CHAPTER - IV

RADIAL DISTRIBUTION FUNCTION FOR THE AZIMUTHAL CONDUCTIVITY OF AN ARC PLASMA.

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INTRODUCTION

The previous chapter has been devoted to show a new radio frequency non-immersive probe method to determine average azimuthal radio frequency conductivity of an arc plasma. It may be mentioned here that adequate electrostatic screening may be utilized to extend the use of the above method for plasma of varied nature. In absence of magnetic field the plasma may be assumed to be isotropic and if the probe frequency be such that electron atom collision frequency exceeds several times the probe frequency (angular), the radio frequency conductivity essentially becomes the d.c. conductivity. Consequently we can change the phrase "average azimuthal radio frequency conductivity" to "average conductivity" only. In those over simplified circumstances it may appear that our experiment claims to be an alternative method for conductivity measurement only. Nevertheless the importance becomes evident when the usual probe method proves inadequate for some reasons or other. The plasma conductivity is mostly determined by the use of ordinary probes. The inadequacy of the usual probe method becomes obvious in several circumstances. It is well known (Lin S.C. et al, 1965) that in hot ionized gas the probe method is accompanied by difficulties arising from the existence of a cold boundary layer around the probe. In the case of cold plasma, the probe current can give little information on the conductivity. In that case an attempt for indirect measurement of conductivity may be made by measuring the electron density; but evaluation of conductivity becomes still difficult due to the fact that no exact method of measurement of the collision frequency has yet been found.

The probe method is not applicable to a field-free plasma such as after-glow plasma, diffusion plasma and so on. Further, in the case of flowing plasma, the probe method should not be used because the inserted probe may appreciably disturb the dynamics of the flow. In some cases the plasma jet may even destroy the diagnostic probe. Only when there exists an electric field in the plasma, the electrical conductivity can directly be determined from the floating potentials of the probes together with the discharge current. One is free to comment at this moment that in a field-embedded plasma where probe method is a reliable diagnostic tool for conductivity measurements, the method mentioned in the previous chapter becomes only an alternative method for conductivity measurement with little significance, if not redundant. But it will be shown in the following that in these cases far from being redundant, our method and the usual probe method may jointly venture to lead most valuable informations regarding the structural behavior of conductivity and electron density of plasma.

In the previous chapter it has been shown that the average azimuthal electrical conductivity of an arc plasma can be determined by studying the change in the band-width of a coil wound around an arc tube due to the presence of the plasma column within it. It was assumed there that the arc column was a cylindrical conductor of uniform conductivity and the loss of radio frequency power was essentially due to eddy current heating of the plasma. It was further shown that if α is the ratio of the radio frequency current in the absence and in the presence of the discharge, the azimuthal conductivity is given by

$$\sigma_S = \frac{\pi}{\ell} \frac{\alpha - 1}{\omega^2 M^2} R_0$$

where ω is the angular frequency of the radio frequency current, ℓ is the length of the coil, M is the average mutual inductance formed between the coil and the plasma and R_0 is the radio frequency resistance of the coil. It has been assumed during the above deduction that the plasma is of uniform conductivity. But it is well known, on the contrary, that a plasma within a tube cannot be regarded as uniform with regard to radial electron density distribution or conductivity. In the case of gas discharge the radial distribution of charge density is cylindrically symmetric and can be represented by the Bessel function which is known as Schottky model. The Schottky model, as applied to glow discharge, can also be assumed to be valid in the case of low pressure arc. At a very low pressure of the order of 10^{-5} torr, the Schottky model is no longer valid and the free-fall model was developed by Tonks and Langmuir (1968) in which it was assumed that ions are lost to the wall due to free-fall in the radial electric field. The validity or otherwise of these assumed models has been put to some experimental tests in the case of glow discharge by the probe method but no elaborate experimental investigation in this regard has been carried out in the case of arc plasma. The existence of this structural behavior of plasma has been encountered or sometimes incorporated in connection to their measurements by several authors who have been using non-immersive coil probe for diagnostics [(Akimov and Konenko, 1966), Basu and Hore, 1977 etc.)].

It has, however, been indicated previously that no calibration is necessary for our measurements in contrast to the methods dealt with by so many authors (Donskoi et al, 1963; Mikoshiba and Smy, 1969; Ciampi and Talini, 1967). It has been shown by Akimov and Konenko, (1966) that the plasma and its simulating electrolytes do not always have the same effect on the measurement circuit. This discrepancy has been attributed, by them, to the radial non-uniformity of the plasma. The increased sensitivity of their measurement called for higher probe frequency. Consequently due to the small depth of penetration at that frequency, this method could give information on the peripheral plasma regions, which have an average conductivity lower than that of the plasma in the rest of cross-sections. Information regarding the structural behavior of the conductivity could be obtained in principle by increasing the depth of penetration (i.e. by decreasing the probe frequency) gradually so that the conductivity profile along the radius might be scanned. This has actually been tried by Akimov and Konenko (1966) but as it is evident that increased depth of penetration was obtained at the expense of the sensitivity of the apparatus ~~thus~~ information regarding the plasma behavior near the axis becomes erroneous if not completely uncertain.

Ciampi and Talini (1967) obtained expressions and measured two distinct average values of conductivity but did not, however, explore the structural behavior of conductivity. Basu and Hore (1977), using a method based on the relation between the plasma parameters and the impedance of an r.f. coil placed co-axially around the plasma, obtained the electron density and electron collision frequency for momentum transfer. They assumed Bessel type electron distribution (Fig. 4.3) along the radius even when the plasma was embedded in an axial magnetic field.

In the present investigation, we have questioned the Bessel distribution of charge density for an arc plasma and have presented a new method for obtaining the conductivity profile for a mercury arc plasma. In the following sections the theory and results have been outlined and it will be shown that the calculation of half-width from the obtained distribution function indicates that the plasma becomes more and more constricted along the axis with the increase of the arc current. The change of this structural behavior of the arc plasma will also be qualitatively explained.

THEORETICAL CONSIDERATIONS

The change in the tuned radio frequency current through a coil wound around an arc tube is effected by the reflected impedance due to eddy currents formed within the plasma. The reflected impedance for a primary coil due to the presence of the plasma, which can be regarded as a secondary coil, can be calculated in the following way.

Let us consider an annular cylinder defined by the radii r and $(r + dr)$ and length ℓ, where ℓ is the length of the coil. The reflected impedance for this annular cylindrical plasma under certain approximations (Ghosal et al, 1976) is given by $\omega^2 M^2(r)/R(r)$ where $R(r)$ is the azimuthal resistance of the annular cylinder and $M(r)$ is the mutual inductance between the coil and the annular cylinder of the plasma and ω is the angular frequency of the applied radio frequency field. In terms of conductivity the reflected impedance of the annular cylinder of the plasma is

$$\frac{\omega^2 l\,[M(r)]^2\,\sigma(r)\,dr}{2\pi r} \qquad .. \quad (A)$$

where $\sigma(r)$ is the azimuthal conductivity of the plasma at a distance r from the axis. Here it has been, however, assumed that the radio frequency field has no effect on the plasma outside the coil on both sides, otherwise in the above expression l should be replaced by l^*, the effective plasma length. The justification of the assumption $l \simeq l^*$ is given in Appendix (4.1).

The total reflected impedance will be the sum of the contributions of all the elementary annular cylinders imagined within the plasma column. Consequently if R_0 is the radio frequency resistance of the primary coil the total effective impedance of the coil will be

$$R' = R_0 + \frac{\omega^2 l}{2\pi}\int_0^R \frac{[M(r)]^2\,\sigma(r)\,dr}{r} \qquad .. \quad (4.1)$$

where R is the radius of the arc tube. $M(r)$ can be written as $M(r) = kr^2$, where k is a constant depending upon the number of turns of the primary coil. If α denotes the ratio of the radio frequency current without and with the plasma, we get from equation (4.1)

$$\alpha - 1 = \frac{\omega^2 k^2 \ell}{2\pi R_0} \int_0^R r^3 \sigma(r)\, dr \qquad .. \ (4.2)$$

If $\sigma(r) = \sigma_0$ = constant for $0 \leqslant r \leqslant R$, equation (4.2) becomes

$$\sigma_0 = \frac{8\pi}{\ell \omega^2} \cdot \frac{\alpha - 1}{[M(R)]^2} \cdot R_0$$

This formula differs by a numerical factor from the expression used in the previous chapter (Ghosal et al, 1976), due to the fact that previously an average value of mutual inductance and current path was taken, whereas $M(r)$ has been assumed here to be a function of r in the form $M(r) = kr^2$. In obtaining the above equation it has, however, been assumed that the skin depth is much greater than the arc radius because it has been calculated in the previous chapter (Ghosal et al, 1976) that for a frequency of 5.1 MHz as used in the present experiment the skin depth is 2 cm.

If I denotes the arc current and E the axial voltage drop per unit length

$$\int_0^R \sigma(r)\, r\, dr = I/2\pi E \qquad .. \ (4.3)$$

then from equation (4.2) and (4.3),

$$\frac{\int_0^R r^3 \sigma(r)\, dr}{\int_0^R r\, \sigma(r)\, dr} = \frac{\alpha - 1}{f^2 k^2 l} \cdot \frac{E}{I} \cdot R_0 \quad .. \quad (4.4)$$

where f $= \omega/2\pi$ is the frequency of the radio frequency current. Since all the terms on the right-hand side of equation (4.4) can be obtained experimentally it is evident that equation (4.4) contains the information regarding the radial distribution of conductivity; but it is evident that from the experimental measurement of the expression on the right-hand side of equation (4.4) it is not possible to determine uniquely the nature of the radial variation of $\sigma(r)$.

However, the utility of the equation lies in the fact that the factor on the right-hand side can be determined experimentally and any proposed form of $\sigma(r)$ will become invalid unless the expression on the left-hand side, calculated on the basis of the proposed form, is equal to that on the right-hand side obtained from experimental measurement.

Regarding the form of $\sigma(r)$ let us make the following assumptions :

(a) $\sigma(r)$ is cylindrically symmetric;

(b) It is a monotonically decreasing function,

(c) $\sigma(r) = 0$ at $r = R$; and

(d) $\sigma(r) = \sigma_0$ at $r = 0$

Thus the general form of $\sigma(r)$ can be written as a polynomial expansion around $r = R$. It is, however, advantageous to assume $\sigma(r)$ of the approximate form

$$\sigma(r) = \sigma_0 \left[1 - \left(\frac{r}{R}\right)^2 \right]^n \qquad .. \quad (4.5)$$

where σ_0 and n are to be determined.

If we denote by 'a' the experimentally determined expression on the right-hand side of equation (4.4), we get from equation (4.4) and (4.5),

$$n = \frac{R^2}{a} - 2 \qquad .. \quad (4.6)$$

Hence, inserting the value of a in equation (4.6), n can be determined and we can obtain an expression for the radial distribution function for $\sigma(r)$ from equation (4.5).

EXPERIMENTAL ARRANGEMENT

The experimental arrangement has been described in a previous paper (Ghosal et al, 1976) and also in chapter III. Measurements were made for a mercury arc plasma formed within an arc tube fitted with two tungsten probes. The oscillator coil was placed near the working coil and the induced radio frequency voltage was tuned with a variable condenser. The tuned currents were measured with a radio frequency milliammeter and the arc current was varied with a rheostat connected in series with the d.c. source used to excite the arc. The circuit constants are given below :

Length of the arc tube = 31 cm.

Anode-cathode spacing = 27 cm.

Outer diameter of the tube = 1.9 cm.

Inner diameter of the tube = 1.5 cm.

Coil length = 4.55 cm.

Coil diameter = 1.9 cm.

Number of turns in the coil = 37.

Probe-to-probe separation = 6.35 cm.

Measurements of the tuned radio frequency currents were made for a frequency of 6.1 MH_z both in the presence and in the absence of the plasma for different arc currents (I = 2.3 amp., 3.1 amp., 4 amp. and 5 amp).

The values of (α - 1) thus obtained are utilized to determine the value of 'a', that is

$$\frac{\alpha - 1}{f^2 k^2 l} \cdot \frac{E}{I} \cdot R_o ,$$

the term on the right-hand side of equation (4.4) for different values of I/E where the field E has been measured by noting the probe-to-probe voltage.

RESULTS AND DISCUSSION

The values of 'a' determined from the expression

$$a = \frac{\alpha - 1}{f^2 k^2 l} \cdot \frac{E}{I} \cdot R_o$$

are entered in Table 4.1 for different values of I/E.

Table 4.1

Values of 'a' for different value of I/E

I/E (amps-cm-volts)	a $(cm)^2$
3.36	0.131
6.56	0.096
9.41	0.094
10.59	0.091

In order to find the distribution function for the radial variation of the conductivity let us assume some simplified models for $\sigma(r)$ and see whether the proposed model gives values consistent with the experimental results.

If it is assumed that $\sigma(r)$ is uniform, then

$$a = \frac{\int_0^R r^3 \sigma(r)\, dr}{\int_0^R r\,\sigma(r)\, dr} = \frac{R^2}{2} = 0.281\ cm^2$$

Thus, comparing with the experimental values of 'a' as entered in Table 4.1, it is evident that $\sigma(r)$ cannot be regarded as uniform along the radial direction, at least for the range of arc currents used here.

It is well known that the distribution function for electron density or the conductivity can be well represented by the Bessel function or more nearly by a parabolic distribution in the case of glow discharge, as has been verified experimentally by measurement with cold probes or by measuring the current distribution flowing to the anode consisting of concentric cylinders. If we assume also that in an arc plasma the distribution is parabolic, then

$$\sigma(r) = \sigma_0\left(1 - \frac{r^2}{R^2}\right) \quad \text{and} \quad a = \frac{\int_0^R r^3\left(1 - \frac{r^2}{R^2}\right) dr}{\int_0^R r\left(1 - \frac{r^2}{R^2}\right) dr} = \frac{R^3}{3} = 0.187$$

Assuming Bessel distribution function i.e. $\sigma(r) = \sigma_0 J_0(2{\cdot}405\, r/R)$

$$a = \frac{\int_0^R J_0(2{\cdot}405\, r/R)\, r^3 dr}{\int_0^R J_0(2{\cdot}405\, r/R)\, r\, dr} = 0{\cdot}159 \text{ cm}^2$$

(See Appendix 4.2).

The value of 'a' thus does not correspond to experimentally observed values but it can be noted that there is a tendency to approach this value at lower arc currents.

Next we turn our attention to equation (4.6) and obtain the values of n for different values of 'a' corresponding to the different values of the parameter I/E as entered in Table 4.1, and the values of n thus obtained are entered in Table 4.2 for corresponding values of I/E.

Table 4.2

Variation of n with I/E

I/E (amps-cm/volts)	n
3.36	2.293
6.563	3.859
9.408	3.984
10.59	4.181

To obtain the nature of the distribution function from equation (4.5) the value of σ_0 has been calculated for different I/E values from equation (4.3) :

$$\int_0^R \sigma_0 \left[1 - \left(\frac{r}{R}\right)^2\right]^n r\,dr = I/2\pi E$$

and after integration we get

$$\sigma_0 = \frac{I/E}{2\pi} \cdot \frac{2(n+1)}{R^2}$$

The distribution function represented by equation (4.5) can now be plotted for different I/E values (Fig. 4.1). The experimentally obtained conductivity distribution function and parabolic and Bessel distribution function are also given in tabular form (Tables 4.4 and 4.5) for comparison. The values of σ_0 as different I/E values are given in table 4.3.

From the nature of the curves it is evident that not only the conductivity at the axis shows a rapid increase with the increase of the arc current but at the same time the nature of the distribution of the azimuthal conductivity undergoes a remarkable change, which is evident from the nature of the curves, indicating that the discharge becomes more and more constricted with the increase of the arc current. The half-widths of the above curves can be regarded as a measure of the constriction of the plasma column at the axis and the variation of half-widths with I/E is shown in Fig. 4.2. There is a rapid fall as the arc current is changed from 2.8 amp. to 3.1 amp. and then the change is slower, and the curve shows a tendency to saturation towards higher currents.

It has been noted that in a mercury vapour tube the arc completely fills the tube for low currents but as the current is increased the arc column contracts and the light becomes more intense at the axis of the tube, which is also corroborated by the present investigation.

The distribution curves obtained here closely resemble the curves obtained by Hoyaux (1968) for a low pressure arc where the magnetic self-constriction is predominant, but the constriction observed in the present investigation cannot be due to the magnetic self-constriction, as the order of the current is much smaller.

The increase of constriction of the plasma column at higher currents is probably due to the fact that the increased energy input causes an increase in gas temperature at the axis, thereby lowering the gas density. Consequently the increased mean free path facilitates the ionization probability, causing a higher charge density at the axis. If, however, the gas density becomes too low an opposing effect may appear. Due to reduction of the gas density the total ionization collision of neutral particles with electrons will be lowered, thereby decreasing the ionization probability. The saturation observed in the present case may be partly due to this effect.

Table 4.3

Values of conductivity at the axis of the arc at different I/E values.

I/E (amps.-cm./volts)	$\sigma_0 = \frac{I/E}{2\pi} \cdot \frac{2(n+1)}{R^2}$ in mhos/cm.
3.36	6.26
6.56	18.05
9.41	26.55
10.69	31.06

Table 4.4

Plotting of experimentally obtained conductivity distribution function at different I/E values

r in cm.	σ (in mhos/cm. at different I/E values given in amps.-cm./volts)			
	I/E=3.36	I/E=6.56	I/E=9.41	I/E=10.59
0	6.26	18.05	26.55	31.06
0.2	5.29	13.58	19.79	28.85
0.4	2.91	4.96	7.00	22.22
0.6	0.60	0.35	0.45	11.18
0.75	0	0	0	0

Table 4.5

Plotting of the parabolic and Bessel distribution function

2.405 r/R	r/R	r (cm)	f para	f Bessel
0	0	0	1	1
0.4	0.166	0.1245	0.972	0.9604
0.8	0.333	0.250	0.889	0.8463
1.2	0.499	0.374	0.751	0.6711
1.6	0.665	0.499	0.558	0.4554
2.0	0.832	0.624	0.308	0.2239
2.405	1	0.75	0	0

Appendix 4.1

Consideration of the effective plasma length (ℓ^*)

Let us consider the effect of the plasma column outside the coil on the reflected impedance. First we consider the magnetic field at the point O (see Fig. 4.4) at a distance Z from the nearest end of the coil carrying current i. The magnetic field due to a small element of the coil at a distance x from the point O is given by

$$\frac{2\pi n i a^2 dx}{(a^2+x^2)^{3/2}}$$

where n and a are the turns per unit length and coil diameter respectively. Thus the total field due to the contribution of all the elements of the coil is given by

$$\mathcal{H} = \int_{z}^{z+\ell} \frac{2\pi n i a^2 dx}{(a^2+x^2)^{3/2}} = \frac{2\pi n i}{a} \cdot \frac{x}{(a^2+x^2)^{1/2}} \Bigg|_{z}^{z+\ell}$$

Taking the long solenoid approximation and considering the field point to be very near to the coil i.e.,

$$\frac{a}{\ell} \ll 1, \text{ and } z \ll \ell$$, the field

is given by

$$\mathcal{H} = \frac{H_0}{2}(1-\cos\theta) = \frac{K_0 i}{2}(1-\cos\theta) \quad \text{(say)}$$

where the angle θ is described in the figure (4.4) and H_0 is the field inside the coil.

The mutual inductance between the coil and an annular plasma cylindrical element of length dz defined by the radii r & $(r + dr)$ considered placed at O is given by

$$M = \frac{K_0}{2}(1-\cos\theta)\pi r^2 = \frac{M(r)}{2}(1-\cos\theta)$$

where M(r) is the same as defined previously. The reflected impedance of the coil due to the presence of the plasma element B is given by

$$\frac{1}{4} \cdot \frac{\omega^2[M(r)]^2\sigma(r)\,dr}{2\pi r}(1-\cos\theta)^2 dz$$

However, in the above deduction the radial variation of magnetic field $\mathcal{H}$ has been neglected. If the plasma is assumed to be infinitely extended, the total contribution of the plasma elements (outside the coil) to the reflected impedance may be given by

$$\mathcal{Z}_{out} = 2 \cdot \frac{1}{4} \cdot \frac{\omega^2[M_0(r)]^2\sigma(r)\,dr}{2\pi r}\int_{\theta=\frac{\pi}{2}}^{0}(1-\cos\theta)^2 dz$$

(The factor 2 in the above expression has been incorporated considering the plasma to be extended on both sides of the coil)

$$= \frac{1}{2} \cdot \frac{\omega^2[M_0(r)]^2\sigma(r)\,dr}{2\pi r} \cdot a \cdot (2-\pi/2)$$

To get the total reflected resistance this is to be added to Z_{in}, i.e., the expression (A) of an earlier section of this chapter.

Thus
$$Z_{out} + Z_{in} = \frac{\omega^2 [M_0(r)]^2 \sigma(r)\,dr}{2\pi r}\left[l + \frac{a}{2}\left(2 - \pi/2\right)\right]$$

giving effective plasma length

$$l^* = \left[l + \frac{a}{2}\left(2 - \pi/2\right)\right] = l + 0{\cdot}16$$

Thus in our case where $l \simeq 6$ cm. the correction is less than 3 percent.

Appendix 4.2

Numerical evaluation of the quantity $a = \dfrac{\int_0^R r^3 J_0(2{\cdot}405\, r/R)\,dr}{\int_0^R r J_0(2{\cdot}405\, r/R)\,dr}$

Let us suppose $r^m J_0\left(2{\cdot}405\; r/R\right) = y^m\left(r/R\right)$

The quantity $\int_0^R y^m\left(\frac{r}{R}\right) dr$ can be evaluated using Scarbourgh's method. The function $y^m(r/R)$ at $r/R = 0, 1/5, 2/5, 3/5, 4/5$ and 1 can be obtained from the graph (Figs. 4.5 and 4.6). These are inserted in table 4.6.

Table 4.6

m	$y^m(0)$	$y^m(1/5)$	$y^m(2/5)$	$y^m(3/5)$	$y^m(4/5)$	$y^m(1)$	$\int_0^R y^m(\frac{r}{R})dr$	a (cm^2)
1	0	0.142	0.235	0.247	0.165	0	0.1178	0.159
3	0	3.189×10^{-3}	2.119×10^{-2}	6.012×10^{-2}	5.94×10^{-2}	0	0.0187	

The quantities $\int_0^R y^m(r/R)\,dr$, obtained by using the formula

$$\int_0^R y^m(r/R)\,dr = \frac{R}{15}\left[y^m(0) + y^m(1) + 2\left\{y^m(2/5) + y^m(4/5)\right\} + 4\left\{y^m(1/5) + y^m(3/5)\right\}\right]$$

are also inserted in table (4.6)

The quantity $$a = \int_0^R y^3(r/R)\,dr \Big/ \int_0^R y^2(r/R)\,dr$$

comes out to be 0.159.

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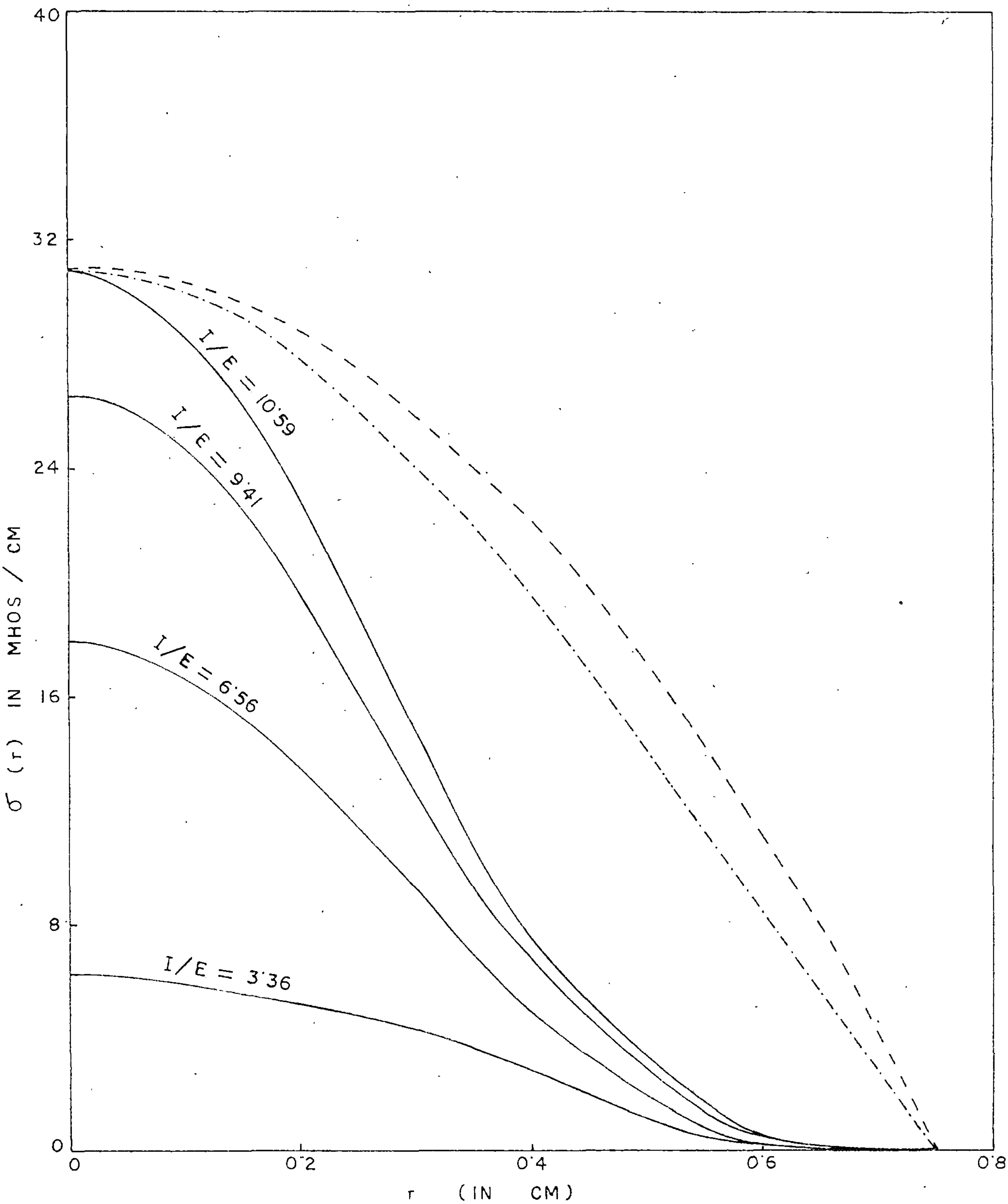

FIG. 4·1. ELECTRICAL CONDUCTIVITY DISTRIBUTION AS FUNCTION OF r FOR DIFFERENT I / E VALUES. PARABOLIC DISTRIBUTION (– – –), BESSEL DISTRIBUTION (–·–·–·–·–·–·–·–)

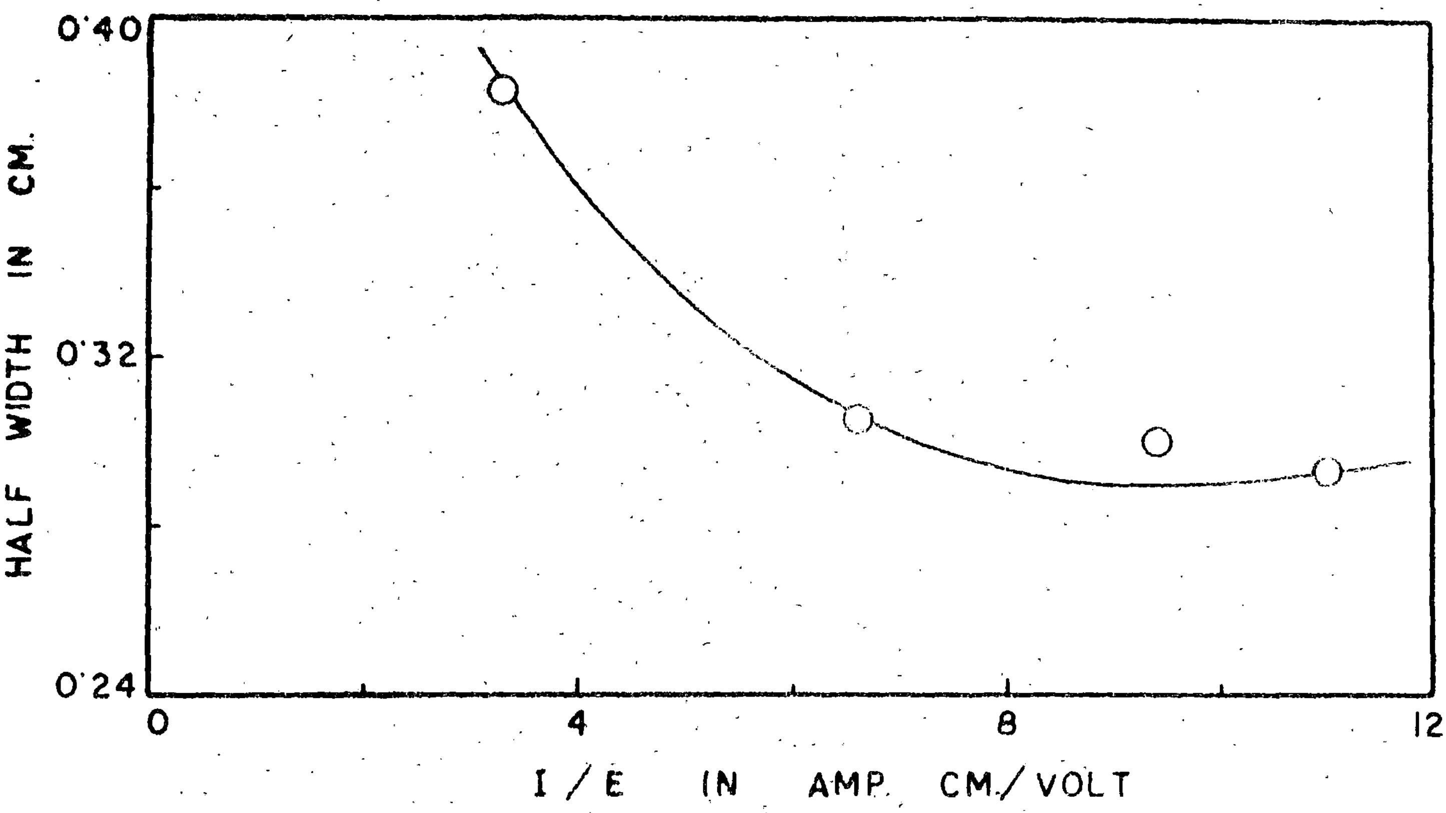

FIG. 4'2 . VARIATION OF HALF -WIDTHS OF CONDUCTIVITY DISTRIBUTION FUNCTION WITH I / E.

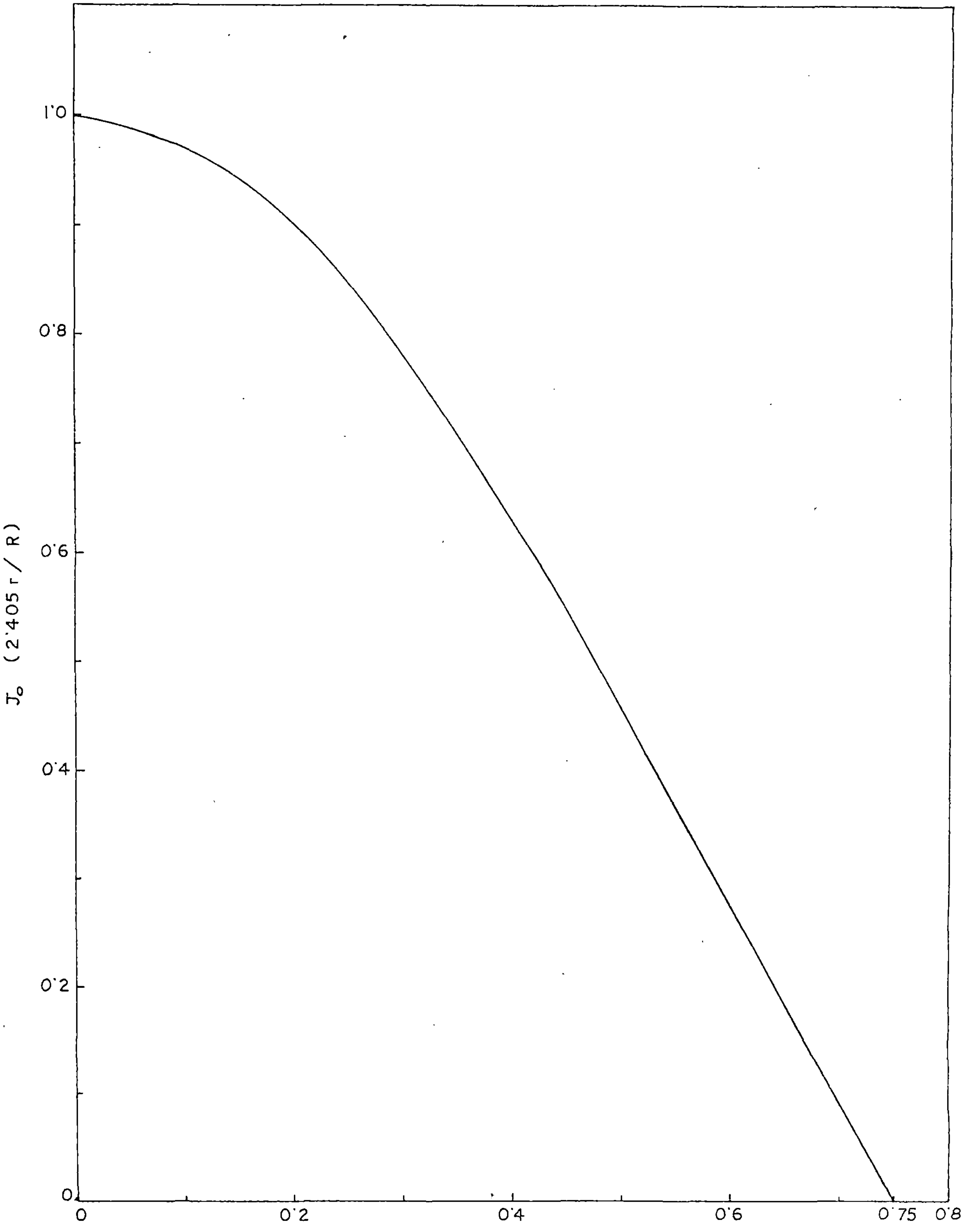

FIG. 4'3. ZERO ORDER BESSEL DISTRIBUTION FUNCTION.

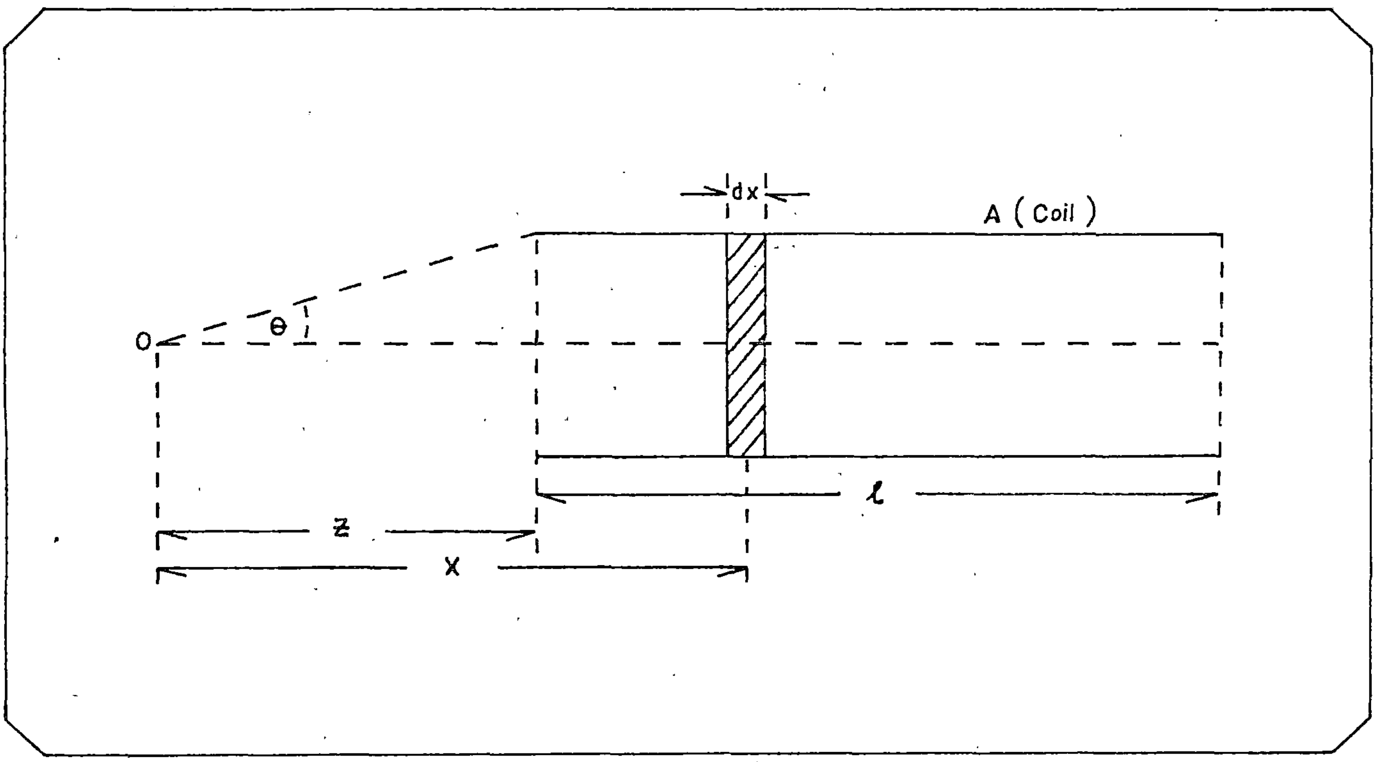

FIG. 4·4. EFFECTIVE PLASMA LENGTH CONSIDERATION.

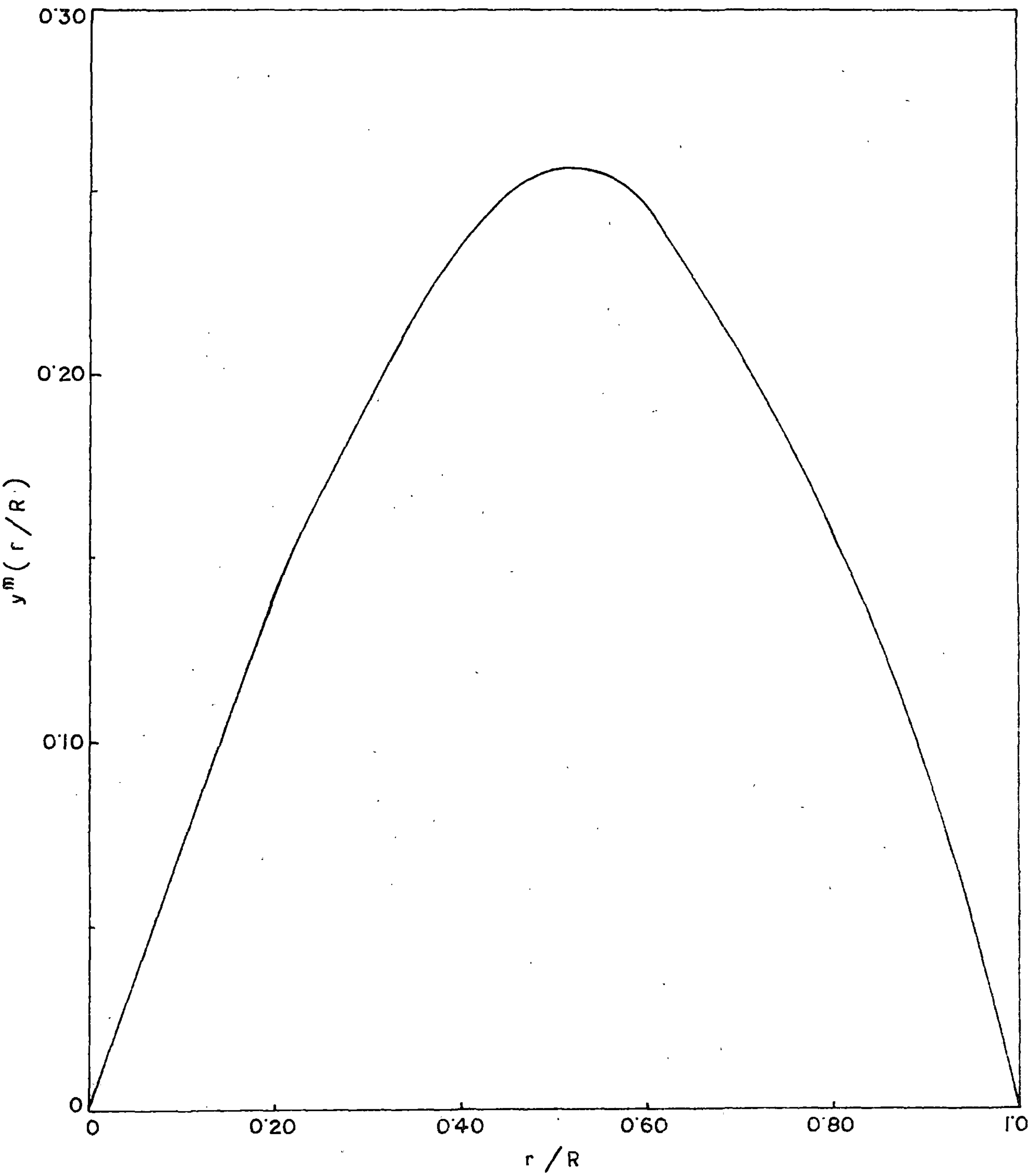

FIG. 4·5. VARIATION OF y^m (r / R) WITH (r / R) IN THE CASE OF m = 1.

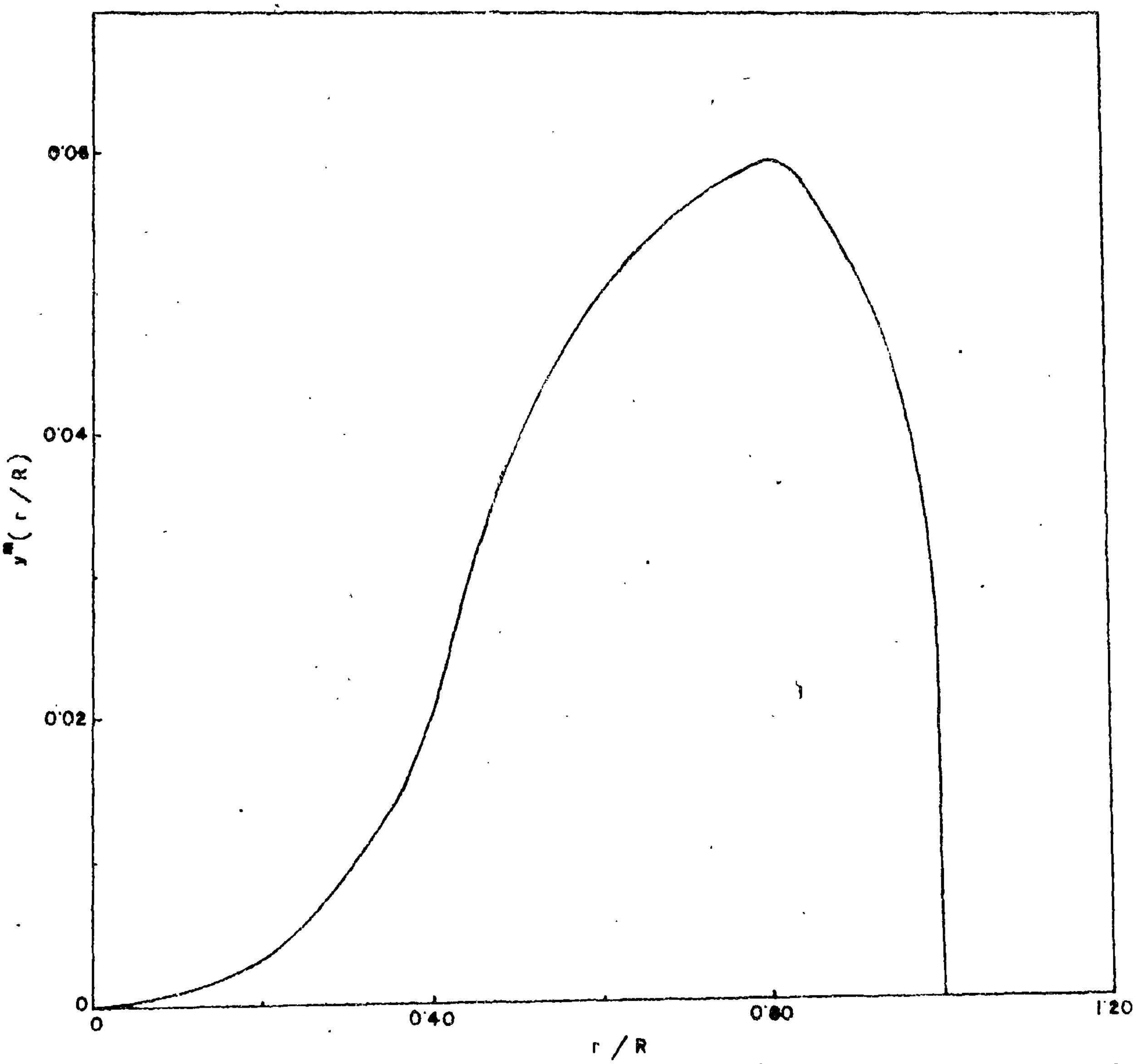

FIG. 4·6 . VARIATION OF $y^m(r/R)$ WITH (r/R) IN THE CASE OF $m = 3$.

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Radial distribution function for the azimuthal conductivity of an arc plasma

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It is shown that the simultaneous measurement of the change in the band width of a coil wound around the positive column of an arc tube and the longitudinal field across the positive column can provide valuable information regarding the structural behaviour of the electrical conductivity or the electron density of the plasma column. A distribution function for the radial variation of the azimuthal conductivity of an arc plasma is proposed and the parameters of the distribution function have been obtained from the above measurements. The calculation of half-widths from the distribution function indicates that the plasma becomes more and more concentrated along the axis with the increase of the arc current. The change of this structural behaviour of the arc plasma is qualitatively explained.

1. Introduction

It is well known that a plasma within a tube cannot be regarded as uniform with regard to radial electron density distribution or conductivity and in the case of glow discharge the radial distribution of the charge density is cylindrically symmetric and can be represented by the Bessel function which is known as a Schottky model. The Schottky model, as applied to glow discharge, can also be assumed to be valid in the case of a low pressure arc. At a very low pressure of the order of 10^{-5} torr the Schottky model is no longer valid and the free-fall model was developed by Tonks and Langmuir (1968) in which it was assumed that ions are lost to the wall due to free fall in the radial electric field. The validity or otherwise of these assumed models has been put to some experimental tests in the case of glow discharge by the probe method but no elaborate experimental investigation in this regard has been carried out in the case of arc plasma. In a previous communication (Ghosal *et al.* 1976) it was shown that the average azimuthal electrical conductivity of an arc plasma can be determined by studying the change in the bandwidth of a coil wound around an arc tube due to the presence of the plasma column within it. It was assumed there that the arc column was a cylindrical conductor of uniform conductivity and the loss of radiofrequency power was essentially due to eddy current heating of the plasma. It was further shown that if α is the ratio of the radiofrequency current in the absence and in the presence of the discharge, the azimuthal conductivity is given by

$$\sigma_s = \frac{\pi}{l}\frac{\alpha-1}{\omega^2 M^2} R_0$$

were ω is the angular frequency of the radiofrequency current, l is the length of the coil, M is the average mutual inductance formed between the coil and the plasma and R_0 is the radiofrequency resistance of the coil. It has been assumed in the above deduction that the plasma is of uniform conductivity.

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Experimental evidence has clearly indicated however that an arc plasma cannot be regarded as a medium of uniform charge density or conductivity and previous attempts, such as those of Schottky or of Tonks and Langmuir are entirely based on some assumed theoretical models. In the present investigation it is our aim to start with some generalized radial conductivity distribution and to measure experimentally a quantity which is a function of this assumed conductivity distribution. The next step will be to find the nearly exact distribution function which gives the closest approach to the experimental results.

2. Theoretical consideration

The change in the tuned radiofrequency current through a coil wound around an arc tube is effected by the reflected impedance due to eddy currents formed within the plasma. The reflected impedance for a primary coil due to the presence of the plasma, which can be regarded as a secondary coil, can be calculated in the following way.

Let us consider an annular cylinder defined by the radii r and $r+dr$ and length l, where l is the length of the coil. The reflected impedance for this annular cylindrical plasma under certain approximations (Ghosal *et al.* 1976) is given by $\omega^2 M^2(r)/R(r)$, where $R(r)$ is the azimuthal resistance of the annular cylinder and $M(r)$ is the mutual inductance between the coil and the annular cylinder of the plasma and ω is the angular frequency of the applied radiofrequency field. In terms of conductivity the reflected impedance of the annular cylinder of the plasma is

$$\frac{\omega^2 l M^2(r)\sigma(r)\,dr}{2\pi r}$$

where $\sigma(r)$ is the azimuthal conductivity of the plasma at a distance r from the axis. The total reflected impedance will be the sum of the contributions of all the elementary annular cylinders imagined within the plasma column. Consequently if R_0 is the radiofrequency resistance of the primary coil the total effective impedance of the coil will be

$$R' = R_0 + \frac{\omega^2 l}{2\pi}\int_0^R \frac{M^2(r)\sigma(r)\,dr}{r} \tag{1}$$

where R is the radius of the arc tube. $M(r)$ can be written as $M(r)=kr^2$, where k is a constant depending upon the number of turns of the primary coil. If α denotes the ratio of the radiofrequency current without and with the plasma, we get from eqn. (1)

$$\alpha - 1 = \frac{\omega^2 k^2 l}{2\pi R_0}\int_0^R r^3\sigma(r)\,dr \tag{2}$$

If $\sigma(r)=\sigma_0=$ constant for $0\leqslant r\leqslant R$, eqn. (2) becomes

$$\sigma_0 = \frac{8\pi}{l\omega^2}\cdot\frac{\alpha-1}{M^2(R)}\cdot R_0$$

This formula differs by a numerical factor from the expression used in the previous paper (Ghosal *et al.* 1976), due to the fact that previously an average value of mutual inductance and current path was taken, whereas $M(r)$ has been assumed here to be a function of r in the form $M(r)=kr^2$. In obtaining the above equations it has, however, been assumed that the skin depth is much greater than the arc radius because it has been calculated in the previous paper (Ghosal *et al.* 1976) that for a frequency of 5·1 MHz as used in the present experiment the skin depth is 2 cm.

If I denotes the arc current and E the axial voltage drop per unit length

$$\int_0^R \sigma(r)r\,dr=\frac{I}{2\pi E} \tag{3}$$

then from eqns. (2) and (3)

$$\frac{\int_0^R r^3\sigma(r)\,dr}{\int_0^R r\sigma(r)\,dr}=\frac{\alpha-1}{f^2k^2l}\cdot\frac{E}{I}\cdot R_0 \tag{4}$$

where $f=\omega/2\pi$ is the frequency of the radiofrequency current. Since all the terms on the right-hand side of eqn. (4) can be obtained experimentally it is evident that eqn. (4) contains the information regarding the radial distribution of conductivity ; but it is evident that from the experimental measurement of the expression on the right-hand side of eqn. (4) it is not possible to determine uniquely the nature of the radial variation of $\sigma(r)$.

However, the utility of the equation lies in the fact that the factor on the right-hand side can be determined experimentally and any proposed form of $\sigma(r)$ will become invalid unless the expression on the left-hand side, calculated on the basis of the proposed form, is equal to that on the right-hand side obtained from experimental measurement.

Regarding the form of $\sigma(r)$ let us make the following assumptions :

(*a*) $\sigma(r)$ is cylindrically symmetric ;
(*b*) it is a monotonically decreasing function,
(*c*) $\sigma(r)=0$ at $r=R$.

Thus the general form of $\sigma(r)$ can be written as a polynomial expansion around $r=R$. It is however advantageous to assume $\sigma(r)$ of the approximate form

$$\sigma(r)=\sigma_0\left[1-\left(\frac{r}{R}\right)^2\right]^n \tag{5}$$

where σ_0 and n are to be determined.

If we denote by a the experimentally determined expression on the right-hand side of eqn. (4), we get from eqns. (4) and (5),

$$n=\frac{R^2}{a}-2 \tag{6}$$

Hence, inserting the value of a in eqn. (6), n can be determined and we can obtain an expression for the radial distribution function for $\sigma(r)$ from eqn. (5).

3. Experimental arrangement

The experimental arrangement has been described in a previous paper (Ghosal *et al.* 1976). Measurements were made for a mercury arc plasma formed within an arc tube fitted with two tungsten probes. The oscillator coil was placed near the working coil and the induced radiofrequency voltage was tuned with a variable condenser. The tuned currents were measured with a radiofrequency milliammeter and the arc current was varied with a rheostat connected in series with the d.c. source used to excite the arc. The circuit constants are given below :

Length of the arc tube = 31 cm.

Anode–cathode spacing = 27 cm.

Outer diameter of the tube = 1·9 cm.

Inner diameter of the tube = 1·5 cm.

Coil length = 4·55 cm.

Coil diameter = 1·9 cm.

Number of turns in the coil = 37.

Wire diameter = 2 mm.

Probe-to-probe separation = 6·35 cm.

Measurements of the tuned radiofrequency currents were made for a frequency of 5·1 MHz both in the presence and in the absence of the plasma for different arc currents ($I = 2{\cdot}3$ amp, 3·1 amp, 4 amp and 5 amp).

The values of $(\alpha - 1)$ thus obtained are utilized to determine the value of a, that is

$$\frac{\alpha - 1}{f^2 k^2 l} \cdot \frac{E}{I} \cdot R_0$$

the term on the right-hand side of eqn. (4) for different values of I/E where the field E has been measured by noting the probe-to-probe voltage.

4. Results and discussion

The values of a determined from the expression

$$a = \frac{\alpha - 1}{f^2 k^2 l} \cdot \frac{E}{I} \cdot R_0$$

are entered in Table 1 for different values of I/E.

I/E (amps-cm/volts)	a (cm²)
3·36	0·131
6·56	0·096
9·41	0·094
10·59	0·091

Table 1.

In order to find the distribution function for the radial variation of the conductivity let us assume some simplified models for $\sigma(r)$ and see whether the proposed model gives values consistent with the experimental results.

If it is assumed that $\sigma(r)$ is uniform, then

$$a=\frac{\int_0^R r^3\sigma(r)\,dr}{\int_0^R r\sigma(r)\,dr}=\frac{R^2}{2}=0{\cdot}281\ \mathrm{cm}^2$$

Thus, comparing with the experimental values of a as entered in Table 1, it is evident that $\sigma(r)$ cannot be regarded as uniform along the radial direction, at least for the range of arc currents used here.

It is well known that the distribution function for electron density or the conductivity can be well represented by the Bessel function or more nearly by a parabolic distribution in the case of glow discharge, as has been verified experimentally by measurement with cold probes or by measuring the current distribution flowing to the anode consisting of concentric cylinders. If we assume also that in an arc plasma the distribution is parabolic, then

$$\sigma(r)=\sigma_0\left(1-\frac{r^2}{R^2}\right)\quad\text{and}\quad a=\frac{\int_0^R r^3\left(1-\frac{r^2}{R^2}\right)dr}{\int_0^R r\left(1-\frac{r^2}{R^2}\right)dr}=\frac{R^2}{3}=0{\cdot}187\ \mathrm{cm}^2$$

The value of a thus does not correspond to experimentally observed values but it can be noted that there is a tendency to approach this value at lower arc currents.

Next we turn our attention to eqn. (6) and obtain the values of n for different values of a corresponding to the different values of the parameter I/E as entered in Table 1, and the values of n thus obtained are entered in Table 2 for corresponding values of I/E.

I/E (amps-cm/volts)	n
3·36	2·293
6·563	3·859
9·408	3·984
10·59	4·181

Table 2. Variation of n with I/E.

To obtain the nature of the distribution function $\sigma(r)$ from eqn. (5) the value of σ_0 has been calculated for different I/E values from eqn. (3) :

$$\int_0^R \sigma_0\left[1-\left(\frac{r}{R}\right)^2\right]^n r\,dr=\frac{I}{2\pi E}$$

and after integration we get

$$\sigma_0 = \frac{I/E}{2\pi} \frac{2(n+1)}{R^2}$$

The distribution function represented by eqn. (5) can now be plotted for different I/E values (Fig. 1).

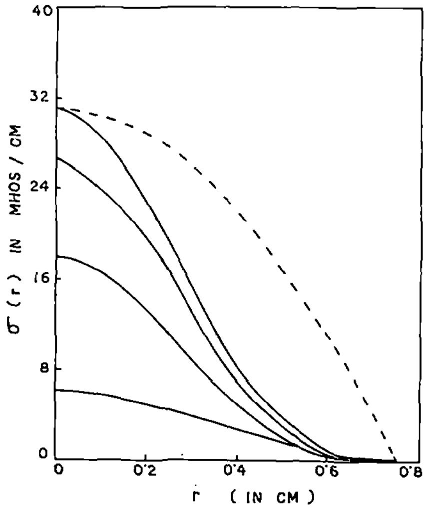

Figure 1. Electrical conductivity distribution as function of r for I/E values (amps-cm/volts) 10·59, 9·408, 6·563 and 3·36 respectively from above downwards (solid lines). Parabolic distribution (- - - - - -).

From the nature of the curves it is evident that not only the conductivity at the axis shows a rapid increase with the increase of the arc current but at the same time the nature of the distribution of the azimuthal conductivity undergoes a remarkable change, which is evident from the nature of the curves, indicating that the discharge becomes more and more constricted with the increase of the arc current. The half-widths of the above curves can be regarded as a measure of the constriction of the plasma column at the axis and the variation of half-widths with I/E is shown in Fig. 2. There is a rapid fall as the arc current is changed from 2·3 amp to 3·1 amp and then the change is slower, and the curve shows a tendency to saturation towards higher currents.

It has been noted that in a mercury vapour tube the arc completely fills the tube for low currents but as the current is increased the arc column contracts and the light becomes more intense at the axis of the tube, which is also corroborated by the present investigation.

The distribution curves obtained here closely resemble the curves obtained by Hoyaux (1968) for a low pressure arc where the magnetic self-constriction is predominant, but the constriction observed in the present investigation cannot be due to the magnetic self-constriction, as the order of the current is much smaller.

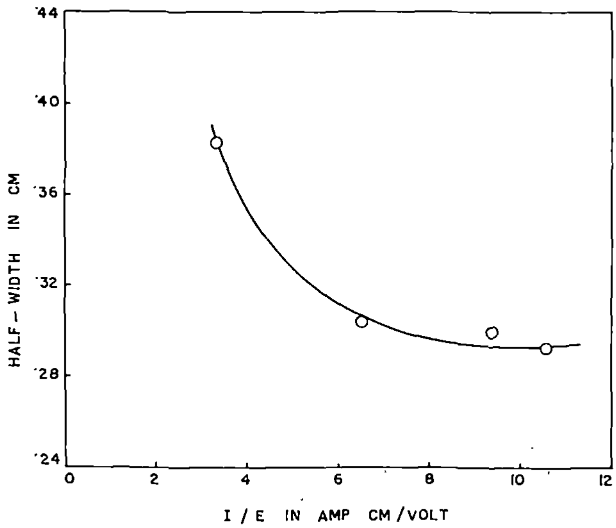

Figure 2. Variation of half-widths of the conductivity distribution function with I/E.

The increase of constriction of the plasma column at higher currents is probably due to the fact that the increased energy input causes an increase in gas temperature at the axis, thereby lowering the gas density. Consequently the increased mean free path facilitates the ionization probability, causing a higher charge density at the axis. If however the gas density becomes too low an opposing effect may appear. Due to reduction of the gas density the total ionization collision of neutral particles with electrons will be lowered, thereby decreasing the ionization probability. The saturation observed in the present case may be partly due to this effect.

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