CHAPTER VI

CURRENT GENERATION PROCESS, HIGH CURRENT DENSITY AND CATHODE PHENOMENA IN AN ARC PLASMA

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INTRODUCTION

It is known that many attempts both theoretical and experimental have been made to explain the high current density and low cathode fall in arcs. It has been generally observed that low boiling point metals have very high cathode current density and the cathode fall of the potential is of the order of least ionization potential of the gas in which the arc burns. Theoretical investigation of the phenomena has been provided by Thomson and Thomson (1933) and by Loeb (1952). Two theories have been advanced namely thermionic emission and field emission to explain the observed experimental results. The most accepted mechanism for the production of large electron current at the cathode is thermionic emission. Because of the high velocity of electrons the space charge region is taken to be due to positive ions and utilizing the space charge equation of Childs and Langmuir and assuming that the cathode drop region to be of the order of one mean free path of the electron or even less the observed cathode current density which varies from a few hundred to a few thousand amperes can be qualitatively explained. Further extension of the theory gives the value of electric field at the cathode and for most of the metals this

comes out to be of the order of 10^5 volts/cm to 10^6 volts/cm. The possibility of such high field strengths has led some workers [Compton (1923) Langmuir (1923)] to suggest that the necessary electrons are produced by field emission. A modification of the field emission theory was suggested by Druyvestyn (1936) where the field is presumed to be produced in an extremely thin layer of relatively high resistance by a layer of positive ions on the outer surface. Ramberg (1932) however deduced from his experimental results that arcs of Cu, Hg, Ag and Au are of the field emission type and those of C, Ca, Mn are thermionic in nature. However, it is noted that the current due to field emission is far less than the arc current, being of the order of 10^{-6} to 10^{-7} amp/sq.cm. However it has been suggested that since the effect of the electric field is to lower the effective value of the work function, both field and thermo ionic emission may work together.

The nature of the electrode regions in the arc is poorly understood inspite of extensive research carried out over the decades. This lack of understanding is a consequence of the complexcities prevailing in these regions caused by the interaction of electrical, magnetic, thermal and fluid dynamic effects which are difficult to assess and sometimes impossible to control. It is also doubtful whether velocity distribution of plasma particles (electrons, ions and neutrals) close to the electrodes is Maxwellian in nature. Finkelnburg and Maecker (1956) and Ecker (1961) have given a comprehensive review of

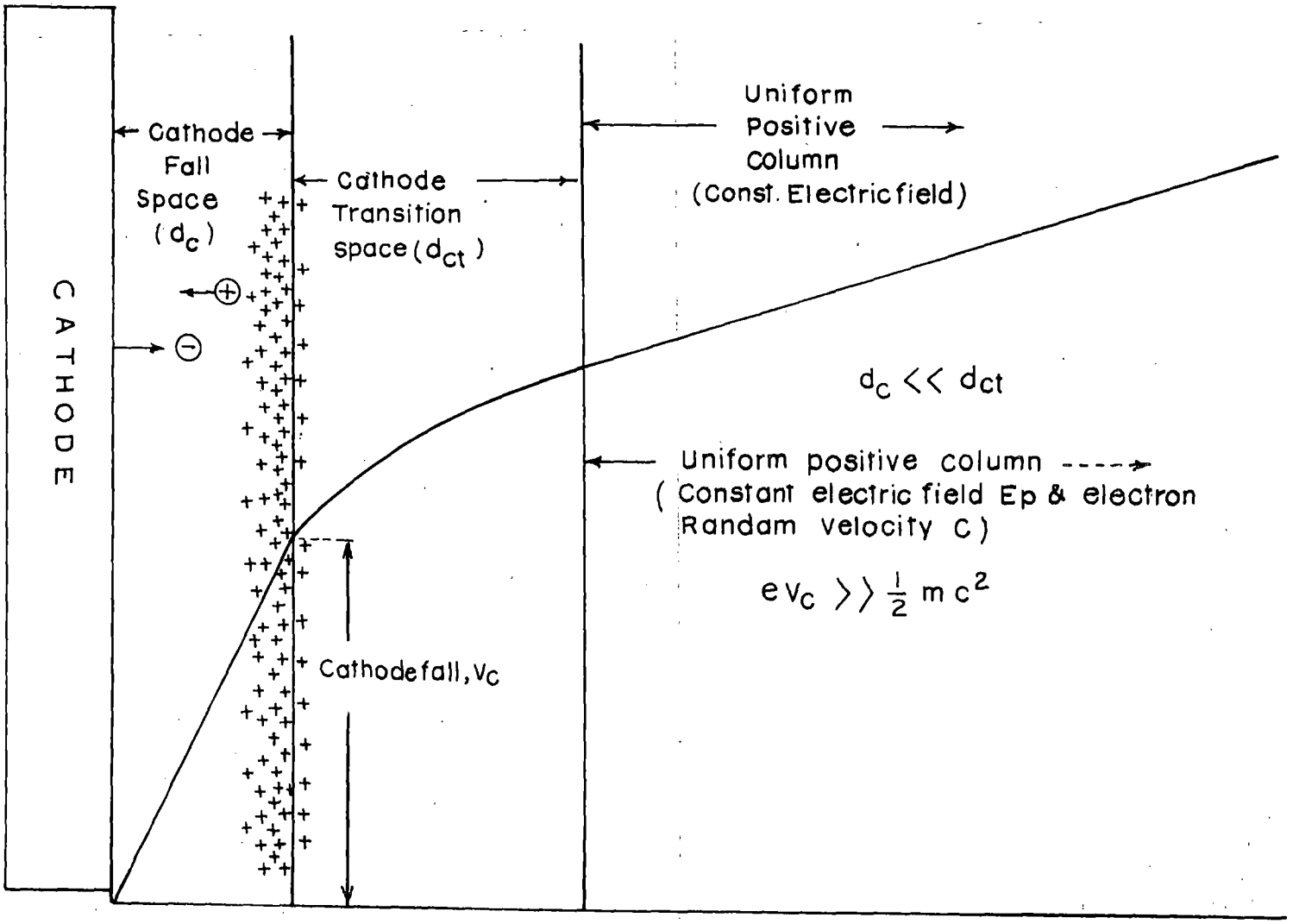


Fig. 6-I: Cathode to positive column of an arc

the experimental results in arcs.

In the present work we have been able to show that all arcs are basically thermionic in nature irrespective of the melting point of the cathode material. Also it has been shown that the space charge layer forming the boundary of the cathode fall space is only at a distance of a mean free path from the cathode surface and ionization of vapour inside the catnode fall space is the main factor for the high current in the arc. A generalised theory has been developed for the high current density and the cathode fall and experiments have been performed to verify the deductions of the theory with observed experimental results.

ANALYTICAL EVALUATION OF THE PROCESS

Fig. 6.1 gives a magnified representation of the cathode, cathode fall space d_c cathode transition space d_{ct} and the positive column of the arc. The cathode transition space is the distance between the cathode fall surface and the beginning of the positive column.

Following assumptions have been made in this theory:

1) Each of the thermally emmitted electrons while coming across the cathode fall space attains a kinetic energy eV_c at the boundary of the fall space when electron velocity is given by

$$v_c = \left(\frac{2eV_c}{m} \right)^{1/2}$$

where v_c is the emerging velocity of the electrons and V_c is the cathode fall.

2) Each such electron when it enters into the cathode transition space ionises one atom within a very short distance from the boundary of the cathode fall space.

3) The excess energy after ionisation over that of the random energy $\frac{1}{2} mC^2$ of the electrons in the uniform positive column is spent by collision until it reaches the uniform position column and attains the random energy of the electrons in the uniform positive column where C is the random velocity of the electron in the positive column.

4) When equilibrium is established, the boundary of the cathode fall space and the positive charge distribution across the fall space boundary remain statistically stationery, so long the external parameters like pressure, and arc current remain unaltered.

5) The field required for the emission of electrons in the fall space is supplied by the positive charge layer across the fall space boundary which also shields the cathode from the rest of the arc.

6) The cathode fall space boundary is at a distance of a mean free path from the cathode surface. So the span of cathode fall space is of the order of a mean free path.

7) The span of the cathode transition space d_{ct}, i.e., the distance from the fall space boundary to the beginning of uniform positive column is much greater than the span of cathode fall space (d_c)

So, $d_c << d_{ct}$

8) By suddenly stopping the arc by some external means like magnetic field, it has been observed that about 2% of the power consumed by the running arc is still flowing in the circuit through the arc giving only violet and ultraviolet radiation between the electrodes so long the electrode is hot. So we assume that a running arc produces considerable ultraviolet radiation in the fall space which is absorbed by the metal vapour in one or two steps. Also due to the collision between on coming vapour and positive ions in the fall space all vapours are ionised and the energy spent by $I_c V_c$ in the fall space is uniformly sprayed through the area of emission of vapour around a small area at the centre where the discharge is taking place. Thus a cumulative ionisation process goes on until all vapours are ionised in the fall space and an uniform energy distribution in the fall space is established.

9) Thus if such process as in (8) is truely taking place, then the positive ion current will gradually increase as we proceed towards the cathode and will definitely be maximum on the cathode surface. Here we must mention the work of Lee and Greenwood (1964) where they found for a 200A carbon arc, a positive ion current to vary from zero at the column end to 15% of the total current at the cathode surface end.

10) Each electron is supposed to be uniformly retarded while moving through the cathode transition space.

11) Ionisation potential V_i may correspond to any of the components of the working fluid in the arc. By component we mean the gas or gases already present, metal vapour or any new gas formed at the cathode by any chemical reaction. V_i is mostly expected to be the lowest of the V_i-S and may be a higher one also in some cases due to the nature of the gases.

12) In fact ionisation by electrons leaving the cathode fall boundary takes place after several mean free paths and the velocity of the ionising electron is supposed to drop to a velocity v_D which must be associated to the characteristics of the fluid in the transition space. So we supposed this velocity to be the geometric mean of the two extreme velocities, e.g. v_c and C, so v_D is given by $v_D = (v_c C)^{\frac{1}{2}}$.

13) Span of cathode emission area and span of cathode fall space change in the same way with pressure such that the ratio of these two is independent of pressure and is constant so long cathode temperature remains at a particular value.

14) Due to high field in the fall space, the thermal emission, here, is Schottky emission.

Theoretical deduction:

Since uniform retardation f is considered in the transition space, so we have

$$f = \frac{v_c^2 - C^2}{2 d_{ct}} \qquad (6.1)$$

where v_c is the velocity of the electron, as it emerges from the cathode fall space and C is the random velocity of the electrons in the positive column. Time taken for this change in velocity will be

$$t = \frac{v_c - C}{f} = \frac{2 d_{ct}}{v_c + C} \quad \cdots (6{\cdot}2) \text{ [from eqn. (6.1)]}$$

when equilibrium in the arc is established, if n_T is the net number of electrons coming out from fall space in one sec. through unit area, then total number of electrons coming out through an area S of fall space and in time t will be

$$n_T \frac{2 d_{ct}}{v_c + C} S$$

For low energy electrons the ionisation efficiency is given by

$$\alpha = ap (V - V_i)$$

when "a" is the number of ion-pairs produced by collision per volt per meter per mm. of Hg pressure. Hence number of positive ions generated by collision in the cathode transition space by these electrons will be

$$n_T \frac{2 d_{ct}}{v_c + C} \cdot s \cdot a \cdot p (V_{ct} - V_i) d_{ct}$$

V_{ct} is the voltage at a distance d_{ct} after the fall space, V_i is the ionisation potential of the gas or vapour in the transition space. Hence number of positive ions per unit volume becomes

$$n_T \frac{2 ap}{v_c + C} (V_{ct} - V_i) d_{ct}$$

In place of d_{ct} if we consider a distance x then charge density at a distance x is given by

$$\rho_x = n_T e \frac{2ap}{v_c + c} (V_x - V_i) x \qquad (6.3)$$

But $v_x^2 = v_c^2 - 2fx$, v_x is the velocity at a distance x .

So, $$v_x^2 = v_c^2 - \frac{v_c^2 - c^2}{d_{ct}} x$$

replacing each of the velocities by equivalent voltages.

We get,

$$V_x = V_c - \frac{V_c - \frac{mc^2}{2e}}{d_{ct}} x, \text{[e is the electronic charge]}$$

So, $$\rho_x = n_T e \frac{2ap}{v_c + c} \left[V_c - V_i - (V_c - \frac{mc^2}{2e}) \frac{x}{d_{ct}} \right] x \qquad (6.4)$$

By Poisson's equation $$\frac{d^2V}{dx^2} = - \rho/\epsilon_o \qquad \text{(in MKS unit)}$$

and boundary conditions are,

$$\text{at } x = 0 \quad V = 0, \ (dV/dx) = 0$$
$$\text{at } x = d_c \quad V = V_c, \ (dV/dx) = E_c$$

Thus we get,

$$- \frac{dV}{dx} = \frac{n_T e}{\epsilon_o} \cdot \frac{2ap}{v_c + c} \left[(V_c - V_i) \frac{x^2}{2} - (V_c - \frac{mc^2}{2e}) \frac{x^3}{3d_{ct}} \right] + C_1$$

So putting boundary conditions, we have

$$\left| -\frac{dV}{dx} \right| = E_c = \frac{n_T e}{\epsilon_o} \cdot \frac{2ap}{\vartheta_c + C} \cdot \frac{d_c^2}{2} \left[(V_c - V_i) - \left(V_c - \frac{mc^2}{2e} \right) \frac{2d_c}{3d_{ct}} \right] \quad (6.5)$$

and $$V_c = \frac{n_T e}{\epsilon_o} \cdot \frac{2ap}{\vartheta_c + C} \cdot \frac{d_c^3}{6} \left[(V_c - V_i) - \left(V_c - \frac{mc^2}{2e} \right) \frac{d_c}{2d_{ct}} \right] \quad (6.6)$$

Ignoring the difference in $\frac{2d_c}{3d_{ct}}$ and $\frac{d_c}{2d_{ct}}$ in (6.5) and (6.6) we have,

$$E_c \approx \frac{3V_c}{d_c} \quad (6.7)$$

Now we assume that each electron coming out from cathode fall space ionises one neutral atom within a short distance ℓ from the cathode fall space boundary, so we can write with K as a loss factor

$$ap\left(V_c - V_c K \frac{\ell}{\lambda} - V_i\right)\ell = 1 \quad (6.8)$$

where $V_c K \frac{\ell}{\lambda}$ is the net energy lost by collisions upto an instant just before ionisation takes place.

Thus K, here, is not a true loss factor but a loss factor which includes the gain in energy by the electron between two consecutive cillisions and loss by collision. Eqn. (6.8) gives

$$\ell = \frac{(V_c - V_i) \pm \sqrt{(V_c - V_i)^2 - \frac{4V_c K}{aL_1}}}{2p\frac{V_c K}{L_1}} \left[\lambda = \frac{L_1}{p} \right]$$

where L_1 is the mean free path of the electron at 1 torr. But under the stated condition ℓ cannot have two values in the transition region since $V_c < 2V_i$. Hence equating the discriminent to zero, we get,

$$K = \frac{aL_1(V_c - V_i)^2}{4V_c} \tag{6.9a}$$

$$\ell = \frac{V_c - V_i}{2p\frac{V_c K}{L_1}} = \frac{2}{ap(V_c - V_i)} \tag{6.9b}$$

Again we consider, when an atom is ionised by an electron through collision, the rest of the energy is lost by collision until equilibrium random velocity C in the uniform positive column is attained by the electron. So we can write by excluding the energy of one atom ionisation,

$$e(V_c - V_i)K \cdot \frac{d_{ct} - d_c}{\lambda} = \frac{1}{2} m(v_D^2 - c^2) \tag{6.10}$$

But $v_c > v_D \gg C$ because E/p in the uniform positive column of an arc for high p, is given by

$$E/p < 1 \text{ Volt/cm. torr}$$

Again $\lambda \simeq d_c$ and $d_c << d_{ct}$

$$\text{So, } \frac{d_c}{d_{ct}} = \frac{2eK(V_c - V_i)}{m v_D^2} = \frac{a L_1 e}{2 m v_D^2} \cdot \frac{(V_c - V_i)^3}{V_c} \qquad (6.11)$$

Thus we have, from eqn. (6.6)

$$V_c = \frac{n_T e}{\epsilon_o} \cdot \frac{ap}{3 v_c} \cdot d_c^3 \cdot (V_c - V_i)\left[1 - \frac{a L_i e}{4 m v_D^2}(V_c - V_i)^2\right]$$

and a mean value of C for $0.5 < E/p < 1$ is taken to be 0.6×10^4 m/sec and a mean value of V_c is taken to be 18 V.

Thus
$$v_c = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 18}{9.1 \times 10^{-31}}} \text{ m/s} = 2.52 \times 10^6 \text{ m/s}$$

and $$v_D = \sqrt{v_c C} = \sqrt{1.51 \times 10^{10}} \text{ m/s}$$

Hence $\frac{a L_1}{4 m v_D^2} = 5.32 \times 10^{-3}$ (in MKS unit)

where a = 26 ion pair /V/m/m.m. of Hg (Air)

$L_1 = 7 \times 10^{-5}$ meter m.m. of Hg (Air)

$m = 9.1 \times 10^{-31}$ Kg (electron)

So we have,

$$V_c = \frac{n_T e}{\epsilon_o} \cdot \frac{ap}{3 v_c} \cdot d_c^3 (V_c - V_i)\left[1 - 5.32 \times 10^{-3}(V_c - V_i)^2\right] \qquad (6.12)$$

upto this we have dealt with n_T, the net no. of electrons emitted from fall space. Now we want to get the value of the ratio n_T/n_e where n_e is the directly generated electron from the cathode. Let α_c be the area of cathode spot from where the electron is basically emitted under the influence of heat and field, which may be called a thermal emission though cathode surface at the actual spot is highly different from the surface of the metal. Actually, the surface at the spot is a mixture of vapour and partially molten metal in case of low melting point metals. And in case of refractory metals like carbon the surface is a mixture of carbon particles and Co, Co_2 for air as a working fluid. Some refractory metals like W, when operated at a low cathode temperature, the emission area may be a solid surface, partly. Thus this is not the surface of a pure thermionic emmitter.

Let A_o be the total area, excluding the α_c at the centre of A_o, from where vapourisation of metal (cathode) takes place.

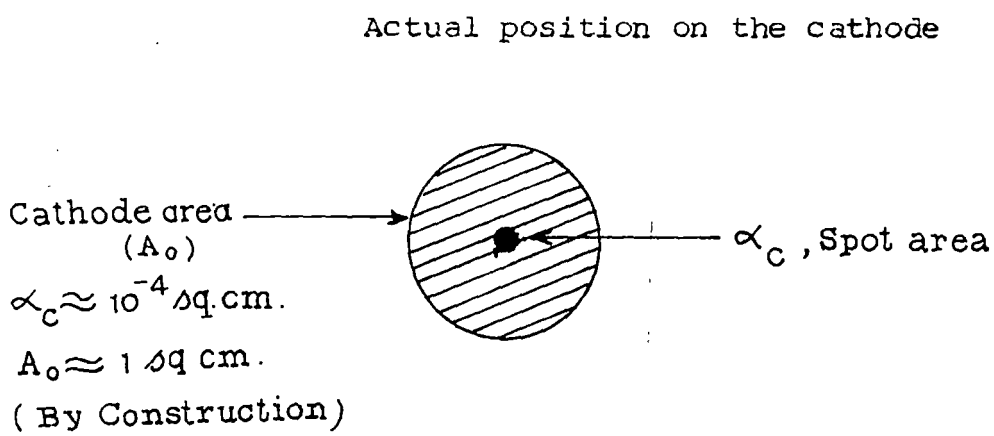
$$A_o \gg \alpha_c$$

Let us assume the power $I_c V_c$ is spent uniformly from every part of the area A_o.

$$\text{Let } P = I_c V_c \text{ joule/Sec} = I_c V_c \times 10^7 \text{ ergs/sec}$$

Let W_e is the energy spent exclusively for electron emmission, i.e., excluding any energy spent against vapourisation and the

ionisation of the vapour, and W_m is the total energy spent for vapourisation followed by ionisation of the vapour. So, as we have assumed the energy spent is uniformly distributed over the area A_o, we can write



$$\frac{We}{\alpha_c} = \frac{Wm}{A_o} \qquad (6.13)$$

and $W_e + W_m = P$ is the total power consumed on the cathode

$$We = \frac{\alpha_c}{\alpha_c + A_o} \; (We + W_m) \simeq \frac{P\alpha_c}{A_o} \qquad (6.14a)$$

and

$$W_m = \frac{A_o}{\alpha_c + A_o} P \simeq P \qquad (6.14b)$$

Electron emmission energy is given by

$$E_e = \phi \times 9.63 \times 10^{11} \text{ ergs/mole} \qquad (6.15)$$

where ϕ is the work function of the metal. Metal vapourisation energy is given by

$$E_v = LM \times 4.2 \times 10^7 \text{ ergs/mole}$$

L is the Lalent heat per gm.

M is the molecular weight

Metal vapour ionisation energy is given by

$$E_i = V_i \times 9.63 \times 10^{11} \text{ ergs/mole}$$

V_i is the ionisation potential of the vapour atom. Hence the metal vapourisation followed by ionisation requires an energy given by

$$E_m = E_v + E_i = (LM + 2.29 \times 10^4 V_i) 4.2 \times 10^7 \text{ ergs/mole} \qquad (6.16)$$

Hence number of electrons emmitted is given by

$$N_e = \frac{W_e}{E_e} = \frac{(P\alpha_c/A_o)\text{ergs/sec}}{\phi \times 9.63 \times 10^{11} \text{ergs/mole}} = \frac{\alpha_c P}{A_o \phi \times 9.63 \times 10^{11}} \text{ mole/sec.}$$

$$\text{So } N_e = \frac{\alpha_c PN}{A_o \phi \times 9.63 \times 10^{11}} \text{ no./sec.} = \frac{\alpha_c P}{A_o \phi} \times 6.25 \times 10^{11} \text{ no./sec.} \qquad (6.17)$$

where N is the Avogadro's number. And vapour ion produced is given by

$$N_m = \frac{W_m}{E_m} = \frac{P(\text{ergs/sec})}{(LM + 2.29 \times 10^4 V_i) \times 4.2 \times 10^7 \text{ ergs/mole}}$$

$$= \frac{P}{(LM + 2.29 \times 10^4 V_i) \times 4.2 \times 10^7} \text{ mole/sec.}$$

$$= \frac{PN}{(LM + 2.29 \times 10^4 V_i) \times 4.2 \times 10^7} \text{ no./sec.}$$

$$\text{So,} \quad N_m = \frac{P \times 1{\cdot}43 \times 10^{16}}{LM + 2{\cdot}29 \times 10^4 V_i} \text{ no./sec.} \qquad (6.18)$$

Hence total number of electrons emitted per sec.,

$$N_T = N_m + N_e \backsimeq N_m$$

Hence total number per unit area is given by

$$n_T = \frac{N_T}{A_o} = \frac{N_m}{A_o} = \frac{P \times 1{\cdot}43 \times 10^{16}}{A_o (LM + 2{\cdot}29 \times 10^4 V_i)} \text{ no./sec./area} \qquad (6.19)$$

And number of electrons emitted directly from metal per unit area per sec. is given by

$$n_e = \frac{N_e}{A_o} = \frac{\alpha_c P}{A_o^2 \phi} \times 6{\cdot}25 \times 10^{11} \text{ no./sec/area} \qquad (6.20)$$

$$\text{So,} \quad \frac{n_T}{n_e} = \frac{(A_o \phi / \alpha_c) \times 2{\cdot}29 \times 10^4}{LM + 2{\cdot}29 \times 10^4 V_i} \qquad (6.21)$$

$$\text{Now let,} \quad \frac{(A_o \phi / \alpha_c) \times 2{\cdot}29 \times 10^4}{LM + 2{\cdot}29 \times 10^4 V_i} = \beta_c \qquad (6.22)$$

$$\text{So,} \quad n_T = \beta_c \, n_e \qquad (6.23)$$

Thus now using (6.23) in (6.12) we get

$$V_C = \frac{n_e e}{\epsilon_o} \beta_C \frac{a p}{3 \vartheta_C} d_C^3 (V_C - V_i)\left[1 - 5{\cdot}32 \times 10^{-3}(V_C - V_i)^2\right] \qquad (6.24)$$

But n_e is the directly emitted electron from metal by the action of temperature T_C and field E_C. So due to Schottkey effect we can write

$$n_e = n_o \exp\left(\frac{0{\cdot}44}{T_C}\sqrt{E_C}\right) \quad \text{(in MKS unit)}$$

Thus we have,

$$V_C = \frac{n_o e}{\epsilon_o} \beta_C \frac{a p}{3 \vartheta_C} d_C^3 (V_C - V_i) \exp\left(\frac{0{\cdot}44}{T_C}\sqrt{E_C}\right)\left[1 - 5{\cdot}32 \times 10^{-3}(V_C - V_i)^2\right] \qquad (6.25)$$

Thus $n_o e = J_o$ is the directly emitted current density on the cathode spot where discharge takes place. So now using eqn. (6.7), and $d_C \simeq \lambda = \frac{L_1}{p}$ we get

$$E_C = \frac{3V_C}{d_C} = \frac{3V_C}{\lambda} = \frac{3pV_C}{L_1} \qquad (6.26)$$

and hence, with $v_c = \left(\frac{2eV_c}{m}\right)^{1/2}$ we have

$$\frac{pV_c^{3/2}}{(V_c - V_i)\left[1 - 5.32\times10^{-3}(V_c - V_i)^2\right]} = \left[\frac{j_o \beta_c d_c a L_1^2}{3\epsilon_o \sqrt{\frac{2e}{m}}}\right] \mathrm{Exp}\left[\frac{0.44}{T_c}\sqrt{\frac{3}{L_1}}\sqrt{V_c p}\right] \quad (6.27)$$

Now α_c and d_c both are controlled by pressure. So we assume d_c and α_c varies in the same way with change in pressure. So d_c/α_c is constant and j_o is constant, so long T_c remains same. So we can take

$\frac{j_o \beta_c d_c L_1^2 a}{3\epsilon_o (2e/m)^{1/2}}$ a constant which depends only on cathode Temperature T_c and nature of the cathode. So taking,

$$A = \frac{J_o \beta_c d_c L_1^2 a}{3\epsilon_o (2e/m)^{1/2}}$$

and $$\frac{0.44}{T_c}\sqrt{\frac{3}{L_1}} \log_{10} e = b$$

We have,

$$\log\left[\frac{pV_c^{3/2}}{(V_c - V_i)\left[1 - 5.32\times10^{-3}(V_c - V_i)^2\right]}\right] = \log_{10} A + b\sqrt{V_c p} \quad (6.28)$$

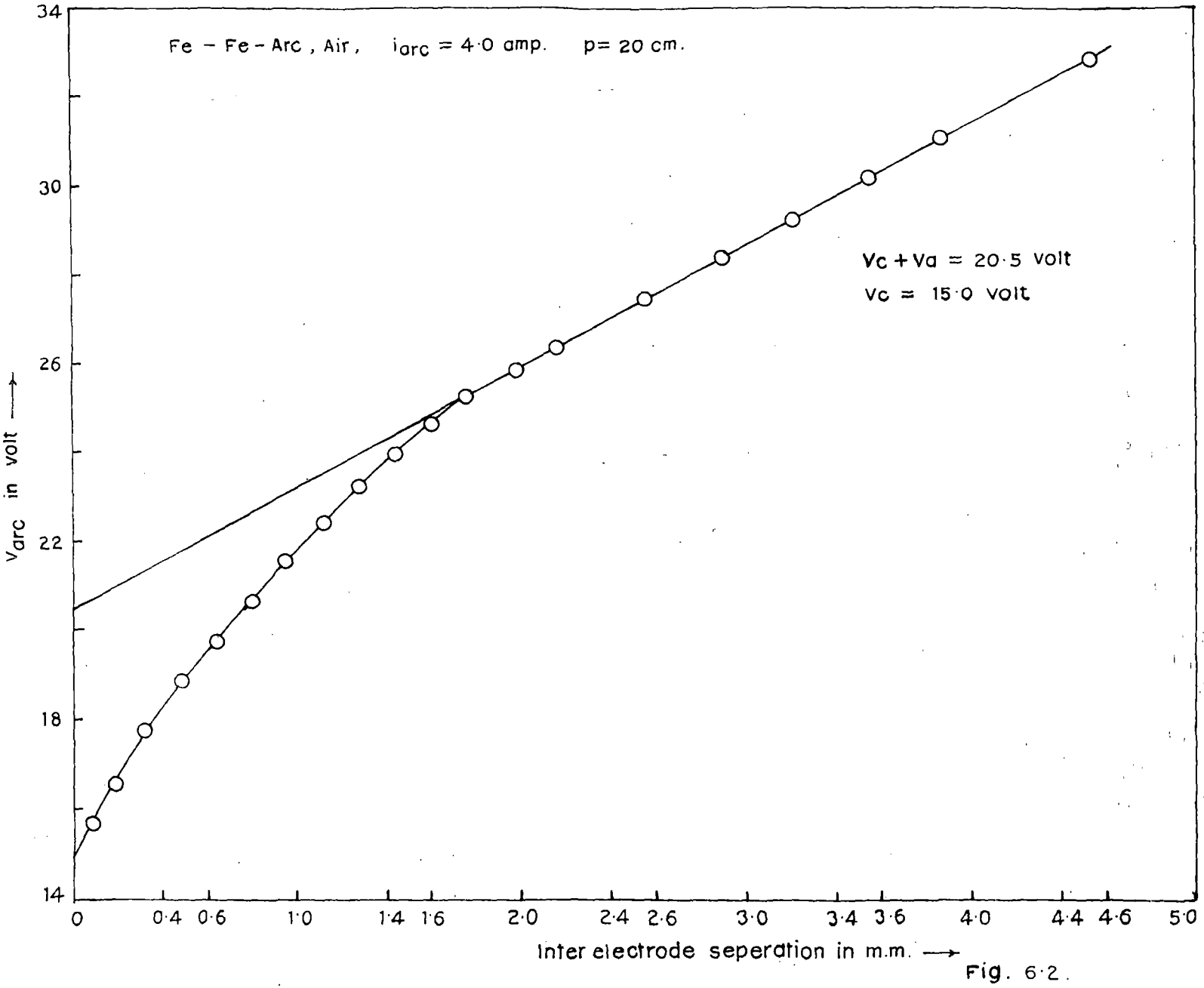

Fe – Fe – Arc , Air , i_{arc} = 4·0 amp. p= 20 cm.
Vc + Va = 20·5 volt
Vc = 15·0 volt
V_{arc} in volt
34
30
26
22
18
14
0
0·4
0·6
1·0
1·4
1·6
2·0
2·4
2·6
3·0
3·4
3·6
4·0
4·4
4·6
5·0
Inter electrode seperation in m.m.

Fig. 6·2.

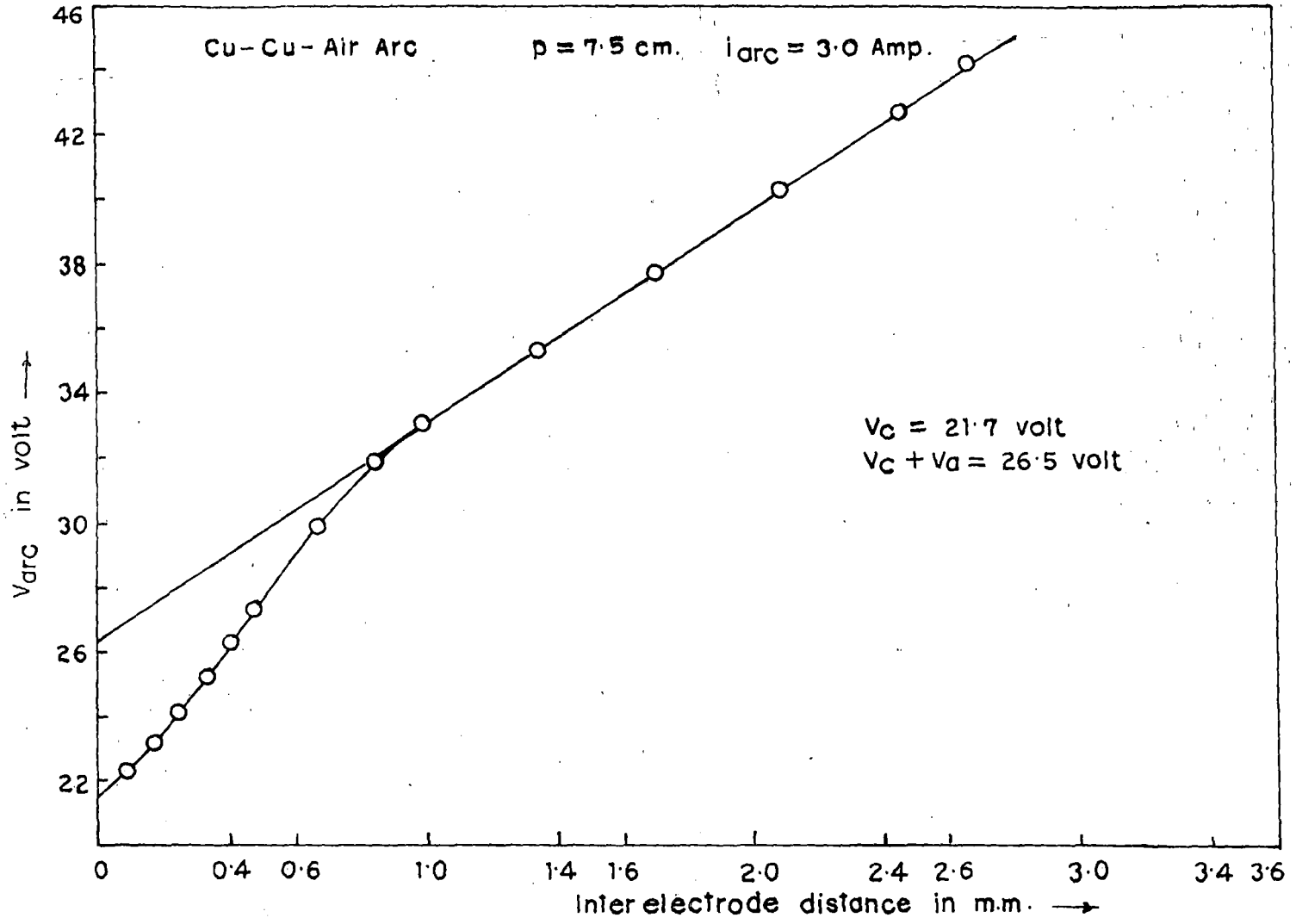

Cu-Cu-Air Arc
p = 7·5 cm.
i_{arc} = 3·0 Amp.
V_c = 21·7 volt
$V_c + V_a$ = 26·5 volt
V_{arc} in volt
46
42
38
34
30
26
22
0
0·4
0·6
1·0
1·4
1·6
2·0
2·4
2·6
3·0
3·4
3·6
Inter electrode distance in m.m.

Fig. 6·3.

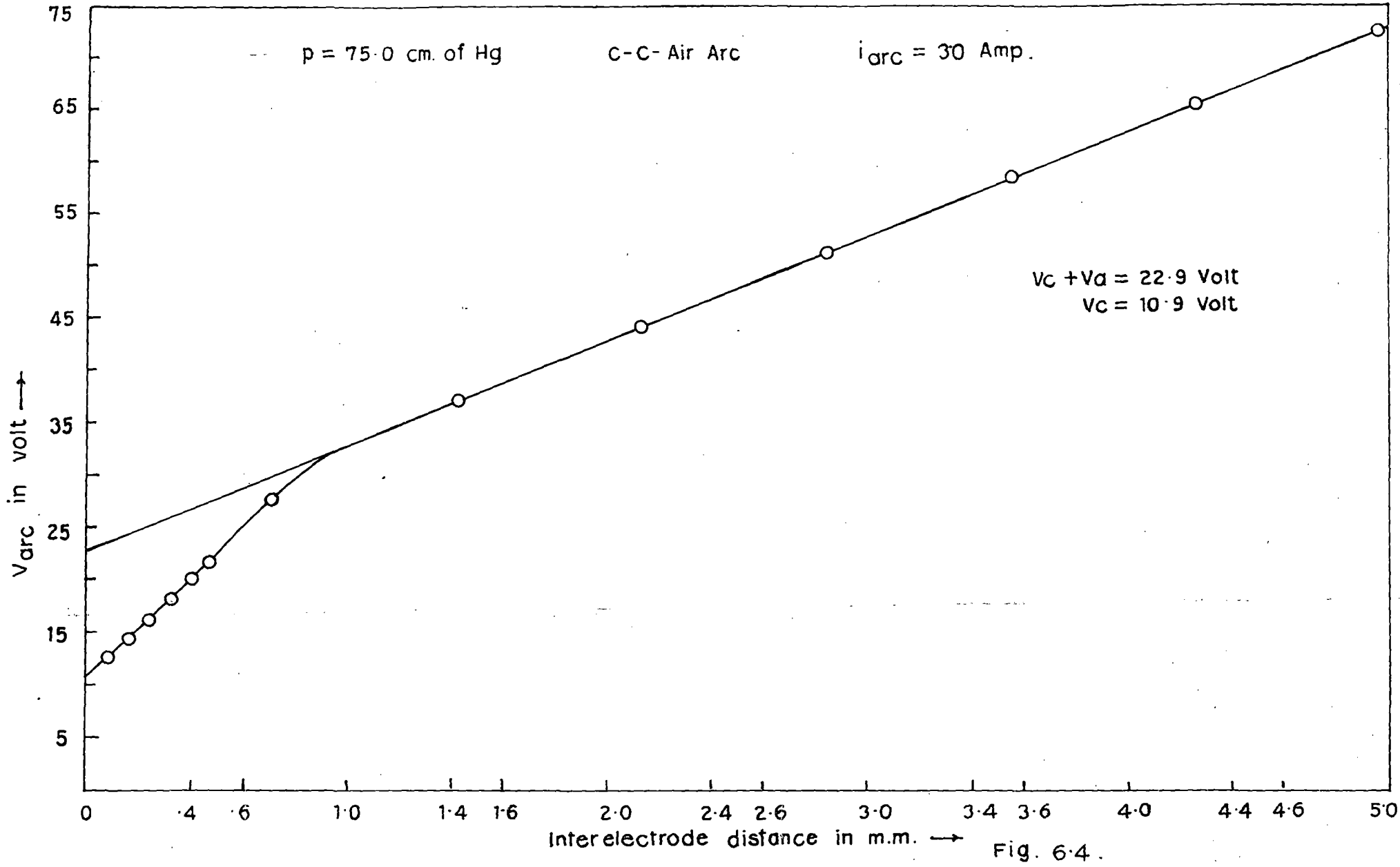

p = 75·0 cm. of Hg
C-C- Air Arc
i_{arc} = 30 Amp.
Vc + Va = 22·9 Volt
Vc = 10·9 Volt
V_{arc} in volt →
75
65
55
45
35
25
15
5
0
·4
·6
1·0
1·4
1·6
2·0
2·4
2·6
3·0
3·4
3·6
4·0
4·4
4·6
5·0
Interelectrode distance in m.m. →

Fig. 6·4.

So from equation (6.28) we can conclude that if

$$\log_{10}\left[\frac{pV_c^{3/2}}{(V_c-V_i)\left\{1-5{\cdot}32\times10^{-3}(V_c-V_i)^2\right\}}\right]$$

is plotted against $\sqrt{V_c}\,p$, the curve should be a straight line if the assumptions are valid.

EXPERIMENTAL ARRANGEMENT AND RESULTS

It is evident from equation (6.28) that to verify the theoretical deduction with the experimental results we have to measure the cathode fall for a wide range of pressure. The arcs investigated in the present work are Cu-Cu, C-C and Fe-Fe arc in air. The arc is struck in a chamber (Fig. 2.7) which can be gradually evacuated. The pressure inside was maintained by a rotary pump and two needle valves one connected at the input and the other at the output. The distance between the two electrodes could be varied and the experiment consists in measuring the arc voltage for various cathode anode separation distances. The variation of arc voltage with separation distance was plotted and the value of the cathode fall was obtained by the standard method as was done previously (Sen, Gantait and Jana, 1988). Three representative curves Fig. 6.2, 6.3 and 6.4 in the present case are shown. The electrode separation was measured with the help of a linear scale and a large circular scale and it was possible to measure a minimum distance of

separation of 0.004 mm. and the arc drop was measured with a digital meter. The results were obtained for three types of arc mentioned above with pressure variation from 50.0 mm to 750 mm. The pressure was measured with a mercury manometer with one end sealed. To prevent fluctuation of the arc current a constant current device was utilized. The results for Cu-Cu, C-C and Fe-Fe arc in air are shown in table 6.1, 6.2 and 6.3.

Table 6.1

i_{arc} = 3.0 amp, Cu-Cu-Air-Arc

P in mm Hg	$V_c + V_a$ in volt	V_c in volt	V_a in volt
50	24.0	21.1	2.9
75	26.5	21.7	4.8
100	29.4	21.4	8.0
125	29.6	20.7	8.9
150	28.5	19.6	8.9
175	26.5	19.0	7.5
200	25.1	18.6	6.5
225	24.4	18.3	6.1
250	23.9	18.0	5.9
350	22.5	17.1	5.4
450	21.5	16.4	5.1
550	20.8	15.8	5.0
650	20.4	15.4	5.0
750	20.0	15.0	5.0

Table 6.2

C-C-Air-Arc i_{arc} = 3.0 Amp

p in mm Hg	Experimental value			Value from extrapolated graph	
	$V_c + V_a$ in volt	V_c in volt	V_a in volt	V_c in volt	V_a in volt
50	27.4	13.2	14.2	13.2	14.2
75	29.6	14.0	15.6	14.0	15.6
100	33.5	15.0	18.5	15.0	18.5
125	35.6	15.4	20.2	15.4	20.2
150	35.6	14.6	21.0	15.3	21.0
175	29.7	12.3	17.4	14.9	21.5
200	25.3	9.3	16.0	14.6	21.6
225	29.4	11.4	18.0	14.1	21.5
250	33.7	13.7	20.0	13.7	21.1
275	33.8	13.4	20.4	13.4	20.6
300	33.1	13.1	20.0	13.1	20.0
350	31.0	12.8	18.2	12.8	18.2
450	27.8	12.0	15.8	12.0	15.8
550	25.7	11.5	14.2	11.5	14.2
650	24.1	11.2	12.9	11.2	12.9
750	22.9	10.9	12.0	10.9	12.0

Table 6.3

Fe-Fe-Air Arc i_{arc} = 4.0 Amp

P in cm Hg	$V_c + V_a$ in volt	V_c in volt	V_a in volt
5.0	15.7	10.4	5.3
7.5	16.9	11.7	5.2
10.0	18.0	12.8	5.2
12.5	19.0	13.7	5.3
15.0	19.8	14.4	5.4
17.5	20.25	14.75	5.5
20.0	20.50	15.0	5.5
22.5	20.2	14.75	5.45
25.0	19.65	14.35	5.30
27.5	18.85	13.80	5.05
30.0	18.20	13.40	4.80
35.0	17.35	12.95	4.40
40.0	16.85	12.70	4.15
45.0	16.6	12.60	4.00

Table 6.4

Fe-Fe-Air-Arc

i_{arc} = 4.0 Amp

V_i = 7.9 volt ($Fe \rightarrow Fe^+$)

$$R = \frac{p V_c^{3/2}}{(V_c - V_i)\{1 - 5.32 \times 10^{-3}(V_c - V_i)^2\}}$$

$T_c = 2400^{\circ}K$

$L_1 = 7 \times 10^{-5}$ meter mm. of Hg.

p in mm of Hg	V_c in volt	$\sqrt{V_c}$	$(V_c - V_i)$ in volt	$[1-5.33\times10^{-3} \times (V_c - V_i)^2]$	$\sqrt{V_c p}$	$\text{Log}_{10} R$	R (in MKS)
50	10.4	3.22	2.5	0.9668	22.80	2.84	692.8
75	11.7	3.42	3.8	0.9232	29.62	2.93	855.5
100	12.8	3.58	4.9	0.8723	35.78	3.03	1072.1
125	13.7	3.70	5.8	0.8210	41.38	3.12	1330.7
150	14.4	3.79	6.5	0.7752	46.48	3.21	1624.7
175	14.75	3.84	6.85	0.7504	50.81	3.29	1928.3
200	15.0	3.87	7.1	0.7318	54.77	3.35	2234.5
225	14.75	3.84	6.85	0.7504	57.61	3.39	2479.3
250	14.35	3.79	6.45	0.7787	59.90	3.43	2707.1
275	13.80	3.71	5.90	0.8148	61.60	3.47	2928.8
300	13.40	3.66	5.50	0.8391	63.40	3.50	3188.1
350	12.95	3.60	5.05	0.8643	67.32	3.57	3738.4
400	12.70	3.56	4.8	0.8774	71.30	3.63	4294.1
450	12.60	3.55	4.7	0.8825	75.30	3.69	4852.9

Table 6.5

Cu-Cu-Air-Arc $i_{arc} = 3.0$ Amp

$T_c = 2200^{\circ}K$ $V_i = 12.1$ volt $(O_2 \rightarrow O_2^-)$

$L_1 = 7.0 \times 10^{-5}$ metre mm. of Hg.

$$R = \frac{P V_c^{3/2}}{(V_c - V_i)\left\{1 - 5.32 \times 10^{-3}(V_c - V_i)^2\right\}}$$

p in mm of Hg	V_c in volt	$\sqrt{V_c}$	$(V_c - V_i)$ in volt	$\left[1 - 5.32 \times 10^{-3} \times (V_c - V_i)^2\right]$	$\sqrt{V_c p}$	$\log_{10} R$	R
50	21.1	4.59	9.0	0.5690	32.48	2.98	945.6
75	21.7	4.66	9.6	0.5097	40.34	3.19	1550.0
100	21.4	4.63	9.3	0.5399	46.26	3.30	1973.0
125	20.7	4.55	8.6	0.6065	50.87	3.35	2257.0
150	19.6	4.43	7.5	0.7008	54.22	3.39	2478.0
175	19.0	4.36	6.9	0.7467	57.66	3.45	2814.0
200	18.6	4.31	6.5	0.7752	60.99	3.50	3182.0
225	18.3	4.28	6.2	0.7955	64.17	3.55	3573.0
250	18.0	4.24	5.9	0.8148	67.08	3.60	3969.0
350	17.1	4.14	5.0	0.8670	77.36	3.76	5716.0
450	16.4	4.05	4.3	0.9016	85.91	3.89	7710.0
550	15.8	3.97	3.7	0.9272	93.22	4.00	10056.0
650	15.4	3.92	3.3	0.9421	100.1	4.10	12622.0
750	15.0	3.87	2.9	0.9553	106.1	4.20	15715.0

Table 6.6

C-C-Air-Arc

$T_c = 3500^{\circ}K$

$i_{arc} = 3.0$ Amp

$V_i = 9.2$ volt (Cyonogen C_2N_2)

$L_1 = 7 \times 10^{-5}$ meter mm of Hg

$$R = \frac{pV_c^{3/2}}{(V_c - V_i)\left\{1 - 5.32 \times 10^{-3}(V_c - V_i)^2\right\}}$$

$L_1 = 7.0 \times 10^{-5}$ meter mm. of Hg

p in mm of Hg	V_c in volt	$\sqrt{V_c}$	$(V_c - V_i)$	$\left[1 - 5.32 \times 10^{-3} \times (V_c - V_i)^2\right]$	$\sqrt{V_c p}$	$\log_{10} R$	R
50	13.2	3.63	4.0	0.915	25.69	2.82	654.6
75	14.0	3.74	4.8	0.877	32.40	2.97	932.9
100	15.0	3.87	5.8	0.821	38.73	3.11	1291.1
125	15.4	3.92	6.2	0.795	43.87	3.19	1530.9
150	15.3	3.91	6.1	0.802	47.91	3.26	1834.2
175	14.9	3.86	5.7	0.827	51.06	3.33	2135.2
200	14.6	3.82	5.4	0.845	54.04	3.39	2444.5
225	14.1	3.75	4.9	0.872	56.32	3.44	2784.3
250	13.7	3.70	4.5	0.892	58.52	3.50	3157.1
275	13.4	3.66	4.2	0.906	60.70	3.55	3544.4
300	13.1	3.62	3.9	0.919	62.69	3.60	3969.4
350	12.8	3.58	3.6	0.931	66.93	3.68	4785.3
450	12.0	3.46	2.8	0.958	73.48	3.84	6965.4
550	11.5	3.39	2.3	0.972	79.53	3.98	9591.1
650	11.2	3.35	2.0	0.979	85.32	4.10	12455.6
750	10.9	3.30	1.7	0.985	90.42	4.21	16110.8

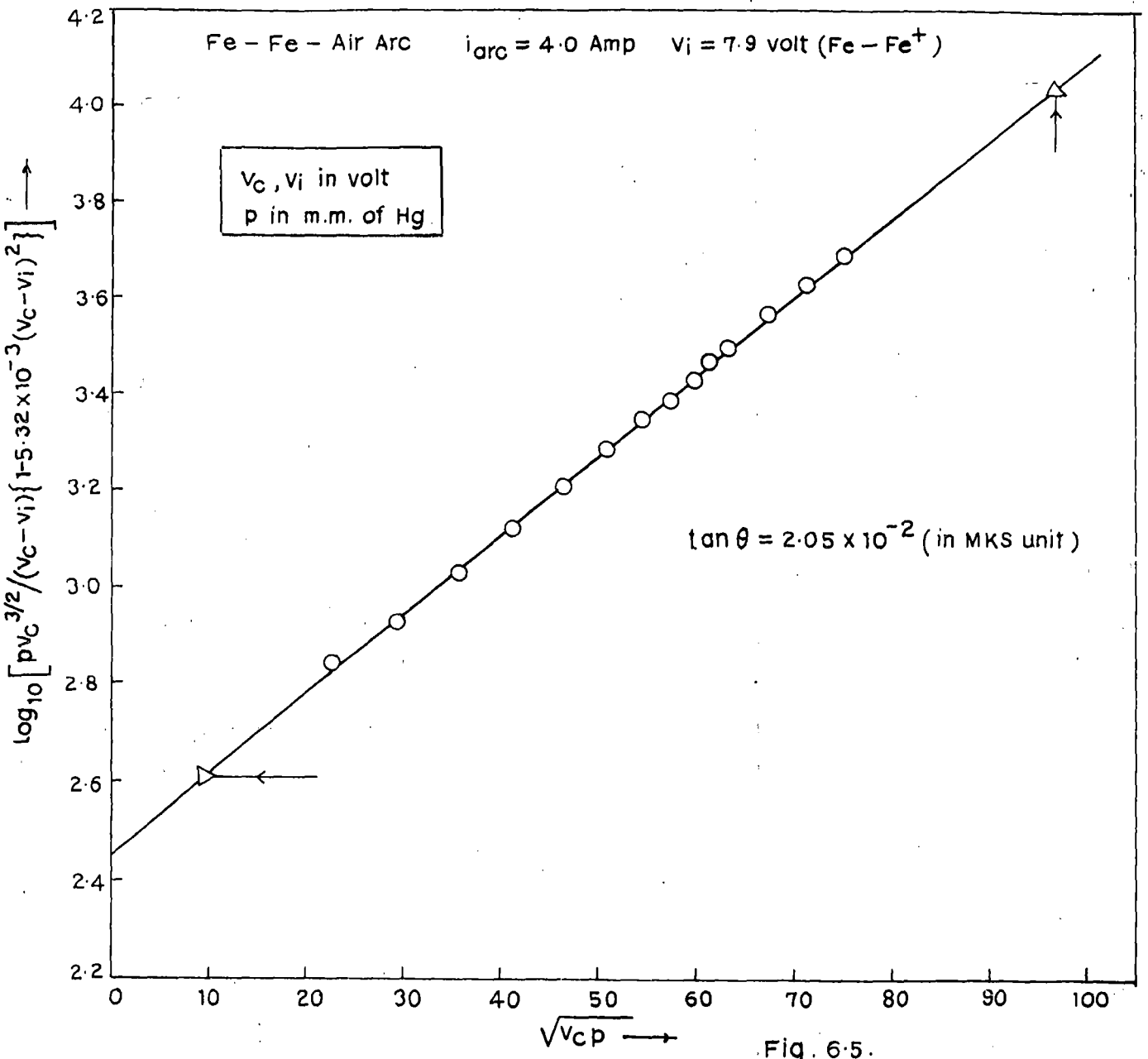

Fe – Fe – Air Arc i_{arc} = 4·0 Amp V_i = 7·9 volt (Fe – Fe^+)
V_c, V_i in volt
p in m.m. of Hg
tan θ = 2·05 x 10^{-2} (in MKS unit)
$\log_{10}\left[pV_c^{3/2}/(V_c-V_i)\{1-5\cdot32\times10^{-3}(V_c-V_i)^2\}\right]$ →
$\sqrt{V_c p}$ →
4·2
4·0
3·8
3·6
3·4
3·2
3·0
2·8
2·6
2·4
2·2
0
10
20
30
40
50
60
70
80
90
100

Fig. 6·5.

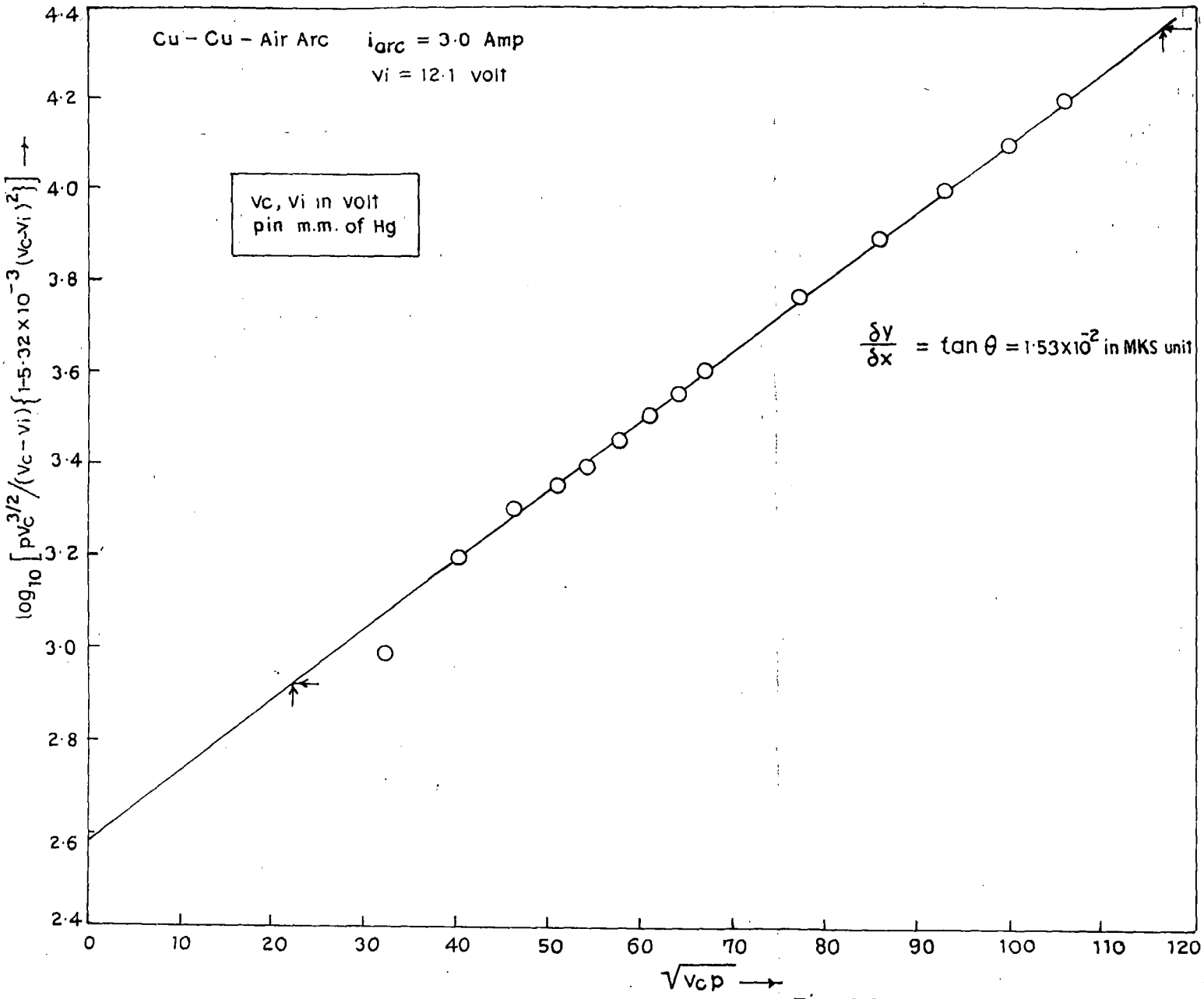

Cu – Cu – Air Arc i_{arc} = 3·0 Amp
Vi = 12·1 volt
Vc, Vi in volt
pin m.m. of Hg
$\frac{\delta y}{\delta x}$ = tan θ = 1·53×10⁻² in MKS unit
$\log_{10}\left[pV_c^{3/2}/(V_c-V_i)\{1-5\cdot32\times10^{-3}(V_c-V_i)^2\}\right]$ →
$\sqrt{V_c p}$ →
4·4
4·2
4·0
3·8
3·6
3·4
3·2
3·0
2·8
2·6
2·4
0
10
20
30
40
50
60
70
80
90
100
110
120

Fig. 6.6.

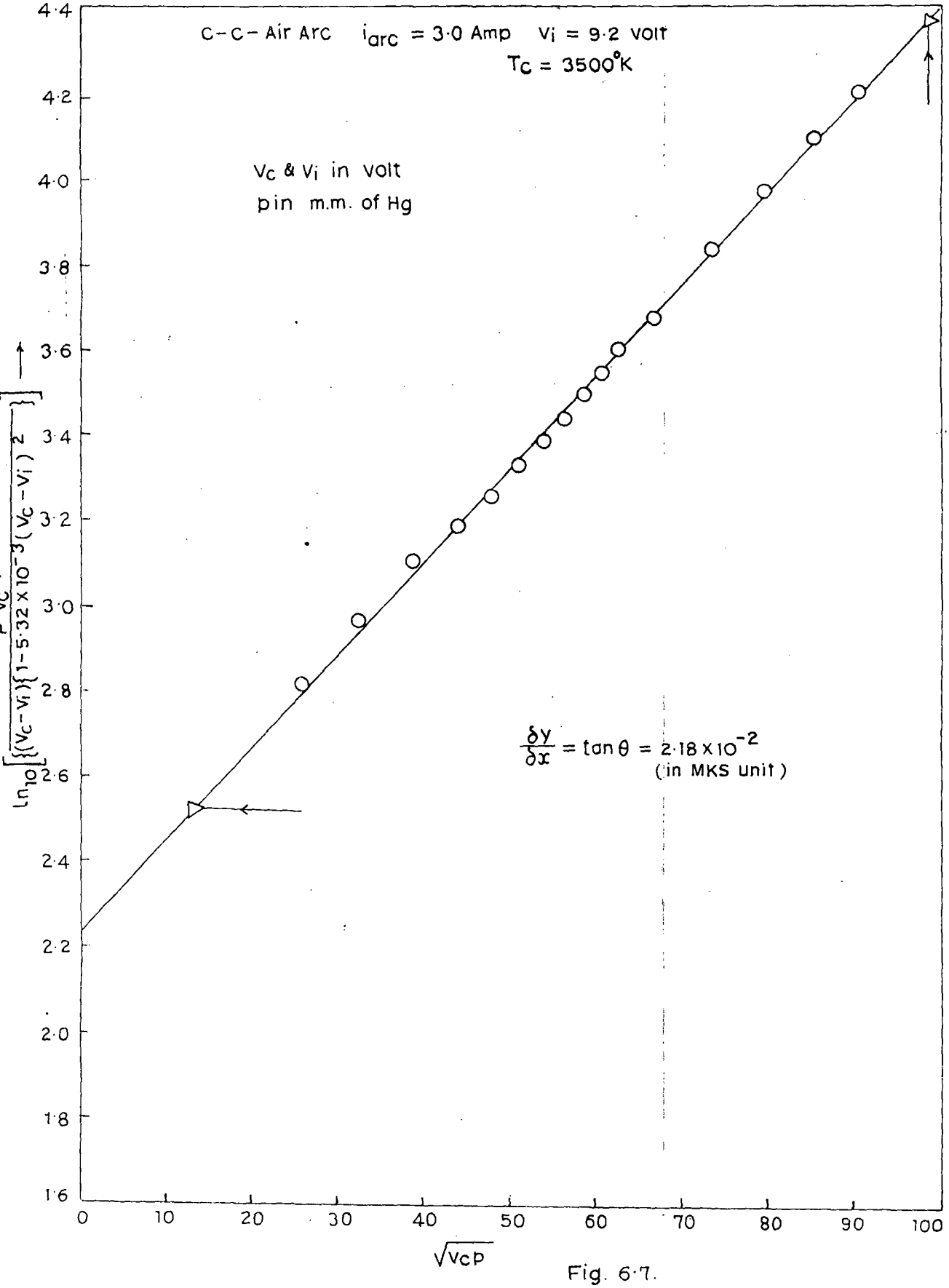

C–C– Air Arc i_{arc} = 3·0 Amp V_i = 9·2 volt
T_c = 3500°K
V_c & V_i in volt
p in m.m. of Hg
$\frac{\delta y}{\delta x} = \tan\theta = 2{\cdot}18 \times 10^{-2}$
(in MKS unit)
$\ln_{10}\left[\frac{p\,V_c^{3/2}}{\{(V_c - V_i)\{1 - 5{\cdot}32 \times 10^{-3}(V_c - V_i)^2\}\}}\right]$
$\sqrt{V_c p}$
4·4
4·2
4·0
3·8
3·6
3·4
3·2
3·0
2·8
2·6
2·4
2·2
2·0
1·8
1·6
0
10
20
30
40
50
60
70
80
90
100

Fig. 6·7.

DISCUSSION

From the values of V_c and the corresponding pressure the values of $\sqrt{V_c p}$ and that of $\log_{10} \frac{p V_c^{3/2}}{(V_c - V_i)\{1 - 5{\cdot}32 \times 10^{-3}(V_c - V_i)^2\}}$ have been calculated where the value of V_i has been taken from the literature (Cobine, 1958) for Cu-Cu, C-C and Fe-Fe arcs and the results are entered in Table 6.5, 6.6 and 6.7. The figures (6.5), (6.6) and (6.7) show that when

$$\log_{10} \frac{p V_c^{3/2}}{(V_c - V_i)\{1 - 5{\cdot}32 \times 10^{-3}(V_c - V_i)^2\}}$$

is plotted against $\sqrt{V_c p}$ the curves are straight lines as predicted by equation (6.28) which shows that the assumptions in the deduction of the theory are valid. Further from the slope of the curves the value of b has been calculated,

$$\text{where } b = \frac{0{\cdot}44}{T_c}\sqrt{\frac{3}{L_1}}\log_{10} e \quad \left[\text{in MKS}\right]$$

and taking the value of T_c and L_1 from literature (Von Engel, 1965) the calculated and experimental values of b are entered in Table 6.7. The agreement for the three arcs is quite satisfactory considering the uncertainty in the values of T_c and L_1.

Table 6.7

Metal	Value of b in MKS units (theoretical)	Value of b in MKS units (experimental)
Cu	1.82×10^{-2}	1.53×10^{-2}
Fe	1.65×10^{-2}	2.07×10^{-2}
C	1.01×10^{-2}	2.18×10^{-2}

Calculation of β_c

From our deduction $$\beta_c = \frac{(A_o\phi/\alpha_c)\, 2{\cdot}29 \times 10^4}{LM + 2{\cdot}29 \times 10^4 V_i}$$

We have taken A_o = 1 sq. cm. and $\alpha_c \doteq 10^{-4}$ sq. cm. [Cobine (1958)] and because of lack of data for variation of α_c with pressure we have taken the value of α_c at p = 760 mm of Hg. so that $(A_o/\alpha_c) \simeq 10^4$. The value of L, M, ϕ, V_i and T_c have been obtained from literature and Cobine (1958) and are shown in table 6.8.

Table 6.8

Metal	Cu	Fe	C
L Cal/gm	730	1071.4	7000
M (Mole) in gm	63.5	56·0	12·0
ϕ in volts	4.45	4.44	8.0
V_i in volts	7.7	7.9	11.3
T_c in ^oK	2200	2400	3500
β_c	4.58×10^3	4.22×10^3	$5{\cdot}34 \times 10^3$

From the relation $A = \dfrac{j_o \beta_c d_c L_1^2 a}{3\epsilon_o \sqrt{2e/m}}$

where a = 26 ion pairs/v/m/mm of Hg, at p = 1 torr. L_1 = 7×10^{-5}m [at $2200^oC (= T_c)$ by Sulhertand's formula], $\epsilon_o = 1.11 \times 10^{-10}$ F/meter, e/m = 1.7593×10^{11} coulombs/kg the value of β_c for the three arcs are obtained from Table 6.8 and the intercept of the curves along the X axis in Fig. 6.5, 6.6 and 6.7 gives the value of $\log_{10}A$ and hence that of A.

Hence the values of J_o have been calculated and entered in table 6.9.

Table 6.9

Metal	$\log_{10}A$	A	$j_o \beta_c$	j_o in MKS unit	j_o in CGS unit
Cu	2.75	562.3	9.45×10^{12}	2.06×10^{9}	2.06×10^{5}
Fe	2.45	281.8	4.73×10^{12}	1.12×10^{9}	1.12×10^{5}
C	2.24	173.8	2.92×10^{12}	5.46×10^{8}	5.46×10^{4}

Further $J = J_o \exp (b\sqrt{V_c p})$

At p = 1 atom.

$b\sqrt{V_c p}$ (for Cu) = 1.63

So $J = 1.057 \times 10^6$ A/cm^2

for Fe, $b\sqrt{V_c p} = 2.02$

So $J = 8.5 \times 10^5$ A/cm^2

for C, $b\sqrt{V_c p} = 2.08$

So, $J = 4.368 \times 10^5$ A/cm^2

From the comparison of experimental results with theoretical deduction we can conclude that the agreement is quite satisfactory and the values of the cathode current density, is in good agreement with published experimental data (Hirsh and Oskam 1978). As mentioned by Cobine (1958) the determinations of the cathooe current density are subject to considerable uncertainty because it is extremely difficult to estimate the exact active area of the cathode spot. The results thus indicate that in the metals studied in this investigation thermionic emission plays a dominant role. We have not considered the effect of field emission in this discussion but as is generally believed both the effects may be simultaneously active.

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