CHAPTER - VIII

ATTENUATION OF ACOUSTIC WAVES THROUGH REFLECTIONS AT THE PLASMA NEUTRAL GAS INTERFACES;WEAKLY IONIZED CASE.

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INTRODUCTION

It is well-known that when a progressive plane acoustic wave travels in a medium, reflection occurs if there is any discontinuity in the characteristic impedance of the medium which is defined as the product of the acoustic wave phase velocity V_p and density ρ of the medium. Several authors (Bhatnagar 1964; Gaur & Saxena 1970; and Saxena & Gaur 1969) have suggested that this idea may be utilised as a plasma diagnostic technique. The experimental technique and the results have been reported by Saxena and Gaur (1969), (Gaur & Saxena 1970 and Saxena & Saxena, 1974). In their experiment, transmission of a plane acoustic wave from air into lossless plasma and into air again has been studied. Due to reflections at the boundaries the transmitted waves were found to be attenuated. The dependence of attenuation on the phase velocity of acoustic wave in a fully ionised plasma can be obtained theoretically and as V_p, the phase velocity is $\left[\gamma k\left(T_e+T_i\right)/m\right]^{1/2}$ (Surdin 1962 a, b; and Bhatnagar et al, 1971), the electron temperature can be calculated by assuming $T = T_1$, where T is the gas temperature from the measured values of attenuation.

Bhatnagar (1964) has also proposed a method by which ion and electron temperature can be obtained by studying reflections at plasma air interfaces at frequencies below and above the electron plasma frequency.

In all the above theoretical descriptions the case of fully ionized plasma has been considered. But if the ionization is weak as in the case in ordinary laboratory plasma the theory should be modified by two major factors : (a) due to the presence of large neutral background the expression of the phase velocity of ion wave obtained earlier will be modified; and (b) since the elasticity of the ion species greatly differs from that of the neutral particle species, the wave in a weakly ionised plasma cannot be described by waves having a single propagation constant. The purpose of the present chapter ~~paper~~ is to obtain theoretically a relation between the incidence and transmitted pressure wave amplitudes considering the above two factors into account.

THEORY

Acoustic Wave Equation in a Weakly Ionized Gas - In the previous chapter (and also Ghosal & Sen 1976) the wave equation has been obtained in terms of the total macroscopic pressure perturbation of the charge particle species. In obtaining the equation it has been assumed that the neutral particles remain at rest and give rise only to an effective frictional force to the ion fluid. It can easily be seen that the waves are effectively carried by ions only. The phase velocity V_p of these waves (called the ion-acoustic waves) has been obtained as

$$V_p^2 = \frac{2\gamma \dfrac{k(T_i + T_e)}{m_i}}{1 + \sqrt{1 + \nu_{pa}^2/\omega^2}} \qquad \text{... (A)}$$

where ν_{pa} has been defined as the plasma-neutral collision frequency. When an acoustic wave impinges on a weakly ionized plasma it is expected that a portion of the perturbation will be taken up by the ions only giving rise to the so called ion acoustic waves and the rest will be taken up by the neutral particles only, which would give rise to an ordinary acoustic mode.

In those experiments (in connection with plasma diagnostics) where both the excitation and the reception of the acoustic wave are done by the transducers, the importance of the knowledge of the total pressure perturbation in the plasma becomes evident since there is no way to sort out the ion-acoustic mode from the total wave with the help of transducers.

In obtaining the wave equation in terms of the total pressure perturbation in a weakly ionised plasma the following major assumptions will be made :

(1) The plasma is composed of non-interacting ion and neutral particle fluid only.

(2) The presence of the electrostatic restoring forces on ion fluid and the effect of the interaction of ions and electrons with neutral particles will effectively augment the temperature of the ion fluids to a value given by

$$T_i' = \frac{2\left(T_i + T_e\right)}{1+\sqrt{1+\nu_{pa}^2/\omega^2}}$$

which is apparent from equation (A).

Let p_i, u_i and n_i represent the small deviation of fluid pressure, fluid velocity and particle density respectively from their respective quiescent values for ion fluids and the corresponding quantities with suffix 'a' represent the same for neutral particle fluid; the linearised hydrodynamic equation in terms of the perturbed variable for a driftless plasma in absence of electric and magnetic field can be written as,

'Equation of motion' i.e.,

$$m_i n_{io} \frac{\partial \vec{u}_i}{\partial t} = -\nabla p_i$$

$$m_a n_{ao} \frac{\partial \vec{u}_a}{\partial t} = -\nabla p_a$$

'Equation of continuity' i.e.,

$$n_{io}\, \nabla . \vec{u_i} = -\frac{\partial n_i}{\partial t} ;$$

$$n_{ao}\, \nabla . \vec{u_a} = -\frac{\partial n_a}{\partial t} ;$$

and

'Equation of state' i.e.,

$$p_i = \gamma_i n_i k T_i' ; \text{ and } p_a = \gamma\, n_a k T_a$$

where n_{io}, n_{ao}, m_i and m_a are the quiescent values of ion density, neutral particles density ion mass and neutral mass respectively. The following macroscopic variables will be defined as :-

(1) Total fluid momentum : $\rho_o \vec{u} = m_i n_{io} \vec{u_i} + m_a n_{ao} \vec{u_a}$

where $\rho_o = m_e n_{ao} + m_i n_{io}$

(2) Total fluid pressure : $p = p_i + p_a$

(3) Ion flux : $\vec{j_i} = n_{io} \vec{u_i}$

(4) Neutral particle flux : $\vec{j_a} = n_{ao} \vec{u_a}$

(5) Total fluid flux : $\vec{j} = \vec{j_i} + \vec{j_a}$

In terms of the new variables $\vec{u}$ and $\vec{j}$ and using all the above equations one can arrive at the following equations :

$$\rho_o \frac{\partial \vec{u}}{\partial t} = -\nabla p \qquad \ldots (8.1)$$

$$\frac{\partial \vec{j}}{\partial t} = -\frac{1}{m}\left(\nabla p_a - \nabla p_i\right) \qquad \ldots (8.2)$$

$$\rho_0 \nabla\cdot\vec{u} = -\frac{1}{V_i^2}\frac{\partial p_i}{\partial t} + \frac{1}{V_a^2}\frac{\partial p_a}{\partial t} \qquad \ldots (8.3)$$

$$\nabla\cdot\vec{J} = \frac{1}{m}\left(\frac{1}{V^2}\frac{\partial p_a}{\partial t} - \frac{1}{V_i^2}\frac{\partial p_i}{\partial t}\right) \qquad \ldots (8.4)$$

$$p = p_i + p_a \qquad \ldots (8.5)$$

where, $V_i^2 = \gamma k T_i'/m = V_p^2$ and $V_a^2 = \gamma T_a/m$

and it is assumed that $m_i \simeq m_a \simeq m$

In terms of the above five equations (8.1 to 8.5) $\vec{u}$, $\vec{J}$, p_i and p_a may be eliminated and the wave equation in terms of the total pressure 'p' can be obtained as :

$$\frac{\partial^4 p}{\partial t^4} = \left(V_i^2 + V_a^2\right)\frac{\partial^2}{\partial t^2}\nabla^2 p - V_i^2 V_a^2 \nabla^2 p \qquad \ldots (8.6)$$

Assuming $p = e^{i(\omega t - \chi x)}$ be a solution of the wave equation (equation 8.6) one gets the required dispersion relation between the propagation constant χ and the excitation frequency ω as :

$$V_i^2 V_a^2 \chi^4 - \left(V_i^2 + V_a^2\right)\omega^2\chi^2 + \omega^4 = 0$$

or

$$\left(\chi^2 - \omega^2/V_i^2\right)\left(\chi^2 - \omega^2/V_a^2\right) = 0$$

which shows that χ^2 has two roots given by,

$$\chi_i^2 = \omega^2/V_i^2$$

and

$$\chi_a^2 = \omega^2/V_a^2$$

Thus the solution of equation (8.6), representing a wave propagating in the positive direction for a particular excitation frequency, is given by

$$p = p_{io}\, e^{i(\omega t - \chi_i x)} + p_{ao}\, e^{i(\omega t - \chi_a x)} \qquad \text{... (8.7)}$$

The phase velocities corresponding to χ_i and χ_a are given by

$$V_{pi}^2 = \omega^2/\chi_i^2 = V_i^2 = \frac{\gamma k T'}{m} \quad \text{and} \quad V_{pa}^2 = \frac{\omega^2}{\chi_a^2} = V_a^2 = \frac{\gamma k T}{m}$$

The solution consists of two modes of waves; one is carried by ion fluid and the other is carried by the neutral particle fluid. The solution of p can be viewed as if it represents the sum of partial pressure of ion fluid wave and neutral particle wave if they were considered separately.

REFLECTION AT THE INTERFACES

Making use of the above idea we shall now consider the transmission of a plane acoustic wave from one medium through a second into a third medium, where the initial and final media are the same (neutral gas medium) and the middle medium is a weakly ionized plasma medium through which two modes of acoustic waves can propagate as shown in the preceding section.

An experimental arrangement which would simulate the above situation is shown in Fig. 8.1. Let MM' and NN' represent two sharp boundaries (Fig. 8.2) which separate medium (2) from medium (1) and (3) and let medium (2) consist of two fluids namely fluid 'a' and fluid 'b'. We define the following quantities :

$(p_i)_1 = A_1 e^{i(\omega t - \chi_1 x)}$

and $(p_r)_2 = \bar{B}_1 e^{i(\omega t - \chi_1 x)}$ representing incident and reflected wave in medium (1).

$(p_t)_{2a} = \bar{A}_{2a} e^{i(\omega t - \chi_a x)}$

and $(p_r)_{2a} = \bar{B}_{2a} e^{i(\omega t - \chi_a x)}$ representing transmitted and reflected wave in fluid 'a' of medium (2).

$(p_t)_{2b} = \bar{A}_{2b} e^{i(\omega t - \chi_b x)}$

and $(p_r)_{2b} = \bar{B}_{2b} e^{i(\omega t - \chi_b x)}$ representing transmitted and reflected wave in fluid 'b' of medium (2).

$(p_t)_3 = \bar{A}_3 e^{i[\omega t - \chi_3(x-l)]}$ representing the transmitted wave in medium (3).,

where

ρ_1 = density of medium (1) and (3)

ρ_a = density of fluid 'a'

ρ_b = density of fluid 'b'

C_1 = phase velocity of acoustic wave in media (1) and (3)

C_a = phase velocity of acoustic wave in fluid 'a'

C_b = phase velocity of acoustic wave in fluid 'b'

ℓ = distance between the two boundary planes.

The pressure amplitude $\bar{B}_1$, $\bar{A}_{2a}$, $\bar{B}_{2a}$, $\bar{A}_{2b}$, $\bar{B}_{2b}$ and A_3 are in general complex, which take into account any change of phase due to reflection and transmission. To obtain a relation between A_1 and $\bar{A}_3$ we shall use the following boundary conditions (Kinsler & Frey 1959):

1) acoustic pressures on two sides of any boundary are equal; and

2) the particle velocities normal to the interface are equal.

Assuming that the first boundary plane is situated at X = 0, the boundary condition of continuity of pressure at X = 0

$$A_1 + \bar{B}_1 = \bar{A}_{2a} + \bar{A}_{2b} + \bar{B}_{2a} + \bar{B}_{2b} \quad \ldots (8.8)$$

continuity conditions of particle velocity give

$$\frac{A_1 - \bar{B}_1}{\rho_1 c_1} = \frac{\bar{A}_{2a} - \bar{B}_{2b}}{\rho_a C_a} = \frac{A_{2b} - B_{2b}}{\rho_b - C_b} \quad \ldots (8.9)$$

Similarly at $x = \ell$, the conditions of continuity of pressure give

$$\bar{A}_{2a} e^{-i\chi_a \ell} + \bar{B}_{2a} e^{i\chi_a \ell} + \bar{A}_{2b} e^{i\chi_b \ell} + \bar{B}_{2b} e^{i\chi_b \ell} = \bar{A}_3 \quad \ldots (8.10)$$

Continuity of particle velocity gives

$$\frac{\bar{A}_3}{\rho_1 C_1} = \frac{A_{2a} e^{-i\chi_a l} - \bar{B}_{2a} e^{i\chi_a l}}{\rho_a C_a} = \frac{\bar{A}_{2b} e^{-i\chi_b l} - \bar{B}_{2b} e^{i\chi l}}{\rho_b C_b} \quad \ldots (8.11)$$

The above six relations (equations 8.8 to 8.11) can be written in a matrix form as

$$\begin{bmatrix} 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -Z_{1a} & 0 & Z_{1a} & 0 \\ 0 & 0 & 1 & -Z_{ab} & -1 & Z_{ab} \\ 0 & 0 & \bar{a} & \bar{b} & a & b \\ 0 & 0 & Z_{1a}\bar{a} & 0 & 1-Z_{1a}a & 0 \\ 0 & 0 & \bar{a} & -Z_{ab}\bar{b} & -a & Z_{ab}\bar{b} \end{bmatrix} \begin{bmatrix} A_1 \\ \bar{B}_1 \\ \bar{A}_{2a} \\ \bar{A}_{2b} \\ \bar{B}_{2a} \\ \bar{B}_{2b} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \bar{A}_3 \\ \bar{A}_3 \\ 0 \end{bmatrix} \quad \ldots (8.12)$$

where $Z_{1a} = \rho_1 C_1 / \rho_a C_a$

and $Z_{ab} = \rho_a C_a / \rho_b C_b$

and $e^{i\chi_a l} = a$ and $a\,\bar{a} = 1$

$e^{i\chi_b l} = b$ and $b\,\bar{b} = 1.$

From equation (8.12) $\bar{B}_1$, $\bar{A}_{2a}$, $\bar{A}_{2b}$, $\bar{B}_{2a}$ and $\bar{B}_{2b}$ can be eliminated using Cramer's rule and the relation between A_1 and $\bar{A}_3$ is obtained as :

$$\bar{A}_3 = \frac{\begin{vmatrix} \bar{a}+a & b+\bar{b} & aZ_{ab}+b \\ (\bar{a}-a)Z_{1a} & 0 & -Z_{1a}Z_{ab}+a \\ (\bar{a}-a) & -Z_{ab}(\bar{b}-b) & Z_{ab}(b-a) \end{vmatrix}}{\begin{vmatrix} -2 & -1 & -Z_{ab}(1-Z_{1a})+1 \\ (1+Z_{1a})(\bar{a}+a) & (\bar{b}+b) & aZ_{ab}(1+Z_{1a})+b \\ (\bar{a}-a) & Z_{ab}(\bar{b}-b) & Z_{ab}(b-a) \end{vmatrix}} A_1$$

$$= S A_1 \quad \text{(say.)} \qquad \text{---- (8.13)}$$

The relation (equation 8.13) contains the information about the phase and absolute amplitude of the transmitted wave in terms of A_1.

The absolute amplitude is given by,

$$|\bar{A}_3|^2 = S S^* |A_1|^2$$

where S^* is complex conjugate to S.

DISCUSSION

(a) Measurement of Electron Temperature - If we consider that the fluid 'a' represents the background neutral particle fluid and the fluid 'b' represents the charge particle fluids or simply ion fluids, we may write $\rho_0 = \rho_i = \alpha \rho_1$ where a x 100 is the percentage of ionization and

$$\rho_a = (1-\alpha)\rho_i \; ; \quad C_1 = V_a = \sqrt{\frac{\gamma k T}{m}} \; ; \; C_b = V_i = V_b = \sqrt{\frac{\gamma k T_i'}{m}}$$

$$Z_{1a} = \frac{1}{1-\alpha} \; ; \quad Z_{ab} = \frac{1-\alpha}{\alpha}\sqrt{\frac{T_i}{T_i'}}$$

and $\chi_b = \chi_i = \frac{\omega}{V_i}$ and $\chi_a = \omega/V_a$

Thus it is seen that the coefficient S of equation (8.13) contains the parameters, α, T_i', T and the excitation frequency. Thus measuring the attenuation $\frac{A_1 - |A_3|}{A_1}$ and α and the gas temperatures T, T_i' can be calculated. If the gas is too rarified so that $\nu_{pa}^2 \ll \omega^2$. and T_i' is simply given by T_e. Thus for a rarefied gas the process of measurement of the electron temperature seems to be straightforward, but in general we have to take an account of the effect of ν_{pa}. ν_{pa} is given by (Ghosal & Sen 1976) $\nu_{pa} = \nu_{ia} + \frac{m_e}{m_i}\nu_{ea}$ where ν_{ea} and ν_{ia} are the effective electron neutral and ion-neutral collision frequencies. In terms of mean free paths, ν_{pa} is given by

$$\nu_{pa} = \frac{m_e}{m_i}\sqrt{\frac{3kT_e}{m_e}}\left(\frac{1}{\lambda_{ea}}\right) + \sqrt{\frac{3kT}{m}}\left(\frac{1}{\lambda_{ia}}\right)$$

If the collision cross-sections are assumed independent of particle energies and are known beforehand, the mean free paths λ_{ea} and λ_{ia} can be calculated in terms of gas pressure and collision cross-sections. Thus, from the knowledge of T_i', T_e can be determined. In any case however, it is necessary to know the percentage of ionization (α X 100%).

It is now worthwhile to mention at this stage that for ordinary laboratory glow discharge plasma the ionization is normally so weak that the major part of the acoustic power is drawn by the neutral particles and according to a numerical estimation it is observed that the amount of attenuation due to reflection of the sonic wave at the interfaces should be very small for very weak ionization. The present author believes that attenuation observed by other authors mentioned earlier should be attributed to the effect of energy transfer from electrons to neutral particles as observed by Ishida and Idehara (1973) whose investigations have been discussed in Chapter I.

(b) Oscillatory Nature of the Attenuation - Since the coefficient contains the terms like $a = e^{i\chi_a l}$ and $b = e^{i\chi_e l}$ the attenuations as in the fully ionised case are expected to be an oscillatory function of ω and l, the length of the plasma slab. This has been experimentally verified by Gaur and Saxena (1970) and explained considering the plasma to be fully ionised. In the latter case the oscillatory nature can be explained in terms of the interference of the incident and reflected waves, but in the weakly ionised case another interference effect resulting from the propagation of two modes of waves within plasma (as explained earlier) also takes place.

(c) Effect of Attenuation within the Plasma Medium - In the present analysis the attenuation of acoustic waves within the plasma itself has been neglected because, the coefficient of attenuation as calculated in ~~the~~ a previous paper (Ghosal & Sen, 1976) shows that it will be extremely small since in most cases the thickness of the plasma column is very small compared to the wavelength of the incident sonic wave.

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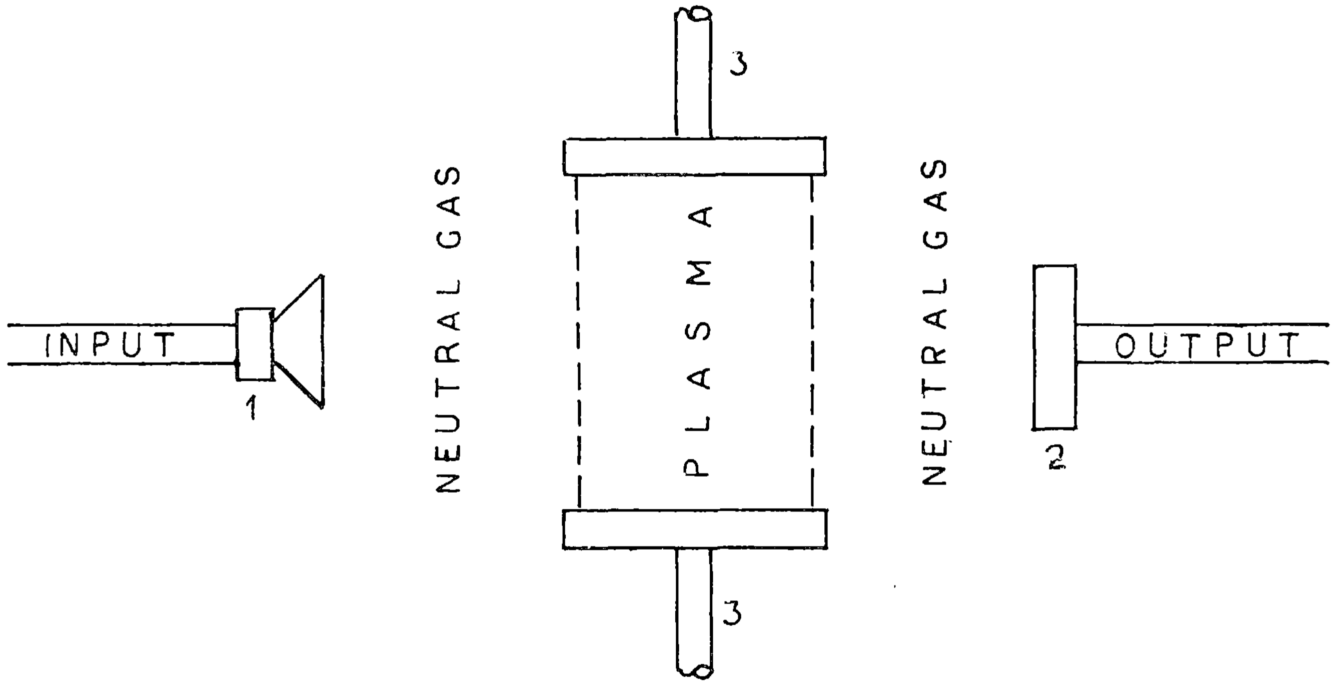

1. SONIC TRANSDUCER FOR EXCITATION
2. SONIC TRANSDUCER FOR RECEPTION
3. PLATE ELECTRODES FOR PLASMA PRODUCTION

Fig. 8.1. Schematic experimental arrangement partaining to theoretical model for the propagation of sonic waves through partially ionised gas.

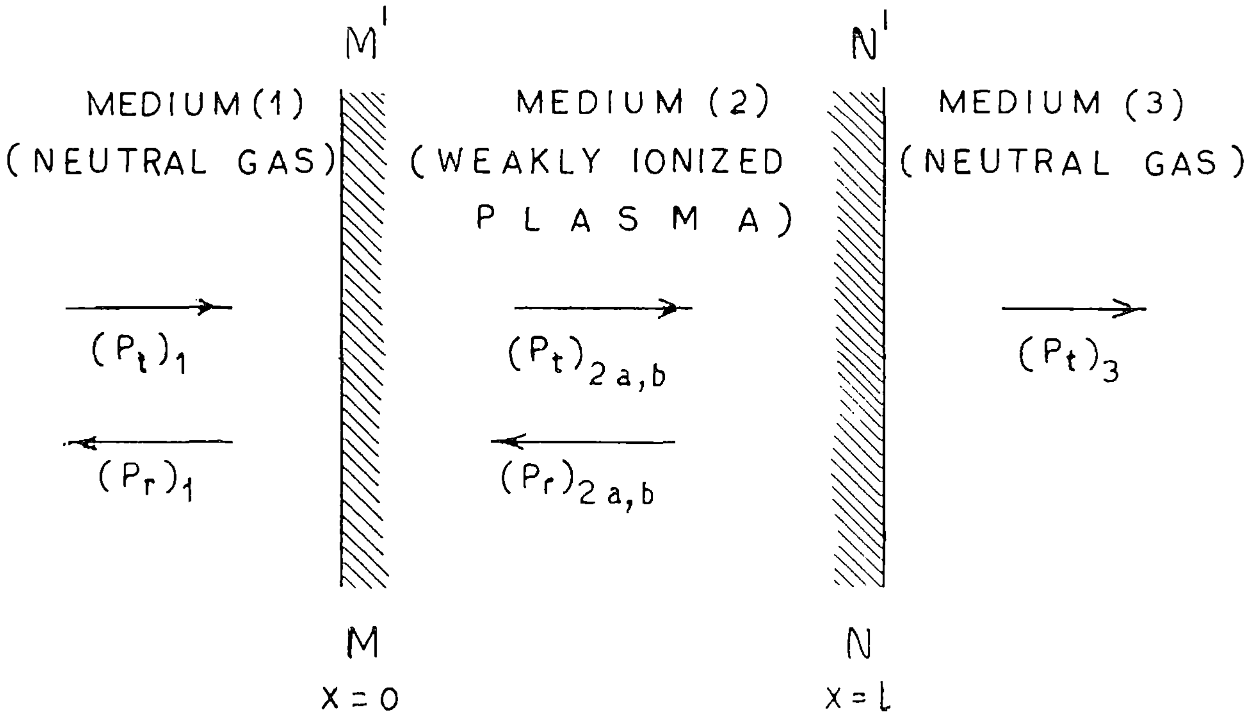

Fig. 8.2. Diagram showing pressure perturbation in incident, reflected and transmitted sonic wave.

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ATTENUATION OF ACOUSTIC WAVES THROUGH REFLECTIONS AT THE PLASMA NEUTRAL GAS INTERFACES : WEAKLY IONIZED CASE

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The problem of transmission of sonic waves through a weakly ionised plasma bounded in each side by a neutral gas medium has been treated by assuming the plasma to be a mixture of two intermingled fluids viz., neutral particle fluid and ion fluid in equilibrium. From a hydrodynamic analysis the wave equation for 'p', the macroscopic pressure perturbation has been obtained and it is shown that two independent wave motions, one due to the neutral particles and the other due to ions are propagated through the plasma with two different phase velocities. Assuming the usual boundary conditions at the interface, the amplitude of the transmitted wave has been calculated in case of weakly ionized plasma; the theory can be utilized for the determination of electron temperature from the measured value of attenuation if the percentage of ionization and collision cross section can be obtained independently.

INTRODUCTION

It is well-known that when a progressive plane acoustic wave travels in a medium, reflection occurs if there is any discontinuity in the characteristic impedance of the medium which is defined as the product of the acoustic wave phase velocity V_p and density ρ of the medium. Several authors (Bhatnagar 1964; Gaur & Saxena 1970; and Saxena & Gaur 1969) have suggested that this idea may be utilised as a plasma diagnostic tech-nique. The experimental technique and the results have been reported by Saxena and Gaur (1969), and Gaur & Saxena 1970). In their experiment, transmission of a plane acoustic wave from air into lossless plasma and into air again has been studied. Due to reflections at the boundaries the transmitted waves were found to be attenuated. The dependence of attenuation on the phase velocity of acoustic wave in a fully ionised plasma can be obtained theoretically and as V_p, the phase velocity is $\sqrt{\frac{\gamma k(T_e+T_i)}{m}}$ (Surdin 1962 *a, b*; and Bhatnagar *et al.* 1971), the electron temperature can be calculated by assuming $T = T_i$, where T is the gas temperature from the measured values of attenuation.

Bhatnagar (1964) has also proposed a method by which ion and electron temperature can be obtained by studying reflections at plasma air interfaces at frequencies below and above the electron plasma frequency.

In all the above theoretical descriptions the case of fully ionized plasma has been considered. But if the ionization is weak as in the case in ordinary laboratory plasma the theory should be modified by two major factors : (a) due to the presence of large neutral background the expression of the phase velocity of ion wave obtained earlier will be modified; and (b) since the elasticity of the ion species greatly differs from that of the neutral particle species, the wave in a weakly ionised plasma cannot be described by waves having a single propagation constant. The purpose of the present paper is to obtain theoretically a relation between the incidence and transmitted pressure wave amplitudes considering the above two factors into account.

THEORY

Acoustic Wave Equation in a Weakly Ionized Gas — In the previous paper (Ghosal & Sen 1976) the wave equation has been obtained in terms of the total macroscopic pressure perturbation of the charge particle species. In obtaining the equation it has been assumed that the neutral particles remain at rest and give rise only to an effective frictional force to the ion fluid. It can easily be seen that the waves are effectively carried by ions only. The phase velocity V_p of these waves (called the ion-acoustic waves) has been obtained as

$$V_p^2 = \frac{2\gamma \dfrac{k(T_i+T_e)}{m_i}}{1+\sqrt{1+\nu_{pa}^2/\omega^2}} \qquad \ldots \text{(A)}$$

where ν_{pa} has been defined as the plasma-neutral collision frequency. When an acoustic wave impinges on a weakly ionised plasma it is expected that a portion of the perturbation will be taken up by the ions only giving rise to the so called ion acoustic waves and the rest will be taken up by the neutral particles only, which would give rise to an ordinary acoustic mode.

In those experiments (in connection with plasma diagnostics) where both the excitation and the reception of the acoustic wave are done by the transducers, the importance of the knowledge of the total pressure perturbation in the plasma becomes evident since there is no way to sort out the ion-acoustic mode from the total wave with the help of transducers.

In obtaining the wave equation in terms of the total pressure perturbation in a weakly ionised plasma the following major assumptions will be made :

(1) The plasma is composed of non-interacting ion and neutral particle fluid only.

(2) The presence of the electrostatic restoring forces on ion fluid and the effect of the interaction of ions and electrons with neutral particles will effectively augment the temperature of the ion fluids to a value given by

$$T_i' = \frac{2\,(T_i+T_e)}{1+\sqrt{1+\nu_{pa}^2/\omega^2}}$$

which is apparent from Eq. (A).

Let p_i, u_i and n_i represent the small deviation of fluid pressure, fluid velocity and particle density respectively from their respective quiescent values for ion fluids and the corresponding quantities with suffix 'a' represent the same for neutral particle fluid ; the linearised hydrodynamic equation in terms of the perturbed variable for a driftless plasma in absence of electric and magnetic field can be written as,

'*Equation of motion*' i.e.,

$$m_i n_{i0} \frac{\partial \vec{u}_i}{\partial t} = -\nabla p_i \quad ; \qquad \ldots (i)$$

$$m_a n_{a0} \frac{\partial \vec{u}_a}{\partial t} = -\nabla p_a \quad ; \qquad \ldots (ii)$$

'*Equation of continuity*' i.e.,

$$n_{i0} \nabla \cdot \vec{u}_i = -\frac{\partial n_i}{\partial t} \; ; \qquad \ldots (iii)$$

$$n_{a0} \nabla \cdot \vec{u}_a = -\frac{\partial n_a}{\partial t} \; ; \text{ and} \qquad \ldots (iv)$$

'*Equation of state*' i. e.,

$$p_i = \gamma_i n_i k T_i' ; \text{ and } p_a = \gamma n_a k T_a,$$

where n_{i0}, n_{a0}, m_i and m_a are the quiescent values of ion density, neutral particles density, ion mass and neutral mass respectively. The following macroscopic variables will be defined as :—

(1) Total fluid momentum : $\rho_0 \vec{u} = m_i n_{i0} \vec{u}_i + m_a n_{a0} \vec{u}_a,$ ... (v)

where $\rho_0 = m_a n_{a0} + m_i n_{i0}$ ... (vi)

(2) Total fluid pressure : $p = p_i + p_a$ ... (vii)

(3) Ion flux : $\vec{j}_i = n_{i0} \vec{u}_i$

(4) Neutral particle flux : $\vec{j}_a = {}^{-}n_{a0} \vec{u}_a$

(5) Total fluid flux : $\vec{j} = \vec{j}_i + \vec{j}_a$

In terms of the new variables $\vec{u}$ and $\vec{j}$ and using all the above equations one can arrive at the following equations :

$$\rho_0 \frac{\partial \vec{u}}{\partial t} = -\nabla p \qquad \ldots (1)$$

$$\frac{\partial \vec{j}}{\partial t} = -\frac{1}{m} (\nabla p_a - \nabla p_i) \qquad \ldots (2)$$

$$\rho_0 \nabla \cdot \vec{u} = -\frac{1}{V_i^2} \frac{\partial p_i}{\partial t} + \frac{1}{V_a^2} \frac{\partial p_a}{\partial t} \qquad \ldots (3)$$

$$\nabla \cdot \vec{j} = \frac{1}{m} \left(\frac{1}{V_a^2} \frac{\partial p_a}{\partial t} - \frac{1}{V_i^2} \frac{\partial p_i}{\partial t} \right) \qquad \ldots (4)$$

$$p = p_i + p_a, \qquad \dots (5)$$

where, $V_i^2 = \gamma\, k\, T_i'/m = V_p^2$ and $V_a^2 = \gamma\, k\, T_a/m$

and it is assumed that $m_i \approx m_a \approx m$

In terms of the above five equations [(1) to (5)] $\vec{u}$, $\vec{j}$, p_i and p_a may be eliminated and the wave equation in terms of the total pressure 'p' can be obtained as :

$$\frac{\partial^4 p}{\partial t^4} = \left(V_i^2 + V_a^2 \right) \frac{\partial^2}{\partial t^2} \nabla^2 p - V_i^2\, V_a^2\, \nabla^2 p \qquad \dots (6)$$

Assuming $p = e^{i(\omega t - \chi x)}$ be a solution of the wave equation (Eq. 6) one gets the required dispersion relation between the propagation constant χ and the excitation frequency ω as :

$$V_i^2 V_a^2\, \chi^4 - \left(V_i^2 + V_a^2 \right) \omega^2 \chi^2 + \omega^4 = 0,$$

or

$$\left(\chi^2 - \omega^2/V_i^2 \right)\left(\chi^2 - \omega^2/V_a^2 \right) = 0,$$

which shows that χ^2 has two roots given by,

$$\chi_i^2 = \omega^2/V_i^2$$

and

$$\chi_a^2 = \omega^2/V_a^2 .$$

Thus the solution of Eq. (6), representing a wave propagating in the positive direction for a particular excitation frequency, is given by

$$p = p_{i0}\, e^{i(\omega t - \chi_i x)} + p_{a0}\, e^{i(\omega t - \chi_a x)}. \qquad \dots (7)$$

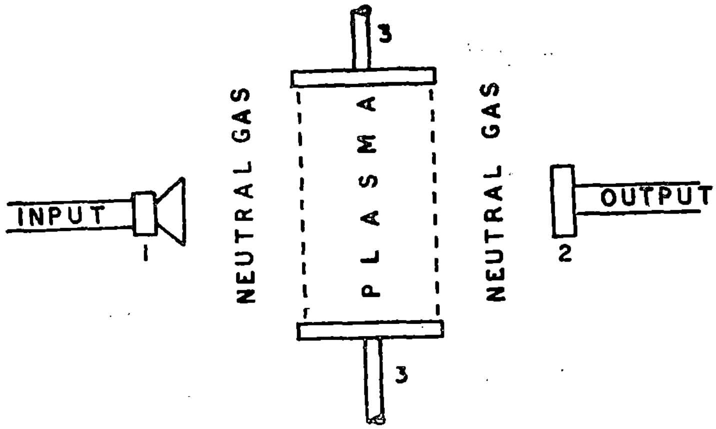
The phase velocities corresponding to χ_i and χ_a are given by

$$V_{pi}^2 = \omega^2/\chi_i^2 = V_i^2 = \frac{\gamma K T'}{m} \text{ and } V_{pa}^2 = \frac{\omega^2}{\chi_a^2} = V_a^2 = \frac{\gamma K T}{m} .$$

The solution consists of two modes of waves; one is carried by ion fluid and the other is carried by the neutral particle fluid. The solution of p can be viewed as if it represents the sum of partial pressure of ion fluid wave and neutral particle wave if they were considered seperately.

Reflection at the Interfaces

Making use of the above idea we shall now consider the transmission of a plane acoustic wave from one medium through a second into a third medium, where the initial and final medium are the same (neutral gas medium) and the middle medium is a weakly ionized plasma medium through which two modes of acoustic waves can propagate as shown in the preceding section.



1. SONIC TRANSDUCER FOR EXCITATION
2. SONIC „ „ RECEPTION
3. PLATE ELECTRODES FOR PLASMA PRODUCTION.

FIG. 1. Schematic experimental arrangement partaining to theoretical model for the propagation of sonic waves through partially ionised gas.

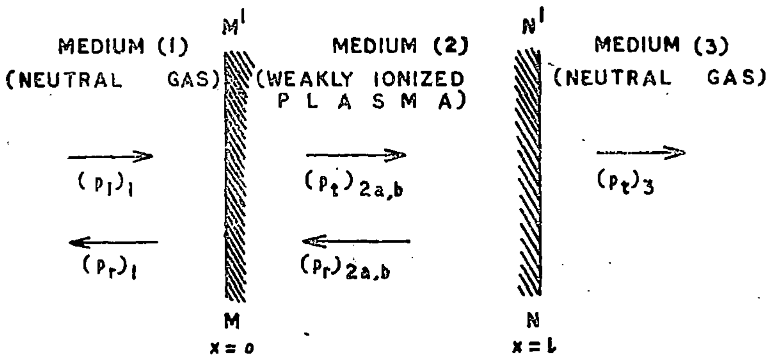


FIG 2. Diagram showing pressure perturbation in incident, reflected and transmitted sonic wave.

An experimental arrangement which would simulate the above situation is shown in Fig. 1. Let MM′ and NN′ represent two sharp boundaries (Fig. 2) which separate medium (2) from medium (1) and (3) and let medium (2) consist of two fluids namely fluid 'a' and fluid 'b'. We define the following quantities :

$(p_i)_1 = A_1\, e^{i(\omega t - \chi_1 x)}$ representing incident and reflected

and $(p_r)_2 = \overline{B_1}\, e^{i(\omega t + \chi_1 x)}$ wave in medium (1).

$(p_t)_{2a} = \overline{A}_{2a}\, e^{i(\omega t - \chi_a x)}$ representing transmitted and

and $(p_r)_{2a} = \overline{B}_{2a}\, e^{i(\omega t + \chi_a x)}$ reflected wave in fluid 'a' of medium (2).

$(p_t)_{2b} = \bar{A}_{2b}\, e^{i(\omega t - \chi_b x)}$ representing transmitted and

and $(p_r)_{2b} = \bar{B}_{2b}\, e^{i(\omega t + \chi_b x)}$ reflected wave in fluid '*b*' of medium (2).

$(p_t)_3 = \bar{A}_3\, e^{i[\omega t - \chi_3 (x - l)]}$ representing the transmitted wave in medium (3).,

where

ρ_1 = density of medium (1) and (3)

ρ_a = density of fluid '*a*'

ρ_b = density of fluid '*b*'

C_1 = phase velocity of accoustic wave in media (1) and (3)

C_a = phase velocity of acoustic wave in fluid '*a*'

C_b = phase velocity of acoustic wave in fluid '*b*'

l = distance between the two boundary planes.

The pressure amplitude $\bar{B}_1$, $\bar{A}_{2a}$, $\bar{B}_{2a}$, $\bar{A}_{2b}$, $\bar{B}_{2b}$ and A_3 are in general complex, which take into account any change of phase due to reflection and transmission, to obtain a relation between A and A_3 we shall use the following boundary conditions (Kinsler & Frey 1959) :

1) acoustic pressures on two sides of any boundary are equal; and
2) the particle velocities normal to the interface are equal.

Assuming that the first boundary plane is situated at $X = 0$, the boundary condition of continuity of pressure at $X = 0$

$$A_1 + \bar{B}_1 = \bar{A}_{2a} + \bar{A}_{2b} + \bar{B}_{2a} + \bar{B}_{2b}$$

continuity conditions of particle velocity give

$$\frac{A_1 - \bar{B}_1}{\rho_1 C_1} = \frac{\bar{A}_{2a} - \bar{B}_{2b}}{\rho_a C_a} = \frac{A_{2b} - B_{2b}}{\rho_b C_b}.$$

Similarly at $x = l$, the conditions of continuity of pressure give

$$\bar{A}_{2a}\, e^{-i\chi_a l} + \bar{B}_{2a}\, e^{i\chi_a l} + \bar{A}_{2b}\, e^{-i\chi_b l} + \bar{B}_{2b}\, e^{i\chi_b l} = \bar{A}_3.$$

Continuity of particle velocity gives,

$$\frac{A_3}{\rho_1 C_1} = \frac{A_{2a}\, e^{-i\chi_a l} - \bar{B}_{2a}\, e^{i\chi_a l}}{\rho_a C_a} = \frac{\bar{A}_{2b}\, e^{-i\chi_b l} - \bar{B}_{2b}\, e^{i\chi_b l}}{\rho_b C_b}.$$

The above six relations can be written in a matrix form as

$$\begin{bmatrix} 1 & 1 & -1 & -1 & -1 & -1 \\ 1 & -1 & -Z_{1a} & 0 & Z_{1a} & 0 \\ 0 & 0 & 1 & -Z_{ab} & -1 & Z_{ab} \\ 0 & 0 & \bar{a} & \bar{b} & a & b \\ 0 & 0 & Z_{1a}\bar{a} & 0 & 1-1Z_{1a}a & 0 \\ 0 & 0 & \bar{a} & -Z_{ab}\bar{b} & -a & Z_{ab}\bar{b} \end{bmatrix} \begin{bmatrix} A_1 \\ \bar{B}_1 \\ \bar{A}_{2a} \\ \bar{A}_{2b} \\ \bar{B}_{2a} \\ \bar{B}_{2b} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \bar{A}_3 \\ \bar{A}_3 \\ 0 \end{bmatrix}$$

where $Z_{1a} = \rho_1 C_1/\rho_a C_a$

and $Z_{ab} = \rho_a C_a/\rho_b C_b$

and $e^{i\chi_a l} = a$ and $a\bar{a} = 1$

$e^{i\chi_b l} = b$ and $b\bar{b} = 1$.

From equation (8) $\bar{B}_1$, $\bar{A}_{2a}$, $\bar{A}_{2b}$, $\bar{B}_{2a}$ and $\bar{B}_{2b}$ can be eliminated using Cramer's rule and the relation between A_1 and A_3 is obtained as :

$$\bar{A}_3 = \frac{\begin{vmatrix} (\bar{a}+a) & (b+\bar{b}) & (aZ_{ab}+b) \\ (\bar{a}-a)Z_{1a} & 0 & -Z_{1a}Z_{ab}+a \\ (\bar{a}-a) & -Z_{ab}(\bar{b}-b) & Z_{ab}(b-a) \end{vmatrix}}{\begin{vmatrix} -2 & -1 & \{-Z_{ab}(1-Z_{1a})+1\} \\ (1+Z_{1a})(\bar{a}+a) & (\bar{b}+b) & aZ_{ab}(1+Z_{1a})+b \\ (\bar{a}-a) & Z_{ab}(\bar{b}-b) & Z_{ab}(b-a) \end{vmatrix}} A_1$$

$$= S\,A_1 \quad \text{(say.)} \qquad \ldots(9)$$

The relation (Eq. 9) contains the information about the phase and absolute amplitude of the transmitted wave in terms of A_1.
The absolute amplitude is given by,

$$|\bar{A}_3|^2 = S S^* |A_1|^2,$$

where S^* is complex conjugate to S.

Discussion

(a) *Measurement of Electron Temperature* — If we consider that the fluid '*a*' represents the background neutral particle fluid and the fluid '*b*' represents the charge particle fluids or simply ion fluids, we may write $\rho_b = \rho_i = \alpha\rho_1$ where $\alpha \times 100$ is the percentage of ionization and

$$\rho_a = (1-\alpha)\,\rho_1 \; ; \qquad c_1 = c_1 = V_a = \sqrt{\frac{\gamma k T}{m}} \; ; \quad c_b = V_i = V_b = \sqrt{\frac{\gamma k T_i}{m}} .$$

$$Z_{1a} = \frac{1}{1-\alpha} \; ; \quad Z_{ab} = \frac{(1-\alpha)}{\alpha} \sqrt{\frac{T}{T_i'}} \; ;$$

and $\chi_b = \chi_i \dfrac{\omega}{V_i}$ and $\chi_a = \omega / V_a$.

Thus it is seen that the coefficient S of Eq. (9) contains the parameters, α, T_i', T and the excitation frequency. Thus measuring the attenuation $\dfrac{A_1 - |\bar{A}_3|}{A_1}$ and α and the gas temperatures T, T_i' can be calculated. If the gas is too rarified so that $\nu_{pa}^2 \ll \omega^2$ and T_i' is simply given by T_e. Thus for a rarefied gas the process of measurement of the electron temperature seems to be straightforward, but in general we have to take an account of the effect of ν_{pa} ν_{pa} is given by (Ghosal & Sen 1976) $\nu_{pa} = \nu_{ea} + \nu_{ia}$, where ν_{ea} and ν_{ia} are the effective electron neutral and ion-neutral collision frequencies. In terms of mean free paths, ν_{pa} is given by

$$\nu_{pa} = \sqrt{\frac{3kT_e}{m_e}} \left(\frac{1}{\lambda_{ea}} \right) + \sqrt{\frac{3kT}{m}} \left(\frac{1}{\lambda_{ia}} \right).$$

If the collision cross sections are assumed independent of particle energies and are known before hand, the mean free paths λ_{ea} and λ_{ia} can be calculated in terms of gas pressure and collision cross sections. Thus, from the knowledge of T_i', T_e can be determined. In any case however, it is necessary to know the percentage of ionization ($\alpha \times 100\%$).

(b) *Oscillatory Nature of the Attenuation*—Since the coefficient contains the terms like $a = e^{i\chi_a l}$ and $b = e^{i\chi_i l}$, the attenuations as in the fully-ionised case are expected to be an oscillatory function of ω and l, the length of the plasma slab. This has been experimentally verified by Gaur and Saxena (1970) and explained considering the plasma to be fully ionised. In the latter case the oscillatory nature can be explained in terms of the interference of the incident and reflected waves, but in the weakly ionised case another interferance effect resulting from the propagation of two modes of waves within plasma (as explained earlier) also takes place.

(c) *Effect of Attenuation within the Plasma Medium* — In the present analysis the attenuation of acoustic waves within the plasma itself has been neglected because, the coefficient of attenuation as calculated in the previous paper (Ghosal & Sen

1976) shows that it will be extremely small since in most cases the thickness of the plasma column is very small compared to the wavelength of the incident sonic wave.

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