Chapter 4

Conventionality of Simultaneity and Tippe Top Paradox

4.1 Introduction

Since the birth of the special theory of relativity paradoxes concerning the theory have always been very common. Even today in the literature newer paradoxes continue to pour in, a latest item in the list being the tippe top paradox. In an interesting paper Basu *et al.* [1] have posed the paradox concerning a fascinating toy known as tippe top. The top has the remarkable property that after giving it an initial vertical spin on a table about its symmetry axis, the top turns itself (after a few rotations) upside down and stands spinning on its stem. A schematic diagram is given in Fig. 4.1. The statement of the paradox goes like this: Not all

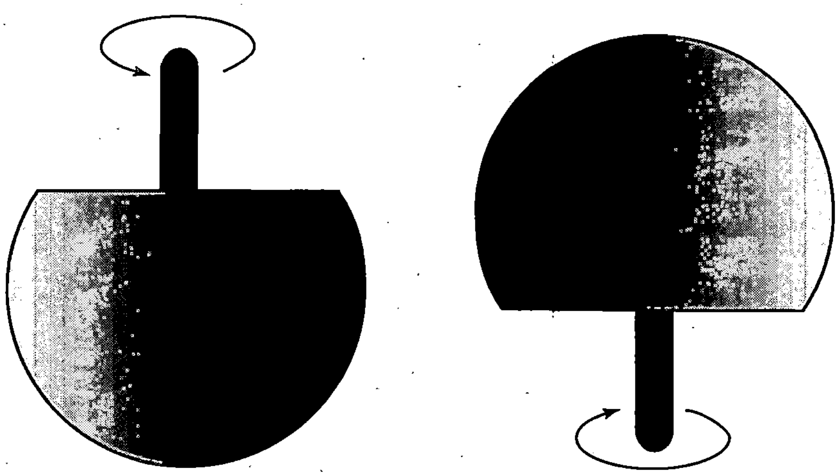

Figure 4.1: A Tippe-top

tops demonstrate this charming feat. In order for this to happen, the ratios of its principal moments of inertia should fall in a certain regime. If now such a top is observed from a rocket frame in which the spinning top appears to recede with a uniform velocity, the cross-section of the top will appear elliptical due to the relativistic length contraction effect along the boost direction. For a sufficiently fast rocket, therefore, it is possible that the ratios of the principal moments of inertia go out of the regime required for the top to tip over. This is certainly paradoxical since a mere change of reference frame cannot alter the fact that the top tips over.

The paper claims to have resolved the paradox first by noting that the paradoxes in relativity often arise because one tends to focus only on one relativistic effect (in this case the length contraction effect) while losing sight of the other kinematical consequences of Special Relativity (SR). The authors then demonstrate, by applying the Lorentz transformations (LT) to the equations of motion of the particles of the top, that the whole body of relativistic effects such as the length contraction, the time dilation, the relativity of simultaneity and the likes find their appearances in the description of the spinning motion of the particles of the top from the reference frame of the rocket. The particle orbits have been studied both analytically and by computer simulation to demonstrate how the classical notion of the rigid body fails in relativity. The origin of the paradox has then been attributed, in different words, to our adherence to the classical notion of rigidity in the relativistic domain.

Although it is interesting as well as instructive to view on the computer screen the motion of the particles of the spinning top as observed from the rocket frame (but referring it to the moving point of contact of the top with the table), we consider the paper to suffer from certain drawbacks which we would like to address. It appears, in accordance with the paper's claim, that the resolution of the paradox relies heavily on the lack of synchrony aspect of SR. This, in our opinion, is a weak point of an otherwise interesting paper. Indeed we will argue (in Sec. 4.4) that the lack of synchrony aspect may well exist in the Galilean world where it is well known that no such paradox should truly exist. The other drawback is the authors' attempt to connect the paradox with one of the corollaries of SR, that the notion of rigid body is inconsistent with it. In Sec. 4.6 below it will be shown that this aspect of SR has no bearing with the current paradox. It is indeed true that many paradoxes can arise if one inadvertently carries the classical notion of rigidity over into relativistic situation [2]. The classical rigid body by definition must move as one entity when it is pushed at one end, i.e., the disturbance at one end of the body would be propagated with infinite velocity through the body. This is in contradiction to the relativity principle that there is a finite upper limit to the speed of transmission of a signal. The analysis of Basu *et al.* considers only uniform rotation of the particles of the top with respect to the table frame; no transients are involved. It is therefore surprizing how the rigidity issue should be connected to the problem. The aim of the present paper is to deliberate on these and related issues.

In recent years a new approach based on Conventionality of Simultaneity (CS), to understanding paradoxes in relativity has been found fruitful. For example, Redhead and Debs [3] have shown that the CS approach provides a means to put an end to the question concerning the notorious twin paradox as to where and when the differential ageing takes place. As another example Selleri [4] has shown that a particular simultaneity convention compared to that of Einstein seems to be more appropriate in explaining the Sagnac effect from the point of view of the rotating turn-table. This study will also follow the CS approach to critically examine the work of Basu *et al.* A brief introduction to the CS thesis of relativity, therefore, is in order. This will be done in Sec. 4.3.[1] The main arguments will be presented in Secs. 4.4 and 4.6 before we summarize all this in Sec. 4.7. However, in order to set the stage we will briefly reproduce in Sec. 4.2

[1]For a fairly comprehensive review for the CS thesis and consequent transformation equations the reader is referred to App. A and App. B.

the basic arguments used in [1] to clarify the paradox [5].

4.2 Equation of Motion and Coordinate System

For simplicity consider a vertical top such that a typical particle P of the top in the table frame Σ^0 executes a horizontal (in the X-Y plane) circular motion. The equation of motion of the particle in the coordinate system of Σ^0 is given by

$$x^0 = R\cos\omega t^0, \qquad y^0 = R\sin\omega t^0, \qquad {x^0}^2 + {y^0}^2 = R^2 \tag{4.2.1}$$

where ω is the angular speed of the top and R represents the distance of P from the axis.

Consider now a frame of reference Σ of the rocket with respect to which the table and the top moves along the positive x-direction common to both Σ and Σ^0. Suppose that the x and t coordinates of the rocket frame are linearly related to the corresponding x^0 and t^0 of Σ^0 through the following transformations:

$$\begin{aligned} x^0 &= ax + bt, \\ t^0 &= gx + ht \end{aligned} \tag{4.2.2}$$

and also suppose $y^0 = y$, where a, b, g and h are independent of x and t. In reference [1] however they have written LT, but here we wish to keep it a bit more general for reasons which will be apparent soon. Clearly from the above transformation equations it follows that the origin of Σ^0 satisfies the following equation of motion with respect to Σ.

$$ax_{\text{orig}} + bt = 0, \tag{4.2.3}$$

where the suffix 'orig' refers to the origin.In other words the translational velocity of the origin, as observed from the rocket frame is given by

$$u = -\frac{b}{a}. \tag{4.2.4}$$

Since this translatory motion is irrelevant to the spinning of the top about its axis, it may be subtracted out from the *apparent* equation of motion of the particles of the top as seen from Σ. We may thus define the spinning coordinates of P as

$$\begin{aligned} x_s &= x - x_{\text{orig}} = x + \frac{b}{a}\,t, \\ y_s &= y, \\ t_s &= t. \end{aligned} \tag{4.2.5}$$

Inserting equations (4.2.2) and (4.2.5) in equation (4.2.1) one obtains

$$x_s = \frac{R}{a}\cos\omega\left[\left(h - \frac{gb}{a}\right)t + gx_s\right], \tag{4.2.6}$$

$$y_s = R\sin\omega\left[\left(h - \frac{gb}{a}\right)t + gx_s\right]. \tag{4.2.7}$$

The trajectory of the particle P in the rocket frame Σ is obtained by eliminating t from the above equations and one thus has

$$a^2{x_s}^2 + {y_s}^2 = R^2. \tag{4.2.8}$$

Now in SR the transformation equation (4.2.2) is nothing but LT:

$$x^0 = \gamma(x - \beta ct), \qquad y^0 = y, \qquad t^0 = \gamma\left(t - \beta c^{-1}x\right) \tag{4.2.9}$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$ with $\beta c = v$ representing the speed of the rocket with respect to the table. In other words for LT, the transformation matrix representing equation (4.2.2) is given by

$$\mathbf{T} = \begin{pmatrix} a & b \\ g & h \end{pmatrix} = \gamma\begin{pmatrix} 1 & -\beta c \\ -\beta c^{-1} & 1 \end{pmatrix}. \tag{4.2.10}$$

Inserting these values of a, b, g and h in (4.2.6), (4.2.7) and (4.2.8) we obtain the equation of motion and that of the trajectory of the particle P as

$$x_s = \gamma^{-1}R\cos\omega\left(\gamma^{-1}t - \gamma\beta c^{-1}x_s\right), \tag{4.2.11}$$

$$y_s = R\sin\omega\left(\gamma^{-1}t - \gamma\beta c^{-1}x_s\right) \tag{4.2.12}$$

and

$$\gamma^2 x_s^2 + y_s^2 = R^2. \tag{4.2.13}$$

Authors of reference [1] rightly point out that the above equations display all the well known effects of relativity. For example equation (4.2.13) shows that what is a circle with respect to the table frame is now an ellipse with respect to the rocket frame (in the spinning coordinates) because of the length contraction effect. Equations (4.2.11) and (4.2.12) show that the angular frequency ω of the top is reduced to $\gamma^{-1}\omega$ because of the time dilation. Finally the relativity of simultaneity (i.e the lack of synchronization of clocks) is manifested in the presence of the spatial coordinate x_s in the phase factors of the sinusoidal functions in equations (4.2.11) and (4.2.12). The authors claim that this feature plays an essential role in the resolution of the paradox. We will however show that this is not quite correct.

The arguments of Basu *et al.* [1] are based on the apparent non-rigid behaviour of the top as seen from the rocket.However, before we go into these in detail let us now study this non-rigid character a bit closely. Equations (4.2.11), (4.2.12) and (4.2.13) display qualitatively two distinct types of non-rigidity viz. type I and type II.

Type I. This type of non-rigidity is manifested in the periodical change of the distance of any particle of the top from the center, as the particle travels along the elliptical path according to equation (4.2.13).

Type II. From equation (4.2.11) and (4.2.12) it will be evident that a chain of particles which lie along the radius of the circle (vide equation (4.2.1)) parallel to the x-axis of Σ^0 at $t^0 = 0$ will appear to take a shape of a bow below the semi-major axis at $t = 0$ with respect to the rocket frame (vide Fig. 4 of reference [1] for a schematic diagram of the positions of the particles in the radial chain at $t = 0$).A particle in the chain which is farther away from the centre will lie farther below the x-axis. This progressive increase of initial phases of the particles is clearly the consequence of the phase factor in equations (4.2.11) and (4.2.12). As time passes, the bow-like chain rotates, straightens itself, again bends down and straightens up and continues like this. (For the stills of the computer movie of the chain of particles in the table and the rocket frames see Fig. 5 of reference [1].) This corresponds to another type of non-rigid behaviour of the material of the top as distinct from the type I. We call it type II. Note that this type II non-rigidity is clearly the result of the relativity of simultaneity.

The authors of reference [1] observe that the spinning top is "more like a visco-elastic fluid in a weird centrifuge subjected to anisotropic external stresses." We reproduce here the "smoothed tracings of the printout of the computer movie" (Fig.5 of Basu et al.)

However, although it is correct that the concept of rigid body dynamics and moments of inertia appears to be no longer valid in the spinning coordinate system, the type II non-rigidity at least has nothing to do with the paradox. In fact this apparent fluid-like motion of the top particles has no connection with the question of the inconsistency of the notion of rigidity in relativity. In the next section we will show, among other things, that the fluid-like motion of the particles of the top can also be seen in the non-relativistic world although it is well known that the notion of rigid body is quite consistent in classical mechanics.

4.3 Conventionality of Simultaneity and Transformation Equations

4.3.1 Relativistic World

In special relativity spatially separated clocks in a given inertial frame are synchronized by light signals. This synchronization is possible provided one

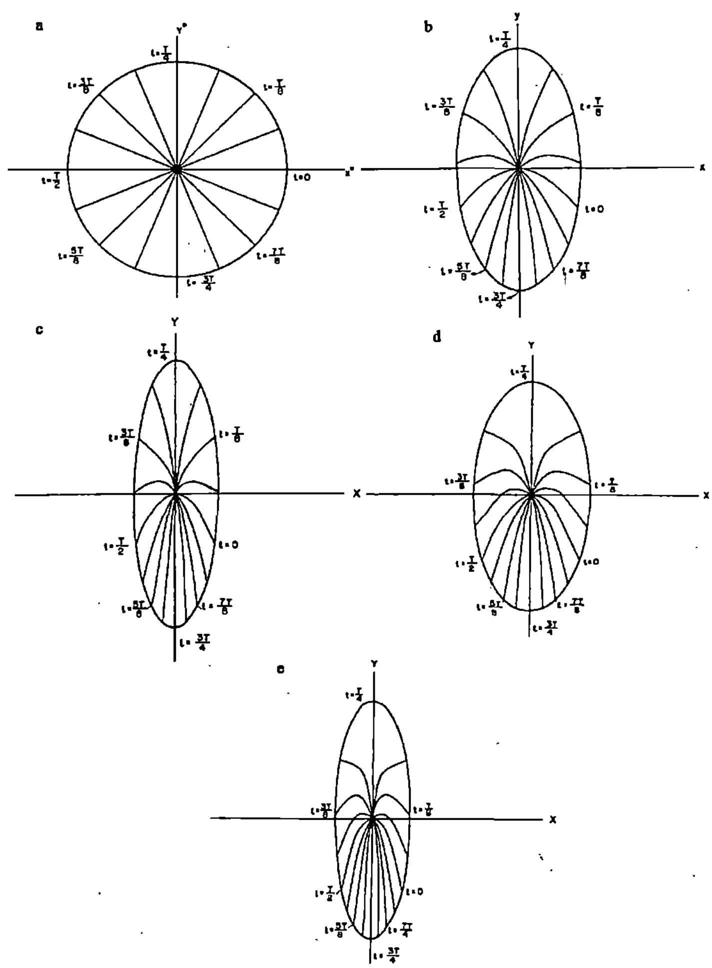

Figure 4.2: Chain of particles (a) In the table frame ($\beta = 0$). (b) In the rocket frame with $\beta = 0.866$ and $\omega = 1.2 \times 10^9 Hz$. (c) In the rocket frame with $\beta = 0.950$ and $\omega = 1.2 \times 10^9 Hz$. (d) In the rocket frame with $\beta = 0.866$ and $\omega = 2.4 \times 10^9 Hz$. (e) In the rocket frame with $\beta = 0.950$ and $\omega = 2.4 \times 10^9$. (From Basu et al.)

knows beforehand the One-Way Speed (OWS) of the signal. But the measurement of OWS requires pre-synchronized clocks and therefore one ends up in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the synchronizing signal within certain bounds. Einstein assumed the OWS of light to be equal to c which is the same as the Two-Way Speed (TWS) of light. The TWS of a signal is an empirically verifiable quantity, as this can be measured by a single clock without requiring any distant clock synchrony. Note that this stipulation (the equality of OWS and TWS) of Einstein has nothing to do with his "constancy of velocity of light" postulate [6, 7]. The assertion that the procedure for distant clock synchrony in SR has an element of convention is known as the CS-thesis, first discussed by Reichenbach [8] and Grünbaum [9]. The synchronization convention adopted by Einstein is commonly known as the Einstein synchrony or the standard synchrony. The possibility of using a synchronization convention other than that adopted by Einstein and consequent transformation equations between inertial frames are much discussed by various authors [4, 6, 7, 10–13]. For example it is known that the relativistic world can well be described by the so called Tangherlini Transformations (TT) by adopting absolute synchrony [4, 6, 7, 12, 14, 15]:

$$x = \gamma \left(x^0 + \beta c t^0 \right), \qquad y = y^0, \qquad t = \gamma^{-1} t^0. \tag{4.3.1}$$

Notice that the absence of spatial coordinate in the time transformation above, means the distant simultaneity is absolute (since $\Delta t^0 = 0 \Longrightarrow \Delta t = 0$). Since, according to the CS-thesis, the question of simultaneity of any two spatially separated events depends on the synchronization convention, the issue of relativity of simultaneity which is often considered as one of the most fundamental imports of SR, has little significance [6]. We will get back to this issue and the transformations (4.3.1) in particular in section 5 in the context of the paradox.

4.3.2 Galilean World

A less well-known fact, however, is that the CS-thesis can also be imported in the classical (Galilean) world. Consider as a fiction that we live in the Galilean world and suppose light travels through ether, stationary with respect to Σ^0. The space-time coordinates of an arbitrary inertial frame Σ moving with speed $v = -\beta c$ with respect to Σ^0 are related to those in Σ^0 by the so called Galilean Transformations (GT)

$$x^0 = x - \beta ct, \qquad y^0 = y, \qquad t^0 = t. \tag{4.3.2}$$

In the Galilean world synchronization issue usually does not come in, since in principle, all the spatially separated clocks can be synchronized instantaneously by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However, ingredients of Einstein synchrony can be incorporated even in this world. Say in a somewhat playful spirit one chooses to synchronize an arbitrarily located clock in any frame Σ with one placed at its origin by sending a light signal from the origin to the clock in such a way that the OWS of light *along any line* passing through the origin is independent of the direction of propagation and is equal to the TWS of the signal along the line. In this case one obtains the so called Zahar Transformation (ZT) [7, 12, 16]:

$$x = x^0 + \beta ct^0, \qquad y = y^0, \qquad t = \gamma^2\left(t^0 + \beta c^{-1}x^0\right), \tag{4.3.3}$$

and its inverse

$$x^0 = \gamma^2\left(x - \gamma^{-2}\beta ct\right), \qquad y^0 = y, \qquad t^0 = t - \gamma^2\beta c^{-1}x. \tag{4.3.4}$$

One may verify from the transformations (4.3.3) and (4.3.4) that the TWS of light along any direction measured in an arbitrary reference frame is given by the same expression as one would have obtained using GT. For example one may verify that the TWS of light along the x-axis and y-axis in Σ are given by the Galilean results, $c\left(1-\beta^2\right)$ and $c\left(1-\beta^2\right)^{\frac{1}{2}}$ respectively [6, 14].[2] Clearly the presence of the spatial coordinate in the time transformations of (4.3.3) and (4.3.4) is the result of the adopted synchrony. The properties of rods and clocks also do not change due to their motions with respect to Σ^0. However, there is an apparent length contraction and time dilation effect with respect to Σ because of different simultaneity criterion used in this frame.

It may also be noted that even in the Galilean world the transformation equations (4.3.3) and (4.3.4) depend on the speed of light c in ether, since light has been chosen as the synchronizing agent. If instead of light the clocks are synchronized by any other signal with speed c' in Σ^0 the transformation equations would have been

$$x = x^0 + \beta' c' t^0, \qquad y = y^0, \qquad t = \gamma'^2 \left(t^0 + \beta' c'^{-1} x^0\right) \tag{4.3.5}$$

where β' and γ' are the same as β and γ except where c is replaced by c'.[3] The synchronization in this case may be called *pseudo-standard synchrony* since the synchronization agent here is not the light signal.

[2]The Galilean world or classical world is thus defined to be a world where the TWS of any signal obeys the transformation law that one would have obtained by using Galilean velocity addition rule. On the other hand the world is said to be relativistic if the space-time admits an invariant TWS [7]. It may be noted, by virtue of the CS thesis, that kinematically different transformations and OWS' may correspond to a same kinematical "world" [14].

[3]Operationally one may consider that Σ^0 is a frame of reference stationary with respect to some fluid which supports an acoustic mode with isotropic speed c'. Clocks in any frame are assumed to be synchronized following Einstein's convention using the signal. The equation (4.3.5) were called Dolphin Transformations (DT) in the Galilean world in reference [7] where it was first derived.

4.4 Tippe top Paradox in the Galilean World

The paradox can now be posed even in the Galilean world. With respect to the observer in the rocket frame Σ, the geometry of the top will appear to have altered because of the length contraction effect which is the outcome of the distant clock synchrony adopted in the rocket frame. This may give rise to the possibility that the ratios of the principal moments of inertia go out of the regime required for the top to tip over! We now proceed to "resolve" the paradox by following the line of arguments used in reference [1]. The matrix of ZT (equations (4.3.3) and (4.3.4)) representing the coordinate transformations for x^0 and t^0 is

$$\mathbf{T} = \begin{pmatrix} \gamma^2 & -\beta c \\ -\gamma^2\beta c^{-1} & 1 \end{pmatrix}. \tag{4.4.1}$$

The equations of motion and the equation of the trajectory of the particle in the spinning coordinates can be obtained by inserting the elements of $\mathbf{T}$ *i.e* a, b, g and h in equations (4.2.6), (4.2.7) and (4.2.8) as

$$x_s = \gamma^{-2} R \cos\omega(\gamma^{-2}t - \gamma^2\beta c^{-1} x_s), \tag{4.4.2}$$

$$y_s = R \sin\omega(\gamma^{-2}t - \gamma^2\beta c^{-1} x_s) \tag{4.4.3}$$

and

$$\gamma^4 x_s^2 + y_s^2 = R^2. \tag{4.4.4}$$

As before, the time dilation, the length contraction and the lack of synchronization of clocks seem to be present in these equations. Only quantitatively these effects differ from those obtained earlier using the LT.

These equations suggest that qualitatively the top material displays the same form of non-rigidity (of both type I and II) as has been observed in the relativistic world as the particles of the top moves in accordance with equations (4.4.2) and (4.4.3) with respect to the observer in the rocket frame. Indeed one only needs to slightly modify the programming codes developed in reference [1] to simulate the motion of the particles of the top and see for oneself the picturesque output on the computer screen displaying as before, the non-rigid character of the body.

The idea of Einstein synchrony in the Galilean world may at a first sight seem to be a bit weird. However, the idea is not as strange as it appears. Consider LT in the non-relativistic regime where $\beta^2 \ll 1$ so that the approximation $\gamma \cong 1$ holds. In this approximation, contrary to common belief, LT does not go over to GT, instead one obtains the so-called Approximate Lorentz Transformation (ALT) [14, 17, 18].

$$x^0 = x - \beta ct, \qquad t^0 = t - \beta c^{-1}x. \tag{4.4.5}$$

As is expected the transformations (4.4.5) do not exhibit length contraction and time dilation. However, by virtue of the presence of the spatial coordinate in the time transformation the simultaneity is not absolute. It can also be verified [14] that ALT represents Einstein Synchrony. It is therefore not surprizing that ZT also reduces to equation (4.4.5) under the same approximation. ALT or Approximate Zahar Transformation (AZT) therefore represents the Galilean world with Einstein Synchrony. Inserting the coefficients of ALT/AZT in equations (4.2.6), (4.2.7) and (4.2.8) one obtains the equations of motion and the trajectory of the spinning particle as

$$x_s = R\cos\omega\left(t - \beta c^{-1}x_s\right), \tag{4.4.6}$$

$$y_s = R\sin\omega\left(t - \beta c^{-1}x_s\right) \tag{4.4.7}$$

and

$$x_s^2 + y_s^2 = R^2. \tag{4.4.8}$$

Clearly according to these equations the material of the top still displays non-rigidity of type II.[4]

In line of reference [1], one may now try to say that the concept of rigid body does not fit in the classical world. One may even be tempted to explain away the paradox by saying that nothing is rigid and all bodies are compressible and failure to comprehend this, leads to the paradox! Clearly this is absurd in the Galilean world or in the non-relativistic regime.

In order to understand the situation more clearly, let us examine the meaning of the spinning co-ordinate system. From the transformation equations (4.2.2) and (4.2.5) one may connect the vectors $\mathbf{x}^0 = \begin{pmatrix} x^0 \\ t^0 \end{pmatrix}$ and $\mathbf{x}_s = \begin{pmatrix} x_s \\ t_s \end{pmatrix}$ as

$$\mathbf{x}^0 = \mathbf{B}\mathbf{x}_s \tag{4.4.9}$$

with

$$\mathbf{B} = \begin{pmatrix} a & 0 \\ g & h - \dfrac{gb}{a} \end{pmatrix} \tag{4.4.10}$$

[4]The equations (4.4.6), (4.4.7) and (4.4.8) could have been obtained using the approximation $\gamma \cong 1$ directly in equations (4.2.11), (4.2.12) and (4.2.13) or alternatively in (4.4.2), (4.4.3) and (4.4.4), however, the present approach is more instructive since it clearly shows the role of the lack of synchrony of clocks in the type II non-rigidity of the top.

where, as before, the y-coordinate has been suppressed although we keep in mind that $y_s = y = y^0$. For Galilean transformation (vide equation (4.3.2))

$$\mathbf{T} = \begin{pmatrix} 1 & -\beta c \\ 0 & 1 \end{pmatrix}, \tag{4.4.11}$$

$\mathbf{B}$ turns out to be the identity matrix.

In this case the equations of motion and trajectory of any particle represented by equation (4.2.1) will not lead to a fluid-like motion of the particles in the spinning coordinate system. However for $\mathbf{B} = \mathbf{1}$, these coordinates are the same as that used in Σ^0!

This only demonstrates that the rigidly rotating top will continue to display its rigid character only in the coordinate system defined in the frame of reference at rest with the table. Therefore, it is obvious that the concept of rigid body dynamics or moments of inertia are applicable only in this unique frame and this knowledge therefore surely resolves the paradox. However, this fact does not depend on whether the world is classical or relativistic. Therefore, in accordance with our earlier assertion, there is no connection of the apparent fluid-like behaviour of the top material in the spinning coordinate system with the issue of incompatibility of the notion of rigidity in SR.

4.5 Lack of Synchronization – Is It Crucial?

Basu *et al.* [1] claimed that the lack of synchronization of clocks i.e the relativity of simultaneity aspect of SR plays an essential role in the resolution of the paradox. But we have already observed in section 3, that the question of relativity of simultaneity is purely conventional and therefore is devoid of any empirical content. Kinematically the relativistic world can be described by TT (4.3.1) representing absolute simultaneity. The transformation matrix representing the inverse of TT in the $x - t$ plane may be written as

$$\mathbf{T} = \begin{pmatrix} \gamma^{-1} & -\gamma\beta c \\ 0 & \gamma \end{pmatrix}. \tag{4.5.1}$$

Inserting the elements of $\mathbf{T}$ in (4.2.6), (4.2.7) and (4.2.8) we get

$$x_s = \gamma R \cos \gamma\omega t, \tag{4.5.2}$$

$$y_s = R \sin \gamma\omega t \tag{4.5.3}$$

and

$$\gamma^{-2} x_s^2 + y_s^2 = R^2. \tag{4.5.4}$$

Notice the absence of the phase terms in the sinusoidal functions. This means that the top does not display type II non-rigidity with respect to the rocket frame. However, the trajectory (4.5.4) of P is still an ellipse (this time the semi-major axis is along the x-direction) manifesting type I non-rigidity of the top. This only reiterates that the rigid rotation has to be defined in the table frame, but for this conclusion to hold the lack of synchronization aspect of SR does not play any role.

4.6 Rigidity and Transcendental Equation

It has been noted in [1] that the transcendental equation (4.2.6) is of the form

$$x_s = f(x_s), \tag{4.6.1}$$

which can be solved by iteration provided

$$|f'(x_s)| < 1. \tag{4.6.2}$$

It is claimed that the condition (4.6.2) when applied to equation (4.3.5) leads to

$$\omega R < c, \tag{4.6.3}$$

which only says that no particle of the top can exceed the speed of light. This result although fascinating, seems to be fortuitous, since, instead of equation (4.2.6) if equation (4.4.2) (which pertains to the Galilean world) is used in the inequality (4.6.2), one obtains the same constraint condition (4.6.3) on the speed of a particle of the top. This is surprizing, since in the Galilean world, there is no such speed limit intrinsically.

For the DT in the Galilean world (vide equation (4.3.5)) the condition (4.6.2) leads to

$$\omega R < c', \tag{4.6.4}$$

which is more surprizing.

On the other hand we have seen that TT which represents the absolute synchrony in the relativistic world does not lead to any transcendental equation (vide equation (4.5.2)) and hence no constraint on the speed of the particle is visible.

4.7 Summary

The present paper discusses the tippe top paradox and different aspects of its resolution proposed in reference [1]. Clock synchronization issues in the relativistic and the Galilean world figured in course of our discussion. A few transformation equations in addition to the LT were discussed in this connection. It is therefore worthwhile to summarize different properties of the transformations in the context of the paradox. This we do in Table 1 so that one is able to get the whole picture at a glance. The table is self-explanatory, however explanations of a few entries may be in order.

Basically we discussed two worlds – relativistic and classical, but an overlapping world "Relativistic/Classical" is included in column 1 as a separate entry. This corresponds to the transformations ALT and AZT (vide column 2) which are the forms of LT and ZT respectively under the approximation $\gamma \simeq 1$. For both the transformations the synchrony type (as shown in column 3) is standard. These transformations do not predict the length contraction and the time dilation effects (vide entries in column 6 and 7). The paradox in this case does not exist prima facie (vide the entry in the last column) in this regime since there is no length contraction effect. However the observer in Σ will find the top material to exhibit non-rigidity of type II.

Note that the entries in the 1st, 5th and 6th rows from column 3 onwards corresponding to the transformations LT, ZT and DT respectively are exactly the same. It means that the paradox and the resolution as suggested in reference [1] completely fits in the classical world too. It therefore dismisses the claim that the paradox has its origin in the incompatibility of the notion of rigidity with SR.

The entries against TT shows that in the relativistic world non-rigidity of Type I of the top exists although the synchrony here is absolute. It therefore follows that "lack of synchronization of clocks" cannot play an essential role in resolving the paradox.

Table 1

World	Transformation	Synchrony Type	Type I Non-rigidity	Type II Non-rigidity	Length Contraction	Time Dilation Effect	Paradox exists? (prima facie)
Relativistic	LT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
Relativistic	TT	Absolute	Yes	No	Yes (w.r.t. Σ^0)	Yes (w.r.t Σ^0)	Yes
Relativistic /classical	ALT/AZT	Standard (Einstein)	No	Yes	No	No	No
	GT	Absolute	No	No	No	No	No
Galilean (Classical)	ZT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
	DT	Pseudo Standard	Yes	Yes	Yes	Yes	Yes

Reference

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