# CHAPTER V

## VOLTAGE CURRENT AND POWER RELATION IN AN ARC PLASMA IN A VARIABLE AXIAL MAGNETIC FIELD

### 5.1. INTRODUCTION

The variation of voltage, current and output power in a mercury arc plasma has been investigated in an axial magnetic field (0 - 1350 G) for three values of discharge current namely 3, 4 and 5A. The variation of voltage, current and output power in an arc plasma under the action of a transverse magnetic field was investigated by Sen and Das (1973). Utilising Beckman's (1948) theory and the modification introduced by Sen and Gupta (1971) regarding the variation of charged particle density and electron temperature in a glow discharge in the presence of a magnetic field the experimental results have been satisfactorily explained. It was further pointed out that the theoretical expressions deduced by Beckman (1948) as modified by Sen and Gupta (1971) are valid over a range of (H/P) values upto 1000 G/torr. Allen (1951) observed that in the case of heavy current pulsed arc discharge in hydrogen the voltage current characteristics showed a slight negative gradient over the arc current range of 25 to 80 A with no magnetic field but became increasingly negative with increase of magnetic field. The effect of an axial magnetic field upto 470G on the arc stability voltage and temperature has been studied by Goldman and White (1965) who observed that the field had no stabilising influence on the arc whereas it led to an increase of voltage drop across the arc. No theoretical interpretation of the results was however, provided. Forrest and Franklin (1966) described a theoretical model for a low pressure arc discharge in a magnetic field in which predictions have been made for radial electron number density profile and radial light emission profile. Current voltage characteristics of glow discharge in longitudinal magnetic field has been investigated by Sen and Jana (1977) and it has been observed that the discharge current increases and the axial voltage drop across the arc decreases with increase of axial magnetic field for the range of pressure investigated (0.685 to 0.925 torr). Assuming, the radial distribution of particles as Bessalian it has been possible to explain the results quantitatively. The results also show that Bessalian distribution holds in the presence of magnetic field as well; electron temperature and its variation in an axial magnetic field (from zero to 1050 gauss) in a mercury arc plasma have been measured by Sadhya and Sen (1980) by a spectroscopic method. A model has been developed in which air plays the role of a quenching gas and it has been found that both atomic and molecular ions of mercury are present in this type of discharge. A distribution function for the radial electron density has been deduced. The results computed on this basis are in agreement with observed experimental data.

The results obtained by Sen and Das (1973) indicate that the theoretical deduction (Beckman, 1948; Sen and Gupta, 1971) regarding the variation of electron density and electron temperature in a transverse magnetic field in the case of glow discharge is valid in the case of an arc plasma as well. It is worthwhile to investigate whether the same model is valid in the case of an arc plasma when subjected to an axial magnetic field. The object is also to find out whether the properties as well as the plasma parameters in an arc plasma are dependent upon the alignment of the magnetic field with respect to direction of flow of arc current. It is hence proposed to investigate in the present work the variation of voltage, current and power relation in a mercury arc plasma in an axial magnetic field and to provide a theoretical treatment of the observed experimental results. In the present investigation the conductivity value in magnetic field has been calculated and an analytical expression presented to represent the variation of conductivity in the magnetic field. Utilising this expression the variation of output power with magnetic field has been explained.

## 5.2. EXPERIMENTAL SETUP

A mercury arc has been investigated, the arc tube (discussed in the article 2.2) of which is cylindrical (length 9 cm and diameter 4.8 cm) and is excited by a stabilised d.c. source with a rheostat to control the current recorded by an ammeter. The mercury arc is cooled by the external circulation of air. The pressure inside the arc maintained at 0.045 torr. To maintain the pressure constant dry air which acts as a buffer gas was introduced by a variable microleak of a needle valve. The mercury arc is placed between the pole pieces of an electromagnet energised by a stabilised d.c. source. The lines of force are parallel to the direction of the flow of arc current and the voltage across the arc is measured by a voltmeter with an accuracy of ± 0.5V. The magnetic field has been accurately measured by a fluxmeter for magnetising current from 1 to 5.5A (Article 2.5, Fig. 2.7). Keeping the distance between the pole pieces at 9.8 cm the magnetic field varies from zero to 1.5 (KG). The schematic experimental arrangement is shown in figure 5.1.

## 5.3. RESULTS AND DISCUSSION

The variation of voltage across the arc, the arc current and power developed across the arc have been plotted in figures 5.2, 5.3 and 5.4, respectively for magnetic field varying from zero to 1.5 kG. It is observed from figure 5.2 that (a) initial voltage across the arc for all the three arc currents investigated (3A, 4A and 5A) is the same when no magnetic field is present which is consistent with our previous observation. (b) When the magnetic field is applied the voltage across the arc increases linearly with magnetic field. The rate of increase is however, different for the three arc currents. Rate of increase is the highest for the

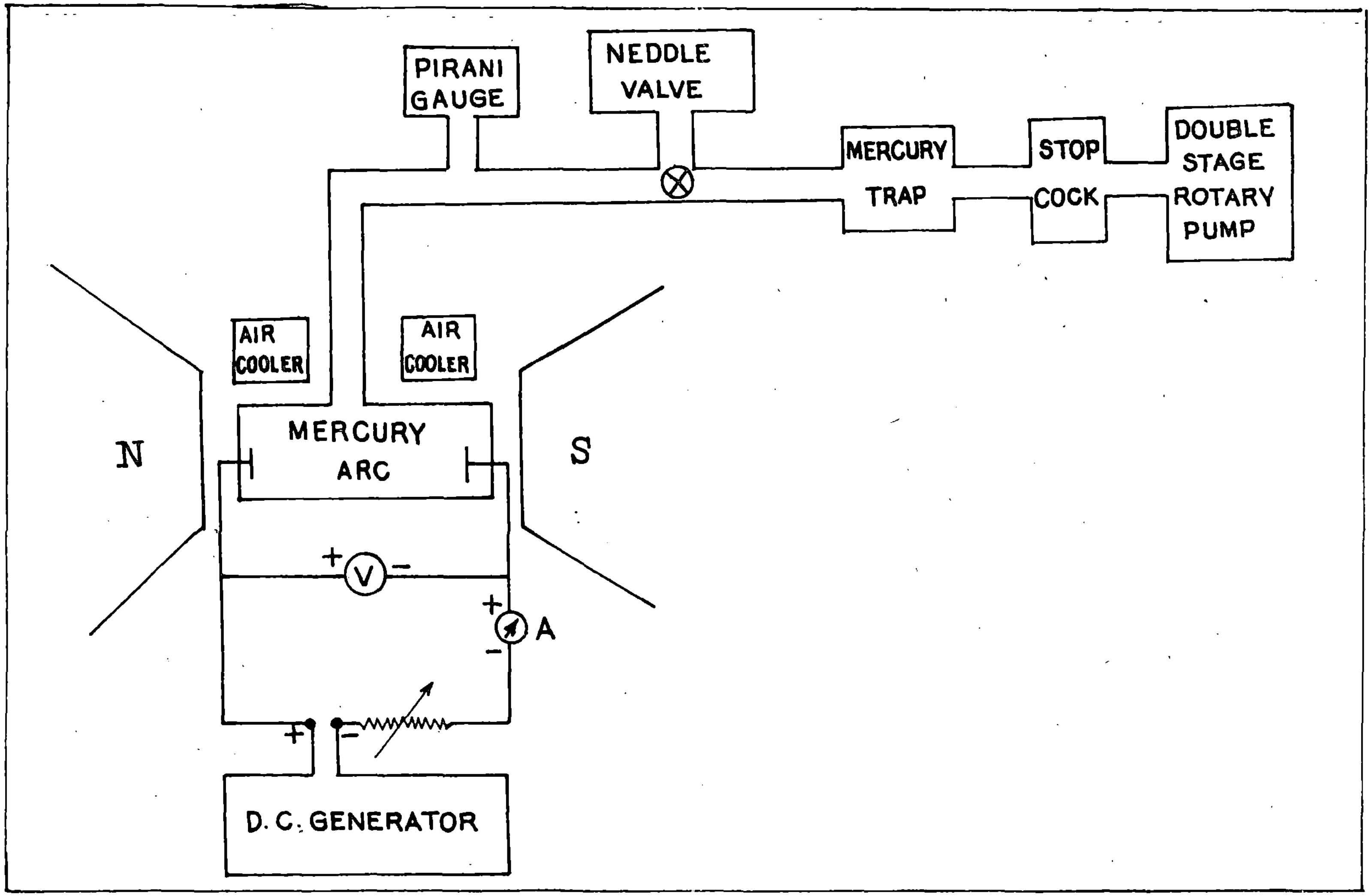


Fig. 5·1. Schematic diagram of experimental set-up in an axial magnetic field.

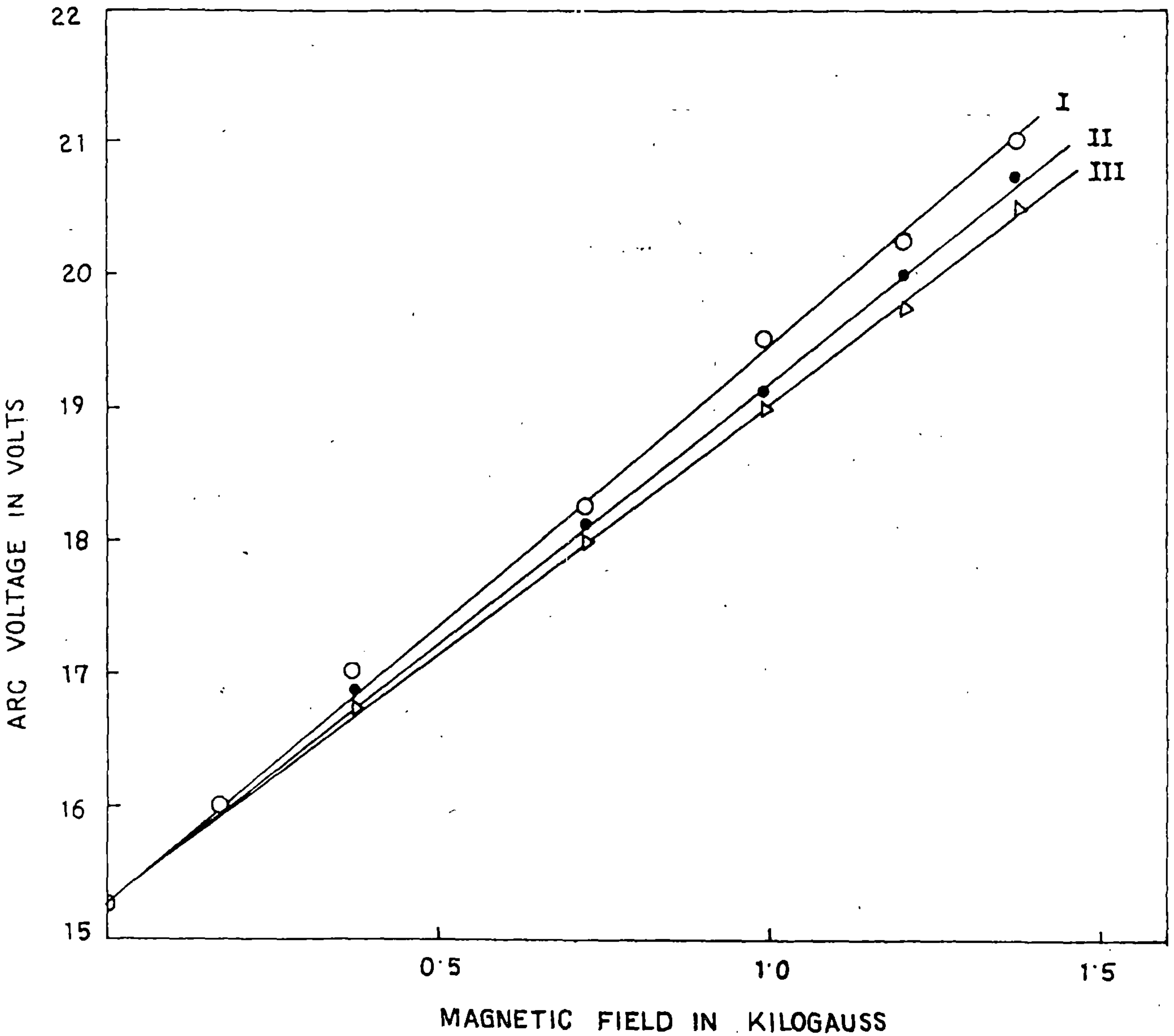


Fig.5·2. Variation of arc current with magnetic field, I 3A $P_{Hg}$ = 0.3032 torr, II 4A $P_{Hg}$ = 0.3342 torr, III 5A $P_{Hg}$ = 0.3658 torr.

lowest initial current and decreases with increase of current.

(c) From figure 5.3, it evident that with increase of the magnetic field the arc current decreases but the rate of decrease depends upon the initial value of arc current.

Let us assume that $i$ is the current when there is no magnetic field, $V_s$ the source voltage of the arc, $R$ the resistance of the current regulating rheostat and $V_a$ the voltage drop across the arc. Then

$$V_S = V_a + iR \qquad (5.1)$$

If $i_H$ is the current on the presence of magnetic field then

$$V_S = V_{aH} + i_H R \qquad (5.2)$$

where $V_{aH}$ is the voltage across the arc in the presence of magnetic field. From equation (5.1) and (5.2) we get

$$(i - i_H) R = (V_{aH} - V_a) \qquad (5.3)$$

Thus it is evident that due to decrease of current in the presence of magnetic field the voltage across the arc increases. To verify eqn. (5.3), $(V_{aH} - V_a)$ has been plotted against $(i - i_H)$ in figure (5.5) for three arc currents and $R$ can be calculated from the slopes of the curves.

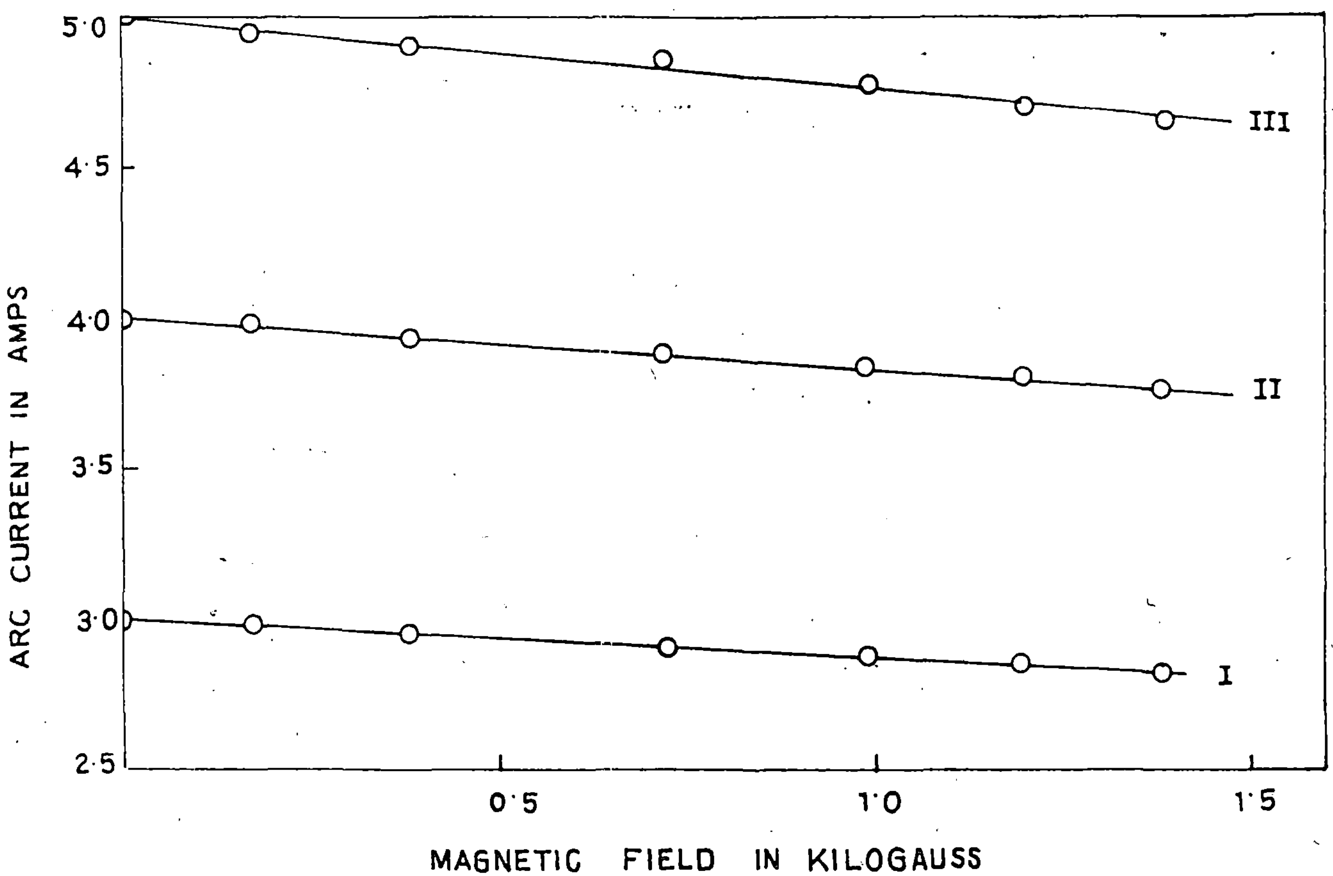


Fig. 5·3. Variation of arc current with magnetic field.

Details of curves as in figure 5.2.

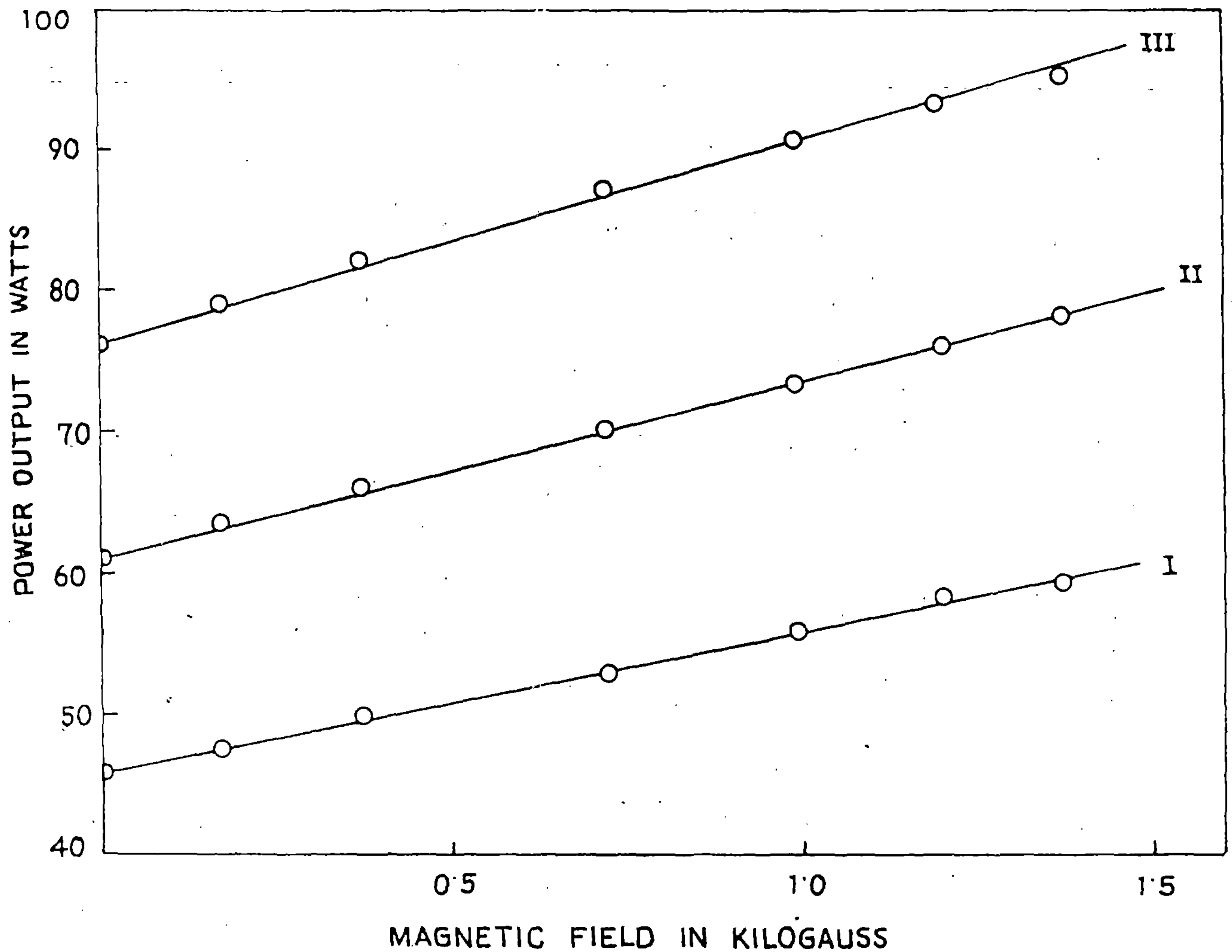


Fig. 5·4. Variation of output power with magnetic field.

Details of curves as in fig. 5.2.

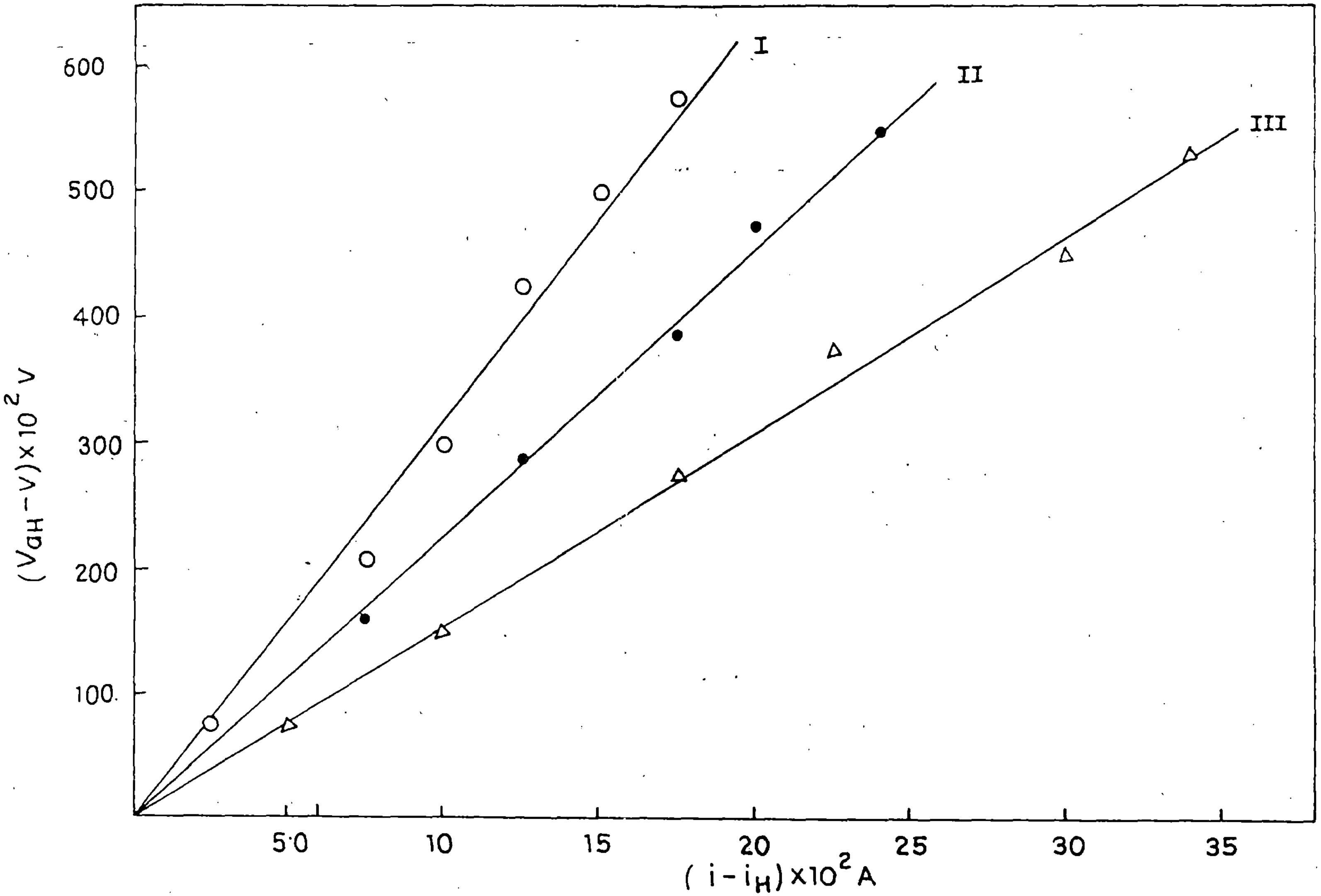


Fig. 5·5. Variation of $(V_{aH} - V_a) \times 10^2$ with $(i - i_{aH}) \times 10^2$.
Details of curves as in figure 5.2.

Table 5.1

| $i = 3A$ | | $i = 4A$ | | $i = 5A$ | |
|---|---|---|---|---|---|
| $i - i_H$ | $V_{aH} - V_a$ | $i - i_H$ | $V_{aH} - V_a$ | $i - i_H$ | $V_{aH} - V_a$ |
| 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| 0.025 | 0.75 | 0.025 | 0.75 | 0.050 | 0.75 |
| 0.065 | 1.75 | 0.075 | 1.60 | 0.10 | 1.50 |
| 0.100 | 3.00 | 0.125 | 2.875 | 0.175 | 2.75 |
| 0.125 | 4.25 | 0.175 | 3.875 | 0.225 | 3.75 |
| 0.150 | 5.00 | 0.20 | 4.75 | 0.30 | 4.50 |
| 0.175 | 5.75 | 0.25 | 5.50 | 0.35 | 5.3 |

The curves are linear and the slope of the curve gives the value of R, the resistance of the series rheostat which is used to adjust the initial current. The R - values for 3, 4 and 5A are respectively $34\,\Omega$, $22.5\,\Omega$ and $16.6\,\Omega$ and are in excellent agreement with the actual resistance values of the rheostat used for adjusting the initial arc currents. The linear variation of arc voltage with magnetic field can be represented by an equation of the form

$$E_H = E + m_0 H = E\left(1 + m_0 \frac{H}{E}\right) = E(1 + mH) \qquad (5.4)$$

where $m$ varies with arc current. For arc currents, of 3, 4 and 5A, the $m_0$ values are respectively 4.23, 3.94, and 3.74 where H is expressed in kG and E is the total voltage across the arc.

We further note that as reported by Sen and Das (1973) almost similar results have been obtained in transverse magnetic field as is now found in the case of a longitudinal magnetic field. But quantitatively there is a difference. In the case of transverse magnetic field the maximum change of current is in the ratio 1.58 whereas in the case of axial magnetic field the ratio is much smaller (1.062) for a magnetic field of the order of 1.35 kG. As a result the ratio of voltage change in the transverse magnetic field is 1.86 whereas in the longitudinal magnetic field it is 1.37. We can thus conclude that in both the cases the effects are similar but the transverse magnetic field will have a more dominant effect on the properties of arc plasma than an axial magnetic field.

Since the voltage across the arc and the current have been measured for an axial magnetic field varying from zero to 1.35 kG it is possible to calculate the average conductivity of the arc plasma for the range of magnetic field investigated. We define the expression for the average conductivity as

$$\sigma_H = \frac{J}{\varepsilon} \qquad (5.5)$$

where $J = \frac{i_H}{\pi r^2}$ , $r$ radius of the arc tube and $\mathcal{E}$ is the corresponding arc voltage/cm. The values of $\sigma_H$ for three arc currents are given in Table 5.2 for values of magnetic field used in the experiment.

Table 5.2

| Magnetic field (kG) | $\sigma_H$ ( in mhos cm) | | |
|---|---|---|---|
| | 3A | 4A | 5A |
| 0.00 | 0.6259 | 0.8347 | 1.0433 |
| 0.17 | 0.5907 | 0.7906 | 0.9845 |
| 0.37 | 0.5475 | 0.7401 | 0.9309 |
| 0.72 | 0.5058 | 0.6803 | 0.8530 |
| 0.99 | 0.4693 | 0.6364 | 0.7997 |
| 1.20 | 0.4478 | 0.6046 | 0.7573 |
| 1.37 | 0.4281 | 0.5751 | 0.7218 |

Let us assume that the variation of $\sigma_H$ with H can be represented by an equation of the form $\sigma_H = \sigma \exp(-\alpha H)$ where $\alpha$ is a constant. We can calculate the value of by a statistical method:

$$\sigma_H = \sigma \exp(-\alpha H),$$

$$\log \sigma_H = \log \sigma - \alpha H,$$

$$S = \sum\left[\log\sigma_H - \log\sigma + \alpha H\right]^2;$$

$$\frac{dS}{d\alpha} = 2\sum H\left[\log\sigma_H - \log\sigma + \alpha H\right] = 0.$$

$$\sum_{i=1}^{i=6} H\log\sigma_H = \log\sigma\sum_{i=1}^{i=6} H - \alpha\sum_{i=1}^{i=6} H^2,$$

$$\alpha = \frac{\log\sigma\sum_{i=1}^{i=6} H - \sum_{i=1}^{i=6} H\log\sigma_H}{\sum_{i=1}^{i=6} H^2}.$$

For a current of 3A,

$$\sigma = 0.6259 \quad \sum H = 4.82 \quad \sum H^2 = 4.9812$$

$$\log\sigma = -0.4658 \quad \sum H\log\sigma_H = -3.684$$

So $$\alpha = \frac{-2.2584 + 3.6784}{4.9812} = 0.2859$$

For a current of 4A,

$$\sigma = 0.8347 \quad \sum H = 4.82 \quad \sum H^2 = 4.9812$$

$$\log\sigma = -0.1807 \quad \sum H\log\sigma_H = -2.237732$$

So $$\alpha = \frac{-0.8710 + 2.237732}{4.9812} = 0.2744.$$

For a current of 5A,

$$\sigma = 1.0433 \quad \sum H = 4.82 \quad \sum H^2 = 4.9812$$

$$\log \sigma = \frac{0.0424 + 1.14776}{4.9812}$$

$$= 0.2714$$

To verify whether the proposed expression for $\sigma_H$ agrees with the experimental results the values of $\sigma_H$ calculated with the above deduced value of $\alpha$ are compared with experimental results in Table 5.3.

Table 5.3

| Magnetic field (kG) | 3A | | 4A | | 5A | |
|---|---|---|---|---|---|---|
| | $\sigma_H$ (expt) | $\sigma_H$ Calculated from eqn.5.6 | $\sigma_H$ (expt) | $\sigma_H$ Calculated from eqn. 5.6 | $\sigma_H$ (expt.) | $\sigma_H$ Calculated from eqn. 5.6 |
| 0.17 | 0.5907 | 0.5963 | 0.7906 | 0.7967 | 0.9845 | 0.9963 |
| 0.37 | 0.5475 | 0.5432 | 0.7401 | 0.7541 | 0.9309 | 0.9436 |
| 0.72 | 0.5058 | 0.5097 | 0.6803 | 0.6851 | 0.8530 | 0.8581 |
| 0.99 | 0.4693 | 0.4720 | 0.6364 | 0.6362 | 0.7997 | 0.7975 |
| 1.20 | 0.4478 | 0.4445 | 0.6046 | 0.6005 | 0.7573 | 0.7533 |
| 1.37 | 0.4281 | 0.4235 | 0.5751 | 0.5732 | 0.7218 | 0.7193 |

The results show good agreement between the experimental and calculated values.

We can thus conclude that the variation of conductivity in an axial magnetic field can be represented as

$$\sigma_H = \sigma_0 \exp(-\alpha H) \qquad (5.6)$$

where the value of $\alpha$ changes slightly with the arc current. Beckman (1948) deduced that in the presence of magnetic field the electron density is reduced. Sen and Gupta (1971) showed that Beckman's expression can be stated as

$$n_H = n_0 \exp(-a H) \qquad (5.7)$$

where $n_H$ and $n_0$ are respectively the electron densities in the presence of and absence of magnetic field, and

$$a = eEC_1^{1/2} r / 2KT_eP \qquad (5.8)$$

where E is the voltage across the discharge tube,

$$C_1 = \left[ (e/M)(L/v_r) \right]^2$$

where L is the mean free path of electron at a pressure of 1.0 torr, $T_e$ the electron temperature and $\sigma$ the conductivity proportional to n, the electron density. Hence comparing (6) and (5) we get

$$\alpha = a = eEC_1^{1/2} r / 2KT_eP$$

The value of $\alpha$ can thus be calculated in an alternative way. In the above expression all the quantities are known except P which is the vapour pressure of mercury. As in an earlier paper (Sadhya and Sen, 1980), the vapour pressure of mercury was determined by noting the temperature of the wall for three different arc currents. The $P_{Hg}$ values for arc currents of 3A, 4A and 5A are 0.3032, 0.3342 and 0.3658 torr respectively. We take E = 15.25 V (experimental result) $C_1 = 2 \times 10^{-6}$ (Sen and Das, 1973) and $T_e = 1.778 \times 10^{3}K$ (Karelina 1942) and r = 1 cm. The calculated value of $\alpha$ is 0.2335 while the value from our experiment is 0.2859. This discrepancy between the two values of $\alpha$ need not be taken too seriously but it shows the agreement of order between the two values. This provides a direct experimental verification of the theoretical deduction of Beckman (1948) as modified by Sen and Gupta (1971).

Figure 5.4 shows the variation of output power at the arc with the magnetic field. It is evident that the variation is linear with the magnetic field in the range of magnetic field investigated here. From our experimental results we note that the variation of $E_H$ , the voltage across the arc in the presence of magnetic field, is linear with the magnetic field and can be represented by an equation of the form $E = E_o + m_o H$ where $m_o$ is the slope of curve, or

$$E_H = E\ (1 + \frac{m_o H}{E}) = E\ (1 + mH)$$

where $m = m_o/E$. The power output at the arc in the presence of the magnetic field is

$$P_H = I E_H = \sigma_H E_H^2 = \sigma \exp(-\alpha H) E^2 (1+mH)^2$$

$$= \sigma E^2 \left[\exp(-\alpha H) + m^2 H^2 \exp(-\alpha H) + 2mH \exp(-\alpha H)\right]$$

$$\frac{dP_H}{dH} = \sigma E^2 \left[-\alpha \exp(-\alpha H) + m^2 2H \exp(-\alpha H) - m^2 H^2 \alpha \exp(-\alpha H) + 2m \exp(-\alpha H) - 2mH\alpha \exp(-\alpha H)\right]$$

Maximising we get

$$H_{max} = (2m^2 - 2m\alpha + 2m^2) / 2m^2\alpha$$

Taking the positive sign before the radical we get

$$H_{max} = (2/\alpha - 1/m)$$

If we take the negative sign before the radical $H_{max}$ becomes negative and hence the negative sign before the radical is discarded.

Taking the values of $\alpha$ and $m$ as obtained here from experimental measurement ( $\alpha = 0.2859$ and $m = 0.2773$) we get $H_{max} = 3606$ G. Since the maximum magnetic field used in the experiment is 1350G the magnetic field for maximum power dissipation will be beyond this range and cannot be observed in the present experimental set-up. We can thus conclude that the variation of conductivity in an arc plasma in an axial magnetic field can be represented by the expression

$$\sigma_H = \sigma \exp(-\alpha H)$$

where $\alpha$ is a constant which shows a small decrease in the value with the increase of the arc current. With this expression for $\sigma_H$ it has been possible to explain the variation of the output power at the arc with magnetic field.

We can thus compare the variation of voltage current and output power of an arc plasma in variable transverse magnetic and axial magnetic fields. In both cases voltage increases and current decreases when the magnetic field is increased but the effect is much more pronounced in a transverse magnetic field. The power output becomes a maximum for a certain value of magnetic field when it is transverse whereas the power output shows almost a linear increase with magnetic field when it is axial. Actual calculation shows that in the case of axial magnetic field the maximum power output will occur at magnetic field which lies beyond the magnetic field investigated in the present experimental set-up.

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## CAPTION FOR FIGURES

Fig.5.1 - Schematic diagram of experimental set-up.

Fig.5.2 - Variation of arc current with magnetic field, I 3A $P_{Hg} = 0.3032$ torr, II 4A $P_{Hg} = 0.3342$ torr, III 5A $P_{Hg} = 0.3658$ torr.

Fig.5.3 - Variation of arc current with magnetic field. Details of curves as in figure 5.2.

Fig.5.4 - Variation of output power with magnetic field. Details of curves as in fig. 5.2.

Fig.5.5 - Variation of $( V_{aH} - V_a) \times 10^2$ with $( i - i_{aH} ) \times 10^2$. Details of curves as in figure 5.2.

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# Voltage current and power relation in an arc plasma in a variable axial magnetic field

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**Abstract.** The variation of voltage, current and output power in a mercury arc plasma has been investigated in an axial magnetic field (0–1350 G) for three values of discharge current namely 3, 4 and 5 A. The voltage increases and current decreases almost linearly and the output power also increases with increase of the magnetic field. The conductivity value in magnetic field has been calculated and an analytical expression presented to represent the variation of conductivity in the magnetic field. Utilizing this expression the variation of output power with magnetic field can be explained.

**Keywords.** Voltage; current; power relation; arc plasma; axial magnetic field.

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## 1. Introduction

The variation of voltage, current and output power in an arc plasma under the action of a transverse magnetic field was investigated by Sen and Das (1973). Utilizing Beckman's (1948) theory and the modification introduced by Sen and Gupta (1971) regarding the variation of charged particle density and electron temperature in a glow discharge in the presence of a magnetic field the experimental results have been satisfactorily explained. It was further pointed out that the theoretical expressions deduced by Beckman (1948) as modified by Sen and Gupta (1971) are valid over a range of (H/P) values 1000 G/torr. Allen (1951) observed that in the case of heavy current pulsed arc discharge in hydrogen the voltage current characteristics showed a slight negative gradient over the arc current range of 25 to 80 A with no magnetic field but became increasingly negative with increase of magnetic field. The effect of an axial magnetic field upto 470 G on the arc stability, voltage and temperature has been studied by Goldman and White (1965) who observed that the field had no stabilizing influence on the arc whereas it led to an increase of voltage drop across the arc. No theoretical interpretation of the results was however provided. Forrest and Franklin (1966) described a theoretical model for a low pressure arc discharge in a magnetic field in which predictions have been made for radial electron number density profile and radial light emission profile. Current voltage characteristics of glow discharge in longitudinal magnetic field has been investigated by Sen and Jana (1977) and it has been observed that the discharge current increases and the axial voltage drop across the arc decreases with increase of axial magnetic field for the range of pressure investigated (0·685 to 0·925 torr). Assuming the radial distribution of particles as Bessalian it has been possible to explain the results qualitatively. The results also show that Bessalian distribution holds in the presence of magnetic field as well; electron temperature and its variation in an axial magnetic field (from zero to 1050 gauss) in a mercury arc plasma have been measured by Sadhya and Sen (1980) by a spectroscopic method. A model has been developed in which air plays the role of a quenching gas and it has been found that both atomic and molecular ions of mercury are present in this type of discharge. A distribution function for the radial electron density has been deduced. The results computed on this basis are in agreement with observed experimental data.

The results obtained by Sen and Das (1973) indicate that the theoretical (Beckman 1948; Sen and Gupta 1971) deduction regarding the variation of electron density and electron temperature in a transverse magnetic field in the case of glow discharge is valid in the case of an arc plasma as well. It is worthwhile to investigate whether the same model is valid in the case of an arc plasma when subjected to an axial magnetic field. The object is also to find out whether the properties as well as the plasma parameters in an arc plasma are dependent upon the alignment of the magnetic field with respect to direction of flow of arc current. It is hence proposed to investigate in the present work the variation of voltage, current and power relation in a mercury arc plasma in an axial magnetic field and to provide a theoretical treatment of the observed experimental results.

## 2. Experimental arrangement

A mercury arc has been investigated, the arc tube of which is cylindrical (length 9 cm and diameter 1·8 cm) and is excited by a stabilized d.c. source with a rheostat to control the current recorded by an ammeter. The mercury arc is cooled by the external circulation of air. The pressure inside the arc is maintained at 0·045 torr. To maintain the pressure constant dry air which acts as a buffer gas was introduced by a variable microleak of a needlevalve. The mercury arc is placed between the pole pieces of an electromagnet energized by a stabilized d.c. source. The lines of force are parallel to the direction of the flow of arc current and the voltage across the arc is measured by a voltmeter with an accuracy of $\pm 0{\cdot}5$ V. The magnetic field has been accurately measured by a fluxmeter for magnetizing current varying from 1 to 5·5 A keeping the distance between the polepieces as 9·8 cm. The magnetic field varies from zero to 1·5 (kG). The schematic experimental arrangement is shown in figure 1.

## 3. Results and discussion

The variation of voltage across the arc, the arc current and power developed across the arc have been plotted in figures 2, 3 and 4, respectively for magnetic field varying from zero to 1·5 kG. It is observed from figure 2 that (a) the initial voltage across the arc for all the three arc currents investigated (3 A, 4 A and 5 A) is the same when no magnetic field is present which is consistent with our previous observation. (b) When the magnetic field is applied the voltage across the arc increases linearly with magnetic field. The rate of increase is however different for the three arc currents. Rate of increase is the highest for the lowest initial current and decreases with increase of current. (c) From figure 3, it is evident that with increase of the magnetic field the arc current decreases but the rate of decrease depends upon the initial value of arc current.

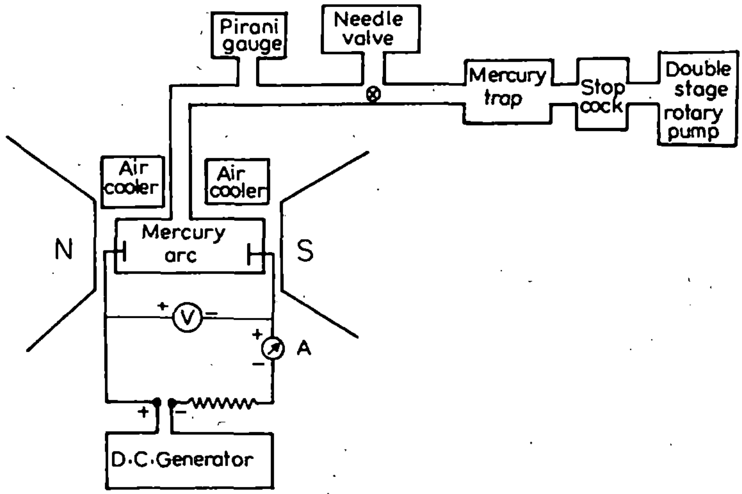


**Figure 1.** Schematic diagram of experimental set-up.

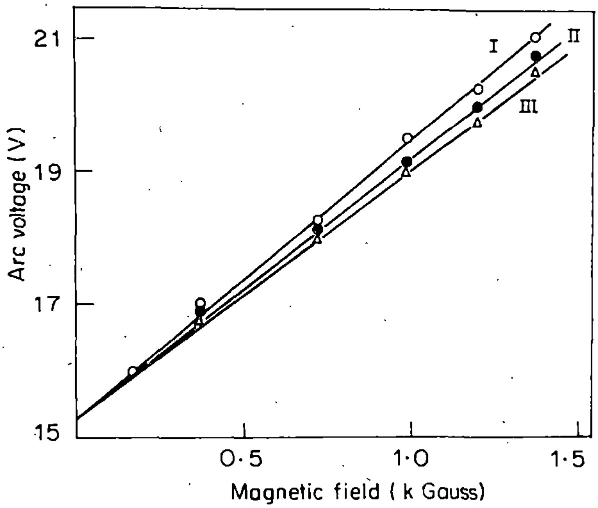


**Figure 2.** Variation of arc current with magnetic field. I 3 A $P_{Hg} = 0{\cdot}3032$ torr, II 4 A $P_{Hg} = 0{\cdot}3342$ torr, III 5 A $P_{Hg} = 0{\cdot}3658$ torr.

Let us assume that $i$ is the current when there is no magnetic field, $V_S$ the source voltage of the arc, $R$ the resistance of the current regulating rheostat and $V_a$ the voltage drop across the arc. Then

$$V_S = V_a + iR. \tag{1}$$

If $i_H$ is the current in the presence of magnetic field then

$$V_S = V_{aH} + i_H R, \tag{2}$$

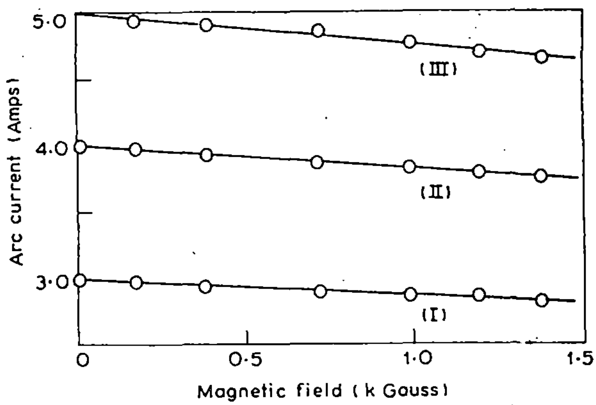


**Figure 3.** Variation of arc current with magnetic field. Details of curves as in figure 2.

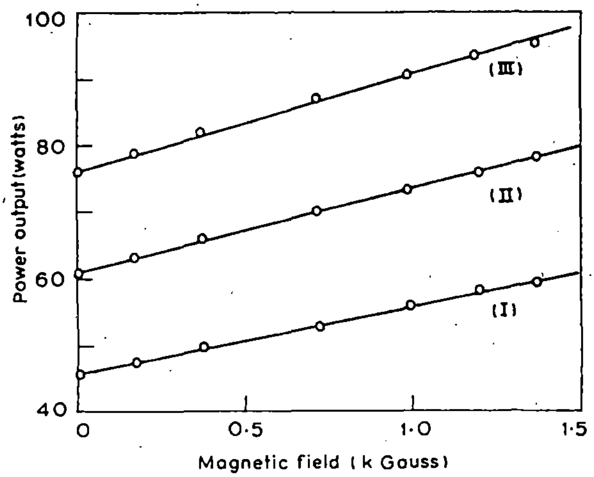


**Figure 4.** Variation of output power with magnetic field. Details of curves as in figure 2.

where $V_{aH}$ is the voltage across the arc in the presence of magnetic field. From (1) and (2) we get

$$(i - i_H)R = (V_{aH} - V_a). \tag{3}$$

Thus it is evident that due to decrease of current in the presence of magnetic field the voltage across the arc increases. To verify equation (3), $(V_{aH} - V_a)$ has been plotted against $(i - i_H)$ in figure 5 for three arc currents and $R$ can be calculated from the slopes of the curves.

The curves are linear and the slope of the curve gives the value of $R$, the resistance of the series rheostat which is used to adjust the initial current. The $R$-values for 3, 4 and

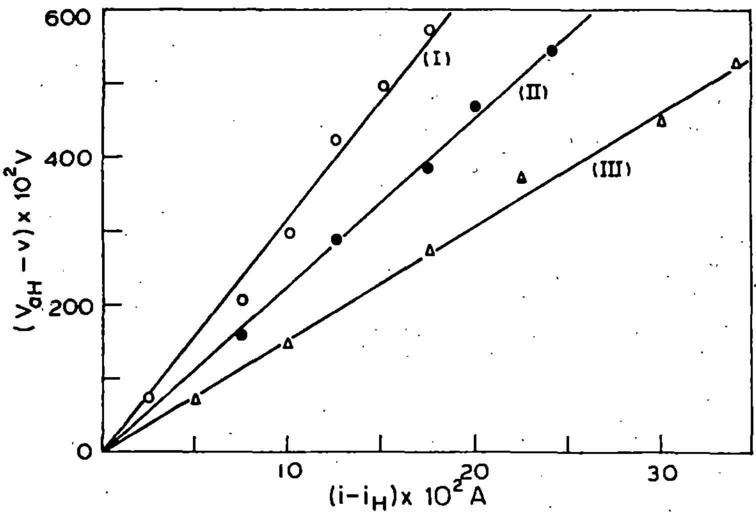


**Figure 5.** Variation of $(V_{aH} - V_a) \times 10^2$ with $(i - i_{aH}) \times 10^2$. Details of curves as in figure 2.

5 A are respectively 34 Ω, 22·5 Ω and 16·66 Ω and are in excellent agreement with the actual resistance values of the rheostat used for adjusting the initial arc currents. The linear variation of arc voltage with magnetic field can be represented by an equation of the form.

$$E_H = E + m_0 H = E\left(1 + m_0 \frac{H}{E}\right) = E(1 + mH), \tag{4}$$

where $m$ varies with arc current. For arc currents of 3, 4 and 5 A, the $m_0$ values are respectively 4·23, 3·94 and 3·74 where $H$ is expressed in kG and $E$ is the total voltage across the arc.

We further note that as reported by Sen and Das (1973) almost similar results have been obtained in transverse magnetic field as is now found in the case of a longitudinal magnetic field. But quantitatively there is a difference. In the case of transverse magnetic field the maximum change of current is in the ratio 1·58 whereas in the case of axial magnetic field the ratio is much smaller (1·062) for a magnetic field of the order of 1·35 kG. As a result the ratio of voltage change in the transverse magnetic field is 1·86 whereas in the longitudinal magnetic field it is 1·37. We can thus conclude that in both cases the effects are similar but the transverse magnetic field will have a more dominant effect on the properties of arc plasma than an axial magnetic field.

Since the voltage across the arc and the current has been measured for an axial magnetic field varying from zero to 1·35 kG it is possible to calculate the average conductivity of the arc plasma for the range of magnetic field investigated. The values of $\sigma_H$ *for three arc currents are given in table 1 for values of magnetic field used in the* experiment.

Let us assume that the variation of $\sigma_H$ with $H$ can be represented by an equation of the form $\sigma_H = \sigma \exp(-\alpha H)$ where $\alpha$ is a constant. We can calculate the value of $\alpha$ by a

**Table 1.** Values of arc conductivity for different magnetic fields.

| Magnetic field (kG) | $\sigma_H$(in mhos cm) | | |
|---|---|---|---|
| | 3 A | 4 A | 5 A |
| 0·00 | 0·6259 | 0·8347 | 1·0433 |
| 0·17 | 0·5907 | 0·7906 | 0·9845 |
| 0·37 | 0·5475 | 0·7401 | 0·9309 |
| 0·72 | 0·5058 | 0·6803 | 0·853 |
| 0·99 | 0·4693 | 0·6364 | 0·7997 |
| 1·2 | 0·4478 | 0·6046 | 0·7573 |
| 1·37 | 0·4281 | 0·5751 | 0·7218 |

statistical method, which is shown for a current of 3 A.

$$\sigma_H = \sigma \exp(-\alpha H),$$

$$\log \sigma_H = \log \sigma - \alpha H,$$

$$S = \sum [\log \sigma_H - \log \sigma + \alpha H]^2;$$

$$\frac{dS}{d\alpha} = 2\sum H[\log \sigma_H - \log \sigma + \alpha H] = 0,$$

$$\sum_{i=1}^{i=6} H \log \sigma_H = \log \sigma \sum_{i=1}^{i=6} H - \alpha \sum_{i=1}^{i=6} H^2,$$

$$\alpha = \frac{\log \sigma \sum_{i=1}^{i=6} H - \sum_{i=1}^{i=6} H \log \sigma_H}{\sum_{i=1}^{i=6} H^2},$$

$$\sigma = 0{\cdot}6259 \quad \sum H = 4{\cdot}82 \quad \sum H^2 = 4{\cdot}9812,$$

$$\log \sigma = -0{\cdot}4658 \quad \sum \mathrm{H} \log \sigma_H = -3{\cdot}684.$$

So

$$\alpha = \frac{-2{\cdot}2584 + 3{\cdot}6784}{4{\cdot}9812} = 0{\cdot}2859.$$

To verify whether the proposed expression for $\sigma_H$ agrees with experimental results the values of $\sigma_H$ calculated with the above deduced value of $\alpha$ are compared with experimental results in table 2.

The results show good agreement between the experimental and calculated values. Following the above procedure the value of $\alpha$ for arc currents of 4 amp and 5 amp has also been calculated and found to be 0·2744 and 0·2714 respectively.

We can thus conclude that the variation of conductivity in an axial magnetic field can be represented as

$$\sigma_H = \sigma \exp(-\alpha H), \tag{5}$$

**Table 2.** Experimental and calculated values of arc conductivity for different values of magnetic field.

| Magnetic field (kG) | $\sigma_H$ (Expt) | $\sigma_H$ (Calculated from equation (5)) |
|---|---|---|
| 0·17 | 0·5907 | 0·5963 |
| 0·37 | 0·5475 | 0·5432 |
| 0.72 | 0·5058 | 0·5097 |
| 0·99 | 0·4693 | 0·4720 |
| 1·20 | 0·4478 | 0·4445 |
| 1·37 | 0·4281 | 0·4235 |

where the value of $\alpha$ changes slightly with the arc current. Beckman (1948) deduced that in the presence of magnetic field the electron density is reduced. Sen and Gupta (1971) showed that Beckman's expression can be stated as

$$n_H = n_0 \exp(-aH), \tag{6}$$

where $n_H$ and $n_0$ are respectively the electron densities in the presence of and absence of magnetic field, and

$$a = eEc_1^{1/2}r/2KT_eP, \tag{7}$$

where $E$ is the voltage across the discharge tube, $C_1 = [(e/m)(\perp/v_r)]^2$, where $L$ is the mean free path of the electron at a pressure of 1 torr, $T_e$ the electron temperature and $\sigma$ the conductivity proportional to $n$, the electron density. Hence comparing (6) with (5) we get

$$\alpha = a = eEC_1^{1/2}r/2KT_eP.$$

The value of $\alpha$ can thus be calculated in an alternative way. In the above expression all the quantities are known except $P$ which is the vapour pressure of mercury. As in an earlier paper (Sadhya and Sen 1980), the vapour pressure of mercury was determined by noting the temperature of the wall for three different arc currents. The $P_{Hg}$ values for arc currents of 3 A, 4 A and 5 A are 0·3032, 0·3342 and 0·3658 torr respectively. We take $E = 15{\cdot}25$ V (experimental result), $C_1 = 2 \times 10^{-6}$ (Sen and Das 1973) and $T_e = 1{\cdot}778 \times 10^3$ K (Karelina 1942) and $r = 1$ cm. The calculated value of $\alpha$ is 0·2335 while the value from our experiment is 0·2859. The discrepancy between the two values of $\alpha$ need not be taken too seriously but it shows the agreement of order between the two values. This provides a direct experimental verification of the theoretical deduction of Beckman (1948) as modified by Sen and Gupta (1971).

Figure 4 shows the variation of output power at the arc with the magnetic field. It is evident that the variation is linear with the magnetic field in the range of magnetic field investigated here. From our experimental results we note that the variation of $E_H$, the voltage across the arc in the presence of magnetic field, is linear with the magnetic field and can be represented by an equation of the form

$$E_H = E_0 + m_0 H,$$

where $m_0$ is the slope of the curve, or

$$E_H = E\left(1 + \frac{m_0 H}{E}\right) = E(1 + mH),$$

where $m = m_0/E$. The power output at the arc in the presence of the magnetic field is

$$P_H = IE_H = \sigma_H E_H^2 = \sigma \exp(-\alpha H) E^2 (1 + mH)^2$$
$$= \sigma E^2 [\exp(-\alpha H) + m^2 H^2 \exp(-\alpha H) + 2mH \exp(-\alpha H)]$$
$$\frac{dP_H}{dH} = \sigma E^2 [-\alpha \exp(-\alpha H) + m^2 2H \exp(-\alpha H)$$
$$- m^2 H^2 \alpha \exp(-\alpha H) + 2m \exp(-\alpha H) - 2mH\alpha \exp(-\alpha H)].$$

Maximising we get

$$H_{\max} = (2m^2 - 2m\alpha + 2m^2)/2m^2\alpha.$$

Taking the positive sign before the radical we get

$$H_{\max} = (2/\alpha) - (1/m).$$

If we take the negative sign before the radical $H_{\max}$ becomes negative and hence the negative sign before the radical is discarded.

Taking the values of $\alpha$ and $m$ as obtained here from experimental measurement ($\alpha = 0{\cdot}2859$ and $m = 0{\cdot}2773$) we get $H_{\max} = 3606$ G. Since the maximum magnetic field used in the experiment is 1350 G the magnetic field for maximum power dissipation will be beyond this range and cannot be observed in the present experimental set-up. We can thus conclude that the variation of conductivity in an arc plasma in an axial magnetic field can be represented by the expression

$$\sigma_H = \sigma \exp(-\alpha H),$$

where $\alpha$ is a constant which shows a small decrease in the value with the increase of the arc current. With this expression for $\sigma_H$ it has been possible to explain the variation of output power at the arc with magnetic field.

We can thus compare the variation of voltage, current and output power of an arc plasma in variable transverse and axial magnetic fields. In both cases voltage increases and current decreases when the magnetic field is increased but the effect is much more pronounced in a transverse magnetic field. The power output becomes a maximum for a certain value of the magnetic field when it is transverse whereas the power output shows almost a linear increase with magnetic field when it is axial. Actual calculation shows that in the case of axial magnetic field the maximum power output will occur at a magnetic field which lies beyond the magnetic field investigated in the present experimental set-up.

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