 0, 275, 410, 550 and 680 G). It was observed that conductivity decreases in presence of magnetic field for all values of pressure and the pressure at which the conductivity becomes a maximum increases with the increase of the magnetic field. The theory put forward by Gilardini (1959) was modified by the authors to explain the experimental results quantitatively.

Sen and Gupta (1969) measured the real part of r.f. conductivity in case of helium, neon and argon over the range of pressure from a few microns to 700 μ and under an external magnetic field varying from zero to 550 gauss. From the data obtained the plasma parameters such as electron density, collision frequency and electron temperature and their variation with magnetic field has been obtained.

Ghosal, Nandi and Sen (1976) measured the azimuthal radio frequency conductivity of an arc plasma by measuring the reflected resistance of a primary coil wound around a mercury arc tube and studied its variation with increasing arc current. It was however, pointed out that the azimuthal conductivity measurement by this method is possible only when the conductivity of the plasma is fairly high.

Most of the measurements reported here refer to investigation of σ_r the real part of radio frequency conductivity and its variation with magnetic field and pressure. Little work has been reported regarding the variation of the r.f. current which is out of phase with the real part of the radio frequency current by $\pi/2$. This current will also vary with pressure and magnetic field. The object of the present investigation is to study the variation of this current in present of a variable transverse magnetic field at a constant pressure.

THEORETICAL ANALYSIS

The radio frequency current I_{rf} in a transverse magnetic field is given by

$$I_{rf} = \frac{ne^2}{m} \frac{\nu_c + J\omega}{(\nu_c + J\omega)^2 + \omega_B^2} E_o e^{J\omega t}$$

$$= \frac{ne^2}{m} E_o e^{J\omega t} \frac{\nu_c(\nu_c^2 + \omega_B^2 + \omega^2) + J\omega(\omega_B^2 - \omega^2 - \nu_c^2)}{(\nu_c^2 + \omega_B^2 - \omega^2)^2 + 4\nu_c^2\omega^2}$$

$$= (I_{rf})_{H\,real} + J(I_{rf})_{iH}$$

So that

$$(I_{rf})_{iH} = \frac{ne^2}{m} E_o e^{J\omega t} \frac{\omega(\omega_B^2 - \omega^2 - \nu_c^2)}{(\nu_c^2 + \omega_B^2 - \omega^2)^2 + 4\nu_c^2\omega^2} \tag{4.10}$$

where $(I_{rf})_{rH}$ and $(I_{rf})_{iH}$ are the real and imaginary parts of the radio frequency current.

where ν_c is the collision frequency, w the frequency of the applied r.f. field, w_B is the electron cyclotron frequency and n is the electron density. At this stage we make some assumptions

(a) the plasma is collision dominated so that $\nu_c \gg w$ if the measurement is carried out at radio frequency.

(b) Due to the presence of the magnetic field the electric field is modified to E_H where $E_H = E\ (1 + C_1H^2/p^2)^{\frac{1}{2}}$ (Beckman, 1948, Sen and Gupta (1971), where $C_1 = \left(\frac{e}{m}\ \frac{L}{V_r}\right)^2$ where L is the mean free path of the electron in the gas at 1 torr and v_r is the random velocity of the electron.

So replacing E_o by E_H,

$$(I_{rf})_{iH} = \frac{ne^2}{m} E_H e^{Jwt} \quad \frac{w(w_B^2 - \nu_c^2)}{(\nu_c^2 + w_B^2)^2 + 4\nu_c^2 w^2} \tag{4.11}$$

and $(I_{rf})_i$ in absence of magnetic field

$$(I_{rf})_i = \frac{ne^2}{m} E_o e^{Jwt} \frac{w}{\nu_c^2 + w^2} \tag{4.12}$$

Putting $E_H = E_o\ (1 + C_1H^2/P^2)^{\frac{1}{2}}$ we get from equations (4.12) and (4.11)

$$\frac{(I_{rf})_{iH}}{(I_{rf})_i} = \left(1 + C_1 \frac{H^2}{p^2}\right)^{1/2} \frac{(w_B^2 - \nu_c^2)\,\nu_c^2}{(\nu_c^2 + w_B^2)^2\ 4\nu_c^2 w^2}$$

now $w_B = \frac{eH}{m}$ so, $H^2 = \frac{w_B^2 m^2}{e^2}$

and $C_1 \frac{H^2}{p^2} = \frac{e^2 L^2}{m^2 v_r^2} \cdot \frac{1}{p^2} \cdot \frac{w_B^2 m^2}{e^2}$

$$= \frac{L^2}{v_r^2 p^2} w_B^2 = \alpha \; w_B^2$$

where L is the mean free path of the electron at a pressure 1 torr.

where $\alpha = L^2 / v_r^2 p^2$ is a constant at a constant pressure.

We can now find the variation of $(I_{rf})_{iH}/(I_{rf})_i$ with w_B. After a detailed calculation it can be shown that

$$\frac{d}{dw_B}\left[(I_{rf})_{iH}/(I_{rf})_i\right] = \frac{7}{2}\alpha w_B^6 + \left(\frac{5}{2}\alpha \nu_c^2 + 3\right) w_B^4$$

$$+\left[6\alpha \nu_c^2 w^2 - \frac{3}{2}\alpha \nu_c^4 + 2\nu_c^2\right] w_B^2$$

$$-\frac{\alpha}{2}\nu_c^6 - 2\alpha \nu_c^4 w^2 - \nu_c^4 + 4\nu_c^2 w \qquad (4.13)$$

To find whether $(I_{rf})_{iH}$ shows any maximum or minimum value with the variation of w_B, i.e., H we can put the equation (4.13) in the form

$$y^3 + py^2 + qy + r = 0 \qquad (4.14)$$

where $w_B^2 = y$, $p = \dfrac{5\alpha\delta_c^2 + 6}{7\alpha}$

$$q = \frac{12\alpha\delta_c^2 w^2 - 3\alpha\delta_c^4 + 4\delta_c^2}{7\alpha}$$

$$r = \frac{8\delta_c^2 w^2 - \alpha\delta_c^6 - 4\alpha\delta_c^4\omega^2 - 2\delta_c^4}{7\alpha}$$

The equation can be reduced to the form $x^3 + ax + b = 0$ by substituting for y the value of $(x - \frac{p}{3})$

$$\text{where } a = \frac{1}{3}\left[3q - p^2\right]$$

$$b = \frac{1}{27}\left[2p^3 - 9pq + 27r\right]$$

and the solutions are

$$X = A+B,\ -\frac{A+B}{2} + \frac{A-B}{2}\sqrt{-3} \text{ and } -\frac{A+B}{2} - \frac{A-B}{2}\sqrt{-3}$$

$$\text{where } A = \left(-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3}$$

$$B = \left(-\frac{b}{2} - \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}\right)^{1/3}$$

To calculate the value of ω_B at which the ratio $(\sigma_{rf})_{iH}/(\sigma_{rf})_i$ becomes a maximum or a minimum. We take the case of air at $P = 0.5$ torr, $E = \frac{400-300}{10}$ volts/cm where 400 volts is the striking voltage and 300 volts is the combined anode and cathode fall and the distance between cathode and anode is 10 cm. So E/P = 20 volts/cm. torr; from Von Engel (1957) for air for E/P = 20 volts/cm torr

$$T_e = 2.5 \text{ eV}$$

$$V_r = 8.414 \times 10^7 \text{ cm/S.}$$

$d = L^2/P^2 v_r^2$ and $L = 1/15$ Von Engel (1957)

So $\alpha = 2.511 \times 10^{-18}$

$$\delta_c = v_r P/L = 6.31 \times 10^8.$$

$$\omega = 10 \text{ Mc/s}$$

and the values of $\mathcal{P}$, q and r have been calculated

$\mathcal{P} = 6.246 \times 10^{17}$ $\quad q = 5.045 \times 10^{34}$, $\quad r = -26.3 \times 10^{51}$.

$a = -7.96 \times 10^{34}$ $\quad b = -18.6 \times 10^{51}$,

$A = 2.598 \times 10^{17}$ $\quad B = 1.022 \times 10^{17}$

$\omega_B = 6 \times 10^8$ $\quad H_{max} = 25$ gauss

and it can be shown $\frac{d^2}{d\omega^2}\left[(\sigma_{rf})_{iH}/(\sigma_{rf})_i\right]$ is positive and variation of σ_{rH} with H will show a minimum at a certain value of H.

DISCUSSION

It is thus possible to calculate analytically the value of the magnetic field at which the radio frequency conductivity (imaginary part) becomes a minimum and such calculations can be carried out for different gases for different pressure and frequency of the exciting radio frequency field. The effect of the magnetic field is to increase the axial electric field and at the same time reduce the charge particle density along the axis, but the rate of increase of the axial electric field predominates the loss of electron density at the axis for higher values of magnetic field and so the radio frequency conductivity increases at higher magnetic field values.

R E F E R E N C E S

Sen, S.N. and Ghosh, A.K., Ind. Jour. Pure & Appl. Phys. 4, 70, 1966.

Gupta, R.N. and Mandal, S.K., Ind. J. Phys., 41, 251, 1967.

Gilardini, A., Nuovo Cimento Supplement. 13, 9, 1959.

Sen, S.N. and Gupta, R.N., Ind. J. Pure & Appl. Phys. 7, 117, 1969.

Ghosal, S.K., Nandi, G.P. and Sen, S.N., International J. Electronics, (G.B) 41, 509, 1976.

Von Engel, A., Ionised Gases (Clarendon Press, Oxford, 1959.

Beckman, L., Proc. Phys. Soc. (Lond) 61, 515, 1948.

Sn, S.N. and Gupta, R.N., Jour. Phys. Appl. Phys. (G.B) 4, 510, 1971.