# THE INFLATIONARY MODELS OF THE EARLY UNIVERSE

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*TO*

*MY PARENTS*

*Preface*

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# CHAPTER I

## INTRODUCTION

## I.1 *INTRODUCTION :*

During the last two decades, the developments in the elementary Particle Physics and in Cosmology have become increasingly dependent on the interplay between the two subjects. The elementary particle physics has entered a very interesting stage with the development of unified theories.[1,2] The superstring theory[2] even raised the hope of unifying gravitational interactions along with the other interactions, viz. electro-weak and strong. However, this ultimate unification can occur only at an energy scale $M_p \sim 10^{19}$ GeV. The standard cosmological model suggests that such an energy scale could have occurred only in the very early universe, close to the Big Bang. The early universe, therefore, may be considered as a unique physical Laboratory for testing the new theories of particle interactions. Fortunately, there has also been considerable progress in the understanding of cosmological models in recent years. It has been realised[3] that the standard Big Bang model is fairly successful in explaining the $2.7^{\circ}$ K cosmic microwave background radiation (in short, MBR), the expansion of the universe (Hubble redshifts), primordial nucleosynthesis and the cosmic abundances of the light elements e.g. the weakly-bound deuteron and $^4$He ( about 23% by mass ). Nevertheless, the model could not be considered as an acceptable model of the early universe because it fails to explain some other observed features of the universe.

Some of these problems are :

**i) the horizon problem :** This is related to the fact that although different distant parts of the universe did not have the opportunity of making causal contact with each other in the past in this model, the present universe is highly isotropic and homogeneous, as may be guessed from the observed uniformity of MBR.

**ii) the flatness problem :** This is related to the observational fact that the present energy density of the universe is very close to the critical density ($\rho_c$), which would correspond to a spatially flat universe. Within the framework of the Big Bang cosmology this leads to a naturalness problem. Thus if the energy density of the universe close to the Planck time is slightly greater than $\rho_c$ say even by one part in $10^{-55}$, the universe would be closed, and would have collapsed millions of years ago. On the other hand, if the density is less than $\rho_c$ by one part in $10^{-55}$, the universe would be open and the present energy density of the universe would be very small. This fine-tuning of the initial conditions finds no explanation in the Big Bang cosmology. The solution of this problem possibly involves a mechanism that produces considerable entropy at a later stage of evolution.

**iii) the small scale inhomogeneity :** Although the universe is homogeneous on a large scale it is very inhomogeneous on the galactic scale and at lower scales. An explanation of the observed structures in the universe may be due to small density perturbations occurring in the very early universe. The origin of these perturbations and the nature of their spectrum seem to need a more sophisticated model.

In addition to these problems there are a few other problems which are in conflict with the concepts of theories of particle physics. It is, therefore, felt essential to look for cosmologies which can solve these basic problems. This led to various inflationary models.[4,5] Although it was Guth[4] who first emphasized the merits of an inflation, ideas similar or close to the inflationary models had been suggested by some other cosmologists.[6-10] Gliner[6] was the first to note that when all four eigenvalues of the stress tensor were equal, the cosmology corresponds to matter with the properties of a vacuum. Gliner and Dymnikova[8] later on showed that this vacuum dominated state makes a transition to a radiation dominated era. This process led to an enormous increase in the scale factor, an idea very similar to the present day inflationary model. However, at present it seems that the equation of state of the superdense baryonic matter taken by them is not appropriate. The possibility of an exponential expansion was also pointed out by Starobinsky[9] from a consideration of the conformal anomaly of the stress tensor, occurring in the semi-classical Einstein equation. Kirzhnits and Linde and Linde studied the vacuum phase transition in an abelian Higgs model.[10] Sato[11] studied the effects of a first order phase transition in the early universe with a motivation to build a consistent baryon number symmetric universe in which CP-violation occurs spontaneously in an exponentially expanding domain. It was, however, Guth[4] who explicitly emphasized the role of a phase transition in solving the cosmological problems.

In the inflationary models gravity is described classically while matter is treated in terms of quantum fields. The basic idea of an inflationary scenario is that in the evolution of the universe there is an epoch when the vacuum energy density dominates the energy density of the universe. During this epoch, the vacuum energy behaves as an effective positive cosmological constant allowing the universe to expand exponentially ( or quasi-exponentially ), $a(t) \sim a_o \exp \left[ \int H \, dt \right]$ where $H = \dot{a}/a$ is the Hubble parameter. This expansion in the scale factor allows a small causally coherent region to grow bigger than the observable universe, provided the exponent $\int H \, dt > 65$ is satisfied. This huge expansion in a very short time is called 'inflation'. A less rapid inflation may be obtained in some other models.[12-15] It has been shown that a non-exponential accelerated phase of expansion

$$\ddot{a}(t) > 0 \qquad (1.1)$$

where a dot denotes differentiation with respect to the proper time t, also leads to an inflation characterized by a period in which a(t) grows as $t^A$ with $A > 1$. An example of this type may be obtained by using an exponential type potential $V(\phi) = V_o \exp(-\lambda \phi)$ with $\lambda \gtrsim 0$, which occurs in the Salam-Sezgin model[16] of N = 2 Supergravity coupled to matter and in some Kaluza-Klein theories.[13] The causal structure of the space-time in all these inflationary models differs considerably from what one gets in the Big Bang model. Consequently, the horizon problem is resolved as the observable universe in the inflationary model had a smaller size compared to a causal horizon volume. In this scenario our observable universe today ( $\sim 10^{28}$ cm ) had a physical size of about 10 cm at the GUT era ( $\sim 10^{-35}$sec. ) which lies well within one of the smooth regions produced by the inflation. The model also helps us to solve the primordial monopole problem which was introduced in cosmology by unified theories. At the end of inflation the density of the primordial monopoles, if any, will be very small. The fresh production of monopoles during reheating will not occur if the reheating temperature is below the temperature of GUT symmetry breaking ( $T < 10^{14}$ GeV ). It may be added that an inflation also provides a mechanism for the origin of the small scale density inhomogeneities. The original fluctuations present in a comoving volume before inflation are expected to be smeared away by the expansion. But the quantum fluctuations generated during inflation may lead to the observed density perturbations.[17,18] The scale invariant Harrison-Zeldovich[19] spectrum emerges naturally within the exponential expansion phase because the de Sitter space is time translation invariant and quantum fluctuations in this space might act as initial seeds for the galaxy formation. However, such a scale invariant spectrum cannot be obtained in power law inflation[12] for a finite exponent ( A ) . The exponential expansion is, therefore, more attractive.

In the original model the idea of an exponential expansion in the early universe was utilized by Guth[4] to resolve the horizon and the flatness problems. However, the model was not successful as it did not permit a universe with sufficient inflation to return gracefully to the usual Friedman-Robertson-Walker universe. There were several other problems, e.g. the domain wall problem, the problem of generation of inhomogeneities which lead to galaxy formation, the problem of large scale inhomogeneities etc.. It was soon realised from the subsequent work of others that the defects in the model[4] (known as old inflation) could not be removed.[20-23] Fortunately, in 1982 Linde[24] and Albrecht and Steinhardt[25] independently suggested a new mechanism to overcome the problems of the old inflation. The model is known as 'new inflation', which will be briefly reviewed in the next section. Subsequently, a more attractive model was advocated by Linde[26] which has apparently eliminated the fine tuning of the potentials needed to get the new inflationary universe scenario. The modified model is known as chaotic inflation. In this scenario, the universe evolves out of a chaotic distribution of initial data. A simple version of this model may be realized by considering a chaotic distribution of the values of the scalar field $\phi$ ( inflaton ) at time $t \sim t_P = M_P^{-1}$, where $t_P$ is the Planck time and $M_P$, the Planck energy. However, Linde proposed the scenario with the assumption of an isotropic inflation of the early universe. The scenario can be realized even in an anisotropic universe as will be shown in chapter II. A scenario may be realized even with a wide variety of interaction potentials. However, the inflationary models in general need an inflaton field with the exception of Starobinsky model.[9] It may be noted that Starobinsky proposed the model long before the advantages of an inflation were fully realized. The motivation for the model was to remove the singularity problem occurring in the standard model. The de Sitter stage in this model is a self-consistent solution of the semi-classical Einstein equation. It is known that the vacuum expectation value of the stress tensor in curved space time develops divergences, which may be removed by adding curvature squared terms to the Einstein action. However, the regularization process[27-31] causes the trace of the stress tensor to develop a non-zero value leading to the well-known conformal anomaly. It is this anomaly which drives inflation in Starobinsky model. But, the original Starobinsky model[9] faces serious problems when confronted with observed limits of the MBR.[32] It was suggested[33,34] later that the inconsistencies in the model may be removed by adding a term $\alpha R^2$ to the Einstein action where $\alpha$ is very large. With this modification, the Starobinsky model reduces to the study of the field equations derived from a Lagrangian

$$L = R + \alpha R^2 \quad . \qquad (1.2)$$

This is the most general Lagrangian including quadratic terms in curvature in four dimensional conformally flat spaces. The cosmological models obtained in this framework have a number of attractive features. A theory with quadratic curvature terms can be made perturbatively renormalizable[35] and asymptotically free[36,37]. The unitarity issue of the theory is, however, yet to be settled. Nevertheless, the higher derivative theory may be considered as an interesting model for testing various ideas related to the quantum properties of gravity.

Considerable work has been done during the last few decades to formulate a quantum theory of gravity.[38] The investigation seems to lead some people to believe that a consistent theory of quantum gravity cannot be obtained within the framework of point-field theories. The advent of the string theory has opened up new and interesting possibilities in this context. String theories[2] may be looked upon as describing the interactions of a few massless and an infinite set of massive states, with masses which are multiples of the Planck mass. The standared low energy physics is now described by the interactions of the massless modes after one integrates over the massive ones. The supergravity theories may be regarded as the low energy regime of superstring theories. The striking discovery[39] that in ten dimensions, a supergravity theory coupled to Yang-Mills fields with a gauge group SO(32) or $E_8 \times E_8'$ is anomaly free had inspired considerable activities in this area. Although the expected breakthrough had not yet come, the world wide hectic activities have served to focus on a number of issues which need further investigation. Since the quantum consistency of the superstring theory is obtained in the critical dimension D = 10, one has to look for a realistic compactification scheme. Candelas *et al*[40] set out to achieve this by requiring that the ten dimensions should compactify to $M^4 \times K$, where $M^4$ is maximally symmetric and K is a compact six dimensional manifold. It was also demanded that the four dimensional theory should have an unbroken N = 1 supersymmetry, so that the hierarchy problem can be tackled. Candelas *et al* obtained a solution with a Minkowskian ($\Lambda$ = 0) space for $M^4$ and a Ricci flat Calabi-Yau manifold for K. While the discovery of Candelas *et al* is striking, the success of the compactification scheme partly rests on its ability to be accommodated in a realistic dynamical theory of evolution. However, attempts[41-45] to build cosmological models based on the N = 1 Yang-Mills supergravity action have not been very successful. Weiss[46] has even questioned the consistency of a Ricci flat compact manifold with observed matter-dominated universe. It is not easy to get inflation in this model. The difficulties stem from the Ricci flatness of the internal space and also from the absence of any dimensionless free parameter. In the absence of SUSY breaking, the naive potential has only one minimum at $g^2_{GUT} = 0$ and the realisation of the cosmological evolution of the compactified theory to the correct vacuum, $g^2_{GUT} \simeq O(1)$, is a non-trivial problem. The available mechanism for SUSY breaking is to invoke gaugino condensation in the hidden $E_8'$ sector of the $E_8 \times E_8'$ theory. Ellis *et al*[41] have considered the tree level potential with some fine-tuning of the condensation temperature and obtained an inflationary phase in the $d = 4$ effective theory. However, the one loop effective potential does not have a minimum and the situation is not very clear. Maeda *et al*[43] have considered another SUSY breaking potential, given by Binetruy and Gaillard[47], which has only asymptotic validity. It is clear that further investigation is called for to understand the detailed dynamics of both the processes, compactification from ten to four dimensions and an inflation in four dimensions.

As the first step to achieve this goal, we intend to look for cosmological models based on a 10-dimensional action choosing the Gauss-Bonnet combination for the quadratic terms in curvature. Keeping in view the possibilities that the conditions necessary for an acceptable compactification may be realised transiently (even approximately), a systematic study of the solution will be undertaken. The calculations of Ellis *et al*[41], Maeda[42] and Maeda *et al*[43] have high-lighted some of the problems in this approach. The major problem is to get an inflationary phase and also a graceful exit from this phase. While the precise mechanism which allows this is not yet understood, it is useful to look afresh at all the possible mechanisms for inflation in 4-dimensions, viz Kaluza-Klein type inflation[48,49] ( $M^4$ expanding, K contracting ), contribution of the higher derivative terms,[9,33,34,50-52] a realistic temperature dependence of the effective potential etc., to see if field equations allow such a solution.

Cosmological models with different higher derivative terms, and, in particular, the Gauss-Bonnet combination have already been studied by various authors.[53-59] But a consistent scenario is yet to emerge. It is clear that cosmological model building in higher dimension is an exercise which may prove useful in our understanding of both cosmology and particle interactions at higher energies. Keeping this objective in view, we have studied a model with Gauss-Bonnet terms in the action, including both matter loop corrections and temperature effects. We have looked for an acceptable scenario of the pre-compactified universe which eventually passes on to a radiation dominated FRW universe. As will be shown in Chapter IV, a mechanism which helps to keep the cosmological constant small emerges naturally in this model.

## I.2 *NEW INFLATION & ITS PROBLEMS :*

In this section we shall discuss briefly the new inflationary model and its shortcomings.[5] Let us recall that in the old inflationary model[4], Guth first made an attempt to resolve the horizon, flatness and monopole problems of the Big Bang model. He obtained the inflationary model by combining the ideas of a phase transition mechanism which the universe might have gone through during the first $10^{-35}$sec, with the effects of general relativity. Inflation in this model ends with a transition to the broken symmetric phase by nucleation and growth of bubbles. However, it was realised soon that the model had a number of problems which lead to unacceptable consequences.[4,20-23] The energy released during the phase transition will thermalize if the bubbles undergo frequent collisions. But this process makes the universe highly anisotropic, contradicting the observed homogeneity of the universe. It is now known that the assumption of a quick phase transition in this model is not tenable. It was also noted that a large expansion factor required for this stage implies that the bubble nucleation rate is slow compared to the expansion rate of the universe.[4] In this scenario the bubbles formed will never percolate. There is no mechanism in this scenario by which the universe may successfully complete the phase transition to the broken symmetric phase and exit gracefully to the usual FRW-universe. To overcome these problems a modified model was suggested by Linde[24] and Albrecht and Steinhardt[25] independently which is known as 'new inflation'.

The basic idea behind this proposal was to construct a model in which inflation occurs after the bubbles of new phase have been formed. The new element introduced in the model is based on a different kind of phase transition commonly referred to as the 'slow rollover' phase transition . Linde and Albrecht and Steinhardt have suggested the model in the framework of the grand unified theories[1] in which the gauge symmetry is broken by the radiative corrections to the effective potential as in the Coleman and Weinberg[60] ( in short, CW ) model. In the abelian Higgs model the effective potential calculated in one loop approximation is given by

$$V_{eff}^{(1)} ( \phi ) = V (0) + B \phi^4 \left[ \ln \left( \frac{\phi}{\phi_0} \right)^2 - \frac{1}{2} \right] \qquad ( 1.4 )$$

where B is a constant determined by the gauge coupling constant and $\phi_0$ corresponds to the global minimum of $V_{eff}^{(1)}$ . In CW-model spontaneous symmetry breaking ( SSB ) is a consequence of quantum corrections to the potential. The effective potential acquires a temperature dependent mass term when finite temperature corrections are taken into account. The effective potential is given by

$$V_{eff}^{(1)} ( \phi, T ) = V_{eff}^{(1)} (\phi) + C T^2 \phi^2 \qquad ( 1.5 )$$

where C is a constant $O(g^2)$ and T is the temperature. This potential provides an energy barrier that can trap the universe in a metastable symmetric phase which occurs during its cooling from a high temperature. At a very high temperature, the absolute minimum of $V^{(1)}_{eff}$ $(\phi, T)$ is located at $\phi = 0$. The position of the absolute minimum does not change until T is less than a critical value $T_c = ( B/2C )^{1/2} \phi_o$ , where $\phi_o$ is the maximal value of the field $\phi$ inside the bubble immediately after its formation.

The general feature of new inflation is as follows : Consider a first order phase transition in the early universe which occurs due to a spontaneous symmetry breaking at an energy scale $M_G$ (GUT scale) . At a temperature $T \gg T_c \approx M_G$ , the vacuum state $\phi = 0$ is the global minimum of the effective potential and the thermal part of the energy momentum tensor dominates. Consequently the universe expands like a radiation dominated phase. As the universe expands, the temperature of the universe drops and falls to $T_c$. When $T \to T_c$ , a second minimum develops at $\phi = \phi_o$ and there are degenerate vacuum states at $T = T_c$ . At this epoch the vacuum energy starts dominating over the energy momentum tensor. For $T < T_c$ , the potential barrier which develops between the two minima of the effective potential does not permit the universe to evolve classicaly from the symmetric state $\phi = 0$ to $\phi = \phi_o$ state instantly. It is known that the quantum mechanical tunneling is also not possible because the transition probability is insignificant.[61] However, at a very low temperature ( $T \approx 0$ ) less than the Hawking temperature $T_H \sim \frac{H}{2\pi}$ , ( H is the Hubble parameter ) the barrier effectively disappears and the $\phi = 0$ state becomes unstable. It is now possible to drive the universe away from $\phi = 0$ by a small amount which may be triggared by a small quantum fluctuation. Let $\tilde{\phi}$ be the value of $\phi$ when it penetrates the barrier. The subsequent evolution from $\phi = \tilde{\phi}$ to $\phi = \phi_0$ can be described by the equation ($\phi$ is assumed to be spatially homogeneous):

$$\ddot{\phi} + 3\,H\,\dot{\phi} + \Gamma\,\dot{\phi} + V'(\,\phi\,) = 0 \qquad (\,1.6\,)$$

where the prime indicates differentiation with respect to $\phi$ and dot that with respect to t. The third term accounts for particle creation due to the time variation of $\phi$. The quantity $\Gamma$ is determined by the nature and coupling of the particles produced. One defines the life time of the $\phi$ particles by $\tau = \Gamma^{-1}$. For $T \ll T_c$, we may write $V_T(\phi) \simeq V_0(\phi)$, and the square of the Hubble parameter $H^2 = \frac{8\pi G}{3}\rho$, can be determined from the expression for $\rho$

$$\rho = \frac{1}{2}\,\dot{\phi}^2 + V(\,\phi\,) + \rho_r\,, \qquad (\,1.7\,)$$

where $\rho_r$ is the radiation energy density produced by the time variation of $\phi$ following the equation

$$\dot{\rho}_r + 4\,H\,\rho_r = \Gamma\,\dot{\phi}^2\,, \qquad (\,1.8\,)$$

the r.h.s. takes into account the effect of particle creation.

The scenario can now be described. Suppose V is very flat in some region between $\phi = \tilde{\phi}$ and $\phi = \phi_0$; $\phi$ will evolve very slowly and the radiation energy density can be neglected at this stage. Under this circumstances the eq. (1.6) can be rewritten as

$$\dot{\phi} = -\,\frac{1}{3}\,H^{-1}\,V'(\phi)\,. \qquad (\,1.9\,)$$

If $V(\phi)$ is sufficiently flat, the time taken by $\phi$ to traverse the flat region may be sufficiently large compared to the expansion time scale $H^{-1}$. During this period

$$H \approx \left\{ 8\pi G\, V(0)/3 \right\}^{1/2} \approx \text{const.} \qquad (1.10)$$

and

$$a(t) \propto \exp(Ht). \qquad (1.11)$$

Thus the scale factor has a large exponential growth during this slow rollover period. This mechanism of transition has been studied in details by Guth and Pi[62], Hawking[63], Brandenberger and Kahn[64] and others[65] considering the evolution of a single inflaton field in the theory. It has been suggested that a single fluctuation region inflates to encompass the entire observable universe. In this scenario the inflation continues to occur even when the universe evolves towards the stable phase. To allow for a natural transition to a FRW universe, the flat portion of $V(\phi)$ is followed by a region where $V(\phi)$ has a sharp drop and the slow rollover transition ceases to occur. The value of $\phi$ changes rapidly in this stage. Around the minimum, it oscillates with a frequency $\omega = (V''(\phi_o))^{1/2} \approx M_G^2 \gg H^2$. These oscillations are damped quickly by both particle creation and the expansion of the universe. Two possibilities exist in this case :

i) If $\Gamma^{-1} \ll H^{-1}$, the rapid time variation of $\phi$ leads to the conversion of the coherent field energy $V(\phi) + \frac{1}{2}\dot{\phi}^2$ into radiation by quantum particle creation,[66] thereby reheating the universe. This is a case of efficient reheating.

ii) If $\Gamma^{-1} \gg H^{-1}$, $\phi$ continues to oscillate, however, the coherent field energy is redshifted away very soon by the expansion of the universe as $a^{-3}$. At a later time $t \simeq \Gamma^{-1}$, the universe is reheated by the radiation energy density leading to a poor reheating.

However, in order to keep the reheating temperature below the GUT scale it is necessary to adjust the parameters in the theory. This fine-tuning is required to avoid fresh production of monopoles. Another success of the new inflation is that it suggests an explanation for the galaxy formation problem. However, alongwith these successes, the new inflationary scenario suffers from some obvious drawbacks, which are outlined below :

a) The CW type potential, which has been found suitable for the scenario needs some fine-tuning of the parameters and is, therefore, not a natural choice.

b) The slow roll equation is not true in general for an interacting field theories.[67]

c) The transition from the symmetric SU(5) may not occur in the direction of the global minimum SU(3) X SU(2) X U(1) . It was pointed out by Guth and Weinberg[20] and Breit *et al*[68] that the universe may instead be trapped in the wrong phase SU(4) X U(1) due to symmetry breaking.

d) Although a scale invariant spectrum of density perturbation is obtained in the new inflation,[18,69] SU(5) GUT produces a very large value for $\frac{\delta\rho}{\rho}$ ( $\sim$ 50 ) which is many orders larger than the experimental value ( $\frac{\delta\rho}{\rho} < 10^{-4}$ ).

To overcome the problem discussed in (d), a proposal has been made by Shafi and Vilenkin[70] which requires an extra inflaton field. It was suggested that the overproduction of the density perturbation may be overcome considering a very weakly interacting field ( $\lambda \sim 10^{-12}$ ). Unfortunately, SU(5) GUT theory does not permit such a weak field. Therefore, it was concluded that the SU(5) GUT is cosmologically unacceptable. However, such fields may occur in supersymmetric and supergravity theories. Inflation of the early universe can be incorporated in a natural way in such theories.[71]

Mazenko *et al*[72] have raised another serious objection against the new inflationary models of the early universe . It was pointed out that due to the violent fluctuations of the field at a very high temperature in the early universe the field may not settle into the false vacuum state ( $\phi = 0$ ) as the universe cools. They have claimed that the models of new inflation are based on wrong physics. It was argued by them that many models previously considered as candidates for new inflation do not enter into an inflationary regime. To see this we note the following: If $\phi$ is only gravitationaly coupled to other fields, at a high temperature the mechanism which confines the field at the value $\phi = 0$ will no longer be valid. In the absence of such confining forces we expect large thermal fluctuations. As a result $\phi(x,t)$ will be inhomogeneous. For $\phi_o < M_p$ the spatial fluctuations in $\phi$ may be much larger than the separation between the minima of $V(\phi)$. The energy momentum tensor $T_{\mu\nu}$ is now dominated by radiation, consequently inflation will not occur. They argued that as the universe expands the scalar field will lose energy. At $T = T_c$, the energy content of the field is insufficient to cross the potential barrier at $\phi = 0$ and, therefore, $\phi(x)$ will rapidly relax to a value $-\phi_0$ if initially $\phi_0(\vec{x}) < 0$. Similarly $\phi(\vec{x}, t)$ will relax to $+\phi_0$ if initially $\phi(\vec{x}) > 0$. Therefore, at $T < T_c$ spatial domains with $\phi(x) = \pm \phi_0$ will be formed. It can be shown that in such domains $V(\phi) = 0$. Consequently, the equation of state necessary for inflation cannot be obtained. However, Albrecht and Brandenberger[73] and Albrecht *et al*[74] almost simultaneously showed that the above argument is not correct in general. The discrepancies can be avoided by a new approach to phase transition in the cosmological models which does not use the concept of an effective potential. They have shown that in many models the expansion of the universe leads to a sufficient Hubble damping of the field configuration which gives rise to a long intermediate phase of inflation. For the CW-model as well as in quartic scalar field potential with double well, the new inflation can be realized. Another objection was pointed by Linde.[26] He pointed out that in theories with a large value of the vacuum potential energy $V(\phi)$ and a sufficiently small coupling constant $\lambda$, the inflationary universe scenario based on a temperature dependent phase transition cannot be realized. This observation provided a strong motivation for the chaotic model suggested by him. This will be discussed in the next chapter.

## I.3 *AIM OF THE WORK :*

The aim of the work is to study some cosmological models of the early universe which are consistent with the recent theories of elementary particle interactions at high energies and also with the constraints imposed by various astrophysical observations. We will study cosmological models within the framework of a quantum field theory in curved space-time. Since an inflationary stage seems to be an essential ingradient of modern cosmological models, we will study various types of inflationary models. Different aspects of any of these models need to be studied carefully before it can be considered as an acceptable theory of the early universe. We have undertaken such a study with particular emphasis on the mechanisms that lead to an inflationary stage and also allows a natural exit from it.

The models which we have chosen for this study are : (i) the chaotic inflation, (ii) the higher derivative theories in 4-dimensions and (iii) higher derivative theories in higher dimensions. We have analysed these models critically and also obtained some new results. Our aim has been to study the efficacy of these models in providing a realistic scenario of the early universe.

## I.4 *SUMMARY OF THE WORK :*

The summary of the chapters contained in this thesis is as follows :

**Chapter I** gives an introduction into the background and nature of the problems studied and also summarizes the results obtained.

**Chapter II :** In this chapter, the chaotic inflationary scenario,[26] as proposed by Linde is briefly reviewed. The chaotic scenario is then studied in an anisotropic universe model. We have considered, in particular, an anisotropic Kantowski-Sachs (KS) metric.[75] For a weakly coupled inflaton field the Einstein's field equation admits here two types of cosmological solutions. For one of these solutions, the initial anisotropy washes out as the universe expands while for the other there is no such transition to an isotropic universe. In the second case, the measure of anisotropy remains constant. We have shown that the chaotic model can be realized in the KS-metric. This observation justifies the claim of Linde that the ideas of chaotic scenario are fairly general. The concept that the present state of the universe should not depend on any specific choice of the initial data gets realized in our results, since KS-model, though simple describes a rather exceptional type of anisotropy.

**Chapter III :** In this chapter, the evolution of the early universe in higher derivative theories has been studied. As a special case, we have studied in detail the modified Starobinsky model, which is described by an action quadratic in curvature :

$$S = \frac{M_P^2}{16 \pi} \int ( R + \alpha R^2 ) \sqrt{-g} \; d^4x \; . \qquad (1.3)$$

Depending on the initial conditions, the early universe in this model may evolve from one of the two different initial stages : (i) de Sitter and (ii) Non-de Sitter (NdS). In the original Starobinsky model[9] the de Sitter phase is an exact solution which is, however, unstable. On the other hand, the de Sitter stage is only an approximate solution in the modified model. The NdS solution noted by us exhibits entirely new features in the initial stages of the evolution. We have emphasized these new features. We also follow the evolution of the universe in different stages, e.g. the initial NdS-phase, quasi-exponential inflation, particle production, reheating etc. . An interesting result obtained[110] in the flat ( $K = 0$ ) model is that the choice of the initial value of the scalar curvature ( $R$ ) determines whether $R$ should increase or decrease. In both the cases $R$ tends asymptotically to the value $\left( \frac{1}{12 \beta^2} \right)^{1/3}$ , where $\beta$ is a constant. However, the magnitude of the inflation is almost independent of this undetermined constant ($\beta$), which is undoubtedly a good feature of the model. In the closed model ( $K = +1$ ), $\beta$ is found to be time dependent. Here, the scale factor of the universe grows linearly with time. The solution is unstable and eventually the universe enters into a different phase of expansion, as in the flat model. The solution found in open ( $K = -1$ ) model is also discussed. However, the scalar curvature (R) here is zero and the solution is not very interesting from a physical point of view. The modification in the Starobinsky model[9] brings in a substantial change in the original scenario in the earlier stages and the new results may have interesting applications in the formulation of quantum cosmology.

**Chapter IV :** In this chapter, we have presented a scenario of the evolution of a ten dimensional universe. As theories of particle interactions often require dimensions more than four for their formulation, it is essential to check if consistent cosmological solutions, which can accommodate these theories are also allowed. Higher derivative terms in the gravitational action and, in particular, the Gauss-Bonnet combinations occur in a natural way in many theories. Cosmological solutions for these theories in ten dimensions have been studied. An attempt has been made to construct a scenario which includes a spontaneous compactification, an inflationary epoch followed by a four dimensional radiation dominated stage. It is noted that with an action containing matter loop correction, including the temperature effect, it is possible to have a realistic scenario, provided there is an inflationary stage of the early

10-dimensional universe. A mechanism is suggested which keeps the 4-dimensional cosmological constant small, close to zero. The study reveals a rich structure of the theory due to the presence of Gauss-Bonnet terms, although one gets back the usual Einstein equation in four dimensions at a large time.

# *CHAPTER II

## CHAOTIC INFLATIONARY MODELS

* A part of this chapter has already been published in *Ref.* 76.

## II.I *INTRODUCTION* :

The inflationary models[5] of the early universe including the new inflationary model[24,25] make use of a phase transition which requires some fine-tuning either in the potential $V(\phi)$ of the inflaton field $\phi$ or in initial conditions of the universe. Linde[26] in 1983 proposed a new approach to obtain an inflationary universe scenario which apparently needs no specific fine-tuning. In the new model , an inflaton field is still needed but the initial state of the field is non-thermal. The advantage of the model is that it is not essential to restrict to a particular initial configuration $\phi = 0$ as was required in the new inflation model. The homogeneous field instead can take any value provided by a random initial distribution satisfying the constraint $V(\phi) \lesssim M_P^4$ . This constraint is necessary so that the quantum behavior of gravity does not dominate and a classical description of space-time remains valid. Since the initial data for the model are randomly distributed, the scenario is called a chaotic inflationary scenario. The basic motivation for the new scenario was to look for a theory of evolution without imposing unnatural constraints on the shape of the effective potential. If successful, it will, therefore, be an attractive scenario of the early universe. The scenario can be implemented in a wide class of theories as has been confirmed by different authors.[77-78] This includes a simple class of theories for the scalar field $\phi$ with a polynomial effective potential, $V(\phi) \sim \phi^n$, $n \lesssim 11$ ;[79] with the assumption that initially $\phi$ satisfies $(\partial_\mu \phi)^2 \lesssim V(\phi) \sim M_p^4$ in a domain of size $l \sim O(M_p^{-1})$. The temperature dependent phase transitions can not be realized since the finite temperature corrections are small compared to the potential at $T \lesssim M_p$ and $\phi \gg M_p$. For a pure $\lambda\phi^4$-theory, quantum fluctuations in the initial universe can drive $\phi$ to a very large value. The field may take considerable time to relax back to the minimum. Thus the chaotic scenario need not be accompanied by any phase transition mechanism. Sufficient inflation may be obtained if the time taken by the process is sufficiently large, obeying $\int H\,dt > 65$.

Let us now briefly describe the model. We assume that the space-time is of the Friedman-Robertson-Walker (FRW) type :

$$ds^2 = dt^2 - a^2(t)\, d\Omega_3^{\,2} \qquad (2.1)$$

where

$$d\Omega_3^{\,2} = \frac{dr^2}{1 - kr^2} + r^2 \left( d\theta^2 + \sin^2\theta\, d\Phi^2 \right),$$

it is the metric of the hypersurfaces $\mathbb{S}^3$, $\mathbb{R}^3$ and $\mathbb{H}^3$ corresponding to $k = +1$, $0$ and $-1$ respectively; and $r$, $\theta$, $\Phi$, $t$ are the space-time co-ordinates. The evolution of the universe is determined by the Einstein's field equation :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - 8\pi G\, T_{\mu\nu} \qquad (2.2)$$

Here $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $T_{\mu\nu}$ represents the energy momentum tensor. Let us now consider a classical scalar field (inflaton) described by a Lagrangian

$$L(\phi) = \frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi - V(\phi) \qquad (2.3)$$

where $V(\phi)$ is the potential energy. The energy momentum tensor is

$$T_{\mu\nu} = \partial_\mu \phi\, \partial_\nu \phi - g_{\mu\nu}\, L \quad . \qquad (2.4)$$

For the metric (2.1), eqs.(2.2) and (2.4) will give two independent equations :

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_P^2} \left[ \frac{1}{2}\, \dot{\phi}^2 + \frac{1}{2}\, a^{-2} (\nabla\phi)^2 + V(\phi) \right] , \qquad (2.5)$$

$$\dot{H} + H^2 = \frac{8\pi}{3M_P^2} \left[ V(\phi) - \dot{\phi}^2 \right] \qquad (2.6)$$

where dot denotes differentiation with respect to proper time t. The evolution of the scalar field is given by the equation

$$\ddot{\phi} + 3\, H\, \dot{\phi} - a^{-2}\, \nabla^2 \phi + \frac{\partial V}{\partial \phi} = 0 \, . \qquad (2.7)$$

Linde[26] in his original version of chaotic inflation made the following assumptions :

a) Space-time is isotropic and homogeneous,

b) the spatial curvature $k/a^2$ of the space-time is negligible compared to other terms in the field equation,

c) the matter part is determined mainly by the scalar field $\phi$ which is weakly interacting,

d) the time and space derivative terms are small compared to the potential $V(\phi)$ in the inflaton Lagrangian $L = \frac{1}{2}\, \partial_\mu \phi\, \partial^\mu \phi - V(\phi)$,

e) the inflaton field is spatially homogeneous and depends on time only.

With the assumption (e), the field equations ( 2.5 ) and ( 2.7 ) reduce to

$$H^2 = \frac{8\pi}{3M_P^2} \left[ V(\phi) + \frac{1}{2}\dot{\phi}^2 \right] , \qquad (2.8)$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 . \qquad (2.9)$$

For simplicity we assume, $V(\phi) = \frac{1}{4} \lambda \phi^4$. At $t \sim t_P$ (Planck time), all values of $\phi$ subject to the condition $V(\phi) \lesssim M_P^4$ are assumed to be equally probable. It can be shown that if the initial value of the field $\phi$ satisfies the constraint $\phi_o >> \frac{1}{3} M_P$ , the kinetic term in the eq.(2.8) can be neglected and the friction term $3H\dot{\phi}$ in the eq.(2.9) makes the variation of $\phi$ very slow so that one can neglect $\ddot{\phi}$. The eqs.(2.8) and (2.9) can be solved and it is found that the domain of the universe in which $\phi$ is initially homogeneous expands exponentially for $\dot{H} << H^2$ :

$$a(t) = a_o \exp ( Ht) \qquad (2.10)$$

where $H = \left( 8\pi\, V(\phi)/3M_P^2 \right)^{1/2} = \left( \frac{2}{3}\pi\lambda \right)^{\frac{1}{2}} \frac{\phi^2}{M_P}$ and the field decreases as

$$\phi = \phi_o exp \left[ - \sqrt{(\lambda/6\pi)}\, M_p t \right] . \qquad (2.11)$$

It can be shown that the standard cosmological problems can be solved if the observed part of the universe emerges from a region having initially $\phi_o ( t \sim M_p^{-1} ) \gtrsim 3.2\, M_p$. This lower bounds on $\phi$ ensures that the magnitude of inflation, ( $\int H\, dt > 65$ ) is sufficient for this purpose. Starting with this high value, the field $\phi$ decreases rapidly and near the minimum of $V(\phi)$ it starts oscillating rapidly. The oscillation of the field transforms the potential energy $V(\phi \sim M_p/3)$ into heat. The reheating temperature of the universe could be $O( \lambda^{-1/4} M_p )$ or lower depending on the strength of coupling of $\phi$ with other matter fields. A good feature of the scenario is that the reheating temperature is independent of $\phi_o$, the initial value of the inflaton field for $\phi_o \gg M_p$. Linde[80] in a subsequent investigation has shown that the scenario can be realized even when $\phi$ is initially highly fluctuating ( $\frac{1}{2} \dot{\phi}^2 \gg V(\phi)$ ) relaxing the assumption (d). In this case the initial value of the kinetic term decreases more rapidly ( $\sim a^{-6}$ ) than the potential term. Consequently a potential dominated regime soon appears leading to inflation. Papantonopoulos *et al*[81] have shown that the favourable situation for inflation, $\dot{\phi}^2 \ll V(\phi)$, emerges quite naturally if one introduces an axion field $\xi$ which has only derivative coupling to $\phi$. The additional field plays a very significant role for realising the situation $\dot{\phi}^2 \ll V(\phi)$ at a slightly later time $t \sim 10\, M_p^{-1}$ even if $\dot{\phi}^2 \sim V(\phi)$ at the Planck time.

Pollock[82] has examined the above result and claimed that the non-minimal couplings of $\phi$ and $\xi$ are not essential. Instead, a fine-tuning in the potential may lead to the conditions required for inflation. Recently , Madsen and Coles[83] have investigated the chaotic inflation in an isotropic Robertson-Walker universe relaxing the assumptions (b), (d), (e). They have shown, by numerical calculations, the viability of the chaotic model even in cases of the dominance of the spatial derivative terms and the dominance of spatial curvature term in a closed ( $k = +1$) universe.

The chaotic inflationary universe scenario is, therefore, an attractive model which has been widely studied in an isotropic Robertson-Walker background. However, the assumption of an isotropic space-time is a special choice. A realistic chaotic model should not depend on any specific choice of initial conditions. It will be particularly useful to study the effects of an initial anisotropy in this chaotic scenario. The problem of isotropization of the universe in some anisotropic cosmology, mostly of the Bianchi types, has already been studied.[84-88] However, the process of isotropization in an inflationary scenario needs further investigation. In the next section of this chapter we shall discuss our work on the effects of an initial anisotropy in the chaotic scenario. We have chosen in particular an anisotropic Kantowski-Sachs (in short, KS) model[75] to realise Linde's chaotic scenario. The KS-model has some interesting features some of which can be studied exactly. Weber[89] has shown that there exists a class of KS-models with a non zero cosmological constant ( $\Lambda \neq 0$ ) which become isotropic asymptotically. Our aim has been to examine the possibility of accommodating a chaotic inflationary scenario within these asymptotically isotropic KS-models. We have seen that it is indeed possible to construct a chaotic model in the anisotropic KS-metric which justifies the claim of Linde that the ideas of the chaotic scenario are fairly general. The concept that the present universe should not depend on any specific choice of initial values gets further support from our results, because KS-models, though simple, describe a rather exceptional type of anisotropy. In sec.II.2, we shall set up the field equations in the KS-model, and in sec.II.3, we discuss the essential features of the solutions which lead to an inflationary regime. Our conclusions are given in sec.II.4 . In sec II.5 we shall briefly review the recent developments of the model introduced by Linde[90-93] and others[94-97].

## II.2 *FIELD EQUATIONS IN THE KS-MODEL :*

The KS-metric is described by the line element :

$$ds^2 = dt^2 - X^2(t)\, dr^2 - Y^2(t) \left[ d\theta^2 + \sin^2\theta\, d\Phi^2 \right] . \qquad (2.12)$$

The metric is locally spherically symmetric and it defines a space-time whose homogeneous spatial hypersurfaces do not admit any 3-parameter simple transitive group of isometries. In fact, the isometry group $G_3$ is isomorphic to $SO(3,\mathbb{R})$ with 2-dimensional orbits. Thus points on the 2-surfaces ($t$ = const., $r$ = const.) are found to be (isometrically) equivalent. Moreover, each homogeneous space sections ( $t$ = const. ) is semi-closed in the sense that the corresponding covering manifold has the topology $R^1 \times S^2$ . The positive scalar curvature on the closed sections is given by the Ricci-scalar $R^* = \dfrac{2}{Y^2} > 0$. The world lines of matter in the KS universe are characterized by the absence of vorticity ($\omega = 0$), but there is non-vanishing shear, $\sigma = \frac{1}{\sqrt{3}} (\dot{X}/X - \dot{Y}/Y)$. Let $H_1 = \dot{X}/X$ and $H_2 = \dot{Y}/Y$ be the expansion rates along the radial and angular directions respectively ; $H$, be the average expansion rate; $R$ and $V$, the mean scale factor and the "volume scale factor", respectively. We have

$$H = \frac{1}{3} (H_1 + 2 H_2) = \frac{\dot{R}}{R} = \frac{1}{3} \frac{\dot{V}}{V} \quad . \qquad (2.13)$$

The average expansion anisotropy is defined by

$$A = \frac{1}{3} \left[ ( \Delta H_1 / H )^2 + 2 ( \Delta H_2 / H )^2 \right] \qquad (2.14)$$

with

$$\Delta H_1 = H_1 - H = \frac{2}{\sqrt{3}}\alpha \text{ and } \Delta H_2 = H_2 - H = -\frac{1}{\sqrt{3}}\alpha . \qquad (2.15)$$

For an isotropic expansion, $H_1 = H_2 = H$ which implies $A = 0$.

Let us now write the Einstein's field equation for the metric (2.12) :

$$2\frac{\dot{X}\dot{Y}}{XY} + \frac{1+\dot{Y}^2}{Y^2} = 8\pi G \rho , \qquad (2.16)$$

$$2\frac{\ddot{Y}}{Y} + \frac{1+\dot{Y}^2}{Y^2} = -8\pi G p , \qquad (2.17)$$

$$\frac{\ddot{Y}}{Y} + \frac{\ddot{X}}{X} + \frac{\dot{X}\dot{Y}}{XY} = -8\pi G p . \qquad (2.18)$$

To determine the energy density ( $\rho$ ) and pressure ( p ) we consider a classical scalar field $\phi$ described by the Lagrangian (2.3). If the field is assumed to be spatially homogeneous, $\rho$ and p which follow from eq. (2.4) are given by

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi) , \qquad (2.19)$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi) . \qquad (2.20)$$

The initial configuration of the field is chosen in such a way that the kinetic term is small compared to the potential term $V(\phi)$. The total energy is then dominated by the potential energy and it acts as a cosmological constant

$$\Lambda = 8\pi G\, V(\phi) . \qquad (2.21)$$

The corresponding inflationary scenario will be discussed in the following section.

## II.3 *CHAOTIC INFLATION IN KS-MODEL :*

Let us now consider the evolution of the vacuum scalar field $\phi$ in a KS-universe with the initial values specified at the Planck time. It has already been pointed out by Linde[26] that in the theories with a large value of the vacuum potential energy $V(\phi)$ and sufficiently small coupling constant $\lambda$, inflationary universe scenario based on a temperature dependent phase transition cannot be realized. The high temperature effects cannot influence the behavior of the inflaton field $\phi$, if the initial value of $\phi$ is large. For a $\frac{1}{4}\lambda\phi^4$ -theory, this happens if $\phi$ at Planck time is greater than $( 10^{-1}\lambda^{1/4} M_P )$. This observation provided another motivation for the chaotic model.[26] We assume that the initial values of $\phi$ and its derivative $\dot{\phi}$ ( at the Planck time ) satisfy $\dot{\phi}_o^2 \lesssim M_P^4$ , $V(\phi) < M_P^4$ . The field $\phi$ satisfies the dynamical equation :

$$\ddot{\phi} + \left( \frac{\dot{X}}{X} + 2\frac{\dot{Y}}{Y} \right)\dot{\phi} + \frac{\partial V(\phi)}{\partial \phi} = 0 . \quad ( 2.22 )$$

We now consider two possibilities :

$$\text{(i)} \quad \frac{1}{2}\dot{\phi}^2 \gg V(\phi) \quad \text{and} \quad \text{(ii)} \quad \frac{1}{2}\dot{\phi}^2 \ll V(\phi).$$

Case (i) : For $\frac{1}{2}\dot{\phi}^2 \gg V(\phi)$ , the energy-momentum tensor has the same form as that for the stiff matter,

$$\rho = p = \frac{1}{2}\dot{\phi}^2 . \qquad (2.23)$$

Since for all $V(\phi) \sim \phi^n$ , $V(\phi) \gtrsim M_p \dfrac{dV}{d\phi}$ , for $\phi \gtrsim M_p$ ; the eq.(2.22) , therefore, reduces to

$$\ddot{\phi} + \left( \dot{X}/X + 2\,\dot{Y}/Y \right) \dot{\phi} = 0 . \qquad (2.24)$$

The solution of the eq.(2.24) is given for a small t by

$$\dot{\phi} = \frac{\text{const.}}{X\,Y^2} \sim \frac{c}{t} \qquad (2.25)$$

where c is a constant determined by the initial behavior of $\dot{\phi}$ . The stiff matter solution[98] is given by $X\,Y^2 \sim t$ . The solution either evolves from :

(a) a point like singularity ,

$$X = X_o\, t^{(1-\beta)/3} , \quad Y = Y_o t^{(2+\beta)/6} , \; -2 < \beta < 1, \qquad (2.26)$$

or, (b) a cigar-like singularity ,

$$X = X_o t^{-(\beta-1)/3} , \; Y = Y_o t^{(2+\beta)/6} , \quad 1 < \beta < 2. \qquad (2.27)$$

The above solutions (2.26) and (2.27) are valid for the time

$$t \ll \left[ \frac{Y_o^2}{12} (4-\beta^2) \right]^{3/(4-\beta)} . \qquad (2.28)$$

The upper limit of time upto which the solutions are valid is determined by $Y_o$ and $\beta$ ; note, however, the constant c is found to be $c = \pm \left[ \frac{4 - \beta^2}{48 \pi G} \right]^{1/2}$ .

There is another type of singular solution which is barrel type : $X \sim$ const. , $Y = Y_o \sqrt{t}$ for $t < Y_o^{\,2}/4$ . The nature of the evolution of $\phi$ and $\dot{\phi}$ in the barrel type case may belong to either of the above two cases. Therefore, we shall discuss the evolution only for the first two cases . Although we are interested in the solutions for $t \gtrsim t_P$ away from the singularity, it will be useful to note that the nature of the singularity determines the sign of the constant c in (2.25), because $\dot{\phi} = \xi \frac{\dot{X}}{X}$ , $\xi > 0$ .Thus, we have $c > 0$ for the point-like case and $c < 0$ for the cigar-like case.

(a) For $c > 0$ :

In this case, we have specifically,

$$\dot{\phi} \sim \dot{\phi}_o M_P^{-1}/t \ , \quad \dot{\phi}_o > 0 \qquad ( 2.29 )$$

which gives

$$\phi \sim \phi_o + \dot{\phi}_o M_P^{-1} \ ln\ t \ . \qquad ( 2.30 )$$

Thus $\phi$ increases slowly (*logarithmically*) for $t \gtrsim M_P^{-1}$ but the kinetic term $\frac{1}{2} \dot{\phi}^2$ decreases rapidly $\sim t^{-2}$ . The kinetic term eventually becomes less important and $V(\phi)$ may begin to dominate. To estimate roughly the time $t = t_1$ when $\frac{1}{2} \dot{\phi}^2 \simeq V(\phi)$ , we note that at $t_o = M_P^{-1}$ , one must have $\dot{\phi}_o \lesssim M_P^2$ and $V(\phi) = \frac{1}{4} \lambda \phi^4 << M_P^4$ . We choose $\lambda \sim 2 \times 10^{-10}$. It can be shown that during the time

$t \sim 3 \times 10^3\ M_P^{-1}$ the field $\phi$ (say $\sim 2\ M_P$ at the Planck time) grows slowly to $\phi_1 \sim 10\ M_P$, while there is a considerable reduction of the value of $\dot\phi$ , from $\dot\phi \sim M_P^2$ to $\dot\phi_1 \sim 10^{-4}\ M_P^2$. If the variation of $\phi$ continues to be small, $V(\phi)$ remains nearly constant and we expect an inflationary era to emerge. The situation is similar to its FRW analogue, studied by Linde[80] .

(b) For $c < 0$ :

We have

$$\dot\phi \sim - |\dot\phi_0|\ M_P^{-1}/t \ , \quad \dot\phi_0 < 0$$

which gives

$$\phi \sim \phi_0 - |\dot\phi_0|\ M_P^{-1}\ \ln t \quad . \qquad (2.31)$$

We have a decreasing mode for $\phi$ , which presents an interesting possibility. Here, the field $\phi$ goes to zero in a time $t_2$ given by

$$t_2 \sim \exp\left[\frac{\phi_0}{|\dot\phi_0|\ M_P^{-1}}\right] . \qquad (2.32)$$

The domination of $V(\phi)$ , if possible must take place at an earlier time $t_1 < t_2$ . If the solution ( 2.31) remains approximately valid, we get the condition

$$\phi_0 - |\dot\phi_0|\ M_P^{-1}\ \ln t_1 > \left(\frac{2}{\lambda}\right)^{1/4} \left[\frac{|\dot\phi_0|\ M_P^{-1}}{t_1}\right]^{1/2} . \qquad (2.33)$$

We consider an example. If $\phi_0$ is $\sim 14\ M_P$ , after a period $t_1 \sim 3 \times 10^3\ M_P^{-1}$ the field reduces to about $6M_P$ , while $\dot\phi$ decreases from $\dot\phi_0$

$\sim M_p^2$ to $3 \times 10^{-4}$ $M_p^2$ and we expect an inflationary scenario. However, if $\phi_o < 8\ M_p$, there will be no inflation.

Thus, in both the modes there are possibilities of an inflationary era. However, in the decreasing mode , one should start with a sufficiently large value of $\phi$ at the Planck time.

Case (ii) : The inflationary stage : Now we consider the situation when $V(\phi) \gg \frac{1}{2}\dot{\phi}^2$ . The situation may be realized right at the planck epoch or through the steps outlined above. Since $\dot{\phi}$ is small, the potential energy $V(\phi)$ acts almost as a cosmological constant given by $\Lambda = 8\pi G\ V(\phi)$. Weber has discussed the general features of the solutions of the KS-field equations with a cosmological constant.[89] It has been shown that there is a class of solutions which tend asymptotically to the isotropic exponential solution, $X$ , $Y \sim exp\ [\ \sqrt{\Lambda/3}\ t\ ]$ for large cosmological times. The solutions may be classified according to their behavior near the singularity ( $t \sim 0$ ). There are three categories : (a) Point-like : $X \sim t$, $Y \sim t$, (b) Cigar-like : $X \sim t^{-1/3}$, $Y \sim t^{2/3}$ and (c) Pancake : $X \sim t$ , $Y \to const.$ . Although we need these classical solutions to describe the inflationary universe only after a nearly constant $V(\phi)$ starts dominating the vacuum energy density, it will be useful to consider the solutions along with their behavior near $t \sim 0$

In the KS-metric, the Einstein field eqs.(2.16-2.18) for a vacuum with a cosmological constant can be integrated to obtain :

$$\dot{Y} = K X \tag{2.34}$$

and

$$\frac{\dot{Y}^2}{Y^2} + \frac{1}{Y^2} - \frac{\alpha}{Y^3} = H_o^2 \tag{2.35}$$

where K and $\alpha$ are integration constants and $H_o = \sqrt{\Lambda/3}$ . If we choose $\alpha = 0$ and $\dot{Y}(0) = 0$ , we get $Y(0) = H_o^{-1}$ and the solutions are given by

$$X = \frac{1}{K} \text{ Sinh } H_o t \ , \tag{2.36}$$

$$Y = \frac{1}{H_o} \text{ Cosh } H_o t \tag{2.37}$$

which has a pancake type initial behavior. The values of H and A are given by

$$H = \frac{1}{3} H_o \left( \text{Coth } H_o t + 2 \tanh H_o t \right) , \tag{2.38}$$

$$A = 2 \left[ \frac{\text{Coth } H_o t - \tanh H_o t}{\text{Coth } H_o t + 2 \tanh H_o t} \right]^2 . \tag{2.39}$$

Clearly the solutions have the asymptotic exponential behavior

$$X , Y \sim \exp \left( H_o t \right) \tag{2.40}$$

and the anisotropy also washes out i.e, for a large t ,

$H \to H_o$ which implies $A \to 0$ .

For $\alpha \neq 0$ , eqs. ( 2.34 ) and ( 2.35 ) cannot be integrated in a closed form. However, the solution shows a cigar like behavior near $t = 0$ . For a large $t$, it behaves like the solution (2.36) – (2.37). If we consider a large $\alpha$, the solution for the range $H_o^{-1} << Y << \alpha$ , can be obtained as

$$X = \frac{(\alpha H_o)^{1/3}}{K} \frac{\text{Cosh}\, \frac{3}{2} H_o t}{\text{Sinh}^{1/3} \frac{3}{2} H_o t} , \qquad ( 2.41 )$$

$$Y = \left[ \frac{\alpha}{H_o^2} \text{Sinh}^2 \frac{3}{2} H_o t \right]^{1/3} , \qquad ( 2.42 )$$

$$A = 2 \left[ \frac{\tanh \frac{3}{2} H_o t - \text{Coth}\, \frac{3}{2} H_o t}{\tanh \frac{3}{2} H_o t + \text{Coth}\, \frac{3}{2} H_o t} \right]^2 . \qquad ( 2.43 )$$

It is interesting to note that for the solutions (2.41)–(2.42) the shear $\sigma$ and hence the anisotropy parameter $A$ turn out to be independent of $\alpha$ . GrØn[99] has shown that inflation may occur in a Bianchi-I anisotropic universe even for an arbitrarily large initial anisotropy as the value of H obtained is independent of the initial value of the shear $\sigma_o$.

The shear is uniquely determined in the above two cases :

$$\sigma = \frac{1}{\sqrt{3}} \left[ \frac{\dot{X}}{Y} - \frac{\dot{Y}}{Y} \right] = \frac{2}{\sqrt{3}} H_o \text{ Cosech } (2H_o t) \quad \text{for } \alpha = 0,$$

$$= -\sqrt{3}\, H_o \text{ Cosech } \left(3H_o t\right) \quad \text{for } \alpha \neq 0. \qquad ( 2.44 )$$

which decays exponentially towards zero for $H_0 t \gg 1$ and the expansion parameter ($\Theta$) is given by :

$$\Theta = \dot{X}/X + 2\,\dot{Y}/Y \qquad (2.45)$$

which tends towards $3H_0$ . GrØn[100], Sahni and Kofman[101] and GrØn and Eriksen[102] obtained results similar to those obtained by us[76]. Since $\sigma \sim t^{-1}$ ,a large $\sigma$ can occur only at an earlier time. Thus, for a classical description of an anisotropic KS-universe around the Planck epoch, we shall have to restrict ourselves to the class of models with $\sigma < 10^{19}$ GeV.

A vacuum solution which is permanently anisotropic also exists in the KS-cosmology. The field equations reduce to the simple form :

$$\frac{\ddot{X}}{X} = 3\,H_0^2 \quad \text{and} \quad \frac{1}{Y^2} = 3\,H_0^2 \qquad (2.46)$$

if one assumes $\dot{Y} = 0$. The solutions for X and Y are given by

$$X = X_0 \, exp\left( \sqrt{3}\, H_0 t \right) , \qquad (2.47a)$$

$$Y = \text{const.} \qquad (2.47b)$$

In this case the measure of anisotropy is a constant $A = 2$ . The above solution corresponds to the infinite barrel type solution of Weber.[89] It is now clear that when $V(\phi)$ acts as a cosmological constant, there is a class of solution which passes through an inflationary stage. Excepting the solution ( 2.47 ), the anisotropy in the initial universe washes out as the universe expands. Thus a description in terms of the inflationary solution of the field equation is possible whatever be the nature of the solution when $V(\phi)$ starts dominating.

## II.5 *CONCLUSIONS* :

To conclude, the results confirm that the chaotic scenario can be realized in the KS- models. This is, of course, not to suggest that the early universe was a KS universe, rather it shows that the chaotic model is fairly general and perhaps could be formulated whatever be the nature of the initial anisotropy. Details of the chaotic model, consistent with other cosmological constraints (e.g., the adiabatic density perturbation generated during inflation, $\delta\rho/\rho < 10^{-4}$) remains to be worked out. Linde has proposed a model with a number of additional scalar fields $\Phi_i$, some of which may have steeper potentials so that $\Phi_i$ may rapidly roll down to the minima of the respective potentials $V(\Phi_i)$. These fields may initiate the inflation, which may be sustained at the last stage by the weakly coupled scalar field $\phi$ ($\lambda \sim 10^{-12}$) . This scenario may provide a useful first step for the construction of a realistic model.

## II.4 *RECENT DEVELOPMENTS IN THE CHAOTIC MODEL* :

Recent developments[90-93,103] in the chaotic model are aimed at making the scenario a more realistic description of the early universe. The formation of global structure due to the quantum effect in the early universe has already been explored by Linde. This has led to the formulation of the eternal chaotic scenario. According to the scenario it is not essential to consider that the universe was created at some particular time t nor it is necessary for it to exist eternally. To understand the eternal scenario let us consider a domain containing a sufficiently large scalar field $\phi$ so that $V(\phi)$ dominates the energy density of the universe. The density fluctuation [ Linde in Ref.5 ] at $\phi_c$ (classical value of the inflaton field) is

$$\frac{\delta\rho}{\rho} \sim \frac{16}{\sqrt{3}} \; \frac{V^{3/2}(\phi_c)}{M_P^3\, V'(\phi_c)} \;. \qquad (2.46)$$

In a $\lambda\phi^4$ -theory, one may choose a $\phi = \phi_* \sim \lambda^{-1/6} M_P$ to make $\delta\rho/\rho \sim O(1)$. The initial configuration of the inflaton field is assumed to be fairly homogeneous so that the spatial gradient can be neglected. The field then rolls towards the *minimum* of the potential during which the universe expands exponentially. The roll down time depends on both classical and quantum effects . For $\lambda\phi^4$-theory, it is easy to show from (2.11) that the classical field $\phi$ in a Hubble time $t = H^{-1}$ decreases by $\Delta\phi \sim 1/\phi$ . However, inhomogeneities $\delta\phi(x)$ are also produced with a wavelength bigger than the size of the event horizon $H^{-1}$ during inflation with an average amplitude $|\delta\phi| \sim \frac{H}{2\pi} \sim \phi^2$ .[104] Therefore, $|\delta\phi| << \Delta\phi$ if $\phi << \phi_*$ . In this case the fluctuations of the field $\phi$ practically do not influence the process of rolling of the field down to the minimum of $V(\phi)$ . However the behavior of the field $\phi$ in domains with $\phi >> \phi_*$ is much more interesting. Let us consider a domain of size $H^{-1}$ with a homogeneous field $\phi >> \phi_*$ of size $H^{-1}$. In the inflationary regime the domain $l \gtrsim O(H^{-1})$ in a typical time $\tau \sim H^{-1}$ grows $e^{H\tau} \sim e$ times and its volume increases by $e^3$ times. As a result, after a time $H^{-1}$ , $O(e^3)$ miniuniverses of size $O(H^{-1})$ are formed. In half of these domains, the fluctuations will increase the magnitude of $\phi$ by $\phi + \delta\phi - \Delta\phi \simeq \phi + \delta\phi$ . These domains in turn will expand independently and will create further domains with fields $\phi >> \phi_*$ . Thus, the inflaton field permanently grows in half of the domains. The increase in $\phi$ increases $V(\phi)$ and $H(\phi)$ . The potential $V(\phi)$ which was many times smaller than $M_P^4$ at $\phi >> \phi_*$ gradually increases. The geometry of the miniuniverses produced may be different from that of the parent universe which was created at some initial moment $t = 0$ and will disappear at $t = t_{max}$. The main part of the universe appears as a result of the expansion of a domain containing a maximal possible field $\phi \sim \lambda^{-1/4} M_P$ at which $V(\phi) = \frac{\lambda}{4} \phi^4 \sim M_P^4$ . It can be shown that the process of the formation of inflationary miniuniverses with a growing field $\phi$ becomes suppressed at $\phi >>$

$\lambda^{-1/4}M_p$ because $V(\phi) \gg M_p^4$. Thus the process of self creation occurs for the range $\lambda^{-1/6}M_p < \phi < \lambda^{-1/4} M_p$. However, in the other half domains where fluctuations have decreased $\phi$ below $\phi_*$ the above process will not occur. Rather each of the domains expands exponentially to a size $M_p^{-1} \exp(\lambda^{-1/3}) \sim 10^{10^4}$ cm., and evolves eventually into a universe like our own. The total physical volume of the universe filled with a permanently growing field $\phi$ increases as $\exp[(3 - \ln 2) Ht]$ and the physical volume of domains in which the field $\phi$ does not decrease, grows as $\frac{1}{2} \exp(Ht)$. This leads to two important consequences :

i) A domain of size $l \gtrsim O(H^{-1})$ containing the field $\phi \gtrsim \lambda^{-1/6}M_p$ reproduces other miniuniverses with $\phi \gtrsim \lambda^{-1/6}M_p$. This process of creation of the miniuniverses occurs without an end. The interesting feature of the scenario is that, it is not essential to assume that the universe was created at some initial moment $t = 0$. The process of the creation and self-reproduction of the universe instead may occur eternally. Consequently, it may have no beginning and no end.

ii) As the mini-universe formation occurs at $\phi \gtrsim \lambda^{-1/6}M_p$ i.e. at $V(\phi) \sim 10^{-4} M_p$ for $\lambda \sim 10^{-12}$, the process of self-reproduction, therefore, occurs at densities much smaller than the Planck density $O(M_p^4)$. The quantum fluctuations of the metric in the miniuniverses at this small density are considered to be small. This leads to an unchanged space-time structure in the new born universe. However, if the field $\phi \sim O(\lambda^{-1/4}M_p)$ then the corresponding energy density is $\sim O(M_P^4)$ . In this case fluctuations of all fields and of metric are very large for a typical time scale $\sim H^{-1} \sim M_P^{-1}$. This behavior leads to the generation of different classical scalar fields $\Phi_i$. The quantum fluctuations may transfer the classical fields $\Phi_i$ in the new born miniuniverse from one minimum of the effective potential $V(\Phi_i, \phi)$ to another minimum. An interesting scenario may result if one considers chaotic inflation in the Kaluza-Klein or superstring theories. When the energy density of the field $\phi$ grows to $O(M_P^4)$, the quantum fluctuations of the metric in the corresponding domain become unity in the Planck scale. It was demonstrated by Linde that in such domains an inflating D-dimensional universe can squeeze locally into a tube of smaller dimension D-n. If the reduced (D-n) dimensional tube of length greater than $M_P^{-1}$ is also inflationary then the subsequent expansion may occur independently of its prehistory and of the fate of the parent universe. In this way the inflationary universe becomes divided into a number of miniuniverses of all possible types of compactification and dimensionalities.

Subsequently it was shown[91] that the eternal chaotic scenario is more generic in which the constraint $\phi \gtrsim C\, M_P$ where $C = O(3)$ for a massless self-interacting scalar field, required to obtain sufficient inflation in the chaotic scenario is not essential in the context of the eternal scenario . It is shown that if $\phi$ is somehow constrained in the interval $0 < \phi \lesssim C\, M_P$ , sufficient inflation may still be obtained by considering a combination of different types of scalar fields $\Phi_i$ . The evolution of the later stage of the universe is driven by the lightest scalar field $\phi_1$ which also determines the large scale structure of the observable universe.

It has been shown[97] that the eternal chaotic inflationary scenario can be realised even in the $R^2$-model. The scenario behaves naturally to fit into a quantum scenario of the universe. Recently, Futamase and Maeda (FM)[94] have shown that if one considers a $\frac{1}{2}\xi R \phi^2$ -theory where $\xi \neq 0$ for non minimal coupling ( $\xi = 1/6$, for conformal coupling ) the chaotic inflationary scenario of the universe will not be realized unless $\xi$ is negative or sufficiently small $\xi \lesssim 10^{-3}$. Futamase *et al*[95] have shown in a subsequent paper that the addition of anisotropy generally rules out the possibility of inflation for $\xi \gtrsim 10^{-2}$ but allows inflation for $\xi < 0$. Recently Maeda *et al*[96] have shown that the presence of $R^2$- term in the gravitational action gives rise to a chaotic inflation smoothly at all values of $\xi$ .

## *CHAPTER III

## HIGHER DERIVATIVE MODELS

*A major part of the contents of this chapter has already been published in *Ref.* (110).

## III.1 *INTRODUCTION :*

An inflationary phase seems to be an essential part of all recent models of the early universe. The idea of an early inflationary phase was proposed by Starobinsky[9] and others[6-8,10] long before the advantages of an inflation were fully realized. The original motivation for the model was to remove the singularity in the cosmological solutions of the classical general relativity. The presence of the singularity may indicate that at length scale smaller than $l_p \sim 10^{-33}$ cm, the classical theory breaks down, making it necessary to use a quantum theory of gravity. However, Einstein's theory cannot be quantized in a simple way. The theory is known to be non-renormalizable. The theory, however, can be made renormalizable by adding curvature squared terms to the usual Einstein-Hilbert action. As we will see, the modified version of the Starobinsky model is based on such a higher derivative theory.

The original model[9] of Starobinsky is described by the self-consistent solution of the semi-classical Einstein equation :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - 8\pi G < T_{\mu\nu} > \qquad ( 3.1 )$$

where $< T_{\mu\nu} >$ is the vacuum expectation value ( VEV ) of the stress energy tensor, $G = M_p^{-2}$ and $M_p$ is the Planck mass. The trace of the energy momentum tensor vanishes classically. In a curved space-time, even in the absence of classical matter or radiation, quantum fluctuations of matter fields give non-trivial contributions to $\langle T_{\mu\nu} \rangle$. The value of $\langle T_{\mu\nu} \rangle$, however, can be expressed as combinations of some of the four possible second order curvature invariants that can be constructed from the scalar (R), Ricci tensor ( $R_{\mu\nu}$ ) and Riemann curvature tensor ( $R_{\mu\nu\alpha\beta}$ ) : these are $R^2$ , $R_{\mu\nu} R^{\mu\nu}$ , $R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}$ and $C^{\mu\nu\alpha\beta} C_{\mu\nu\alpha\beta}$ where $C_{\mu\nu\alpha\beta}$ is the Weyl tensor. If we take a conformally flat space-time, the Weyl tensor vanishes and in the case of free, massless, conformally invariant fields the VEV of the stress tensor takes the simple form[27-31]:

$$\langle T_{\mu\nu} \rangle = q \, {}^{(1)}H_{\mu\nu} + w \, {}^{(3)}H_{\mu\nu} \tag{3.2}$$

where q and w are the numerical coefficients and

$${}^{(1)}H_{\mu\nu} = 2 R_{;\mu;\nu} - 2 \Box R g_{\mu\nu} + 2 R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^2 , \tag{3.3}$$

$${}^{(3)}H_{\mu\nu} = R_{\mu}{}^{\sigma} R_{\nu\sigma} - \frac{2}{3} R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^{\sigma\tau} R_{\sigma\tau} + \frac{1}{4} g_{\mu\nu} R^2 . \tag{3.4}$$

We note that

$$R^{\alpha}_{\beta\mu\nu} = \partial_{\nu} \Gamma^{\alpha}_{\beta\mu} - \partial_{\mu} \Gamma^{\alpha}_{\beta\nu} + \Gamma^{\alpha}_{\sigma\nu} \Gamma^{\sigma}_{\beta\mu} - \Gamma^{\alpha}_{\sigma\mu} \Gamma^{\sigma}_{\beta\nu} , \tag{3.5}$$

$$R_{\mu\nu} = R^{\sigma}{}_{\mu\sigma\nu} , \tag{3.6}$$

$$R = R^{\mu}{}_{\mu}. \qquad (3.7)$$

The tensor ${}^{(1)}H_{\mu\nu}$ is identically conserved, ${}^{(1)}H_{\mu}{}^{\nu}{}_{;\nu} = 0$ and can be obtained by varying a local action :

$$\frac{2}{\sqrt{-g}} \frac{\delta}{\delta g^{\mu\nu}} \int R^2 \sqrt{-g}\, d^4x \quad .$$

The infinities in the $\langle T_{\mu\nu} \rangle$ can be absorbed by introducing counter terms in the gravitational Lagrangian. A counter term of the form $C R^2 \sqrt{(-g)}$ is also needed, where C is a logarithmically divergent constant. The coefficient q , therefore, can take any value since an arbitrary finite part can always be added to C . Perhaps, the coefficient can be determined by some experiment. The tensor ${}^{(3)}H_{\mu\nu}$ is conserved only in a conformally flat space-time and cannot be obtained by varying a local action. Its coefficient w is, however, uniquely determined by

$$w = \left(N_0 + (11/2)\, N_{1/2} + 31\, N_1\right) / 1440\,\pi^2 \qquad (3.8)$$

where $N_i$ ( i = 0,1/2, 1 ) give the number of quantum fields having spin- 0, 1/2, 1 respectively. The VEV of the trace of the stress energy tensor is now non-zero

$$\langle T_{\nu}{}^{\nu} \rangle = - 6\, q\, \Box R + w \left( \frac{1}{3} R^2 - R^{\sigma\tau} R_{\sigma\tau} \right) . \qquad (3.9)$$

This is known as the conformal anomaly. Starobinsky[9] has shown that the de Sitter phase is entirely driven by this anomaly. However, the de Sitter stage is unstable for $w > 0$ and $q < 0$, allowing eventually an exit from the inflationary epoch.

The Starobinsky model, however, runs into problems when confronted with the measured limits on the anisotropy of the Micro-wave Background Radiation[32] ( in short, MBR ), $\left( \frac{\Delta T}{T} \right)^2 \lesssim 3 \times 10^{-8}$. This inequality will hold in the model only if $w \gtrsim 10^{10}$. The model, therefore, needs a substantial change unless one admits the existence of more than $10^{10}$ matter fields in the early universe. Starobinsky[33], Pollock[34] and Kofman *et al*[78] have shown that if a local term proportional to $R^2$ is added to the Einstein action, the constant $q$ is effectively renormalized. The renormalized value of $q$ may then be treated as a free parameter with $q M_P^{-2} \sim M^{-2}$. If one assumes that $M \ll M_P$ and $R_{\mu\nu\rho\delta} R^{\mu\nu\rho\delta} \ll M_P^4$, [ which is necessary for the validity of the one loop quantum correction ] the term containing $w$ may be neglected. In some supergravity models even $w = 0$ occurs naturally. With these modifications, the Starobinsky model reduces to the study of the field equation corresponding to the higher derivative theory with an action :

$$S = \frac{M_P^2}{16 \pi} \int \left( R + \alpha R^2 \right) \sqrt{-g} \; d^4x \quad . \qquad (3.10)$$

It is known that with suitable counter terms viz. $C_{\mu\nu\rho\delta}\, C^{\mu\nu\rho\delta}$, $R^2$ and $\Lambda$ added to the Einstein action, one gets a perturbation theory which is well behaved and formally renormalizable.[35] The theory is asymptotically free.[36,37] Nevertheless, the theory was not considered earlier as an acceptable theory of quantum gravity because the theory was thought to be non-unitary, the bare graviton propagator containing an additional spin-2 ghost. Recently Antoniadis and Tomboulis[105] have argued that since the position of the ghost is unstable, a perturbation theory with bare propagators cannot be formulated and a modified perturbation expansion involving dressed propagators, which exhibits the unstable nature of the pole explicitly should be used. The position of the complex pole is found to be gauge dependent. They suggested that the S-matrix defined only for physical amplitudes with real external momenta and external transverse massless graviton interacting with matter fields should be unitary. Jhonstone[106], however, has pointed out that the argument given by Antoniadis and Tomboulis was not correct for all models, and, therefore, the unitarity of the $R^2$-theory has not been proved. Nevertheless, the $R^2$-theory may be considered as an effective low energy theory, which acts as a theoretical laboratory in which new ideas could be tested. In particular, the theory has some advantages as a testing ground of the ideas of quantum cosmology : (i) the higher derivative theory extends the range of applicability beyond the WKB approximation because it is renormalizable , at least in a perturbative sense, (2) it is also possible to obtain a finite result for a large oscillation of the conformal degrees of freedom of $g_{\mu\nu}$. The modified Starobinsky model is a special case of the generalized theory with curvature squared terms which are particularly important in the quantum gravity domain.

The action (3.10) may also be obtained from a general gravitational action containing terms linear and second order in curvature $[8\pi G = 1]$

$$I_G = \frac{1}{2}\int d^4x \sqrt{-g}\,\left[ R + \alpha R^2 + b\, R_{\mu\nu} R^{\mu\nu} + c\, R_{\mu\nu\rho\delta} R^{\mu\nu\rho\delta} \right] \tag{3.11}$$

where $\alpha$, $b$ and $c$ are constants. We can make use of the identity which is valid for any 4-dimensional space-time :

$$\frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{-g}\,\left[ R^2 - 4 R_{\alpha\beta} R^{\alpha\beta} + R_{\alpha\beta\gamma\delta} R^{\alpha\beta\gamma\delta} \right] = 0 \;. \tag{3.12}$$

An additional identity exists for an isotropic and homogeneous space-time

$$\frac{\delta}{\delta g^{\mu\nu}} \int d^4x \sqrt{-g}\,\left[ R^2 - 3 R_{\alpha\beta} R^{\alpha\beta} \right] = 0 \;. \tag{3.13}$$

Using the identities (3.12)-(3.13), the action (3.11) can be always written as :

$$I_G = \frac{1}{2}\int d^4x \sqrt{-g}\, L(R) \tag{3.14}$$

where $L(R) = R + \alpha R^2$. The action (3.14) is thus the most general action, quadratic in curvatures in conformally flat space-time, excluding the cosmological term. If matter is present the energy momentum tensor of the matter will be given by

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \quad \frac{\delta I_M}{\delta g^{\mu\nu}} \tag{3.15}$$

where $I_M$ is the matter action. The field equations can now be derived by varying the total action $I = I_G + I_M$ ;

$$L' \; R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} L + L'' \; \left( \nabla_\mu \nabla_\nu R - \Box R \; g_{\mu\nu} \right) + L''' \; \left( \nabla_\mu R \; \nabla_\nu R \right.$$

$$\left. - \nabla^\sigma R \; \nabla_\sigma R \; g_{\mu\nu} \right) + T_{\mu\nu} = 0 \tag{3.16}$$

where $\Box = g_{\mu\nu} \; \nabla^\mu \; \nabla^\nu$, $\nabla$ is the covariant differential operator and the prime denotes a derivative w.r.t. R.

We consider a Robertson-Walker metric

$$ds^2 = dt^2 - a^2(t) \left[ \frac{dr^2}{1 - k \; r^2} + r^2 \; ( \; d\theta^2 + \mathrm{Sin}^2\theta \; d\Phi^2 ) \right] \tag{3.17}$$

where $k = +1, \; 0 \; , -1$ for closed, flat and open space respectively. The symmetry of the space-time tells us that only two of the equations (3.16) are independent. We, therefore, consider the trace of (3.16) and its time-time component only. These are

$$R \; L' - 2 \; L - 3 \; L'' \; \left( \ddot{R} + 3 \frac{\dot{a}}{a} \dot{R} \right) - 3 \; L''' \; \dot{R}^2 + T = 0 \; , \tag{3.18}$$

$$L' R_{00} - \frac{1}{2} g_{00} L + 3 L'' \left( \frac{\dot{a}}{a} \right) \dot{R} + T_{00} = 0 \quad . \qquad ( 3.19 )$$

It is sufficient to consider the second equation, as the first equation can be derived from it by differentiating with respect to time.

Cosmological models corresponding to the action ( 3.10 ) have been investigated by Mijic *et al*[107], Pollock[108] and Paul *et al*[109,110] in a flat FRW -metric. These authors studied the classical evolution of the early universe leading to particle creation, their thermalization and a subsequent transition to a radiation dominated universe. Kofman *et al*[78], Müller and Gottlöber[111] and Starobinsky[33], have considered an extra inflaton field in the $R^2$-theory. The results of their investigations show that the presence of an inflaton field increases the inflationary period. Another motivation[78] for introducing the inflaton field was to realise the chaotic scenario within the Starobinsky model. Kofman *et al*[78] have noted that the effect of vacuum polarisation near the singularity changes the effective equation of state to that of a radiation. The inclusion of the field is, however, not an essential ingradient of the model. It has been found[50,112] that the de Sitter stage which corresponds to a constant $R$, is a solution of a general Lagrangian in the absence of matter if the condition

$$L' R - 2 L = 0 \qquad ( 3.20 )$$

is satisfied. The stability of this Stage in the higher derivative gravity was studied in a different context by Barrow and

Ottewill[50], Müller *et al*[112], Vilenkin[113], Barrow and Cotsakis[114] and Chimento[115]. It is found that for a positive coupling constant $\alpha > 0$, and no matter, there is a de Sitter phase which is stable. In this case the inflation is an asymptotic stage and not an intermediate one. However, in the other case $\alpha < 0$, the de Sitter stage is unstable but the Friedmann radiation solution is stable. This result generalizes the idea of Ruzmaikin and Ruzmaikina.[116] In 1970, they noted some interesting properties of the spatially flat, radiation filled, homogeneous and isotropic solutions of the quadratic Lagrangian theory. The curvature squared terms were utilized by them to remove the singularity problem but when they did so the model failed to approach towards the Friedmann era. It is now clear that the above observation is related to the instability of flat Minkowski space. This result suggests that such solutions are physically inadmissible. Gurovich[117] has examined the effect of logarithmic terms on the quadratic Lagrangian theory. He found that by adding a term of the form $R^2 \ln (R/R_o)$ where $R_o$ is a constant, models can be constructed that are singularity free as $t \to 0$ and also approaches the Friedmann cosmology at late times. This is because the logarithmic term changes its sign when $R \lessgtr R_o$. The de Sitter stage eventually leads to a radiation dominated FRW-universe. Another interesting observation in this connection was made by Whitt.[118] He has shown that by conformally transforming the metric, the quadratic theory may be written in the form of gravity with a minimally coupled scalar field interacting with a potential which is flat enough to produce sufficient inflation. Pollock[108] has utilized this result to calculate the density perturbation amplitude ( $\frac{\delta\rho}{\rho}$ ). Anderson[119] has suggested that the vacuum energy density in a curvature squared theory plays an important role even now and could be large enough to close the universe. In the original Starobinsky model there exists a de Sitter stage whereas in the modified model in addition to the de Sitter solution there exists another type of solution which we call non-de Sitter (NdS) solution. The evolution of the de Sitter phase has been studied extensively in the literature.[50,112-115] Vilenkin[113] has shown that the de Sitter stage in the Starobinsky model produces sufficient inflation to solve the cosmological problems. For an initial Hubble parameter H < $H_0$ where $H_0 \sim M_P$, H(t) will decrease in time. However, when H << $H_0$ , the decrease in H(t) will be linear with time and the subsequent evolution should be the same as in $R^2$-model. The NdS solution obtained by us will be discussed shortly. Inflation in this model occurs in different stages beginning with a non-exponential inflationary phase. Mijic *et al* [107] have noted similar pre-exponential stage in the case of a flat model. Considering a closed universe, Gottlöber and Müller[120] have also noted a pre-inflationary stage. In comparison to the original Starobinsky model,[9] these results show that an initial de Sitter phase is not essential. However, these models differ from our model in the choice of initial conditions necessary for the classical evolution. In this chapter, we will emphasize on these new features. In sec. III.2, we shall set up the field equations for the modified model and discuss the new solutions. In sec. III.3, we discuss the cosmological solutions. Our conclusions will be given in section III.4 .

## III.2 *FIELD EQUATIONS IN THE MODIFIED STAROBINSKY MODEL :*

We consider the FRW-metric (3.17) and obtain the field equation from (3.16) in the modified Starobinsky model as :

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \frac{1}{6M^2} \left[ 2 R_{;\mu;\nu} - 2 g_{\mu\nu} R^{;\sigma}_{;\sigma} + 2 R R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^2 \right] - T_{\mu\nu} \qquad (3.21)$$

where $M^2 = 1/6\alpha$ . For the metric (3.17), the field equation (3.21) may be written in the absence of matter as

$$12 H^2 = \frac{R^2 - 12 H \dot{R}}{R + 3 M^2} - 12 \frac{k}{a^2} \qquad (3.22)$$

where

$$R = 6 \dot{H} + 12 H^2 + 6 \frac{k}{a^2} \qquad (3.23)$$

with $H = \dot{a}/a$ . The scalar curvature $\mathbf{R}$ may be looked upon as a homogeneous scalar field (scalaron) which satisfies the equation :

$$\ddot{R} + 3 H \dot{R} + M^2 R = 0 \quad , \qquad (3.24)$$

which is obtained by taking the trace of the eq.(3.21).

## III.3 *COSMOLOGICAL SOLUTIONS* :

We assume that at a time $t \sim t_P$, the classical universe somehow emerges out of the quantum foam and starts evolving following the equations (3.22) and (3.24). We assume that at the beginning $R \gg 3M^2$. This assumption reduces the field equations to :

$$12 H^2 = R - 12 H \frac{\dot{R}}{R} - 12 \frac{k}{a^2} \quad , \qquad (3.25)$$

$$\ddot{R} + 3 H \dot{R} = 0 \quad . \qquad (3.26)$$

The reduced field equations (3.25) and (3.26) admit two different types of solutions at an early stage of evolution : *(a) de Sitter and (b) Non-de Sitter*, Which will be discussed in the following.

(a) de Sitter Solution :

The de Sitter solution corresponds to $\dot{R} = 0$. In this case eqs.(3.25) and (3.26) lead to

$$R = 12 \left[ H^2 + \frac{k}{a^2} \right] = \text{const.} \quad . \qquad (3.27)$$

For different k the solutions are given below

$$a(t) = \begin{cases} H_o^{-1} \text{ Cosh } H_o t & \text{for } k = +1 , \\ H_o^{-1} \exp H_o t & \text{for } k = 0 , \\ H_o^{-1} \text{ Sinh } H_o t & \text{for } k = -1 . \end{cases} \quad (3.28)$$

The de Sitter solution in the modified Starobinsky model is only an approximate solution. The stability of the de Sitter stage in the modified model has already been studied.[50,112-115] It has been noted that the de Sitter stage is unstable and eventually the universe enters into the FRW-radiation dominated phase. As the de Sitter phase of the $R^2$-theory has been discussed in the literature[50,107,108,112-115] in details, we will not elaborate on this.

**(b) Non de Sitter solutions :**

The important feature of this early stage of evolution is that $\dot{R} \neq 0$. The existence of a non-de Sitter solution in $R^2$-model is a new solution noted by us[109,110], Mijic *et al*[107] and Müller and Gottlöber[120]. The model exhibits entirely new features in the initial stage of the evolution of the early universe. The scenario evolves in different stages and one may arrange to get sufficient inflation so as to solve the cosmological problems.

The NdS solution is obtained by noting that the field equation (3.25) permits a general solution for $k = +1,\ 0$ of the form

$$H = \beta(t)\ R^2 \qquad (3.29)$$

where $\beta(t)$ is a time dependent parameter for $k = +1$, and constant for $k = 0$. The scalar curvature R satisfies the equation :

$$\dot{R} = \frac{1}{12\ \beta(t)} - \beta(t)\ R^3 - \frac{\dot{\beta}(t)}{\beta(t)}\ R \qquad (3.30)$$

where

$$\dot{\beta}(t) = k/(aR)^2 \ . \qquad (3.31)$$

The above relation (3.31) covers both the cases :

(i) $\dot{\beta}(t) = 0$, for $k = 0$ and (ii) $\dot{\beta}(t) \neq 0$, for $k = +1$.

The case of $k = -1$ will be considered separately .

We will first discuss the cosmological model for a flat ( $k = 0$ ) universe. We have studied the evolution for the early universe in three different stages :

**First stage :**

The post quantum era is characterised by the conditions $R \gg M^2$ and $\frac{3}{2}\dot{H} \gg M^2$. For $k = 0$, we have $\beta$ = constant and the field equations permit solution

$$H = \beta\ R^2 \ . \qquad (3.32)$$

The scalar curvature R satisfies the equation :

$$\dot{R} = \frac{1}{12\ \beta} - \beta\ R^3 \qquad (3.33)$$

We now show that for a given $\beta$, the evolution at a later stage is almost independent of the initial value, $R_o$, provided $R_o \gg 3M^2$. This is a desirable feature of the theory, since one needs, in that case, no explanation for the initial value chosen. From (3.33), it is evident that the choice $R_o \lessgtr \left[ \frac{1}{12\beta^2} \right]^{1/3}$ leads to $\dot{R}_o \gtrless 0$. Thus the initial value of R itself determines whether R will be in the increasing or decreasing mode. In both the cases, R tends asymptotically to the constant value $\left( 1/12\beta^2 \right)^{1/3}$. The evolution of R(t) and H(t) in both the cases with $\beta$ = 288.68 are shown in *fig* (1) and *fig* (2).

A relation connecting the age of the universe in this stage and the value of the scalar curvature may be obtained from (3.33) :

$$t = \left( 16\,\beta/3 \right)^{1/3} \left[ \frac{1}{2} \ln \frac{R^2 - \sigma R + \sigma^2}{( R + \sigma )^2} - \sqrt{3} \tan^{-1} \frac{R\sqrt{3}}{2\sigma - R} \right] - C \qquad ( 3.34 )$$

where $\sigma = - ( 1/12\,\beta^2 )^{1/3}$ and C is an integration constant. The integration constant may be utilised to fit the initial values. From the *figs* (1) and (2), it is clear that both R and H approach their asymptotic values with a gradually diminishing rate. Thus the universe undergoes an inflation with H varying slowly near a value determined by the constant $\beta$. However, this stage will continue till the condition $\frac{3}{2}\dot{H} \gg M^2$ is violated, when the universe exits from this stage.

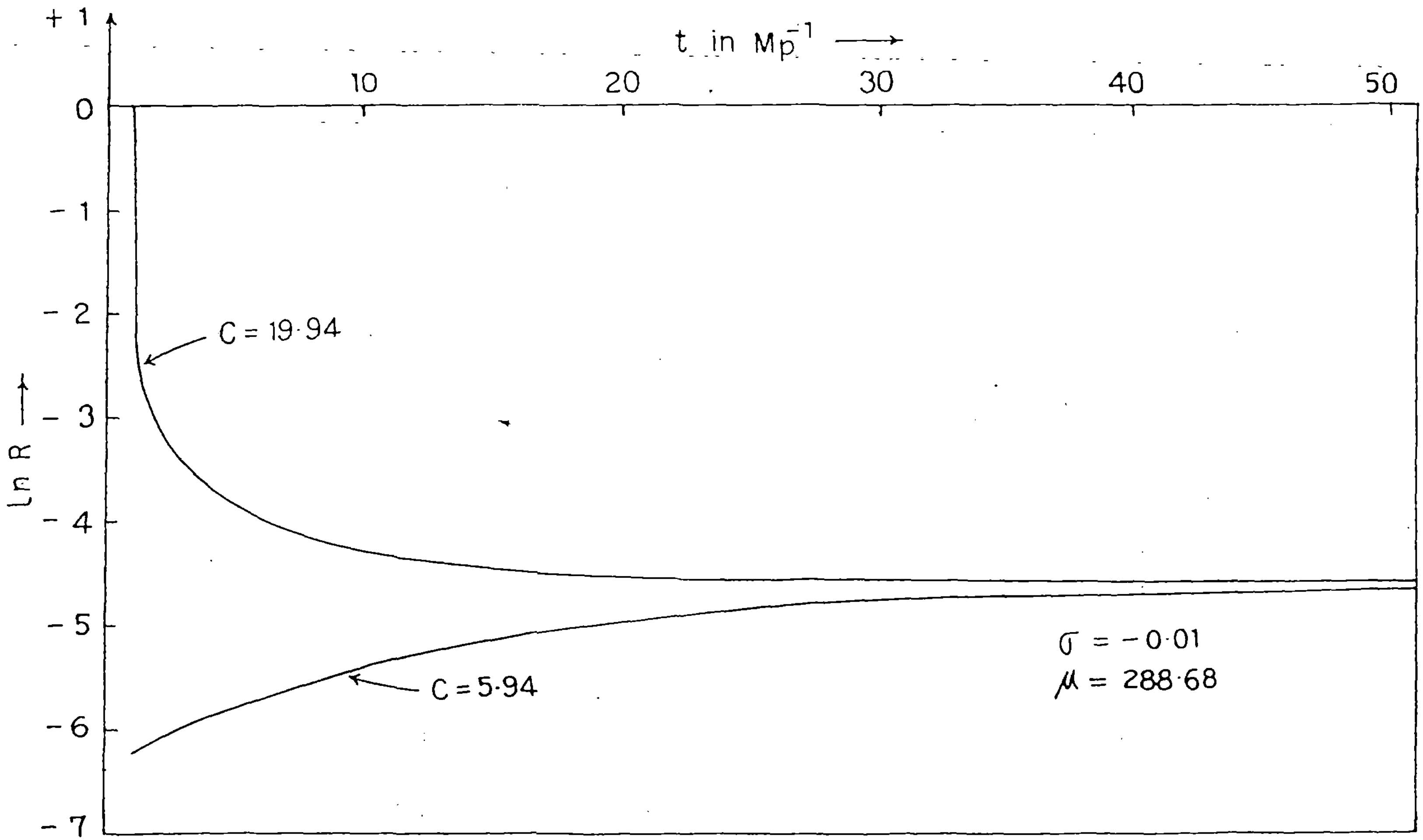


*fig.*(1) : Variation of the scalar curvature (R) in $\log_e$ scale with time (t) for $\beta$ = 288.68 and $R_o$ = 0.001 $M_p^2$ ( for increasing mode ) and $R_o$ = 1.0 $M_p^2$ ( for decreasing mode ).

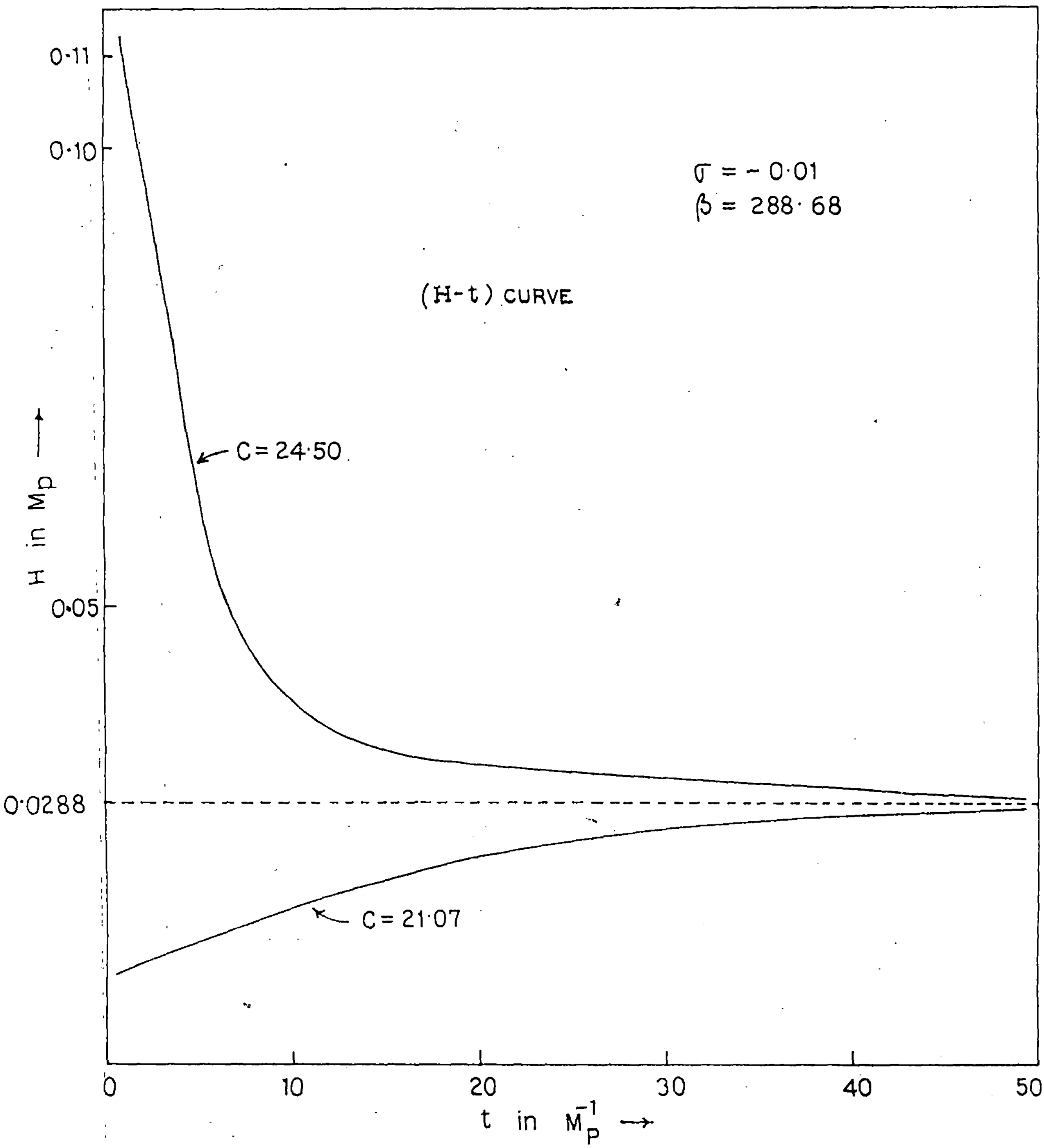


*fig.* (2) : Early evolution of the Hubble parameter (H) with time (t) in Planck scale.

It is possible to estimate roughly the inflation of the universe in this stage. From (3.23), (3.25) and (3.26) we obtain the equation for $H$ :

$$2 \frac{\ddot{H}}{\dot{H}} + 6 H - \frac{\dot{H}}{H} = 0 \qquad (3.35)$$

which may be integrated to obtain

$$\int_{t_0}^{\tilde{t}} H \, dt = \frac{1}{6} \ln \frac{H_1}{H_0} - \frac{1}{3} \ln \frac{\dot{H}_1}{\dot{H}_0} . \qquad (3.36)$$

In the above, $\tilde{t}$ gives the time when the first stage terminates and $H_1$ and $\dot{H}_1$ indicate the values of the quantities at time $\tilde{t}$. Since $H_1$ and $H_0$ are nearly of the same order, the first term on the r.h.s. of (3.36) is small. However, we may choose $\dot{H}_1 \approx 10^{-8} \dot{H}_0$, so that the second term gives an inflation $\sim e^6$. This value cannot be altered much, since the orders of values of $H$ and $\dot{H}$ at both the limits are almost determined in this scenario. It is also evident that in this stage, the magnitude of inflation ( $\sim e^6$ ) is almost independent of the undetermined constant $\beta$, which may be considered as another good feature of the scenario.

**Second stage :**

The field equation for this stage may be derived from eqs.(3.23), (3.25) and (3.26) respectively :

$$2 H \ddot{H} + 6 H^2 \dot{H} - \dot{H}^2 + M^2 H^2 = 0 . \qquad (3.37)$$

While it is difficult to solve this equation exactly, we note that as $\dot{H}$ decreases to a value $\sim M^2$, $H(t)$ becomes slowly varying, and one may get $\dot{H} \ll H^2$ and $\ddot{H} \ll H\dot{H}$. The solution is now given by

$$a = a^* \exp\left[ - \frac{M^2}{12} \left( t^* - t \right)^2 \right] \qquad (3.38)$$

and

$$H = \frac{M^2}{6} ( t^* - t ) \qquad (3.39)$$

where $t^*$ is the time when the second stage terminates and $a^* = a(t^*)$. It is evident that during the transition period, i.e. starting from $\tilde{t}$ till the onset of the second stage, $H$ decreases smoothly from a value $H \approx ( 1/12\, \beta^2)^{1/3}$ to a value $\sim \frac{M^2}{6} t^*$. The smoothness being guaranteed by the fact that $\dot{H}$ remains small ( $\sim M$ ) throughout this period. This stage may be attained provided the earlier evolution creates the necessary conditions $\ddot{H} \ll H\dot{H}$ and $\dot{H} \ll H^2$, which of course the present scenario readily permits. This stage terminates when the above conditions are violated by $H$ becoming too small ( $\ll M$ ) The duration of this stage may be estimated from the requirement

$$\int_{\tilde{t}}^{t^*} H\, dt \gtrsim 60 \qquad (3.40)$$

so that $t^* \gtrsim 10^5\, t_P$ if $M = 10^{-4}\, M_P$, giving a rather long inflationary stage.

**Third stage :**

When $H \ll M$, we expect another phase of evolution to commence. Since t is already very large ( $\gg M^{-1}$ ), a power series expansion in $(Mt)^{-1}$ may be useful here in determining an approximate solution of the equation (3.37). An oscillatory solution has already been studied in this connection by some authors.[9,107,113] Upto terms of order $O(M^{-2}\, t^{-2})$ , one has

$$H = \frac{4}{3\,t} \,\mathrm{Cos}^2 \frac{Mt}{2} \left[ 1 - \frac{\mathrm{Sin}\, Mt}{Mt} \right] \qquad (3.41)$$

where the scale factor a(t) is given by

$$a(t) = \mathrm{const.}\;\; t^{2/3} \left[ 1 + \frac{2}{3\,Mt} \,\mathrm{Sin}\, Mt \right] . \qquad (3.42)$$

The oscillation of the expansion rate can be thought of as coherent oscillation of a massive field, the 'scalaron' . Following Vilenkin[113] and Zeldovich and Starobinsky,[121] the rate of particle production can now be calculated. As the oscillation of the scale factor gets damped, the particles produced thermalise leading to a radiation dominated universe with a temperature

$$T_{th} \sim |\tilde{m}^2| \left( M\, M_p \right)^{-1/2} \qquad (3.43)$$

where $\tilde{m}^2 = m^2 - 3\,(\xi - 1/6)\,M^2$ , m ( $<$ M/2 ) being the most massive particle created and $\xi$ the conformal parameter. If $M \lesssim 10^{14}$ GeV, the reheating temperature $T_{th} \lesssim 10^{12}$ GeV, which is sufficiently low to solve the monopole problem. The choice $M \lesssim 10^{14}$ GeV also leads to a small density perturbation amplitude as expected. At a still later time, the universe enters into a matter dominated phase with $a \sim t^{2/3}$, as can be seen from (3.42).

Let us now consider the evolution in the case of a closed universe , $k = +1$ .It can be seen that the only choice for $\beta(t)$ permitted by the field equation is

$$\beta = \frac{a^3}{64\sqrt{3}} . \qquad (3.44)$$

With the choice $R_o \gg 3 M^2$, i.e. for a pure $R^2$-theory, the relation

$$H = \frac{1}{\sqrt{3}\, a} \qquad (3.45)$$

is exactly satisfied. The scale factor of the universe in this phase of evolution grows linearly in time as

$$a(t) = a_o + \frac{1}{\sqrt{3}} t . \qquad (3.46)$$

The scalar curvature (R) and the Hubble constant (H) are related as $R = 24 H^2$ . However, this solution is unstable. Consequently, this phase ends eventually and the universe enters in a different phase which closely follows the evolution outlined above for a flat universe. An interesting aspect of this NdS solution is that one needs initial conditions $a_o \neq 0$ and $\dot{a}_o \neq 0$ for the evolution of the early universe. The initial conditions are ,therefore, different from those that have been studied in the context of quantum cosmology of the $R^2$-theory by Hawking and Luttrell[122] and Vilenkin[113], where the classical universe emerges in a de Sitter phase with $\dot{a}_o = 0$ , $a_o \neq 0$.

Note that the exact relation between the Hubble parameter and the scale factor a(t),

$$H = \frac{1}{\sqrt{3}\ a} \qquad (3.47)$$

is true as long as $H \gg M$ i.e. when $R^2$-term dominates in the action. However, with an increase in a(t), the magnitude of the Hubble parameter (H) decreases and the solution will no longer be valid as the neglected terms become important. The universe, therefore, switches over to another phase of expansion. The different phases of evolution in this case will be similar to those followed by a flat universe, as outlined above. The non-de Sitter solution may be interesting from the quantum cosmological point of view, because of the special initial conditions in which the classical universe may emerge in the $R^2$ quantum cosmology .

For an open universe (k = − 1) also there is a NdS solution of the equations(3.22)-(3.24), the relevant field equation can be written as

$$2\,H\,\ddot{H} + 6\,H^2\,\dot{H} - \dot{H}^2 + \frac{2}{a^2}\,H^2 + \frac{1}{a^4} + M^2\left(H^2 - \frac{1}{a^2}\right) = 0\ . \qquad (3.51)$$

The equation permits an exact solution

$$H = a^{-1} \qquad (3.52)$$

which also begins with a linearly expanding scale factor. But the scalar curvature R = 0 in this case. The solution is not very interesting from a physical point of view.

## III. 4 *CONCLUSIONS :*

To conclude, the modification in the Starobinsky model suggested brings in a substantial change in the earlier stages of the scenario. The modified Starobinsky model in an isotropic Robertson-Walker metric admits two different types of evolution (i) de Sitter and (ii) Non-de Sitter ( NdS ). The de Sitter stage is an approximate solution of the field equation which eventually makes a transition to a FRW -universe. On the otherhand, NdS evolution of the early universe has different stages : (1) pre-inflationary , (2) quasi-exponential and (3) particle production. The initial data for the NdS evolution is different from those in the de Sitter evolution. This may be interesting from the quantum cosmological point of view. The quantum creation of a universe with NdS initial condition will be studied elsewhere.[123]

*CHAPTER IV*

*HIGHER DIMENSIONAL MODELS*

*A major part of the contents of this chapter has already been reported in *Refs.* (124) & (125).

## IV.1 *INTRODUCTION :*

Model building in higher dimensions was initiated by Kaluza and Klein, who tried to unify gravity with electromagnetic interaction by introducing an extra dimension.[126] The topology of the 5-dimensional world was chosen to be $M^4 \times S^1$, where $M^4$ is the 4 D Minkowski space and $S^1$ is a circle. A dimensional reduction mechanism was invoked to realise the 4 D theory. The basic philosophy of the approach has been to realise the gauge symmetries of the effective four dimensional theory from the isometries of the extra-space. Inspite of its aesthetic appeal, the theory was beset with a number of problems. The basic question why the universe treats three of its spatial dimensions differently from the rest was also not answered.

The Kaluza-Klein philosophy has been revived[127] and considerably generalized during the last two decades, once it was realised that many interesting theories of particle interactions require more than four dimensions for their formulation. It is, therefore, essential to check if consistent cosmological solutions, which can accommodate these theories are also allowed. Attempts have been made to build models in higher dimensions which may undergo a spontaneous compactification,[128] i.e. to a product space $M^4 \times M^d$ , $M^d$ describing the compact 'inner' space. One of the first attempts to build a higher dimensional cosmological model was made by Chodos and Detweiler.[129] It was shown that a

5 dimensional universe admits an anisotropic Kasner type behavior in which the 3-space scale factor expands while the extra space scale factor contracts. This model, however, is not able to solve the cosmological probleme. Later, Sahdev[48] obtained an inflationary scenario in a higher dimensional universe. Considering a higher-dimensional radiation filled universe, he finds a solution in which the 3-space expands forever while the scale factor for the extra space, b(t) grows at the beginning, attains a maximum value and then decreases to attain a stable value of the Planck order. The stability may be due to some unknown quantum gravity effects. When the inner space reaches the static configuration, the universe may appear as the usual FRW universe in 4 dimensions. Abbott *et al*[49] have shown that for sufficient inflation and entropy generation it is essential to consider either a large number of the extra dimension $D \gtrsim 40$ or initially a large internal size of the universe. Several other attempts[130] were made to construct models in which the presence of extra dimensions leads to inflation. But the results of Okada[131] have made it clear that extra dimensions alone are not sufficient to get a realistic cosmology. One has also to consider specific particle interactions which are important in the very early universe. Alvarez and Gavela[132] (AG) have shown that in an adiabatic process, a large amount of entropy may be generated due to the shrinking of the extra space. It was also noted[133] that sufficient heating occurs due to entropy generated by AG process.

The revival of the Kaluza-Klein theory in its modern version[127] on the otherhand is due to its possible connection with the supergravity theories. In particular, the spontaneous compactification mechanism has made it possible to obtain[134] N = 8, d = 4 ( N is number of supersymmetric generators, d is the dimension of space-time ) supergravity starting from N = 1, d = 11 supergravity. The advent of superstring theories[39] saw a spurt in model building activities in higher dimensions. Since the quantum consistency of superstring theories can be obtained only in D = 10 dimensions, one has to look for a suitable compactification scheme. Candelas *et al*[40] obtained a solution in which the universe compactifies to $M^4 \times K$, where $M^4$ is a Minkowskian space and K is a Ricci flat Calabi-Yau manifold. They considered the quadratic term $R_{ABCD}\ R^{ABCD}$ . However, a simple minded string-corrected action is not expected to give satisfactory results. Zwiebach[51] suggested that the string corrections to Einstein action upto first order in the slope parameter $\alpha'$ and fourth power of momenta should be a term of the type $\alpha'$(GB). It was, however, soon realised that due to the very nature of the slope expansion, the field-redefinition theorem of 't Hooft and Veltman[135] is applicable to this case. Thus, on Einstein shell ($R_{\mu\nu} = 0$), an action of the form $R + a\ R^2_{\mu\nu} + b\ R^2$ can be transformed into R itself (neglecting higher order terms) by field redefinition

$$g_{\mu\nu} \rightarrow g_{\mu\nu} + a\ R_{\mu\nu} + g_{\mu\nu}\ \frac{a + 2b}{2 - D}\ R \qquad (\ 4.1\ )$$

where D gives the number of dimensions. It has been shown by Deser and his collaborators[58,136] that because of the above results, on the linearized Einstein shell, one cannot tell the action $R + \alpha'(GB)$ and $R + \alpha' R^2_{\mu\nu\alpha\beta}$ apart and this result generalizes to all higher order ghost terms. Thus the quadratic curvature gravity theories are actually unitary.

Some attempts[53-56] have already been made to build cosmological models with various higher derivative terms including the Gauss-Bonnet ( GB, henceforth ) terms in the action. Boulware and Deser[58] have pointed out that the inclusion of dilaton in the Einstein action with the GB terms removes the de Sitter ground state permitted in its absence. Bailin *et al*[45] found that the GB terms permit an approximate stable Friedmann solution with a slowly varying compact internal space which is unstable with the $R^{ABCD} R_{ABCD}$ term only. Wetterich[59] has obtained the ground state $M^4 \times C$ for theories with a linear combination of the quadratic terms under certain restrictions. Lorentz-Petzold[53] has recently obtained cosmological solutions with the GB terms in the presence of matter. Henriques[54] has studied the action with GB terms in order to build a cosmological scenario but the field equation considered by him is not correct. Accetta *et al*[137] studied the stability of compactification by balancing the contributions of antisymmetric fields against one loop correction. It is obvious that a consistent scenario of the early universe and its subsequent evolution into a FRW stage remains to be fully developed. Cosmological model building in higher dimensions is an exercise which may provide a clear understanding of the special role of the GB terms. With this object in view, we have presented in this chapter a cosmological model with the GB terms taking into account various effects, e.g. the matter loop correction and the thermal effects. We considered an action in which the coefficients of R and GB terms are arbitrary to begin with. These will, however, be determined when the solution is tailored to fit the expected cosmological scenario. Also , we will not restrict to the Einstein shell and, therefore, the complete structure of the GB terms will be fully utilized.

We start with a 10-dimensional universe. To get a realistic four dimensional scenario at a later stage we assume that the ten dimensional universe is a hypersphere and its scale factor becomes large $\sim e^{22} M_P^{-1}$ even before the space dimensions 3 and 6 start evolving differently. This initial expanding phase of the 10-dimensional early universe is somehow arrested and the universe goes over to a stage when particles are produced, which eventualy thermalize. We thus get a hot symmetric universe containing heavy particles of different species. Perhaps the mechanism which makes the 10-D universe exit from the initial expanding phase, also forces the space dimensions to behave differently. We assume that the initial 10 D universe now has a geometry $( R^1 \times M^3 ) \times M^6$ , where the six dimensions undergo a spontaneous compactification. In section IV.2 , we shall study this model based on an appropriate action, and set up the field equations. The solution will be discussed in section IV.3 . The solution of the cosmological constant problem will be discussed in section IV.4 . In section IV.5 we will summarize our results. The last section will contain some concluding remarks.

## IV.2 *FIELD EQUATIONS WITH GAUSS-BONNET TERMS :*

We will consider a higher dimensional gravitational action which contains the Einstein term, the GB-terms and a matter field term :

$$S = \int \sqrt{-g}\ d^4x \left[ C_o + C_1 R + C_2 \left( R_{ABCD}R^{ABCD} - 4\, R_{AB}R^{AB} + R^2 \right) + L_m \right] \qquad (4.2)$$

where A,B, ... are ten dimensional indices ; $C_o$ , $C_1$ and $C_2$ are dimensional constants ; R, $R_{AB}$ and $R_{ABCD}$ are the Ricci scalar, the Ricci tensor and the Riemann tensor respectively, g is the determinant of the 10 dimensional metric. We choose units so that $\hbar = c = 1$ . The spatial manifold is assumed to be maximally symmetric and the line element is

$$ds^2 = - dt^2 + a^2(t)\ d\Omega_3^{\ 2} + b^2(t)\ d\Omega_6^{\ 2} \qquad (4.3)$$

with

$$d\Omega_3^2 = \frac{dr^2}{1 - K_3 r^2} + r^2 ( d\theta^2 + \mathrm{Sin}^2\theta \; d\phi^2 ) ,$$

$d\Omega_6^2$ is a generalization of the FRW form, where a(t) is the scale factor for 3-physical space and b(t) is that for the internal space and the spaces are characterised by $K_3$ and $K_6$. We shall choose both $K_3$ and $K_6$ to be equal to + 1 (closed model).

A variation of the action (4.2) with $g_{AB}$ yields the field equation :

$$- \frac{C_0}{2} g_A^{\ C} + C_1 \left[ R_A^{\ C} - \frac{1}{2} g_A^{\ C} R \right] - C_2 \left[ \frac{1}{2} \left( R_{BDEF} R^{BDEF} - 4 R_{BD} R^{BD} + R^2 \right) \delta_A^{\ C} - \left( 2 R_{BDEA} R^{BDEC} + 2 R R_A^{\ C} - 4 R_D^{\ B} R_{BA}^{\ \ DC} - 4 R_B^{\ C} R_A^{\ B} \right) \right] = T_A^{\ C} . \qquad (4.4)$$

The energy momentum tensor can be written as

$$T_{AB} = \{ \rho , p \, g_{ij} , p' g_{mn} \} \qquad . \qquad (4.5)$$

The field equations (4.4) corresponding to the metric (4.3) are given by

$$- C_1 \left[ 3 \left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right) + 15 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) + 18 \frac{\dot{a} \dot{b}}{a b} \right]$$

$$= 2 C_2 \left[ 90 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right)^2 + 180 \left( \frac{\dot{a} \dot{b}}{a b} \right)^2 + 36 \frac{\dot{a} \dot{b}}{a b} \right.$$

$$\left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right) + 360 \frac{\dot{a}\,\dot{b}}{a\,b} \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) + 90 \left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right)$$

$$\left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) \Big] + \Lambda - \rho \qquad , \qquad (4.6)$$

$$- C_1 \left[ 2 \frac{\ddot{a}}{a} + 6 \frac{\ddot{b}}{b} + 12 \frac{\dot{a}\,\dot{b}}{a\,b} + \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} + 15 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) \right]$$

$$= 2\, C_2 \Big[ 90 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right)^2 + 60 \left( \frac{\dot{a}\,\dot{b}}{a\,b} \right)^2 + 24 \frac{\ddot{a}}{a} \frac{\dot{a}\,\dot{b}}{a\,b}$$

$$+ 120 \frac{\ddot{b}}{b} \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) + 120 \frac{\ddot{b}}{b} \frac{\dot{a}\,\dot{b}}{a\,b} + 240 \frac{\dot{a}\,\dot{b}}{a\,b}$$

$$\left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b} \right) + 60 \frac{\ddot{a}}{a} \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) + 12 \frac{\ddot{b}}{b}$$

$$\left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right) + 30 \left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right) \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) \Big]$$

$$+ \Lambda + p \qquad , \qquad (4.7)$$

$$- C_1 \left[ 3 \frac{\ddot{a}}{a} + 5 \frac{\ddot{b}}{b} + 3 \left( \frac{K_3}{a^2} + \frac{\dot{a}^2}{a^2} \right) + 10 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right) + 15 \frac{\dot{a}\,\dot{b}}{a\,b} \right]$$

$$= 2\, C_2 \Big[ 30 \left( \frac{K_6}{b^2} + \frac{\dot{b}^2}{b^2} \right)^2 + 120 \left( \frac{\dot{a}\,\dot{b}}{a\,b} \right)^2 + 6 \frac{\ddot{a}}{a}$$

$$\left(\frac{K_3}{a^2}+\frac{\dot a^2}{a^2}\right)+30\,\frac{\dot a\,\dot b}{a\,b}\left(\frac{K_3}{a^2}+\frac{\dot a^2}{a^2}\right)+60\,\frac{\ddot a}{a}\,\frac{\dot a\,\dot b}{a\,b}$$

$$+\,60\,\frac{\ddot b}{b}\left(\frac{K_6}{b^2}+\frac{\dot b^2}{b^2}\right)\;+\,120\,\frac{\ddot b}{b}\,\frac{\dot a\,\dot b}{a\,b}\;+\,180\,\frac{\dot a\,\dot b}{a\,b}$$

$$\left(\frac{K_6}{b^2}+\frac{\dot b^2}{b^2}\right)+30\,\frac{\ddot b}{b}\left(\frac{K_3}{a^2}+\frac{\dot a^2}{a^2}\right)+60\,\frac{\ddot a}{a}\left(\frac{K_6}{b^2}+\frac{\dot b^2}{b^2}\right)$$

$$+\,60\left(\frac{K_3}{a^2}+\frac{\dot a^2}{a^2}\right)\left(\frac{K_6}{b^2}+\frac{\dot b^2}{b^2}\right)\Big]+\Lambda+p' \,. \qquad (4.8)$$

The conservation equation for $\rho$, $p$ and $p'$ is given by

$$\frac{d}{dt}\left[\,\Omega_3\Omega_6\,\rho\,\right]+p\,\Omega_6\,\frac{d\Omega_3}{dt}+p'\,\Omega_3\,\frac{d\Omega_6}{dt}=0 \qquad (4.9)$$

where $\Omega_3 = 2\,\pi^2\,a^3\,,\quad \Omega_6=\dfrac{2\,\pi^{7/2}}{\Gamma(7/2)}\,b^6.$

To determine $T_{AB}$, we must have a knowledge of the matter Lagrangian $L_m$. We presume that we are considering an effective theory which is free from all anomalies, gauge as well as gravitational. While such a theory will involve interacting fields, both bosons and fermions, we will make some simplifying assumptions in the results that follow. We require only the general structure of the free energy for matter fields which we assume to be non-interacting and in thermal equilibrium. We also use an adiabatic approximation, i.e. the free energy is determined in a static background. For non-interacting scalar particles, the free energy (F) in the one loop approximation is given by

$$\beta F = \frac{1}{2} \ln \mathrm{Det} \left[ - \Box + m^2 \right] \qquad (4.10)$$

where $\beta = T^{-1}$. This has been evaluated by a number of authors.[131,139] We have, in our model also fermions and gauge particles and the exact form of the free energy is not known for the entire system. Some results are available for odd dimensions only.[138] For our problem, we may hopefully follow Okada[131] and assume a general form, as in the scalar case, [for $b \ll a$]

$$F = \frac{\Omega_3}{b^4} h(x) , \qquad (4.11)$$

where $h(x) = h_1 - h_2 x^4 - h_3 x^{10}$ and $x = 2\pi bT$. In above, $h_1$ represents the one loop quantum effect and $h_2$ and $h_3$ give the thermal effects determined by the nature and number of particles in the theory. Recently calculations of the effective potential in even dimensions have been done by Gleiser *et al*,[140] following Witten's[141] proposal of two internal product spaces, each of an odd dimension. This leads to the same form for the free energy.

The entropy (S) and the internal energy (U) are defined by

$$S = - \frac{\partial F}{\partial T}\Big|_{a,b} = \frac{2\pi\Omega_3}{b^3} \left[ 4 h_2 x^3 + 10 h_3 x^9 \right] , \qquad (4.12)$$

$$U = F + T\,S = \frac{\Omega_3}{b^4}\left[\, h(x) - x\, h'(x) \,\right] \qquad (4.13)$$

where $h'(x) = \dfrac{\partial h}{\partial x}$ . The quantities $\rho$ , p and p' can now be determined from the thermodynamic relations . Substituting these results, we look for solutions which may be relevant for a realistic cosmological model. This will be considered in the next section.

## IV.3 *COSMOLOGICAL SOLUTIONS* :

We follow the evolution of a universe described by equations (4.6)-(4.8) at an initial time $t = t_o$. We assume that the initial temperature is high and the scale factors $a(t)$ and $b(t)$ are large enough so that the $h_3$ term in the equation (4.11) dominates. Since the universe is already large, the curvature terms ( $\sim a^{-2}$ , $\sim b^{-2}$) in the field equations can be neglected to begin with. With these approximations, the field equations(4.6)-(4.8) admit a solution in which the 3-space scale factor $a(t)$ increases exponentially while that of 6-space $b(t)$ decreases as follows :

$$a(t) = a_{oi}\ exp\left[\alpha\ (t - t_o)\right]\ ,\ b(t) = b_{oi}\ exp\left[-\frac{1}{2}\alpha\ (t - t_o)\right] \qquad (4.14)$$

where $a_{oi}$ and $b_{oi}$ are their initial values and $\alpha$ is a constant to be determined by the algebraic equations

$$\frac{9}{4} C_1\ \alpha^2 - \frac{81}{4} C_2\ \alpha^4 = \Lambda - 9\ Q_o\ , \qquad (4.15)$$

$$-\frac{9}{4} C_1\ \alpha^2 + \frac{27}{4} C_2\ \alpha^4 = \Lambda + Q_o \qquad (4.16)$$

where $\Lambda = C_o/2$ and $Q_o = \dfrac{h_3\ (2\pi)^{10}}{2\ \pi^{7/2}}\ T^{10}$ . Since $T \propto (a^3 b^6)^{-1/9}$ , $T$ is a constant. The total volume of the universe during this regime remains unchanged although its shape changes. However, the solution (4.14) needs a modification when b(t) eventually becomes smaller and the terms $O(b^{-2})$ in the field equation become important. Considering the terms upto $O(b^{-2})$ in the field eqs.(4.6)-(4.8), a solution may still be obtained in which a(t) continues expanding at the same rate but the contraction rate of b(t) is slightly reduced. The solution is given by

$$a(t) = a_{oi}\ exp\left[\alpha\ (t - t_o)\right]\ , \qquad (4.17)$$

$$b(t) = b_o\left[1 + \beta'\ exp\left[-\alpha\ (t - t_o)\right]\right]^{1/2} \qquad (4.18)$$

where $\beta'$ is a large constant and $b_o = b_{oi}/\sqrt{\beta'}$ . The temperature of the universe , however, decreases in this stage. Keeping terms upto $b^{-2}$ order, we find

$$T = T_o\left\{1 - \frac{10}{3}\ b_o^2/b^2\right\} \qquad (4.19)$$

where $T_o$ is the initial temperature. Existence of the solutions (4.17)-(4.18) impose the relations

$$\Omega_o = -C_1\left[\frac{9}{10}\ \alpha^2 - \frac{15}{4\ b^2}\right] \qquad (4.20)$$

where $\alpha^2 = -\frac{1}{6}\ C_1/C_2\ ,\quad b_o = \sqrt{15/(12\ \alpha^2)}$ and

$$\frac{33}{80}\ \frac{C_1^2}{C_2} = \frac{C_o}{2} = \Lambda\ . \qquad (4.20a)$$

Thus the 10-D cosmological constant $\Lambda$ is not a free parameter but is determined in terms of the ratio $C_1^2/C_2$ . This stage of evolution continues till either $O(b^{-4})$ terms become important in the field equations or $b(t) \sim \left( 4.1/\alpha^2 \right)^{1/2}$. Since we expect the expansion in the three space to explain the observable universe, $\alpha^2$ must be positive. This suggests two possibilities for the signs of $C_1$ and $C_2$. However, the only choice which gives a realistic model is $C_1 < 0$ and $C_2 > 0$ . The higher dimensional models considered so far have $C_1 > 0$ and $C_2 \gtrless 0$ in the action (4.2). Thus the sign of the R-term considered in our model is unconventional. As will be shown later on, this still leads to the usual Einstein equations in four dimensions.

The inflationary scenario outlined above will cause a fall in the temperature and eventually a time will come when $h_2$ and $h_1$ can no longer be neglected in comparison to $h_3$. We expect a variation of the scenario here. The value of the scale factor $b(t)$ which underwent a prolonged contraction is now of the order of Planck length and we intend to look for solutions with $\dot{b}/b$ very small. We are interested in recovering a universe which becomes radiation dominated at a large t . We assume that it is now possible to neglect terms of order $\left( \dot{b}b^{-1}t^{-1} \right)$ , $\dot{b}^2/b^2$ and $\ddot{b}/b$ in the field equations (4.6)-(4.8) to get

$$3 \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{x} \left( \lambda - 3 \eta T^4 \right) , \qquad (4.21)$$

$$2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} = \frac{1}{x}\left(\lambda + \eta T^4\right) , \qquad (4.22)$$

$$-12 C_2 \frac{\ddot{a}}{a} \frac{\dot{a}^2}{a^2} + y\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = F(b) \qquad (4.23)$$

where

$$\lambda = \Lambda + \frac{15 C_1}{b^2} + \frac{180 C_2}{b^4} - \frac{\sigma}{b^{10}} , \qquad (4.24)$$

$$F(b) = \Lambda + \frac{10 C_1}{b^2} + \frac{60 C_2}{b^4} + \frac{2}{3}\frac{\sigma}{b^{10}} , \qquad (4.25)$$

$$x = - C_1 - \frac{60 C_2}{b^2} , \qquad (4.26)$$

$$y = - 3 C_1 - \frac{120 C_2}{b^2} , \qquad (4.27)$$

$$\eta = \frac{2\pi}{\Omega_6} h_2 , \quad \sigma = \frac{h_1}{(2\pi)^{7/2}} \Gamma(7/2) . \qquad (4.28)$$

Note that $\Lambda$, the ten dimensional cosmological constant can have a value different from that calculated in (4.20a), because of the vacuum energies released during various phase transitions the universe passed through. To see the consistency of the solution corresponding to a 4-D radiation dominated universe, we note that if $\lambda/x$, the effective 4-D cosmological constant, is zero or negligibly small, the equations (4.21) and (4.22) become identical to the Einstein's equation for a radiation-dominated 4D universe. We, therefore, look for a large time solution of (4.23) with the scale factor a(t) expanding as $t^{1/2}$. The equation tells us that b(t) must adjust itself so that $F(b) \to 0$, because the first term on the R.H.S. goes as $t^{-4}$ and hence is negligible, while the second term vanishes identically. For any $b < \left( 60\, C_2 / |C_1| \right)^{1/2}$, x is negative. Thus although we started out with a negative $C_1$, the 4-D field equation reduces to the Einstein's equation. The equation (4.23) provides the constraint which ensures that $\lambda$ remains small, as will be shown below.

## IV.4 *A SOLUTION OF THE COSMOLOGICAL CONSTANT PROBLEM :*

The problem of cosmological constant is sought to be solved here by the dynamics of the compact space. We notice that at a large t, F(b) should go to zero if $a \sim t^{1/2}$. It is easy to see that for given $h_1$ and $\Lambda$ there is a range of b values that can make both F(b) and $\lambda$ very small. However, the value of $\Lambda$ may change whenever there is a phase transition causing b to adjust itself to make $F(b) \sim 0$ again. The cosmological constant problem is solved if we can show that this change in b also keeps $\lambda$ small. The arguments that such a situation does prevail are given in the following :

1. Let $\Lambda$ be increased a little, $\Lambda \to \Lambda + \delta$. To satisfy $F(b) \sim 0$, b changes to $b + \varepsilon$. To first order in $\varepsilon$, one can show that

$$\delta = \frac{2\,\varepsilon}{b} \left[ -\Lambda + \frac{60\, C_2}{b^4} + \frac{8}{3}\, \frac{\sigma}{b^{10}} \right] . \qquad (4.29)$$

Thus if $\frac{60\ C_2}{b^4} + \frac{8}{3}\frac{\sigma}{b^{10}} > \Lambda$ , a reasonable demand, small increase in $\Lambda$ will cause a small increase in b.

2. One can show from (4.24) and (4.25) that

$$\lambda = F(b) - \frac{b}{6}\frac{d\lambda}{db} . \qquad (4.30)$$

At a large t, $F(b) \to 0$ , giving

$$\lambda \simeq - \frac{b}{6}\frac{d\lambda}{db} . \qquad (4.31)$$

Since $\varepsilon$ is positive for an increase in $\Lambda$ , eq.(4.31) shows that if $\lambda$ is positive, it will decrease and if $\lambda$ is negative it will increase. $\lambda$ thus has the tendency to move towards the zero value. Moreover, the rate of change is proportional to its value, making $\lambda$ confined to values close to zero, once the value of $\lambda$ is small.*

The four dimensional gravitational constant G will be given by

$$8\pi G = - \frac{3\eta}{x\sigma_o} = \frac{6\pi h_2}{\sigma_o \Omega_6 \left( C_1 + 60\ C_2/b^2 \right)} .$$

where $\sigma_o$ is the Stefan's constant. This relation has some interesting features, we note that if the compactified scale factor $b^2 < 60\ C_2/|C_1|$ , we have the usual attractive force of gravity. However, if b hovered around the values $b^2 > 60\ C_2/|C_1|$ , we could have seen a case of repulsive four dimensional



* It can be shown that for $b^2 < 40 C_2/|C_1|$ , $\lambda/x$ approaches zero for a small increase of b.

gravitational force. Another interesting feature of the relation is the possible variation of G as b changes. We have no indication of a significant variation of G at a later stage of evolution, and this is consistent with the possibility that the last phase transition ( QCD ? ) determined the present value of the 4-D gravitational constant, there being no significant change in the value of b thereafter.

## IV.5 *SUMMARY OF THE RESULTS* :

To summarise, we have presented a scenario of the evolution of the higher dimensional universe taking into account the quadratic curvature terms in the GB-form. A consistent scenario of the early universe is obtained in which initially the scale factor for the three space expands exponentially while that of the extra space compactifies to a small size of the Planck order. However, we need an initially expanding phase of the 10-D universe so as to have sufficient inflation in the physical 3-space required for solving the cosmological problems. This requires $a_{oi} \sim e^{22} M_P^{-1}$, which shows that the 10 D universe itself must have a sizable inflation. This is in general agreement with the results obtained by Abbott *et al*[49] and Shafi and Wetterich[130]. A good feature of our model is that it allows a mechanism for solving the four dimensional cosmological constant problem. Moreover, the universe emerges eventually as a four dimensional radiation-dominated FRW-universe. However, we have no suggestion for the mechanism which triggers the onset of the compactification. These are problems which may depend on the nature of particle interactions at the relevant temperature. The purpose of our exercise is to bring out the special features of the higher derivative terms in the action which is slightly different from the string corrected action considered by other workers.[53-57,130] In a limited way, we have seen the rich structure of this type of theories with quadratic curvature. No doubt, these terms can be made to play an important role in the cosmological evolution of the universe, even if the string approach is not successful.

## IV.6 *CONCLUDING REMARKS* :

We have considered so far three different cosmological models which incorporate an inflationary phase as a part of the evolution of the early universe. It should, however, be mentioned that it is by no means conclusively proved that an inflation is the only way to solve the cosmological problems. There are already some dissident voices. Padmanabhan and Seshadri[142] claim that the horizon problem is not completely resolved by an inflation. The necessity of assuming a large quantity of dark matter, so as to obtain $\Omega \sim 1$ , is another weak point of the scenario.

A more important question to be answered is how the universe behaved at times earlier than the Planck time. For such a description we will need a quantum description of space-time, and possibly an understanding of the origin of the classical time. The quantum gravity effects here will naturally involve very large fluctuations of metric and of all matter fields and will require a complete quantum treatment. It is now understood that the quantum cosmology that we need should be a theory describing consistently both the dynamics of the universe and the boundary conditions to be imposed on it. While some progress has already been made in formulating a simple-minded quantum cosmology on a minisuperspace, considerable confusion persists in this field. The two alternative proposals for the boundary conditions, one by Hartle and Hawking[143] and the other by Vilenkin[144], lead to divergent results for the probabilities of the creation of a universe out of nothing. There are also serious technical difficulties in evaluating the relevant path integrals. It is obvious that quantum cosmology badly needs a breakthrough. When it comes, the quantum description of the early universe will also help in making the right choice for the scenario in the post-Planckian era. Very exciting physics in this field is expected in near future.

# CHAPTER V

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