CHAPTER - II

CRACK PROBLEMS IN ELASTODYNAMICS

HIGH FREQUENCY SCATTERING OF ANTIPLANE SHEAR WAVES BY AN INTERFACE CRACK

1. INTRODUCTION.

Scattering of elastic waves by a crack of finite length at the interface of two dissimilar elastic materials is important in view of its application in Geophysics and in Mechanical engineering problems. The extensive use of composite materials in modern technology has created interest in the wave propagation problems in layered media with interfacial discontinuities. The diffraction of Love waves by a crack of finite width at the interface of a layered half space was studied by Neerhoff [1979]. Kuo [1984] carried out numerical and analytical studies of transient response of an interfacial crack between two dissimilar orthotropic half spaces. Following the method of Mal [1970], Srivastava et al. [1980] also considered the low frequency aspect of the interaction of antiplane shear waves by a Griffith crack at the interface of two bonded dissimilar elastic half space.

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But high frequency solution of the diffraction of elastic waves by a crack of finite size is interesting in view of the fact that transient solution close to the wave front can be represented by an integral of high frequency component of the solution. Green's function method together with a function-theoretic technique based upon an extended Wiener-Hopf argument has been developed by Keogh [1985 a], [1985 b] for solving the problem of high frequency scattering of elastic waves by a Griffith crack situated in an infinite homogeneous elastic medium.

In the present paper, we have derived the high frequency solution of the diffraction of SH-wave when it interacts with a Griffith crack located at the interface of two bonded dissimilar elastic half spaces. To solve the problem, following the method of Chang [1971], the problem has been formulated as an extended Wiener-Hopf equation and the asymptotic solutions for high frequencies or for wavelengths short compared to the length of the crack have been derived. Expressions for the dynamic stress intensity factor and the crack opening displacement have been obtained and the results have been illustrated graphically for two pairs of different types of material.

2. FORMULATION OF THE PROBLEM

Let (x,y,z) be a rectangular Cartesian coordinates. Let an open crack of finite length 2L be located at the interface of two bonded dissimilar elastic semi-infinite solids lying parallel to x-axis. The x-axis is taken along the interface, y-axis vertically upwards into the medium and z-axis is perpendicular to the plane of the paper. (μ_1,ρ_1) and (μ_2,ρ_2) are coefficients of rigidity and density respectively of the upper and lower semi-infinite medium. The crack is subjected to a normally incoming antiplane shear wave originating at $y = -\infty$.

We are interested in finding the high frequency solution of the diffraction problem i.e. the solution when the length of the crack is large compared to the wave length of the incident wave.

Accordingly we shall have to solve the problem when the crack is subject to the following boundary conditions:

$$\sigma_{yz}^{(1)}(x,0+) = \sigma_{yz}^{(2)}(x,0-) = -P_S - P_0 e^{-i\omega t}, \quad |x| < L \qquad (1)$$

$$\sigma_{yz}^{(1)}(x,0+) = \sigma_{yz}^{(2)}(x,0-), \quad |x| > L \qquad (2)$$

$$w_1(x,0+) = w_2(x,0-), \quad |x| > L \qquad (3)$$

where ω is the circular frequency and P_S is the static pressure.

Assume

$$w_1(x,y,t) = W_1(x,y)\, e^{-i\omega t} \tag{4}$$

$$w_2(x,y,t) = W_2(x,y)\, e^{-i\omega t} \tag{5}$$

where W_1 and W_2 satisfy the following two wave equations

$$\nabla^2 W_1(x,y) + k_1^2 W_1(x,y) = 0 \tag{6}$$

$$\nabla^2 W_2(x,y) + k_2^2 W_2(x,y) = 0 \tag{7}$$

with $$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

The shear wave number k_1 and k_2 are related to the two shear wave velocities C_1 and C_2 of medium (1) and (2) respectively by

$$k_1 = \omega/C_1 \quad (8) \qquad k_2 = \omega/C_2 \quad (9)$$

Without any loss of generality we assume that $k_2 > k_1$.

Let $$\sigma_{yz}^{(1)}(x,y,t) = \tau_{yz}^{(1)}(x,y)\, e^{-i\omega t} \tag{10}$$

$$\sigma_{yz}^{(2)}(x,y,t) = \tau_{yz}^{(2)}(x,y)\, e^{-i\omega t} \tag{11}$$

In the boundary condition (1), P_S is the static pessure assumed to be sufficiently large so that crack faces do not come in contact during vibration. Since we are interested in the dynamic part of the stress distribution, so the boundary conditions (1), (2) and

(3) may be written as

$$\tau_{yz}^{(1)}(x,0+) = \tau_{yz}^{(2)}(x,0-) = -P_o, \quad |x| < L \tag{12}$$

$$\tau_{yz}^{(1)}(x,0+) = \tau_{yz}^{(2)}(x,0-), \quad |x| > L \tag{13}$$

and

$$W_1(x,0+) = W_2(x,0-), \quad |x| > L \tag{14}$$

that is

$$\mu_1 \frac{\partial W_1}{\partial y} = \mu_2 \frac{\partial W_2}{\partial y} = - P_o, \quad |x|<L, \quad y = 0 \tag{15}$$

$$\mu_1 \frac{\partial W_1}{\partial y} = \mu_2 \frac{\partial W_2}{\partial y}, \quad |x|>L, \quad y = 0 \tag{16}$$

and $\quad W_1(x,0+) = W_2(x,0-), \quad |x|>L \quad (17)$

In order to obtain solutions of wave equations (6) and (7) we introduce Fourier transform defined by

$$\bar{W}(\alpha,y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} W(x,y)\, e^{i\alpha x}\, dx \tag{18}$$

Thus we obtain the transformed wave equations as

$$\frac{d^2 \bar{W}_1}{dy^2} - (\alpha^2-k_1^{\,2})\bar{W}_1 = 0 \tag{19}$$

$$\frac{d^2 \bar{W}_2}{dy^2} - (\alpha^2 - k_2^2)\bar{W}_2 = 0 \tag{20}$$

The solutions of (19) and (20), bounded as y tends to infinity, are

$$\bar{W}_1(\alpha, y) = A_1(\alpha)\, e^{-\gamma_1 y}, \quad y \geq 0 \tag{21}$$

$$\bar{W}_2(\alpha, y) = A_2(\alpha)\, e^{\gamma_2 y}, \quad y \leq 0 \tag{22}$$

where $\gamma_1 = (\alpha^2 - k_1^2)^{1/2}$ (23) $\gamma_2 = (\alpha^2 - k_2^2)^{1/2}$ (24)

Introducing for a complex α

$$G_+(\alpha) = \frac{1}{\sqrt{2\pi}} \int_L^{\infty} \tau_{yz}^{(1)}(x,0)\, e^{i\alpha (x-L)}\, dx \tag{25}$$

$$G_-(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-L} \tau_{yz}^{(1)}(x,0)\, e^{i\alpha (x+L)}\, dx \tag{26}$$

and

$$G_1(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-L}^{L} \tau_{yz}^{(1)}(x,0)\, e^{i\alpha x}\, dx \tag{27}$$

the transformed stress at the interface y = 0 can be written as

$$\bar{\tau}_{yz}^{(1)}(\alpha,0) = G_+(\alpha)\, e^{i\alpha L} + G_1(\alpha) + G_-(\alpha)\, e^{-i\alpha L} \tag{28}$$

Using the boundary condition (12) we note that

$$G_1(\alpha) = -\frac{P_0}{\sqrt{2\pi}\ i\alpha}\left[e^{i\alpha L} - e^{-i\alpha L}\right] \tag{29}$$

Further using the fact that

$$\bar{\tau}^{(1)}_{yz}(\alpha,0) = -\mu_1\gamma_1 A_1(\alpha) \tag{30}$$

we obtain from (28)

$$-\mu_1\gamma_1 A_1(\alpha) = G_+(\alpha)e^{i\alpha L} + G_-(\alpha)e^{-i\alpha L} - \frac{P_0}{\sqrt{2\pi}\ i\alpha}\left[e^{i\alpha L} - e^{-i\alpha L}\right] \tag{31}$$

Since from (12) and (13) stress τ_{yz} is continuous at all points of the interface, so we obtain

$$A_2(\alpha) = -\frac{\mu_1\gamma_1}{\mu_2\gamma_2} A_1(\alpha). \tag{32}$$

So (21) and (22) take the forms

$$\bar{W}_1(\alpha,y) = A_1(\alpha)\ e^{-\gamma_1 y}, \quad y \geq 0 \tag{33}$$

$$\bar{W}_2(\alpha,y) = -\frac{\mu_1\gamma_1}{\mu_2\gamma_2} A_1(\alpha)\ e^{\gamma_2 y}, \quad y \leq 0 \tag{34}$$

Now $\bar{W}_1(\alpha,0+) - \bar{W}_2(\alpha,0-) = \frac{1}{\sqrt{2\pi}} \int_{-L}^{L} \left[W_1(x,0+) - W_2(x,0-) \right] e^{i\alpha x}\, dx$

$$= B(\alpha), \qquad \text{(say)} \tag{35}$$

wich is the measure of the discontinuity of displacement along the surface of the crack. From (35) we get

$$A_1(\alpha) = \frac{\mu_2 \gamma_2 B(\alpha)}{\mu_1\gamma_1 + \mu_2\gamma_2} \tag{36}$$

Eliminating $A_1(\alpha)$ from (31) and (36) we obtain an extended Wiener-Hopf equation, namely

$$G_+(\alpha)\, e^{i\alpha L} + G_-(\alpha)\, e^{-i\alpha L} + B(\alpha)K(\alpha) = \frac{P_0}{\sqrt{2\pi}\ i\alpha}\left[e^{i\alpha L} - e^{-i\alpha L}\right] \tag{37}$$

where

$$K(\alpha) = \frac{\mu_1\mu_2\gamma_1\gamma_2}{\mu_1\gamma_1 + \mu_2\gamma_2} = \frac{\mu_1\mu_2(\alpha^2-k_1^{\,2})^{1/2}}{(\mu_1+\mu_2)} R(\alpha) \tag{38}$$

$$R(\alpha) = \frac{(\mu_1+\mu_2)\ (\alpha^2-k_2^{\,2})^{1/2}}{\mu_1(\alpha^2-k_1^{\,2})^{1/2} + \mu_2(\alpha^2-k_2^{\,2})^{1/2}} \tag{39}$$

In order to solve the Wiener-Hopf equation given by (37) we assume that the branch points $\alpha = k_1$ and k_2 of $K(\alpha)$ possess a small imaginary part such that

$$k_1 = k_1 + ik_1' \quad \text{and} \quad k_2 = k_2 + ik_2'$$

where k_1' and k_2' are infinitesimally small positive quantities which would ultimately be made to tend to zero.

Now we write $K(\alpha) = K_+(\alpha)K_-(\alpha)$ where $K_+(\alpha)$ is analytic in the upper half plane $\text{Im}\,\alpha > -k_2'$ whereas $K_-(\alpha)$ is analytic in the lower half plane given by $\text{Im}\,\alpha < k_2'$. Since $\tau_{yz}(x,0)$ decreases exponentially as $x \longrightarrow \pm\infty$, $G_+(\alpha)$ and $G_-(\alpha)$ have the same common region of regularity as $K_+(\alpha)$ and $K_-(\alpha)$.

Now (37) can easily be expressed as two integral equations relating $G_+(\alpha)$, $G_-(\alpha)$ and $B(\alpha)$ as follows:

$$\frac{G_+(\alpha)}{K_+(\alpha)} - \frac{P_0}{\sqrt{2\pi}\, i\alpha}\left[\frac{1}{K_+(\alpha)} - \frac{1}{K_+(0)}\right] +$$

$$+ \frac{1}{2\pi i}\int_{C_+} \frac{e^{-2isL}}{(s-\alpha)K_+(s)}\left[G_-(s) + \frac{P_0}{\sqrt{2\pi}\, is}\right] ds$$

$$= -B(\alpha)\,K_-(\alpha)\,e^{-i\alpha L} + \frac{P_0}{\sqrt{2\pi}\, i\alpha\, K_+(0)} -$$

$$- \frac{1}{2\pi i}\int_{C_-} \frac{e^{-2isL}}{(s-\alpha)K_+(s)}\left[G_-(s) + \frac{P_0}{\sqrt{2\pi}\, is}\right] ds \qquad (40)$$

and

$$\frac{G_-(\alpha)}{K_-(\alpha)} + \frac{P_0}{\sqrt{2\pi}\ i\alpha\ K_-(\alpha)} + \frac{1}{2\pi i}\int_{C_-} \frac{e^{2isL}}{(s-\alpha)\ K_-(s)}\left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\ is}\right]ds$$

$$= -B(\alpha)K_+(\alpha)\ e^{i\alpha L} - \frac{1}{2\pi i}\int_{C_+} \frac{e^{2isL}}{(s-\alpha)\ K_-(s)}\left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\ is}\right]ds \quad (41)$$

where C_+ and C_- are the straight contours below the pole at s = 0 and situated within the common region of regularity of $G_+(s)$, $G_-(s)$, $K_+(s)$, and $K_-(s)$ as shown in Fig. 1.

In (40), the left-hand side is analytic in the upper half plane whereas the right-hand side is analytic in the lower-half plane and both of them are equal in the common region of analyticity of these two functions. So by analytic continuation, both sides of (40) are analytic in the whole of the s-plane. Now since

$$\tau_{yz} \sim (x \mp L)^{-1/2} \qquad \text{as } x \longrightarrow \pm L$$

so,

$$G_{\pm}(\alpha) \sim \alpha^{-1/2} \qquad \text{as } |\alpha| \longrightarrow \infty$$

and also

$$K_{\pm}(\alpha) \sim \alpha^{1/2} \qquad \text{as } |\alpha| \longrightarrow \infty$$

so it follows that

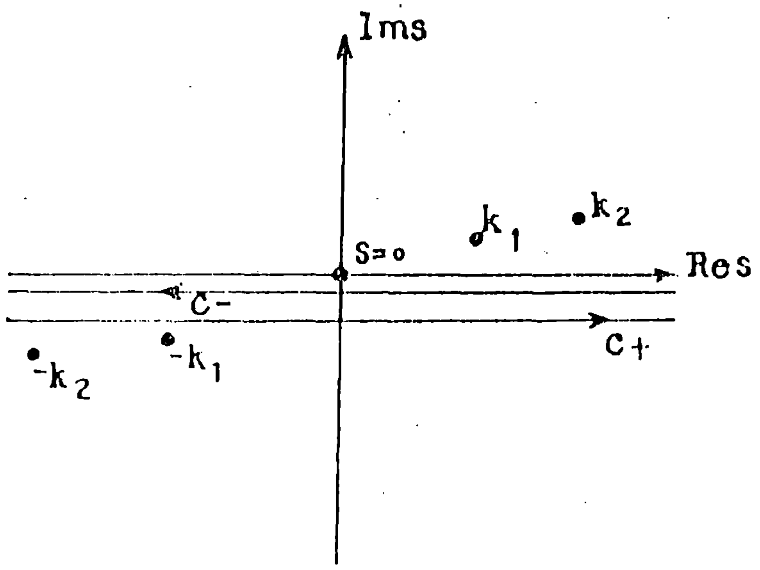

Fig. 1. Path of integration in the complex s-plane.

$$\frac{G_{\pm}(\alpha)}{K_{\pm}(\alpha)} \sim \alpha^{-1} \quad \text{as } |\alpha| \longrightarrow \infty.$$

Therefore by Liouville's theorem, both sides of (40) are equal to zero. Equation (41) can be treated similarly.

Therefore from (40) and (41) we obtain the system of integral equations given by

$$\left[G_{+}(\alpha) - \frac{P_{0}}{\sqrt{2\pi}\; i\alpha} \right] \frac{1}{K_{+}(\alpha)} + \frac{P_{0}}{\sqrt{2\pi}\; i\alpha\; K_{+}(0)} +$$

$$+ \frac{1}{2\pi i} \int_{C_{+}} \frac{e^{-2isL}}{(s-\alpha)\; K_{+}(s)} \left[G_{-}(s) + \frac{P_{0}}{\sqrt{2\pi}\; is} \right] ds = 0 \qquad (42)$$

and

$$\left[G_{-}(\alpha) + \frac{P_{0}}{\sqrt{2\pi}\; i\alpha} \right] \frac{1}{K_{-}(\alpha)} +$$

$$+ \frac{1}{2\pi i} \int_{C_{-}} \frac{e^{2isL}}{(s-\alpha)\; K_{-}(s)} \left[G_{+}(s) - \frac{P_{0}}{\sqrt{2\pi}\; is} \right] ds = 0 \qquad (43)$$

Since $\tau_{yz}^{(1)}(x,0)$ is an even function of x, so from (25) and (26) it can be shown that $G_{+}(-\alpha) = G_{-}(\alpha)$ and it has been shown in the Appendix that $K_{+}(-\alpha) = iK_{-}(\alpha)$. Using these results and replacing α by $-\alpha$ and s by $-s$ in (42) it can easily be shown that equations (42) and (43) are identical. So $G_{+}(\alpha)$ and $G_{-}(\alpha)$ are to be determined from any one of the integral equation (42) or (43).

3. HIGH FREQUENCY SOLUTION OF THE INTEGRAL EQUATION

To solve the integral equation (43) in the case when normalized wave number $k_1 L \gg 1$, the integration along the path C_- in (43) is replaced by the integration round the circular contour C_0 round the pole at $s = 0$ and by the integration along the contours C_{k_1} and C_{k_2} round the branch cuts through the branch points k_1 and k_2 of the function $K_-(s)$ as shown in Fig. 2.

Thus equation (43) takes the form

$$\left[G_-(\alpha) + \frac{P_0}{\sqrt{2\pi}\; i\alpha} \right] - \frac{P_0\; K_-(\alpha)}{\sqrt{2\pi}\; i\alpha\; K_-(0)} +$$

$$+ \frac{K_-(\alpha)}{2\pi i} \int_{C_{k_1}+C_{k_2}} \frac{e^{2isL}}{(s-\alpha)K_-(s)} \left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\; is} \right] ds = 0 \qquad (44)$$

Now

$$\int_{C_{k_1}} \frac{e^{2isL}}{(s-\alpha)K_-(s)} \left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\; is} \right] ds$$

$$= \frac{1}{\mu_1} \int_{C_{k_1}} \frac{e^{2isL}\; K_+(s)}{(s-\alpha)(s^2-k_1^2)^{1/2}} \left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\; is} \right] ds$$

which can easily be evaluated when $k_1 L \gg 1$ and is found to be equal to

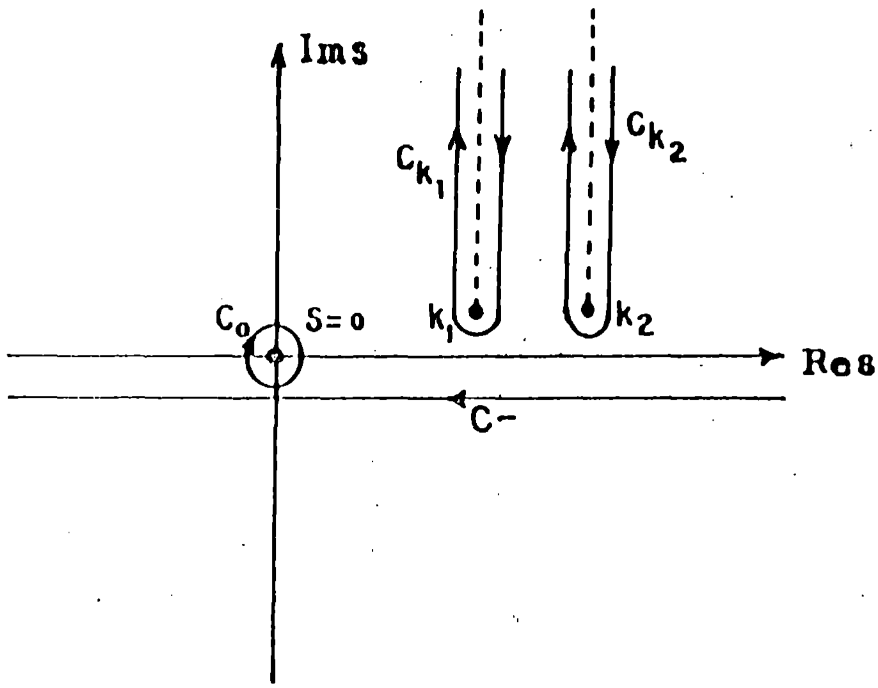

FIG. 2. Path of integration C_0, C_{k_1}, C_{k_2}.

$$- \frac{1}{\mu_1} \sqrt{\frac{\pi}{k_1 L}} \frac{e^{2ik_1 L} K_+(k_1) e^{i\pi/4}}{(k_1-\alpha)} \left[G_+(k_1) - \frac{P_0}{\sqrt{2\pi}\, ik_1} \right] \qquad (45)$$

Similarly for $k_1 L >> 1$

$$\int_{C_{k_2}} \frac{e^{2isL}}{(s-\alpha)\, K_-(s)} \left[G_+(s) - \frac{P_0}{\sqrt{2\pi}\, is} \right] ds$$

$$= - \frac{1}{\mu_2} \sqrt{\frac{\pi}{k_2 L}} \frac{e^{2ik_2 L} K_+(k_2) e^{i\pi/4}}{(k_2-\alpha)} \left[G_+(k_2) - \frac{P_0}{\sqrt{2\pi}\, ik_2} \right] \qquad (46)$$

Using the results (45) and (46) and also the relations $G_+(-\alpha) = G_-(\alpha)$ and $K_-(-\alpha) = -iK_+(\alpha)$, we obtain from (44)

$$F_+(-\alpha) + \frac{A(k_1)F_+(k_1)e^{2ik_1 L}}{\mu_1(k_1-\alpha)\sqrt{k_1 L}} + \frac{A(k_2)F_+(k_2)e^{2ik_2 L}}{\mu_2(k_2-\alpha)\sqrt{k_2 L}} = C(\alpha). \qquad (47)$$

where

$$F_+(\xi) = \frac{1}{K_-(-\xi)} \left[G_+(\xi) - \frac{P_0}{\sqrt{2\pi}\, i\xi} \right] \qquad (48)$$

$$A(\xi) = \frac{[K_+(\xi)]^2 e^{i\pi/4}}{2\sqrt{\pi}} \qquad (49)$$

and

$$C(\xi) = \frac{P_0}{\sqrt{2\pi}\, iK_-(0)\xi} \qquad (50)$$

Substituting $\alpha = -k_1$ and $\alpha = -k_2$ in (47), we obtain respectively the equations

$$\left[1 + \frac{A(k_1)e^{2ik_1L}}{2\mu_1 k_1\sqrt{k_1L}}\right]F_+(k_1) + \frac{A(k_2)F_+(k_2)e^{2ik_2L}}{\mu_2(k_1+k_2)\sqrt{k_2L}} = -C(k_1) \quad (51)$$

and

$$\frac{A(k_1)e^{2ik_1L}}{\mu_1(k_1+k_2)\sqrt{k_1L}}F_+(k_1) + \left[1 + \frac{A(k_2)e^{2ik_2L}}{2\mu_2 k_2\sqrt{k_2L}}\right]F_+(k_2) = -C(k_2) \quad (52)$$

Now solving (51) and (52) we get

$$F_+(k_1) = C(k_1)\left[\frac{A(k_2)\ (k_1-k_2)e^{2ik_2L}}{2\mu_2 k_2(k_1+k_2)\sqrt{k_2L}} - 1\right]U(k_1,k_2) \quad (53)$$

and

$$F_+(k_2) = C(k_2)\left[\frac{A(k_1)\ (k_2-k_1)e^{2ik_1L}}{2\mu_1 k_1(k_1+k_2)\sqrt{k_1L}} - 1\right]U(k_1,k_2) \quad (54)$$

where

$$U(k_1,k_2) = \left[1 + \frac{A(k_1)\ e^{2ik_1L}}{2\mu_1 k_1\sqrt{k_1L}} + \frac{A(k_2)\ e^{2ik_2L}}{2\mu_2 k_2\sqrt{k_2L}} + \frac{A(k_1)A(k_2)(k_1-k_2)^2 e^{2i(k_1+k_2)L}}{4\mu_1\mu_2 k_1 k_2(k_1+k_2)^2\sqrt{Lk_1}\ \sqrt{Lk_2}}\right]^{-1} \quad (55)$$

Now expanding $U(k_1,k_2)$ and neglecting higher order terms of $(k_1L)^{-1/2}$ and $(k_2L)^{-1/2}$ and using (47) we get

$$G_-(\alpha) = - C(\alpha)\, K_-(0) + C(\alpha)\, K_-(\alpha) +$$

$$+ \frac{K_-(\alpha)A(k_1)e^{2ik_1L}.C(k_1)}{\mu_1(k_1-\alpha)\sqrt{k_1L}} \left[1- \frac{A(k_1)e^{2ik_1L}}{2\mu_1k_1\sqrt{k_1L}} - \frac{A(k_2)k_1e^{2ik_2L}}{\mu_2k_2\sqrt{k_2L}\,(k_1+k_2)}\right] +$$

$$+ \frac{K_-(\alpha)A(k_2)e^{2ik_2L}.C(k_2)}{\mu_2(k_2-\alpha)\sqrt{k_2L}} \left[1- \frac{A(k_1)k_2e^{2ik_1L}}{\mu_1k_1\sqrt{k_1L}\,(k_1+k_2)} - \frac{A(k_2)e^{2ik_2L}}{2\mu_2k_2\sqrt{k_2L}}\right]$$

(56)

Now replacing α by $-\alpha$ and using $C(-\alpha) = - C(\alpha)$. We have

$$G_+(\alpha) = C(\alpha)\, K_-(0) - C(\alpha)K_-(-\alpha) +$$

$$+ \frac{K_-(-\alpha)A(k_1)e^{2ik_1L}.C(k_1)}{\mu_1(k_1+\alpha)\sqrt{k_1L}} \left[1- \frac{A(k_1)e^{2ik_1L}}{2\mu_1k_1\sqrt{k_1L}} - \frac{A(k_2)k_1e^{2ik_2L}}{\mu_2k_2\sqrt{k_2L}\,(k_1+k_2)}\right]+$$

$$+ \frac{K_-(-\alpha)A(k_2)e^{2ik_2L}.C(k_2)}{\mu_2(k_2+\alpha)\sqrt{k_2L}} \left[1- \frac{A(k_1)k_2e^{2ik_1L}}{\mu_1k_1\sqrt{k_1L}\,(k_1+k_2)} - \frac{A(k_2)e^{2ik_2L}}{2\mu_2k_2\sqrt{k_2L}}\right].$$

(57)

4. STRESS INTENSITY FACTOR AND CRACK OPENING DISPLACEMENT NEAR THE CRACK TIPS

Now as $\alpha \longrightarrow \infty$

$$K_-(-\alpha) = -\, iK_+(\alpha) = -\, i(\alpha+k_1)^{1/2}\left[\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right]^{1/2} \simeq -\, i\alpha^{1/2}\left[\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right]^{1/2}$$

$$\frac{K_-(-\alpha)}{\alpha + k_1} \approx -\, i\alpha^{-1/2}\left[\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right]^{1/2}$$

$$\frac{K_-(-\alpha)}{\alpha+k_2} \approx -i\alpha^{-1/2}\left[\frac{\mu_1\mu_2}{\mu_1+\mu_2}\right]^{1/2}$$

So as $\alpha \rightarrow \infty$ we get from (56) and (57)

$$G_+(\alpha) \approx S\,\alpha^{-1/2} + \frac{P_0}{\sqrt{2\pi}\; i\alpha}$$

and

$$G_-(\alpha) \approx -\, iS\,\alpha^{-1/2} - \frac{P_0}{\sqrt{2\pi}\; i\alpha} \qquad (58)$$

where

$$S = \frac{P_0}{\sqrt{2\pi}\, K_-(0)} \left[1 - \frac{A(k_1)\, e^{2ik_1L}}{\mu_1 k_1 \sqrt{k_1 L}} + \frac{A(k_2)\, e^{2ik_2L}}{\mu_2 k_2 \sqrt{k_2 L}} + \right.$$

$$+ \frac{1}{2} \left(\frac{A^2(k_1)\, e^{4ik_1L}}{\mu_1^2\, k_1^2\, k_1\, L} + \frac{A^2(k_2)\, e^{4ik_2L}}{\mu_2^2\, k_2^2\, k_2\, L} \right) +$$

$$\left. + \frac{A(k_1)\, A(k_2)\, e^{2i(k_1+k_2)L}}{\mu_1\, k_1\, \mu_2\, k_2 \sqrt{k_1 L\, k_2 L}} \right] \left(\frac{\mu_1\, \mu_2}{\mu_1 + \mu_2} \right)^{1/2} \qquad (59)$$

Now from equation (37) using (58) and also the fact that

$$K(\alpha) \longrightarrow \pm\, \alpha \frac{\mu_1 \mu_2}{\mu_1 + \mu_2} \quad \text{as } \alpha \longrightarrow \pm\, \infty \qquad (60)$$

we get

$$B(\alpha) = \frac{\pm\, S}{\alpha\sqrt{\alpha}} \left[ie^{-i\alpha L} - e^{i\alpha L} \right] \frac{\mu_1 + \mu_2}{\mu_1 \mu_2} \quad \text{as } \alpha \longrightarrow \pm\, \infty \qquad (61)$$

Taking inverse Fourier-Transform of (35) and using the results of Fresnel integrals viz.

$$\int_0^{\infty} \frac{\begin{matrix}\sin\\ \cos\end{matrix}(x+L)\alpha}{\sqrt{\alpha}}\, d\alpha = \left(\frac{\pi}{2(x+L)} \right)^{1/2} \qquad (62)$$

We get the displacement jump across the surface of the crack as

$$\Delta W = W_1(x,0+) - W_2(x,0-) = 2S_1(1-i)\sqrt{(L-x)} \quad \text{for } x \to L-0 \tag{63}$$

and

$$\Delta W = W_1(x,0+) - W_2(x,0-) = 2S_1(1-i)\sqrt{(x+L)} \quad \text{for } x \to -L+0 \tag{64}$$

where $$S_1 = \frac{(\mu_1+\mu_2)}{\mu_1\ \mu_2} S \tag{65}$$

Next in order to find the value of τ_{xy} near about the crack tip we use (61) in (36) and (32) and to obtain

$$A_j(\alpha) = \frac{(-1)^{j+1} S}{\mu_j\ \alpha\sqrt{\alpha}} \left[ie^{-i\alpha L} - e^{i\alpha L} \right], \ (j = 1,2) \text{ as } \alpha \to \infty \tag{66}$$

and

$$A_j(\alpha) = \frac{(-1)^{j+1} S}{\mu_j\ \alpha\sqrt{-\alpha}} \left[e^{-i\alpha L} - ie^{i\alpha L} \right], \ (j = 1,2) \text{ as } \alpha \to -\infty \tag{67}$$

Now

$$\tau_{yz}(x,y) = \mu_j \frac{\partial W_j(x,y)}{\partial y}, \quad j=1,2$$

$$= \mu_j \frac{\partial}{\partial y} \left[\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} A_j(\alpha) \exp\left\{- \gamma_j|y| - i\alpha x\right\} d\alpha \right] \tag{68}$$

Substituting the values of $A_j(\alpha)$ as $|\alpha| \to \infty$, we can write the stress near about the crack tip as

$$\tau_{yz}(x,y) = \frac{S}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\alpha|y|}}{\sqrt{\alpha}} \Big[e^{i\alpha(x+L)} - ie^{i\alpha(x-L)} - ie^{-i\alpha(x+L)} + e^{-i\alpha(x-L)} \Big] d\alpha$$

$$= \frac{S(1-i)}{\sqrt{2\pi}} \int_0^\infty \frac{e^{-\alpha|y|}}{\sqrt{\alpha}} \Big[\cos\alpha(x+L) - \sin\alpha(x+L) + \cos\alpha(x-L) + \sin\alpha(x-L) \Big] d\alpha$$

$$= S\,(1-i) \left[\frac{1}{\sqrt{r_2}} \sin\frac{\phi_2}{2} + \frac{1}{\sqrt{r_1}} \cos\frac{\phi_1}{2} \right] \qquad (69)$$

near about the crack tips, where

$$r_1 = \left[(x-L)^2 + y^2 \right]^{1/2}, \quad \phi_1 = \sin^{-1}\frac{|y|}{r_1} \qquad (70)$$

$$r_2 = \left[(x+L)^2 + y^2 \right]^{1/2}, \quad \phi_2 = \sin^{-1}\frac{|y|}{r_2} \qquad (71)$$

Therefore at the interface (y = 0) we obtain

$$\tau_{yz} \longrightarrow \frac{S(1-i)}{\sqrt{(x-L)}} \quad \text{as } x \longrightarrow L+0 \tag{72}$$

and

$$\tau_{yz} \longrightarrow \frac{S(1-i)}{\sqrt{-(x+L)}} \quad \text{as } x \longrightarrow -L-0 \tag{73}$$

Now the stress intensity factor is defined by

$$K = \frac{|(1-i)\ S|\sqrt{2\pi k_1}}{P_0} \tag{74}$$

The absolute value of the complex stress intensity factor defined by (74) has been plotted against k_1L in Fig.3 for values of $k_1L > 1$ for the following two sets of materials, given by

First Set:

Steel	$\rho_1 = 7.6$ gm/cm^3	$\mu_1 = 8.32 \times 10^{11}$ dyne/cm^2
Aluminium	$\rho_2 = 2.7$ gm/cm^3	$\mu_2 = 2.63 \times 10^{11}$ dyne/cm^2

Second Set:

Wrought iron	$\rho_1 = 7.8$ gm/cm^3	$\mu_1 = 7.7 \times 10^{11}$ dyne/cm^2
Copper	$\rho_2 = 8.96$ gm/cm^3	$\mu_2 = 4.5 \times 10^{11}$ dyne/cm^2.

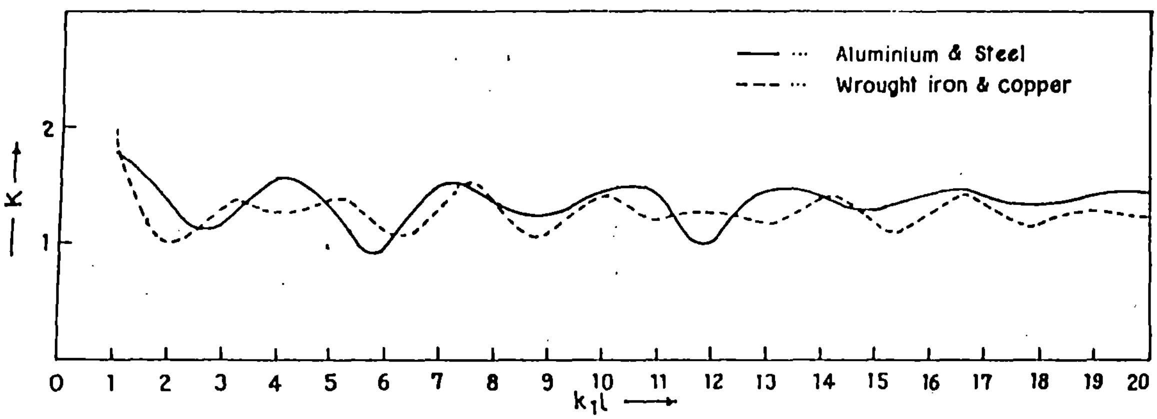

FIG. 3. Stress intensity factor K versus dimensionless frequency $k_1\,l$.

5. CRACK OPENING DISPLACEMENT AT POINTS AWAY FROM THE CRACK TIPS

Next in order to obtain the displacement jump for the large values of $k_1(L-x)$ and $k_1(L+x)$ we write $G_+(\alpha)$ and $G_-(\alpha)$ from (57) and (56) respectively as

$$G_+(\alpha) = \frac{p}{\alpha} - \frac{QK_-(-\alpha)}{\alpha} + \frac{R(k_1,k_2)\ K_-(-\alpha)}{k_1+\alpha} + \frac{R(k_2,k_1)\ K_-(-\alpha)}{k_2+\alpha} \tag{75}$$

and

$$G_-(\alpha) = -\frac{P}{\alpha} + \frac{QK_-(\alpha)}{\alpha} + \frac{R(k_1,k_2)\ K_-(\alpha)}{k_1-\alpha} + \frac{R(k_2,k_1)\ K_-(\alpha)}{k_2-\alpha} \tag{76}$$

where $$P = \frac{P_0}{\sqrt{2\pi}\ i} \tag{77}$$

$$Q = \frac{P_0}{\sqrt{2\pi}\ iK_-(0)} = \frac{P}{K_-(0)} \tag{78}$$

and

$$R(k_m,k_n) = \frac{QA(k_m)\ e^{2ik_m L}}{\mu_m k_m \sqrt{Lk_m}}\left[1 - \frac{e^{2ik_m L}\ A(k_m)}{\sqrt{Lk_m}\ 2\mu_m k_m} - \frac{e^{2ik_n L}\ A(k_n)k_m}{\sqrt{Lk_n}\ \mu_n k_n (k_m+k_n)}\right] \tag{79}$$

where $m = 1$ when $n = 2$

and $m = 2$ when $n = 1$.

Again using $K_-(-\alpha) = -iK_+(\alpha)$ we get from (37)

$$B(\alpha) = -\frac{Qie^{i\alpha L}}{\alpha K_-(\alpha)} + \frac{iR(k_1,k_2)\, e^{i\alpha L}}{(k_1+\alpha)\, K_-(\alpha)} + \frac{iR(k_2,k_1)\, e^{i\alpha L}}{(k_2+\alpha)\, K_-(\alpha)} -$$

$$- \frac{Q\, e^{-i\alpha L}}{\alpha\, K_+(\alpha)} - \frac{R(k_1,k_2)\, e^{-i\alpha L}}{(k_1-\alpha)\, K_+(\alpha)} - \frac{R(k_2,k_1)\, e^{-i\alpha L}}{(k_2-\alpha)\, K_+(\alpha)} \qquad (80)$$

From (35) we get the displacement jump across the surface of the crack as

$$W_1(x,0+) - W_2(x,0-) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} B(\alpha)\, e^{-i\alpha x}\, d\alpha. \qquad (81)$$

Now substituting the expression of $B(\alpha)$ from (80) in (81) and approximately evaluating the integrals arising in (81) term by term for large values of $k_1(L-x)$, $k_2(L-x)$, $k_1(L+x)$ and $k_2(L+x)$ and neglecting terms of order higher than $(k_1L)^{-3/2}$ and $(k_2L)^{-3/2}$, we obtain finally the crack opening displacement across the cracked-surface in the following form:

$$\Delta W = W_1(x,0+) - W_2(x,0-) = 2\pi\ Qi\ K_+(0)\left(\frac{1}{\mu_1 k_1} + \frac{1}{\mu_2 k_2}\right) +$$

$$+ \sqrt{2}\ Q\ e^{-i\pi/4}\left[\left(\frac{e^{ik_1(L-x)}}{\sqrt{k_1(L-x)}} + \frac{e^{ik_1(L+x)}}{\sqrt{k_1(L+x)}}\right) \times\right.$$

$$\times \left\{R_1 + \frac{R_1\ R_{11}\ e^{2ik_1L}}{\sqrt{2k_1L}} + \frac{R_2\ R_{21}\ e^{2ik_2L}}{\sqrt{2k_2L}} + \frac{R_1(R_{11})^2\ e^{4ik_1L}}{\sqrt{2k_1L}\ \sqrt{2k_1L}} +\right.$$

$$\left. + \frac{R_2R_{22}R_{21}e^{4ik_2L}}{\sqrt{2k_2L}\ \sqrt{2k_2L}} + \frac{R_1R_{12}R_{21}e^{2i(k_1+k_2)L}}{\sqrt{2k_1L}\ \sqrt{2k_2L}} + \frac{R_2R_{21}R_{11}e^{2i(k_1+k_2)L}}{\sqrt{2k_1L}\ \sqrt{2k_2L}}\right\} +$$

$$+ \left(\frac{e^{ik_2(L-x)}}{\sqrt{k_2(L-x)}} + \frac{e^{ik_2(L+x)}}{\sqrt{k_2(L+x)}}\right) \times$$

$$\times \left\{R_2 + \frac{R_2R_{22}e^{2ik_2L}}{\sqrt{2k_2L}} + \frac{R_1R_{12}e^{2ik_1L}}{\sqrt{2k_1L}} + \frac{R_2(R_{22})^2\ e^{4ik_2L}}{\sqrt{2k_2L}\ \sqrt{2k_2L}} +\right.$$

$$\left.\left. + \frac{R_1R_{11}R_{12}e^{4ik_1L}}{\sqrt{2k_1L}\ \sqrt{2k_1L}} + \frac{R_2R_{21}R_{12}e^{2i(k_1+k_2)L}}{\sqrt{2k_1L}\ \sqrt{2k_2L}} + \frac{R_1R_{12}R_{22}e^{2i(k_1+k_2)L}}{\sqrt{2k_1L}\ \sqrt{2k_2L}}\right\}\right]$$

(82)

where

$$R_1 = \frac{K_+(k_1)}{\sqrt{2}\,\mu_1 k_1} \qquad R_2 = \frac{K_+(k_2)}{\sqrt{2}\,\mu_2 k_2}$$

$$R_{11} = \frac{D\,[K_+(k_1)]^2}{\mu_1\,(k_1+k_1)} \qquad R_{22} = \frac{D\,[K_+(k_2)]^2}{\mu_2\,(k_2+k_2)}$$

$$R_{21} = \frac{D\,K_+(k_1)\,K_+(k_2)}{\mu_1\,(k_1+k_2)} \qquad R_{12} = \frac{D\,K_+(k_1)\,K_+(k_2)}{\mu_2\,(k_1+k_2)}$$

$$D = (-1)\,\frac{e^{i\pi/4}}{\sqrt{2\pi}}\,. \qquad (83)$$

Expressions in (63) and (64) give the displacement jump nearabout the crack tips where as the displacement jump at points away from the crack tips are given by (82).

From these two results we can obtain the crack opening displacement at any point of the crack surface $-L < x < L$, $y = 0$.

Here also normalized crack opening displacement has been plotted against normalized distance x/L from the centre of the crack for two different sets of materials in Fig. 4. It is interesting to note that oscillatory nature of the crack opening displacement increases with the increase of frequencies as a result of the interference of waves inside the crack. Further we note that amplitude of the crack opening displacement decreases with the increase of frequency.

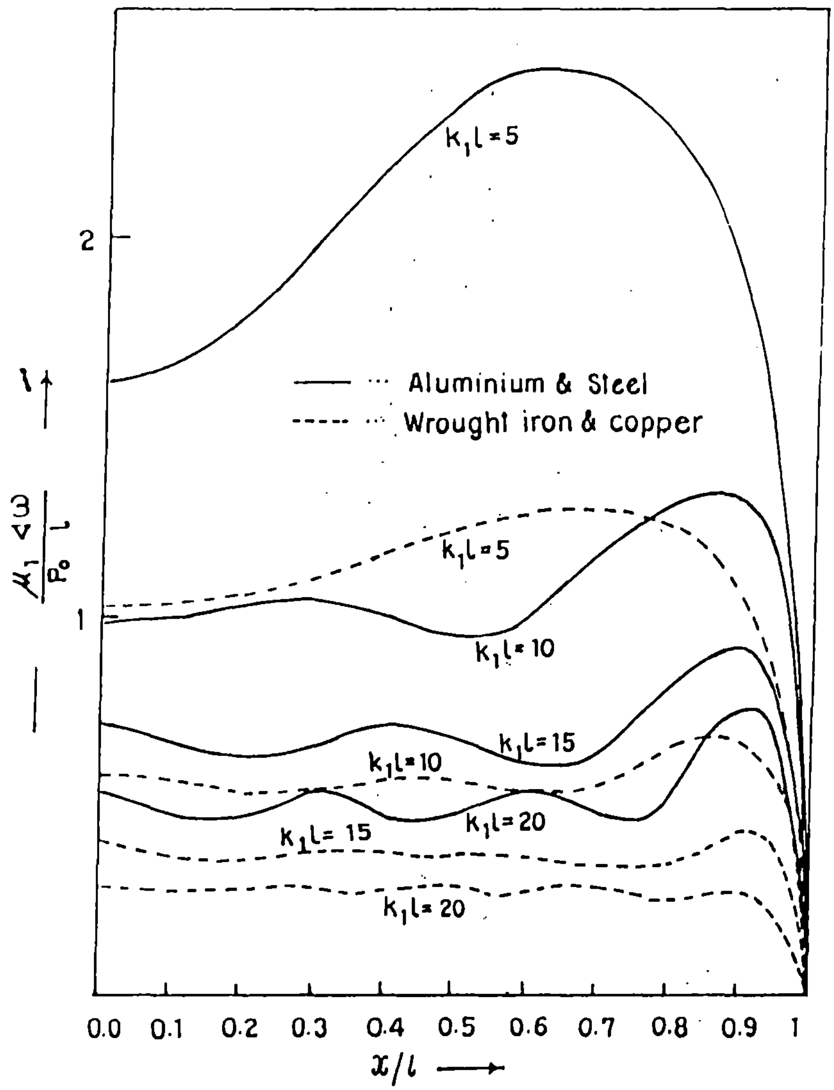

FIG. 4. Normalized crack opening displacement versus normalized distance x/l from the centre of the crack.

Appendix

$$K(\alpha) = \frac{\mu_1\mu_2(\alpha^2 - k_1^2)^{1/2}}{(\mu_1 + \mu_2)} R(\alpha)$$

where $$R(\alpha) = \frac{(\mu_1+\mu_2)(\alpha^2-k_2^2)^{1/2}}{\mu_1(\alpha^2-k_1^2)^{1/2}+\mu_2(\alpha^2 - k_2^2)^{1/2}}$$

put $$m = \frac{\mu_2}{\mu_1}$$

Therefore

$$K(\alpha) = \frac{\mu_2(\alpha^2-k_1^2)^{1/2}}{1 + m} R(\alpha) \tag{A1}$$

where

$$R(\alpha) = \frac{(1+m)\ (\alpha^2-k_2^2)^{1/2}}{(\alpha^2-k_1^2)^{1/2}+m(\alpha^2-k_2^2)^{1/2}} \rightarrow 1 \quad \text{as} \quad |\alpha| \rightarrow \infty$$

Now $$R_+(\alpha)\ R_-(\alpha) = \left[\frac{m}{1+m} + \frac{(\alpha^2-k_1^2)^{1/2}}{(m+1)(\alpha^2-k_2^2)^{1/2}} \right]^{-1}$$

Therefore

$\log R_+(\alpha) + \log R_-(\alpha) =$

$$= \mathrm{Log} \left[\frac{m}{1+m} + \frac{(\alpha^2-k_1^2)^{1/2}}{(m+1)(\alpha^2-k_2^2)^{1/2}} \right]^{-1} = \log R(\alpha)$$

Therefore

$$\log R_+(\alpha) = \frac{1}{2\pi i}\int_{C_L} \frac{\log R(z)}{(z-\alpha)}\,dz = \frac{1}{2\pi i}\int_{-ic-\infty}^{-ic+\infty} \frac{\log R(z)}{(z-\alpha)}\,dz$$

where the path of integration C_L is shown in Fig. 5.

Putting z = -z and using the fact that R(z) = R(-z), we get

$$\log R_+(\alpha) = -\frac{1}{2\pi i}\int_{ic-\infty}^{ic+\infty} \frac{\log R(z)}{(z+\alpha)}\,dz$$

$$= -\frac{1}{2\pi i}\int_{C_1} \frac{\log R(z)}{(z+\alpha)}\,dz$$

where C_1 is the contour round the branch points k_1 and k_2 as shown in Fig. 6.

So,

$$\log R_+(\alpha) = \frac{1}{2\pi i}\int_{C_1} \frac{\log\left[\dfrac{m}{m+1} + \dfrac{(z^2-k_1^2)^{1/2}}{(m+1)(z^2-k_2^2)^{1/2}}\right]}{(z+\alpha)}\,dz$$

$$= \frac{1}{2\pi i}\int_{k_1}^{k_2} \frac{\log\left[1 + \dfrac{i(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}}\right]}{(z+\alpha)}\,dz \quad -$$

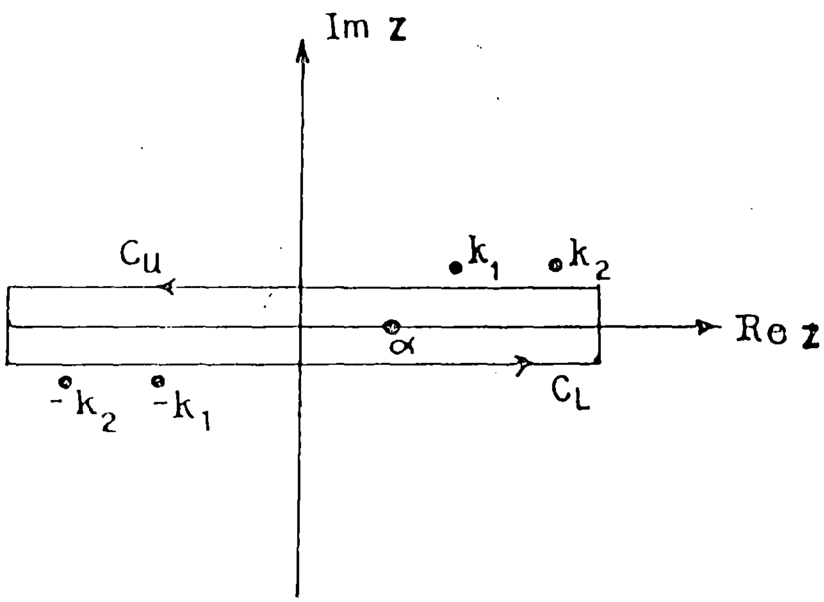

Fig. 5. Complex z-plane.

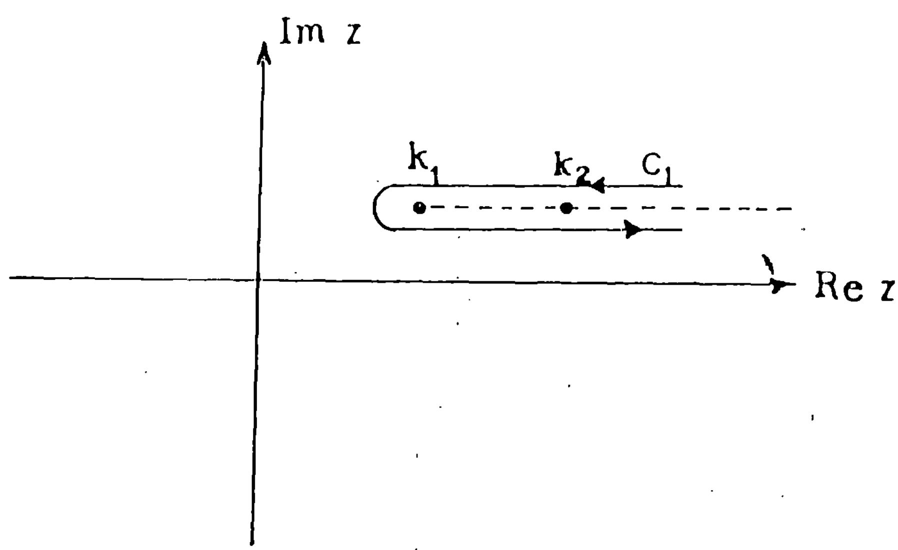

Fig. 6. Path of integration round the branch points.

$$- \frac{1}{2\pi i} \int_{k_1}^{k_2} \frac{\log \left[1 - \frac{i(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}} \right]}{(z + \alpha)} dz$$

$$= \frac{1}{\pi} \int_{k_1}^{k_2} \frac{\tan^{-1}\left[\frac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}} \right]}{(z + \alpha)} dz$$

Therefore $$R_+(\alpha) = \exp \left[\frac{1}{\pi} \int_{k_1}^{k_2} \frac{\tan^{-1}\left[\frac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}} \right]}{(z + \alpha)} dz \right]$$

Similarly $$R_-(\alpha) = \exp \left[\frac{1}{\pi} \int_{k_1}^{k_2} \frac{\tan^{-1}\left[\frac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}} \right]}{(z - \alpha)} dz \right]$$

Therefore from (A1) we can write

$$K_+(\alpha) = \frac{\sqrt{\mu_2}\ (\alpha+k_1)^{1/2}}{\sqrt{(1+m)}} \exp\left[\frac{1}{\pi}\int_{k_1}^{k_2} \frac{\tan^{-1}\left[\dfrac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}}\right]}{(z+\alpha)}\,dz\right] \qquad (A2)$$

and

$$K_-(\alpha) = \frac{\sqrt{\mu_2}\ (\alpha-k_1)^{1/2}}{\sqrt{(1+m)}} \exp\left[\frac{1}{\pi}\int_{k_1}^{k_2} \frac{\tan^{-1}\left[\dfrac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}}\right]}{(z-\alpha)}\,dz\right] \qquad (A3)$$

Hence from (A2) and (A3) we get

$$K_+(-\alpha) = \frac{\sqrt{\mu_2}\ i(\alpha-k_1)^{1/2}}{\sqrt{(1+m)}} \exp\left[\frac{1}{\pi}\int_{k_1}^{k_2} \frac{\tan^{-1}\left[\dfrac{(z^2-k_1^2)^{1/2}}{m(k_2^2-z^2)^{1/2}}\right]}{(z-\alpha)}\,dz\right]$$

$$= iK_-(\alpha)$$

i.e. $$K_+(-\alpha) = iK_-(\alpha) \qquad (A4)$$

---x---

HIGH FREQUENCY SCATTERING OF PLANE HORIZONTAL SHEAR WAVES BY A GRIFFITH CRACK PROPAGATING ALONG THE BIMATERIAL INTERFACE

1. INTRODUCTION

Scattering of elastic waves by a stationary or a moving crack of finite length at the interface of two dissimilar elastic materials is important in view of its application in fracture mechanics as well as in seismology. Recently, Takei, Shindo and Atsumi [1982] considered the problem of diffraction of transient horizontal shear waves by a finite crack lying on a bimaterial interface. The method of solution was extended by Ueda, Shindo and Atsumi [1983] to solve the problem of torsional impact response of a penny shaped interface crack. Srivastava et al [1980] also considered the low frequency aspect of the interaction of an antiplane shear wave by a Griffith crack at the interface of two bonded dissimilar elastic half spaces.

In the case of cracks of finite size, travelling at a constant velocity, loads, for mathematical simplicity, are usually assumed to be independent of time. However, in practice, structures

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are often required to sustain oscillating loads where the dynamic disturbances propagate through the elastic medium in the form of stress waves. The problem of diffraction of plane harmonic polarized shear wave by a half plane crack extended under antiplane strain was first studied by Jahanshahi [1967]. Later Chen and Sih [1973] considered the interaction of stress waves with a semi-infinite running crack under either the plane strain or the generalized plane stress condition. Sih and Loeber [1970] and Chen and Sih [1975] also considered the problem of scattering of plane harmonic waves by a running crack of finite length. In both the cases the problem was reduced to a system of simultaneous Fredholm integral equations which were solved numerically.

In the present paper, we have investigated the high frequency solution of the problem of diffraction of horizontally polarized shear waves by a finite crack moving on a bimaterial interface. The high frequency solution of the diffraction of elastic waves by a crack of finite size is important in view of the fact that transient solution close to the wave front can be represented by an integral of the high frequency component of the solution. In order to solve the problem, following the method of Chang [1971], the problem has been formulated as an extended Wiener-Hopf equation and the asymptotic solutions for high frequencies or for wave lengths which are short compared to the length of the crack have been derived. Expressions for the dynamic stress intensity factor at the crack tip and the crack opening displacement have been derived. The dynamic stress intensity factor for high frequencies has been illustrated graphically for two pairs of different types of materials for different crack velocities and angles of incidence.

2. FORMULATION OF THE PROBLEM AND ITS SOLUTION

Let a plane crack of width 2L move at a constant velocity V at the interface of two bonded dissimilar elastic semi-infinite media due to the incidence of the plane horizontal SH-wave

$$W_i = A \exp[-\{k_1(X \cos\theta_1 + Y \sin\theta_1) + \Omega T\}] \quad (1)$$

in the medium. The crack lies on the bimaterial interface along Y=0 with respect to the fixed rectangular co-ordinate system (X,Y,Z) as shown in Fig.1.

We assume that the displacement and stress fields W_j, τ_{yz_j} (j=1,2) are

$$W_j = W_j(X,Y,T) \quad (2)$$

$$\tau_{yz_j} = \mu_j \frac{\partial W_j(X,Y)}{\partial Y}, \quad (3)$$

in which subscripts j=1,2 refer to the upper and lower half planes,

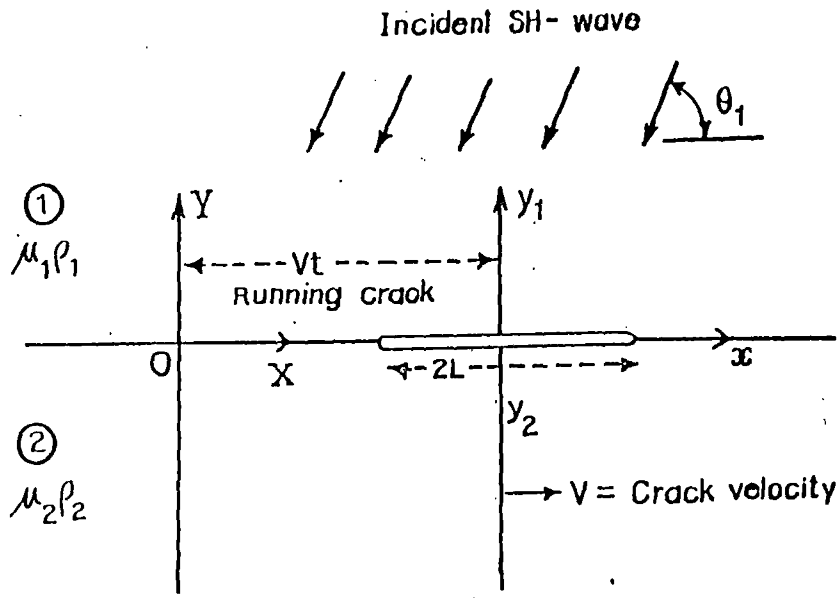

Fig. 1. Running interface crack.

respectively, T denotes time and μ_j is shear modulus of elasticity. The displacement W_j is governed by the classical wave equation

$$\frac{\partial^2 W_j}{\partial X^2} + \frac{\partial^2 W_j}{\partial Y^2} = \frac{1}{c_j^2} \frac{\partial^2 W_j}{\partial T^2}, \qquad (j=1,2) \tag{4}$$

where $c_j=(\mu_j/\rho_j)^{1/2}$ is shear wave velocity and ρ_j is the density of the material. Without any loss of generality, we further assume that $c_1 > c_2$.

Due to the incident wave given by (1), reflected and transmitted waves in the absence of the crack may be written in the form

$$W_r = B \exp [-i\{k_1(X \cos\theta_1 - Y \sin\theta_1) + \Omega T \}] \tag{5}$$

and

$$W_T = C \exp [-i\{k_2(X \cos\theta_2 + Y \sin\theta_2) + \Omega T \}], \tag{6}$$

where

$$B = \frac{k_1 \sin\theta_1 - m k_2 \sin\theta_2}{k_1 \sin\theta_1 + m k_2 \sin\theta_2} A \tag{7}$$

$$C = \frac{2k_1 \sin\theta_1}{k_1 \sin\theta_1 + m k_2 \sin\theta_2} A \tag{8}$$

$$m = \mu_2/\mu_1 \qquad \text{and} \qquad k_1 \cos\theta_1 = k_2 \cos\theta_2 \tag{9}$$

A,B,C are incident, reflected and transmitted wave amplitude, k_j is the wave number, $\Omega = k_j c_j$ is the circular frequency and θ_1, θ_2 are the angles of incidence and refraction, respectively.

A set of moving co-ordinates (x, y_j, z, t) attached to the centre of the crack moving at a constant velocity V is introduced in accordance with

$$x = X - Vt, \qquad y_j = s_j Y, \qquad z = Z, \qquad t = T \tag{10}$$

where $s_j = (1-M_j^2)^{1/2}$ and $M_j = V/c_j$ is the Mach number.

In terms of the translating co-ordinates x, y_j, equation (4) becomes

$$\frac{\partial^2 W_j}{\partial x^2} + \frac{\partial^2 W_j}{\partial y_j^2} + \frac{1}{c_j^2 s_j^2}\frac{\partial}{\partial t}\left[2M_j c_j\frac{\partial W_j}{\partial x} - \frac{\partial W_j}{\partial t}\right] = 0 \tag{11}$$

In the moving system (x,y,z,t) equations (1),(5) and (6) take the form

$$e^{-i\omega t}\begin{bmatrix} W_i \\ W_r \\ W_T \end{bmatrix} = \begin{bmatrix} A\exp[-i\{k_1(x\cos\theta_1 + \frac{y_1}{s_1}\sin\theta_1) + \omega t\}] \\ B\exp[-i\{k_1(x\cos\theta_1 - \frac{y_1}{s_1}\sin\theta_1) + \omega t\}] \\ C\exp[-i\{k_2(x\cos\theta_2 + \frac{y_2}{s_2}\sin\theta_2) + \omega t\}] \end{bmatrix}, \tag{12}$$

where $\omega = \Omega\alpha$ and $\alpha = (1+M_1\cos\theta_1) = (1+M_2\cos\theta_2)$.

In view of the equation (12) we take the solution of (11) as

$$W_j(x,y_j)e^{-i\omega t} = w_j(x,y_j)\exp[i(M_j\lambda_j x - \omega t)]. \tag{13}$$

Substitution of equation (13) into equation (11) yields the

Helmholtz equation governing w_j :

$$\frac{\partial^2 w_j}{\partial x^2} + \frac{\partial^2 w_j}{\partial y_j^{\,2}} + \lambda_j^{\,2} w_j = 0 , \qquad (j = 1,2) \qquad (14)$$

where $\lambda_j = \dfrac{k_j \alpha}{s_j^{\,2}}$

Applying Fourier transform, equation (14) can be solved and the result is

$$w_1(x,y_1) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_1(\xi)\exp[-i\xi x-(\xi^2-\lambda_1^{\,2})^{1/2} y_1]d\xi , \qquad y_1>0 \qquad (15)$$

$$w_2(x,y_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} B_2(\xi)\exp[-i\xi x+(\xi^2-\lambda_2^{\,2})^{1/2} y_2]d\xi , \qquad y_2<0 \qquad (16)$$

From (13),(15) and (16) we obtain the displacement components due to scattered field as

$$W_1 = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_1(\xi)\exp[-i\xi x - \nu_1 y_1]d\xi , \qquad y_1>0 \qquad (17)$$

and

$$W_2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} A_2(\xi)\exp[-i\xi x + \nu_2 y_2]d\xi , \qquad y_2<0, \qquad (18)$$

where

$$\nu_j = [(\xi + \lambda_j M_j)^2 - \lambda_j^{\,2}]^{1/2}, \qquad j=1,2 \qquad (19)$$

$A_1(\xi)$ and $A_2(\xi)$ are the unknown quantities to be determined from the following boundary conditions:

$$\mu_1 s_1 \frac{\partial W_1}{\partial y_1} = \mu_2 s_2 \frac{\partial W_2}{\partial y_2}, \quad \text{for all } x, \quad y=0 \tag{20}$$

$$W_1 = W_2, \qquad |x|>L, \ y=0 \tag{21}$$

$$\frac{\partial W_1}{\partial y_1} + \frac{\partial W_i}{\partial y_1} + \frac{\partial W_r}{\partial y_1} = 0, \qquad |x|<L, \qquad y=0+ \tag{22}$$

From the boundary condition (22) we obtain

$$\frac{\partial W_1}{\partial y_1} = A_1 \exp[-ik_1 x \cos\theta_1], \quad |x|<L, \quad y=0, \tag{23}$$

where $$A_1 = \frac{i(A-B)k_1 \sin\theta_1}{s_1} \tag{24}$$

Using (17), the above equation can be written as

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} A_1(\xi)\nu_1 \exp[-i\xi x]d\xi = -A_1 \exp[-ik_1 x \cos\theta_1], \qquad -L<x<L$$

$$= P(x), \quad x>L \quad \text{(say)}$$

$$= Q(x), \quad x<-L \quad \text{(say)}$$

Therefore,

$$A_1(\xi)\nu_1 = \exp[i\xi L]\, G_+(\xi) + \exp[-i\xi L]\, G_-(\xi) -$$

$$- \frac{A_1}{i(\xi-\xi_0)}\left[\exp\{i(\xi-\xi_0)L\} - \exp\{-i(\xi-\xi_0)L\} \right], \tag{25}$$

where

$$G_+(\xi) = \int_L^{\infty} P(x)\ \exp[i\xi(x-L)]dx \tag{26}$$

$$G_-(\xi) = \int_{-\infty}^{-L} Q(x)\ \exp[i\xi(x+L)]dx \tag{27}$$

$$\xi_0 = k_1 \cos\theta_1. \tag{28}$$

From the boundary condition (20) we obtain

$$A_2(\xi) = -\frac{M\,\nu_1 A_1(\xi)}{\nu_2} \tag{29}$$

where $M = \dfrac{\mu_1 s_1}{\mu_2 s_2}.$ (30)

Next using the boundary condition (21), we obtain

$$A_1(\xi) - A_2(\xi) = \int_{-\infty}^{\infty} (W_1-W_2)\ \exp[i\xi x]dx$$

$$= \int_{-L}^{L} P_1(x)\ \exp[i\xi x]dx$$

$$= N(\xi) \qquad \text{(say)}, \tag{31}$$

which is the measure of the discontinuity of displacement along the surface of the crack. Now with the aid of (29) and (31), we find

$$A_1(\xi) = \frac{\nu_2 N(\xi)}{\nu_2 + M\nu_1} \tag{32}$$

Eliminating $A_1(\xi)$ from (25) and (32) we obtain an extended Wiener-Hopf equation, namely

$$\exp[\, i\xi L\,]G_+(\xi) + \exp[-i\xi L\,]G_-(\xi) - N(\xi)K(\xi)$$

$$= \frac{A_1}{i(\xi-\xi_0)}\left[\exp\{i(\xi-\xi_0)L\} - \exp\{-i(\xi-\xi_0)L\}\right], \tag{33}$$

where $$K(\xi) = \frac{\nu_1\nu_2}{\nu_2 + M\,\nu_1} = \frac{\nu_1}{1+M}R(\xi) \tag{34}$$

$$R(\xi) = \frac{(1+M)\nu_2}{\nu_2 + M\,\nu_1} \tag{35}$$

In order to solve the Wiener-Hopf equation given by (33) we assume that branch points $\xi=\lambda_1(1-M_1)$, $\lambda_2(1-M_2)$, $-\lambda_1(1+M_1)$ and $-\lambda_2(1+M_2)$ of $K(\xi)$ possess small imaginary parts, which would ultimately be made to tend to zero.

Now we write $K(\xi) = K_+(\xi)K_-(\xi)$, where $K_+(\xi)$ is analytic in the upper-half plane $\mathrm{Im}\,\xi>\mathrm{Im}\,[-\lambda_1(1+M_1)]$, whereas $K_-(\xi)$ is analytic in the lower-half plane given by $\mathrm{Im}\,\xi<\mathrm{Im}\,[\,\lambda_1(1-M_1)]$. The expressions of $K_+(\xi)$ and $K_-(\xi)$ are derived in the Appendix. Since $\frac{\partial W_1}{\partial y_1}$ decreases exponentially as $x \to \pm\infty$, $G_+(\xi)$ and $G_-(\xi)$ have the same common region of regularity as $K_+(\xi)$ and $K_-(\xi)$.

Now equation (33) can easily be expressed as two integral equations involving $G_+(\xi)$, $G_-(\xi)$ and $N(\xi)$ as follows:

$$\frac{G_+(\xi)}{K_+(\xi)} - \frac{A_1 e^{-i\xi_0 L}}{i(\xi-\xi_0)}\left[\frac{1}{K_+(\xi)} - \frac{1}{K_+(\xi_0)}\right] +$$

$$+ \frac{1}{2\pi i}\int_{C_+} \frac{e^{-2isL}}{(s-\xi)K_+(s)}\left[G_-(s) + \frac{A_1 e^{i\xi_0 L}}{i(s-\xi_0)}\right]ds$$

$$= N(\xi)K_-(\xi)e^{-i\xi L} + \frac{A_1 e^{-i\xi_0 L}}{i(\xi-\xi_0)K_+(\xi_0)} -$$

$$- \frac{1}{2\pi i}\int_{C_-} \frac{e^{-2isL}}{(s-\xi)K_+(s)}\left[G_-(s) + \frac{A_1 e^{i\xi_0 L}}{i(s-\xi_0)}\right]ds, \qquad (36)$$

where c_+ and c_- are the straight contours below the pole at $\xi = \xi_0$ and situated within the common region of regularity of $G_+(\xi)$, $G_-(\xi)$, $K_+(\xi)$ and $K_-(\xi)$ as shown in Fig.2.

The left hand side of (36) is analytic in the upper-half plane whereas the right hand side is analytic in the lower-half plane and both of them are equal in common region of analyticity of these two functions. Therefore, by analytic continuation, both sides of (36) are analytic in the whole of the s-plane. Next, by Liouville's theorem, it can be shown that both sides of (36) are equal to zero. Thus we obtain

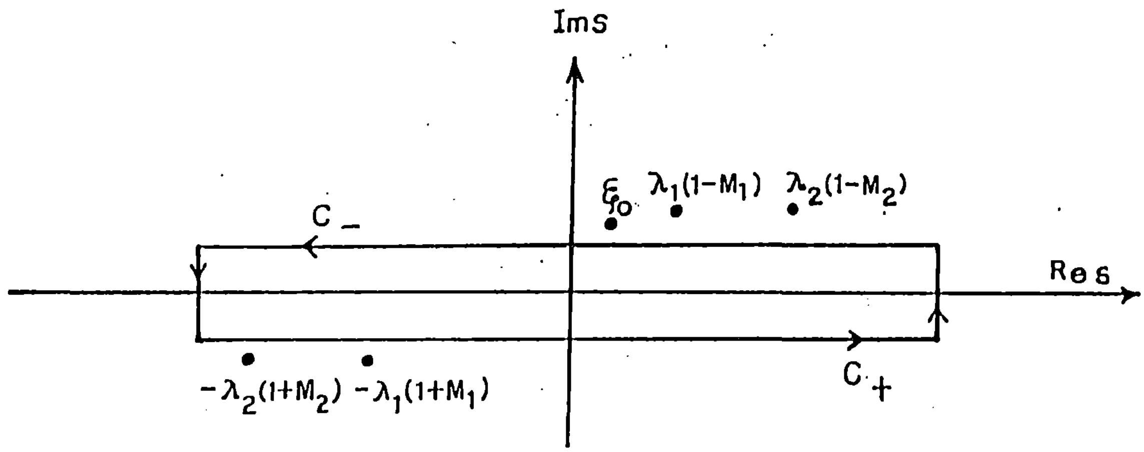

Fig. 2. Path of integration in the complex s-plane.

$$\frac{1}{K_+(\xi)}\left[G_+(\xi) - \frac{A_1 e^{-i\xi_0 L}}{i(\xi-\xi_0)}\right] + \frac{A_1 e^{-i\xi_0 L}}{i(\xi-\xi_0)K_+(\xi_0)} +$$

$$+\frac{1}{2\pi i}\int_{C_+}\frac{e^{2isL}}{(s-\xi)K_+(s)}\left[G_-(s) + \frac{A_1 e^{i\xi_0 L}}{i(s-\xi_0)}\right]ds = 0 \qquad (37)$$

similarly, we also obtain

$$\frac{1}{K_-(\xi)}\left[G_-(\xi) + \frac{A_1 e^{i\xi_0 L}}{i(\xi-\xi_0)}\right] +$$

$$+\frac{1}{2\pi i}\int_{C_-}\frac{e^{2isL}}{(s-\xi)K_-(s)}\left[G_+(s) - \frac{A_1 e^{-i\xi_0 L}}{i(s-\xi_0)}\right]ds = 0 \qquad (38)$$

3. HIGH FREQUENCY SOLUTION OF THE INTEGRAL EQUATIONS

In order to obtain $G_+(\xi)$ and $G_-(\xi)$ from the integral equations (37) and (38) in case when the normalized wave number $\lambda_1(1+M_1)L>>1$, the integration along the path c_+ in (37) is replaced by the integration along the loops $L_{-\lambda_1}$ and $L_{-\lambda_2}$ round the branch points $-\lambda_1(1+M_1)$ and $-\lambda_2(1+M_2)$ of $K_+(s)$, respectively. Also, the integration along the path c_- in (38) is replaced by the integration round the circular contour L_0, round the pole $s=\xi_0$ and by the integrations along the loops L_{λ_1} and L_{λ_2} round the branch cuts through the branch points $\lambda_1(1-M_1)$ and $\lambda_2(1-M_2)$ of the function $K_-(s)$ as shown in Fig. 3.

Finally evaluating the integrals along the straight line paths round the branch points for large values of frequency, we obtain two equations given by

$$F_\pm(\xi)+C_\pm(\xi)+\sum_{j=1}^{2}\frac{\sigma_j e^{2i\lambda_j(1\pm M_j)}\ A_\mp[\mp\lambda_j(1\pm M_j)]\ F_\mp[\mp\lambda_j(1\pm M_j)]}{2\{\lambda_j(1\pm M_j)-\xi\}\ (\lambda_j L)^{1/2}} = 0, \quad (39)$$

where $\sigma_1=1$ and $\sigma_2=M$, and

$$F_\pm(\xi) = \frac{1}{K_\pm(\xi)}\left[G_\pm(\xi) \mp \frac{A_1 e^{\mp i\xi_0 L}}{i(\xi-\xi_0)}\right]$$

$$A_\pm(\xi) = \frac{ie^{i\pi/4}}{\pi^{1/2}}[K_\pm(\xi)]^2$$

$$C_\pm(\xi) = \frac{A_1 e^{\mp i\xi_0 L}}{i(\xi-\xi_0)K_\pm(\xi_0)} \quad (40)$$

Now substituting $\xi = \lambda_1(1-M_1)$ and $\lambda_2(1-M_2)$ and $\xi = -\lambda_1(1+M_1)$ and $-\lambda_2(1+M_2)$ in (39) a system of linear equations of $F_+[\lambda_1(1-M_1)]$, $F_+[\lambda_2(1-M_2)]$, $F_-[-\lambda_1(1+M_1)]$ and $F_-[-\lambda_2(1+M_2)]$ are obtained. Now solving them and neglecting higher order terms of $(\lambda_1 L)^{-1/2}$ and $(\lambda_2 L)^{-1/2}$ we obtain, finally, after some algebraic manipulation:

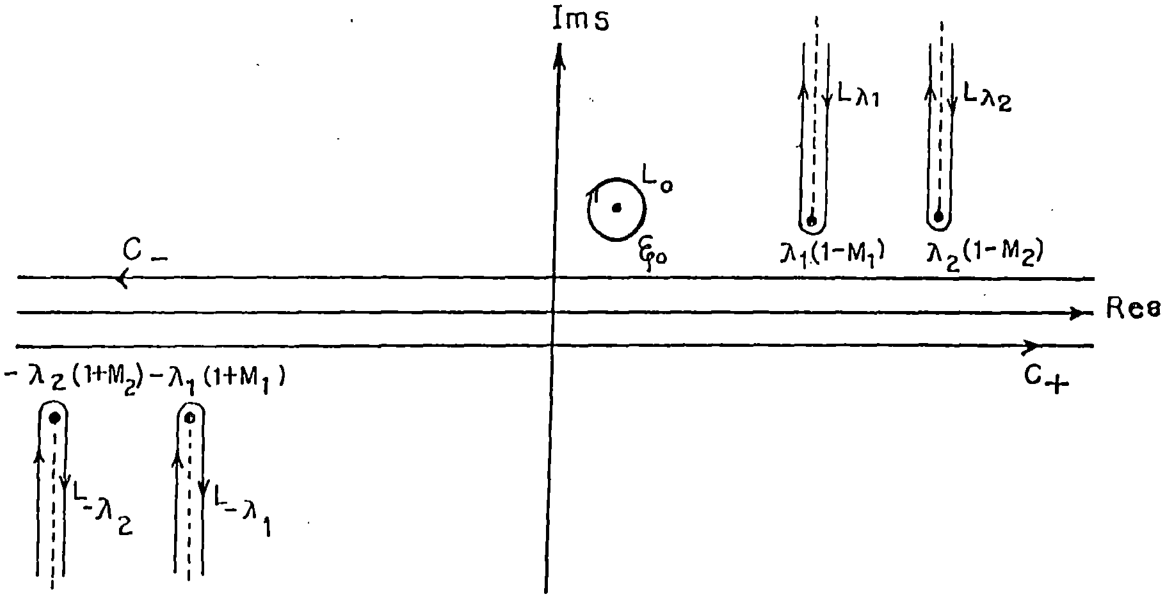

Fig. 3. Path of integration L_0, L_{λ_1}, L_{λ_2} and $L_{-\lambda_1}$, $L_{-\lambda_2}$.

$$F_{\pm}[\pm\lambda_k(1\mp M_k)] = - C_{\pm}[\pm\lambda_k(1\mp M_k)] \times$$

$$\times \left[1-\sum_{j=1}^{2} \frac{\sigma_j e^{2i\lambda_j(1\mp M_k)L} A_{\mp}[\mp\lambda_j(1\pm M_j)] C_{\mp}[\mp\lambda_j(1\pm M_j)]}{2(\lambda_j L)^{1/2}\{\lambda_j(1\pm M_j)+\lambda_k(1\mp M_k)\}C_{\pm}[\pm\lambda_k(1\mp M_k)]} \right], \quad k=1,2 \quad (41)$$

Now using (39) we get from (41)

$$G_{\pm}(\xi) = \pm \frac{A_1 e^{\mp i\xi_0 L}}{i(\xi-\xi_0)} \mp \frac{A_1 e^{\mp i\xi_0 L} K_{\pm}(\xi)}{i(\xi-\xi_0)K_{\pm}(\xi_0)} +$$

$$+ \sum_{k=1}^{2}\left[\frac{\sigma_k e^{2i\lambda_k(1\pm M_k)L} A_{\mp}[\mp\lambda_k(1\pm M_k)] C_{\mp}[\mp\lambda_k(1\pm M_k)] K_{\pm}(\xi)}{2(\lambda_k L)^{1/2}\{\lambda_k(1\pm M_k)\pm\xi\}} \times \right.$$

$$\left. \times \left\{1-\sum_{j=1}^{2} \frac{\sigma_j e^{2i\lambda_j(1\mp M_j)L} A_{\pm}[\pm\lambda_j(1\mp M_j)] C_{\pm}[\pm\lambda_j(1\mp M_j)]}{2(\lambda_j L)^{1/2}\{\lambda_j(1\mp M_j)+\lambda_k(1\pm M_k)\}C_{\mp}[\mp\lambda_k(1\pm M_k)]}\right\}\right] \quad (42)$$

4. CRACK OPENING DISPLACEMENT AT POINTS AWAY FROM THE CRACK TIPS

In order to obtain the displacement jump for the large values of $\lambda_1(L-x)$, $\lambda_2(L-x)$, $\lambda_1(L+x)$ and $\lambda_2(L+x)$, we can write $G_+(\xi)$ and $G_-(\xi)$ from (42) as

$$G_{\pm}(\xi) = \pm \frac{P_{\pm}}{\xi-\xi_0} \mp \frac{Q_{\pm}K_{\pm}(\xi)}{\xi-\xi_0} + \sum_{k=1}^{2} \frac{K_{\pm}(\xi)\, R_{\pm}^{(k)}}{\{\lambda_k(1\pm M_k)\pm\xi\}}, \quad (43)$$

where

$$P_{\pm} = \frac{A_1 e^{\mp i\xi_0 L}}{i}, \tag{44}$$

$$Q_{\pm} = \frac{A_1 e^{\mp i\xi_0 L}}{iK_{\pm}(\xi_0)} = \frac{P_{\pm}}{K_{\pm}(\xi_0)} \tag{45}$$

$$R_{\pm}^{(k)} = \frac{\sigma_k e^{2i\lambda_k(1\pm M_k)L} A_{\mp}[\mp\lambda_k(1\pm M_k)]\, C_{\mp}[\mp\lambda_k(1\pm M_k)]}{2(\lambda_k L)^{1/2}} \times$$

$$\times \left[1 - \sum_{j=1}^{2} \frac{\sigma_j e^{2i\lambda_j(1\mp M_j)L} A_{\pm}[\pm\lambda_j(1\mp M_j)]\, C_{\pm}[\pm\lambda_j(1\mp M_j)]}{2(\lambda_j L)^{1/2}\{\lambda_j(1\mp M_j)+\lambda_k(1\pm M_k)\}\, C_{\mp}[\mp\lambda_k(1\pm M_k)]}\right] \tag{46}$$

Now we obtain from (33)

$$N(\xi) = -\frac{Q_+ e^{i\xi L}}{(\xi-\xi_0)K_-(\xi)} + \frac{R_+^{(1)} e^{i\xi L}}{\{\xi+\lambda_1(1+M_1)\}K_-(\xi)} + \frac{R_+^{(2)} e^{i\xi L}}{\{\xi+\lambda_2(1+M_2)\}K_-(\xi)} +$$

$$+ \frac{Q_- e^{-i\xi L}}{(\xi-\xi_0)K_+(\xi)} - \frac{R_-^{(1)} e^{-i\xi L}}{\{\xi-\lambda_1(1-M_1)\}K_+(\xi)} - \frac{R_-^{(2)} e^{-i\xi L}}{\{\xi-\lambda_2(1-M_2)\}K_+(\xi)}\,. \tag{47}$$

From (31) we obtain the displacement jump across the surface of the crack as

$$W_1(x,0+) - W_2(x,0-) = \frac{1}{2\pi}\int_{-\infty}^{\infty} N(\xi)e^{-i\xi x}\, d\xi \tag{48}$$

Substituting the expression of $N(\xi)$ from (47) in (48) and approximately evaluating the integrals arising in (48) term by term for large values of $\lambda_1(L-x)$, $\lambda_2(L-x)$, $\lambda_1(L+x)$, and $\lambda_2(L+x)$, and neglecting terms of order higher than $(\lambda_1 L)^{-3/2}$ and $(\lambda_2 L)^{-3/2}$, we finally obtain the crack opening displacement across the cracked surface at points away from the crack tips in the following form:

$$\Delta W = W_1(x,0+) - W_2(x,0-)$$

$$= - iQ_+ K_+(\xi_0) e^{i\xi_0(L-x)} \times$$

$$\times \left[\frac{1}{\{(\xi_0+\lambda_1 M_1)^2 - \lambda_1^2\}^{1/2}} + \frac{M}{\{(\xi_0+\lambda_2 M_2)^2 - \lambda_2^2\}^{1/2}} \right] -$$

$$- \frac{e^{-i\pi/4}}{\pi^{1/2}} \left[T_+ - T_- \right], \qquad (49)$$

where

$$T_\pm = \sum_{k=1}^{2} \frac{\sigma_k e^{i\lambda_k(1\mp M_k)(L\mp x)}}{\{\lambda_k(L\mp x)\}^{1/2}} \left[\frac{Q_\pm K_\pm[\pm\lambda_k(1\mp M_k)]}{2^{1/2}[\lambda_k(1\mp M_k)\mp\xi_0]} - \right.$$

$$- \sum_{j=1}^{2} \frac{\sigma_j \; A_\mp[\mp\lambda_j(1\pm M_j)] \; K_\pm[\pm\lambda_k(1\mp M_k)]}{2(2\lambda_j L)^{1/2} \{\lambda_k(1\mp M_k)+\lambda_j(1\pm M_j)\}} \left(\frac{Q_\mp e^{2i\lambda_j(1\pm M_j)L}}{\{\lambda_j(1\pm M_j)\pm\xi_0\}} - \right.$$

$$\left.\left. - \sum_{r=1}^{2} \frac{\sigma_r A_\pm[\pm\lambda_r(1\mp M_r)] \; Q_\pm \; e^{2i[\lambda_r(1\mp M_r)+\lambda_j(1\pm M_j)]L}}{2(\lambda_r L)^{1/2}\{\lambda_r(1\mp M_r)+\lambda_j(1\pm M_j)\}\{\lambda_r(1\mp M_r)\mp\xi_0\}} \right) \right]. \qquad (50)$$

5. STRESS INTENSITY FACTOR AND CRACK OPENING DISPLACEMENT NEAR THE CRACK TIPS

Now considering the behaviour of ξ at infinity we obtain from (42)

$$G_{\pm}(\xi) \simeq \pm \frac{A_1 e^{\mp i\xi_0 L}}{i(\xi-\xi_0)} + S_{\pm}\xi^{-1/2} \quad \text{as } \xi \longrightarrow \infty, \tag{51}$$

where
$$S_{\pm} = \frac{1}{(1+M)^{1/2}} \left[\mp \frac{A_1 e^{\mp i\xi_0 L}}{iK_{\pm}(\xi_0)} \pm \right.$$

$$\pm \sum_{k=1}^{2} \frac{\sigma_k e^{2i\lambda_k(1\pm M_k)L} A_{\mp}[\mp\lambda_k(1\pm M_k)]\, C_{\mp}[\mp\lambda_k(1\pm M_k)]}{2(\lambda_k L)^{1/2}} \times$$

$$\left. \times \left\{ 1 - \sum_{j=1}^{2} \frac{\sigma_j e^{2i\lambda_j(1\mp M_j)L} A_{\pm}[\pm\lambda_j(1\mp M_j)]\, C_{\pm}[\pm\lambda_j(1\mp M_j)]}{2(\lambda_j L)^{1/2}\{\lambda_j(1\mp M_j)+\lambda_k(1\pm M_k)\} C_{\mp}[\mp\lambda_k(1\pm M_k)]} \right\}\right]. \tag{52}$$

Now, from equation (33), using (51) and also the fact that

$$K(\xi) \longrightarrow \pm \frac{\xi}{1+M} \quad \text{as } \xi \longrightarrow \pm\infty, \tag{53}$$

we obtain

$$N(\xi) = \frac{1+M}{\pm\xi(\xi)^{1/2}} \left[S_{+} e^{i\xi L} + S_{-} e^{-i\xi L} \right] \quad \text{as } \xi \longrightarrow \pm\infty. \tag{54}$$

Taking inverse Fourier transform of (31) and using the results of Fresnel integrals, viz.

$$\int_0^{\infty} \frac{\begin{matrix}\sin\\ \cos\end{matrix}(x+L)\alpha}{(\alpha)^{1/2}}\, d\alpha = \left[\frac{\pi}{2(x+L)} \right]^{1/2}, \tag{55}$$

we obtain the displacement jump across the surface of the crack as

$$\Delta W = W_1(x,0+) - W_2(x,0-)$$

$$= -(1+M)(1+i)S_- \left[\frac{2(x+L)}{\pi} \right]^{1/2} \quad \text{for } x \longrightarrow -L+0 \tag{56}$$

$$= -(1+M)(1-i)S_+ \left[\frac{2(L-x)}{\pi} \right]^{1/2} \quad \text{for } x \longrightarrow L-0. \tag{57}$$

Expressions (56) and (57) give the displacement jump near to the crack tips, whereas the displacement jump away from the crack tips is given by (49).

Next, in order to find the value of τ_{yz} near to the crack tip we use (54) in (32) and (29) and obtain

$$A_j(\xi) = \frac{(-1)^{j+1}\sigma_j}{\xi(\xi)^{1/2}} \left[S_+ e^{i\xi L} + S_- e^{-i\xi L} \right], \quad j=1,2 \quad \text{as } \xi \longrightarrow \infty \tag{58}$$

$$A_j(\xi) = \frac{i(-1)^{j+1}\sigma_j}{\xi(-\xi)^{1/2}} \left[S_+ e^{i\xi L} - S_- e^{-i\xi L} \right], \quad j=1,2 \quad \text{as } \xi \longrightarrow -\infty \tag{59}$$

Now $$\tau_{yz}(x,y_j) = \mu_j \frac{\partial w_j(x,y_j)}{\partial y} = \mu_j s_j \frac{\partial w_j(x,y_j)}{\partial y_j}$$

$$= \frac{\mu_j s_j}{2\pi} \frac{\partial}{\partial y_j} \left[\int_{-\infty}^{\infty} A_j(\xi)\, e^{-i\xi x - \nu_j |y_j|}\, d\xi \right]. \quad (60)$$

Now substituting the values of $A_j(\xi)$ as $|\xi| \longrightarrow \infty$ in (60) and integrating, we obtain the stress near to the crack tip as

$$\tau_{yz}(x,y_1) = -\frac{\mu_1 s_1}{(2\pi)^{1/2}} \left[(1-i)S_+ \frac{\cos(\psi_1/2)}{r_1^{1/2}} + (1+i)S_- \frac{\sin(\psi_2/2)}{r_2^{1/2}} \right] \quad (61)$$

and

$$\tau_{yz}(x,y_2) = -\frac{\mu_1 s_1}{(2\pi)^{1/2}} \left[(1-i)S_+ \frac{\cos(\phi_1/2)}{d_1^{1/2}} + (1+i)S_- \frac{\cos(\phi_2/2)}{d_2^{1/2}} \right], \quad (62)$$

where $$r_1 = \{(x-L)^2+y_1^2\}^{1/2}, \quad \psi_1 = \sin^{-1} \frac{|y_1|}{r_1}$$

$$r_2 = \{(x+L)^2+y_1^2\}^{1/2}, \quad \psi_2 = \sin^{-1} \frac{|y_1|}{r_2} \quad (63)$$

$$d_1 = \{(x-L)^2+y_2^2\}^{1/2}, \quad \phi_1 = \sin^{-1} \frac{|y_2|}{d_1}$$

$$d_2 = \{(x+L)^2+y_2^2\}^{1/2}, \quad \phi_2 = \sin^{-1} \frac{|y_2|}{d_2}.$$

Therefore at the interface (y=0) near to the right-hand crack vertex, we obtain

$$\tau_{yz} \longrightarrow - \frac{\mu_1 s_1 (1-i) S_+}{\{ 2\pi(x-L)\}^{1/2}} \qquad \text{as } x \longrightarrow L+0. \tag{64}$$

Now the normalized dynamic stress intensity factor K at the crack tip x = L is defined by

$$K = \left| \frac{[2\pi k_1 (x-L)]^{1/2} \tau_{yz}}{\mu_1 A_1} \right| = s_1 \left| \frac{(1-i) S_+ (k_1)^{1/2}}{A_1} \right| \quad \text{for } x \longrightarrow L+0, \tag{65}$$

where A_1 is given by (24).

The absolute values of the complex stress intensity factor defined by (65) has been plotted against k_1L in Fig.4 for values $k_1L>1$ for different values of the Mach number M_2 and the angle of incidence for the following sets of materials:

first set:

Steel	$\rho_1 = 7.6$ gm/cm^3,	$\mu_1 = 8.32 \times 10^{11}$ dyne/cm^2
Aluminium	$\rho_2 = 2.7$ gm/cm^3,	$\mu_2 = 2.63 \times 10^{11}$ dyne/cm^2

second set:

Wrought iron	$\rho_1 = 7.8$ gm/cm^3,	$\mu_1 = 7.7 \times 10^{11}$ dyne/cm^2
Copper	$\rho_2 = 8.96$ gm/cm^3,	$\mu_2 = 4.5 \times 10^{11}$ dyne/cm^2

As the Mach number $M_2 \longrightarrow 0$ the stress intensity factor K tends to the value of the stress intensity factor corresponding to the stationary crack. The problem for $\theta_1 = \pi/2$ and $M_2=0.0$ was solved earlier by Pal and Ghosh [1990]. The graph of stress intensity factor vs K_1L corresponding to $\theta_1=\pi/2$ and $M_2=0.0$ as given in Fig.4a is found to coincide exactly with that given by Pal and Ghosh [1990]. It is interesting to note that for both pairs of materials, as M_2 increases, the peaks of the curves of stress intensity factors decrease in magnitude and occur at lower values of K_1L. Further, it may be noted that for any fixed value of M_2 the stress intensity factor decreases with the decrease in the value of the angle of incidence.

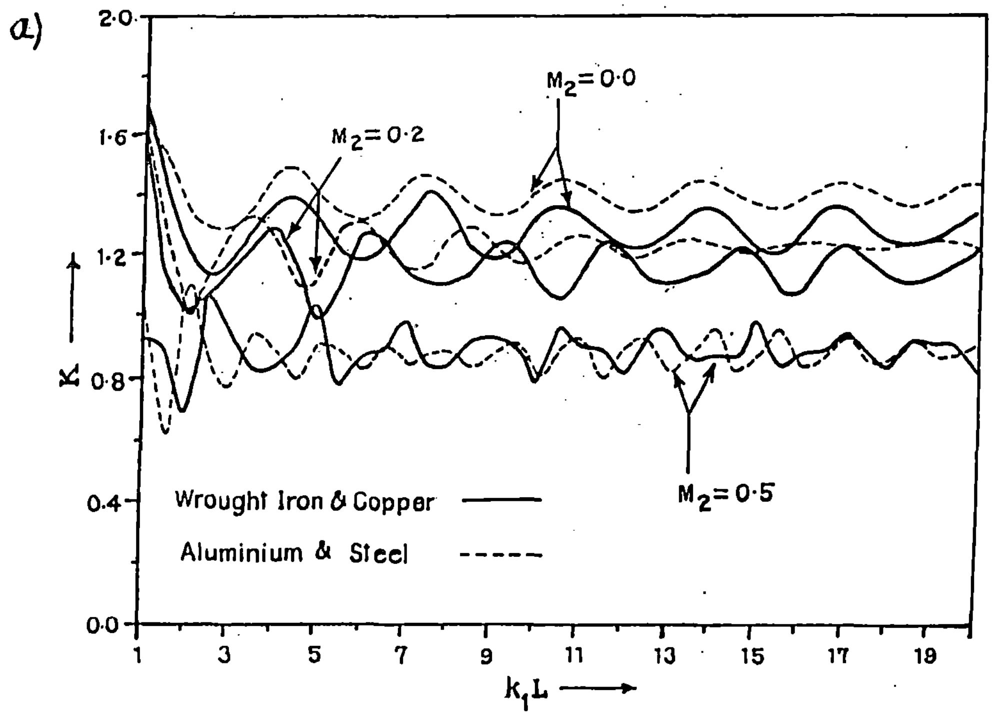

Fig.4(a). Stress intensity factor K versus dimensionless k_1L for $\theta_1 = \pi/2$.

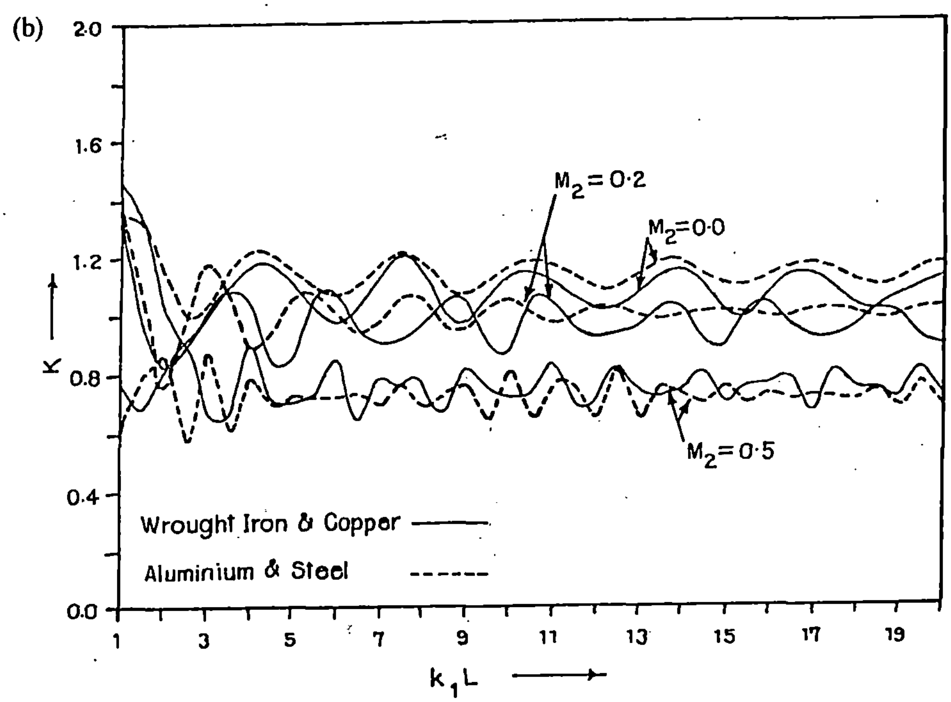

Fig.4(b). Stress intensity factor K versus dimensionless k_1L for $\theta_1 = \pi/3$.

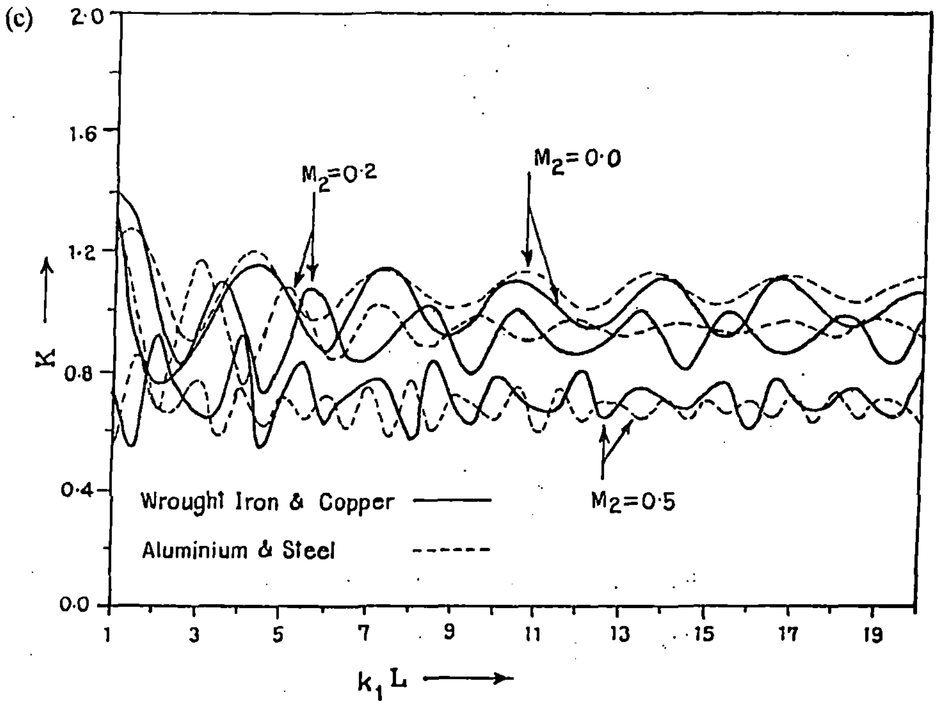

Fig.4(c). Stress intensity factor K versus dimensionless k_1L for $\theta_1 = \pi/4$.

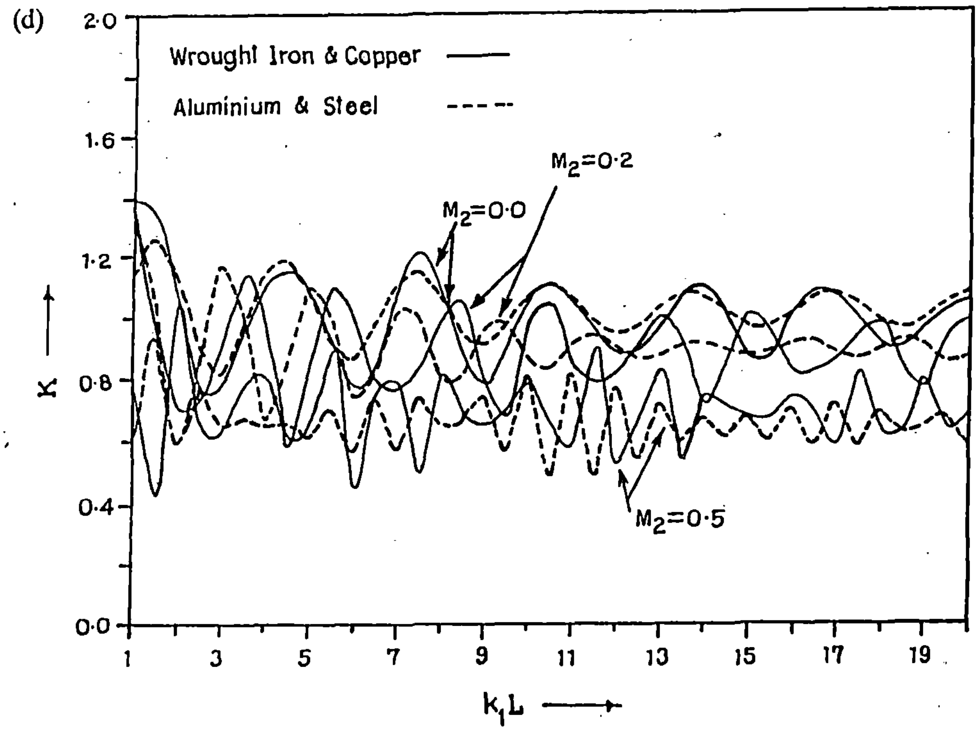

Fig.4(d). Stress intensity factor K versus dimensionless k_1L for $\theta_1 = \pi/6$.

APPENDIX

$$K(\xi) = \frac{\{(\xi+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{1+M} R(\xi) \tag{A1}$$

where $$M = \frac{\mu_1 s_1}{\mu_2 s_2}$$

and $$R(\xi) = \frac{(1+M)\{(\xi+\lambda_2 M_2)^2-\lambda_2^2\}^{1/2}}{M\{(\xi+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}+\{(\xi+\lambda_2 M_2)^2-\lambda_2^2\}^{1/2}} \longrightarrow 1$$

as $|\xi| \longrightarrow \infty$

Now $$R_+(\xi)R_-(\xi) = \frac{1}{\dfrac{1}{1+M} + \dfrac{M\{(\xi+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{(1+M)\{(\xi+\lambda_2 M_2)^2-\lambda_2^2\}^{1/2}}}$$

Taking log on both sides

$$\log R(\xi) = \log R_+(\xi) + \log R_-(\xi) = \frac{1}{2\pi i} \int_{c_L+c_U} \frac{\log R(\eta)}{\eta-\xi} d\eta$$

where the paths of integration c_L and c_U are shown in Fig.A1.

Therefore $$\log R_+(\xi) = \frac{1}{2\pi i} \int_{c_L} \frac{\log R(\eta)}{\eta-\xi} d\eta$$

$$\log R_-(\xi) = \frac{1}{2\pi i}\int_{c_U} \frac{\log R(\eta)}{\eta-\xi}\, d\eta$$

or $$\log R_+(\xi) = \frac{1}{2\pi i}\int_{-ic-\infty}^{-ic+\infty} \frac{\log R(\eta)}{\eta-\xi}\, d\eta$$

Putting $\eta = -\eta$

$$\log R_+(\xi) = \frac{1}{2\pi i}\int_{ic+\infty}^{ic-\infty} \frac{\log R(-\eta)}{\eta+\xi}\, d\eta$$

$$\log R_-(\xi) = \frac{1}{2\pi i}\int_{ic+\infty}^{ic-\infty} \frac{\log R(\eta)}{\eta-\xi}\, d\eta,$$

therefore

$$\log R_-(\xi) = \frac{1}{2\pi i}\int_{c_1} \frac{1}{(\eta-\xi)} \log\left[\frac{1}{\dfrac{1}{1+M} + \dfrac{M\{(\eta+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{(1+M)\{(\eta+\lambda_2 M_2)^2-\lambda_2^2\}^{1/2}}}\right] d\eta$$

where c_1 is the contour round the branch points $\lambda_1(1-M_1)$ and $\lambda_2(1-M_2)$ as shown in Fig. A2.

Therefore

$\log R_-(\xi) =$

$$= \frac{1}{2\pi i} \int_{\lambda_1(1-M_1)}^{\lambda_2(1-M_2)} \frac{1}{(\eta-\xi)} \left[\log\left(\frac{1}{1+M} + \frac{iM\{(\eta+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{(1+M)\{\lambda_2^2-(\eta+\lambda_2 M_2)^2\}^{1/2}} \right) - \right.$$

$$\left. - \log\left(\frac{1}{1+M} - \frac{iM\{(\eta+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{(1+M)\{\lambda_2^2-(\eta+\lambda_2 M_2)^2\}^{1/2}} \right) \right] d\eta$$

$$= \frac{1}{\pi} \int_{\lambda_1(1-M_1)}^{\lambda_2(1-M_2)} \frac{1}{(\eta-\xi)} \tan^{-1} \left[\frac{M\{(\eta+\lambda_1 M_1)^2-\lambda_1^2\}^{1/2}}{\{\lambda_2^2-(\eta+\lambda_2 M_2)^2\}^{1/2}} \right] d\eta,$$

and therefore

$$R_-(\xi) = \exp\left[\frac{1}{\pi} \int_{\lambda_1(1-M_1)}^{\lambda_2(1-M_2)} \frac{1}{(\eta-\xi)} \tan^{-1} \left(\frac{M\,[(\eta+\lambda_1 M_1)^2-\lambda_1^2\,]^{1/2}}{[\,\lambda_2^2-(\eta+\lambda_2 M_2)^2\,]^{1/2}} \right) d\eta \right].$$

Similarly

$$R_+(\xi) = \exp\left[\frac{1}{\pi} \int_{\lambda_1(1+M_1)}^{\lambda_2(1+M_2)} \frac{1}{(\eta+\xi)} \tan^{-1} \left(\frac{M\,[(\eta-\lambda_1 M_1)^2-\lambda_1^2\,]^{1/2}}{[\,\lambda_2^2-(\eta-\lambda_2 M_2)^2\,]^{1/2}} \right) d\eta \right].$$

Therefore from (A1) we can write

$$K_{+}(\xi) = \left[\frac{\xi+\lambda_1(1+M_1)}{(1+M)}\right]^{1/2} \times$$

$$\times \exp\left[\frac{1}{\pi}\int_{\lambda_1(1+M_1)}^{\lambda_2(1+M_2)} \frac{1}{(\eta+\xi)}\tan^{-1}\left(\frac{M\,[(\eta-\lambda_1 M_1)^2-\lambda_1^2\,]^{1/2}}{[\,\lambda_2^2-(\eta-\lambda_2 M_2)^2\,]^{1/2}}\right)d\eta\right] \quad \text{(A2)}$$

and

$$K_{-}(\xi) = \left[\frac{\xi-\lambda_1(1-M_1)}{(1+M)}\right]^{1/2} \times$$

$$\times \exp\left[\frac{1}{\pi}\int_{\lambda_1(1-M_1)}^{\lambda_2(1-M_2)} \frac{1}{(\eta-\xi)}\tan^{-1}\left(\frac{M\,[(\eta+\lambda_1 M_1)^2-\lambda_1^2\,]^{1/2}}{[\,\lambda_2^2-(\eta+\lambda_2 M_2)^2\,]^{1/2}}\right)d\eta\right]. \quad \text{(A3)}$$

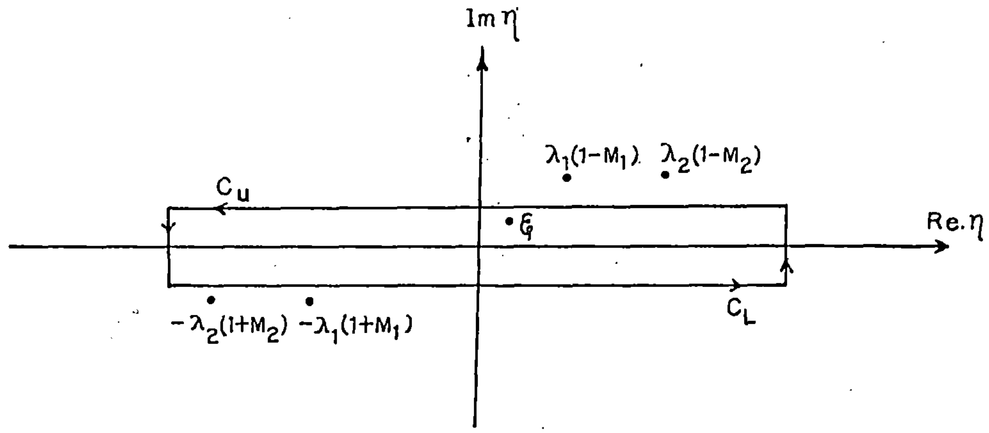

Fig. A1. Complex η-plane.

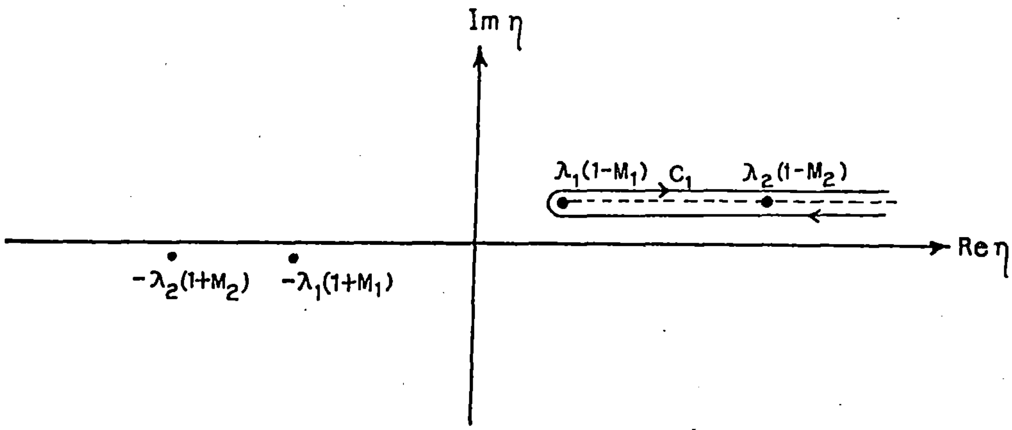

Fig. A2. Path of integration round the branch points.