INTRODUCTION

The theory of thermo-elasticity is concerned with the influence of the thermal state upon the distribution of stress and strain and with the inverse effect, that of deformation upon the thermal state of an elastic medium, Duhamel[58] in 1838, initiated the subject deriving equation for the distribution of strain in an elastic medium containing temperature gradients. Subsequently, these results were rediscovered by several authors but Neumann gave the present from known as Duhamel-Neumann relations. The basic theory was applied by Duhamel to a number of problems and later used by Neumann and other authors as the basis of the detailed study. These investigations were instrumental in developing techniques for the solution of thermo-elastic problems but not untill the present century did the subject received the practical stimulus. There have been a rapid development of thermo-elasticity stimulated by various engineering sciences in the post war years. A considerable progress in the field of air-craft and machine structures, mainly with gas and steam turbines, highway engineering especially in the preparation of air base, and the emergence of new topics in chemical and nuclear engineering have given rise to numerous problems in which thermal stress play an important role and frequently even a primary role.

For most practical problems the effect of the stresses and deformations upon the temperature distribution is quite small and can be neglected. The procedure allows the determination of the temperature distribution in the solid resulting from prescribed thermal condition to become first, an independent step of a thermal stress analysis; the second step is then the determination of the stresses and deformation of the body due to this temperature distribution. Before prceeding further, it will be worthwhile mentioning briefly equation of heat conduction and steady state, dynamic state of thermo-elasticity.

EQUATION OF HEAT CONDUCTION

Let in the space (X_r) a solid body B be bounded by the surface S and $T(X_r,t)$ denote the temperature at point (X_r) and at the time t. Then temperature differences between the points of the region B results in a flow of heat. Across a surface element $d\sigma$ at the point (X_r) the quantity of heat flowing in the time interval Δt is

$$\nabla Q = -\lambda T_{,n}\, d\sigma \Delta t$$

Where λ is the coefficient of internal heat conduction, $T_{,n} = \dfrac{\partial T}{\partial n}$ is the normal derivative of the temperature at the point (X_r) of the surface element,in the direction of heat flow.

Now we investigate the equilibrium due to heat in a region B_1 bounded by S_1 constituting a part of B. The quantity of heat flowing into the region B_1 across the boundary S_1 in the time Δt is given by

$$\Delta Q' = \lambda \int_{S_1} T_{,n}\, d\sigma \Delta t$$

If W denotes the quantity of heat generated in unit volume in unit time, then the quantity of heat generated inside the region under consideration is

$$\Delta Q'' = \int_{B_1} W\, d\sigma \Delta t$$

On the other hand, $\Delta Q'' = \Delta Q' + \Delta Q''$ can be determined from

$$\Delta Q = \int_{B_1} c\rho \dot{T}\, dv \Delta t$$

Where ρ is the density and c is the specific heat of the body. The condition $\Delta Q = \Delta Q' + \Delta Q''$ implies the equation

$$\int_{B_1} (c\rho \dot{T} - W)dv - \lambda \int_{S_1} T_{,n}\, d\sigma = 0$$

Which by divergence theorem becomes

$$\int_{B_1} (c\rho\dot{T} - W - \lambda T_{,kk})dv = 0$$

Since this is true for all arbitrary region B_1 hence

$$T_{,kk} - \dot{T}/s = -Q/s \tag{1}$$

Where $s = \lambda/\rho c, W = Qc\rho$

We have used tensor notation, i.e.

$$T_{,i} = \frac{\partial T}{\partial X_i}, T_{,kk} = \nabla^2 T$$

In a cartesian coordinate system. Dots represent derivatives with respect to time.

Solution of equations (1) determine temperature as a function of position and time. If the temperature is independent of time and if there are no heat sources inside the region B, then (1) can be by Laplace equation

$$T_{,kk}=0 \tag{2}$$

and hence in this case, temperature function is a potential function.

EQUATIONS OF THERMO-ELASTICITY.

Generation of stress and strain in a body takes place due to non-uniform distribution of temperature. The temperature T represents the increment of the temperature from the initial stress less state. We assume that the change in temperature is small and therefore it has no influence on the mechanical and thermal properties of the body.

We shall confine ourselves to an isotropic homogeneous body with respect to both its mechanical and thermal properties. Let $u_i(i=1,2,3)$ be the components of displacement vector $\vec{u}$, $e_{ij}(i,j=1,2,3)$ be the components of displacement of strain tensor and $\sigma_{ij}\,(i,j=1,2,3)$, the components of stress tensor.

In the linear theory of elasticity, the strain tensor e_{ij} is considered with the displacement vector by the relation

$$e_{ij}=(u_{i,j} + u_{j,i})/2, \qquad i,j=1,2,3 \qquad (3)$$

The strain tensor is symmetric, i.e. $e_{ij} = e_{ji}$, The components of strain tensor can not be arbitrary, since they should have the following six relations - the so called comparability conditions :

$$e_{ij,\,kl} + e_{kl,\,ij} - e_{jl,ik} - e_{ik,\,jl} = 0 \qquad i,j,k,l = 1,2,3 \qquad (4)$$

Which are satisfied identically if e_{ij} is expressed by u_i in accordance with (3) when the displacement field is continuous.

In thermo-elasticity strain tensors are made up of two parts. The first part e^0_{ij} is a uniform expansion proportional to the temperature rise T. Since this expansion is the same in all directions for an isotropic body, only normal strains and no shearing strains arise in this manner. If α_t is the coefficient of linear expansion and δ_{ij} is the kronecker's symbol, then

$$e^0_{ij} = \alpha_t T\delta_{ij}, \qquad i,j=1,2,3 \qquad (5)$$

The second part e'_{ij} comprises the strains required to maintain the continuity of the body as well as those arising because of external loads. These strains are related to the stresses by means of the Hooke's law of linear isothermal elasticity. Hence

$$e'_{ij} = [\sigma_{ij} - \frac{\nu}{1+\nu}\theta\delta_{ij}]/2\mu_1, \qquad i,j=1,2,3 \qquad (6)$$

Where μ_1 is the shear modulus, ν is the poission's ratio and $\theta = \sigma_{kk}$ is the sum of the normal stresses. Hence finally we have

$$e_{ij} = e_{ij}^0 + e_{ij}^l = \alpha_t T\delta_{ij} + [\sigma_{ij} - \frac{\nu}{1+\nu}\theta\delta_{ij}]/2\mu_1 \tag{7}$$

the so called Duhamel-Neumann relation.

Denoting $\theta = e_{kk}$, we have from (7)

$$\theta - 3\alpha_t T = \frac{1-2\nu}{E}\theta, \quad E = 2\mu_1(1+\nu)$$

Where E is the young's modulus

Solving (7) for stresses, we have

$$\sigma_{ij} = 2\mu_1 e_{ij} + (\lambda\theta - \gamma T)\delta_{ij}, \qquad i,j=1,2,3 \tag{9}$$

Where λ, γ are Lame's elastic constants given by the relations

$$\nu = \frac{\lambda}{2(\lambda + \mu_1)}, \quad \gamma = (3\lambda + 2\mu_1)\alpha_t$$

Now, it order to find the equations of elastic equilibrium, let us consider a body B with boundary S loaded in an arbitrary way and placed in a stationary temperature field. Let us consider the equilibrium of a sub-domain B_1 with boundary S_1. If F_i denotes the components of the body force per unit volume and P_i the components of surface tractions acting on the surface S_1 then from the condition of equilibrium we obtain the following three equations for the region B_1:

$$\int_{B_1} F_i dv + \int_{S_1} P_i d\sigma = 0, \qquad i=1,2,3$$

Taking into account that $P_i = \sigma_{ij} n_j$, where n_j denotes the components of unit normal vector of surface S_1, we get, on making use of devergence theorem

$$\int_{B_1} (F_i + \sigma_{ij,j}) dv = 0$$

Since this is true for an arbitrary region B_1, the equilibrium equations take the form

$$\sigma_{ij,j} + F_i = 0, \qquad i= 1,2,3 \tag{10}$$

If in these equilibrium equations, we express stresses by strains and then by displacements, we obtain a system of three equations in which the unknown functions are the components of displacement vector :

$$\mu_1 u_{i,kk} + (\lambda + \mu) u_{k,ki} + F_i - \gamma T_{,i} = 0 \qquad i,k = 1,2,3. \tag{11}$$

In cylindrical coordinates (r,θ,z), let u,v,w represent the components of displacement vector $\vec{u}, \sigma_{rr}, \sigma_{\theta\theta}, \sigma_{zz}$ represent normal stresses and $\tau_{rz}, \tau_{\theta z}, \tau_{r\theta}$ represent shear stresses. In the case of axial symmetry about the z-axis, equations (11) reduce to two equations.

$$\nabla^2 u - r^{-2} u + \frac{1}{(1-2\nu)} \theta_{,r} - \frac{2(\nu+1)}{(1-2\nu)} \alpha_t T_{,r} = 0$$

$$\nabla^2 w + \frac{1}{1-2\nu} \theta_{,z} - \frac{2(\nu+1)}{(1-2\nu)} \alpha_t T_{,z} = 0 \tag{12a,b}$$

Where

$$\theta = \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}, \qquad \nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2}$$

To solve the equations (11) in the absence of body forces i.e. F_i=0 Goodier[65] introduced a thermoelastic potential ϕ in terms of which the displacement vector is defined by the relation

$$u_i = \partial\phi / \partial x_i \tag{13}$$

and ϕ is a particular solution of the poission's equation

$$\nabla^2\phi = T(x_r) \tag{14}$$

A well known particular integral of (14) is

$$\phi(x_r) = -\frac{1}{4\pi}\int_V \frac{T(\xi_r)dV(\xi_r)}{R(x_r,\xi_r)} \tag{15}$$

Where $R(x_r,\xi_r)$ is the distance between the points (x_r) and (ξ_r).

Integrals of the type (15) were employed by Borchardt [44] in a general discussion of the theory of thermo elasticity and also to solve certain special problems involving asymmetric distribution of temperature in solids with spherical on circular boundaries. Problems concerning spheres and cylinders are dealt in (16, PP. 362-67). The problems of thin elastic plates, under fairly general distributions of temperatures have been considered by Galarkin. Nadai, Marguerre[79]. Sokolnikoff[123] and Pell[97]. Several approximate solutions of the engineering problems concerned with thermal stresses in plates and rods are discussed in chapter 14 of Timoshenko and Goodier's "Theory of Elasticity"[35].

The calculation of the steady-state thermal stresses in an isotropic elasstic half-space or slab with traction free faces has been the subject of several investigations. The distribution of thermal stress due to special temperature distribution in infinite and semi-infinite solids have been discussed by a variety of authors, i.e. Mindlin and Cheng[85], Myklested[86], Sternberg and Mc'Dowel[112], using an extension of Boussinesq - Papkovich method of isothermal elasticity solved the problem of half-space. The basis of the method is that the solution of the equation of equilibrium (11) may be expressed in terms of the four Boussinesq-Papkovich functions, one of which is the solution of poisson's equation and remaining three are of Laplace equation. These equations have been studied extensively, particularly in potential theory, and general procedures of their solutions are known. Sneddon and Locket[121] approached this class of problems by direct solution of the equations of thermo-elasticity using a double Fourier integral transform method, the results being transformed to Hankel type integral in the case of axial symmetry. A further approach due to Nowinski[93] exploits the fact that in steady-state thermo-elasticity each component of the displacement vector is a bi-harmonic function which can be expressed as a combination of harmonics. Possibly the most economical method of solutions of the type of problems is that of Williams[131] who expressed the displacement vector in terms of two scalar potential functions, one of which is directly related to the temperature field. Further, Muki[22] has introduced the displacement and stress components in the form of Hankel transform for the particular solution of the thermo-elastic equations.

It is to note that Nowacki[25] has made thorough survey of the problems of both elaso-static and elasto-dynamic in presence of the temperature excellently.

ABOUT THE THESIS

In the present age of science and technology it is inevitable to have a study on the problems of thermo-elasticity because of the increasing range of applications of the theory and analysis of the thermal stresses in industry, and especially in advanced technologies such as Aerospace Engineering. Laser Engineering. Design of Turbines, Micro Electronics Industry. The subject has tremendous importance in compliance with its application in Geophysical and Seismological problems. The interest in this field of science has been increasing among mechanical engineers, semi-conductor engineers and chemical engineers.

The works of this thesis occupy some important and interesting problems of thermoelasticity. It is largely involved in the determination of basic and fundamental objectives viz. displacement, stresses and stress intensity factors due to the presence of temperature and its impact on the elastic body with endeover to obtain results which will be practically important in applications to applied mathematics, engineering and technology. Here basically statical problems of thermoelasticity have been dealt with. The complete work is divided into four chapters and the problems in each chapter are relevant to each other.

Chapter 1 contains two important and highly interesting penny-shaped crack problems comprising mixed boundary conditions. The first problem suggests penny - shaped flaw subjected to uniform internal heat flux. The second mixed boundary value problem is designed to evaluate the timperature displacement relationship for the penny - shaped inclusion and the stress intensity factor at the boundary of the externally cracked region.

The stress analysis of a penny - shaped crack located in an isotropic elastic solid is a classical problem in linear elastostatics. These problems are of fundamental interest to the study of initiation and propagation of facture in brittle solids. The classical studies of the penny - shaped flaw problem are given by Sneddon[31], Sack[100], Sneddon and Lowengrub[29], Kassir and Sih[17] and Cherepanov[8]. Recently, Selvadurai and Singh[107] have discussed the problem where the surfaces of the penny - shaped crack are indented by aflat penny-shaped rigid inclusion which is important in indentation testing of brittle ceramic materials. In a problem Selvadurai and Singh[107] have examied a penny-Shaped flaw located in the plane of an external crack in an isotropic elastic solid subjected to uniform internal pressure. This paper is concerned with the problem of a penny - shaped crack subjected to uniform internal heat flux located in an isotropic elastic solid which is weakened by an external crack situated in the plane on the penny - shaped crack. The analysis of axisymmetric mixed boundary value problem is achived by employing a Hankel transform development of the governing field equations. The analysis of this problem concerntrates on the evaluation of the stress intensity factors at the boundary of the penny - shaped crack and at the boundary of the externally cracked region. These stress intensity factors are evaluated in power series form in terms of a non - dimensional parameter which involves the ratio of the radius of the penny - shaped crack to the radius of externally crack region. Thermoelastic stress distribution in presence of either rigid or elastic inclusions plays an important role in the mathematical theory of elasticity. Detailed accounts of inclusion problems in classical elasticity are given by Mura[23], Willis[133] and Walpole[130]. Flat disc Shaped inclusions are particular limiting cases of the general class of three-dimension ellipsoidal or Spheroidal inclusions. Works of Collins[50], Keer[72] and Selvadurai[102] are primarily concerned with the study of discinclusions which are embeded in bonded contact with isotropic and transversely isotropic elastic solids. In fact, studies of the interaction between thermal cracks and inclusions located in elastic media has received only limited attention. Recently, Selvadurai and Singh[107] examined the problem of the internal indentation of a penny -shaped crack by a rigid penny - shaped inclussion. This particular problem is of interest to the study of thermally influenced fracturing and degradation of multiphase composits. In this paper, we examine the displacements and stress-intensity factor when axisymmetric temperature distribution acts on a penny-shaped rigid inclusion which is located in the plane of an external circular crack. The mathematical formulation of the mixed boundary value problem employs a Hankel transform development of the governing field equations. The integral equations associated with the mixed boundary conditions are reduced to a system of Abel - type integel equations which are solved approximately involving expansion of the governing functions in power series in terms of a small non-dimensional parameter. The small parameter corresponds to the ratio of the radius of the penny - shaped inclusion to the radius of the external circular crack. Analytical results along with graphical representations are provided for the stiffness of the inclusion embeded in a completely cracked interface and stress intensity factor at the boundary of the externally cracked region.

Chapter II Contains two very useful problems of thermoelasticity (i) a double layered problem (ii) a three layered problem.

The design of highways and airport runways as well as the foundation problems in soil mechanics, especially when the earth mass supporting a heavy structure has different soil strate over it, it is highly needful to look to the endurance of the solid or land on which there is generated a thermal stress either due to impulse shock or owing to some local heating nucleus. Investigation of the stress distribution in a layered system was made by Burnister[40] in a series of papers. In a later paper, Acum and Fox[38] attacked a problem in a three layered system only by the method of Burnister. R.D. Mindlin and D.H. Cheng[85] who employed Galorkin vector for the center of dilatation to obtain stresses for semi-space. Paria[26] in this paper, determined elastic stress distribution in a three layered system due to a concenrated force. But when a plane bombs from above on the surface of the two layered system or a three layered system such as highway or airport land, an emmense heat is generated on the surface and as the heat is assumed to be distributed through layers, therefore the stresses are generated and in each layer and the underlying mass also. the upper layer of the double layered system, as in high ways is considered to be concrete pavement and the underlying mass is natural soil and in case of three layered system, the upper layer is of concrete pavement, the middle layer is of gravel base course and the lower mass is the natural soil as in the case of airport runways. The method of solution consists in taking the Hankel transform[27] of the stress function instead of stress function itself. Stresses in each layer due to the distribution of temperature and ultimately total stress in the underlying mass have been determined for both the cases. The type of heat flux function, if possible from physical point of view, is taken to be linear and graphical representation of the stress distribution in the underlying mass are shown in the figures. Experimental results of the elastic constants are taken from international critical tables[15].

In Chapter III, an attempt has been made to solve three boundary value problems in which thermal stresses have been derived due to sudden heating of the boundary of a circular hole, uniform temperature distributed over a band of the cylindrical hole in infinite bodies and with a constant heat flow over a circular area on the plane boundary of a semi - infinite elastic medium. The high stress concentration found at the edge of a hole is of great practical significance as in the cases of holes in ship's decks. When the hull of a ship is bent, tention or compression is produced in the decks and there by a high stress concentation is developed at the holes. It is often necessary to reduce the stress concentration at holes, such as access holes in airplane wings and fuselages. In the case of circular hole near the straight boundary of a semi - infinite plate under tension parallel to this boundary was analysed by G.B.Jeffery[69]. The circualr hole in a strip is discussed by H.C.J. Howland and A.C.stevensen. Solutions have been obtained for the infinite plate with a circular hole when forces are applied to the boundary of the hole and for the corresponding problem of the strip. In a thermoelastic problem R.Lorenz has discussed a case of a cylender with a concentric hole in it. P.Choudhury[48] in his work, has determined thermal stresses due to temperature distributed over a band of the cylindrical hole in an infinite body. In this paper, the thermal stresses in an infinite elastic solid have been obtained when a band of the internal surface of a cylindrical hole in the solid is kept under uniform distribution of temperature. Temperature potential function is considered in terms of modified Bessel function. The stresses and temperature distribution of the the axi - symmatric boundary value problem are determined and numerical evaluations are made for a suitable component of stress. The other problem deals with the determination of thermal stresses in an infinitely extended thin plate due to sudden heating of the boundary of a circular hole in it. A majority of the investigation in thermoelasticity employs the classical equation of heat conduction which assumes that the flow of heat an elastic material is independent of the variation of strain. Since this assumption is not concordant with the laws of thermodynamics, the effect of rate of strain on the flow of heat is to be taken into account. A modified heat conduction equation has now been considered by several workers in a number of problems of which a major portion is in three dimensions(c.f.Naribole 1961). The present paper is concerned with the determination of temperature and stresses in an infinite plate with a circular hole due to sudden heating of the boundary of the hole. The differential equations of heat conduction coupled with elastic deformation are solved simultaneously as they do not admit of independent solution. Laplace transform is applied as a mathematical tool. The last paper of this chapter solves a semi - infinite thermoelastic problem with constant heat flow over a circular area on the plane boundary. In a paper, Nowacki[25] has solved the problem of thermal stresses in an elastic half space, the bounding surface of which is kept at a constant temperature $T = T_0$ inside a circle of radius a, the exterior of the circle being thermally insulated. In this paper, the solution has been obtained in which there is a constant supply of heat over a circular area of radius a on a bounding plane surface, the rest being kept at a constant temperature T = O. The solution involves Bessel function and the analysis deserves dual integral equations which have been solved to give the temperature distribution and the displacement potential as employed by J.N.Goodier[65]. Numerical work is made accordingly.

Chapter IV contains three relevant coupled problems of thermoelasticity which have solved approximately by way of the coupled equations. The first problem deals with the determination of the distribution of temperature and displacement in a thin semi - infinite elastic rod when its free end is subjected to periodic heating. It has been pointed out by P.Chadwick (1960) that the rigorous approach, i.e. the approach by way of the coupled equations, to the thermal boundary value problems, meets with severe analytical difficulties and upto 1960, only one partial solution [Paria 1958] had been published. The situation has not changed appreciably since; it has been possible to study the behaviour of transient, only for very small or very large values of the time. In view of this situation, it would seem profitable to study the approximate solutions of the coupled thermo - elastic equations, a procedure suggested by Lessen [1956]. In this note the one dimensional problem of the periodic heating of the free surface of a semi-infinite rod has been solved by a perturbation procedure, approximations upto the first order being retained. The second paper derives an approximate expression for the temperature distribution in a uniform semi - infinite rod due to an applied pressure step at on end. Though the effect of temperature changes on the motion of elastic media has long been considered in elasticity, the effect of the compression and dilatation of elastic substances on their temperature changes has recently been formulated in the coupled thermoelastic equations by Biot and Lessen. The exact solution of coupled thermo - elastic equations, even in their simplest forms, had not hither to been possible owing to their complexity. Recently, however Nariboli and Nyayadhish have given a method by which it is possible to solve one - dimensional coupled problems. As their solutions are expressed in terms of two finite and four infinite integrals, a large amount of numerical computation is necessary before it is possible to obtain from their solutions an idea of the nature of variation of the temperature, displacement etc. For this reason, an approximate solution of such a problem has been obtained; this involves the approximate evaluation of temperature change of a semi-infinite elastic rod due to the application of a pressure step at one end. The solution has been obtained by solving the equations of motion in the absence of temperature change and then substituting this in the coupled heat conduction equation. The procedure is in effect a decoupling but it is worth noting that the resultant temperature change is proportional to the coupling constant [illegible]; this is in accordance with the fact that the uncoupled theory does not take into account the effect of the volume changes on the temperature. The approximate solution has been numerically evaluated and its form for small and large values of time has also been obtained. Finally, the theory of coupled thermoelasticity is applied to solve the problem of determination of the distribution of temperature and the thermoelastic deformation in a half space under the action of a thermal shock on the bounding surface. Nariboli and Nyaydhish (1963) have determined the distribution of temperature and thermo - elastic deformation in a half - space, the plane boundary of which is held rigidly fixed and subjected to an instantaneous temperature rise. Solutions have been obtained by the authors for small values of time. In the present paper, the plane boundary of the half-space is free of stress and is subjected to a thermal shock. Moreover, the perturbation method is employed with the thermo - elastic coupling factor e as the perturbation parameter. The solution is obtained by the application of Laplace transform. In order to find the inverse Laplace transform for the temperature, we have assumed further the thermo - elastic coupling factor to be very small. The deformation field is, however, obtained for small values of time. It may be mentioned here that the problem with the formulation of a different type of thermal boundary condition was solved by paria (1968).