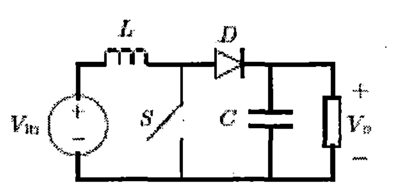
Chapter 3

POWER ELECTRONICS AND MODEL OF B00ST CONVERTER

3.1 Power Electronics:

Power electronics uses the semiconductor devices as switching element which are developing technically rapidly[18]-[21],[120]. It has a variety of nonlinear behavior due to its nonlinear operation and dynamics. It helps to study the modeling strategies ,the chaotic dynamics and to observe the bifurcation scenarios [15],[23],[24]. In the past decades power electronics has gone through intense development in many aspects of technology including power devices, control methods, circuit design, computer-aided analysis, passive components, packaging techniques, etc. Power electronics is mainly motivated by practical applications. However good analytical models allowing better understanding and systematic circuit design were only developed in late 1970's and in-depth analytical and modeling work is still being actively pursued today[25]-[58]. Our aim is to give an account of methods and techniques that can be applied to study the many "strange" phenomena previously observed in power electronics. Simple DC/DC boost converters (fig.3.1) are used to illustrate the modeling approaches that are capable of retaining the essential qualitative properties. Such properties peculiar to switching and non-smooth systems are systematically analyzed in terms of possible bifurcation scenarios and nonlinear phenomena. A circuit of simple dc/dc boost converter is presented for discussion of the modeling methods for characterizing nonlinear phenomena such as bifurcation and chaos in power electronics.



Simple DC-DC boost converter .

3.2 Power Electronics Circuits and the DC-DC Boost converters :

A power electronics circuit operates by toggling its topology among a set of linear or nonlinear circuit topologies under the control of a feedback system. They can be considered as piecewise switched circuits. In the above circuit an inductor is 'switched' between the input and the output through an appropriate switching element (*S*)[121]. The way in which the inductor is switched determines the output voltage level and transient behavior. Semiconductor switches like SCR / IGBT / FET / BJT etc. are used to implement the switching and through the use of a feedback control circuit the relative durations of the various switching intervals can be continuously adjusted. Such feedback action effectively controls the dynamics and steady-state behavior of the circuit. Thus both the circuit topology and the control method determines the dynamical behavior of the circuit.

In a typical period-1 operation the switch *S* and the diode *D* are turned on and off in a cyclic and complementary fashion under the command of a pulse-width modulator. When the switch *S* is closed (diode *D* is open) the inductor current rises up and when the switch *S* is open (diode *D* closed) the inductor current decays. The duty ratio continuously controlled by a feedback circuit , maintain the output voltage at a fixed level even under input and load variations in the mode either *continuous conduction mode* (CCM) or *discontinuous conduction mode* (DCM). In CCM the inductor current is maintained non-zero throughout the entire repetition cycle. This happens when the inductance *L* is relatively large or the load current demand is relatively high. In DCM the inductor current is zero for an interval of time within a cycle. This happens when the inductance *L* is relatively small or the load current demand is low causing the inductor current to fall to zero as the inductor is being discharged. During the interval of zero inductor current both switch *S* and diode *D* are open. For both CCM and DCM the output voltage has a fixed relationship with the input voltage as determined by the duty ratio. A steady-state relationship can be easily found. The expression (equation no. 3.1& 3.2) shows the steady-state output voltage expressions for stable period-1 operation which is preferred for most industrial applications. Here we denote steady-state duty ratio by δs and the repetition period by T. It represents only one particular operating regime. Because of the existence of many possible operating regimes it would be of practical importance to have a thorough understanding of what determines the behavior of the circuit so as to guarantee a desired operation or to avoid an undesirable one.

3.3 Typical control strategies for DC-DC Boost converters :

The dc-dc Boost converters are designed to deliver a regulated output voltage. The control of it takes on two approaches namely voltage feedback control and current-programmed control also known as *voltage-mode* and *current-mode* control respectively . In voltage-mode control the output voltage is compared with a reference to generate a control signal which drives the pulse-width modulator via some typical feedback compensation configuration. For current-mode control an inner current loop is used in addition to the voltage feedback loop, the aim of which is to force the peak inductor current to follow a reference signal which is derived from the output voltage feedback loop . The result of current-mode control is a faster response. This kind of control is mainly applied to boost and buck-boost converters which suffer from an undesirable non-minimum phase response. The simplified schematics is given below

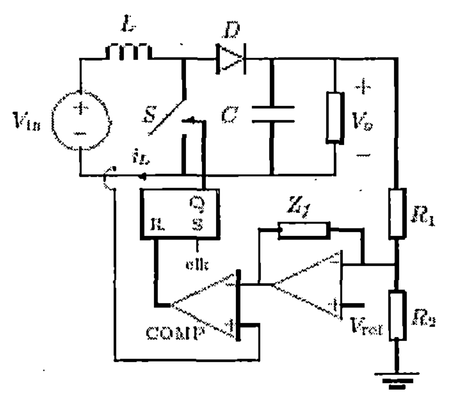


Fig. 3.2 Typical control approaches for DC-DC current-mode control.

$$V_o = \frac{1}{1-\delta} V_i \quad \text{-------- (3.1)}$$

$$V_o = \frac{V_i}{2}\left(1+\sqrt{1+\frac{2\delta^2 L}{RT}}\right) \quad \text{--------(3.2)}$$

Steady state relationship of input and output (1) for CCM and (2) for DCM for stable period-1 operation. Different control are explained elaborately in [59]-[62], [65],[66],[96].

3.4 Conventional Treatments for the dc/dc Boost converters :

Power electronics circuits are essentially piecewise switched circuits . The number of possible circuit topologies is fixed and the switching is done in a cyclic manner but may not necessarily periodically because of the feedback action. This results in a nonlinear time-varying operating mode demanding the use of nonlinear methods for analysis and design. The methods like state-space averaging, phase-plane trajectory analysis, Lyapunov based control, Volterra series approximation etc. are introduced to study the non linearities. For many practical reasons the design of power electronics systems demands "adequate" simplifying models. As closed-loop stability and transient responses are basic design concerns in practical power electronics systems models that can permit the direct application of conventional frequency-domain approaches will present obvious advantages. Most engineers are trained to use linear methods is also a strong motivation for developing linearized models like the averaging approach [68] .

3.5 Bifurcations and Chaos in dc/dc Boost converters :

Power electronics engineers frequently encounter phenomena such as sub harmonics oscillations , jumps, quasi-periodic operations, sudden broadening of power spectra, bifurcations and chaos. Power supply engineers have experienced bifurcation phenomena and chaos in switching regulators when some parameters like input voltage and feedback gain are varied but usually do not examine the phenomena in detail they avoid these phenomena by adjusting component values and parameters. So the phenomena remain somewhat mysterious and rarely examined in a formal manner. So it immediate needs for investigating such nonlinear phenomena as chaos and bifurcation. The study of nonlinear[45],[48],[119] phenomena offers the opportunity of rationalizing the commonly observed behavior. Thus knowing how and when chaos occurs will certainly help to avoid it or to exploit for useful engineering applications.. That is why the study of bifurcations and chaos in dc/dc Boost converters has recently attracted much attention from both the power electronics and the circuits and systems communities.

3.6 A Survey of Research Findings:

The occurrence of bifurcations and chaos in power electronics was first reported in the literature in the late eighties by Hamill *et al.* Experimental observations regarding boundedness, chattering and chaos were also made by Krein and Bass back in 1990. These early reports did not contain any rigorous analysis rather they seriously pointed out the importance of studying the complex behavior of power electronics and its likely benefits for practical design. Since then much interest has been taken in the power electronics and circuits research communities in pursuing formal studies of the complex phenomena commonly observed in power electronics. In 1990, Hamill *et al* presented a paper at the IEEE Power Electronics Specialists Conference reporting an attempt to study chaos in a simple buck converter which became a subject of intensive research in the following decade. Using an implicit iterative map the occurrence of period-doublings, sub harmonics and chaos in a simple buck converter was demonstrated by numerical analysis, PSPICE simulation, MATLAB simulation, Multisim simulation and laboratory measurement. The derivation of a closed-form iterative map for the boost converter under a current-mode control scheme was presented later by the same group of researchers . This closed-form iterative map allowed the analysis and classification of bifurcations and structural instabilities of this simple converter. Since then, a number of authors have contributed to the identification of bifurcation patterns and strange attractors in a wider class of circuits and devices of relevance in power electronics. Some key publications are summarized below.

The occurrence of period-doubling for a simple dc/dc converter was reported in 1994 by Tse fig.3.3 . By modeling the dc/dc converter as a first-order iterative map, the period-doubling bifurcations can be located analytically. The idea is based on evaluating the Jacobian of the iterative map about the fixed point corresponding to the solution undergoing the period-doubling bifurcation . Simulations and laboratory measurements have confirmed the findings. Formal theoretical studies of conditions for the occurrence

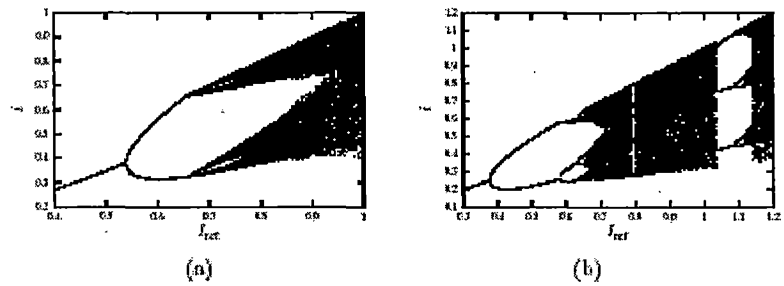


Fig. 3.3 Bifurcation diagrams from a current-mode controlled boost converter with For (a) *T/CR* = 0.125 and for (b) *T/CR* = 0.625.

of period-doubling in dc/dc converter were reported subsequently. Chakrabarty *et al.* who specifically studied the bifurcation behavior under variation of a range of circuit parameters including storage inductance, load resistance, output capacitance, etc. In 1996, Fossas and Olivar presented a detailed analytical description of dc/dc converter dynamics identifying the topology of its chaotic attractor and studying the regions associated with different system evolutions. Various possible types of operation of the converter were also investigated through the so-called stroboscopic map obtained by periodically sampling the system states. The bifurcation behavior of dc/dc Boost converters under current-mode control has been studied by a number of authors. Deane first reported on the route to chaos in a current-mode controlled boost converter. Chan and Tse studied various types of routes to chaos and their dependence upon the choice of bifurcation parameters. Figure two bifurcation diagrams numerically obtained from a current-mode controlled boost converter with output current level being the bifurcation parameter. In 1995, the study of bifurcation phenomena was extended to a fourth-order 'Cuk DC/DC converter under a current-mode control scheme. The four dimensional system is represented by an implicit fourth order iterative map, from which routes to chaos are identified numerically. It can be seen from figures (fig. 3.4) for the case of the boost converter the bifurcation behavior contains transitions where a "sudden jump from periodic solutions to chaos" is observed. These transitions cannot be explained in terms of standard bifurcations such as period-doubling" and "saddle-node".

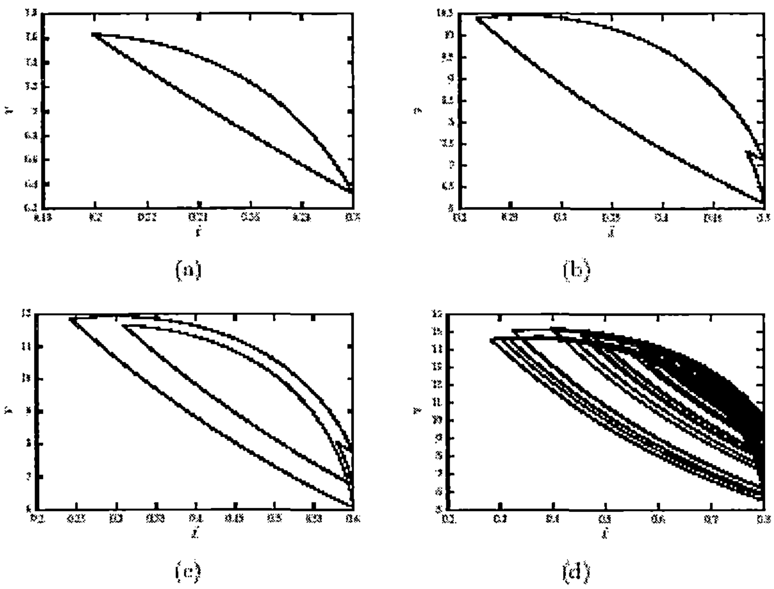


Fig. 3.4 Trajectories from a current-mode controlled boost converter. (a) Stable period-1 operation (b) stable period-2 operation (c) stable period-4 operation and (d) chaotic operation.

In fact,as studied by Banerjee *et al an*d Di Bernardo these transitions are due to a novel class of bifurcation known as "grazing" or "border-collision" bifurcation which is unique to switched dynamical systems. Since most power electronics circuits are non-autonomous systems driven by fixed-period clock signals, the study of the dynamics can be effectively carried out using appropriate discrete-time maps. In addition to the aforementioned stroboscopic maps). Di Bernardo *et al.* studied alternative sampling schemes and their applications to the study of bifurcation and chaos in power electronics. It has been found that non-uniform sampling can be used to derive discrete-time maps (termed A-switching maps) which can be used to effectively characterize the occurrence of bifurcations and chaos in both autonomous and non-autonomous systems. These maps turned out to be particularly useful for the investigation of nonsmooth bifurcation in power electronics circuits and systems. Also, the occurrence of periodic chattering was explained in terms of sliding solutions . When external clocks are absent and the system is "free-running", for example,under a hysteretic control scheme, the system is autonomous and does not have a . fixed switching period. Such free-running converters were indeed extremely common in the old days when fixed-period integrated-circuit controllers were not available. For this type of autonomous converters, chaos cannot occur if the system order is below three. Power electronics circuits other than dc/dc converters have also been examined in recent years. Dobson *et al.* reported "switching time bifurcation" of diode and thyristor circuits. Such bifurcation manifests as jumps in the switching times. Some attempts have been made to study higher order parallel-connected systems of converters which are becoming popular design choice for high current applications.

3.7 Modeling Strategies:

Different modeling strategies are presented in[36],[40],[52],[53],[67]-[69],[86],[91]. Averaging techniques retains the low-frequency properties while ignoring the detailed dynamics within a switching cycle. Usually, the validity of averaged models is only restricted to the low-frequency range up to an order of magnitude below the switching frequency. Averaging techniques can be useful to analyze those bifurcation phenomena which are confined to the low-frequency range. An effective approach for modeling power electronics circuits with a high degree of exactness is to use appropriate discrete-time maps obtained by uniform or non-uniform sampling of the system states. The aim is to derive an iterative function that expresses the state variables at one sampling instant in terms of those at an earlier sampling instant. The basic concepts of discrete-time maps and how they can be used to explore nonlinear phenomena in power electronics are explained.

3.8 Discrete-time maps

As is often the case in nonlinear dynamical systems, the analysis of complex phenomena that are of relevance to engineering requires "adequate" models. As mentioned previously, discrete-time maps [43],[93] obtained by suitable sampling of the system dynamics can be extremely useful for characterizing the occurrence of bifurcations and chaos in power electronics circuits. We distinguish two different classes of maps, *Poincare maps* and *normal form maps*. *Poincare maps* is useful for describing the global dynamics of the system under investigation from one sampling instant to the next. N*ormal form maps* is valid locally to a bifurcation point, is an invaluable tool for classifying the system behavior of a bifurcation. Several kinds of

Poincar'e maps have been defined for the analysis of nonlinear phenomena in power electronics circuits.

Derivation of the most commonly used maps, the *stroboscopic* map, the *S-switching* (synchronous switching) map, and the *A-switching* (asynchronous switching) map are performed. The boost converter can be described by a piecewise smooth system of the form:

$$\frac{di_L}{dt} = -\frac{1}{L}v_c + \frac{\delta t}{L}V_i \qquad \text{.........(3.3)}$$

$$\frac{dv_c}{dt} = \frac{1}{C}i_L - \frac{1}{RC}V_c \qquad \text{.........(3.4)}$$

where $i(t)$ is the inductor current, $v(t)$ the capacitor voltage, E a constant input voltage, and $\delta(t)$ a modulated signal. In general, discrete-time maps can be categorized into the following three classes .

- *Stroboscopic map:* obtained by sampling the system states *periodically* at time instants which are multiples of the period of the clock signal (stroboscopic instants). Note that the system states are sampled at each stroboscopic instant irrespectively of whether the system configuration switches or not at that instant.

- *S-switching map (synchronous switching):* obtained by sampling the system states at those stroboscopic time instants when the system commutes from phase 2 to phase 1.

- *A-switching map (asynchronous switching):* obtained by sampling the system states at time instants within each clock cycle when the system commutes from phase 1 to phase 2.

In order to derive the relevant maps, the following two simplifying assumptions are introduced which can be removed later.

i. No more than one commutation takes place during each period of the modulating signal.

ii. The commutation from phase 2 to phase 1 can only take place at time instants which are multiples of the ramp cycle T. It is always true when the converter is under current-mode control.

3.9 Derivation of stroboscopic and S-switching maps

The stroboscopic map is the most widely used type of discrete-time maps for modeling dc/dc boost converters. It is obtained by sampling the system dynamics every T seconds, (at the beginning of each clock cycle).We use suffix k as the counting index for this map. Applying a simple iterative procedure to the solutions of the state equations for phases 1 and 2, the stroboscopic map is obtained.

i. Derivation of A-switching maps

The A-switching map is obtained when the state vector is asynchronously sampled at switching times internal to the modulating period. All the maps introduced above can be used to obtain analytical conditions for the existence of given periodic orbits and standard bifurcations such as period-doubling and saddle-node. Many reports of standard bifurcations, as surveyed earlier, have effectively employed the stroboscopic and S-switching maps . The A-switching maps are particularly useful for the analysis of multi-switching behavior, and can be constructed more conveniently for operations where skipped cycles are frequent such as when the converter is operating in a chaotic regime.

ii. Analysis and Classification of Non-smooth Bifurcations

Focus on the non-smooth bifurcations, which are relevant to power electronics.

iii. System formulation

As discussed earlier, because of their switching nature, power electronics systems are often modeled by sets of ordinary differential equations (ODEs) or maps which are piecewise-smooth (PWS). Particularly whenever the circuit under investigation switches to a different configuration (from ON to OFF or vice versa), the vector field of the corresponding model changes from one functional form to another. According to the properties of the system along its discontinuity boundaries, we can identify three main classes of PWS dynamical systems:

- Systems with discontinuous states (jumps);
- Systems with a discontinuous vector field, i.e., the first derivative of the system states is discontinuous across the phase-space boundaries;
- Systems with a discontinuous Jacobian, i.e., the second derivative of the system states is discontinuous across the phase-space boundaries.

The discontinuity boundaries between different phase-space regions can themselves be smooth or piecewise smooth.

iv. Bifurcation possibilities

Power electronics systems can exhibit standard bifurcations such as period doubling or saddle-node. Such bifurcations are indeed frequently observed both analytically and experimentally as surveyed earlier. Some of the most common dynamical transitions observed in power electronics circuits, such as the "sudden jump to chaos" mentioned earlier, cannot be explained in terms of standard bifurcations. Power electronics systems, being *switched dynamical systems,* exhibit an interesting class of bifurcations which are not observed in their smooth counterparts. For switched dynamical systems, a dramatic change of the system behavior is observed when a part of the system trajectory hits tangentially one of the boundaries between different regions in phase space. When this occurs, the system is said to undergo a *grazing* bifurcation which is also known as C-bifurcation.

3.10 Derivation of normal-form map near a grazing bifurcation.

When the system undergoes a grazing bifurcation, fixed point of such a normal-form map crosses transversally some boundary in the phase space.). This is also called *border-collision bifurcation* whose occurrence in one-dimensional and two-dimensional maps was first reported in the western literature by Nusse *et al.* It is worth noting that such normal-form maps are not always piecewise linear. It can be shown that the form of the normal-form map at a grazing depends on the discontinuity of the system vector field, and is piecewise linear only if the discontinuity boundary between the ON and OFF zones is itself discontinuous such as in the cases of many power electronics circuits[100],[101],[110],[115]. Knowing the form of the normal-form map associated with a border collision becomes important when the aim is to predict the dynamical behavior of the system following such a bifurcation. i.e., from a fixed point associated with an orbit which does not cross the boundary to one corresponding to a solution which crosses the boundary. Linearizing the system flow about each of these fixed points, it can then be obtained a piecewise linear map. This approximate map is valid if the switching hyper plane which itself non-smooth. An acceptable method for classifying and predicting the dynamical scenarios following a border-collision bifurcation is given in Di Bernardo *et al.* It is worth mentioning that power electronics systems can exhibit a peculiar type of solution, termed *sliding*, which lies within the system discontinuity set. Intuitively, this can be seen as associated with an infinite number of switching between different phase-space regions which keep the trajectory

3.11 Chaos in Power Electronics

Chaos in power electronics are presented and explained in [32]-[35], [37]-44],[87]-[90], [95],[96]. The presence of sliding bifurcation can give rise to the formation of so-called sliding orbits, i.e., periodic solutions characterized by sections of sliding motion (or chattering). These solutions play an important role in organizing the dynamics of a given power electronics circuit . Research is still on-going in identifying a novel class of bifurcations, called *sliding bifurcations,* which involve interactions between the system trajectories and discontinuity sets where sliding motion is possible.

3.12 Current Status and Future Work

Research in nonlinear phenomena of power electronics is heading first most of the work reported so far has focused on identifying the phenomena and explaining them in the language of the nonlinear dynamics literature. The engineers commonly observed "strange" phenomena (e.g., chaos and bifurcation) and its become the topics that can be scientifically approached, rather than just being "bad" laboratory observations not relevant to the main technical interest. The chaos and system theorists published and demonstrated the rich dynamics of power electronics, offering a new area for theoretical study. It seems that identification work will continue to be an important area of investigation to know when and how a certain bifurcation occurs and how to avoid it. Power electronics is an emerging discipline with new circuits and applications creation every day. The lack of general solutions for nonlinear problems makes it necessary for each application to be studied separately and the associated nonlinear phenomena identified independently. Future research will inevitably move toward any profitable exploitation of the nonlinear properties of power electronics. As a start, some applications of chaotic power electronics systems and related theory have been identified, for instance, in the control of electromagnetic interference by "spreading" the noise spectrum , in the application of "targeting" orbits with less iterations (i.e., directing trajectories to certain orbits in as little time as possible)and in the stabilization of periodic operations.

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