CHAPTER VII

BREAKDOWN OF ARGON UNDER RADIOFREQUENCY EXCITATION IN TRANSVERSE MAGNETIC FIELD

7.1. Introduction:-

In a high frequency gas breakdown the electrons gain energy from the field by suffering collisions with the gas atoms and having their ordered oscillatory motion changed to random kinetic motion. Thus when this random kinetic energy exceeds the ionization potential, electron multiplication occurs. The breakdown of gases in high frequency electrical fields has been studied by many workers. In presence of the magnetic field, either longitudinal or transverse, the breakdown of a gas excited by a radio-frequency voltage has been studied by Townsend and Gill (1938) for two frequencies, 48 MHz and 30 MHz and within a range of pressure varying from a few m torr to 0.24 torr. In 1951 Llewellyn et al (1951) investigated the high frequency breakdown between wire and coaxial cylinders in air over the range of frequencies from 3.5 Mc/S to 70 Mc/S at pressure below 20 mm Hg. To interpret the results they assumed that the electronic mean free path is small compared with the linear dimension of the discharge tube and the collision frequency is very much greater than the frequency of the applied field. Bayet (1951) studied the breakdown mechanism in alternating fields, including u.h.f. fields and the role of deionization mechanisms (recombination, diffusion and the effect of electron attachment) was discussed. Haefer (1953) gave a detailed experimental and theoretical account of breakdown voltage measurements in coaxial cylindrical tubes in presence of an axial magnetic field. Using cylindrical electrodes Ferritti and Veronesi (1955) showed that the magnetic field has a strong influence on the breakdown voltage of the discharge. Measurements were made in air at 0.1, 0.5, 1.0 mm Hg. pressure and at 10, 15, 20, 25, 30 Mc/S and the magnetic field varying from 0 to 650 gauss. Hale (1947) computed the magnitude of breakdown voltage in rare gases when excited by a radio frequency voltage (frequency a few Mc/S to 400 Mc/S) for some fixed values of pressure such as 100μ , 150μ and 200μ. The main assumptions in this theory are that (a) the amplitude of oscillation of the electron must be equal to the mean free path of the electron at that pressure (b) energy gained by the electron between two successive collisions must be equal to or greater than the ionization energy of the gas. The values of breakdown voltage thus calculated are in fair agreement with his experimental results. It is the object of the present investigation to calculate the breakdown voltage of argon, following the procedure adopted by Hale (1947) where a steady transverse magnetic field is applied and calculate its variation over a range of radiofrequency for some chosen values of pressure. It will be of interest to see how the frequency at which the breakdown field becomes a minimum shifts with the increase of the magnetic field. The results are expected to provide information regarding the nature of interaction of a magnetic field with the motion of electrons in a radio frequency field.

7.2. Theoretical treatment:-

It is well known that the breakdown potential for high frequency fields is measured by those electrons in the gas which are able in acquiring the ionising energy in one mean free path. It is known as well that in presence of external transverse magnetic field electronic mean free path also decreases [Blevin and Haydon (1958), Sen and Ghosh (1963)]. Thus, the breakdown potential should be a function of the gas, gas pressure, electrode separation, frequency of the applied field and the magnetic field.

Under this consideration the solution of the equation of motion of the electron which moves in the high frequency field and magnetic field in the gas can be achieved provided before breakdown takes place the electric field is not distored by space charge. The applied radio frequency field is along the X-axis and the magnetic field is assumed along Z axis. The motion of electron under this assumption is given by

$$m \frac{dV_x}{dt} + HeV_y = eE \sin 2\pi ft \quad \cdots (7.1)$$

$$m \frac{dV_y}{dt} - HeV_x = 0 \quad \cdots (7.2)$$

According to Hale (1947) the electron is assumed to be at rest when the instanteneous value of the applied field is zero and all collisions which do not result in the formation of an ion pair are neglected because such collisions do not lead the breakdown of the gas concerned. The conditions are so imposed that the electrons which are at rest when the instantaneous field is zero will acquire ionising potential at the end of one electronic mean free path. However, this simplification does not eventually alter the results because the electrons which acquire the ionizing energy in gas in one mean free path cause the initial break-down.

In presence of magnetic field the equation of motion of electron after differentiating equation(7.1)

$$m \frac{d^2 v_x}{dt^2} + He \frac{dv_y}{dt} = eE\, 2\pi f \cos 2\pi ft$$

or

$$\frac{d^2 v_x}{dt^2} + \frac{He}{m} \frac{dv_y}{dt} = \left(\frac{eE}{m}\right) 2\pi f \cos 2\pi ft$$

or

$$\frac{d^2 v_x}{dt^2} + \omega_B \frac{dv_y}{dt} = \left(\frac{eE}{m}\right) 2\pi f \cos 2\pi ft \qquad \ldots(7.3)$$

From eqn. (7.2)

$$m \frac{dv_y}{dt} = \left(\frac{He}{m}\right) v_x$$
$$= \omega_B v_x$$

substituting this into eqn.(7.3) we have

$$\frac{d^2 v_x}{dt^2} + \omega_B^2 v_x = (e/m) E \omega \cos \omega t$$

Hence

$$v_x = A \cos \omega_B t + B \sin \omega_B t + \frac{1}{\omega_B^2 - \omega^2} (e/m) E \omega \cos \omega t$$

Boundary condition

$$V_x = 0 \quad \text{at} \quad t = 0$$

$$\therefore \; A = -\frac{1}{(\omega_B^2 - \omega^2)} (e/m)\,\omega E$$

and from $\frac{dV_x}{dt} = 0$ at $t = 0$, $B = 0$.

$$\therefore \; V_x = \frac{1}{\omega^2 - \omega_B^2}(e/m)\omega E \cos\omega_B t + \frac{1}{\omega_B^2 - \omega^2}(e/m) E\omega \cos\omega t$$

...(7.4)

when $H = 0$, $\omega_B = 0$

then

$$V_x = \frac{1}{\omega^2}(e/m)\omega_E - \frac{1}{\omega^2}(e/m)E\omega\cos\omega t$$

$$\therefore \; V_x = (e/m)\frac{E}{\omega}(1 - \cos\omega t)$$

or $$V_x = (e/m)\frac{E}{2\pi f}(1 - \cos\omega t)$$

or, $$f = (e/m)\frac{E}{2\pi V_x}(1 - \cos 2\pi f t) \quad \ldots\ldots (7.4a)$$

From eqn. (7.4)

$$x = \frac{1}{\omega^2 - \omega_B^2}(e/m)\frac{\omega}{\omega_B}E \sin\omega_B t + \frac{1}{\omega_B^2 - \omega^2}(e/m)E\frac{\omega \sin\omega t}{\omega} + C$$

...(7.5)

At $x = 0$, $t = 0$ then $C = 0$.

$$\therefore \; x = \lambda_H = \frac{1}{\omega^2-\omega_B^2}(e/m)\frac{\omega}{\omega_B}E\sin\omega_B t + \frac{1}{\omega_B^2-\omega^2}(e/m)E\sin\omega t \quad \ldots(7.6\text{ a})$$

$$E = \frac{\lambda_H}{\dfrac{(e/m)}{(\omega^2-\omega_B^2)}\left[\dfrac{\omega}{\omega_B}\sin\omega_B t - \sin\omega t\right]} \quad \cdots(7.6\text{ b})$$

Now

$$x = \frac{(\omega/\omega_B)(e/m)E\sin\omega_B t}{\omega^2(1-\omega_B^2/\omega^2)} + \frac{1}{(\omega_B^2-\omega^2)}(e/m)E\sin\omega t$$

$$= (e/m)\frac{1}{\omega^2}(\omega/\omega_B)\left(1-\frac{\omega_B^2}{\omega^2}\right)^{-1}E\left(\omega_B t - \frac{\omega_B^3 t^3}{3!} + \cdots\cdots\right) + \frac{1}{(\omega_B^2-\omega^2)}(e/m)E\sin\omega t$$

$$= (e/m)\frac{1}{\omega^2}(\omega/\omega_B)E\left[\omega_B t + \text{in terms of } \omega_B + \cdots\cdots\right] + \frac{1}{(\omega_B^2-\omega^2)}(e/m)E\sin\omega t$$

$$= (e/m)\frac{1}{\omega^2}(\omega/\omega_B)E\omega_B t + \frac{1}{\omega_B^2-\omega^2}(e/m)E\sin\omega t$$

$$\therefore\ x = (e/m)\frac{1}{\omega}Et + \frac{1}{\omega_B^2 - \omega^2}(e/m)E\sin\omega t$$

when $H = 0$, $\omega_B = 0$

then

$$x = (e/m)\frac{1}{\omega}Et - \frac{1}{\omega^2}(e/m)E\sin\omega t$$

$$x = \lambda = (e/m)\frac{E}{4\pi^2 f^2}(2\pi ft - \sin 2\pi ft)$$

Substituting f from eqn. (7.4a) we get,

$$E = \frac{v_x^2}{\lambda(e/m)}\ \frac{2\pi ft - \sin 2\pi ft}{(1 - \cos 2\pi ft)^2} \qquad \ldots(7.8)$$

eqn. (7.5) and (7.7) are identical with those provided by Hale (1947) in no magnetic field.

In light of eqn.(7.4) and (7.6) it is assumed that the gas under interest will undergo breakdown when the values of E and f are such that λ will be one electronic mean free path and the electron will acquire ionizing energy at the end of its mean free path.

Since $(1/2)mv^2 = eV_i$ where V_i is the ionization potential of a gas under study, ν is calculated.

With the help of eqn. (7.8) for an assigned value of λ which is a function of pressure, E the peak voltage has been computed taking ft as $\frac{1}{16}$, $\frac{2}{16}$, $\frac{3}{16}$ and so on for pressure 100μ, 150μ and 200μ and entered in table 7.1, and 7.2 and 7.3 respectively. Hence the value of f is calculated from eqn.(7.5) The value of ft for which E is minimum (in absence of magnetic field) comes out to be around 5/16 (Hale, 1947).

In presence of magnetic field, λ decreases with magnetic field. From the concept of equivalent pressure as provided by Blevin and Haydon (1958), Sen and Ghosh (1963) λ is given by

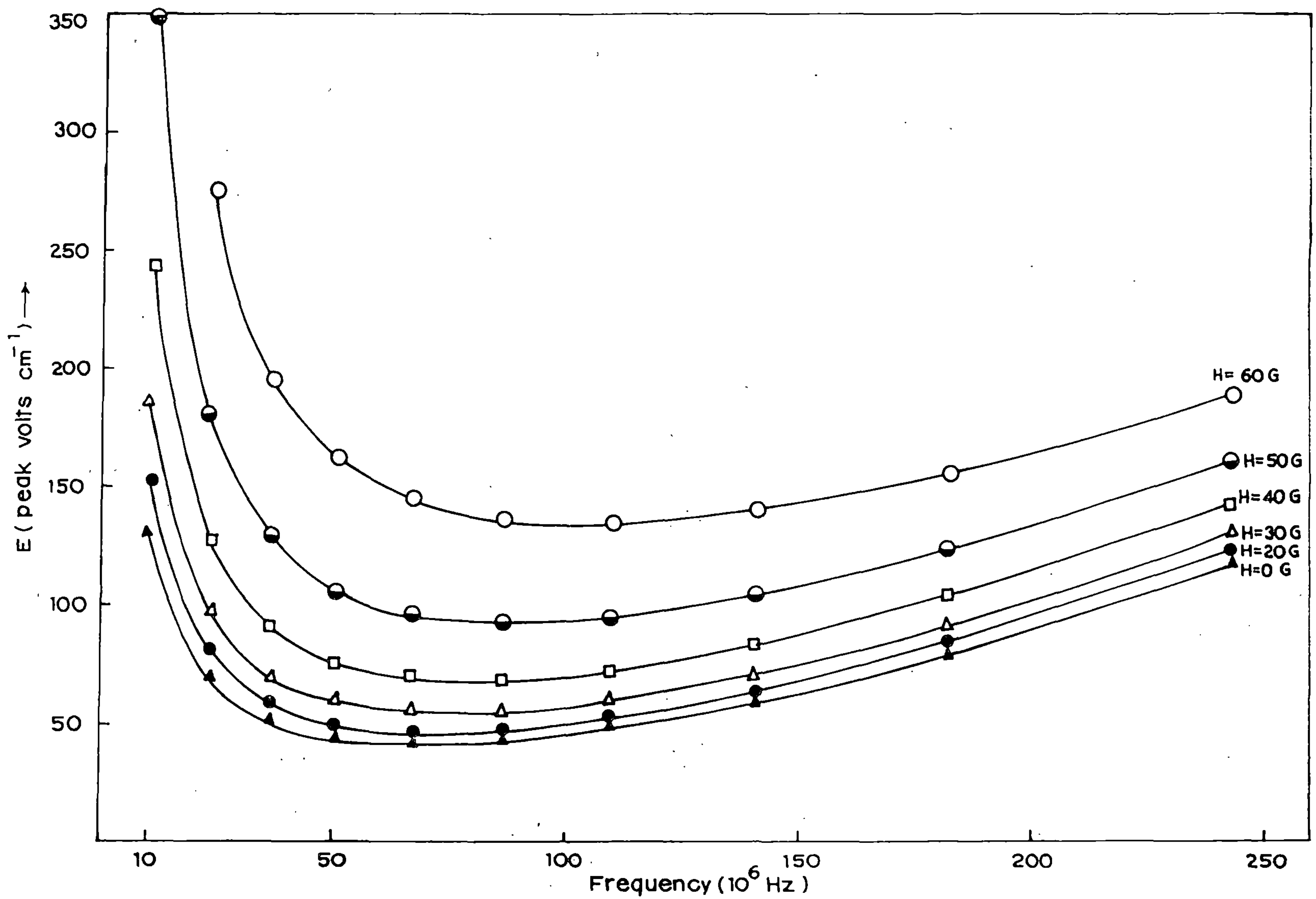
$$\lambda_H = \frac{\lambda}{(1 + C_1 H^2/P^2)^{1/2}} \qquad \ldots(7.9)$$

where $C_1 = (e/m \cdot L/v_r)$ and v_r is the random velocity and L is the mean free path of electron at 1 torr. From literature value for respective parameters in eqn. (7.9) values for λ_H have been computed and are entered in Table 7.4. Similarly E has been estimated from eqn.(7.6) for three pressures

100μ , 150μ , 200μ and entered into the tables 7.1 7.2 and 7.3 and the results are plotted in figures (7.1), and (7.2), and (7.3) respectively. It is seen that minimum breakdown field increases with increasing magnetic field and the minimum shifts to higher frequencies when magnetic field is increased.

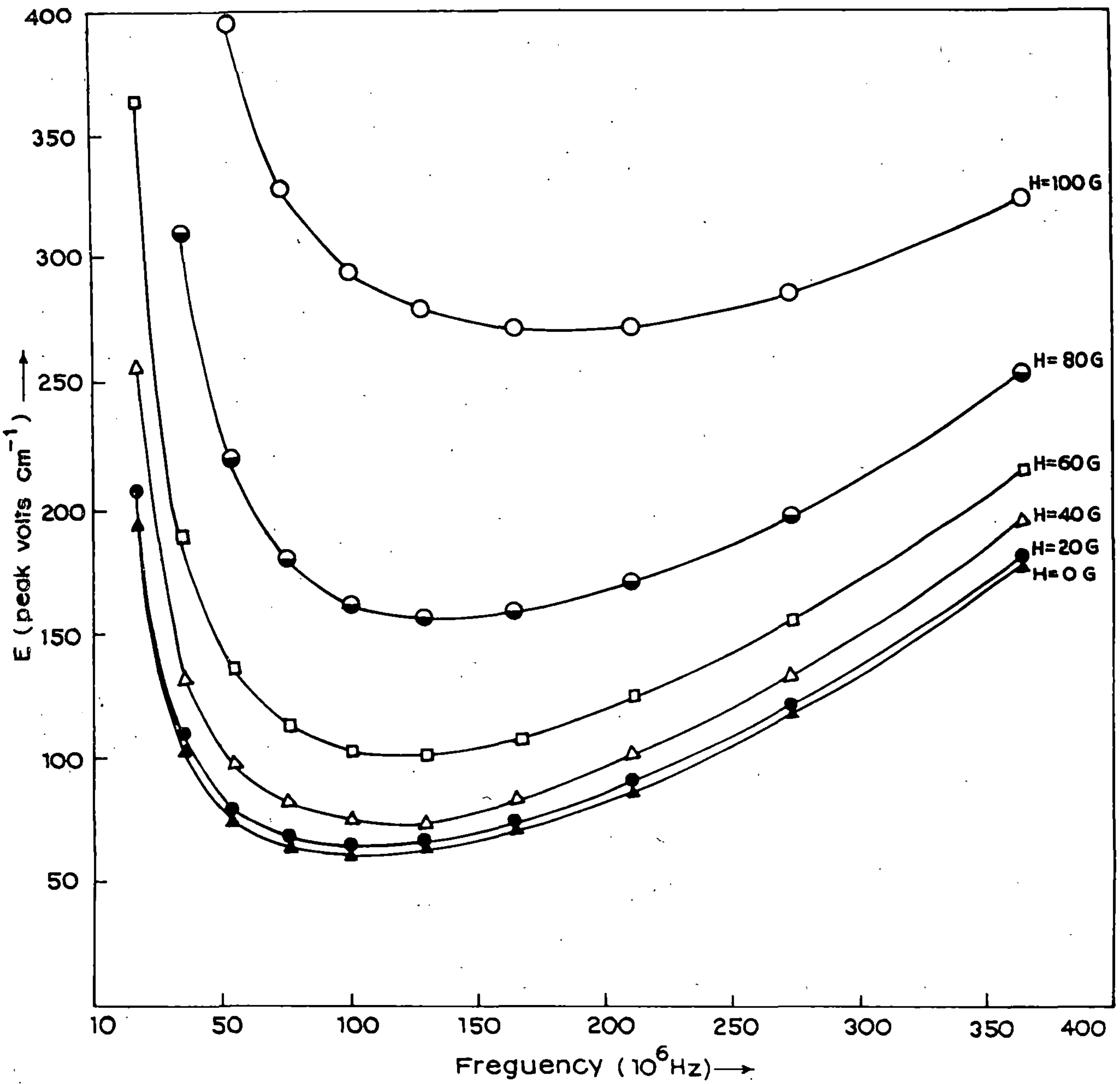
7.3. Discussion:-

On the basis of this model it is also possible to calculate the minimum breakdown field (V/cm) in presence of magnetic field. The theoretical investigation has been carried out for three pressures namely 100μ 150μ and 200μ and magnetic field upto 60 gauss in 100μ ; 80 gauss in 150μ and 120 gauss in 200μ Fig. 7.1, 7.2 and 7.3 show the general characteristics of E vs. frequency of the applied electric field where magnetic field is a parameter. It is observed that all the characteristics show that the minimum breakdown potential increases with magnetic field and also with frequency. From another approach the increase in break down voltage with magnetic field in high frequency field has been observed by Deb and Goswami (1964). They used electrodeless high frequency discharge at low pressure in transverse magnetic field. It is shown that with increase in α , the ratio of the cyclotron frequency to the frequency of the applied field, the breakdown



350
300
250
200
150
100
50
E (peak volts cm^{-1}) ⟶
H= 60 G
H=50 G
H=40 G
H=30 G
H=20 G
H=0 G
10
50
100
150
200
250
Frequency (10^6 Hz)

Fig. 7·1.



400
350
300
250
200
150
100
50
E (peak volts cm^{-1}) ⟶
H=100 G
H= 80G
H=60G
H=40G
H=20G
H=0 G
10
50
100
150
200
250
300
350
400
Frequency (10^6Hz) ⟶

Fig. 7·2

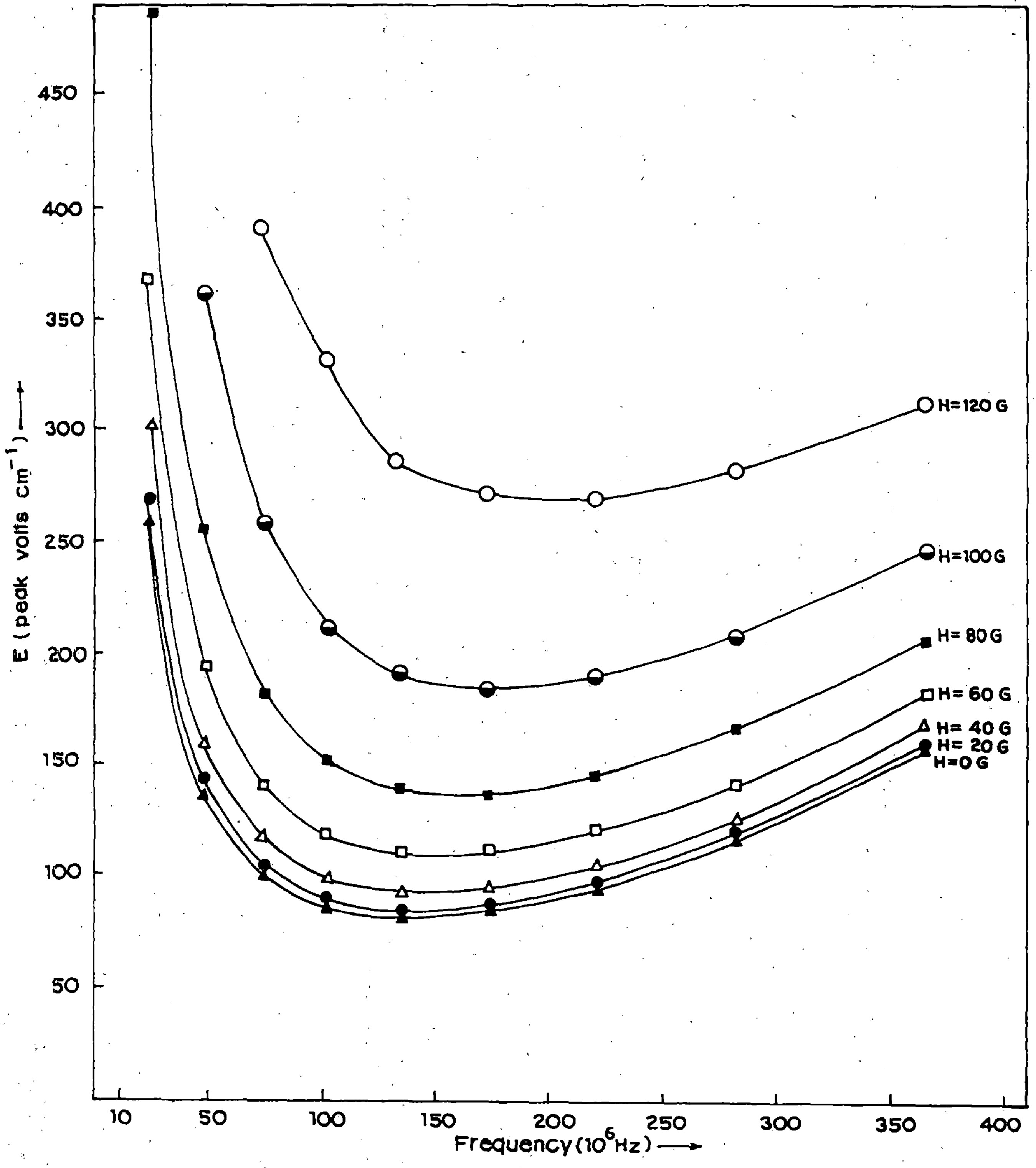

E (peak volts cm^{-1})
450
400
350
300
250
200
150
100
50
10
50
100
150
200
250
300
350
400
Frequency (10^6 Hz)
H=120 G
H=100G
H= 80G
H= 60 G
H= 40 G
H= 20 G
H=0 G

Fig. 7·3

field tends to increase and the main region of the curve is displaced towards longer wavelength. The increase of breakdown voltage of a low pressure ac discharge in neon with frequencies ranging from 100 Hz. to 20 KHz. has been observed by Kelkar (1970). He used neon glow lamp protected by a series resistance R_s.

The limitations of this model which we have described is linked with validity of the relation for λ_H . As it is strictly restricted for low value of H/P, the whole model is valid for small values of H/P. The values of breakdown voltages have been calculated by making the same assumptions as those of Hale (1947). The effect of magnetic field on the motion of the electrons has been taken into consideration by the concept of equivalent pressure which is the direct outcome of the calculation that the mean free path of the electron in presence of magnetic field is related with the mean free path, in absence of magnetic field by the relation

$$\lambda_H = \frac{\lambda}{(1+C_1 H^2/P^2)^{1/2}}$$

It has been shown by Sen and Ghosh (1963) that their experimental results for measurement of radiofrequency breakdown voltages in a non-resonant magnetic field in air and nitrogen can be quantitatively explained satisfactorily by introducing the concept of equivalent pressure. Unfortunately we could not get any experimental data in literature of breakdown voltages over this range of frequencies pressure and magnetic field for rare gases so that a comparision between the two could be made. But it was noted by Sen and Ghosh (1963) that the breakdown voltage increases with magnetic field for all values of pressure for the frequency of radio frequency voltage used. It is worthwhile to make some experimental measurements in this frequency region.

From the curves in figures (7.1), (7.2) and (7.3), the values of ω_b^2 and ω^2_{min} where ω_B is the electron cyclotron frequency and ω_{min} is the frequency at which the breakdown voltage becomes a minimum have been calculated for each curve for the three pressures and the results are entered in Table (7.5). The variation of ω^2_{min} against ω_B^2 has been plotted for three pressures (100μ, 150μ, and 200μ) in figures (7.4), (7.5) and (7.6). They are all straight lines making different intercepts along the X-axis and can be represented by the relation

$$\omega^2_{min} = C - m\omega_B^2$$

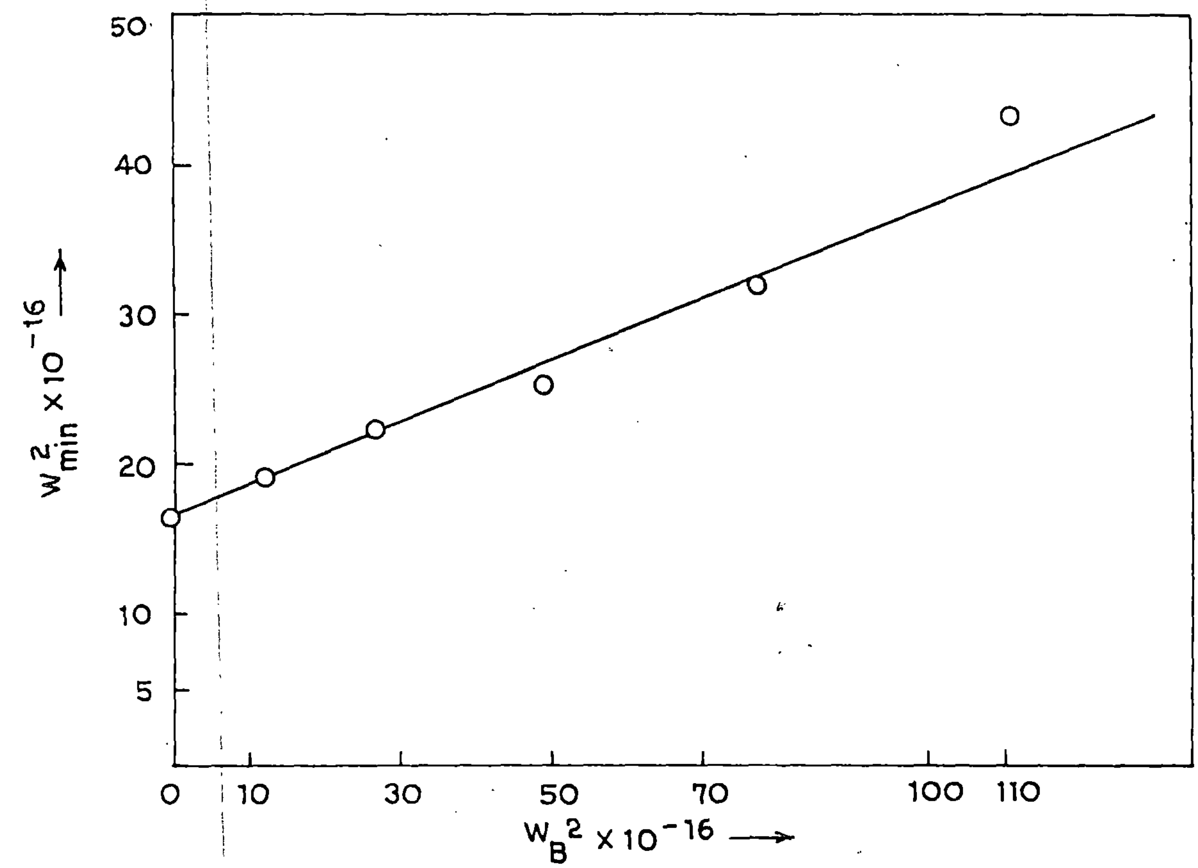
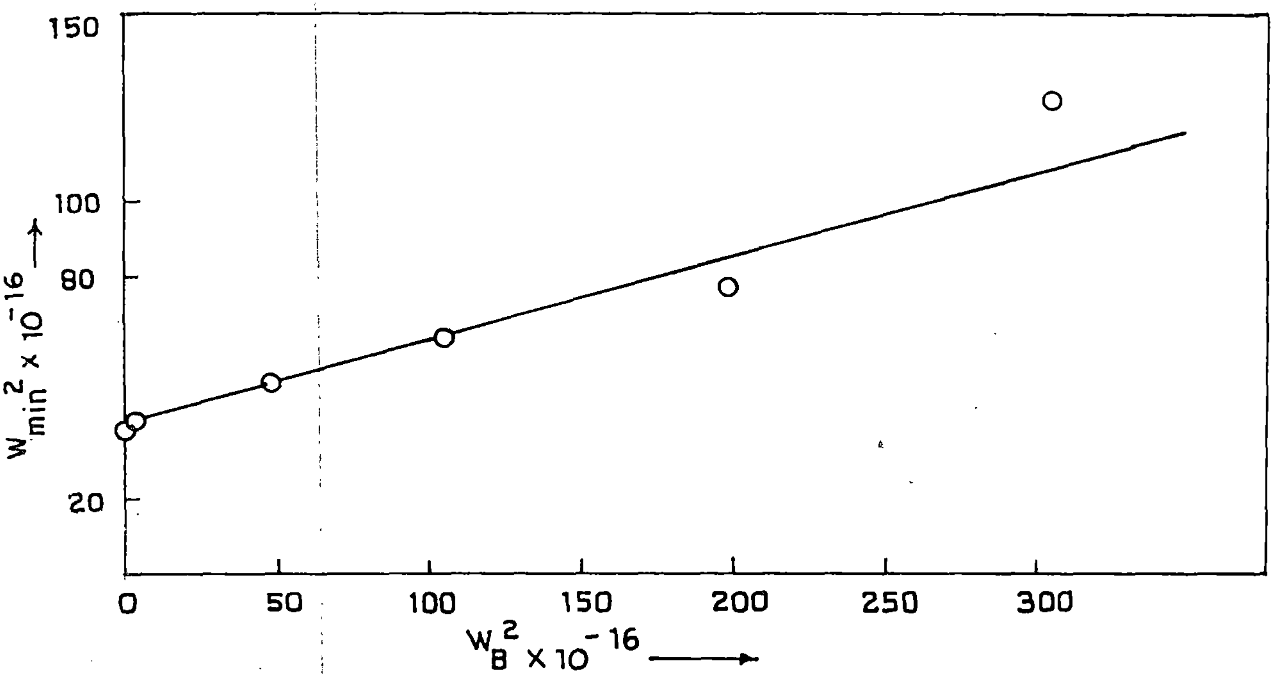

50
40
30
20
10
5
$W^2_{min} \times 10^{-16}$
0
10
30
50
70
100
110
$W_B^2 \times 10^{-16}$

Fig. 7·4



150
100
80
20
0
50
100
150
200
250
300
$W_{min}^2 \times 10^{-16}$
$W_B^2 \times 10^{-16}$

Fig. 7·5

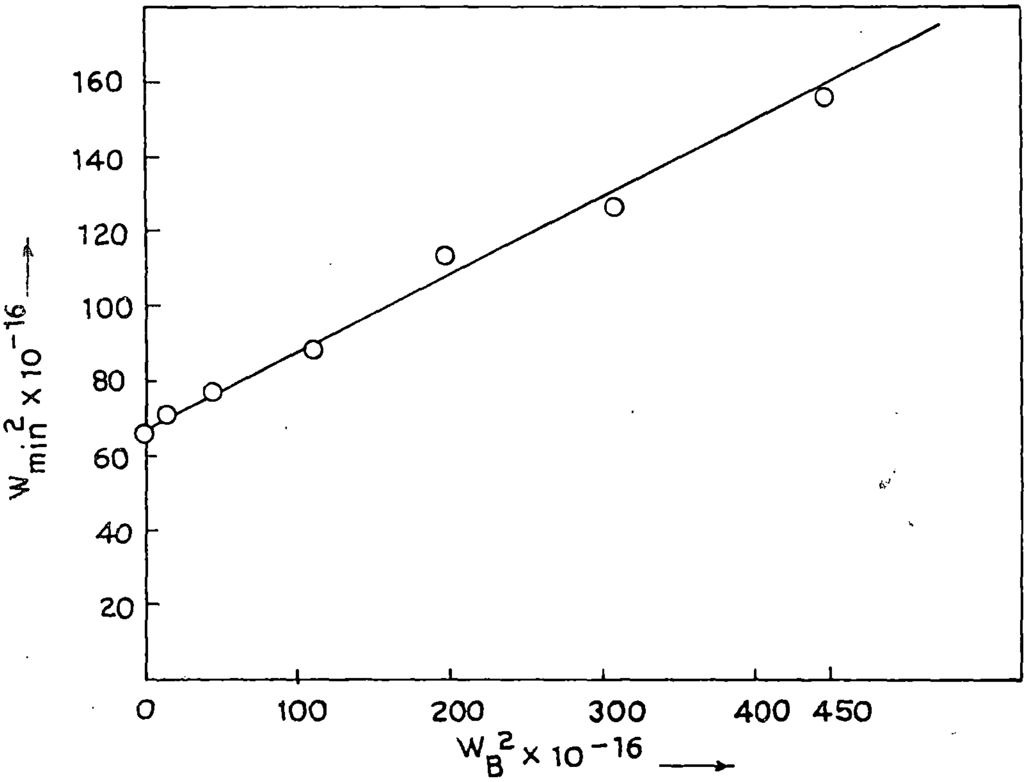

160
140
120
100
80
60
40
20
0
100
200
300
400
450
$W_{min}^2 \times 10^{-16}$
$W_B^2 \times 10^{-16}$

Fig. 7·6

$$\text{or} \quad \omega_B^2 = \frac{C}{m} - \frac{\omega_{min}^2}{m}$$

if it can be assumed that c/m is the collision frequency in analogy with the term for lower highbrid frequency as in the theory of plasma oscillations then δ_c^2 = c/m where C is the intercept and m is the slope of the respective curve. As C and m can be obtained from the curves the values of δ_c can be calculated and results are entered in table (7.6). The values of δ_c are entered in the fifth column of table (7.6) and the results are consistent with the values reported by McDaniel (1964) The last column gives the ratio δ_c /p which is almost constant as it should be.

An alternative theory for the breakdown of gases under high frequency field has been put forward by Holstein (1946) and later on developed by Brown and his coworkers (1947, 1948). The theory assumes that a gas breaks down when the rate of generation of electrons by collision of electrons with gas molecules is just compensated by the loss of electrons by diffusion. The theory predicts that when a transverse magnetic field is superimposed in addition to high frequency electric field, then for certain values of magnetic field and applied frequency a resonance will occur when $\omega_B = \omega$. Physically this means that the magnetic field reverses the direction of the electron without loss of energy and as the applied field reverses the electron can rapidly gain energy from the field provided its motion is not interrupted by frequent collisions of electrons with gas molecules and when collision is not taken into consideration the resonance condition is expected when

$$\omega_B = \omega_{min}$$

The above mathematical analysis thus shows that resonance condition is modified when collision of electrons with molecules of the gas is taken into consideration specially when collision frequency is greater than the frequency of the applied field.

The analysis further shows that Hale's theory should be modified by taking into consideration the collision of electrons with neutral molecules. However, the two main assumptions that the amplitude of oscillation should be equal to mean free path and the energy gained in traversing a mean free path should be equal to ionising energy of the gas are valid in presence of a transverse magnetic field as well. As the concept of equivalent pressure is valid for low values of (H/P) the above theoretical calculation will hold for low values of (H/P) as well.

Table 7.1

Values of E with and without magnetic field at pressure P = 100μ

ft	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16	9/16	10/16
$E_H = 0$	129.660	68.490	50.042	42.818	40.792	42.449	47.956	58.916	79.385	119.285
$f \times 10^{-6}$	11.801	23.962	36.947	51.197	67.440	86.646	110.286	140.892	182.615	243.482
$t \times 10^{9}$	5.296	5.217	5.076	4.883	4.634	4.328	3.967	3.549	3.080	2.567
$E_H = 10$	134.713	71.032	51.910	44.307	42.119	43.690	49.145	60.147	80.648	120.599
$E_H = 20$	151.494	79.668	57.966	49.224	46.419	47.691	53.052	64.068	84.651	124.621
$E_H = 30$	184.526	96.694	69.900	58.807	54.780	55.417	60.495	71.423	92.045	131.958
$E_H = 40$	243.825	127.260	91.273	75.922	69.614	68.967	72.000	83.845	104.181	143.684
$E_H = 50$	348.091	181.251	129.118	106.260	95.875	92.728	95.399	104.582	123.691	161.743
$E_H = 60$	526.225	274.431	195.701	160.738	145.446	136.301	135.312	140.813	155.751	189.571

Table 7.2

Values of E with and without magnetic field at pressure P = 150 μ

ft	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/16	9/16	10/16
$E_H = 0$	194.495	102.736	75.063	64.230	61.188	63.673	71.934	88.374	119.078	178.929
$f \times 10^{-6}$	17.703	35.943	55.406	76.796	101.16	129.969	165.430	211.339	273.925	365.226
$t \times 10^{9}$	3.531	3.478	3.384	3.255	3.089	2.885	2.645	2.366	2.054	1.711
$E_H = 20$	208.320	109.785	79.529	68.322	64.789	67.031	75.200	91.671	122.443	182.399
$E_H = 40$	256.857	134.771	97.685	82.453	77.148	71.464	86.284	102.760	133.706	196.783
$E_H = 60$	365.692	190.893	136.912	113.916	104.416	103.458	109.407	125.748	156.279	215.530
$E_H = 80$	596.573	310.494	220.982	181.528	163.055	156.501	159.045	171.606	198.910	254.464
$E_H = 100$	1045.433	549.103	395.374	328.262	295.643	279.389	271.925	271.813	285.634	325.229

Table 7.3

Values of E with and without magnetic field at pressure P = 200 μ

ft	1/16	2/16	3/16	4/16	5/16	6/16	7/16	8/18	9/16	10/16
E_H = 0	259.324	136.980	100.083	85.635	81.583	84.897	95.911	117.832	158.769	238.570
f x 10^{-6}	23.603	47.924	73.874	102.384	134.879	173.291	220.291	281.783	365.231	486.965
t x 10^9	2.648	2.608	2.538	2.442	2.317	2.163	1.984	1.774	1.540	1.284
E_H = 20	269.365	143.532	103.820	88.630	84.247	87.378	98.264	120.303	161.301	241.174
E_H = 40	2.993	159.483	115.934	98.399	92.840	95.472	106.103	128.126	169.294	249.225
E_H = 60	369.021	193.495	139.801	117.592	109.529	110.706	121.009	142.851	184.091	263.931
E_H = 80	487.677	254.632	182.545	151.822	139.222	137.963	145.895	167.669	208.370	287.327
E_H = 100	696.078	362.447	258.209	212.567	191.752	185.475	190.785	209.002	247.357	323.392
E_H = 120	1052.353	548.962	391.331	321.632	287.603	272.641	272.681	281.628	311.673	379.216

Table 7.4

Values of λ_H at different pressures and magnetic fields.

Value of C_1	Magnetic field in gauss	P = 100 μ $\lambda_H = \frac{\lambda}{(1+C_1H^2/P^2)^{1/2}}$	P = 150 μ $\lambda_H = \frac{\lambda}{(1+C_1H^2/P^2)^{1/2}}$	P = 200 μ $\lambda_H = \frac{\lambda}{(1+C_1H^2/P^2)^{1/2}}$
	0	0.4152	0.2768	0.2076
	20	0.4074	0.2745	0.2066
	30	0.3982	-	-
0.97×10^{-6}	40	0.3863	0.2677	0.2037
	50	0.3525	-	-
	60	0.3575	0.2575	0.1991
	80	-	0.2451	0.1932
	100	-	0.2314	0.1862
	120	-	-	0.1787

Values of λ has been taken from vacuum technology by A.Roth (North-Holland Publishing Company, 1976).

<u>Table 7.5.</u>

Magnetic field in Gauss	$\omega_B^2 \times 10^{-16}$	P = 100μ $\omega_{min}^2 \times 10^{-16}$	P = 150μ $\omega_{min}^2 \times 10^{-16}$	P = 200μ $\omega_{min}^2 \times 10^{-16}$
0	0	16.65	39.44	66.75
20	12.39	19.27	42.64	70.89
30	27.77	22.18	—	—
40	49.42	25.30	52.99	77.26
50	77.26	31.92	—	—
60	111.30	43.43	62.73	88.74
80	197.68		77.26	114.06
100	309.06		127.91	127.92
120	444.79			157.75

Table 7.6.

Pressure in μ	$C \times 10^{-16}$	m	$\delta_c^2 \times 10^{-16}$	$\delta_c \times 10^{-8}$	$\frac{\delta_c}{p} \times 10^{-9}$
100	16.5	0.205	80.49	8.97	8.97
150	40	0.2327	171.89	13.11	8.74
200	66	0.2105	313.54	17.71	8.86

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