Conceptual Questions in Relativistic Sagnac Effect and Related Issues

Thesis Submitted for a Degree of Doctor of Philosophy in Science (Physics) of the University of North Bengal

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আমার স্বর্গতঃ পিতা
বিমান রায়চৌধুরির স্মৃতির উদ্দেশ্যে

Dedicated to the memory of my father
Late Biman Raychaudhuri

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TO WHOM IT MAY CONCERN

This to certify that the investigations reported in this thesis entitled "**Conceptual Questions in Relativistic Sagnac Effect and Related Issues**" by Mr. **Biplab Raychaudhuri** has been carried out here by the candidate himself under my supervision and guidance. He has fulfilled all the requirements for the submission of the thesis for Ph.D. degree of the University of North Bengal. Although the research work presented in this dissertation has been performed in collaboration with others, much of the work has been performed by him and his contribution is quite substantial. In character and disposition Mr. **Biplab Raychaudhuri** is fit to submit the thesis for Ph. D. degree.

Prof. S. K. Ghosal
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Preface

The present dissertation, as the title suggests deals with among other things some conceptual issues concerning relativistic Sagnac effect. Historically perhaps no other experiment posed so strong a conceptual challenge to special relativity, since its advent in 1905, as the Sagnac experiment did. This experiment was first conceived by O. Lodge, A. A. Michelson and few others and finally was successfully performed by G. Sagnac in 1913. This is essentially an interference experiment performed with counter-propagating light beams where the whole set up is mounted on a rotating turntable. The idea of the experiment as conceived was pre-relativistic and its main aim was to prove the existence of ether. The Sagnac result unlike that of Michelson-Morley experiment is a non-null effect. The experimental result was accurate up to the first order in v/c (it still remains so) and thus was easily explained through classical kinematics. When one considers the effect of relativity, one finds that the correction is only in the second order in v/c. In the disc type Sagnac experiments although this second order effect can be neglected, the considerations of relativity in explaining the effect from the theoretical standpoint is highly challenging in various ways. The existence of the rich ramifications of the problem can be appreciated from the fact that not one but at least twenty different derivations of the Sagnac phaseshift formula was reported to exist. In 1996, Franco Selleri pointed out that even though there are so many relativistic derivations of the Sagnac formula, there seems to exist no derivation from the perspective of the rotating frame. Our interest in Sagnac effect stemmed from this observation of Selleri. The special theory of relativity is challenged even today in the context of the Sagnac result. Some authors still claim that the use of the Lorentz transformation, and for that matter, standard relativity gives null result in the derivation of the Sagnac result. Again on the contrary some believe that classical physics gives null result for Sagnac effect with matter waves. Also one finds in the literature that even there is no unanimity among the authors about the correct relativistic formula of Sagnac effect and at least two different relativistic formulae exist for the Sagnac result. These are higher order corrections and present day precision may not resolve this dilemma experimentally, but from the theoretical point of view, this dilemma should not be left unresolved. I decided to take up some of these issues hoping to address various question related to relativistically rotating discs, Ehrenfest and other paradoxes of relativity, including some aspect of general relativistic (optical) Sagnac effect. The present thesis is a compilation of the findings of the present author with his collaborators following investigations in these basic issues.

In recent times the Sagnac effect is much discussed int the literature. In global positioning system whose purpose is accurate navigation on or near earth's surface and to provide accurate worldwide clock synchronization and timing system, the correction due to the Sagnac effect is essential for accuracy. It has also been proposed in recent times to make the Sagnac effect the basis of gravitational wave detection using a Laser Interferometer Space Antenna (LISA). In fact number of websites related to the Sagnac effect listed by search engines increased enormously in recent years from what we found when we started our work on it. This shows the intensity of interest of physics community in the Sagnac effect. Applications of the Sagnac effect in newer fields are emerging day by day. Newer experimental techniques are evolving taking the measurement of the Sagnac result more and more precise. The Sagnac experiment performed with matter wave has opened a new perspective of the Sagnac effect. In a very interesting experiment performed in 2003, Wang, Zheng, Aiping and Langley claim to have observed Sagnac effect in a fibre optic conveyer set up, which may be interpreted as an observation of two different speeds of light in opposite directions by an *inertial* observer! People used to believe that the Sagnac phaseshift is proportional to the area of the optical loop of the set up. Wang et. al. experiment refutes this claim showing that the effect can be observed in the zero area configuration too. The basic theory of this Wang et. al. effect can be found in our linear Sagnac thought experiment set up discussed in the main text. This thought experiment was however proposed earlier by the present authors.

The present dissertation is an investigation on the Sagnac effect from a purely theoretical point of view. The aim here is to discuss some foundational questions and paradoxes related to rotating frame in general and the Sagnac effect in particular. Indeed for the present author and his collaborators, the study of the theoretical aspects of the Sagnac effect converged into the study of the relativity theory in rotating frames. The *conventionality of simultaneity* (CS) thesis of special relativity has been found to be an essential tool in this study. The conceptual difficulties posed by the Sagnac effect are found to be related to improper handling of physics in rotating frames in special relativity. The correction to the Sagnac formula due to the curvature of spacetime is also discussed for the sake of completeness.

The main text of the present study comprises of Chap. 3 to Chap. 7 which report the observations and results obtained by me along with my collaborators during our studies in the last few years. Some of these observations have been published and some have been reported in the national and international meetings.

The whole volume is organized as follows: In Chap. 1, after a brief introduction, we give a topicwise summary of the main chapters (Chap. 3 through Chap. 7). The reader will find a bird's eye view of the topics discussed in the main text therein; this will act as a gateway for to the main contents. Mathematical language is by and large avoided therein. In Chap. 2, a review of the Sagnac effect is given. The readers familiar with the Sagnac effect may wish to skip this chapter. However, going through it will be beneficial because this will act as a ready reference since particularly those aspects of the Sagnac effect are covered which are relevant for the present study.

The next chapter (*i.e.* Chap. 3) discusses the resolution of the long standing problem of Ehrenfest paradox thereby offering a solution of the controversy over the form of the Sagnac effect formula. A recently posed paradox called Tippe-top paradox concerning rotating frames is analyzed and its resolution is offered in Chap. 4. Another recent paradox which apparently challenges the very foundation of SR has been discussed with a proposed resolution which finds its place in Chap. 5. In Chap. 6 we discuss the possibilities of having a transformation equation from an inertial frame to a rotating frame and uphold F. Selleri's claim that absolute synchrony is the natural synchrony in a rotating frame. In the process we point out a weak point in a work by Post which, to our knowledge has not received proper attention anywhere till date. In Chap. 7, the analysis of the Sagnac effect is extended in curved spacetimes and some interesting results have been obtained in the pre-horizon regime.

All the chapters are self contained assuming some prior knowledge of the reader in the CS thesis of relativity and of different relativistic transformations corresponding to different procedures of synchronization of clocks. However a brief introduction to the CS thesis is given in Chap 1 to start with. Also, whenever the CS thesis is used in a chapter, a brief introduction suitable for that purpose is given for the sake of completeness. A fairly comprehensive account of the CS thesis and relativistic transformations are given in appendices A and B, respectively with the hope that this will provide an introduction to those who are not much familiar with this aspect of SR. The reason behind adding this appendix is that textbooks on SR seldom discuss the CS thesis and consequent transformations and the present thesis heavily relies on this Reichenbach-Grünbaum thesis of CS.

The Chap. 4 in the dissertation discusses an issue which as such is not connected with Sagnac effect, however it involves rotations. The importance of the inclusion of this treatise and placing it before another Sagnac topic is that the resolution of the tippe top paradox makes use of a different coordinate transformation (Zahar transformation) in the Galilean (classical) world where the clocks are synchronized following Einstein's method. This kind of approach is used to resolve a paradox of SR is done for the first time. This novel method of posing a problem in a hypothetical classical world and resolving it through the Zahar transformation to understand a paradoxe appearing in SR is proposed by us and hitherto unavailable in the literature. This chapter which discusses the Tippe top paradox apart from its intrinsic importance, therefore provides a preparation for the reader to pass smoothly to Chap. 5, apart from its intrinsic importance. Indeed, this exercise will help the reader to understand the resolution of the Selleri paradox without any difficulty. Moreover our intellectual journey in this study is reflected in the order of appearance of the topics (apart from the fact that this is the chronological order of publication too). We wish to share this experience with our readers, as well.

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and comfortable even in my toughest and most strenuous moments.

Notations and Abbreviations

ALSE	antiparallel LSE
ALT	Approximate Lorentz Transformation
CS thesis	Conventionality of Simultaneity thesis
DT	Dolphin transformation
GPS	Global Positioning System
GR	General theory of Relativity
LSE	Linear Saganc effect
LT	Lorentz transformation
OWS	One-way speed (velocity) (of light)
PLSE	parallel LSE
SD	Sagnac Delay
SP	Selleri Paradox
SR	Special Relativity
TE	Transformation equation(s)
TT	Tangherlini transformation
TWS	Two way speed (velocity) (of light)
ZT	Zahar transformation

Chapter 1

Introduction

1.1 Scope and Motivation

Since the time of Newton, physics on rotating frames remains a subject of fascination for the physicists: in addition to physical forces there appear some pseudo forces peculiar to rotating frames which shows, in classical context the absoluteness of rotation. The interest of physicists about rotation did not subside in the last century. With the development of newer physics, rotation stimulated further interests. The rotating disc problem, discussed by Paul Ehrenfest [1] in 1909 and now popularly known as Ehrenfest paradox apparently challenged the internal consistency of the theory of relativity. Einstein was interested in the effect of rotation on the spacetime geometry [2]. In recent past the desynchronization of clocks on a rotating frame became a subject of discussion.

In the early days of SR when it was not universally accepted by the scientists, the null result of Michelson-Morley experiment was not considered conclusive enough (along with the success of SR) to throw away the concept of all-pervading luminiferous ether. Georges Sagnac in 1913 very cleverly used rotation in his quest for the existence of ether. He considered his experiments conceptually similar to Michelson-Morley experiment and published his results in two papers (in french) [3] entitled *The existence of the luminiferous ether demonstrated by means of the effect of a relative ether wind in an uniformly rotating interferometer* and *On the proof of reality of the luminiferous ether with the experiment of the rotating interferometer*. In the Sagnac type experiments, one essentially sends out two coherent beams of light in opposite directions along the edge of a rotating disc.[1] The two beams are allowed to interfere at a point after they complete a full

[1]Sagnac's original experimental arrangement was different. This will be discussed in Chap. 2. By *the Sagnac type experiments* we refer to the set of experiments which use the basic principle of Sagnac's original experiment [4].

circle. When the disc is set into rotation, a fringe shift is observed which is found to depend on the angular velocity of the spinning disc. This is commonly referred to as the the Sagnac effect.

The Sagnac type experiments were repeated by several authors using photon (called optical Sagnac type experiment) and the accuracy is much improved by using laser and fibre optic interferometer. The effect has been observed in interferometers built for electrons, neutrons, atoms, superconducting Cooper pairs, radio wave and X-rays.

There are several applications of the Sagnac effect in recent years. Based on the Sagnac effect, ring laser and fibre optic gyros are being used as navigational tools. Importance of this effect also lies in connection with the question of time keeping clocks of clock-stations around the earth and Global Positioning System (GPS). Here the earth plays the role of rotating platform on which clocks between clock stations are synchronized by sending lights via satellites.

In spite of successful applications of the Sagnac effect in many fields, the conceptual challenges it put forward are no less interesting. Sagnac himself considered his experiment a proof of existence of luminiferous ether and thus explained it using classical kinematics. He was successful in his endeavour because his experimental result was of the first order in v/c. In a recent paper commenting on the early histories of theoretical studies in the Sagnac effect Selleri [3] remarks – "Surprisingly theoreticians were little interested in the Sagnac effect, as if it did not pose a conceptual challenge. As far as I know Einstein's publications never mentioned it, for example." Langevin [5] was the first to discuss its theoretical explanation. He was of the opinion that the Sagnac effect being a first order experiment cannot give judgement for or against any theory [3]. Langevin explained the empirical observations of the Sagnac experiment with Galilean kinematics which is "...slightly veiled in relativistic form by some words and symbols ..." [3]. In another publication in 1937 Langevin [6] tried to provide two relativistic explanations of the Sagnac effect. One of the explanations was based on the idea to adopt time of the centre of rotation (fixed in the laboratory) everywhere on the disc. Though for the standard relativity it looks queer, physicists acquainted with the CS thesis will recognize this for adopting absolute synchrony on the rotating frame. This will be made clear in the main text (Chap. 5).

Analysis of the Sagnac effect is usually done from the perspective of inertial frame and surprisingly there are several derivations for that. Hasselbach and Nicklaus [7] list twenty different derivations that exist in the literature and commented "This great variety (if not disparity) in the derivation of the Sagnac phase shift constitutes one of the several controversies ... that have been surrounding the Sagnac effect since the earliest days of studying interferences in rotating frames of reference." (This once again shows the great difficulty of explaining physics in rotating frame in SR.) Though so many derivations exists for the Sagnac formula, Selleri [8] claims that no calculation is available which is done truly from the perspective of the rotating frame. To provide one, he used inertial transformation (introduced by him) which is in conformity with the CS thesis to provide a calculation from the perspective of a rotating frame. He observes that the LT representing standard synchrony gives null result while the TT representing absolute synchrony gives the correct result for the Sagnac phase shift formula. (On the basis of this, Selleri termed absolute synchrony as the 'nature's choice' of synchronization.) The present author along with his collaborators has been fascinated by this very interesting observation of Selleri. The authors then decided to check the veracity of Selleri's observation that the

Sagnac effect has not been explained from the perspective of an observer on board the rotating platform. A brief literature survey brought attention to a very influential work by E. J. Post [9]. At the first sight it appears that Post has precisely done this. But a close scrutiny reveals some drawbacks of Post's considerations which has been explained in Chap. 6. There it has been explained how Post intuitively introduced a co-ordinate transformation from inertial to rotating frame (manifestly displaying absolute simultaneity – thereby contradicting *standard SR*) to use it in his metrical derivation of the Sagnac effect from the rotating frame perspective. Further, not being satisfied with the intuitive introduction of a coordinate transformation, Post 'derives' that TE from the LT under some kinematic condition due to rotation. The time transformation part of the TE is independent of space thus representing absolute synchrony. It is therefore surprising how, imposition of simple kinematical condition in the LT imply a TE which represent totally different synchrony!

There are other rich theoretical ramifications concerning the Sagnac effect. For example, we discovered that there exist two relativistic Sagnac phase shift formulae (one without a relativistic contraction factor and one with it) in the literature arising from the consideration (or lack of it) of length contraction of the periphery of the disc in rotation respectively. Present day experimental set up can measure upto the first order of precision in v/c and thus is unable to give judgment in this dilemma. It has been shown [10, 11] that this problem is intimately connected with the Ehrenfest paradox [1]. A detailed study of this aspect of the phase-shift formula and the Ehrenfest paradox forms the text of Chap. 3.

There are many interesting issues related to the behaviour of clocks and their synchronization in a rotating frame. In any frame, if the clocks are synchronized in the method of standard SR, they become desynchronized if the frame is accelerated. Moreover, it can be shown that in rotating frames each clock gets desynchronized with itself. This desynchronization is regarded, by some authors, as the 'root cause of the Sagnac phaseshift'. Refuting this claim, it has been shown [12, 13] (Chap. 5) that even if the clocks are synchronized in an absolute way in conformity with the CS thesis of SR (discussed below), the Sagnac effect still exists.

A few year ago Selleri put forward a very interesting paradox [3, 14] concerning the Sagnac effect. The paradox concerns the theoretical prediction of an anisotropy in the speed of light in a reference frame comoving with the edge of rotating disc even in the limit of zero acceleration. The counter intuitive problem not only challenges the internal consistency of SR but also undermines the basic tenet of the CS thesis of relativity. In one of the chapters of the main text (Chap. 5 we have taken up the issue, where the CS thesis is used, as a tool, in a novel way to recast the paradox in the classical world facilitating the resolution of the issue. A value judgement is given in favour of absolute synchronization in rotating frame upholding Selleri's observation.

To prepare the stage for the discussion on Selleri paradox, another interesting paradox of SR, namely the Tippe-top paradox has been addressed (Chap. 4). A tippe top is a children's toy which, when spun on a flat base tips over after a few rotations and eventually stands spinning on its stem. The ability of the top to demonstrate this feat depends on its geometry *i.e.* all tops are not tippe tops. To a sufficiently fast moving observer the geometry of the top may get altered due to Lorentz contraction to such an extent that the top may not tip over! This is certainly paradoxical since a mere change of perspective cannot alter the fact that the top tips over on the base. Based on the CS thesis and different possible transformation equations in relativity (in classical world too) a novel method will be used to resolve the paradox by posing it in the classical world. This method indeed has been used to resolve Selleri paradox discussed in the earlier paragraph.

So far we have considered the Sagnac effect or issues involving rotation in flat spacetime. However in view of the fact that all the real experiments are performed in the curved spacetime of the gravitational field of earth, it will be interesting to study the effect of curvature (gravity) of the spacetime. In Chap. 7 we have taken up the issue in detail. We also ask what happen if the whole earth (or a rotating star) acts as the turn table. The Sagnac experiment may be suitably modified to be performed where the observers (light source) and the mirrors are carried by rockets orbiting round a gravitating source. The rockets may be replaced by satellites when they are moving in their natural (free falling) orbits. The interesting results of this investigation have been reported in the last chapter of the main text.

In most of the discussions we extensively make use of the CS thesis and possible different relativistic transformations. Therefore before we conclude, a brief introduction to the CS thesis may be in order. This we provide in the following section.

1.2 The CS Thesis: A Brief Introduction

The role of conventionality regarding synchronization of spatially distant clocks in a given reference are much discussed in the literature. In appendix A we review the Conventionality of simulataneity (CS) thesis for readers who are unfamiliar with this idea. In this section we briefly sketch the main philosophy behind the CS thesis.

The role of convention in the definition of the simultaneity of distant events is one of the most debated issues (problems) in special relativity. The source of this problem lies in the fact that in SR distant clocks in a *given* inertial frame are synchronized by light signals, the one-way speed of light (OWS) has to be known beforehand for the purpose. To know the OWS on the other hand one requires to have presynchronized clocks and the whole process of synchronization ends up in a logical circularity which forces us to introduce a degree of arbitrariness in assigning the value for the OWS of light. However, Einstein synchronized spatially distant clocks by assuming the equality of the velocity of light in two opposite directions. This is known in the literature as the *standard synchronization.* Einstein's procedure to synchronize clocks at different space points is but one of several possible alternative conventions (referred to in the literature as *non-standard synchronization*) and many of the results he obtained depended on his special choice of synchrony. As an example the question of the discordant judgments of simultaneity by two inertial observers moving with respect to each other is also matter of such a simultaneity convention.

Though Einstein identified the problem, the role of convention in the synchronization of clocks has been advocated especially by Reichenbach [15] in 1928 and later by Grünbaum [16]. They claimed that the relation of simultaneity *within* an inertial frame of reference contains an ineradicable element of convention and the conventionality lies in the assumption regarding the OWS of light. To clarify this point further recall that Einstein originally proposed that the criterion for the synchrony of distant clocks be that the time of arrival and reflection of a light ray as determined at one clock be precisely halfway between the time of its departure and its return upon reflection as determined at the other clock. This criterion clearly presupposes that light has the same speed in all directions. Indeed, since the specification of a value for the OWS of light enables directly a simple light-signal procedure for the synchronization of distant clocks, any assumption of the OWS values is equivalent to the assumption of a criterion for synchrony. It follows that the specifications of either distant simultaneity criterion or OWS of light will alike be referred to as synchrony conventions.

Einstein himself referred to the distant simultaneity criterion he proposed as a free stipulation of the empirical meaning of distant simultaneity [17], and the issue is whether other criteria leading to different distant simultaneity judgments and consequently different OWS might not have been chosen without compromising the empirical success of the theory. The conventionalist thesis holds that a range of choices are possible, all fully equivalent with respect to experimental outcome. According to the conventionalist thesis, any synchrony convention will be admissible so long as it is consistent with the round-trip principle, the principle which holds that the average speed of a light ray over any closed path has a constant value. In fact, one can restate the second relativity postulate by replacing *the velocity of light* by *the TWS of light*. A convention within the SR must be consistent with the round-trip principle since this principle is a consequence of the theory prior to the adoption of any criterion for distant simultaneity and may in principle be tested with a single clock. It is precisely this thesis that is known as the CS thesis. According the CS thesis the conventional ingredient of SR which logically cannot have any empirical content, gives rise to results that are often erroneously construed as the new philosophical imports of special relativity theory.

1.3 General Transformation for SR

There has now been a substantial amount of clarification of the CS thesis due to a number of authors (referred at suitable places in this dissertation). Possibility of using synchronization conventions other than that adopted by Einstein has also been much discussed. In appendix B we present a derivation of a transformation equations which is general enough to incorporate all the possible conventions of the OWS of light and thus, engulf all the possible synchronization procedures. We largely take a pedagogical approach with sufficient details in deriving these equations starting from a set of general transformation equations. Throughout the derivation matrix algebra is used since with matrix algebra it is easier to find the inverse transformation, composite transformation and velocity addition formulae. The derivation presented here closely follows the derivation of Sjödin [18].The equivalence (relation) of this general transformation with the transformation derived by Selleri [8] has also been discussed. At the end we discuss a set of transformation equations where acoustic signal is used synchronization agent and the TE is found out in a material subtratum. The transformation is called the Dolphin transformation (DT) [19]. In this very important transformation, derived by Ghosal, Mukhopadhyay and Chakraborty [19], two roles of speed of light – one as a synchronizing agent and the other as a physical constant – which is mingled up beyond recognition in standard SR are explicitly split up. That some important transformation equations in the relativistic as well as in the classical world can be obtained from the DT under suitable synchronization conditions is also discussed.

1.4 Topic-wise Summary

1.4.1 Relativistic Sagnac Effect and Ehrenfest Paradox

In Chap. 3 a long standing problem of rotating disc, known as Ehrenfest paradox [1] is discussed and a resolution showing its intimate relation with the dilemma as to the correct relativistic formula for Sagnac phase-shift formula is offered. This chapter will address the issue in the light of a novel, kinematically equivalent linear Sagnac-type thought experiment, which provides a vantage point from which the effect of rotation in the usual Sagnac effect can be analyzed. These thought experiment in its two versions will be discussed in Secs. 3.5 and 3.6. The relativistic formula for the Sagnac phase-shift seems to depend on the way the Ehrenfest paradox is resolved. Kinematic resolution of the Ehrenfest paradox proposed by some authors predicts the usually quoted formula for the Sagnac delay but the resolution itself is shown to be based upon some implicit assumptions regarding the behaviour of solid bodies under acceleration. In order to have a greater insight into the problem, a second version of the thought experiment involving linear motion of a "special type" of a nonrigid frame of reference (Sec. 3.6 is discussed. It is shown, in Sec. 3.7 and in Sec. 3.8, by analogy that the usually quoted special relativistic formula for the Sagnac delay follows, provided the material of the disc matches the "special type."

1.4.2 Conventionality of Simultaneity and Tippe Top Paradox

In this chapter we critically examine a recently posed paradox (tippe top paradox [20] in relativity) and its suggested resolution. A tippe top when spun on a table, tips over after a few rotations and eventually stands spinning on its stem. The ability of the top to demonstrate this charming feat depends on its geometry (all tops are not tippe tops). To a rocket-bound observer the top geometry should change because of the Lorentz contraction. This gives rise to the possibility that for a sufficiently fast observer the geometry of the top may get altered to such an extent that the top may not tip over! This is certainly paradoxical since a mere change of the observer cannot alter the fact that the top tips over on the table. In an effort to resolve the issue the authors of the paradox compare the equations of motion of the particles of the top from the perspective of the inertial frames of the rocket and the table and observe among other things that (1) the relativity of simultaneity plays an essential role in resolving the paradox and (2) the puzzle in some way is connected with one of the corollaries of special relativity that the notion of rigidity is inconsistent with the theory. We show here that the question of the incompatibility of the notion of rigidity with special relativity has nothing to do with the current paradox and the role of the lack of synchronization of clocks in the context of the paradox is grossly over-emphasized. The conventionality of simultaneity of special relativity and the notion of the standard (Einstein) synchrony in the Galilean world have been used to throw light on some subtle issues concerning the paradox

With a brief introduction to the content of the chapter, we shall briefly reproduce in Sec. 4.2 the basic arguments of Ref. [20] to clarify the paradox, in order to set the stage. The study will follow the CS approach to critically examine the work of Basu *et al.* Though a fairly comprehensive account of CS thesis at possible transformations alternative to LT is given in appendix A and appendix B, a brief introduction to this is given in Sec. 4.3 for ready reference and also to make the content of the chapter self-contained. The main arguments will be presented from Sec. 4.4 through Sec. 4.6. We summarize all this in Sec. 4.7. The table of comparison given at the end of the chapter gives the summary of our findings at glance.

1.4.3 On the Anisotropy of the Speed of Light on a Rotating Platform and Selleri Paradox

In this chapter(Chap. 5) we discuss a recently posed paradox in relativity concerning the speed of light as measured by an observer on board a rotating turn-table. The counter-intuitive problem put forward by F. Selleri [14] concerns the theoretical prediction of an anisotropy in the speed of light in a reference frame comoving with the edge of a rotating disc even in the limit of zero acceleration. The paradox not only challenges the internal consistency of the special relativity theory but also undermines the basic tenet of the conventionality of simultaneity thesis of relativity. The present paper resolves the issue in a novel way by recasting the original paradox in the Galilean world and thereby revealing, in a subtle way, the weak points of the reasonings leading to the fallacy. As a background the standard and the non-standard synchronies in the relativistic as well as in the Galilean world are discussed. In passing, this novel approach also clarifies (contrary to often made assertions in the literature) that the so-called "desynchronization" of clocks cannot be regarded as the root cause of the Sagnac effect. Finally in spite of the flaw in the reasonings leading to the paradox Selleri's observation regarding the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

The chapter starts with a brief introduction to the Selleri paradox in Sec. 5.1. The paradox is discussed in Sec. 5.2. Selleri paradox is critically analysed from the view point of CS thesis and absolute synchrony in Sec. 5.3. Discussion of the paradox in the Galilean world can be found in Sec. 5.4. The phenomenon of desynchronization of clocks in rotating frame and its relation to Selleri paradox and Sagnac effect are critically examined in Sec. 5.5. A value judgement about different synchronization schemes applied to rotating frame will be given in Sec. 5.6 before the conclusions are summarized in Sec. 5.7.

1.4.4 Sagnac Formula from the Perspective of Rotating Platform and Absolute Synchrony

A recent claim by Selleri that the analysis of the Sagnac effect is usually done from the perspective of the laboratory frame and no derivation is available from the perspective of rotating frame is examined. Selleri further claims that a straight forward application of Lorentz transformation (LT) gives null result for the effect; however if instead of LT, Tangherlini tranformation (TT) representing absolute synchrony is used, then one can correctly predict the Sagnac effect from the perspective of a rotating observer. A calculation provided by E. J. Post in one of the most influential papers on the Sagnac effect might be construed as one done from the stand point of a rotating observer. The present paper adresses the issues raised by Selleri vis-a-vis Post's treatment. In Post's calculation a standard flat spacetime metric is transformed, by a proposed transformation representing the coordinates of a rotating frame. A critical examination reveals that Post's treatment has some inherent weaknesses and his intuitive transformation which gives the Sagnac result indeed leads to TT. This upholds Selleri's claim that in a rotating frame the absolute synchrony is the preferable one.

After a brief introduction on the calculation of Sagnac effect from the perspective of rotating frame we reviewed the calculation offered by Post. In the next section we critically examined Post's transformation to show that it predicts TT. In the last section some weaknesses of another 'derivation', given by Post in the appendix of his paper, of the proposed transformation are pointed out and it has been shown that this derivation predicts absolute synchrony (TT) too.

1.4.5 Sagnac effect in Curved Spacetime

The general relativistic correction to the Sagnac effect is obtained for a disc type experiment where the turntable is placed in an axisymmetric or spherically symmetric gravitational field. It will be shown that special cases of the present gedanken set up include a scenario where the beam splitter is placed on the surface of the earth which then acts as a turn table. Special cases where the observers move in geodetic or non-geodetic orbits have been discussed. The discussion of Sagnac experiment in the pre-horizon regime of the deep field of a neutron star or a black hole is done to gain an insight into the nature of such field in that region. The appearance of arbitrarily large Sagnac phase shift under certain conditions is noticed and these things have been discussed analytically and graphically in details.

After a brief introduction the Sagnac phaseshift formula in curved spacetime has been obtained in Schwarzschild field in Sec. 7.2. Sagnac effect for equatorial orbit is discussed in Sec. 7.3. Sagnac effect in Kerr-Newman field is discussed in Sec. 7.4. Experiment with satellites has been discussed in this case too. Some interesting results arising in the pre-horizon regime are discussed analytically and graphically.

Reference

[1] Paul Ehrenfest, *Phys. Z.*, **10**, (1909) p. 918.

[2] Albert Einstein, *The Meaning of Relativity* (Oxford University Press, New Delhi, 1951).

[3] F. Selleri, "Sagnac effect: End of a mystery", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frame", Fundamental Theories of Physics (Kluwer Academic, New York, 2004). Series Editor: A. van der Merwe.

[4] G. Sagnac, *Comp. Rend. Acad. Sci.*, **157**, (1913) pp. 708–710.

[5] P. Langevin, *Compt. Rend.*, **173**, (1921) p. 831.

[6] P. Langevin, *Compt. Rend.*, **205**, (1937) p. 51.

[7] H. Hasselbach and M. Nicklaus, *Phys. Rev. A*, **48**, (1993) pp. 143–151.

[8] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[9] E. J. Post, "Sagnac effect", *Rev. Mod. Phys.*, **39**, (1967) p. 475.

[10] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[11] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Rotating disc problem and sagnac phase-shift formula", in M. C. Duffy (Ed.), "Physical Interpretations of Relativity Theory, VIII", (P. D. Publication, Liverpool, 2000). Held at Imperial College London.

[12] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.*, **17**(5), (2004) pp. 457–477.

[13] S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarker and Saroj Nepal, "On synchronization and desynchronization in rotating frames and a recent paradox", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (IX)", (2004). Held at Imperial College, London (To be Published).

[14] F. Selleri, in F. Selleri (Ed.), "Open Question in Relativistic Physics", pp. 69–80 (Apeiron, Montreal, 1998).

[15] H. Reichenbach, *The Philosophy of Space and Time* (Dover, NewYork, 1957), 2nd edition.

[16] Adolf Grünbaum, *Philosophical Problems of Space and Time* (Alfred Knopf, New York, 1973), 2 edition.

[17] A. Einstein, *Relativity: the Special and the General Theory* (Methuen, London, 1960).

[18] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979) pp. 229–245.

[19] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994) pp. 21–28.

[20] Aniket Basu, R. S. Saraswat, Kedar B. Khare, G. P. Sastry and Sougato Bose, *Eur. J. Phys.*, **23**, (2002) pp. 295–305.

1.5 List of Publications

1. **Relativistic Sagnac Effect and Ehrenfest Paradox** – S. K. Ghosal, *Biplab Raychaudhuri*, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys.* **33**(6), 981-1001 (2003).

2. **Rotating Disc Problem and Sagnac Phase-Shift formula** – S.K. Ghosal, *Biplab Raychaudhuri*, Anjan Kumar Chowdhuri and Minakshi Sarker **in** *Physical Interpretation of Relativity Theory VII, Late Papers*, Ed. M. C. Duffy, (PD Publications, Liverpool, 2000).

3. **Conventionality of Simultaneity and Tippe Top Paradox** – S. K. Ghosal, *Biplab Raychaudhuri*, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.* **16**(6), 549-563 (2003).

4. **On the Anisotropy of the Speed of Light on a Rotating Platform** – S. K. Ghosal, *Biplab Raychaudhuri* , Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.*, **17**(5), 457–477, (2004).

5. **Synchronization and Desynchronization in rotating frames** – S.K. Ghosal, *Biplab Raychaudhuri*, Anjan Kumar Chowdhuri and Minakshi Sarker **in** *Physical Interpretation of Relativity Theory IX*, held at Imperial College, London: September 2004, Ed. M. C. Duffy (To be published).

6. **Sagnac Formula from the Rotating Frame Perspective and Absolute Synchrony** – *Biplab Raychaudhuri* and S. K. Ghosal, (*Ready for communication*).

7. **Sagnac Effect in Curved Spacetime** – *Biplab Raychaudhuri* , Anjan Kumar Choudhuri and S. K. Ghosal. (*Ready for communication*).

8. *Absolute synchrony in Microwave Background and Sagnac Effect* – S. K. Ghosal, Minakshi Sarker, *Biplab Raychaudhuri* and Anjan Kumar Chowdhuri *Proc. of International Conference on Gravitation and Cosmology (ICGC-2000)* held at IIT Kharagapur, 4-7 January, 2000 (Abstracted).

9. *Synchrony Gauge in Classical and Relativistic Sagnac Effect and related issues* – S. K. Ghosal, *Biplab Raychaudhuri*, Minakshi Sarker and Saroj Nepal, *Proc. of International Conference of Gravitation and Cosmology (ICGC-2004)* held at Kochi, 5-10 January, 2004 (Abstracted).

Chapter 2

Sagnac Effect

2.1 Introduction

As the title suggests, the present thesis concerns different aspects of the Sagnac effect. Thus, it is worthwhile to give an account of the effect from its historical and conceptual perspectives. With this aim we present a brief review of earlier works (both theoretical and experimental) in the subject in this chapter. The discourse presented here is by no means exhaustive, although we have tried our best to make it self-contained so that this article can stand by itself as an introduction to the more profound studies on the Sagnac effect for the readers who are unfamiliar with this very important phenomenon. In particular we have left behind discussions over matter wave Sagnac effect (Sagnac experiment performed with matter waves) since our work involves only optical Sagnac effect.

2.2 Sagnac Effect

There has been a great deal of interest in recent years in the Sagnac effect. This is not only for its practical importance in navigational application for sensing rotation but also for its rich theoretical ramifications. In 1913 Sagnac [1] in his life long quest for ether devised an experiment where he compared round-trip times of two light signals traveling in opposite directions along a closed path on a rotating disc. It was observed that the time required by a light signal to make a close circuit on the plane of the disc differed depending on the sense (direction) of the signal's round-trip with respect to the spin of the disc. The essentials of the experiment consist of a monochromatic source of light, interferometer and a set of mirrors mounted on a turn-table. Light from the source is split into two beams by a beam splitter (half-silvered mirror) allowing them to propagate in opposite directions. These beams then are constrained (by suitably placed mirrors) to make round trips and are then reunited at the beam splitter to produce a fringe pattern. The difference in round-trip times for these counter propagating beams leads to a phase difference with a consequent shift in the fringe pattern when the turn table is put into rotation. This phenomenon is commonly known as the Sagnac effect (see Fig. 2.1).

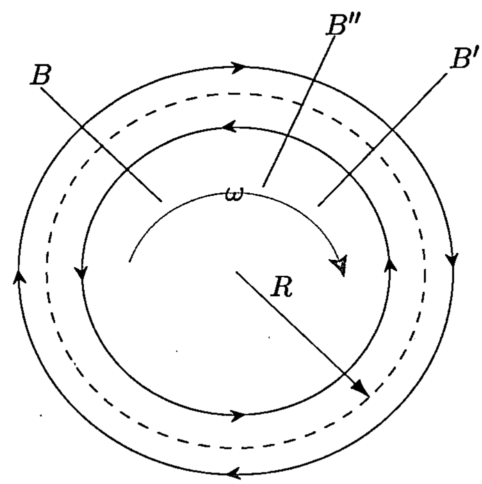


Figure 2.1: A diagram for Sagnac experiment

The Sagnac experiment performed using light beam is called the optical Sagnac effect in contrast with Sagnac experiment performed with other matter waves. The effect is universal and is also manifested for matter waves. Sagnac type experiments were done with electrons [2], neutrons [3], atoms [4], Superperconducting Cooper pairs [5], Radiowaves [6] and X-rays [7].

Accuracy of the result in Sagnac-type experiments is much improved by using laser beams (instead of usual light beams) and fibre optic interferometers using fibre optic cables to guide light beam along the periphery of the disc (instead of mirrors placed on the periphery). Traveling-wave laser in a ring cavity as the sensor is used to detect 'absolute rotation' using the Sagnac effect. This device, known as *ring laser gyroscopes* replaces the spinning-mass gyroscopes with a purely optical sensor in the field of navigation and guidance [8]. Importance of this effect also lies in connection with the question of time keeping clocks of clock-stations around the earth [9]. Here the earth plays the role of rotating platform from which clocks between clock stations are synchronized by sending lights via satellites [10, 11]. Among many applications of Sagnac effect we shall give a very brief account of global positioning system (GPS) in Sec. 2.7.

2.3 Historical Perspective

All the experiments involving interference with the help of rotation is in general termed as 'Sagnac-type experiments' [12]. But Sagnac was not the first one to conceive such an experiment, as recent historical surveys [13] reveal.

Oliver Lodge [14, 15] was possibly the first, as far as present knowledge goes, to suggest a Sagnac-type experiment. His aim was to provide a means to detect the ether drag by heavy bodies by an immobile Fizeau interferometer. An observed band shift would have suggested the ether drag by the Earth. The null result made him to conclude that ether is not dragged by the Earth's rotation and this rotation could not be measured with respect to the immobile ether. Lodge [15] wrote that "if the system as a whole, *i.e.* a lamp, an optical system [interferometer], a telescope [instrumental set up to observe fringe shift] and an observer had been set upon a rotating table and put into rotation, then a relative shift of interfering bands could have been observed at different directions of the rotations."

Albert Michelson [16] suggested construction of large sized circular interferometers with dimension of 1×1km to measure the angular rate of the

Earth's rotation about its axis. The experiment was not done immediately but performed much later (in 1923). In 1925 he, in collaboration with H. Gale and F. Pearson constructed another one of dimension 630 $\times$ 340m and placed in a system of 305mm steel tubes. The air pressure maintained inside the tubes was very low (12mm Hg) because in open air, the fringes were blurred due to airflows. With this they were the first to measure the angular rate of the Earth's rotation. Michelson [16] suggested another set of experiment with dimension 10 $\times$ 10km to measure the angular rate of the Earth's rotation about the Sun. But the project was never realized.

Another important contribution to the Sagnac-type experiments was made by Francis Harres in 1911 although his objective, quite different from Sagnac, was to study dispersion properties of glasses by substituting the linear motion in Fresnel-Fizeau experiment by a circular motion. The dispersion data obtained by Harress did not agree well with data available from others methods. "By substituting a circular motion for a linear motion[1] Harress tacitly assumed the absence of exactly the effect" that Sagnac was looking for later on [12]. Harress' interpretation was that the interference band shifts were solely associated with dragging of light by the rotating glass inside the interferometer rather than with the rotation of the interferometer as a whole [13]. Harress died a few years later (in 1915) and could not do further work towards the solution of discrepancy [12]. Harzer [17] reworked on Harress' data from a different kinematic perspective accounting properly for the rotation [12]. His most important conclusion was that the fringe shift does not depend on the presence of a co-moving (with the table) refracting medium along the path of the light beams. He expressed surprise that his analysis suggests that dispersion had no influence on the final fringe shift.

[1] *i.e.* Ring interferometer instead of linear Fresnel-Fizeau experiment

Einstein [18] showed that this is not so surprising as the dispersion term in Fresnel-Fizeau drag co-efficient really stems from a Doppler shift due to a motion between source and medium.

Sagnac conceived his experiment well before they were performed. As far as recent knowledge goes, Sagnac discussed the possibility of measurement of angular velocity with a circular interferometer with G. Lipman in 1909 [13]. He published the results of his experiment in 1913 [1, 19, 20].[2] We shall discuss further on Sagnac experiment in the next section.

One of the few text books on relativity which discusses the Sagnac effect is the book by Ugarov [22]. There he commented that he is describing "Garess's experiment (1912), subsequently repeated by Sagnac." [22, page 344]. In the same page the diagram refers to it as Sagnac-Garess experiment. In the index the experiment is also referred to as Sagnac-Garess experiment. The name "Garess" used thrice in this manner carried no further reference in the book, neither this name was found in any of the other literatures quoted earlier. It seems Ugarov has mistaken this name for 'Harress'.

2.4 Sagnac's Experiment

The interferometer in Sagnac's experiment essentially look like Fig. 2.2 Light comes out of a light source, gets reflected by a mirror and falls upon a half silvered mirror. This is the beam splitter. One of the beams goes through the half silvered mirror, gets reflected by four mirrors before finally entering the telescope passing through the beam splitter. The other part of the beam gets reflected by

[2] Unfortunately the present author does not have access to the original works of Sagnac, nor does he have enough knowledge of the French language to know whether Ref. [20] and Ref. [21] are the same or not. These citations are gathered from secondary sources.

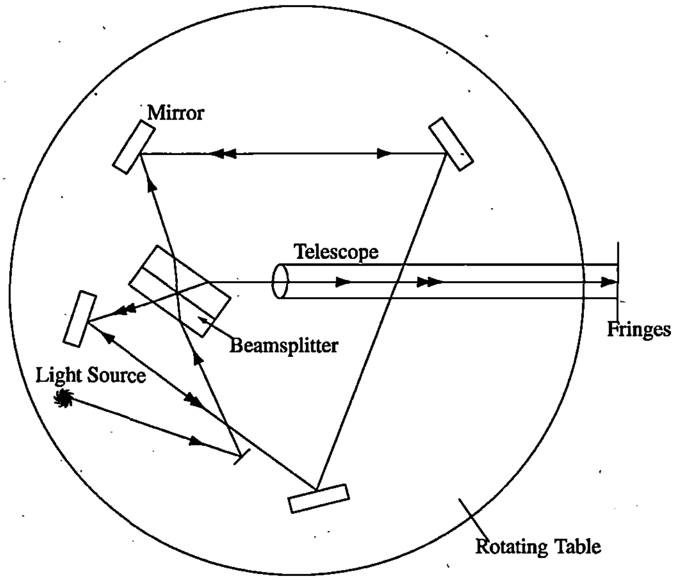


Figure 2.2: A schematic diagram of Sagnac Experiment

the half silvered mirror, goes to one of the four mirrors (the last one for the previous case), the gets reflected by the other three mirrors tracing the same path as the former beam did, but in the opposite direction before finally reaching the telescope. Through the telescope one can observe the interference fringes. If the interferometer is now given a constant angular motion, a fringe shift can be observed. The fringe shift is observed experimentally to be [12]

$$\Delta Z = \frac{4\omega \cdot \mathbf{A}}{\lambda c}, \tag{2.4.1}$$

where

$\vec{\omega} =$ angular velocity,

$\vec{A} =$ area vector of the disc,

$\lambda =$ wave length of light used,

$c =$ velocity of light.

If the disc is rotated once clockwise and then anticlockwise, then the fringe shift between these will be double of that in each case. After taking the photographs of interference bands upon rotation, the platform is brought to rest, the photo plate is displaced. Then the platform was set in rotation in the opposite direction and another photograph is taken. Thus on one hand a random initial shift of the bands is excluded, on the other hand the fringe shift is doubled. Sagnac used indigo mercury light, plane polarized by Nickol prism to make bands sharper. The interferometer geometry was 866cm^2. Rotation rate was 120rpm or 2Hz and fringe shift observed was 0.07 ± 0.01. Sagnac's instrument was precise enough to measure this fringe shift clearly [23].

Though the increase of rotational speed increases fringe shift imposing less percentage of error, the distortion of the interferometer geometry (change of mirror positions, in particular) due to strong centrifugal force would not give *pure fringe shift.*

Hungarian physicist Bel Pogany [24–26] performed Sagnac experiment with a greater precision in 1926. The parameters were [12]

$$
\begin{aligned}
A &= 1178 \times \text{cm}^2, \\
\omega &= 157.43\text{rad/s}(\sim 50\text{Hz or } 3000\text{rpm}), \\
\lambda &= 5460 \times 10^{-8}.
\end{aligned}
$$

He reproduced the theoretically expected (double) fringe shift $\Delta Z = 0.906$ with an error of 2 per cent. His interferometer was in the form of a square[3]. It consisted of four mirrors and a prism of total internal reflection. The prism was used as the

[3]There is a discrepancy here between the specifications given by Post [12] and Malykin [13]. According to Malykin the sides of the interferometer was 35cm thus making $A = 1225\text{cm}^2$, rotation rate is 1000-2000rpm. Unfortunately the present author does not have access to Pogany's original works to confirm the data.

beam splitter. Light from the source came in and went out as that did in Harress' arrangement.

In 1928 Pogany performed a modified experiment with two 32cm long massive glass rods in the path of the light beam to increase the optical length of the interferometer. With this he demonstrated with high precision that the Sagnac result is independent of the refractive index of the medium in the light path (see also Ref. [6]). Due to the development of wave guide circular interferometer this issue was much discussed in late 70's [27, 28].

Dufour and Prunier [29, 30] showed that the Sagnac effect is practically independent of where the light source is placed – on the platform (Sagnac) or outside the platform (Harress, Pogany). They also showed that if the interferometer rotates but not the medium, then the observed fringe shift increases. Effect of medium vanishes when the relative velocity of medium and the interferometer vanishes [12].

Scheme of a guide circular interferometer was patented by Aaron Wallace on 3 September, 1963 (submitted for consideration on 7 July, 1958). Interestingly, this was first constructed on 1976 [27]. This document first presented the idea of construction of Sagnac experiment using X-rays [7], electrons [2] and γ-rays (the last one is not realized uptill now). But construction of circular lasers [31] made scientists to lose interest in circular interferometers [13].

The whole world of optical interferometers changed after the invention of laser in 1958. In 1961 Javan, Benett and Herriot produced first gas laser. With this the foundation for the creation of the laser gyroscope was laid. Main advantage of laser is that it is possible to maintain a constant phase difference very precisely between two beams. This can be used to measure the optical beat frequencies at the interference as an alternative to fringe shift experiments. Rosenthal [32] in

1962 first suggested the possibility of making a self oscillations version of Sagnac ring by using lasers. The device was made at Sperry Gyroscope Company by Macek and Davis (1963) [31]. Here a helium-neon discharge as the amplifying medium was used to produce the laser. Its wave length was 1150nm and a 1m square optical cavity was used. This kind of experimental set up is, in general, called *ring lasers* [33].

In ring lasers clockwise and counter clockwise modes of laser beams occur in the same optical cavity. Thus an "unusually stable beat between two optical frequencies" [12] can be generated almost ideally. Due to mechanical instabilities of the frequency the individual modes may fluctuate many MHz. But both of the modes have almost the identical fluctuation, so the frequency difference can be stable to within a few Hz.

At high rotation rates, the linear relationship between the beat frequency and rotation rate is maintained. But the problem begins when the rotation rate is reduced – the response becomes non-linear. The beat disappeared altogether at sufficiently low rates. This is called the 'lock-in' region or 'dead-band' region and surpasses the sensitivity required in typical navigation applications by several orders of magnitude. One of the ways to circumvent this problem is to apply "rotation, real or apparent" to " 'bias' the gyroscope away from the lock-in region" [33]. Most favours to date, is to apply an actual rotation, in the form of a mechanical oscillation or 'dither'.

Ring laser is a subject by its own. We restrain to make further discussion of this because it lies outside the basic purview of this dissertation. The readers are referred to an excellent review article (running 103 pages) under the title '*Ring Lasers*' by J. R. Wilkinson [33].

Sagnac effect was performed using rotating superconducting interferometers where the de Broglie waves of a Cooper pair of superconducting electrons in a metal replaces the optical signal in original Sagnac experiment. Here an annulus of superconducting material interrupted at one point by a Josephson junction acts an interferometer. This is called 'superconducting quantum interference device' or SQUID [5, 34–37]. The SQUID allowed a new demonstration of the Bohm-Aharonov effect.

Sagnac type experiment was performed using neutron interferometer. In this case, thermal neutrons with $\lambda \approx 1.3 \times 10^{-10}m$ and $v/c \cong 1.1 \times 10^{-5}$ was used. Three 'ears' sculptured from a single crystal of silicon act as successive Bragg reflectors [35, 38, 39]. The beams interfere at the third 'ear'. Due to Bragg reflection the beams are aligned to an accuracy of 10^{-6}rad, and imported a coherent length of the order of $10^9\lambda$. According to Greenberger [38] the neutron interferometer forms explicit and striking illustrations; twin paradox, sign change of spinors under a 2π rotation, uncertainty, Schrödinger cat paradox etc. [35].

2.5 Formal Derivations of Sagnac Formula

2.5.1 Classical Derivation of Sagnac Effect

The optical Sagnac effect can be suitably analysed by assuming the light circuits to be circular. This can be achieved by constraining the light beams to propagate tangentially to the internal surface of a cylindrical mirror. Within the framework of Newtonian physics it is quite straightforward to calculate the Sagnac phase-shift in terms of the difference of arrival times of the corotating and counterrotating light signals when they are reunited at the beam splitter. We give here a simple derivation [12, 40]. Let us assume that the circular interferometer experiment ('interferograph' in Sagnac's language [13]) is mounted on a turntable (vide

Fig. 2.1 of radius R. The turntable is rotating with a uniform angular velocity ω. It is possible to compute Sagnac shift for an arbitrary angular velocity, "although the analysis becomes somewhat complicated" [41].

In Sagnac's experiment (Fig. 2.2) four mirrors were used and thus the light beams did not follow a path grazing to the periphery of the disc. But as stated earlier, without any problem one can assume that light is traveling a circumferential path. This experiment can indeed be done (and has been already done) with optical fiber as the light guide, where light macroscopically will follow an average circular path.

Let us suppose that a light pulse leaves the beam splitter at position B' and meets the beam splitter again at time t_1 (Fig. 2.1). During this period the beam splitter has moved to a new position B'; hence light has to travel an extra distance (BB')

$$x = vt_1 \tag{2.5.1}$$

with respect to the inertial frame of the laboratory which is at rest with the rotation. One therefore has

$$L + x = ct_1. \tag{2.5.2}$$

In order to write (2.5.2) it is implicitly assumed that the speed of light is c in the laboratory and in accordance with special relativity it is independent of the motion of the source. There is no harm in assuming this as classically this is equivalent to the hypothesis of ether, through which light propagates and which is believed to be stationary with respect to the inertial frame of the laboratory. Eliminating x

from Eqs. (2.5.1) and (2.5.2) one obtains

$$t_1 = \frac{L}{c - v}. \tag{2.5.3}$$

A similar set of arguments for the counter rotating beam gives its round trip time t_2. In this the beam splitter moves to its new position B'', so that with respect to the laboratory, the counter rotating light pulse travels a path shorter than L by the amount $BB'' = vt_2$. For light propagation one may therefore write

$$l - vt_2 = ct_2,$$

$$t_2 = \frac{L}{c + v}. \tag{2.5.4}$$

Similar set of results (Eqs. (2.5.3) and (2.5.4)) may also be obtained from a classical idea that light travels with velocities $c-v$ and $c+v$ for the co and counter rotating beams. This result gives correct prediction upto the limit $v^2/c^2 \gg 1$. The difference in these times is therefore given by

$$\Delta t = t_1 - t_2 = \frac{2lv}{c^2 - v^2}.$$

Substituting $L = 2\pi R$ and $v = \omega R$ we obtain

$$\Delta t = \frac{4\pi r^2 \omega}{c^2}\gamma^2 \tag{2.5.5}$$

where $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$.

This is known as classical Sagnac Delay (SD). This is the difference of time of arrival (some times called 'the time of flight') of the co and counter rotating beams at the beam splitter after making a round trip. The phase difference between the two light beams at the beam splitter is thus given by

$$\Delta\phi = \frac{2\pi c}{\lambda}\Delta t, \tag{2.5.6}$$

where λ is the wavelength of used light beam[4]. Explicitly written, this gives [23]

$$\begin{aligned}\Delta\phi &= \frac{2\pi c}{\lambda}\frac{4\pi R^2\omega}{c^2}\gamma^2 \\ &= \frac{8\pi R^2\omega}{\lambda c}\gamma^2.\end{aligned}$$

Thus the fractional fringe shift [33] due to rotation is

$$\Delta Z = \frac{\Delta\phi}{2\pi} = \frac{4\pi R^2\omega}{\lambda c}\gamma^2. \tag{2.5.7}$$

For all practical purposes, $\gamma \approx 1$ and thus the fractional fringe shift reduces to

$$\Delta Z = \frac{4\pi R^2\omega}{\lambda c}. \tag{2.5.8}$$

This is the result in Scalar form given by Post [12].

The result was predicted by Sagnac in some of his earlier papers [23, 42] and some of his papers in 1901. He gave a full account in Ref. [21].

2.5.2 Relativistic Sagnac effect

The theoretical derivation of Sagnac result given earlier (Sec. 2.5.1) is derived from classical mechanics. Also, the whole treatment was done from the perspective of laboratory frame where the disc axis is stationary. Let us now use SR to discuss the time delay. The SD given by Eq. (2.5.5) is obtained from the stationary laboratory frame. As Sagnac effect is essentially an effect on the rotation platform (A linear version of Sagnac effect [40, 43, 44] has been proposed too). The analysis of SD must be done from be view point of an observer on board the platform. The SD obtained in the earlier section is dilated. In $\Delta\tau_{\text{rot}}$ corresponds to the time difference as observed from the rotating frame, this is

[4]The phase difference is given by $2\pi c/\lambda$ (a constant for a given wave length) times the SD. Thus, it is customary to use SD in discussions of Sagnac effect.

related to that on the stationary frame (2.5.5) by the time dilation factor

$$\Delta\tau_{\text{rot}} = \gamma^{-1}\Delta t. \tag{2.5.9}$$

For Eqs. (2.5.5) and (2.5.9) we obtain the expression

$$\Delta\tau_{\text{rot}} = \frac{4\pi\omega R^2}{c^2}\gamma. \tag{2.5.10}$$

This expression is obtained by several authors in a variety of ways. Post deduced this from metrical consideration by using first arbitrary transformation and then specifying it. Dieks and Nienhuis [45] calculated the result by direct use of LT.

An interesting point of the above derivation is that only time dilation effect of SR is used. The authors who derived Eq. (2.5.10) never showed explicitly any cause of not considering the length contraction effect of SR, with the exception of Selleri [46]. The periphery of the disc ($2\pi R$) as seen from the laboratory is the contracted length. Thus if L is the periphery of the disc for the rotating observer (proper length and $L_0 = 2\pi R$ is the length measured by the laboratory observer, then they are related by

$$L_0 = \gamma^{-1}L. \tag{2.5.11}$$

Using this relation with Eq. (2.5.10) we obtain

$$\Delta\tau_{\text{rel}} = \frac{2Lv}{c^2}. \tag{2.5.12}$$

This is the same expression that was experimentally obtained. Thus we obtained two expressions, one is Eq. (2.5.10) and the other one is Eq. (2.5.12) and upto the first order of v/c they agree. The implication of this will be addressed in a separate chapter 3. We refrain from discussing this here any further.

2.5.3 Metrical Treatment of Sagnac Effect

In the previous section a simplistic derivation of relativistic Sagnac effect has been given. In this section we shall give a treatment based on spacetime metric. This formulation will be generalized when we discuss Sagnac effect in curved spacetime in Chap. 7.

The metric of flat spacetime in spherical polar co-ordinate is given by

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 - r^2 \sin^2 d\phi^2. \tag{2.5.13}$$

For Sagnac experiment

$$r = \text{constant}, \qquad \theta = \text{constant} = \pi/2 \text{ (say)}.$$

With these values, the metric becomes,

$$ds^2 = c^2 dt^2 - r^2 d\phi^2. \tag{2.5.14}$$

For the motion of light $\phi = \Omega t$ where Ω is the angular velocity of light. Thus we obtain

$$ds^2 = c^2 \left(1 - \frac{r^2 \Omega^2}{c^2} dt^2\right). \tag{2.5.15}$$

As light follows null geodesics, we may solve Eq. (2.5.15)for Ω and obtain

$$\Omega_{\pm} = \pm \frac{c}{r}. \tag{2.5.16}$$

The $\pm$ sign here in the right hand side indicates two different directions of light. In Sagnac effect we take the difference of arrival time of two counter rotating beams at the beam splitter. Thus, the $\pm$ sign is unimportant here and ignoring it we obtain

$$\Omega_{\pm} = \frac{c}{r}. \tag{2.5.17}$$

The angular displacement of light in time t is thus given by

$$\phi_{\pm} = \Omega_{\pm} t. \tag{2.5.18}$$

Now, if the observer is on the platform rotating with an angular velocity ω_0, the rotation angle can be written as

$$\phi_0 = \omega_0 t. \tag{2.5.19}$$

Eliminating t from Eqs. (2.5.18) and (2.5.19) we obtain

$$\phi_{\pm} = \Omega_{\pm}\left(\frac{\phi_0}{\omega_0}\right). \tag{2.5.20}$$

Now, the co-rotating light meets the source after traversing an angle of $2\pi + \phi_{0+}$ angle. Thus

$$\Omega_{+}\left(\frac{\phi_{0+}}{\omega_0}\right) = 2\pi + \phi_{0+}. \tag{2.5.21}$$

or,

$$\phi_{0+} = \frac{2\pi\omega_0}{\Omega_{+} - \omega_0}. \tag{2.5.22}$$

The counterrotating beam meets the beamsplitter after traversing an angle $2\pi - \phi_{0-}$. Thus we can write

$$\phi_{0-} = \frac{2\pi\omega}{\Omega_{-} + \omega}. \tag{2.5.23}$$

These can be written in a combined form

$$\phi_{0\pm} = \frac{2\pi\omega}{\Omega_{\pm} \mp \omega}. \tag{2.5.24}$$

Now the proper time of the observer is given by

$$\mathrm{d}\tau = \sqrt{g_{00}}\,\frac{\mathrm{d}x^0}{c}. \tag{2.5.25}$$

or,

$$\mathrm{d}\tau = \sqrt{\left(1 - \frac{r^2\omega^2}{c^2}\right)}\ \frac{\mathrm{d}\phi}{\omega}. \tag{2.5.26}$$

Thus, the Sagnac delay in the frame of the observer is found by integrating it from ϕ_{0-} to ϕ_{0+}

$$\delta\tau = \int_{\phi_{0-}}^{\phi_{0+}} \mathrm{d}\tau = \left(1 - \frac{r^2\omega^2}{c^2}\right)^{1/2} \frac{\phi_{0+} - \phi_{0-}}{\omega}.$$

Now

$$\phi_{0+} - \phi_{0-} = \frac{4\pi\omega^2 r^2}{c^2}\left(1 - \frac{r^2\omega^2}{c^2}\right)^{-1}.$$

Thus the Sagnac delay is given by

$$\delta\tau = \frac{4\pi\omega r^2}{c^2}\left(1 - \frac{r^2\omega^2}{c^2}\right)^{-1/2} \tag{2.5.27}$$

This is the Sagnac delay in the rotating frame. One must notice here that while we used 'proper time' of the rotating observer, we have not used 'proper length' of the disc. Thus, quite expectedly we obtain the result as in Eq. (2.5.10) with $v = \omega r$. If one now recognizes that the periphery is a contracted one when observed from the laboratory, using Eq. (2.5.11) one obtains the same Eq. (2.5.12).

Thus one may write relativistic SD as follows

$$\Delta\tau_{\mathrm{rel}} = \frac{4\pi\omega r^2}{c^2}\,\Gamma \tag{2.5.28}$$

where $\Gamma = 1$ or $\Gamma = \gamma$. Later we shall see that value of Γ depends on the material of the disc and intimately connected with Ehrenfest paradox. This will be addressed in details in Chapter 3.

2.6 Sagnac Effect in Curved Spacetime : Formal Approach

So far we have reviewed the theoretical aspect of the experiment performed by Sagnac (and others). The experiment is either done in flat spacetime or done in so weak a gravitational field that the correction due to gravitation is negligible. But if the experiment is performed in a strong gravitational field then the correction terms will be considerable. In Chap. 7 we shall address precisely that issue.

In regard to general relativistic Sagnac effect, there are two methods of derivation available. One is metrical derivation first used by Post [12]. This is extended to Kerr field by Tartaglia [47] and we shall discuss a version of this in Chap. 7. Another method of derivation based on differential geometry as mathematical tool has been given by Ashtekar and Magnon [41] and later "elaborated" by Anandan [48]. Here we give a brief review of the treatment given by Ashtekar and Magnon.

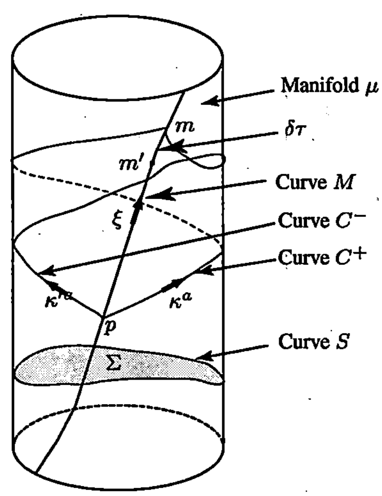


Figure 2.3: Sagnac Experiment in a general spacetime

Ashtekar and Magnaon considers a hollow toroid as the path of light. In practice one may consider the use of optical fibre. At some point there is a half silvered morror as beam splitter. They considered a spacetime $(\mathcal{M}, g_{ab})$ where $\mathcal{M}$ is a 4 manifold and g_{ab} is a metric with signature $(+2)$. In this spacetime, Sagnac tube is represented by a 2D timelike submanifold μ of $\mathcal{M}$. $(G = c = 1)$. The mirror is represented by a timelike curve M. p denotes the event when light is first emitted by the source. Two counter rotating light beams are represented by two null curves C^+ and C^-. ξ^2, κ^a, κ'^a are tangent vectors to the curves M, C^+ and C^- respectively. The rotational frequency of the light rays are thus given by

$$\nu = \kappa_a \xi^a = \kappa'_a \xi^a. \tag{2.6.1}$$

The two events when the light beams meet the mirror again after traversing the round trip are denoted by m and m'. They are represented by first intersecting points of C^+ and C^- with M. Thus, on the spacetime diagram 2.3, SD is the distance mm' measured along the world line M of the mirror.

During the experiment the whole set up should not be deformed (due to centrifugal force). This amounts to demand, in the present context, that the Sagnac tube is rigid, or 4-velocity field ξ^a to be Born rigid on μ (*i.e.* $\mathcal{L}_\xi(g_{ab} + \xi_a \xi_b) = 0$). If now the tube is assumed to be in *stationary* motion (equivalent to *uniform rotation* in Newtonian analysis which simplifies our calculation) and the 2-manifold μ admits a timelike killing vector field and if the tube moves along the trajectories of this Killing field, then the above requirement is naturally satisfied.

The metric induced on μ by the metric g_{ab} on $\mathcal{M}$ is denoted by h_{ab} (signature 0 or $-+$), and the timelike Killing vector field on (μ, h_{ab}) is denoted by t^a. The tube follows the integral curves t^a. The 4-velocity ξ^a of the tube is given by

$$\xi^a = \lambda^{-1/2} t^a \tag{2.6.2}$$

where $t_a t^a = -\lambda$.

μ being a 2 manifold with h_{ab} (sig.0) there are exactly two null directions at each point of ν. Once the event p is fixed, the null curves C^+ and C^-, events m and m' and the Sagnac shift $\Delta\tau$ are completely determined. However, for convenience, the authors introduce two null vector fields κ^a and κ'^a tangent to

C^+ and C^- such that

$$\kappa^a t_a = \kappa'^a t_a = -1 \tag{2.6.3}$$

From, Eqs. (2.6.2) and (2.6.3) one obtains the value of ν (2.6.1)

$$\nu = -\lambda^{1/2} \tag{2.6.4}$$

As these vectors are curl free, the integrals

$$I = \oint_c \kappa_a \mathrm{d}s^a \qquad \text{and} \qquad I' = \oint_{c'} \kappa'_a \mathrm{d}s^a$$

are independent of the particular choice of the closed curves c and c' on μ. The curves are defined in the following way (Fig. 2.3)

$c \Rightarrow$ closed curve pmp along C^+.

$c' \Rightarrow$ closed curve $pm'p$ along C^-.

As κ^a and κ'^a are null

$$I = \int_{\substack{m \\ \mathcal{M}M}}^{p} \kappa_a \mathrm{d}s^a \qquad \text{and} \qquad I' = \int_{\substack{p \\ \mathcal{M}M}}^{m'} \kappa'_a \mathrm{d}s^a. \tag{2.6.5}$$

and the integrals are evaluated along $\mathcal{M}M$. Using Eqs. (2.6.2) and (2.6.3) in Eq. (2.6.5) one may write

$$I + I' = (\lambda_M)^{-1/2}\ \Delta\tau$$

where λ_M is the value of the scalar field along the worldline M of the mirror and $\Delta\tau$ is the distance between the events m and m' evaluated along M.

Now, in place of c in the integrals if any closed curve S on μ is chosen, then

$$I + I' = 2\oint_S \lambda^{-1}\ t_a \mathrm{d}s^a.$$

Thus the Sagnac shift is given by

$$\Delta\tau = 2\,(\lambda_M)^{1/2} \oint\limits_S \lambda^{-1}\, t_a \mathrm{d}s^a, \tag{2.6.6}$$

or alternatively,

$$\Delta\tau = 2\,(\nu_M)^{-1} \oint\limits_S \nu\, \xi_a \mathrm{d}s^a, \tag{2.6.7}$$

where ν (2.6.1) is the frequency of the light as seen by ξ^a and ν_M is that along the world line M of the mirror.

One can notice that the integration does not depend on the choice of S, and SD depends upon ν_M. If the position of the mirror is changed SD will also change unlike the Newtonian case where it is independent of the location of the mirror on the tube. Ashtekar and Magnon comments on this difference –

> Whereas in general relativity the frequency of the light rays remains constant only along the world line of a point on the tube, but changes from one point of the tube to another, in the Newtonian frame work the frequency is constant everywhere on the tube.

Ashtekar and Magnon then considered two cases . For the first, it is stationary object. It is, as usual in GR, expressed by a timelike Killing vector field t^a and its state of rotation by a twist t^a. Using Stoke's law in Eq. (2.6.6), they obtained the SD as

$$\begin{aligned} \Delta\tau &= 2\,(\lambda_M)^{1/2} \int\limits_\Sigma \nabla_{[a}\lambda^{-1} t_{b]} \mathrm{d}s^{ab} \\ &= (\lambda_M)^{1/2} \int\limits_\Sigma \lambda^{-3/2} \omega^a\, \epsilon_{abc}\, \mathrm{d}s^{bc}, \end{aligned} \tag{2.6.8}$$

where,

$$\nabla_a = \text{any derivative operator on } \mathcal{M},$$

$$\epsilon_{abc} = \text{alternating tensor},$$

$$\epsilon_{abc} = \epsilon_{abc}\lambda^{-1/2}t^d,$$

$$\omega^a = \epsilon^{abcd}t_b\nabla_c t_d, \text{ the twist of Killing field } t^a.$$

Thus Sagnac shift is interpreted as a measure of the flux of ($\lambda^{-3/2}$ times) the twist of Killing field through the tube.

In the second case the object is considered to be a stationary axisymmetric. Such an object in GR is described by two commuting Killing vector fields; one timelike, in some neighbourhood of spatial infinity and one rotational, denoted by T^a and R^a, respectively. The rotation is thus represented by $T^a R_a$. Thus they obtained, from Eq. (2.6.6) as

$$\Delta\tau = \left[2\lambda^{-1/2}\left(R^a R_a\right)^{-1/2}(\text{length of } S)\right]\left(T^a R_a\right). \tag{2.6.9}$$

In Chap. 7 we shall however, follow a simpleminded approach by directly using the explicit forms of the metric possessing physically interisting symmetries. The approach will be the extended version of metrical treatment in flat spacetime [12, 49–51]. Tartaglia [52] also followed this approach. Indeed we shall see that Tartaglia's results agree with ours in the same physical situations as special cases.

2.7 GPS and Sagnac Effect

As described earlier, there are several application of Sagnac effect. Here we briefly mention one such application due to its immense importance at the present age. This is called global position system (GPS). A GPS is a system constructed to make navigation on or near the surface of earth, and to provide accurate worldwide clock synchronization and timing system. The whole procedure is achieved by three principal "segments', a space segment, a control segment and a user segment [53].

The space segment is consisted of some satellites carrying atomic clocks. Presently 24 satellites are working. Spare satellites and spare clocks are also provided.

To provide at least four satellites always above the local horizon, "four satellites in each of six orbital planes inclined at 55^o with respect to earth's equatorial plane"[53]. Navigation and timing signals are provided with the clocks.

The data from satellites are collected by several ground-based monitoring stations. From these stations data are then sent to a Master Control station (MCS) situated at Colorado Spring, CO. This MCS then "analyzes the constellation and projects the satellite ephemerides and clock behaviour forward for the next few hours. This information is then uploaded into the satellites for retransmission to users". The whole segment is called the control segment.

The user segment consists of all the users. These users receive signals retransmitted from the satellites and can determine their position, velocity and the time on their local clocks.

In the GPS the primary reference frame is Earth centred, Earth fixed system, the WGS-84 frame, because users are interested to know their positions on the surface (or very near it) of earth. This makes it clear that due to the rotation of earth one must consider the Sagnac effect to do any calculations accurate. As Ashby [53] points out that a very small Sagnac correction of one nanosecond can save a navigational error 30 cm, while Sagnac correction amount to hundreds of nanoseconds. The reader is referred to the article by Neil Ashby [53] for detailed discussion, where he describes Sagnac (or "Sagnac like") corrections "from the point of view of the local inertial frame and also, from the point of view of an earth-centred rotating frame in which Sagnac effect is described by terms in the fundamental scalar invariant that couple space and time.

2.8 The Area factor: the LSE and the FOC Experiments

It has long been believed that Sagnac phase shift depends on the projection of the area of the light contour on the plane of the disc. The first order Sagnac phase shift form is usually quoted in terms of that (vide Eq. (2.4.1)). Consequently if a modified Sagnac type experiment is performed in the laboratory which contains the essential elements of Sagnac experiment but have zero projection area, we should expect zero phase. Recent experiments (gedanken and laboratory) suggest that there is no reason for obtaining a zero phase shift in this case. Indeed, it has been shown [40, 43, 44] that the result depends on the length of the light path and not on the projection area, because the thought experiment corresponds to a zero area configuration.

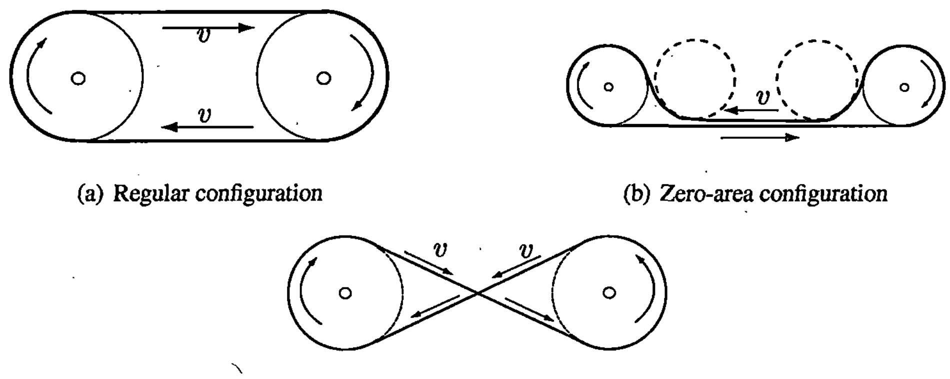
2.8.1 Linear Sagnac Effect (LSE)

The basic ingredients of Sagnac experiment are that the light signals moving parallel and antiparallel to the motion of the platform throughout its journey complete round trip with the help of mirrors on a platform moving with uniform "speed" in the laboratory. Let us now consider a set of two experiments which modifies Sagnac experiment retaining its essential ingredients. We consider a rigid platform with a light source and a facing mirror at the two extreme ends. Light is emitted from the source, reaches the mirror to be reflected back and reaches the source point again. This is essentially the round trip feature of motion of light beams in the Sagnac type experiment. A mechanism is attached to the platform which reverses the journey of it at the instant when light falls on the mirror. For the first experiment, when light is emitted from the source, the platform moves in the same direction as light travels with respect to the laboratory while reverses its direction when light is reflected by the mirror. Thus throughout the journey the motion of light remains parallel to the motion of the platform mimicking the paralel propagation of light beam in Sagnac experiment. For the second experiment the same mechanism is adopted except for the fact that the direction of light is antiparallel to direction of platform throughout its round trip journey mimicking the antiparallel propagation of light in in the Sagnac type experiment. The difference between the round-trip times gives the Sagnac formula as function of the length of the platform, *i.e.* length of the light contour. This is referred to as linear Sagnac effect [40, 43, 44]. This has been discussed in detail in Sec. 3.4. One should note that though the area enclosed by light contour is zero, the Sagnac phase shift is still non-zero and the length of the light contour determines the Sagnac phase shift formula.

2.8.2 FOC Experiment

An experiment has been recently performed by Wang, Zheng, Yao and Langley [54] and must be referred to in connection with the LSE. The authors modified a Fibre optic gyroscope (FOG) by adding an extra 50m of single mode fibre to the original fibre loop in their experiment with a new device which they call fibre optic conveyor(FOC). They wrapped the extra fibre onto a polyester ribbon that went around two wheels with diameter of 30cm. A mechanical conveyor is used to rotate one of the wheels which makes the fibre move. As an FOG is sensitive to rotational motion, even though the FOG travels with the mechanical conveyor its uniform motion does not cause any phase shift.

The group performed the experiment with 24 different arrangements of conveyor speeds, fibre lengths and three different configurations as shown in the figures.



(a) Regular configuration

(b) Zero-area configuration

(c) Figure-8 configuration

Figure 2.4: FOC Experiments of Wang, Zheng, Yao and Langley

Fig. 2.4(a) is an FOC experiment in a *regular configuration.* Several fibre lengths were considered. The setup in Fig. 2.4(b) is called the *zero-area configuration* since the area enclosed by the fibre cable (light contour) is almost zero except for the areas enclosed at the two ends because of the wheel. The experimental setup shown in Fig. 2.4(c) is called *figure-8 configuration* for obvious reasons. Their contention is that from rotational view point the two enclosed areas having opposite directions cancel other's effect simulating a zero-area experiment. The whole arrangement and its conclusions fit well with LSE (gedanken) experiment although with minor adjustment in the reasoning.

The group observed the Sagnac effect, obtained the result as first order effect and concluded –

1. Sagnac phase shift depends on the lengths and speed of the moving fibre and these are the fundamental factors, rather than the enclosed area determining

the SD, as customarily quoted (say, in Ref. [12]) – because they observed Sagnac effect even when their FOC has zero-area.

2. Sagnac effect is also observable for uniform motion – as they authors observed Sagnac effect when the interferometer is in linear motion.

We make the following observations regarding their conclusion. First, they rightly observe that Sagnac effect is dependent on the length of the path light travels and the velocity of the platform. The difference of distances traveled by the two counter rotating beams before reaching the beam splitter sets a phase difference between them. This difference depends on the length of the path and the velocity of the platform. Note that in the limit where the radii of the two wheels vanishes, the FOC experiments simulates LSE and becomes ideally a zero-area experiment. The LSE (thought) and FOC (laboratory) experiment prove beyond doubt that Sagnac formula has nothing to do with the area enclosed by the light contour. However, one is certainly free to choose his/her own way of quoting Sagnac formula – in terms of the length of light path or the area enclosed by the light contour – although, in view of these experiments the former is preferable.

Their second conclusion is rather superficial although this is essentially their guiding principle in designing the zero-area experiment. The essential ingredient of the Sagnac effect lies in the facts that 1) Mutually counter rotating light beams complete a round-trip before reunion and 2) the platform suffers acceleration at any point on the path of the light beams. It does not matter if the measuring device is moving in uniform motion at the time of measurement *locally* because *globally* the platform is not an inertial frame. This is evident from our analysis of LSE. In fact, one of the most important contributions of the FOC experiment to the understanding of the Sagnac effect is that rotation of the platform is not an essential ingredient, rather it is a *special case*.

Reference

[1] G. Sagnac, *Comp. Rend. Acad. Sci.*, **157**, (1913) pp. 708–710.

[2] H. Hasselbach and M. Nicklaus, *Phys. Rev. A*, **48**, (1993) pp. 143–151.

[3] R. Colella, A. W. Overhauser, J. L. Staudenmann and S.A. Werner, *Phys. Rev. A*, **21**, (1980) pp. 1419–1438.

[4] F. Riehle, Th. Kister, A. Wittie, J. Helmcke and ch. J. Borde, *Phys. Rev. Lett.*, **67**, (1991) pp. 177–180.

[5] D. Fargion, L. Chiatti and A. Aiello, "Quantum mach effect by sagnac phase shift on cooper pair in rf-squid", Preprint astro-ph/9606117, babbage.sissa.it.

[6] I. L. Bershtein, *Dokl. Akad. Nauk. SSSR*, **75**, (1950) p. 635.

[7] V. Vyschtskii *et al.*, *Phys. Usp.*, **62**, (1994) p. 289.

[8] S. Merhab, *Aerospace Sensor Systems and Applications* (Springer, New York, 1996).

[9] D. W. Allan *et al.*, *IEEE Trans. Instrum. Meas.*, **IM-34**(2), (1985) pp. 118–125.

[10] M. Yamamoto N. Saburi and K. Harada, *IEEE Trans. Instr. Meas.*, **IM-25**, (1976) pp. 473–477.

[11] G. Petit and G. Wolf, *Astron. Astrophys.*, **286**, (1994) pp. 971–977.

[12] E. J. Post, "Sagnac effect", *Rev. Mod. Phys.*, **39**, (1967) p. 475.

[13] G. B. Malykin, "Sagnac effect", *Physics-Uspekhi*, **40**(3), (1997) pp. 317–321.

[14] Oliver Lodge, *Philos. Trans. R. Soc. (London)*, **184A**, (1893) p. 727.

[15] Oliver Lodge, *Philos. Trans. R. Soc. (London)*, **189A**, (1897) p. 149.

[16] A. A. Michelson, *Philos. Mag*, **8**(56), (1904) p. 716.

[17] P. Harzer, *Astron. Nach.*, **198**, (1914) p. 378.

[18] A. Einstein, *Astron. Nach.*, **198**, (1914) p. 378.

[19] G. Sagnac, *Compt. Rend.*, **157**, (1913) p. 1410.

[20] G. Sagnac, *J. Phys. (Paris)*, **4**, (1914) p. 177.

[21] G. Sagnac, *J. Phys. Radium Ser.*, **4**, (1914) pp. 177–195.

[22] V. A. Ugarov, *Special Theory of Relativity* (Mir Publisher, Moscow, 1979). Translated from Russian by Yuri Atanov.

[23] R. Anderson, H. R. Bilger and G. E. Stedman, *Am. J. Phys.*, **62**(11), (1994) pp. 975–985.

[24] B. Pogany, *Ann. der Phys.*, **80**, (1926) p. 217.

[25] B. Pogany, *Naturwissenschaften*, **15**, (1927) p. 177.

[26] B. Pogany, *Ann. der Phys.*, **85**, (1928) p. 244.

[27] V. Vali, R. W. Shorthill and M. F. Berg, *Appl. Opt.*, **16**, (1977) p. 2605.

[28] G. Schiffner W. R. Leeb and E. Scheiterer, *Appl. Opt.*, **18**, (1979) p. 1293.

[29] A. Dufour and F. Prunier, *Compt. Rend.*, **204**, (1937) p. 1322.

[30] A. Dufour and F. Prunier, *Compt. Rend.*, **205**, (1937) p. 658.

[31] W. M. Macek and D. T. M Davis., *Appl. Phys. Lett.*, **2**, (1963) p. 67.

[32] A. H. Rosenthal, *J. Opt. Soc. Am.*, **52**, (1962) p. 1143.

[33] J. R. Wilkinson, "Ring lasers", *Prog. Quant. Electr.*, **11**, (1987) pp. 1–103.

[34] B. W. Petley, *Contemp. Phys.*, **21**, (1980) p. 607.

[35] G. E. Stedman, *Contemp. Phys.*, **26**, (1985) p. 311.

[36] J. E. Zimmerman, *Cryogenics*, **20**, (1980) p. 3.

[37] J. E. Zimmerman and J. E. Mercereau, *Phys. Rev. Lett.*, **14**, (1965) p. 887.

[38] D. M. Greenberger, *Rev. Mod. Phys.*, **55**, (1983) p. 875.

[39] S. A. Werner, *Phys. Today*, **33**, (1980) p. 24.

[40] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[41] A. Ashtekar and A. Magnon, *J. Math. Phys.*, **16**(2), (1975) pp. 341–344.

[42] G. Sagnac, *Compte. Rendus.*, **141**, (1905) p. 1220.

[43] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Rotating disc problem and sagnac phase-shift formula", in M. C. Duffy (Ed.), "Physical Interpretations of Relativity Theory, VIII", (P. D. Publication, Liverpool, 2000). Held at Imperial College London.

[44] S.K. Ghosal and Minakshi Sarker, "Cosmologically preferred frame and sagnac effect", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (VII)", volume Late Papers (1998). Held at Imperial College, London.

[45] D. Dieks and G. Nienhuis, *Am. J. Phys.*, **58**, (1990) pp. 650–655.

[46] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[47] A. Tartaglia, *Found. Phys. Lett.*, **12**(1), (1999) pp. 17–28.

[48] J. Anandan, *Phys. Rev. D*, **24**, (1981) pp. 338–346.

[49] A. A. Logunov and Yu. V. Chugree, *Sov. Phys. Uspekhi*, **31**, (1988) p. 861.

[50] P. Langevin, *Compt. Rend.*, **173**, (1921) p. 831.

[51] P. Langevin, *Compt. Rend.*, **205**, (1937) p. 51.

[52] A. Tartaglia, *Phys. Rev. D*, **58**, (1998) p. 064009.

[53] Neil Ashby, "The sagnac effect in the global positioning system", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frame", Fundamental Theories of Physics (Kluwer, Dordrecht, 2004).

[54] R. Wang, Yi. Zheng, Aiping Yao and D. Langley, *Phys. Lett. A*, **312**, (2003) pp. 7–10.

Chapter 3

Relativistic Sagnac effect and Ehrenfest Paradox

3.1 Introduction

In the last chapter, where we have reviewed some aspects of Sagnac effect we have shown that two relativistic formulae for Sagnac effect exists in the literature involving the appearance of the relativistic γ factor in the formula. The present day precision in measurement of Sagnac effect is unable to decide between the two formulae and the issue may remain inconclusive for years to come. Nevertheless from the theoretical standpoint the question cannot be lightly dismissed. In this chapter we shall see that the issue is intimately connected with the Ehrenfest paradox concerning the spatial geometry of a rotating disc.

3.2 Ehrenfest Paradox

Rotation has always been a subject of fascination by its own for the relativist. It is the subject of vast and growing literature. Sometimes it has also been some source of misunderstanding leading to conceptual problems. Ehrenfest paradox is one of them. The paradox can be described as follows:

When a body moves, it is contracted along its direction of motion by a factor of γ, the Lorentz contraction factor. There will be no contraction along the directions perpendicular to the direction of motion.

Now let us consider a disc rotating with a velocity $v = \omega R$, ω and R being its angular velocity and radius respectively. The circumference of the disc (when at rest in an inertial frame) is $2\pi R$, according to Euclidean geometry. What will happen to the disc when it is rotating? Let us discuss the issue from the point of view of an observer in the inertial frame. At any point on the circumference, the linear velocity of the disc $v = \omega R$ is tangential. Thus the radius of the disc is always perpendicular to the velocity. So, according to SR, There will be no change of the radial length of the disc.

Any elementary circumferential length segment of the disc is parallel to the linear velocity, and thus will suffer Lorentz-Fitzerald contraction. The contracted length then will be $2\pi R\gamma^{-1}$. This means that circumference to diameter ratio of a disc at rest will not be maintained for a disc under rotation, violating Euclidean geometry in an *inertial frame*! Essentially this is known as Ehrenfest Paradox (EP) [1].

Paul Ehrenfest was the first to identify this problem. His intention was to understand Max Born's notion of relativistic rigidity [2]. Though generally this is called 'rotating disc problem', originally Ehrenfest proposed it for a *relativistically rigid* rotating cylinder.[1] This problem was termed by Varićak [3] as Ehrenfest Paradox(EP).

Grøn [4] is of the view that EP *motivated* Einstein *to search for a geometrical theory of gravity*. Einstein never participated in the EP debate [4] yet he proposed a resolution of EP in a letter to J. Petzold in 1919 (quoted by J. Stachel [5]).

> *A rigid circular disk at rest must break up if it is set into rotation, on account of the Lorentz contraction of the tangential fibres and the non-contraction of the radial ones. Similarly, a rigid disc in rotation (produced by casting) must explode as consequence of the inverse changes in length, if one attempts to put it at rest.*

A detail discussion of rotating disc problem and its historical development is given by Ø. Grøn in Ref. [4].

[1] Born's definition of relativistic rigidity concerns a motion where the proper length of the moving body is constant. In a latter section of this chapter we comment on this issue and its relation to EP.

3.3 Sagnac effect and Ehrenfest Paradox

In Sec. 2.5.2 we have given a relativistic analysis of Sagnac effect and mentioned that there is a mild controversy as to what is the correct relativistic formula for Sagnac effect. The formula can written as (2.5.28)

$$\delta\tau = \frac{4\pi\omega r^2}{c^2}\Gamma. \tag{3.3.1}$$

There are two claims about the value of Γ depending on whether one considers length contraction of the periphery of the disc or not. These are,

1. $\Gamma = 1$
2. $\Gamma = \gamma$

respectively, γ being the Lorentz factor. Selleri [6] and Goy [7] support the first claim pointing out the discrepancy ($\Gamma = \gamma$ is generally believed to be the *true relativistic formula* [8]). The present day precision in measurement of the Sagnac effect may be unable to decide between the two formulae, nevertheless from the theoretical and pedagogical standpoint the question cannot be ignored[2]. We shall see below that the issue is intimately connected with the EP. However for this moment we refrain from discussing the paradox any further or from giving any verdict right away regarding the discrepancy over the correct relativistic formula for Sagnac effect. In order to appreciate the question, the real physics behind the

[2]To appreciate a current perspective of this pedagogical question consider the following: The Sagnac experiment is often regarded as fundamental as the Michelson-Morley experiment so much so that in some recent papers it is claimed that one may even rederive relativity theory from some new postulates based on the Sagnac effect [9, 10]. Unfortunately the Sagnac effect unlike the Michelson-Morley result is a verification of a first order effect. To hope to derive relativity theory from the Sagnac result therefore requires the exact Sagnac formula and one cannot remain content with the approximate one.

Sagnac effect may be brought out first by delinking any other effect that may be present due to the rotation of the disc from some *pure* Sagnac effect.

Let us first recognize that the essential content of the Sagnac experiment lies in its ability to detect acceleration of the experimental platform by comparing the round-trip times for lights traveling parallel or antiparallel to the motion of the platform. It is therefore expected that the acceleration need not have to be due to rotation alone; a suitably modified Sagnac type experiment should as well be able to detect the change of the direction of motion of a platform which is allowed to move or shuttle along a straight line. In the next section we shall propose such a thought experiment which will mimic the optical Sagnac experiment in almost all respect but with the difference that the motion of the experimental platform will not involve rotation. The outcome of the experiment will be called the *pure* Sagnac effect. This will give us a perspective which will enable us to appreciate the connection between the Sagnac effect and the Ehrenfest paradox. We shall see below that the formula obtained for the *pure* Sagnac effect may or may not be modified when rotation is introduced. This modification or the lack of it will depend on the way the Ehrenfest paradox is resolved [11, 12].

It is interesting to note that no author has ever explicitly mentioned any role of the EP in the derivation of the SD. In Sec. 3.8 we shall argue that the so-called kinematic resolution of the EP is based on some implicit assumptions regarding the behaviour of the solid discs when set into rotation. In order to prepare the background of this argument, in Secs. 3.6 and 3.7 we shall consider another version of the linear Sagnac experiment and analysis involving a non-rigid frame of reference of a special kind. We shall see that the Sagnac type experiment performed on such platforms gives the usually quoted formula for the SD. Significance of these observations among other things will be discussed in

Sec. 3.8 and finally will be summarized in Sec. 3.9.

3.4 Sagnac effect in a Linear Platform

The title of this section may appear to be misleading at the first reading, because Sagnac experiment detects, and is used to detect rotation. The derivation also involves rotation. But a close scrutiny of the derivations will reveal that what the Sagnac effect essentially detects is the acceleration of the platform, of which rotation is a special case. Sagnac experiment detects acceleration of the experimental platform by comparing times for lights traveling parallel or antiparallel to the motion of the platform. Thus the acceleration need not have to be due to rotation alone; a suitably modified form of Sagnac experiment should be able to detect the change of direction of platform with linear motion as well. Here we describe such a gedanken experiment [13, 14] which will mimic the optical Sagnac experiment in almost all respect but with the difference that the motion of the experimental platform will not involve rotation. Though Sagnac effect without rotation may seem to be a contradiction in terms one must recognise that the basic ingredients of Sagnac experiment that will also be included in our modified (gedanken) Sagnac experiment essentially are

1. Light signal complete round trips with the help of mirrors.
2. The experimental platform moves with a uniform *speed* (in case of rotational platform, this speed is tangential to the circumference)
3. Light travels parallel (or antiparallel) to the direction of motion of the platform throughout during their round trips.
4. The difference of the round trip times (the SD) for the parallel and the antiparallel light signals will be measured on board the platform.

The outcome of this experiment is called *pure* Sagnac effect, as no other effect than the acceleration that may have their present in case of its rotational analogue, such as Coriolis force or centrifugal force (both are essentially non-inertial forces) will not be present in this modification. The reason behind introducing this linear analogue of Sagnac experiment is that delinking this rotation-specific forces will enable us to appreciate the connection between the Sagnac effect and the EP. It will also be shown later that the formula obtained for linear version of the Sagnac effect (we call it *linear Sagnac effect* (LSE)) may or may not be modified when rotation is introduced depending on the way EP is resolved. In next two sections we shall discuss two versions of LSE [13, 14]. In the following sections we shall discuss a resolution of EP and show that the resolution EP will determine the value of Γ (3.3.1) to find the correct SD formula.

It is interesting to note that no author has ever explicitly mentioned any role of EP in the derivation SD. Indeed we shall argue in a later section that the so-called kinematic resolution of the EP is based on some implicit assumptions regarding the behaviour of the solid discs when set into rotation.

3.5 Linear Sagnac Effect – I

In optical Sagnac experiment, one essentially compares the round trip times of two light signals one of which propagates parallel and the other travel antiparallel to the direction of motion of the edge of the rotating disc on which all the measurements are carried out. The gedanken linear version of the Sagnac effect is proposed so that it essentially contains this characteristic of Sagnac effect. Two separate experiments will be considered for the parallel and the antiparallel journeys.

Let us consider a *rigid* linear platform A is made such that it can move in

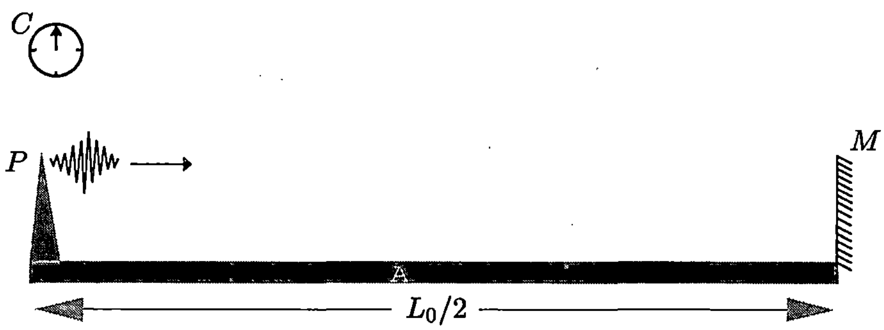


Figure 3.1: Experimental Arrangement

either direction with a constant linear velocity $\mathbf{v}$. Also, it is possible to change the motion to the opposite direction as and when needed. The length of the rigid rod, in its rest frame is assumed to be $L_0/2$. At one end of the rigid rod (right end in Fig. 3.1) there is a mirror M. A sensor is attached to the mirror. Whenever light falls on the mirror, the sensor passes a massage to a mechanism set to the platform and the platform at once reverses its direction of motion and starts to move in the opposite direction. This reversal of direction is assumed to occur in no time, *i.e.* there will be no time lag. This mechanism will assure that the light motion will be parallel or antiparallel throughout its journey to the direction of motion.

At the other end, there is a light source P. A certain mechanism is set so that whenever light pulse is emitted from P, the platform starts to move in a predefined direction.

There is a clock (C) attached to the source which records the round trip time of the light pulse. Each experiment is divided in two phases. At the first phase of the first experiment (Fig. 3.2) when light is emitted from the source and moves toward the mirror, the rod starts to move toward right (along the positive $x-$direction) with a constant linear velocity $\mathbf{v}$. After a certain time, the light pulse reaches the mirror and is reflected by it. At this phase, the velocity of light is along the

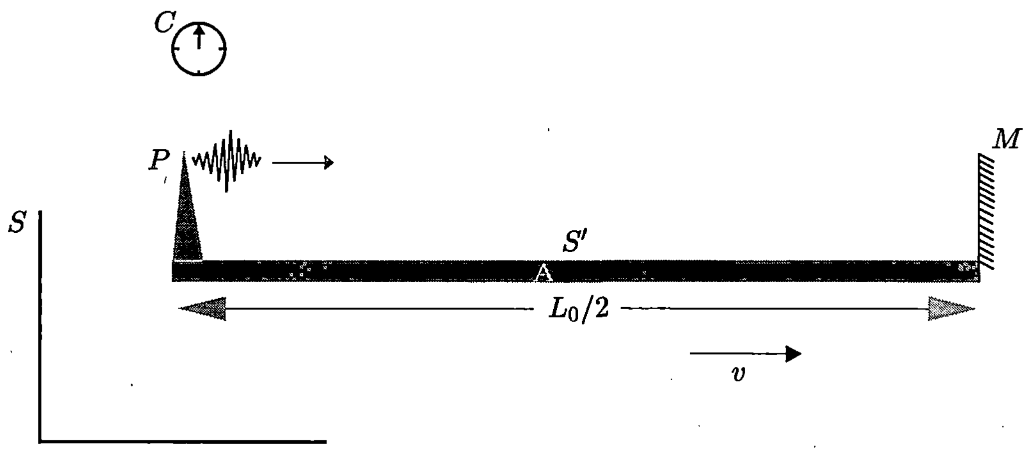


Figure 3.2: Linear Sagnac Effect–I phase I

negative x-direction. By some mechanism attached to the mirror, just the moment the light beam reaches the mirror, the whole platform reverses its direction of motion and starts to move along the negative $x-$axis (Fig. 3.3) with a constant velocity $-\mathbf{v}$. This is the second phase of the first experiment.

From Figs. 3.2 and 3.3 it is clear that the motion of light is always parallel to the direction of motion. Let us call the whole experiment (Figs. 3.2 and 3.3) parallel linear Sagnac experiment (PLSE). This is analogous to the co-rotating beam of light in the rotational optical Sagnac effect.

The frame S in the figures is the laboratory frame, S' and S'' are the frames comoving with the platform throughout the first phase and the second phase of the experiment respectively. A frame K (not shown in the figures) may be considered to be attached to the platform A. The frame S is obviously inertial from definition. S' and S'' are also inertial frames because throughout each phase, the platform moves with a constant velocity $\pm\mathbf{v}$. But the frame K attached to the platform throughout both the phases is certainly a non-inertial frame because at one point

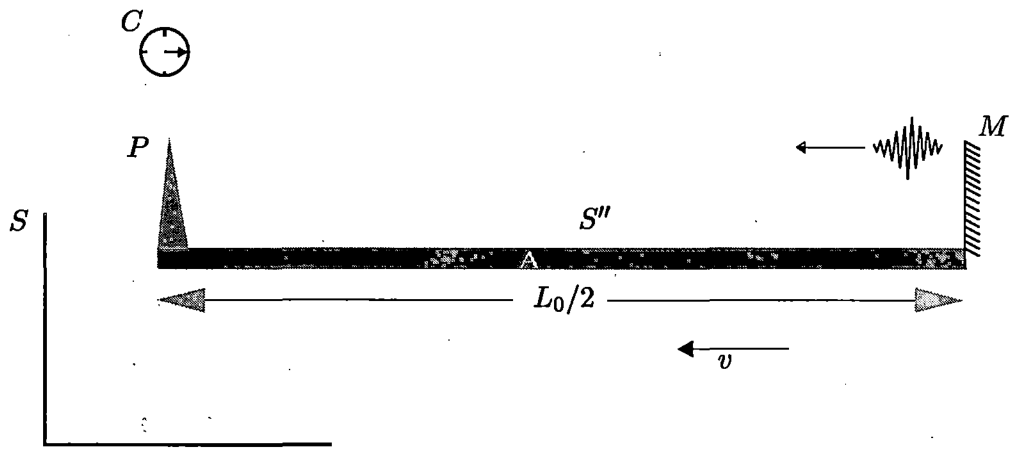


Figure 3.3: Linear Sagnac Effect–I phase II

of the experiment it suffers an acceleration, velocity changes from $+\mathbf{v}$ to $-\mathbf{v}$, though for an infinitesimal time.

A similar arrangement is made for the antiparallel linear Sagnac effect (ALSE). (see Fig. 3.4 and Fig. 3.5) Here in the first phase of the second experiment when light is emitted by the source and traverses towards the mirror M along the positive $x-$axis, the platform A moves along negative $x$direction with constant velocity $-\mathbf{v}$ (Fig. 3.4)

In the second phase, light after the reflection moves towards negative $x-$axis while the platform moves along the positive $x-$axis (Fig. 3.5)

From Figs. 3.4 and 3.5 it is clear that light motion is antiparallel to the motion of the platform throughout this experiment. This is analogous to the counterrotating beam of the rotational Sagnac effect. As in the previous case, S, S' and S'' are inertial, K is non-inertial. Though we used S' and S'' in both the cases they are certainly not the same.

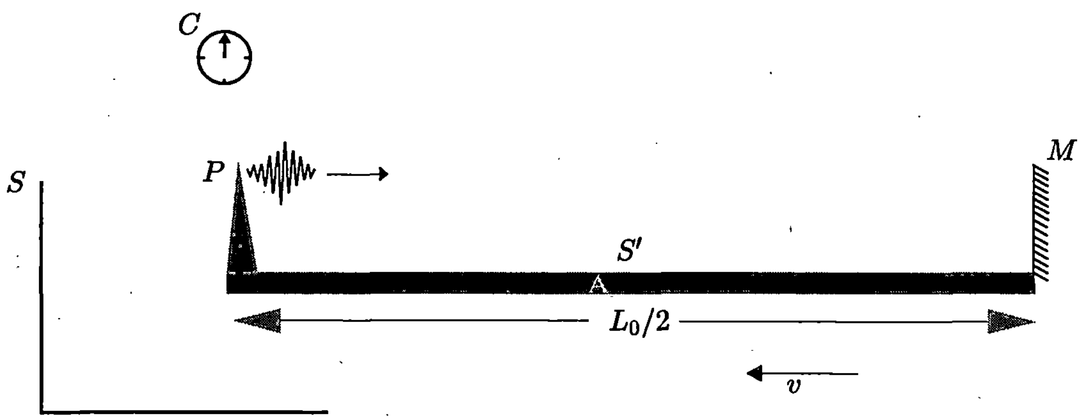


Figure 3.4: Linear Sagnac Effect–II phase I

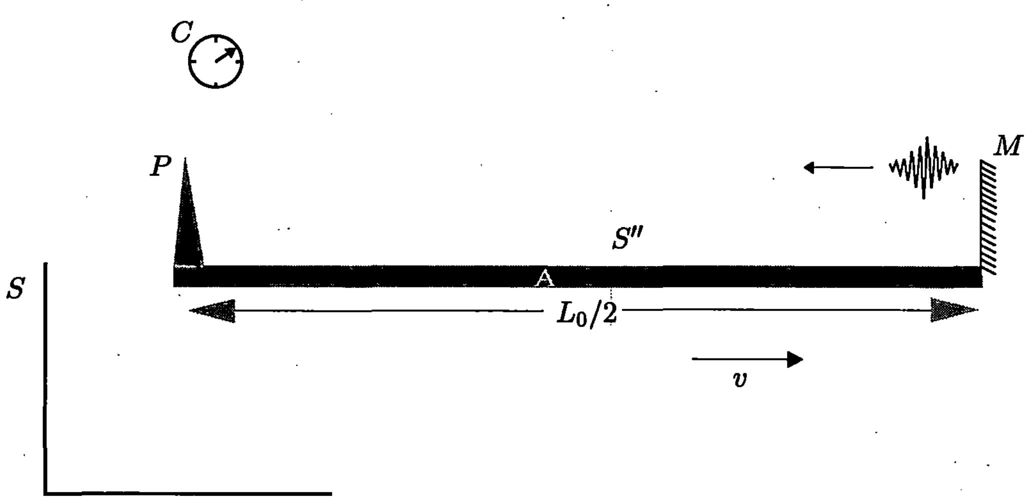


Figure 3.5: Linear Sagnac Effect–II phase II

Now, we can write for the motion of the light pulse, using Lorentz transformation

$$\mathrm{d}t_{\text{lab}} = \gamma \left(\mathrm{d}t \pm \frac{v \mathrm{d}x}{c^2} \right), \tag{3.5.1}$$

where t_{lab} is the co-ordinate time of the global inertial frame of the laboratory and t and x refer to the local time and spatial co-ordinate of the instantaneous inertial frames S' and S'' and the positive and negative sign apply for the forward and reverse journeys of the rod respectively.

For the first experiment where light is always moving parallel to the motion of the rod, we obtain the round trip time $\Delta t_{\text{lab}}(1)$ by integrating Eq. (3.5.1)

$$\begin{aligned} \gamma^{-1}\, \Delta t_{\text{lab}}(1) &= \int \mathrm{d}t + \int_0^{\frac{L_0}{2}} \frac{|v| \mathrm{d}x}{c^2} - \int_{\frac{L_0}{2}}^{0} \frac{|v| \mathrm{d}x}{c^2} \\ &= \int \mathrm{d}t + \frac{L_0 |v|}{c^2}. \end{aligned} \tag{3.5.2}$$

Similarly we can find the round trip time $\Delta t_{\text{lab}}(2)$ for the experiment where light moves antiparallel to the motion of the rod throughout the experiment

$$\begin{aligned} \gamma^{-1}\, \Delta t_{\text{lab}}(2) &= \int \mathrm{d}t - \int_0^{\frac{L_0}{2}} \frac{|v| \mathrm{d}x}{c^2} + \int_{\frac{L_0}{2}}^{0} \frac{|v| \mathrm{d}x}{c^2} \\ &= \int \mathrm{d}t - \frac{L_0 |v|}{c^2}. \end{aligned} \tag{3.5.3}$$

A look at Eq. (3.5.1) will clarify the nature of $\int \mathrm{d}t$ in Eqs. (3.5.2) and (3.5.3). S' being an inertial frame, the observe in S' will measure the time of arrival of light from the source to the mirror to be $L_0/2c$ because the velocity of light is c in S' , $L_0/2$ being the length of the rod. The same is true for an observer in S'' who measures the time of flight of light from the mirror to the source. Thus the total time of flight of light measured by two independent observers in S' and S'' in two different phases of the experiment is

$$2 \times \frac{L_0}{2c} = \frac{L_0}{c}.$$

This is $\int \mathrm{d}t$ and is equal for both the experiments.

Now, if $\Delta\tau = \int \mathrm{d}\tau$ is the time of flight of light for the round trip as measured by an observer in K (remember, K is comoving with rod and suffers direction reversal) then, we may write, by virtue of time dilation effect

$$\Delta\tau = \gamma^{-1}\Delta t_{\text{lab}}$$

Eqs. (3.5.2) and (3.5.3) may now be written as

$$\Delta\tau(1) = \int \mathrm{d}t + \frac{L_0|v|}{c^2}, \tag{3.5.4}$$

$$\Delta\tau(2) = \int \mathrm{d}t - \frac{L_0|v|}{c^2}. \tag{3.5.5}$$

Difference of these times gives for Sagnac delay for this linear gedanken version of Sagnac experiment

$$\delta\tau = \frac{2L_0|v|}{c^2}. \tag{3.5.6}$$

Note that $L_0/2$ is the proper length of the rod thus L_0 is a constant. Thus the SD in this case is linear in v.

Also note that all the mathematical steps that are involved in this derivation smoothly go over to that leading to rotational Sagnac effect. Except there instead of two inertial frames S' and S'', one has to consider an infinite number of momentary inertial frames for the calculations [15, 16]. Since no other effect (that might be present because of rotational motion of the disc in the usual Sagnac experiment) is involved in this linear arrangement the time delay expressed by the above relation may be said to be the result of *pure* Sagnac effect.

3.6 Linear Sagnac Effect – II

For linear Sagnac experiment type II, we shall slightly modify the experiment type I. Here instead of considering a light source and a mirror fixed on a rigid rod of proper length $L_0/2$, we consider two aircrafts (unbonded, *i.e.* not tied together) separated by a distance $L/2$ initially at rest with the laboratory.

Suppose that these aircrafts are programmed in such a way that they can move in any direction always preserving a constant separation $L/2$ with respect to the laboratory. To a casual observer in the laboratory there will appear to be a bond between the objects because of the programmed constant separation of the two, but in reality they are unbonded. One may term this apparent bond between the aircrafts as *software bond* as distinguished from the bond that exists between any molecules in a solid body [17].

The two experiments in two phases described in the previous section can now be repeated by placing the light source to one aircraft and the reflector to the other. For the first phase of the experiment assume that the aircrafts are accelerated from rest and finally the system moves with constant velocity v along the positive $x-$direction.

Suppose now a light pulse from a source attached to the first aircraft travel towards the second one on the right and falls on the right and falls on the mirror attached to it. As soon as the light pulse falls on the mirror, the *software bonded* system starts moving in the negative $x-$direction. The reflected pulse of light also travels in this direction and the time of transit for the light pulse for parallel propagation is recorded upon return to the source.

In the second experiment almost the whole programme repeated except now, at the time of emission of the light pulse, the motion of the aircraft system is along the negative $x-$direction although the light pulse travels toward the right that subsequently falls on the mirror attached to the second aircraft. The direction of motion of the unbonded system is reversed as light falls on the mirror and travels to the left to be recorded at the position of the source again. In this arrangement, propagation of light remains antiparallel throughout its entire journey.

One may take a look at all the figures in the previous section. But the rigid rod will not be their and the separation of the source and the mirror (mounted on two aircrafts) is $L/2$ in the S frame. As the aircrafts are programmed frame S, the calculation for round trip time of flight for the light pulse, unlike in the previous rigid platform experiment, should be done in the laboratory frame.

Though the experiment is performed for linear motion, the kinematical considerations leading to Eqs. (2.5.2) and (2.5.1) in Sec. 2.5.1 still remain valid. For example, for the parallel propagation experiment, in order to complete the round trip in time t_1, the light pulse, as viewed from the laboratory, has covered in addition to the distance L, an extra distance $x = vt_1$ because of the to and fro motion (with speed v) of the aircraft. This means $ct_1 = L + vt_1$, or,

$$t_1 = \frac{L}{c - v}.$$

The round trip time for the antiparallel experiment in the S frame is given by

$$t_2 = \frac{L}{c + v},$$

and their difference can be written as

$$\Delta t = t_1 - t_2 = \frac{2Lv}{c^2}\,\gamma^2.$$

If $\delta\tau$ denotes the corresponding time difference by an observer in the first aircraft one obtains, on account of time dilation

$$\delta\tau = \gamma^{-1}\,\Delta t = \frac{2Lv}{c^2}\,\gamma. \tag{3.6.1}$$

Note that $\delta\tau$ is now *not* linearly related to v. This formula (3.6.1) differs from the formula (3.5.6). While the SD for the rigid platform (3.5.6) depends linearly on

v, the SD for unbonded (software bonded) system has a non-linear relation with v.

One may however argue that in Eq. (3.5.6) L_0 (actually $L_0/2$) is the proper length of the rod and in contrast L (actually $L/2$) in Eq. (3.6.1) represents the distance of the aircrafts as measured from the laboratory frame. If the Eq. (3.5.6) were expressed in terms of L the formulae (3.5.6) and (3.6.1) would have agreed.

However, there is subtle point here. Certainly one is at liberty to quote the expressions in terms of proper distance L_0 or of the co-ordinate distance L between the source and the reflector. But if the issue is the question of dependence of SD on the speed of the platform, the formulae (3.5.6) and (3.6.1) predict different results. One must know here, while quoting the results, which of the lengths, L_0 or L is independent of speed of the platform v. In the type I experiment, the source and the reflector maintains a constant separation in its rest frame (S', S'', K). Thus the proper length of the separation is independent in v. In the type II experiment, however, the source and the reflector (mounted on two separate untied aircrafts) maintains their separation by software programme in the laboratory frame S. Thus their proper separation is stretched according to Lorentz formula and depends on v. The difference will be clear if one looks at the graph (Fig. 3.6) where the linear (dashed) one represents Eq. (3.5.6) (all the constants are set to unity).

3.7 Coordinate System of the Unbonded Frame

Consider an one-dimensional array of some software bonded particles that constitutes a frame of reference K and suppose from its state of rest at $t = 0$, the system is set in motion. If the space-time co-ordinates of the laboratory frame S are denoted by x and t, the equation of motion of the particle at the origin of K

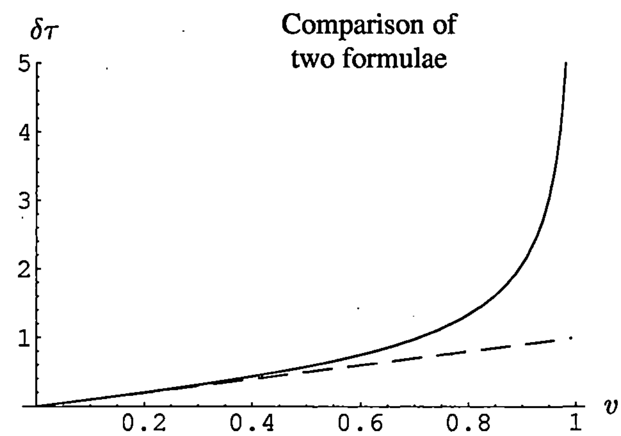


Figure 3.6: Comparison of two formulae for two types of LSE. Dashed one represents LSE1

may be expressed as

$$x = f(t),$$

where $f(t)$ is some function of time which is zero at $t = 0$.

For any particle of the array this may be written as

$$x = x' + f(t),$$

where x' is the spatial coordinate of the particle with respect to S, when it was at rest with the laboratory (at $t = 0$). The variable x' may be used to label the array of points and these may act as spatial coordinates in K. Taking the coordinate time t' of K same as t one may write the following transformations between S and K in terms of the coordinate differentials

$$\mathrm{d}x' = \mathrm{d}x - \dot{f}(t)\mathrm{d}t, \qquad \mathrm{d}t' = \mathrm{d}t, \tag{3.7.1}$$

where $\dot{f}(t) = v = \dfrac{\mathrm{d}f(t)}{\mathrm{d}t}$ is the instantaneous velocity of the aircraft.

The line-element in natural unit ($c = 1$) of the 2-dimensional Minkowskian space in the coordinate system S

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}x^2$$

may be transformed accordingly so that with respect to x' and t' one may write

$$\mathrm{d}s^2 = \gamma^{-2}\mathrm{d}t'^2 - 2v\mathrm{d}x'\mathrm{d}t' - \mathrm{d}x'^2 \tag{3.7.2}$$

where $\gamma = (1 - v^2)^{-\frac{1}{2}}$.

Again for any line-element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

the proper distance dl between two points x^i and $x^i + dx^i$ is given by (vide reference [18] for instance)

$$\mathrm{d}l = \sqrt{\left(\frac{g_{oi}g_{ok}}{g_{oo}} - g_{ik}\right)\mathrm{d}x^i\mathrm{d}x^k}, \tag{3.7.3}$$

where $i, k = 1, 2, 3$. For the line element (3.7.2) this is given by

$$\mathrm{d}l = \frac{\mathrm{d}x'}{\sqrt{1 - v^2}} \tag{3.7.4}$$

If the points x' and $x' + \mathrm{d}x'$ refer to the co-ordinates of two neighbouring particles of K, by definition $\mathrm{d}x'$ is an invariant. In that case the proper distance should stretch according to equation (3.7.4) honouring the well-known special relativistic effect of length contraction as the unbounded array of particles is set into motion. Note that expression (3.7.2) represents a perfectly legitimate co-ordinate description of the 2-dimensional Minkowsky space and although the transformations (3.7.1) have Galilean structure, special relativity is taken care of when we used equation (3.7.3) to obtain the proper distance.

3.8 Discussion

Let us now return to the original question as to the correct relativistic formula for the usual Sagnac effect. As pointed out earlier there are two contesting claims for Sagnac delay:

Claim 1: $\Delta\tau \propto \dfrac{\beta}{\sqrt{1-\beta^2}}$,

Claim 2: $\Delta\tau \propto \beta$.

If the special relativistic correction due to time dilatation is incorporated in the classical expression (Eq. (2.5.5)) one obtains Eq. (2.5.10) which corresponds to claim 1. On the other hand if not only time dilatation but also Lorentz contraction of the circumference of the rotating disc is taken into account one gets equation (2.5.12)) and claim 2 appears to be the true one. Barring a few exceptions [7, 19] most authors adhere to claim 1 without stating explicitly the reason behind not considering the length contraction effect. However as we have seen there was no ambiguity as to the correct formula for time delay for the linear Sagnac effect (thought) experiment with rigid rod. There it was evident that the Lorentz contraction of the rigid platform ought to be taken into account in addition to the time dilatation effect.

For the usual Sagnac experiment on a rotating disc, the Lorentz contraction of the disc's circumference is not generally considered perhaps to avoid the Ehrenfest paradox. On the contrary, favouring claim 2, Selleri [19] and Goy [7] assume relativistic contraction of the edge of the disc without addressing any possible paradox that may arise due to such an assumption. Indeed the result is inconsistent unless it is explicitly assumed that the disc does not remain a disc and becomes a non-flat object. It is therefore amply clear that in order to decide between claim 1 and claim 2 it is necessary to understand how the Ehrenfest problem is resolved. Ehrenfest's problem concerns the mechanical behaviour of a material disc set in rotation from rest. The paradox remains a paradox as long as one implicitly assumes that the disc is Born rigid [2, 20]. By definition Born rigid motion of a body leaves the proper lengths of the body unchanged. Grøn [11] showed that the transition of the disc from rest to rotational motion in a Born rigid way is a kinematic impossibility. It is the recognition of this fact which is known as the kinematic resolution of the Ehrenfest paradox [11, 12].

Cavalleri [20] on the contrary observes that the Ehrenfest paradox cannot be solved from a purely kinematical point of view and the solution of the paradox is intrinsically dynamical. This was refuted by Grøn [11] who rather endorsed a remark by Phipps [21] that to think that dynamics can exist "without the foundation of logically consistent kinematics" is an absurdity.

The present authors believe that both the viewpoints are correct in the present context. To recognise that Born rigid rotation is an impossibility and an implicit assumption on the contrary is the source of the paradox, may follow from pure kinematics; but if it is asked – "what exactly will happen to the solid disc?" the answer will lie in the realm of dynamics. It appears that there is no unanimity in the literature as to this precise question. Synge [22] and Pounder [23] introduce the concept of superficial rigidity [20] according to which the circumference and radius of the disc when put into rotation, undergo change in accordance with special relativity but suggest that the flat disc changes to a surface of revolution symmetric about the axis of rotation. In this way the possible violation of Euclidean geometry in the inertial system is avoided. In the case of uniform rotation this allows radial contraction without any change of meridian arc-length. Some specific prescriptions were also suggested by a few earlier authors who proposed bending of the surface of the rotating disc in the form of a paraboloid of revolution (vide [20] for further references). The rotating disc or wheel taking the shape of spherical segment when in rotation was suggested by Sokolovsky [24] as a resolution of the 'wheel paradox'.

Eddington [25] also investigated the problem of the rotating disc. He studied the question of alteration of the radius of a disc made of homogeneous incompressible material when caused to rotate with angular velocity ω. He showed that the radius of the disc is a function of the angular velocity ω and is approximately given by

$$a' = a\left(1 - \frac{1}{8}\omega^2 a^2\right)$$

where a is the rest radius of the disc. A similar view has also been expressed by Weinstein [26] who holds that the disc under rotation will be in torsion with a consequent reduction of both the radius and the circumference.

Of recent interest is the so called kinematic resolution of the Ehrenfest paradox as discussed by Grøn [11] and Weber [12]. According to the authors, it follows from purely kinematic considerations, that the radius of the disc remains unaltered but the proper measure of the circumference is increased in such an extent that the Lorentz contraction effect just gets compensated. In other word although there is a Lorentz contraction of the periphery with respect to the laboratory frame it is not visible because of the stretching of the periphery [3]. However, the stretching of the disc's circumference in its proper frame is a dynamical effect (related to the property of the solid material of the disc). How can one hope to get this dynamical effect purely from the kinematic considerations? Clearly the result must have been assumed implicitly. To clarify this let us consider a rotating co-ordinate system which is often discussed in connection

[3]Recently Klauber [9, 10] and Tartaglia [27] based upon different arguments also conclude that there will be no contraction of the circumference of the disc. The authors believe that relativistic contraction effect will not at all take place for rotating discs.

with the Ehrenfest paradox. Suppressing one spatial dimension Grøn considers the following transformation:

$$r' = r, \qquad \theta' = \theta - \omega t, \qquad t' = t \tag{3.8.1}$$

where r and θ refer to the radial and angular co-ordinates and t refers to the time co-ordinate of laboratory frame and the primed quantities refer the same in the rotating system.

The rotating frame of reference is equated with that of the rotating disc. It is precisely this equation where lies the implicit assumption that the disc's periphery is stretched due to rotation. Based on these transformations the line-element may be written as

$$\mathrm{d}s^2 = \mathrm{d}r'^2 + r'^2\mathrm{d}\theta'^2 + 2\omega r'^2\mathrm{d}\theta'\mathrm{d}t' - \left(1 - \frac{\omega^2 r'^2}{c^2}\right) c^2\mathrm{d}t'^2. \tag{3.8.2}$$

Using the formula (3.7.3) for proper spatial distance for the line element (3.8.2) the tangential proper spatial line-element is obtained as

$$\mathrm{d}l = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} r\mathrm{d}\theta'. \tag{3.8.3}$$

Note that while using Eq. (3.7.3) a minus sign under the radical is required since now the metric (3.8.2) has a different signature.

Integrating Eq. (3.8.3) along the whole element one obtains the proper length L_0

$$L_0 = L\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}, \tag{3.8.4}$$

where $L = 2\pi R$ is the circumference of the disc as observed from the inertial frame of the laboratory. However it will be wrong to assume, from the above argument, that the proper circumference of the disc has increased by a Lorentz factor. To conclude from (3.8.4) that the periphery of the disc is stretched by a

γ-factor due to rotation is to assume that L was also the circumference of the disc when it was at rest and remains the same, as it is brought up to rotation from its state of rest. Any transformation (representing rotation) must reflect the validity of this assumption. Clearly Eq. (3.8.1) does not guarantee this.

The transition of the disc to its rotational motion from its state of rest can be expressed by writing the transformations (3.8.1) in a slightly modified form

$$r' = r, \qquad \theta' = \theta - f(t), \qquad t' = t \tag{3.8.5}$$

where the function $f(t)$ is assumed to have the following properties: $f(t)$ and $\dfrac{\mathrm{d}f}{\mathrm{d}t} = \omega(t) = 0$ at $t = 0$ and $\omega(t)$ thereafter increases and finally approaches a constant value. Obviously the transformations (3.8.5) then represent a rotating coordinate system with constant angular velocity after a period of angular acceleration from its state of rest, which in terms of the differentials read

$$\mathrm{d}r' = \mathrm{d}r, \qquad \mathrm{d}\theta' = \mathrm{d}\theta - \omega(t)\mathrm{d}t, \quad \mathrm{d}t' = \mathrm{d}t. \tag{3.8.6}$$

Note that relations (3.8.2) and (3.8.3) still remain valid. Now, if we recall our discussions in Sec. 3 and 4, and the transformations (3.7.1), we see by analogy that the transformations (3.8.6) represent the motion of the disc which is composed of an unbonded arrangement of particles that are programmed to move in a particular way so that their mutual separations with respect to the laboratory frame of reference remain constant. Therefore the constancy of L (and not L_0) in other words, is the outcome of he assumed programme of motion of the particles of the disc governed by equation (3.8.5).

3.9 Summary

We are now in a position to summarize our findings. We have seen that there is a scope for confusion regarding the correct relativistic expression for the Sagnac delay. Although the oft-quoted result is that given by equation (2.5.10), no role of the so-called Ehrenfest paradox in arriving at the result is usually discussed. It is expected that the special relativistic result for the Sagnac effect will differ from its classical counterpart, usually due to two kinematic effects of special relativity – the length contraction and the time dilatation. The inclusion of the length contraction effect in the circumference (and not in the radius) of a rotating disc invites a paradox that there is an apparent violation of the Euclidean geometry in an inertial frame. On the other hand, the non-inclusion of the Lorentz contraction effect will violate special relativity. To understand and clarify these issues a Sagnac-type thought experiment (without rotation) performed on a linear rigid platform has been presented. Since no paradox is associated with this arrangement although kinematically all aspects of the usual Sagnac experiment are incorporated in it, the linear experiment sets the right kind of perspective against which the role of the Ehrenfest paradox in the rotating disc experiment can be discussed.

Although the resolution of the Ehrenfest paradox lies in appreciating the fact that 'Born rigid' rotation of the disc from its state of rest is a kinematic impossibility, people differ when trying to be specific about the exact deformation of the disc brought about by rotation. We give below just two opposite viewpoints that are found in the literature.

According to the so-called kinematic resolution of the paradox there should not be any contraction of the circumference as observed from the inertial frame of the laboratory that is at rest with the axis but, the periphery should stretch in terms of proper measure so that the Lorentz contraction effect of special relativity is automatically taken care of. The conclusion apparently follows from the widely discussed line element representing a rotating co-ordinate system [11, 18, 28]. It has been however shown, by drawing analogy with the version II linear Sagnac type (thought) experiment, that the kinematic resolution presupposes that the disc material is composed of 'unbonded' particles that are programmed to rotate in such a way that the distances between the particles remain fixed with respect to the laboratory as the system passes to a rotational motion from its state of rest. If this happens, the formula for the Sagnac delay will be given by Eq. (2.5.10).

The other view point is to suppose that the disc material obeys the Synge-Pounder criterion of superficial rigidity. In this case the disc should bend and take a shape of a paraboloid so that at any radial point, the circumference is Lorentz contracted but there is no contraction of the meridian. However the distance of the periphery from the centre will be shortened. Therefore the paradox does not exist. In this case too the resolution of the paradox is based on a specific postulate regarding the behaviour of the material of the disc undergoing rotation. As a consequence, the Sagnac delay should be given by equation (2.5.12) that corresponds to the result obtained for the linear Sagnac effect of the first form (Eq. (3.5.6)). Some authors [7, 19] have quoted this result too however not addressing any role of the Ehrenfest paradox in their derivation. If instead of a disc, the rotating platform is assumed to be a massive solid cylinder, the deformations of the kind just mentioned are perhaps excluded and the usual formula (Eq. (2.5.10)) pertains to this case. However, in this case, the constraint imposed on the particles of the cylinder by the form of the solid body would work in such a way, that the particles of the body can be thought of as 'unbonded' as the cylinder is set into rotation (vide Sec. 3.8). Indeed for a disc there cannot be one right formula; for example, the deformation of the kind considered by Eddington [25] as mentioned in Sec. 3.8 would give a result different from Eq. (2.5.10) or Eq. (2.5.12).

Reference

[1] Paul Ehrenfest, *Phys. Z.*, **10**, (1909) p. 918.

[2] Max Born, *Ann. Physik*, **30**, (1909) p. 840.

[3] V. Varićak, *Phys. Z.*, **12**, (1911) p. 169.

[4] Ø. Grøn, "Space geometry in rotating reference frames: A historical appraisal", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frames", Fundamental Theories of Physics (Kluwer, New York, 2004). Series Editor: A. van der Merwe.

[5] J. Statchel, "Einstein and the rigidly rotating disk", in A. Held (Ed.), "General Relativity and Gravitation: One Hundred Years after the Birth of Alber Einstein", volume I, pp. 1–15 (Plenum, New York).

[6] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[7] F. Goy, "Non-invariant velocity of light and clock synchronization in accelerated systems", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (V)", pp. 129–138 (1996). Held at Imperial College, London.

[8] E. J. Post, "Sagnac effect", *Rev. Mod. Phys.*, **39**, (1967) p. 475.

[9] R. D. Klauber, *Found. Phys. Lett.*, **11**, (1998) pp. 405–444.

[10] R. D. Klauber, *Am. J. Phys.*, **67**, (1998) pp. 158–159.

[11] Ø Grøn, *Am. J. Phys.*, **43**, (1975) pp. 869–876.

[12] T. A. Weber, *Am. J. Phys.*, **65**, (1997) pp. 486–487.

[13] S.K. Ghosal and Minakshi Sarker, "Cosmologically preferred frame and sagnac effect", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (VII)", volume Late Papers (1998). Held at Imperial College, London.

[14] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[15] D. Dieks and G. Nienhuis, *Am. J. Phys.*, **58**, (1990) pp. 650–655.

[16] D. Dieks, *Eur. J. Phys.*, **12**, (1991) pp. 253–259.

[17] C. W. Sherwin, "Three theories of relative measurement", (Physical Interpretation of Relativity Theory Meeting (III), Imperial College, London, 1992).

[18] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Butterworth Heinemann, Oxford, 2002), 4th revised english edition edition.

[19] F. Selleri, *Found. Phys. Lett.*, **10**, (1997) pp. 73–83.

[20] G. Cavalleri, *Nuovo Cimento B*, **53**, (1968) pp. 415–431.

[21] Jr. T. E. Phipps, *NOLTR*, **47**, (1973) pp. 73–79.

[22] J. L. Synge, "Studies in mathematics and mechanics: Presented to richard von mises by friends, colleagues and pupils", (1954). New York.

[23] J. R. Pounder, *Comm. Dublin Institute for Advanced Studies A*, **11**.

[24] Yu. I. Sokolvsky, *The Special Theory of Relativity* (Hindustan Publishing, New Delhi, 1962).

[25] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, 1965).

[26] D. H. Weinstein, *Nature*, **232**, (1971) p. 548.

[27] A. Tartaglia, *Found. Phys. Lett.*, **12**(1), (1999) pp. 17–28.

[28] C. Møller, *The Theory of Relativity* (Oxford University Press, New Delhi, 1972), second edition edition.

Chapter 4

Conventionality of Simultaneity and Tippe Top Paradox

4.1 Introduction

Since the birth of the special theory of relativity paradoxes concerning the theory have always been very common. Even today in the literature newer paradoxes continue to pour in, a latest item in the list being the tippe top paradox. In an interesting paper Basu *et al.* [1] have posed the paradox concerning a fascinating toy known as tippe top. The top has the remarkable property that after giving it an initial vertical spin on a table about its symmetry axis, the top turns itself (after a few rotations) upside down and stands spinning on its stem. A schematic diagram is given in Fig. 4.1. The statement of the paradox goes like this: Not all

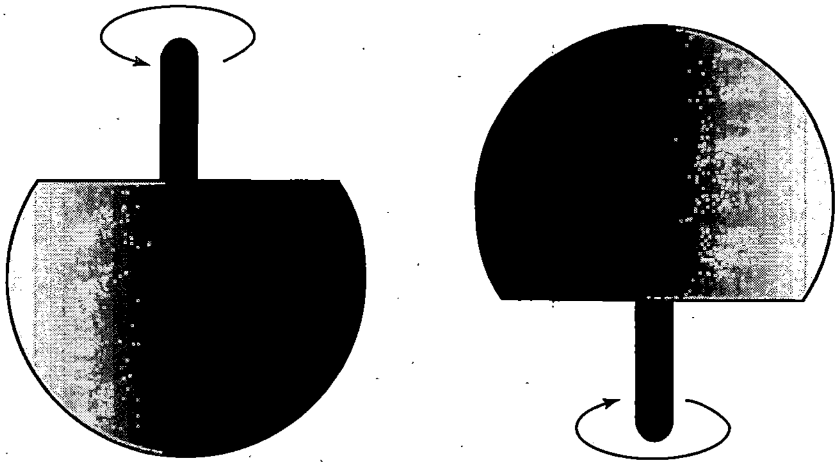


Figure 4.1: A Tippe-top

tops demonstrate this charming feat. In order for this to happen, the ratios of its principal moments of inertia should fall in a certain regime. If now such a top is observed from a rocket frame in which the spinning top appears to recede with a uniform velocity, the cross-section of the top will appear elliptical due to the relativistic length contraction effect along the boost direction. For a sufficiently fast rocket, therefore, it is possible that the ratios of the principal moments of inertia go out of the regime required for the top to tip over. This is certainly paradoxical since a mere change of reference frame cannot alter the fact that the top tips over.

The paper claims to have resolved the paradox first by noting that the paradoxes in relativity often arise because one tends to focus only on one relativistic effect (in this case the length contraction effect) while losing sight of the other kinematical consequences of Special Relativity (SR). The authors then demonstrate, by applying the Lorentz transformations (LT) to the equations of motion of the particles of the top, that the whole body of relativistic effects such as the length contraction, the time dilation, the relativity of simultaneity and the likes find their appearances in the description of the spinning motion of the particles of the top from the reference frame of the rocket. The particle orbits have been studied both analytically and by computer simulation to demonstrate how the classical notion of the rigid body fails in relativity. The origin of the paradox has then been attributed, in different words, to our adherence to the classical notion of rigidity in the relativistic domain.

Although it is interesting as well as instructive to view on the computer screen the motion of the particles of the spinning top as observed from the rocket frame (but referring it to the moving point of contact of the top with the table), we consider the paper to suffer from certain drawbacks which we would like to address. It appears, in accordance with the paper's claim, that the resolution of the paradox relies heavily on the lack of synchrony aspect of SR. This, in our opinion, is a weak point of an otherwise interesting paper. Indeed we will argue (in Sec. 4.4) that the lack of synchrony aspect may well exist in the Galilean world where it is well known that no such paradox should truly exist. The other drawback is the authors' attempt to connect the paradox with one of the corollaries of SR, that the notion of rigid body is inconsistent with it. In Sec. 4.6 below it will be shown that this aspect of SR has no bearing with the current paradox. It is indeed true that many paradoxes can arise if one inadvertently carries the classical notion of rigidity over into relativistic situation [2]. The classical rigid body by definition must move as one entity when it is pushed at one end, i.e., the disturbance at one end of the body would be propagated with infinite velocity through the body. This is in contradiction to the relativity principle that there is a finite upper limit to the speed of transmission of a signal. The analysis of Basu *et al.* considers only uniform rotation of the particles of the top with respect to the table frame; no transients are involved. It is therefore surprizing how the rigidity issue should be connected to the problem. The aim of the present paper is to deliberate on these and related issues.

In recent years a new approach based on Conventionality of Simultaneity (CS), to understanding paradoxes in relativity has been found fruitful. For example, Redhead and Debs [3] have shown that the CS approach provides a means to put an end to the question concerning the notorious twin paradox as to where and when the differential ageing takes place. As another example Selleri [4] has shown that a particular simultaneity convention compared to that of Einstein seems to be more appropriate in explaining the Sagnac effect from the point of view of the rotating turn-table. This study will also follow the CS approach to critically examine the work of Basu *et al.* A brief introduction to the CS thesis of relativity, therefore, is in order. This will be done in Sec. 4.3.[1] The main arguments will be presented in Secs. 4.4 and 4.6 before we summarize all this in Sec. 4.7. However, in order to set the stage we will briefly reproduce in Sec. 4.2

[1]For a fairly comprehensive review for the CS thesis and consequent transformation equations the reader is referred to App. A and App. B.

the basic arguments used in [1] to clarify the paradox [5].

4.2 Equation of Motion and Coordinate System

For simplicity consider a vertical top such that a typical particle P of the top in the table frame Σ^0 executes a horizontal (in the X-Y plane) circular motion. The equation of motion of the particle in the coordinate system of Σ^0 is given by

$$x^0 = R\cos\omega t^0, \qquad y^0 = R\sin\omega t^0, \qquad {x^0}^2 + {y^0}^2 = R^2 \tag{4.2.1}$$

where ω is the angular speed of the top and R represents the distance of P from the axis.

Consider now a frame of reference Σ of the rocket with respect to which the table and the top moves along the positive x-direction common to both Σ and Σ^0. Suppose that the x and t coordinates of the rocket frame are linearly related to the corresponding x^0 and t^0 of Σ^0 through the following transformations:

$$\begin{aligned} x^0 &= ax + bt, \\ t^0 &= gx + ht \end{aligned} \tag{4.2.2}$$

and also suppose $y^0 = y$, where a, b, g and h are independent of x and t. In reference [1] however they have written LT, but here we wish to keep it a bit more general for reasons which will be apparent soon. Clearly from the above transformation equations it follows that the origin of Σ^0 satisfies the following equation of motion with respect to Σ.

$$ax_{\text{orig}} + bt = 0, \tag{4.2.3}$$

where the suffix 'orig' refers to the origin.In other words the translational velocity of the origin, as observed from the rocket frame is given by

$$u = -\frac{b}{a}. \tag{4.2.4}$$

Since this translatory motion is irrelevant to the spinning of the top about its axis, it may be subtracted out from the *apparent* equation of motion of the particles of the top as seen from Σ. We may thus define the spinning coordinates of P as

$$\begin{aligned} x_s &= x - x_{\text{orig}} = x + \frac{b}{a}\,t, \\ y_s &= y, \\ t_s &= t. \end{aligned} \tag{4.2.5}$$

Inserting equations (4.2.2) and (4.2.5) in equation (4.2.1) one obtains

$$x_s = \frac{R}{a}\cos\omega\left[\left(h - \frac{gb}{a}\right)t + gx_s\right], \tag{4.2.6}$$

$$y_s = R\sin\omega\left[\left(h - \frac{gb}{a}\right)t + gx_s\right]. \tag{4.2.7}$$

The trajectory of the particle P in the rocket frame Σ is obtained by eliminating t from the above equations and one thus has

$$a^2{x_s}^2 + {y_s}^2 = R^2. \tag{4.2.8}$$

Now in SR the transformation equation (4.2.2) is nothing but LT:

$$x^0 = \gamma(x - \beta ct), \qquad y^0 = y, \qquad t^0 = \gamma\left(t - \beta c^{-1}x\right) \tag{4.2.9}$$

where $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$ with $\beta c = v$ representing the speed of the rocket with respect to the table. In other words for LT, the transformation matrix representing equation (4.2.2) is given by

$$\mathbf{T} = \begin{pmatrix} a & b \\ g & h \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta c \\ -\beta c^{-1} & 1 \end{pmatrix}. \tag{4.2.10}$$

Inserting these values of a, b, g and h in (4.2.6), (4.2.7) and (4.2.8) we obtain the equation of motion and that of the trajectory of the particle P as

$$x_s = \gamma^{-1}R\cos\omega\left(\gamma^{-1}t - \gamma\beta c^{-1}x_s\right), \tag{4.2.11}$$

$$y_s = R\sin\omega\left(\gamma^{-1}t - \gamma\beta c^{-1}x_s\right) \tag{4.2.12}$$

and

$$\gamma^2 x_s^2 + y_s^2 = R^2. \tag{4.2.13}$$

Authors of reference [1] rightly point out that the above equations display all the well known effects of relativity. For example equation (4.2.13) shows that what is a circle with respect to the table frame is now an ellipse with respect to the rocket frame (in the spinning coordinates) because of the length contraction effect. Equations (4.2.11) and (4.2.12) show that the angular frequency ω of the top is reduced to $\gamma^{-1}\omega$ because of the time dilation. Finally the relativity of simultaneity (i.e the lack of synchronization of clocks) is manifested in the presence of the spatial coordinate x_s in the phase factors of the sinusoidal functions in equations (4.2.11) and (4.2.12). The authors claim that this feature plays an essential role in the resolution of the paradox. We will however show that this is not quite correct.

The arguments of Basu *et al.* [1] are based on the apparent non-rigid behaviour of the top as seen from the rocket.However, before we go into these in detail let us now study this non-rigid character a bit closely. Equations (4.2.11), (4.2.12) and (4.2.13) display qualitatively two distinct types of non-rigidity viz. type I and type II.

Type I. This type of non-rigidity is manifested in the periodical change of the distance of any particle of the top from the center, as the particle travels along the elliptical path according to equation (4.2.13).

Type II. From equation (4.2.11) and (4.2.12) it will be evident that a chain of particles which lie along the radius of the circle (vide equation (4.2.1)) parallel to the x-axis of Σ^0 at $t^0 = 0$ will appear to take a shape of a bow below the semi-major axis at $t = 0$ with respect to the rocket frame (vide Fig. 4 of reference [1] for a schematic diagram of the positions of the particles in the radial chain at $t = 0$).A particle in the chain which is farther away from the centre will lie farther below the x-axis. This progressive increase of initial phases of the particles is clearly the consequence of the phase factor in equations (4.2.11) and (4.2.12). As time passes, the bow-like chain rotates, straightens itself, again bends down and straightens up and continues like this. (For the stills of the computer movie of the chain of particles in the table and the rocket frames see Fig. 5 of reference [1].) This corresponds to another type of non-rigid behaviour of the material of the top as distinct from the type I. We call it type II. Note that this type II non-rigidity is clearly the result of the relativity of simultaneity.

The authors of reference [1] observe that the spinning top is "more like a visco-elastic fluid in a weird centrifuge subjected to anisotropic external stresses." We reproduce here the "smoothed tracings of the printout of the computer movie" (Fig.5 of Basu et al.)

However, although it is correct that the concept of rigid body dynamics and moments of inertia appears to be no longer valid in the spinning coordinate system, the type II non-rigidity at least has nothing to do with the paradox. In fact this apparent fluid-like motion of the top particles has no connection with the question of the inconsistency of the notion of rigidity in relativity. In the next section we will show, among other things, that the fluid-like motion of the particles of the top can also be seen in the non-relativistic world although it is well known that the notion of rigid body is quite consistent in classical mechanics.

4.3 Conventionality of Simultaneity and Transformation Equations

4.3.1 Relativistic World

In special relativity spatially separated clocks in a given inertial frame are synchronized by light signals. This synchronization is possible provided one

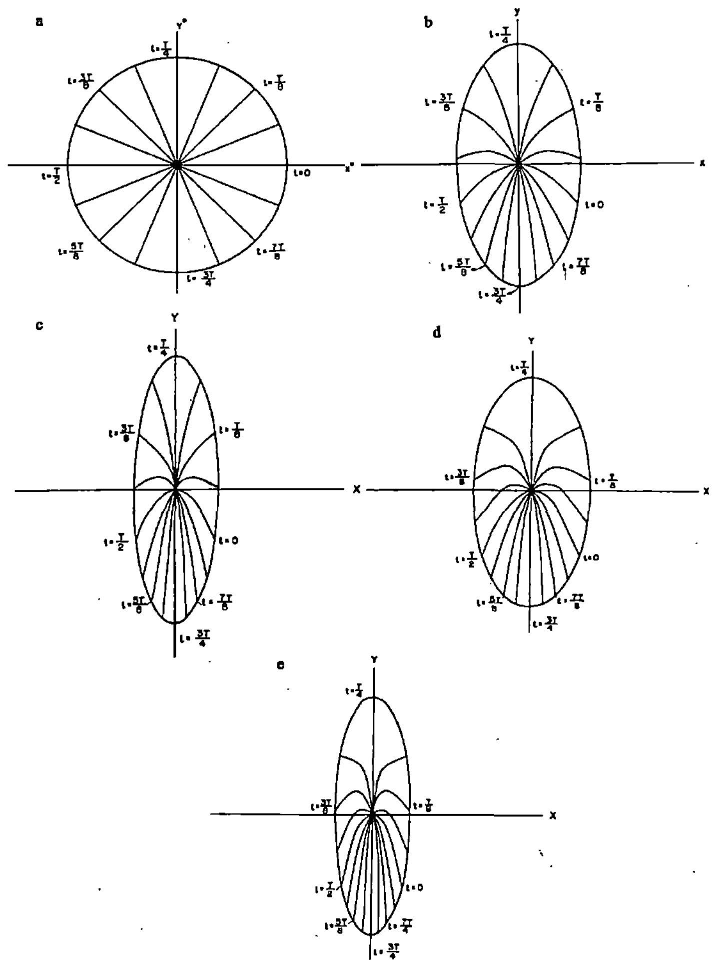


Figure 4.2: Chain of particles (a) In the table frame ($\beta = 0$). (b) In the rocket frame with $\beta = 0.866$ and $\omega = 1.2 \times 10^9 Hz$. (c) In the rocket frame with $\beta = 0.950$ and $\omega = 1.2 \times 10^9 Hz$. (d) In the rocket frame with $\beta = 0.866$ and $\omega = 2.4 \times 10^9 Hz$. (e) In the rocket frame with $\beta = 0.950$ and $\omega = 2.4 \times 10^9$. (From Basu et al.)

knows beforehand the One-Way Speed (OWS) of the signal. But the measurement of OWS requires pre-synchronized clocks and therefore one ends up in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the synchronizing signal within certain bounds. Einstein assumed the OWS of light to be equal to c which is the same as the Two-Way Speed (TWS) of light. The TWS of a signal is an empirically verifiable quantity, as this can be measured by a single clock without requiring any distant clock synchrony. Note that this stipulation (the equality of OWS and TWS) of Einstein has nothing to do with his "constancy of velocity of light" postulate [6, 7]. The assertion that the procedure for distant clock synchrony in SR has an element of convention is known as the CS-thesis, first discussed by Reichenbach [8] and Grünbaum [9]. The synchronization convention adopted by Einstein is commonly known as the Einstein synchrony or the standard synchrony. The possibility of using a synchronization convention other than that adopted by Einstein and consequent transformation equations between inertial frames are much discussed by various authors [4, 6, 7, 10–13]. For example it is known that the relativistic world can well be described by the so called Tangherlini Transformations (TT) by adopting absolute synchrony [4, 6, 7, 12, 14, 15]:

$$x = \gamma\left(x^0 + \beta c t^0\right), \qquad y = y^0, \qquad t = \gamma^{-1} t^0. \tag{4.3.1}$$

Notice that the absence of spatial coordinate in the time transformation above, means the distant simultaneity is absolute (since $\Delta t^0 = 0 \implies \Delta t = 0$). Since, according to the CS-thesis, the question of simultaneity of any two spatially separated events depends on the synchronization convention, the issue of relativity of simultaneity which is often considered as one of the most fundamental imports of SR, has little significance [6]. We will get back to this issue and the transformations (4.3.1) in particular in section 5 in the context of the paradox.

4.3.2 Galilean World

A less well-known fact, however, is that the CS-thesis can also be imported in the classical (Galilean) world. Consider as a fiction that we live in the Galilean world and suppose light travels through ether, stationary with respect to Σ^0. The space-time coordinates of an arbitrary inertial frame Σ moving with speed $v = -\beta c$ with respect to Σ^0 are related to those in Σ^0 by the so called Galilean Transformations (GT)

$$x^0 = x - \beta ct, \qquad y^0 = y, \qquad t^0 = t. \tag{4.3.2}$$

In the Galilean world synchronization issue usually does not come in, since in principle, all the spatially separated clocks can be synchronized instantaneously by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However, ingredients of Einstein synchrony can be incorporated even in this world. Say in a somewhat playful spirit one chooses to synchronize an arbitrarily located clock in any frame Σ with one placed at its origin by sending a light signal from the origin to the clock in such a way that the OWS of light *along any line* passing through the origin is independent of the direction of propagation and is equal to the TWS of the signal along the line. In this case one obtains the so called Zahar Transformation (ZT) [7, 12, 16]:

$$x = x^0 + \beta ct^0, \qquad y = y^0, \qquad t = \gamma^2 \left(t^0 + \beta c^{-1} x^0\right), \tag{4.3.3}$$

and its inverse

$$x^0 = \gamma^2 \left(x - \gamma^{-2} \beta ct\right), \qquad y^0 = y, \qquad t^0 = t - \gamma^2 \beta c^{-1} x. \tag{4.3.4}$$

One may verify from the transformations (4.3.3) and (4.3.4) that the TWS of light along any direction measured in an arbitrary reference frame is given by the same expression as one would have obtained using GT. For example one may verify that the TWS of light along the x-axis and y-axis in Σ are given by the Galilean results, $c\left(1-\beta^2\right)$ and $c\left(1-\beta^2\right)^{\frac{1}{2}}$ respectively [6, 14].[2] Clearly the presence of the spatial coordinate in the time transformations of (4.3.3) and (4.3.4) is the result of the adopted synchrony. The properties of rods and clocks also do not change due to their motions with respect to Σ^0. However, there is an apparent length contraction and time dilation effect with respect to Σ because of different simultaneity criterion used in this frame.

It may also be noted that even in the Galilean world the transformation equations (4.3.3) and (4.3.4) depend on the speed of light c in ether, since light has been chosen as the synchronizing agent. If instead of light the clocks are synchronized by any other signal with speed c' in Σ^0 the transformation equations would have been

$$x = x^0 + \beta' c' t^0, \qquad y = y^0, \qquad t = \gamma'^2\left(t^0 + \beta' c'^{-1} x^0\right) \tag{4.3.5}$$

where β' and γ' are the same as β and γ except where c is replaced by c'.[3] The synchronization in this case may be called *pseudo-standard synchrony* since the synchronization agent here is not the light signal.

[2]The Galilean world or classical world is thus defined to be a world where the TWS of any signal obeys the transformation law that one would have obtained by using Galilean velocity addition rule. On the other hand the world is said to be relativistic if the space-time admits an invariant TWS [7]. It may be noted, by virtue of the CS thesis, that kinematically different transformations and OWS' may correspond to a same kinematical "world" [14].

[3]Operationally one may consider that Σ^0 is a frame of reference stationary with respect to some fluid which supports an acoustic mode with isotropic speed c'. Clocks in any frame are assumed to be synchronized following Einstein's convention using the signal. The equation (4.3.5) were called Dolphin Transformations (DT) in the Galilean world in reference [7] where it was first derived.

4.4 Tippe top Paradox in the Galilean World

The paradox can now be posed even in the Galilean world. With respect to the observer in the rocket frame Σ, the geometry of the top will appear to have altered because of the length contraction effect which is the outcome of the distant clock synchrony adopted in the rocket frame. This may give rise to the possibility that the ratios of the principal moments of inertia go out of the regime required for the top to tip over! We now proceed to "resolve" the paradox by following the line of arguments used in reference [1]. The matrix of ZT (equations (4.3.3) and (4.3.4)) representing the coordinate transformations for x^0 and t^0 is

$$\mathbf{T} = \begin{pmatrix} \gamma^2 & -\beta c \\ -\gamma^2\beta c^{-1} & 1 \end{pmatrix}. \tag{4.4.1}$$

The equations of motion and the equation of the trajectory of the particle in the spinning coordinates can be obtained by inserting the elements of $\mathbf{T}$ *i.e* a, b, g and h in equations (4.2.6), (4.2.7) and (4.2.8) as

$$x_s = \gamma^{-2} R \cos\omega(\gamma^{-2}t - \gamma^2\beta c^{-1} x_s), \tag{4.4.2}$$

$$y_s = R \sin\omega(\gamma^{-2}t - \gamma^2\beta c^{-1} x_s) \tag{4.4.3}$$

and

$$\gamma^4 x_s^2 + y_s^2 = R^2. \tag{4.4.4}$$

As before, the time dilation, the length contraction and the lack of synchronization of clocks seem to be present in these equations. Only quantitatively these effects differ from those obtained earlier using the LT.

These equations suggest that qualitatively the top material displays the same form of non-rigidity (of both type I and II) as has been observed in the relativistic world as the particles of the top moves in accordance with equations (4.4.2) and (4.4.3) with respect to the observer in the rocket frame. Indeed one only needs to slightly modify the programming codes developed in reference [1] to simulate the motion of the particles of the top and see for oneself the picturesque output on the computer screen displaying as before, the non-rigid character of the body.

The idea of Einstein synchrony in the Galilean world may at a first sight seem to be a bit weird. However, the idea is not as strange as it appears. Consider LT in the non-relativistic regime where $\beta^2 \ll 1$ so that the approximation $\gamma \cong 1$ holds. In this approximation, contrary to common belief, LT does not go over to GT, instead one obtains the so-called Approximate Lorentz Transformation (ALT) [14, 17, 18].

$$x^0 = x - \beta ct, \qquad t^0 = t - \beta c^{-1} x. \tag{4.4.5}$$

As is expected the transformations (4.4.5) do not exhibit length contraction and time dilation. However, by virtue of the presence of the spatial coordinate in the time transformation the simultaneity is not absolute. It can also be verified [14] that ALT represents Einstein Synchrony. It is therefore not surprizing that ZT also reduces to equation (4.4.5) under the same approximation. ALT or Approximate Zahar Transformation (AZT) therefore represents the Galilean world with Einstein Synchrony. Inserting the coefficients of ALT/AZT in equations (4.2.6), (4.2.7) and (4.2.8) one obtains the equations of motion and the trajectory of the spinning particle as

$$x_s = R\cos\omega\left(t - \beta c^{-1} x_s\right), \tag{4.4.6}$$

$$y_s = R\sin\omega\left(t - \beta c^{-1} x_s\right) \tag{4.4.7}$$

and

$$x_s^2 + y_s^2 = R^2. \tag{4.4.8}$$

Clearly according to these equations the material of the top still displays non-rigidity of type II.[4]

In line of reference [1], one may now try to say that the concept of rigid body does not fit in the classical world. One may even be tempted to explain away the paradox by saying that nothing is rigid and all bodies are compressible and failure to comprehend this, leads to the paradox! Clearly this is absurd in the Galilean world or in the non-relativistic regime.

In order to understand the situation more clearly, let us examine the meaning of the spinning co-ordinate system. From the transformation equations (4.2.2) and (4.2.5) one may connect the vectors $\mathbf{x}^0 = \begin{pmatrix} x^0 \\ t^0 \end{pmatrix}$ and $\mathbf{x}_s = \begin{pmatrix} x_s \\ t_s \end{pmatrix}$ as

$$\mathbf{x}^0 = \mathbf{B}\mathbf{x}_s \tag{4.4.9}$$

with

$$\mathbf{B} = \begin{pmatrix} a & 0 \\ g & h - \dfrac{gb}{a} \end{pmatrix} \tag{4.4.10}$$

[4]The equations (4.4.6), (4.4.7) and (4.4.8) could have been obtained using the approximation $\gamma \cong 1$ directly in equations (4.2.11), (4.2.12) and (4.2.13) or alternatively in (4.4.2), (4.4.3) and (4.4.4), however, the present approach is more instructive since it clearly shows the role of the lack of synchrony of clocks in the type II non-rigidity of the top.

where, as before, the y-coordinate has been suppressed although we keep in mind that $y_s = y = y^0$. For Galilean transformation (vide equation (4.3.2))

$$\mathbf{T} = \begin{pmatrix} 1 & -\beta c \\ 0 & 1 \end{pmatrix}, \tag{4.4.11}$$

$\mathbf{B}$ turns out to be the identity matrix.

In this case the equations of motion and trajectory of any particle represented by equation (4.2.1) will not lead to a fluid-like motion of the particles in the spinning coordinate system. However for $\mathbf{B} = \mathbf{1}$, these coordinates are the same as that used in Σ^0!

This only demonstrates that the rigidly rotating top will continue to display its rigid character only in the coordinate system defined in the frame of reference at rest with the table. Therefore, it is obvious that the concept of rigid body dynamics or moments of inertia are applicable only in this unique frame and this knowledge therefore surely resolves the paradox. However, this fact does not depend on whether the world is classical or relativistic. Therefore, in accordance with our earlier assertion, there is no connection of the apparent fluid-like behaviour of the top material in the spinning coordinate system with the issue of incompatibility of the notion of rigidity in SR.

4.5 Lack of Synchronization – Is It Crucial?

Basu *et al.* [1] claimed that the lack of synchronization of clocks i.e the relativity of simultaneity aspect of SR plays an essential role in the resolution of the paradox. But we have already observed in section 3, that the question of relativity of simultaneity is purely conventional and therefore is devoid of any empirical content. Kinematically the relativistic world can be described by TT (4.3.1) representing absolute simultaneity. The transformation matrix representing the inverse of TT in the $x - t$ plane may be written as

$$\mathbf{T} = \begin{pmatrix} \gamma^{-1} & -\gamma\beta c \\ 0 & \gamma \end{pmatrix}. \tag{4.5.1}$$

Inserting the elements of $\mathbf{T}$ in (4.2.6), (4.2.7) and (4.2.8) we get

$$x_s = \gamma R \cos \gamma\omega t, \tag{4.5.2}$$

$$y_s = R \sin \gamma\omega t \tag{4.5.3}$$

and

$$\gamma^{-2} x_s^2 + y_s^2 = R^2. \tag{4.5.4}$$

Notice the absence of the phase terms in the sinusoidal functions. This means that the top does not display type II non-rigidity with respect to the rocket frame. However, the trajectory (4.5.4) of P is still an ellipse (this time the semi-major axis is along the x-direction) manifesting type I non-rigidity of the top. This only reiterates that the rigid rotation has to be defined in the table frame, but for this conclusion to hold the lack of synchronization aspect of SR does not play any role.

4.6 Rigidity and Transcendental Equation

It has been noted in [1] that the transcendental equation (4.2.6) is of the form

$$x_s = f(x_s), \tag{4.6.1}$$

which can be solved by iteration provided

$$|f'(x_s)| < 1. \tag{4.6.2}$$

It is claimed that the condition (4.6.2) when applied to equation (4.3.5) leads to

$$\omega R < c, \tag{4.6.3}$$

which only says that no particle of the top can exceed the speed of light. This result although fascinating, seems to be fortuitous, since, instead of equation (4.2.6) if equation (4.4.2) (which pertains to the Galilean world) is used in the inequality (4.6.2), one obtains the same constraint condition (4.6.3) on the speed of a particle of the top. This is surprizing, since in the Galilean world, there is no such speed limit intrinsically.

For the DT in the Galilean world (vide equation (4.3.5)) the condition (4.6.2) leads to

$$\omega R < c', \tag{4.6.4}$$

which is more surprizing.

On the other hand we have seen that TT which represents the absolute synchrony in the relativistic world does not lead to any transcendental equation (vide equation (4.5.2)) and hence no constraint on the speed of the particle is visible.

4.7 Summary

The present paper discusses the tippe top paradox and different aspects of its resolution proposed in reference [1]. Clock synchronization issues in the relativistic and the Galilean world figured in course of our discussion. A few transformation equations in addition to the LT were discussed in this connection. It is therefore worthwhile to summarize different properties of the transformations in the context of the paradox. This we do in Table 1 so that one is able to get the whole picture at a glance. The table is self-explanatory, however explanations of a few entries may be in order.

Basically we discussed two worlds – relativistic and classical, but an overlapping world "Relativistic/Classical" is included in column 1 as a separate entry. This corresponds to the transformations ALT and AZT (vide column 2) which are the forms of LT and ZT respectively under the approximation $\gamma \simeq 1$. For both the transformations the synchrony type (as shown in column 3) is standard. These transformations do not predict the length contraction and the time dilation effects (vide entries in column 6 and 7). The paradox in this case does not exist prima facie (vide the entry in the last column) in this regime since there is no length contraction effect. However the observer in Σ will find the top material to exhibit non-rigidity of type II.

Note that the entries in the 1st, 5th and 6th rows from column 3 onwards corresponding to the transformations LT, ZT and DT respectively are exactly the same. It means that the paradox and the resolution as suggested in reference [1] completely fits in the classical world too. It therefore dismisses the claim that the paradox has its origin in the incompatibility of the notion of rigidity with SR.

The entries against TT shows that in the relativistic world non-rigidity of Type I of the top exists although the synchrony here is absolute. It therefore follows that "lack of synchronization of clocks" cannot play an essential role in resolving the paradox.

Table 1

World	Transformation	Synchrony Type	Type I Non-rigidity	Type II Non-rigidity	Length Contraction	Time Dilation Effect	Paradox exists? (prima facie)
Relativistic	LT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
Relativistic	TT	Absolute	Yes	No	Yes (w.r.t. Σ^0)	Yes (w.r.t Σ^0)	Yes
Relativistic /classical	ALT/AZT	Standard (Einstein)	No	Yes	No	No	No
Galilean (Classical)	GT	Absolute	No	No	No	No	No
	ZT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
	DT	Pseudo Standard	Yes	Yes	Yes	Yes	Yes

Reference

[1] Aniket Basu, R. S. Saraswat, Kedar B. Khare, G. P. Sastry and Sougato Bose, *Eur. J. Phys.*, **23**, (2002) pp. 295–305.

[2] Aurthur Evett, *Understanding the Spacetime Concepts of Special Relativity* (Publishers Creative Services, New York, 1982).

[3] Talal A. Debs and Michael L. G. Redhead, *Am. J. Physics*, **64**(4), (1996) pp. 384–391.

[4] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[5] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhury and Minakshi Sarker, *Found. Phys. Lett.*, **16**(6), (2003) pp. 549–563.

[6] S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.*, **15**(4), (1991) pp. 369–374.

[7] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994) pp. 21–28.

[8] H. Reichenbach, *The Philosophy of Space and Time* (Dover, NewYork, 1957), 2nd edition.

[9] Adolf Grünbaum, *Philosophical Problems of Space and Time* (Alfred Knopf, New York, 1973), 2 edition.

[10] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 497–513.

[11] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 515–524.

[12] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979) pp. 229–245.

[13] A. Winnie, *Phil. Sc.*, **37**, (1970) pp. 81–99.

[14] S.K. Ghosal, K. K. Nandi and P. Chakraborty, *Z. Naturforsch*, **46a**, (1991) pp. 256–258.

[15] F. Tangherlini, *Nuovo Cimento Suppl.*, **20**, (1961) pp. 1–86.

[16] E. Zahar, *British Journal of Philosophy of Science*, **28**, (1977) pp. 195–213.

[17] P. Di Mauro, "La formula di diffusione compton con al meccanica newtoniana", Atti del XVI Congresso Nazionale di Storia della Fisica e dell' Astronomia, Como 28-29 , 179-184 (1999).

[18] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Butterworth Heinemann, Oxford, 2002), 4th revised english edition edition.

Chapter 5

On the Anisotropy of the Speed of Light on a Rotating Platform

5.1 Introduction

The term "inertial frame of reference" in physics refers to an idealised concept. Our knowledge of physics in inertial frames has always been obtained in frames having small but non-zero acceleration. Indeed it is well-known that no perfectly inertial frame can be identified in practice. It is therefore expected that physics in non-inertial frames will go over smoothly to that in inertial frames in the mathematical limit of zero acceleration. In some recent papers [1, 2] Selleri observes that the existing relativity theory fails our expectations on that count. In this connection Selleri poses a paradox concerning the speed of light as measured by an observer on board a rotating turn-table. If two light beams from a common source are sent along the rim of a rotating disc in opposite directions and the round-trip speeds (c_+ for counter-rotating and c_- for co-rotating beams) for these two light beams are measured, it will be found from simple kinematics, that the ratio of these speeds $\rho = c_+/c_-$ is only a function of the linear speed v of disc at its edge and it differs from unity if $v \neq 0$. This observation finds its support in the well known Sagnac effect [3, 4] which is manifested in the experimentally observed asymmetry in the round-trip times of light signals co-rotating and counter-rotating with the interferometer.However, since the rotating turn-table is not an inertial frame, one might initially not be inclined to consider the observed anisotropy of light propagation with respect to this frame a startling result as such. But Selleri then considers a situation where one gradually increases the radius of the disc and at the same time allows the angular velocity ω of the same to get decreased proportionately in such a way that the linear speed $v = \omega R$ of the periphery remains constant. The edge of the disc can then be thought of approaching (locally) an inertial frame since, in the limit the centripetal acceleration $a = v^2/R$, tends to zero. [1] In an inertial frame ρ must strictly be unity since, light propagation is considered to be isotropic in such a frame according to the special relativity (SR). However Selleri shows that the ratio ρ on board the rotating disc does not change in the limit process provided v remains constant and therefore will continue to differ from unity as long as v is finite. A discontinuity in the behaviour of ρ as a function of acceleration is thus predicted. This is certainly paradoxical in the light of the observations made in the beginning of this section. We hereafter refer to it as the Selleri paradox (SP). SP has so far met with evolving but inadequate responses. For example in one paper Rizzi and Tartaglia [6] observe that the "calculations of Selleri are quite careful" and the "paradox cannot be avoided if it is maintained that the round-trip on the turn-table corresponds to a well defined circumference whose length is univocally defined". It appears that the authors of Ref. [6] cannot accept the global anisotropy of light speed in the frame of reference of the rotating disc and hold that because of the "impossibility of a symmetrical and transitive synchronization at large", the notion of whole physical space on the platform at a given instant is conventional. Their final conclusion is that the counter-rotating and co-rotating light beams travel different distances with respect to the *frame of the disc* in such a way that the *global* ratio ρ remains unity. The view point towards the resolution of SP also finds its endorsement in a later paper by Tartaglia [7]; although in a subsequent

[1] There is a scope for confusion here. Although an element of the disc will have zero acceleration in the limit considered, an observer on the turn table would be able to detect its rotation since the latter is an absolute concept. In an article Klauber [5] even claimed that there would be, for example a change of mass of a particle on the disc due to a general relativistic effect which can be seen to depend only on the circumferential velocity and not on the acceleration. This effect would even enable one to determine in principle the rotational motion of the platform from local measurements alone!

paper Rizzi and Tartaglia [8] somewhat retract from the past position and allow an observer at rest on the disc to consider a notion of its unique circumference in the "relative space of the disc" and hence endorse the view that light propagation can be anisotropic in the reference frame of the rotating turn-table. In conjunction with Budden's observation [9] the authors then correctly identify the root of the paradox and hold that the basic weak point of Selleri's arguments lies in equating the global ratio ρ of the speeds of light propagating in opposite directions along the rim with the local ratio ρ' of the same at any point on the edge of the disc. The latter ratio is always equal to unity if Einstein synchrony is used in any local inertial frame instantaneously comoving with the element of the rim at the point concerned (such frames will hereafter be referred to as momentarily comoving inertial frames (MCIF))and therefore SP does not pose any harm to SR.

However, Selleri's argument regarding the equality of two ratios ρ and ρ' is based on a symmetry argument (rotational invariance) but the authors of Ref. [8] do not clearly state what is precisely wrong with Selleri's symmetry reasoning. Further the arguments by the authors although correct, are blurred by their ambivalent observations (in the same paper) that the global ratio ρ itself comes out to be unity (a) if the time measuring clock is suitably corrected to "account for the desynchronization effect" or (b)if the space is suitably defined according to "geometry of Minkowskian spacetime". Note that (b) is the reiteration of their earlier stand in this regard [6, 7].

In Refs. [1, 2] Selleri raises another matter in connection with SP. In light of conventionality of (distant) simultaneity (CS) thesis of SR, the author discusses the conventionality issue on a rotating turn-table and argues that not the Lorentz transformation (LT) but the relativistic transformation with absolute synchrony (which is one of the many possible synchronization conventions for which light propagation is anisotropic) only correspond to the correct expression for ρ (see Eq. (5.2.16) later). In a recent paper Minguzzi [10], whose view we share, briefly addresses the issue. The author agrees that isotropic convention (standard synchrony) can be unsuitable in certain situations but maintains that the possibility of anisotropic conventions does not imply any inconsistency of SR. However SP has not been discussed therein in its entirety.

To sum up it may be said that the responses to SP so far available in the literature are not fully satisfactory. We therefore hold that the paradox which poses a challenge to the very foundations of SR by questioning its self consistency, deserves to be given a fuller treatment. Indeed there are many subtle issues concerning SP. For example it will be seen in Sec. 5.3 that the paradox not only undermines the standard relativity theory but also denies the basic tenet of the CS thesis. The purpose of the present paper is to re-examine Selleri's arguments in the light of the CS thesis and provide a resolution of SP in a novel way by recasting the paradox in the classical world (see Sec. 5.4). It will however be argued that while both the self-consistency of SR and the CS thesis remain unchallenged, SP has a merit in that if properly interpreted in the light of reasonings presented in this paper, the whole issue will throw new light on various related issues like the question of time on rotating platform, desynchronization and its debated role in the explanation of Sagnac effect [11, 12].

We organize the paper as follows. Before we present our main arguments in Sec. 5.4 and onwards, the CS thesis will be discussed (in Sec. 5.3) in the context of the paradox. However in order to set the stage we will briefly reproduce in Sec. 5.2 the arguments of Selleri leading to SP. In Sec. 5.5 the issue of desynchronization vis-a-vis the Sagnac effect will be addressed and finally in Sec. 5.6 the standard synchrony and absolute synchrony will be compared upholding Selleri's point of view in this regard [13, 14].

5.2 The Paradox

Suppose a light source is placed at some fixed position Σ on the rim of the turn-table and two light signals start from Σ at the laboratory time t_{01}, and are constrained (by allowing them for example, to graze a suitably placed cylindrical mirror on the rim) to travel in opposite directions in a circular path along the periphery of the disc. Let that, after making the round trips, the counter-rotating and co-rotating light flashes reach Σ at times t_{02} and t_{03} respectively.

As seen from the laboratory, the counter-rotating light signal travels a distance shorter than the circumference L_0 by the amount

$$x = v(t_{02} - t_{01}), \tag{5.2.1}$$

where $v = \omega R$ is the linear speed of the disc at its periphery. Similarly the co-rotating light beam has to travel a distance larger than L_0 by the amount

$$y = v(t_{03} - t_{01}). \tag{5.2.2}$$

From simple kinematics it therefore follows that

$$L_0 - x = c(t_{02} - t_{01}) \tag{5.2.3}$$

$$L_0 + y = c(t_{03} - t_{01}), \tag{5.2.4}$$

where L_0 is the disc's circumference as seen from the laboratory. From equations (5.2.3) and (5.2.4) and using equations (5.2.1) and (5.2.2) one readily obtains the round-trip times for counter-rotating and co-rotating signals respectively as

$$t_{02} - t_{01} = \frac{L_0}{c(1+\beta)} \tag{5.2.5}$$

and

$$t_{03} - t_{01} = \frac{L_0}{c(1-\beta)}. \tag{5.2.6}$$

By taking the difference of these times, one may note here that the delay between the arrival of the two light signals at the point Σ is obtained as

$$\Delta t_s = t_{03} - t_{02} = \frac{2L_0\beta}{c}\gamma^2, \tag{5.2.7}$$

where $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$. As an aside remark, it may be noted that (5.2.7) is nothing but the well-known delay time of classical Sagnac Effect. The relativistic formula for Sagnac delay can easily be obtained by noting that Δt_s in Eq. (5.2.7) is not the time as measured on board the platform and hence time dilation effect has to be considered. By multiplying both sides by $\gamma^{-1} = (1-\beta^2)^{-\frac{1}{2}}$ one obtains the relativistic formula for Sagnac delay as

$$\Delta\tau_s = \frac{2L_0\beta}{c}\gamma, \tag{5.2.8}$$

where $\Delta\tau_s = \gamma^{-1}\Delta t_s$ denotes the delay time as measured on board the turntable.[2].

Suppose now that a clock C_Σ is placed on the disc's rim at Σ so that it co-rotates with the platform and also let t denotes the time of C_Σ. When the disc is in motion, according to Selleri, the laboratory time t_0 and t may be assumed to be related, most generally as

$$t_0 = tF_1(v, a). \tag{5.2.9}$$

Similarly for the circumference also Selleri assumes a relation between L_0 and the proper circumference L as

$$L_0 = LF_2(v, a), \tag{5.2.10}$$

[2]The controversial issue of the appearance of the relativistic γ-factor in the Sagnac formula has been discussed in detail vis-a-vis the Ehrenfest paradox in Chap. 3. However Eq. (5.2.8) is the most widely quoted one. See also Ref. [15, 16]

where F_1 and F_2 are some functions of the linear velocity $v = \omega R$ and the acceleration $a = v^2/R$ of the edge of the disc.

Although, from the widely accepted hypothesis of locality [6, 11] it is evident that these functions are nothing but the usual time dilation and length contraction factors

$$F_1 = F_2^{-1} = \gamma, \tag{5.2.11}$$

however, Selleri keeps open the possibility that F_1 and F_2 may depend on the acceleration as well.

Inserting equations (5.2.9) and (5.2.10) in equation (5.2.5) and (5.2.6) one gets the following times of flight of the counter-rotating and co-rotating light signals as measured on board the disc,

$$t_2 - t_1 = \frac{L}{c(1+\beta)} \frac{F_2}{F_1}, \tag{5.2.12}$$

$$t_3 - t_1 = \frac{L}{c(1-\beta)} \frac{F_2}{F_1}. \tag{5.2.13}$$

The round-trip speeds for these beams are therefore given by

$$c_+ = \frac{L}{t_2 - t_1} = c(1+\beta)\frac{F_1}{F_2}, \tag{5.2.14}$$

$$c_- = \frac{L}{t_3 - t_1} = c(1-\beta)\frac{F_1}{F_2}. \tag{5.2.15}$$

Consequently the ratio of these light speeds turn out to be

$$\rho = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{5.2.16}$$

Selleri now argues that since no point on the rim is preferred, the instantaneous velocities of either signals at any point of the rim must be the same, and therefore, the above ratio ρ is true not only for the global light velocities but also for the instantaneous velocities at any point on the rim. Now, as pointed out in section 1, if we consider that $R \to \infty$ and $\omega \to 0$ in such a way that v, the linear speed of any element of the circumference remains constant, so that the centripetal acceleration $a \to 0$, any short part of the circumference can be thought of as an inertial frame of reference in the limit. However, the ratio ρ does not change as long as v is kept constant. Hence, a discontinuity results in the behaviour of ρ as a function of acceleration ($\rho = \rho(a)$), since as $a \to 0$, but not equal to zero, ρ continues to differ from unity, but if $a = 0$, SR predicts that ρ must be equal to unity! In Fig. 5.1, the ratio ρ is plotted as a function of acceleration for rotating platforms of constant peripheral velocity and decreasing radius. The black dot ($\rho = 1$) represents the prediction of the SR and this is discontinuous with the values of ρ of the rotating platforms [17].

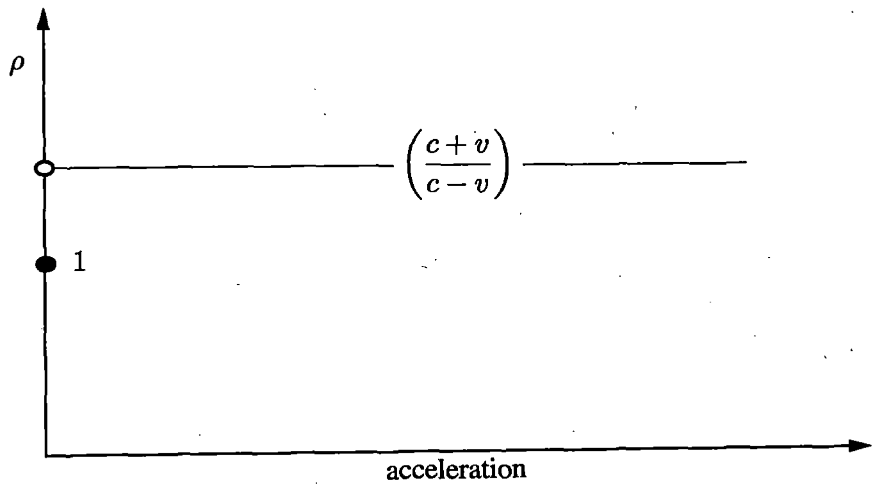


Figure 5.1: The ratio ρ versus acceleration of rotating platform

It may be argued that the above gedanken experiment with infinitely sized disc is impossible to perform since the times of flight of the co-rotating and counter-rotating light beams whose ratio we are currently interested in, would become infinite and therefore unmeasurable [18]. However it is enough to note that if the radius of the disc is increased arbitrarily from a finite value and at the same time v is kept constant by suitably adjusting the angular velocity no tendency for the ratio ρ getting diminished would be seen although the acceleration of a point on the circumference gets reduced arbitrarily in the process.

It is worthwhile to mention in this context that recently Wang et-al [19] has obtained a travel time difference $\Delta t = \frac{2vL}{c^2}$ between two counter-propagating light beams (indicating $\rho \neq 1$) in a uniformly moving fibre where v is the speed of the light source or the detector (comoving with the fibre) with respect to the laboratory and L in the length of the fibre (see Sec. 2.8.2). The experiment has been performed using a fibre optic conveyer(FOC) where two light beams leaving a source travels in opposite directions through an optical fibre loop which is made to move with uniform speed like a conveyer belt by a couple of rotating wheels separated by a distance. The interesting feature of the FOC arrangement is that here the observer (*i.e.* the source or the detector) is attached to one of the straight-fibre segments and therefore moves with *uniform velocity* along a straight line. Experimental observation together with a symmetry argument (similar to that used by Selleri in the rotating disc context) may lead one to infer that the statement $\rho \neq 1$ is also valid *locally* in a segment of uniformly moving fibre indicating *local* anisotropy in the speed of light in vacuum[3] with respect to an inertial observer! Such an outcome which apparently follows from a symmetry argument is also paradoxical if one believes in SR. Although the purported scope of the present paper restricts us to deliberating on SP in its original form and consequent issues following a few responses it has received, the arguments that will be used in the following sections will equally apply to the paradox in the FOC context as well.

[3]As suggested by Wang et-al, here we have assumed that experiment using FOC with a hollow core would give the same result. Indeed the result $\rho = \frac{1+\beta}{1-\beta}$ remains valid in this case since the simple minded analysis presented in this section leading to the equation also applies to this situation

Indeed all Selleri wanted to achieve was to obtain an inertial observer with respect to whom $\rho = \dfrac{1+\beta}{1-\beta}$. In the FOC arrangement this comes naturally dispensing with the trick of letting the radius of the disc go to infinity and the angular speed to zero while the peripheral velocity is kept constant.

Before we leave this section, we write explicitly the expressions for c_+ and c_- in the full relativistic context:

$$c_+ = \frac{c}{1-\beta}, \tag{5.2.17}$$

$$c_- = \frac{c}{1+\beta}, \tag{5.2.18}$$

which follow from Eqs. (5.2.14) and (5.2.15) where the expressions for F_1 and F_2 as given in Eq. (5.2.11) have been substituted. Here we may point out that the genesis of SP relates to these equations since in the limit of infinite radius and zero angular velocity, the above results do not change indicating (as if) the violation of second relativity postulate (isotropy and constancy of light speed). As we have mentioned earlier, the above results although correct, are so counter-intuitive that the authors of Refs. [6, 7] in their initial reactions discarded the results altogether only to retract from their position later in a sort of a rejoinder [8].

5.3 CS Thesis and Absolute Synchrony

In the relativity theory distant simultaneity is conventional. In order to synchronize spatially separated clocks in a given inertial frame one should know the one-way speed (OWS) of the synchronizing signal, however to know OWS one needs pre-synchronized clocks. One therefore is caught in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the light signal within certain bounds. The CS thesis, first discussed by Reichenbach and Grünbaum [20, 21],[4] is the assertion that the procedure for distant clock synchrony is conventional. Einstein therefore assumes as a convention that the OWS of light is isotropic and is equal to the two-way speed (TWS) c of the signal. Note that the latter is an empirically verifiable quantity since it does not depend on the convention regarding the synchronization of spatially separated clocks since the TWS can be measured by a single clock. The synchrony is commonly known as the Einstein synchrony or the standard synchrony. However since the clock synchronization procedure is conventional, conventions other than the standard one may equally be chosen [23–26]. Selleri [1, 2] has shown that the space-time transformation between a preferred inertial frame S_0 (where clocks are standard-synchronized so that OWS is isotropic in the frame) and any other frame S may generally be written as

$$\begin{aligned} x &= \gamma(x_0 - \beta c t_0) \\ y &= y_0 \\ t &= \gamma^{-1} t_0 + \epsilon(x_0 - \beta c t_0) \end{aligned} \tag{5.3.1}$$

which represents a set of theories *equivalent* to SR. The free parameter ϵ which can at most be a function of the relative velocity of S with respect to S_0, depends on the simultaneity criterion adopted in S. For the standard synchrony however,

$$\epsilon = -\frac{\beta\gamma}{c}. \tag{5.3.2}$$

For this value of ϵ equation (5.3.1) reduce to Lorentz transformation. The OWS' of light in S, c'_+ and c'_- along the negative and positive x-directions respectively may easily be obtained from the transformation (5.3.1) as

$$\frac{1}{c'_+} = \frac{1}{c} - \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right] \tag{5.3.3}$$

[4]For a comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman [22].

and

$$\frac{1}{c'_-} = \frac{1}{c} + \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right] \tag{5.3.4}$$

If S_0 is assumed to be the inertial frame of reference at rest with the axis of the rotating disc and S be an MCIF, c'_+ and c'_- would then mean the local speeds of light counter-rotating and co-rotating with the disc respectively as measured on board the rotating platform. From Eqs. (5.3.3) and (5.3.4) one may thus obtain the ratio for these local speeds of light ρ' in terms of the ϵ-parameter

$$\rho' = \frac{c'_+}{c'_-} = \frac{1+\beta+\epsilon c\gamma^{-1}}{1-\beta-\epsilon c\gamma^{-1}}, \tag{5.3.5}$$

which agrees with the ratio ρ given by Eq. (5.2.16) provided $\epsilon = 0$. But as mentioned in the last section, the equality of ρ and ρ' according to Selleri as if follows from the symmetry of the situation. Therefore, in the rotating disc context, $\epsilon = 0$ appears to be the only allowed convention according to which the speed of light is anisotropic. Note that for this value of ϵ only the local speeds of light as given by Eqs. (5.3.3) and (5.3.4) reduce to the expressions (5.2.17) and (5.2.18). The transformation (5.3.1) with $\epsilon = 0$ is known as the Tangherlini transformation(TT) [27]

$$\begin{aligned} x &= \gamma\left(x_0 - \beta c t_0\right), \\ y &= y_0, \\ t &= \gamma^{-1} t_0. \end{aligned} \tag{5.3.6}$$

The transformation represents the relativistic world with absolute synchrony [24, 25].[5]

We now have a ramification of the original paradox. The value of ρ (and hence ρ') represented by equation (5.2.16), which implies anisotropic propagation of light in the rotating frame, is obtained theoretically from the perspective of the

[5]Notice that in view of the absence of the spatial coordinate x in the above transformation for time, the simultaneity is independent of the frame of reference considered and is therefore absolute.

inertial frame S_0. The result also finds its empirical support in the Sagnac effect. It therefore appears that as if a particular (non-standard) synchrony is dictated both by theory and experiment. This is absurd since, if it were true it not only would reject the Lorentz transformation but also would contradict the basic tenet of the CS thesis that clock synchronization is conventional.

Before offering a resolution of the SP we ask ourselves if such a paradox could exist in the classical (Galilean) world too. The initial reaction would be to answer in the negative since nothing seems to be mysterious or enigmatic in this world and as is well-known, counter intuitive problems by contrast exist in the relativity theory probably because of its new philosophical imports. However we answer the question in the affirmative. One of the so-called new philosophical imports of SR is the notion of relativity of simultaneity. It can be shown that this notion can also be introduced in the Galilean world. Indeed in the next section it will be shown that by doing so the paradox can be artificially created even in this world where normally one would not expect it to exist.[6] The perspective of the paradox will hopefully provide deeper understanding of the problem and other related issues.

5.4 Selleri Paradox in the Galilean World

Let us consider a fiction that we live in the Galilean (classical) world and suppose light travels through ether stationary with respect to an inertial frame S_0. The space-time coordinates of an arbitrary inertial frame S moves with respect to S_0 are related to those in S_0 by the so-called Galilean transformation (GT):

$$x = x_0 - \beta t_0, \qquad y = y_0, \qquad t = t_0. \tag{5.4.1}$$

[6] Such an approach has been found fruitful elsewhere in understanding a recent paradox in relativity [28]. For a detail account see Chap. 4

In the Galilean world synchronization issue usually does not figure in, since in principle all the clocks in any given inertial frame can be synchronized by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However the ingredients of the CS thesis can also be incorporated in this world. For example, one may employ the Einstein synchrony to describe the kinematics in this world. Suppose one sends out a light signal from the origin of S outwards along a line which makes an angle θ with the x-axis and the signal comes back to the origin along the same line after being reflected by a suitably placed mirror, the TWS can be obtained by measuring the time of flight of the round-trip by a clock placed at the origin. The expression for this TWS can be obtained from Eq. (5.4.1) and is given by [25]

$$\overleftrightarrow{c}(\theta) = \frac{c(1-\beta^2)}{(1-\beta\sin^2\theta)^{1/2}}. \tag{5.4.2}$$

Now in a somewhat playful spirit one may choose to synchronize arbitrarily located clocks with one placed at the origin by sending light by *stipulating* the OWS of light to be equal to the TWS (in fact none can prevent one in doing so), the relevant transformation that would honour such a stipulation would be given by

$$\begin{aligned} x &= x_0 - \beta c t_0 \\ t &= \gamma^2\left(t_0 - \tfrac{\beta x_0}{c}\right) \end{aligned} \tag{5.4.3}$$

which was originally obtained by E. Zahar in 1977 [29] and is now commonly known as the Zahar transformation(ZT). For a quick check one may readily verify that the TWS of light along the x-axis and y-axis in S,that follow from Eq. (5.4.3) are given by the well-known classical results, $c(1-\beta^2)$ and $c(1-\beta^2)^{1/2}$, respectively [23, 30].

In the context of the rotating disc, x and t denote the coordinate and time of an event in an MCIF at any point on the edge of the disc, while x_0 and t_0 refer to the same in the inertial frame S_0 which is stationary with the axis of rotation. Let us now write the inverse of ZT (Eq. (5.4.3)) for time in the differential form as

$$dt_0 = dt \pm \frac{\gamma^2 \beta \, dx}{c} \tag{5.4.4}$$

where dx refer to the length of the infinitesimal element of the disc which is covered by the light signal in time dt when the signal is co-rotating ($+$ sign) or counter-rotating ($-$ sign) with the disc. Note that the phase term (space dependent term) in (5.4.4) was absent in the GT. Clearly the term is an artefact of the Einstein synchrony. For the complete revolution for the counter-rotating light signal, the round-trip time in the laboratory is thus obtained by integrating (5.4.4) as

$$\Delta t_{0+} = \oint dt - \oint \frac{\gamma^2 \beta}{c} \, dx$$

or,

$$\Delta t_{0+} = \oint dt - \frac{\gamma^2 \beta L_0}{c}, \tag{5.4.5}$$

and similarly for the co-rotating signal

$$\Delta t_{0-} = \oint dt + \frac{\gamma^2 \beta L_0}{c}. \tag{5.4.6}$$

Notice that $\oint dt$ in Eqs. (5.4.5) and (5.4.6) are the same because of the adopted synchrony which is given by

$$\oint dt = \frac{L_0}{c(1-\beta^2)}, \tag{5.4.7}$$

since (from Eq. (5.4.2)), for $\theta = 0$,

$$\overset{\leftrightarrow}{c}(0) = c(1-\beta^2) \tag{5.4.8}$$

which has been assumed to be the same as the OWS following the synchrony. That the Zahar transformation and hence Eqs. (5.4.5) and (5.4.6) are consistent with the classical world can be checked by calculating $c_\pm$ ($= L_0/\Delta t_{0\pm}$) from Eqs. (5.4.5) and (5.4.6) and by making use of Eq. (5.4.7). They are obtained as

$$c_\pm = c(1 \pm \beta) \tag{5.4.9}$$

which could have been obtained from elementary kinematics using GT. This agreement is expected since the global round-trip speeds are observables independent of the synchrony gauge. Further by taking the difference of Eqs. (5.4.5) and (5.4.6), by virtue of the cancellation of the $\oint dt$ terms one obtains the usual classical expression for the Sagnac delay quoted earlier

$$\Delta t_s = \frac{2L_0\beta}{c}\gamma^2. \tag{5.4.10}$$

From Eq. (5.4.9) it is evident that in the classical world too

$$\rho_{\text{classical}} = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{5.4.11}$$

Clearly we are confronted with the same apparent paradox that the ratio of the round-trip speeds of the two counter-propagating light signals differ from unity ($\rho \neq 1$) although locally the one-way speeds of light in opposite directions have been assumed to be the same ($\rho' = 1$). (This is manifested in the cancellation of $\oint dt$ terms while taking the difference of (5.4.5) and (5.4.6) in arriving at the classical Sagnac effect formula (5.4.10)).The rather tortuous way of deriving the Eqs.(5.4.9), (5.4.10) and (5.4.11) serves two things. It demonstrates how the Sagnac effect can be construed as an effect of "desynchronization" of clocks (due to the contribution of the phase terms(5.4.4)) on the rotating platform even in the classical world. This effect is usually regarded as a 'real' physical phenomenon in the context of the relativistic Sagnac effect [31]. But the present derivation demonstrates that the desynchronization cannot be an objective phenomenon since here we clearly see it as an artifact of standard synchrony which is nothing but a stipulation. The other utility of this scheme of the derivation is that it allows us to understand clearly that the two apparently contradictory results ($\rho = 1$ and $\rho \neq 1$) follow from the same transformation (5.4.3). The contradiction is therefore a logical one. It means that the trouble lies in the arguments (and not in the physical theory — in this case it is the classical kinematics) leading to the paradoxical conclusions.

Further, not only Zahar transformation, the Galilean world can also be represented by the following transformation [25]

$$
\begin{aligned}
x &= x_0 - \beta c t_0, \\
y &= y_0, \\
t &= t_0 + \epsilon \left(x_0 - \beta c t_0 \right),
\end{aligned} \tag{5.4.12}
$$

where, as before, ϵ is a free parameter which depends on the choice of synchrony. GT and ZT are recovered for $\epsilon = 0$ and $\epsilon = -\gamma^2\beta/c$ respectively. Note that these are the classical analogues of Selleri's transformation (5.3.1). The OWS' of light that follow from (5.4.12) are given by

$$\frac{1}{c'_+} = \frac{1}{c(1-\beta)} + \epsilon, \tag{5.4.13}$$

$$\frac{1}{c'_-} = \frac{1}{c(1-\beta)} - \epsilon \tag{5.4.14}$$

and the corresponding ratio of these velocities is given by

$$\rho'_{\text{classical}} = \frac{c'_+}{c'_-} = \frac{1-\beta}{1+\beta} \cdot \frac{1 - c\epsilon(1+\beta)}{1 + c\epsilon(1-\beta)}. \tag{5.4.15}$$

As before, here also we see that $\rho'_{\text{classical}}$ corresponds to $\rho_{\text{classical}}$ (5.4.11) provided $\epsilon = 0$. For ZT $\left(\epsilon = -\gamma^2\beta/c\right)$, $\rho'_{\text{classical}} = 1$ which agrees with the stipulation of standard synchrony used to derive the transformation. But now $\rho'_{\text{classical}} \neq \rho_{\text{classical}}$, although the latter ratio also has been obtained using the same transformation *i.e.* ZT. We thus see that Selleri's arguments, if carried over into the classical world, also lead to the paradox.

As remarked earlier, in order to address the paradox one needs to look into the reasonings leading to it rather than expecting any flaw in the theories (relativistic or classical). One may ask why Selleri expects that ρ should be equal to ρ' (or equivalently why $\rho_{\text{classical}}$ should be equal to $\rho'_{\text{classical}}$)? The primed ratios are measured in the MCIF whereas the unprimed ratios are global, *i.e.* they are based on the measurements of the average speeds of light signals when they make complete round-trips. Selleri's argument goes somewhat like this: Since the stationary inertial reference frame at rest with the centre of the disc is isotropic in every sense, the isotropy of space should ensure that the instantaneous velocities of light are the same at all points on the rim of the disc and therefore the average velocities should coincide with the instantaneous ones.

It is interesting that there is nothing wrong even in Selleri's observation regarding the symmetry of the situation, however the conclusion that the two ratios (ρ and ρ') are equal does not necessarily follow from the symmetry arguments. Below we give an example and explain why and how the local speeds of light may differ from their global values in spite of the symmetry.

Consider the motion of a rigid rod AB of length $L_0/2$ with respect to the inertial frame S_0.[7] Suppose that the rod initially moves with uniform velocity βc towards the right parallel to the x-axis of S_0 (vide Fig. 5.2).

The left end A of the rod is assumed to coincide with the origin of S_0 at the laboratory time $t_0 = 0$, when an observer at A on board the rod who carries a clock C_A sends out a light pulse towards B where another observer sitting on the rod holds a mirror (M) facing A. As soon as the light pulse reaches the observer at B and is reflected back and starts to travel towards A, the rod is also made to

[7]This is a reconstruction of linear Sagnac effect described in Sec. 3.4 to suit the represention of the present problem.

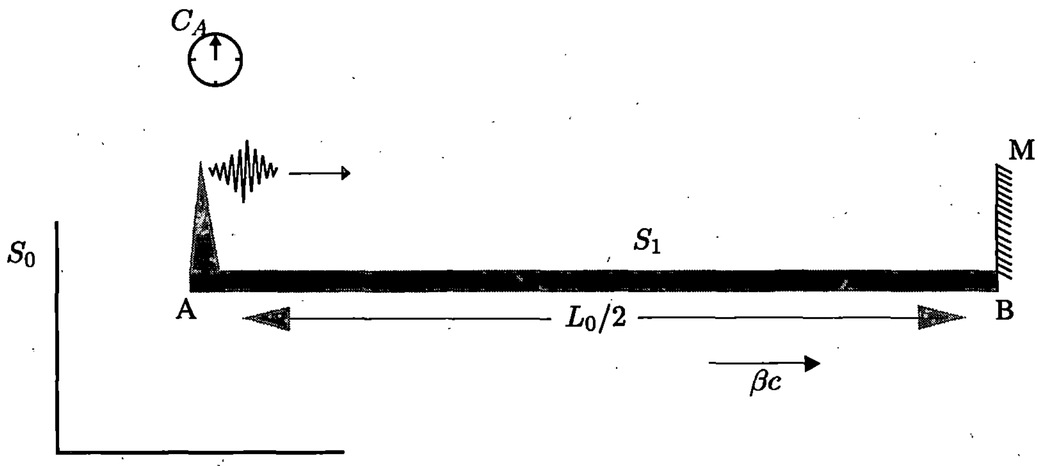


Figure 5.2: Rod moving towards the right

change its direction of motion and travel towards the left with the same uniform speed βc.[8] (vide Fig 5.3)

Suppose now that the observers in the laboratory record the times of the following three events:

Event 1: The light pulse sent out from A at the laboratory time $t_0 = t_{01} = 0$.

Event 2: The light pulse received at B at the laboratory time $t_0 = t_{02}$.

Event 3: The reflected light pulse received at A at the laboratory time $t_0 = t_{03}$.

From simple kinematics one obtains

$$t_{02} = \frac{L_0}{2c(1-\beta)} \quad \text{and} \quad t_{03} = \frac{L_0}{c(1-\beta)}. \tag{5.4.16}$$

If Galilean transformation is used for any event, there is no distinction between the laboratory times and the corresponding times measured by observers on board

[8] The present analysis of this thought experiment, which essentially corresponds to a linear Sagnac effect discussed elsewhere [15, 16, 32] by the present authors can be seen to fit well (with minor adjustment in the reasonings) with the FOC experiment [19] in the limit when the size of the wheels at the two ends tend to zero.

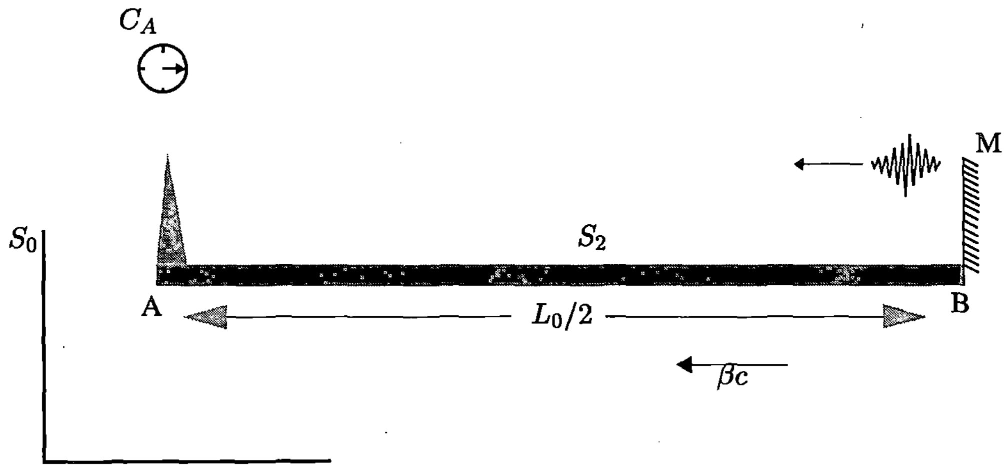


Figure 5.3: Rod moving towards the right

the rod.

However if the observers wish to adopt the Einstein synchrony (*i.e.* for light TWS= OWS) in the inertial frames of the moving rod (we label them S_1 for the rod moving towards the right and S_2 when it moves towards the left say), they may do it by correcting the times for the event 2 in the respective frames. Let us denote these corrected times by t_{12} for S_1 and t_{22} for S_2.[9] The corrected times will be given by

$$t_{12} = t_{01} + \frac{L_0}{2\overset{\leftrightarrow}{c}(0)} = \frac{L_0}{2c}\gamma^2 \tag{5.4.17}$$

and

$$t_{22} = t_{03} - \frac{L_0}{2\overset{\leftrightarrow}{c}(0)} = \frac{L_0(1+2\beta)}{2c}\gamma^2, \tag{5.4.18}$$

where we have made use of Eq. (5.4.16) and inserted Eq. (5.4.8)

Note that for the derivation of Eq. (5.4.17) and (5.4.18), it has been implicitly

[9]The symbol t_{ik} refers to the time of the k-th event according to an observer of the inertial frame S_i.

stipulated that the times recorded on rod-observer's clock, t_{11} and t_{23} (which are the times recorded on C_A) are the same as the laboratory times t_{01} and t_{03} respectively. In the classical situation this is possible because no rate-correction is necessary. In the relativistic situation this stipulation is also possible by making the rate correction to the moving clocks by an appropriate Lorentz factor.

For event 2 the total disagreement of times between observers in S_1 and S_2 is therefore given by

$$\delta t_{\text{gap}} = t_{22} - t_{12} = \frac{L_0 \beta}{c} \gamma^2. \tag{5.4.19}$$

Now in this example, physics is the same whether light propagates forward or backward with respect to S_0, but still the global speed $L_0/(t_{03} - t_{01}) = c(1-\beta)$ is different from the local speeds

$$\frac{L_0/2}{(t_{12} - t_{01})} = \frac{L_0/2}{(t_{03} - t_{22})} = c\left(1 - \beta^2\right), \tag{5.4.20}$$

since the total discrepancy in synchrony given by Eq. (5.4.19) remains unaccounted for in such a comparison if Einstein synchrony is used. Thus we see that in spite of the symmetric situation the global speed of light ought to be different from its local counterpart in this synchrony.

It is interesting to note that in the rotating disc situation this discrepancy in synchronization between any two adjacent MCIFs can be evenly distributed throughout its circumference by honouring the symmetry of this situation. It is therefore evident that the global ratio ρ is in general not the same as the local ratio ρ'. In fact it should be amply clear by now that while the former is an empirically verifiable quantity (based on the measurements of times of flight of light by a *single* clock) the latter quantity depends only on one's own choice of synchrony (see Eq. (5.4.17) or (5.4.18) to understand how the times of the event 2 in S_1 and S_2 are required to be adjusted in order to synchronize the clocks in the Einstein way). Note that in this respect the classical kinematics is no different from its relativistic counterpart.

5.5 Desynchronization

From the above analysis it is evident that if in order to calculate the round-trip time for light in the (non-inertial) frame of the rod, one adds up the times of flight of the same in the inertial frames S_1 and S_2, where the Einstein synchrony has been employed, the result will be wrong by the amount δt_{gap}. This happens since $t_{22} \neq t_{12}$. It only means that S_1 and S_2 cannot be meshed together. However in seeking to dovetail these frames one may set $t_{22} = t_{12} = \frac{L_0}{2c}\gamma^2$. But in that case t_{23} has to be altered by the amount δt_{gap} to preserve the Einstein synchrony in S_2. However since according to our stipulation t_{23} is the time measured by C_A, any possibility of alteration in the value of t_{23} would mean C_A is desynchronized with itself.

In the literature this phenomenon is known as the "desynchronization" in the context of synchronization of clocks in a rotating platform. It is not difficult to show that the measure of this desynchronization in the case of a rotating disc, which is often termed as the "time lag" [12, 33] (for the corotating light signal) is the same as δt_{gap} obtained in the shuttling rod example above. Note that this δt_{gap} is just half of the classical Sagnac delay (see Eq. (5.2.7)). If the same effect is calculated for the counter propagating beam, the total time lag $\Delta\tau_{\text{lag}}$ comes out to be $2\delta t_{\text{gap}}$. As mentioned earlier, people tend to regard this desynchronization ($\Delta\tau_{\text{lag}}$) as the real cause of the Sagnac effect in the relativistic context [6, 7, 12, 33]. For example in Ref. [7], Tartaglia observes that the "simplest explanation for this effect attributes it to the time lag accumulated along any round trip ...". Earlier, Rizzi and Tartaglia [6] expressed a similar view in order to give the "true" relativistic explanation for the Sagnac time difference by ascribing it to the non-uniformity of time on the rotating platform and to the "time lag" arising in synchronizing clocks along the rim of the disc. Selleri also remarks (while not sharing this view) that "an "orthodox" approach to dealing with the rotating platform problem is to consider a position dependent desynchronization ... as an objective phenomenon."

The present analysis of the classical Sagnac effect using Einstein synchrony reveals that this desynchronization is only an artefact of the Einstein synchrony and hence is devoid of any empirical content. Since, if instead of ZT, one uses the Galilean transformation, there is no "desynchronization" but still there is Sagnac effect. Therefore "desynchronization" is conventional in nature and hence cannot be considered an "objective phenomenon". For future reference we call this desynchronization as desync1.

In a recent paper Rizzi and Serafini [12] acknowledges Selleri and Klauber (see footnote on page 4 of Ref. [12]) who have brought to their attention this fact that the much talked about "desynchronization" is merely a "theoretical artefact". However the present paper reveals this in a more convincing way by explicitly showing how this "desynchronization" can be manufactured in the classical world too.

The authors of Ref. [12] however somewhat supporting the orthodox view regarding the connection of the Sagnac effect and the "desynchronization", redefines the latter in the following way: Starting from any point Σ on the rim of a rotating disc if two synchronized clocks are slowly transported in opposite directions along the periphery and are brought back to the same position, they will be found to be out of synchrony by the amount which is equal to that obtained for desync1 *i.e.* $\Delta\tau_{\text{lag}}$. This desynchronization will hereafter be referred to as desync2.

The desynchronization, thus defined, is the result of the comparison of two clocks at the same space point and hence is independent of the distant synchrony convention. The authors therefore claim that they have revealed the "deep physical" and "non-conventional nature" of the time lag. However it is enough to point out the fallacy of this claim by mentioning that these two desynchronizations (desync1 and desync2) are two different things altogether, since if something is conventional, it can be changed or removed by altering the convention, but the "time lag" or time difference in the readings of the two slowly transported clocks after their round trips cannot be altered by redefining the synchrony on the rotating disc.

The equality of these time lags (*i.e.* desync1 and desync2), therefore, is itself conventional and is true accidentally (as opposed to logically) in the relativistic situation if the Einstein synchrony is used in the rotating frame. If instead, the absolute synchrony is used desync1 $=$ 0 while desync2 still remains non-zero. In the classical case the situation is reversed, since in this case desync2 is always zero since there is no time dilation of clocks with respect to the laboratory frame however for the Einstein synchrony in the disc (which corresponds to ZT) desync1 $\neq$ 0. These are however equal in the absolute synchrony (which corresponds to GT). Rizzi and Serafini also claim that desync2 brings to light the "dark physical root of the Sagnac effect". However this claim is also in error too. It is obvious that desync2 cannot be regarded as the physical cause of the Sagnac effect, since we observe that in the classical world desync2 is always zero but still the Sagnac effect exists. This reveals that desync2 and Sagnac effect are unconnected entities. The equality of these two different entities in the relativistic world is at best fortuitous.

5.6 Synchrony – A Value Judgement

One is now in a position to inquire if it is possible to consistently synchronize clocks on the rim of the turn-table so that no gap in synchrony arises. Let us call such a synchrony as "good synchrony". To answer this consider the following scheme for synchronization due to Cranor *et. al.* [34]. In this scheme before the disc is set into motion with respect to S_0 all observers on the rim of the disc and those in the laboratory set their clocks according to the Einstein synchrony. The disc is then set into rotation uniformly (here 'uniformly' means all the points of the rim are treated identically [34]) which after some time may be assumed to attain a constant angular velocity. Alternatively one may set all the clocks on the rim (as well as those adjacent to them in S_0) a common time (say $t = 0$) as soon as the observers on the rim receive a flash of light sent out from a light source at the center of the disc.

Clearly the symmetry of the problem demands that the observers in the laboratory as well as those on the rim of the disc should continue to agree on the question of simultaneity as the synchronization process "favours no particular observer" [34]. This symmetry argument is evidently true in the classical as well as in the relativistic world. Only in the latter case although the observers in the laboratory frame and in the rotating frame agree on simultaneity, the clock *rates* in these frames differ due to the time dilation effect of relativity.

It is evident that there will be no gap in the synchrony between two successive MCIFs (in the linear example between S_1 and S_2) if the observers in these frames agree on simultaneity with those in S_0. Again if there is no synchrony gap the global ratio ρ should be equal to the local ratio ρ'. The agreement on simultaneity between the frames in turn requires ϵ to be equal to zero in Eqs. (5.3.1) and (5.4.12). In the classical world this implies GT, on the other hand in the relativistic world this corresponds to TT (Eq. (5.3.6)).

It means if the clocks of the disc were synchronized according to the scheme discussed above when the latter was at rest with respect to S_0, nothing has to be done further to synchronize them in order to have consistent synchrony throughout the rim when the disc picks up its uniform angular speed. The synchrony is thus "automatic". Any other synchrony (which corresponds to $\epsilon \neq 0$) including the Einstein Synchrony is to be achieved through human intervention. Selleri [1] therefore singled out the absolute synchrony by calling it as "nature's choice".

One may however ask at this point if it is at all possible to synchronize the clocks on the rim in the absolute way (so that $\epsilon = 0$) without referring to the underlying inertial frame *i.e.* by means attached to the turn table itself. Indeed this can be done in practice. For instance an observer with a clock on the rim at a point Σ can start the process by sending a light pulse to an adjacent clock in the anticlockwise direction and synchronize the latter with his own clock first by assuming the OWS of light to be equal to c. In the same way the third clock adjacent to the second one in the same direction can be synchronized with the latter and the synchronization procedure may continue in this way until finally one arrives at the first clock. The observer then discovers that the clock at Σ is not synchronized with itself. The desynchronization, *i.e.* the defect in synchrony will again be different if checked clockwise rather than counter-clockwise. By trial however the observer will be able to discover that the defect in synchrony disappears if the one-way speeds in the two different directions correspond to two different numerical values c_1 and c_2 (say). With these obtained empirical values for the OWS of light, not only the clocks on the rim are synchronized in the absolute way but also the linear speed of the βc and hence the angular velocity $\omega = \beta c/R$ of the rotating disc are determined if c_1 and c_2 are substituted for c_+

and c_- in Eqs. (5.2.17) and (5.2.18) (or alternatively in Eq. (5.4.9) if one considers the Galilean world). All this however refers to the question of synchronization in the large and does not mean that in MCIFs it is mandatory to adopt the absolute (non-isotropic) synchronization.

One may now question our nomenclature "good synchrony" for the one for which light propagation is anisotropic (remember that for $\epsilon = 0$ light propagation is anisotropic in the classical as well as in the relativistic world). Let us clarify this: In the classical world people would be immediately happy to know that the demand for consistent synchronization in the large requires $\epsilon = 0$, which recovers GT. They would say, "After all we get back our old time tested transformation, the Einstein synchrony (leading to ZT) is a bad one, since it is not automatic and natural and it leads to inconsistent synchronization in the large." What should be our reaction who live in the relativistic world? If one carries on the same sort of arguments in the relativistic world, one may give a value judgment in favour of the absolute synchrony ($\epsilon = 0$) hence may call it the "good synchrony" by contrast, unless one seeks to indulge in double standard.

5.7 Conclusion

SP refers to a theoretical prediction regarding the OWS of light grazing the circumference of a rotating disc. The essential content of SP is that simple kinematics together with some appropriate symmetry arguments predict an anisotropy in the speed of light with respect to an "inertial observer". The claim apparently is substantiated by the Sagnac effect. (In the recent FOC experiment [19] the "inertial observer" is obtained automatically (see Sec 5.2 while in the original rotating disc context one needs to take the limit $R \to \infty$ and $\omega \to 0$ while preserving the linear speed of the rim of the disc so that any point on the rim can be thought of as an inertial observer.)

Some earlier responses to the issue are either incomplete or they suffer from certain drawbacks. Here we have shown that by adopting the Einstein synchrony SP can be recast in the Galilean world (see Sec. 5.4. This facilitates in understanding the weak point of the reasoning leading to the fallacy.

It has been argued that SP hinges on the assumption that the (global) ratio of the round-trip speeds of the light beams co-rotating and counter-rotating with the disc (as if) ought to be the same as the local ratio of the OWS in the MCIFs since no point on the rim is preferred. The present analysis in the classical world reveals how in spite of symmetry of the situation the two ratios can be different.

The issue of the "desynchronization of clocks" which is often regarded as the physical cause of the Sagnac effect has been put under the scanner. It is held that of the two types of desynchronization discussed here, desync1 is a theoretical artifact while desync2, although a convention-free entity, is also unable to qualify itself as the root cause of the effect. Finally, in spite of the lacunae in the reasonings leading to SP, the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

Postscript: After the publication of the content of this chapter [13], Selleri has given an answer [35]. There the author has agreed with us in many ways. However, he has disagreed with us regarding our claim of consistency of SR in spite of his forceful paradox by the following words: "In conclusion the GRCS[10] way of dealing with the rotating platform problem introduces a useless complication." We consider that this is not strictly a refuting statement as such and indeed by the phrase "...*useless complication*" the author has supported our view in a way. We keep further discussion on it out of the present scope.

[10]Selleri referred our paper by this name.

Reference

[1] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[2] F. Selleri, in F. Selleri (Ed.), "Open Question in Relativistic Physics", pp. 69–80 (Apeiron, Montreal, 1998).

[3] E. J. Post, "Sagnac effect", *Rev. Mod. Phys.*, **39**, (1967) p. 475.

[4] G. Sagnac, *Comp. Rend. Acad. Sci.*, **157**, (1913) pp. 708–710.

[5] R. D. Klauber, *Found. Phys. Lett.*, **11**, (1998) pp. 405–444.

[6] G. Rizzi and A. Tartaglia, *Found. Phys.*, **28**, (1998) pp. 1663–1683.

[7] A. Tartaglia, *Found. Phys. Lett.*, **12**(1), (1999) pp. 17–28.

[8] G. Rizzi and A. Tartaglia, *Found. Phys. Lett.*, **12**(2), (1999) pp. 179–186.

[9] T. Budden, *Found. Phys. Lett.*, **11**, (1998) p. 4.

[10] E. Minguzzi, *Found. Phys. Lett.*, **15**(2), (2002) pp. 153–169. Available at arXiv:gr-qc/0103049 v3.

[11] R. Klauber, "Toward a consistent theory of relativistic rotation", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frames", Foundations of Physics (Kluwer, Dordrecht, 2004). Series Editor: A. van der Merwe.

[12] G. Rizzi and A. Serafini, "Synchronization and desynchronization in rotating platforms", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frames", Foundations of Physics (Kluwer Academic, Dordrecht, 2004). Series Editor: A. van der Merwe.

[13] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.*, **17**(5), (2004) pp. 457–477.

[14] S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarker and Saroj Nepal, "On synchronization and desynchronization in rotating frames and a recent paradox", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (IX)", (2004). Held at Imperial College, London (To be Published).

[15] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[16] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Rotating disc problem and sagnac phase-shift formula", in M. C. Duffy (Ed.), "Physical Interpretations of Relativity Theory, VIII", (P. D. Publication, Liverpool, 2000). Held at Imperial College London.

[17] F. Selleri, "Sagnac effect: End of a mystery", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frame", Fundamental Theories of Physics (Kluwer Academic, New York, 2004). Series Editor: A. van der Merwe.

[18] A. Tartaglia and M. L. Ruggiero, "Sagnac effect and pure geometry", `gr-qc/0401005v1` (2004).

[19] R. Wang, Yi. Zheng, Aiping Yao and D. Langley, *Phys. Lett. A*, **312**, (2003) pp. 7–10.

[20] Adolf Grünbaum, *Philosophical Problems of Space and Time* (Alfred Knopf, New York, 1973), 2 edition.

[21] H. Reichenbach, *The Philosophy of Space and Time* (Dover, NewYork, 1957), 2nd edition.

[22] R. Anderson, I. Vetharaniam and G. E. Stedman, "Conventionality of synchronisation, gauge depencence and test theories of relativity", *Physics*

Reports, **295**, (1998) pp. 93–180. This review article has an extensive list of references (240 in number).

[23] S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.*, **15**(4), (1991) pp. 369–374.

[24] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994) pp. 21–28.

[25] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979) pp. 229–245.

[26] A. Winnie, *Phil. Sc.*, **37**, (1970) pp. 81–99.

[27] F. Tangherlini, *Nuovo Cimento Suppl.*, **20**, (1961) pp. 1–86.

[28] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhury and Minakshi Sarker, *Found. Phys. Lett.*, **16**(6), (2003) pp. 549–563.

[29] E. Zahar, *British Journal of Philosophy of Science*, **28**, (1977) pp. 195–213.

[30] S.K. Ghosal, K. K. Nandi and P. Chakraborty, *Z. Naturforsch*, **46a**, (1991) pp. 256–258.

[31] D. Dieks, *Eur. J. Phys.*, **12**, (1991) pp. 253–259.

[32] S.K. Ghosal and Minakshi Sarker, "Cosmologically preferred frame and sagnac effect", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (VII)", volume Late Papers (1998). Held at Imperial College, London.

[33] J. Anandan, *Phys. Rev. D*, **24**, (1981) pp. 338–346.

[34] Maria B. Cranor, Elizabeth M. Heider and Richard H. Price, *Am. J. Phys.*, **68**(11), (2000) pp. 1016–1020.

[35] F. Selleri, *Found. Phys. Lett.*, **17**, (2004) pp. 599–606.

Chapter 6

Sagnac Formula from the Rotating Frame Perspective and the Absolute Synchrony

6.1 Introduction

In an interesting paper Franco Selleri [1] remarks that the calculations of SD available in the literature is mostly done from the perspective of the laboratory frame, and they "... say nothing about the description of phenomenon given by an observer placed on the rotating platform." Selleri even remarked that standard SR predicts null result for the Sagnac effect from the perspective of rotating frame. While the last statement is not strictly correct as we have discussed in detail in Chap. 5 [2, 3] in connection with the so called Selleri paradox, the former one may prompt one to survey the literature to find if there exists the derivation of Sagnac effect from the standpoint of a rotating observer. [1]

In some of our earlier chapters we presented some of the common derivations of the Sagnac effect. More than one derivations were presented to highlight different aspects of the effect in the appropriate contexts.

For example, in Sec. 2.5.1, classical Sagnac effect was obtained from simple kinematics. Indeed under relevant approximations no distinction between classical Sagnac effect and relativistic Sagnac effect is usually made. In the section following it this treatment is extended to the realm of relativity. However, the result has been obtained essentially in the laboratory frame and then the effect on board the rotating platform is obtained by introducing appropriate relativistic factor to account for time dilation.

A special relativistic metrical treatment of Sagnac effect is given in Sec. 2.5.3

[1]It is often remarked that such a thing is not possible in fully special relativistic framework since the observer is non-inertial, as if the full treatment may require GR. This kind of argument is often given in connection with the calculation of asymmetric aging of the twin paradox from the perspective of the traveler twin. However introduction of GR for these problems in a flat spacetime is decidedly misleading [4].

(some times these types of treatments are erroneously considered as 'general relativistic treatments'). There, a flat spacetime metric in the laboratory (inertial) frame is expressed in terms of coordinates which directly represent measurement by the standard clocks and rods. The calculation of the effect is performed by asserting that both a photon and the observer (beam splitter) rotate in the rest frame along a circular path, although with different angular velocities. Since the spacetime metric is written in terms of standard time and length coordinates of the inertial frame, here also the calculation is essentially done from the perspective of the laboratory.

Dieks [5] offered another calculation of SD which directly makes use of the LT and this has been used by us in Sec. 3.5, but a close scrutiny will reveal that the calculation too is done from the perspective of the laboratory frame.

Observe that all these justify Selleri's contention that the calculations in general (including the textbook ones [6]) are done from the *perspective of the laboratory frame*. Since the Sagnac effect is essentially a phenomenon on a rotating frame, it is proper to think that a calculation should be available which is done from the *perspective of the rotating frame*. To provide one, Selleri proposes a calculation based on *inertial transformation* introduced by him. The transformation contains a free parameter ϵ which, in conformity with the CS thesis, is capable of describing relativistic physics. The author then considers a set of localized inertial frames moving tangentially to the disc which are instantaneously comoving with the small circumferential elements of the disc. The OWS' of light moving parallel and antiparallel to the direction in these frames predicted by the inertial transformation are found out. The difference between the times of flight of the light beams in the two directions to traverse the circumferential length of the disc gives the SD in terms of the ϵ parameter. It comes out that the laboratory and the disc observer would agree on the SD if and only if $\epsilon = 0$, for which the inertial transformation implies TT which corresponds to absolute synchrony. On the contrary the value of ϵ corresponding to LT gives *null result* as already mentioned.

Note that by considering absolute synchrony Selleri allows anisotropy of OWS' of light in a locally chosen instantaneously comoving inertial frame and is thus able to extend this local feature to a global one when he finds correctly the difference of time of flight. This amounts to meshing up all the instantaneously comoving inertial frames without any problem. We believe that Selleri is indeed correct in this regard.[2] Indeed for various reasons we have given a value judgment in favour of the absolute synchrony on a rotating platform (vide Sec. 5.6). We shall show below that an important treatment of the Sagnac effect (by E. J. Post [7]) implicitly assumes absolute synchrony thus confirming Selleri's claim.

6.2 Post's Calculation

Coming back to the question of the existence of treatments of the Sagnac effect with respect to an observer on board the rotating platform, let us consider the well-known calculation by E. J. Post [7]. Post's approach to finding the SD from the rotating frame perspective is to follow the metrical treatment with a transformed (to rotating frame) flat spacetime metric. A lesson that one learns from GR is that any co-ordinate transformation, even the Galilean one will be able to obtain physical effects provided the co-ordinates used are properly interpreted in the

[2] That LT gives null result in this treatment essentially means that this kind of straightforward extension is not valid while the clocks are Einstein synchronized. The calculation of SD within the standard relativistic frame work is still possible provided the synchrony gap which has been exemplified in Sec. 5.5 is taken into consideration.

context of the rotating frame. Such an approach was intiated by P. Langevin [8, 9]. Using Galilean type transformation he was able to predict the experimentally observable first order effect. However a treatment will be considered to be one done truly from the perspective of rotating frame provided it includes coordinates which directly relates measurements by standard rods and standard clocks and Langevin's treatments fail on that count.

Post in 1967, in a very comprehensive review of Sagnac effect (and perhaps the most cited in this field) tried to obtain the expression for SD from a more suitable metric. Surely for such a calculation one would need a coordinate system which is rotating with respect to the system in the underlying inertial frame. He therefore looks for a suitable transformation which may represent a rotating frame and would have given the correct relativistic Sagnac formula

$$\Delta\tau = \frac{4\pi R^2}{c^2}\gamma. \tag{2.5.10}$$

Starting from the expression of flat line element in the cylindrical co-ordinate in laboratory frame

$$\mathrm{d}s^2 = c^2\mathrm{d}t_0^2 - \mathrm{d}r_0^2 - r_0^2\mathrm{d}\phi_0^2, \tag{6.2.1}$$

Post subjects it to a (somewhat arbitrarily) Galilean-type transformation,

$$\begin{aligned} \mathrm{d}t_0 &= \epsilon\mathrm{d}t, \\ \mathrm{d}r_0 &= \mathrm{d}r, \\ \mathrm{d}\phi_0 &= \mathrm{d}\phi + \epsilon\omega\mathrm{d}t. \end{aligned} \tag{6.2.2}$$

where ϵ is upto now an undetermined factor (our notation is different from that used by Post). The co-ordinates without a suffix represents the coordinates in rotating frame. For circular symmetric path of the light beams $\mathrm{d}r_0 = \mathrm{d}r = 0$. The line element expressed in the rotating frame under this transformation now reads

$$\mathrm{d}s^2 = \epsilon^2c^2\left(1 - \frac{R^2\omega^2}{c^2}\right)\mathrm{d}t^2 - R^2\mathrm{d}\phi - 2R\,\epsilon\,\omega\,\mathrm{d}\phi\mathrm{d}t, \tag{6.2.3}$$

where R is the radius of the disc. Imposition of null geodesic condition $\mathrm{d}s = 0$ for the light beams gives two solutions for round trip times of two counter rotating beams. Their difference gives the SD

$$\Delta\tau = \frac{4\pi R^2\omega}{\epsilon\left(c^2 - \omega^2 R^2\right)} \tag{6.2.4}$$

which still contains the undetermined factor ϵ. $\epsilon = 1$ gives the classical Sagnac formula

$$\Delta\tau = \frac{4\pi R^2\omega}{c^2}\,\gamma^2. \tag{2.5.5}$$

Again if the assumed formula is the relativistic one, already obtained theoretically one gets a value for $\epsilon = \gamma$. Now since in the calculation ϵ remains a free parameter, only to be determined using an assumed Sagnac formula one can hardly call the exercise to be truly a derivation; at least it does not serve our original objective, *i.e.* obtaining the Sagnac effect from the rotating frame perspective.

6.3 Derivation of TT from Post's Transformation

However, Post's approach will constitute a proof if instead of finding out the value ϵ from the known relativistic formula for SD one could somehow guess the expression for ϵ beforehand. We now suppose that intuitively Post postulated (from the consideration of time dilation effect or otherwise) that $\epsilon = \gamma$. In that case Post's derivation is based on the postulate of the coordinate transformation representing the *relativistic* rotation of the disc

$$\mathrm{d}t_0 = \gamma \mathrm{d}t, \tag{6.3.1}$$

$$\mathrm{d}\phi_0 = \mathrm{d}\phi + \gamma\omega \mathrm{d}t. \tag{6.3.2}$$

Multiplying the second equation (6.3.2) by R one may write

$$R\,\mathrm{d}\phi_0 = R\,\mathrm{d}\phi + \gamma v\,\mathrm{d}t, \tag{6.3.3}$$

where we have replaced ωR by v, the speed of an element of the circumference of the disc. Writing $R\,\mathrm{d}\phi_0 = \mathrm{d}x_0$ and $R\,\mathrm{d}\phi = \mathrm{d}x$ one obtains

$$\mathrm{d}x_0 = \mathrm{d}x + v\gamma\,\mathrm{d}t \tag{6.3.4}$$

where the term $\mathrm{d}x_0$ represents the linear length of the element of the disc as seen from the rest frame. Regarding the meaning of the coordinate length we shall return soon.

It is interesting to note that in the above transformation although intuitively somehow time dilation has been taken into consideration (see the appearance of Lorentz factors as coefficients of the time differentials in the TE), the other relativistic effects, *viz* the length contraction has not been considered. It is therefore evident that the transformation takes care of the Ehrenfest paradox by considering the disc to be composed of *unbonded* (software bonded) particles (see Sec. 3.7. Also see Refs. [10, 11].). Indeed we have shown in details in Chap. 3(Ref. [10, 11]) that SD formula with a γ factor (which Post has assumed in the context) pertains to such cases.

Inverting the transformation Eqs. (6.3.1) and (6.3.2) one writes

$$\mathrm{d}t = \gamma^{-1}\mathrm{d}t_0, \tag{6.3.5}$$

$$\mathrm{d}x = \mathrm{d}x_0 - v\mathrm{d}t_0. \tag{6.3.6}$$

We already observed that the absence of any space term in Eq. (6.3.5) implies absolute synchrony!

The physical meaning of $\mathrm{d}x$ is now very clear. Eq. (6.3.6) does not predict any length contraction, this is in conformity with what we have said earlier that the distance between the particles remain the same when the disc picks up its uniform motion from its state of rest. It has already been discussed that in such a situation the rest length of the elements of the disc (material) will have to be *stretched* by a Lorentz factor. Thus $\mathrm{d}x$ is Eq. (6.3.6) represents stretched length. However there is no *stretching* in relativistic transformations in standard coordinates. Therefore, according to the standard coordinate (say $\mathrm{d}x'$), $\mathrm{d}x' = \gamma^{-1}\mathrm{d}x$. The final transformation in standard coordinates can then be written as

$$\begin{aligned} \mathrm{d}x' &= \gamma\left(\mathrm{d}x_0 - v\mathrm{d}t_0\right), \\ \mathrm{d}t' &= \gamma^{-1}\mathrm{d}t_0, \end{aligned} \tag{6.3.7}$$

which is nothing but the Tangherlini transformation. Note that we have changed the notation $\mathrm{d}t \to \mathrm{d}t'$ for symmetry.

6.4 Derivation of TE (Appendix of Post's Paper)?

It seems that Post himself was not satisfied with his intuitive approach. He thus presents a interesting 'derivation' of his TE (Eq. (6.2.2)) in the appendix of the paper. He starts from the generalized LT where the relative motion is along any arbitrary direction:

$$\begin{aligned} t_0 &= \gamma\left(t + \frac{\mathbf{v.r}}{c^2}\right), \\ \mathbf{r}_0 &= \mathbf{r} - \mathbf{v}\left[(1-\gamma)\frac{\mathbf{v.r}}{v^2} - \gamma t\right], \end{aligned} \tag{6.4.1}$$

where, again, the co-ordinates without suffix represent rotating frame. He then sets $\mathbf{v.r} = 0$ on the basis that at every point on a rotating disc $\mathbf{v}\perp\mathbf{r}$ to show that the transformation in differential term assumes the form

$$\begin{aligned} \mathrm{d}t_0 &= \gamma\mathrm{d}t, \\ \mathbf{dr}_0 &= \mathbf{dr} - \mathbf{v}\gamma\mathrm{d}t. \end{aligned} \tag{6.4.2}$$

Assuming coordinate differential perpendicular to the radius (along the unit vector $\mathbf{n}$ say)

$$\mathbf{dr} = r\mathrm{d}\phi\mathbf{n}, \qquad \mathbf{dr}_0 = r\mathrm{d}\phi_0\mathbf{n}, \tag{6.4.3}$$

and

$$\mathbf{v} = \omega r \mathbf{n}, \tag{6.4.4}$$

he obtains the transformation in his desired form (6.2.2). In this manner *adjusting* a free parameter to obtain the transformation is avoided, as if the transformation is *derivable* for LT!

Before we point out an error in the above derivation we once again note that the absence of any space term in the time transformation equation implies absolute synchrony, where two events simultaneous in one frame are simultaneous in the other frame (vide App. A and App. B). In LT, on the contrary, simultaneity is relative, since in standard SR the clocks are synchronized according to the Einstein procedure. Surprisingly, Post finds a TE obeying absolute synchrony starting from a TE obeying Einstein synchrony which is absurd.

The flaw in Post's calculation resides in the way the term $\mathbf{v}.\mathbf{r}$ is set to zero before writing the TE in differential form. One should be very cautious in using LT in the case of rotation since LT is valid here *only locally*. Thus, for a circumferential element of the disc one must write down the transformation in differential form first and the constraint condition $dr = 0$ has to be put afterwards. Indeed the differential of the time transformation would be

$$\mathrm{d}t_0 = \gamma\left(\mathrm{d}t + \frac{\mathbf{r}.\mathrm{d}\mathbf{v}}{c^2}\right), \tag{6.4.5}$$

where we have put $\mathrm{d}\mathbf{r} = 0$ *after* the differention. Note that $\dfrac{\mathbf{r}.\mathrm{d}\mathbf{v}}{c^2} \neq 0$ since $\mathrm{d}\mathbf{v}$ is *not* perpendicular to $\mathbf{r}$.

The similar mistake has been committed in arriving at the equation

$$\mathrm{d}\phi_0 = \mathrm{d}\phi + \gamma v\,\mathrm{d}t. \tag{6.4.6}$$

That there is a drawback in the arguments can be understood in the following way.

Multiplying Eq. (6.4.6) by R, the radius of the disc, and noting that $R\,\mathrm{d}\phi_0$ and $R\,\mathrm{d}\phi$ are the elements of length of the circumference with respect to the rest frame and the instantaneously comoving inertial frame respectively, one observes that there is no length contraction effect. We have already noted that the time transformation also does not display relativity of simultaneity. It is therefore surprising that starting from LT which generates these important special relativistic effects does not display them when written in terms of time and space differentials.

To our knowledge these drawbacks of Post's treatment almost remain unnoticed. However, Selleri [12], noting that Post's review paper is *very influential*, once perfunctorily remarked that [Post] ... *hastens to make the second term disappear with the (arbitrary) choice of* $\mathbf{r}$ *perpendicular to* $\mathbf{v}$. According to him the time transformation Post used in the main text is arbitrary. However, Selleri fails to recognize that Post's transformation only confirms his own thesis that only TT can explain the Sagnac effect from the *rotation frame perspective*.

Reference

[1] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[2] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.*, **17**(5), (2004) pp. 457–477.

[3] S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarker and Saroj Nepal, "On synchronization and desynchronization in rotating frames and a recent paradox", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (IX)", (2004). Held at Imperial College, London (To be Published).

[4] Talal A. Debs and Michael L. G. Redhead, *Am. J. Physics*, **64**(4), (1996) pp. 384–391.

[5] D. Dieks, *Eur. J. Phys.*, **12**, (1991) pp. 253–259.

[6] V. A. Ugarov, *Special Theory of Relativity* (Mir Publisher, Moscow, 1979). Translated from Russian by Yuri Atanov.

[7] E. J. Post, "Sagnac effect", *Rev. Mod. Phys.*, **39**, (1967) p. 475.

[8] P. Langevin, *Compt. Rend.*, **173**, (1921) p. 831.

[9] P. Langevin, *Compt. Rend.*, **205**, (1937) p. 51.

[10] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[11] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Rotating disc problem and sagnac phase-shift formula", in M. C. Duffy (Ed.), "Physical Interpretations of Relativity Theory, VIII", (P. D. Publication, Liverpool, 2000). Held at Imperial College London.

[12] F. Selleri, "Sagnac effect: End of a mystery", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating Frame", Fundamental Theories of Physics (Kluwer Academic, New York, 2004). Series Editor: A. van der Merwe.

Chapter 7

Sagnac Effect in Curved Spacetime

7.1 Introduction

In the last chapter we remarked that introduction of GR in explaining the Sagnac effect from the rotating observer's perspective is misleading from conceptual standpoint since essentially here we are dealing with flat spacetime. However this does not means that there cannot be such an effect in a curved spacetime, and on the contrary the full machinery of GR is required to calculate the effect in presence of a gravitational field. Therefore, apart from the special relativistic discussion of Sagnac effect, there have been several endeavours to discuss this effect in a curved spacetime [1–3]. Ashtekar and Magnon [1] provides an elegant mathematical treatment of this effect in a general co-ordinate systems. In their description which discusses the problem from a geometrical point of view, the path of light beams in the Sagnac experiment travel in a toroidal tube with perfectly reflecting internal walls, called the Sagnac tube. The Sagnac tube is described by a two dimensional timelike submanifold embedded in the usual four dimensional spacetime. In this submanifold the beam splitter is represented by a timelike curve and light rays travel in two null geodesics. The emission of light beams and the superposition of light beams on the mirror after their (single) roundtrip journey are represented by two events on the curve. The events are the first intersections of the null geodesics and the worldline of the mirror. The Sagnac phase shift is given by the distance between the two events along the worldline of the mirror. The authors avoided mathematical complications by assuming that 1) *the submanifold allows a timelike Killing field vector field* and 2) *the Sagnac tube has a stationary motion along the trajectories of the Killing field.* They also contended that general relativistic Sagnac effect represents a gravitational analog of Aharonov-Bohm experiment in electrodynamics and the Sagnac shift "may be regarded as a measure of the flux, through the tube, of the natural magnetic field" associated with the Killing vector. We have discussed the treatment of Ashtekar and Magnon in Sec. 2.6. Anandan [2] provides analyses of the Sagnac effect in 'relativistic and non-relativistic' world and also discusses a group-theoretic treatment.

Although elegant, these analyses seem to be too sophisticated to be used directly in physically interesting situations. From the experimental side, the accuracy of the measurements of the Sagnac effect has reached an unprecedented precision due to the introduction of ring lasers. The great accuracy of these measurements poses the problem of higher order corrections to SD which have been sought usually through special relativistic approach.[1] In view of possibility of achieving greater accuracies, it will be interesting to study the general relativistic corrections to the Sagnac effect performed on rotating disc on the surface of a massive body. After all, all the Sagnac experiments are performed on the surface of the earth which is a massive body. In fact the detail calculation will show the general relativistic corrections is often more than the special relativistic one due to the γ factor or its absence.

Earlier Tartglia [3] has studied the effect in a situation where the observer of the effect is a satellite like object orbiting round a massive body (in one situation the turn table itself is the source of the gravitational field). In the present chapter the effect will be derived for a *disc-type* experiment where the turn table is placed in an axisymmetric gravitational field. Instead of spherical symmetry the term 'axisymmetry' has been chosen to highlight the fact that the earth is a rotating massive body. Primarily this rotation of the source will have influence on the Sagnac effect only through the stationarity of the gravitational field (Kerr field).

[1]From the experimental point of view whether the SD formula contains a γ factor (honouring the Ehrenfest paradox) or not is still an open question. However, from theoretical point of view the issue has been discussed in detail in Chap. 3

However, it will be shown that special cases of the present gedanken set up include a scenario where the beam splitter is placed on the surface of the earth which then acts as a turn table. Other special cases which include observers (beam splitter) moving in natural (geodetic) or unnatural (non-geodetic) orbits will also be taken up. To make the calculations more general, we shall work in Kerr-Newman spacetime. The terrestrial situation can then be understood as a special case. However, in order to not to lose forest in the trees we will do the calculations in the Schwarzschild metric first before making the generalizations. Although the primary intention has been to understand small general relativistic corrections to the disc type[2] Sagnac effect in the far field, one cannot but explore interesting general relativistic effects in the deep field. Some interesting such effects will be discussed in the connection. Although these things may be just academic – indeed nobody is going to perform a Sagnac experiment around neutron star or a black hole, however the effort will give us a lot of insight into the nature of such deep fields in the pre-horizon regime.

7.2 Sagnac effect in a Schwarzschild field

Let us now discuss the general relativistic correction to Sagnac effect. Consider a Sagnac experiment performed in a Schwarzschild field. Approximately this is the terrestrial gravitational field which will influence the light propagation on the Sagnac disc. Expressed in geometrical unit ($G = c = 1$) the metric is given by

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - r^2\,\sin^2\theta\,\mathrm{d}\phi^2, \quad (7.2.1)$$

[2]The term 'disc' includes in its meaning also the 'Sagnac tube' mentioned in the derivation of Ashtekar and Magnaon since here also the light path defines a 2-manifold embedded in the spacetime.

where M is the source mass. For simplicity let us consider the axis of the massless Sagnac disc to coincide with the polar axis of the field.[3] In Sagnac experiment light travels along the periphery of the plane disc. This gives r = constant and θ = constant. Writing $k = \sin\theta$ and $F(r) = \left(1 - \frac{2M}{r}\right)$ in Eq. (7.2.1) we obtain

$$\mathrm{d}s^2 = F(r)\,\mathrm{d}t^2 - r^2k^2\mathrm{d}\phi^2.$$

If we consider the angular momentum of light beam moving in a circular path to be Ω then, $\phi = \Omega t$. Substituting,

$$\mathrm{d}s^2 = \left[F(r) - r^2k^2\Omega^2\right]\mathrm{d}t^2. \tag{7.2.2}$$

For light, $\mathrm{d}s^2 = 0$, giving

$$\Omega_\pm = \pm\frac{\sqrt{F(r)}}{rk}. \tag{7.2.3}$$

The $\pm$ sign in Eq. (7.2.3) indicates two different directions of the motion of light. This is insignificant in the present context because in the Sagnac effect we take the difference of times of reaching of two counter-rotating beams. Thus the angular displacement of light in time t is given by

$$\Omega_+ = \Omega_- = \Omega_\pm = \frac{F(r)^{1/2}}{rk}. \tag{7.2.4}$$

The motive behind still retaining the $\pm$ sign in $\Omega_\pm$ will be evident in the forthcoming sections when we generalize the calculations to Kerr-Newmann field. If the angular velocity of the rotating observer is ω_0, the rotation angle will be

$$\phi_0 = \omega_0 t. \tag{7.2.5}$$

[3]In the Schwarzschild case the choice of the polar axis is arbitrary, hence in the context of a terrestrial experiment the analyses presented in this section is equivalent to assuming the turn table to be horizontal at any point on the earth surface. This conclusion would however not be valid if an axissymetric metric were used since the polar axis in this case would represent that of the rotation of the source producing the field.

Eliminating t from Eqs. (7.2.4) and (7.2.5), we obtain

$$\phi_{\pm} = \Omega_{\pm}\left(\frac{\phi_0}{\omega_0}\right). \tag{7.2.6}$$

The co-rotating light meets the beam-splitter after traversing an angle $2\pi + \phi_{0+}$ where ϕ_{0+} is the extra path traversed by the co-rotating beam. Thus

$$\Omega_{+}\left(\frac{\phi_{0+}}{\omega_0}\right) = 2\pi + \phi_{0+}.$$

Solving for ϕ_{0+} we obtain

$$\phi_{0+} = \frac{2\pi\omega_0}{\Omega_+ - \omega_0}.$$

Similar argument for counter-rotating light leads to,

$$\phi_{0-} = \frac{2\pi\omega_0}{\Omega_- + \omega_0}.$$

In a compact form, we can write

$$\phi_{0\pm} = \frac{2\pi\omega_0}{\Omega_{\pm} \mp \omega_0} \tag{7.2.7}$$

where $\Omega_+ = \Omega_- = \dfrac{F(r)^{1/2}}{rk}$.

The proper time of the rotating observer moving with an angular velocity ω_0 in a Schwarzschild metric is given by

$$d\tau = \left[F(r) - r^2k^2\omega_0^2\right]^{\frac{1}{2}}\frac{d\phi}{\omega_0}. \tag{7.2.8}$$

Thus the SD in the frame of rotating observer is found by integrating from ϕ_{0-} to ϕ_{0+}

$$\delta\tau = \left[F(r) - r^2k^2\omega_0^2\right]^{\frac{1}{2}}\frac{\phi_{0+} - \phi_{0-}}{\omega_0}. \tag{7.2.9}$$

Putting the values. we obtain the SD for Schwarzschild metric for a polar orbit

$$\delta\tau = \frac{4\pi\omega_0\rho^2}{\left(1 - \dfrac{2M}{r} - \omega_0^2\rho^2\right)^{1/2}} \tag{7.2.10}$$

where $\rho = rk = r\sin\theta$. Note that ρ is effectively the radius of the polar orbit, *i.e.* the radius of the disc. Also note that this formula reduces to special relativistic one, Eq. (2.5.10) if M is put equal to zero.

Evidently, this calculation holds even if the field is the Reissner-Norstrøm one:

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1}\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 - r^2\,\sin^2\theta\,\mathrm{d}\phi^2, \tag{7.2.11}$$

where Q is the charge of the star. The calculations will be unchanged except that in this case $F(r)$ will be given by

$$F(r) = \left(1 - \frac{2M}{r} - \frac{Q^2}{r^2}\right)$$

in all the expression upto Eq. (7.2.9) and the expression will be

$$\delta\tau = \frac{4\pi\omega_0\rho^2}{\left(1 - \frac{2M}{r} - \frac{Q^2}{r^2} - \omega_0^2\rho^2\right)^{1/2}}. \tag{7.2.12}$$

However this exercise is purely academic without any immediate observational importance. As a special case, if this ω_0 is assumed to be the angular velocity of the source the results (Eqs. (7.2.10) and (7.2.12) would correspond to the massive turn table scenario as mentioned in Sec. 7.1 where the beam splitter rotates with the gravitating body.

7.3 Sagnac Effect in an Equatorial Orbit

7.3.1 Experiment with Rockets

One may now put $\theta = \pi/2$ to get the result for an equatorial orbit of light. To simulate this case one has to construct an experiment using a system of rockets which forces the observer to move in a circular path with the massive source as the centre. One orbiting rocket-system will be the observer *i.e.* the beam-splitter.

Light will be forced to follow the same circular orbit by some mirrors, orbiting with other such rocket systems. Alternately one may think that as if the disc is now an annular one which contains the earth at the center. A special case would be that with annular thickness equal to zero and the disc rotating with the planet. The situation is equivalent to that with the observer on the earth observing fringe shift by sending and receiving the counter propagating light beams around the earth.

Coming back to the rocket arrangement, note that, all the calculations done in Sec. 7.2 smoothly go over to the equatorial case with $\theta \to \pi/2$. The result is thus given by

$$\delta\tau = \frac{4\pi\omega_0 r^2}{\left(1 - \frac{2M}{r} - r^2\omega_0^2\right)^{1/2}}. \tag{7.3.1}$$

This, again, reduces to the usual special relativistic Sagnac formula (with a γ factor) in the special relativistic limit, *i.e.* $M = 0$ for unbonded particle system (2.5.10), (vide Ref. [4, 5]). This is expected as rocket type experiment with the orbiting mirrors obviously consists of an unbonded system.

7.3.2 Natural Orbit, the "Free Falling Observer"

In the previous case, both the observer (rocket controlled) and the light beams are forced to move in the same circular orbit by some mechanism. Therefore these are not geodetic or natural orbits. In a geodetic orbit observer follows a particular path as soon as the initial conditions are set. As long as the coordinate radius $r > 3M$, geodetic 'circular' orbits for a test particle (observer) exists – hence in principle a Sagnac experiment can be performed by these free falling observers and mirrors although the circular trajectories of the light beams may no correspond to null geodesics. However, at $r = 3M$, both the light beams and the observer move in a common circular trajectory, *i.e.* both the beam splitter and a photon can (in principle) follow natural orbits without any rocket and mirror arrangements. For equatorial natural orbit, clearly ρ is to be replaced by r in Eq. (7.3.1). Also ω_0 is then not a free parameter, indeed as will be shown below that it is a function of r. Hence the Sagnac formula in this case is a function of the coordinate radius only. Imposition of the circular orbit condition on the geodesic fixes one of the conserved quantities U_3 and U_0 which is obtained as [6, pp. 106]

$$\frac{U_3}{U_0} = \left[\frac{Mr^3}{(r-2M)^2}\right]^{1/2}, \tag{7.3.2}$$

where U_μ = the μ-th component of the covariant velocity, *i.e.* $U^3 = \dfrac{\mathrm{d}\phi}{\mathrm{d}s}$ and $U^0 = \dfrac{\mathrm{d}t}{\mathrm{d}s}$, $\mathrm{d}s$ being the arc length.

In this case the coordinate angular speed of the observer can be calculated as [6, 7]

$$\omega_0 = \frac{\mathrm{d}\phi}{\mathrm{d}t} = -\frac{U_3}{U_0}\frac{1-\dfrac{2M}{r}}{r^2}. \tag{7.3.3}$$

Plugging in Eq. (7.3.2) in Eq. (7.3.3) gives

$$\omega_0 = -\left[\frac{Mr^3}{(r-2M)^2}\right]^{1/2} \frac{1-\dfrac{2M}{r}}{r^2},$$

i.e.

$$\omega_0^2 r^2 = \frac{M}{r}. \tag{7.3.4}$$

Using this, the Sagnac formula reduces to

$$\delta\tau = \frac{4\pi\sqrt{Mr}}{\left(1-\dfrac{3M}{r}\right)^{1/2}} \tag{7.3.5}$$

which is now for a given source a function of coordinate radius alone. An interesting point here is that there is a singularity at $r = 3M$. The equation $r = 3M$ stands for the last circular orbit for a particle, while this is the only natural orbit for light. Another interesting matter concerning the orbit is that if we want to carry on the experiment in this orbit, we do not need any rocket arrangement for the satellite to make it revolve round the gravitating source. Also for light propagation no mirror system is required to guide it. However in this case $\delta\tau$ is arbitrarily large.

The reason for this singularity is not far to seek. One intuitively understands that infinite Sagnac effect can take place if light after its emergence from the source takes arbitrarily large time to reunite with the beam splitter in one of its journeys. Surely in this case the coordinate speed of light must be equal to that of the observer. From the global point of view as if, in this case, the wavefromt of light for the corotatin beam should appear to stand still with respect to the beam splitter. That this stipulation is correct can be shown as follows:

From Eq. (7.2.4), the coordinate speed of a photon is

$$v_{\text{photon}} = \Omega r = \sqrt{1 - \frac{2M}{r}}.$$

For a particle the same is given by (see Eq. (7.3.4))

$$v_{\text{particle}} = \sqrt{\frac{M}{r}}.$$

The equality of these speeds now give the condition

$$\sqrt{1 - \frac{2M}{r}} = \sqrt{M} r$$

which gives $r = 3M$.

Carrying on this sort of argument in the disc type arrangement one also expects a singularity. In this case, as

$$\begin{aligned} v_{\text{particle}} &= \omega_0 \rho \\ v_{\text{photon}} &= \Omega\rho = \sqrt{1 - \frac{2M}{r}} \end{aligned} \tag{7.3.6}$$

(vide Eq. (7.2.4)). The equality of these speeds in this case gives

$$\omega_0 = \frac{1}{\rho}\sqrt{1 - \frac{2M}{r}}. \tag{7.3.7}$$

This condition if put in Eq. (7.2.10) gives infinite value for $\delta\tau$. Thus for a given disc size ρ one expects a singularity if the angular velocity of the disc has the value given by the expression (7.3.7).

Coming back to the free falling situation, arbitrarily large Sagnac effect does not pertain to observation on Earth or on a normal star. In order to access $r = 3M$ (a region where the Schwarzschild exterior solution is valid), one must consider a very compact star or a black hole. We shall hence use the term black hole instead of a "star" in the following discussion.

7.4 Sagnac Effect in Kerr-Newman Field

In the last section we have discussed two types of Sagnac experiment in the field of a Schwarzschild black hole (viz. disc type and sattelite based). The same calculation has been extended easily in the Reissner-Nordstrøm field. Let us now perform these calculations in the field of a Kerr-Newman black hole. At first we shall discuss the disc type experiment.

The Kerr-Newman geometry expressed in Boyer-Lindquist coordinate is given by [8, page 877-878] (in geometrical units)

$$\mathrm{d}s^2 = \frac{\Delta}{\rho^2}\left(\mathrm{d}t - a\sin^2\theta\mathrm{d}\phi\right)^2 - \frac{\sin^2\theta}{\rho^2}\left[(r^2+a^2)\mathrm{d}\phi - a\mathrm{d}t\right]^2 - \frac{\rho^2}{\Delta}\mathrm{d}r^2 - \rho^2\mathrm{d}\theta^2, \tag{7.4.1}$$

where

$$\begin{aligned}
\Delta &= r^2 - 2Mr + a^2 + Q^2,\\
\rho^2 &= r^2 + a^2\cos^2\theta,\\
a &= J/M = \text{angular momentum per unit mass},\\
Q &= \text{charge of the source}.
\end{aligned}$$

Please note that the special cases are

$Q = 0,$	Kerr geometry,
$J = 0$	Reisner-Nordstrøm geometry,
$Q = J = 0$	Schwarzschild geometry,
$M^2 = Q^2 + a^2$	"Extreme Kerr-Newman geometry."

Also note that in Boyer-Lindquist co-ordinate the black hole rotates in the ϕ direction [8]. In Sagnac experiment light travels along the edge of a plane disc. As before we define the disc

$$\begin{aligned} r &= \text{constant}, \\ \theta &= \text{constant}, \\ \sin\theta &= \text{constant} = k. \end{aligned} \tag{7.4.2}$$

Unlike the Schwarzschild case however the disc orientation (perpendicular to the polar axis) has an absolute significance, since the polar axes represents the axis of rotation. Putting the conditions (7.4.2) in Eq. (7.4.1) we obtain

$$\begin{aligned} \mathrm{d}s^2 = {} & \frac{r^2 - 2Mr + a^2 + Q^2}{r^2 + a^2(1-k^2)}(\mathrm{d}t - ak^2\mathrm{d}\phi)^2 \\ & - \frac{k^2}{r^2 + a^2(1-k^2)}\left[(r^2+a^2)\mathrm{d}\phi - a\mathrm{d}t\right]^2. \end{aligned} \tag{7.4.3}$$

Let us make the following substitutions to simplify our calculation

$$\begin{aligned} A &= r^2 - 2Mr + a^2 + Q^2, \\ B &= r^2 + a^2(1-k^2), \\ D &= r^2 + a^2. \end{aligned}$$

With this substitutions the metric (7.4.3) becomes

$$\mathrm{d}s^2 = \frac{A}{B}(\mathrm{d}t - ak^2\mathrm{d}\phi)^2 - \frac{k^2}{B}(D\mathrm{d}\phi - a\mathrm{d}t)^2. \tag{7.4.4}$$

If the coordinate angular velocity of light is Ω, then, for light $\phi = \Omega t$. Substituting

$$\mathrm{d}s^2 = \frac{A}{B}(1 - ak^2\Omega)^2\mathrm{d}t^2 - \frac{k^2}{B}(D\Omega - a)^2\mathrm{d}t^2. \tag{7.4.5}$$

Using the condition $ds = 0$, we may write

$$A(1 - ak^2\Omega)^2 - k^2(D\Omega - a)^2 = 0. \tag{7.4.6}$$

Solving for Ω we obtain

$$\Omega_\pm = \pm(X \pm Y), \tag{7.4.7}$$

where

$$\begin{aligned} X &= \frac{\sqrt{a^2(D-A)^2 - \frac{1}{k^2}(Aa^2k^2 - D^2)(A - k^2a^2)}}{Aa^2k^2 - D^2}, \\ Y &= \frac{a(A-D)}{Aa^2k^2 - D^2}. \end{aligned}$$

As in the Schwarzschild case, the $\pm$ sign at the beginning (outside the bracket) in Eq. (7.4.7) indicates two different directions of the motion of light and this is insignificant because in the Sagnac effect we take essentially the difference of time of reaching of two counter-rotating beams. Note that unlike the Schwarzschild case (Eq. (7.2.4)) here $\Omega_+ \neq \Omega_-$. This is because of the rotating nature of the Kerr black hole. The angular displacement of light in time t is thus given by

$$\phi_\pm = \Omega_\pm t. \tag{7.4.8}$$

If the angular velocity of the observer is ω_0, the rotation angle $\phi_0(t)$ at any time t will be

$$\phi_0 = \omega_0 t. \tag{7.4.9}$$

Eliminating t from Eqs. (7.4.8) and (7.4.9) we obtain (following the notations of Sec. 7.2)

$$\phi_\pm = \Omega_\pm \left(\frac{\phi_0}{\omega_0}\right). \tag{7.4.10}$$

The co-rotating light meets the beam-splitter after traversing an angle equal to $2\pi + \phi_{0+}$, (say), thus

$$\Omega_+ \left(\frac{\phi_{0+}}{\omega_0}\right) = 2\pi + \phi_{0+}.$$

Solving, we obtain

$$\phi_{0+} = \frac{2\pi\omega_0}{\Omega_+ - \omega_0}.$$

Similar reasoning leads to, for counter-rotating light

$$\phi_{0-} = \frac{2\pi\omega_0}{\Omega_- + \omega_0}.$$

Combining, we obtain

$$\phi_{0\pm} = \frac{2\pi\omega_0}{\Omega_\pm \mp \omega_0}. \tag{7.4.11}$$

Now, the proper time of the rotating of observer is, from Eq. (7.4.5)

$$d\tau = \frac{1}{\sqrt{B}}\left[A(1 - ak^2\omega_0)^2 - k^2(D\omega_0 - a)^2\right]^{1/2} \frac{d\phi}{\omega_0}. \tag{7.4.12}$$

The Sagnac delay in the frame of the rotating observer is found by integrating from ϕ_{0-} to ϕ_{0+},

$$\delta\tau = \frac{1}{\sqrt{B}}\left[A(1 - k^2\omega_0)^2 - k^2(D\omega_0 - a)^2\right]^{1/2} \frac{\phi_{0+} - \phi_{0-}}{\omega_0}. \tag{7.4.13}$$

Putting the values of $\Omega_\pm$ from Eq. (7.4.7) into Eq. (7.4.11) and values of $\phi_{0\pm}$ from Eq. (7.4.11) we obtain the Sagnac result in the proper frame of the observer

$$\delta\tau = \frac{4\pi}{\sqrt{B}}\left[A(1 - ak^2\omega_0)^2 - k^2(D\omega_0 - a)^2\right]^{1/2} \times \frac{-a(A - D) + \omega_0(Aa^2k^2 - D^2)}{-\frac{1}{k^2}(A - k^2a^2) + 2\omega_0 a(A - D) - \omega_0^2(Aa^2k^2 - D^2)}. \tag{7.4.14}$$

This is the Sagnac result for disc experiment in the proper frame of the observer in a Kerr-Newmann gravitational field. To our Knowledge this is new result and is not available in the literature to date.

It is interesting to note here that if the disc rotates with an angular velocity

$$\omega_{\rm n}(r,k) = \frac{\left(2Mr - Q^2\right)a}{(r^2+a^2)^2 - (r^2 - 2Mr + a^2 + Q^2)a^2k^2}, \tag{7.4.15}$$

one obtains a null result *i.e.* the Sagnac delay becomes zero. This means that the observer here will receive both the co-rotating and counter-rotating light beams simultaneously. The observer will consider both the directions (of light travel) as equivalent in his local geometry. Thus the disc in this case is *nonrotating relative to the local spacetime geometry* [8]. The observer on board the rotating platform is called 'locally nonrotating observer". Any observer on the disc rotating in Kerr-Newmann field with an angular velocity $\omega_{\rm n}(r,k)$ is a locally nonrotating observer. This is equivalent to a disc with zero angular velocity in Schwarzschild geometry (putting $a = 0$ in Eq. (7.4.15) one obtains $\omega_{\rm n} = 0$) in which case no Sagnac effect will be observed.

7.5 Experiments with Rockets and Sattelites in Kerr-Newman Field

Let us now discuss the Sagnac experiment with rockets and satellites[4] in a Kerr-Newman field. For simplicity we assume the satellite to move in an equatorial orbit. Thus with the condition

$$\theta = \pi/2 \Rightarrow k = \sin\theta = 1,$$

[4]We use the term *sattelite* when the observer is in natural (free falling) orbits for which there is no human control on its angular velocity; on the contrary the term rocket is used to highlight that ω can be handput.

we obtain

$$A = r^2 - 2Mr + a^2 + Q^2,$$
$$B = r^2,$$
$$D = r^2 + a^2,$$

With these values we obtain the Sagnac result from Eq. (7.4.14) as

$$\delta\tau = 4\pi \left[a\left(-\frac{2M}{r} + \frac{Q^2}{r^2}\right) + \omega_0 \left(r^2 + a^2 + \frac{2M}{r}a^2 - \frac{Q^2}{r^2}a^2\right)\right] \times \left[\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) - 2\omega_0 a\left(-\frac{2M}{r} + \frac{Q^2}{r^2}\right) - \omega_0^2 \left(\frac{2M}{r}a^2 + r^2 + a^2 - \frac{Q^2}{r^2}a^2\right)\right]^{-1/2}. \tag{7.5.1}$$

In ordinary Kerr field however, $Q = 0$ and the corresponding result was deduced by Tartaglia [3] and is given by

$$\delta\tau = 4\pi \frac{\omega_0 \left(\frac{2M}{r}a^2 + a^2 + r^2\right) - \frac{2M}{r}a}{\left[\left(1 - \frac{2M}{r}\right) + 2a\omega_0 \frac{2M}{r} - \omega_0^2 \left(\frac{2M}{r}a^2 + a^2 + r^2\right)\right]^{1/2}}. \tag{7.5.2}$$

Here also the Sagnac delay vanishes for *local non-rotating* observers whose angular velocity is given by

$$\omega_{\mathrm{n}} = \frac{2Ma}{2Ma^2 + a^2 r + r^3}, \tag{7.5.3}$$

where we have assumed the Kerr geometry ($Q = 0$).

For a sattelite the observer is moving in a geodetic path and the angular velocity is given by [3]

$$\omega_{\pm} = \frac{2aM \pm \sqrt{3a^2M^2 + Mr^3}}{a^2M - r^3} \tag{7.5.4}$$

Putting Eq. (7.5.4) in Eq. (7.5.2) we obtain

$$\delta\tau = \frac{4\pi}{r} \frac{S_{\pm}(2Ma^2 + a^2 r + r^3) - 2MaT}{Z}, \tag{7.5.5}$$

where

$$\omega_\pm = \frac{S}{T}, \qquad S_\pm = 2aM \pm \sqrt{3a^2M^2 + Mr^3} \qquad T = a^2M - r^3,$$

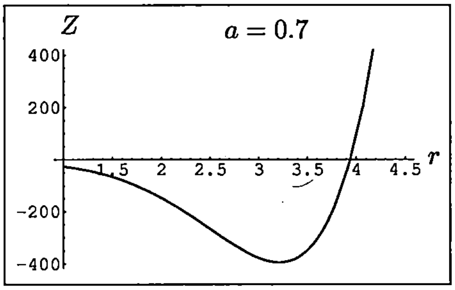
and

$$Z = \left[T^2\left(1 - \frac{2M}{r}\right) + \frac{4Ma}{r}S_\pm T - S_\pm^2\left(\frac{2M}{r}a^2 + a^2 + r^2\right)\right]^{1/2} \quad (7.5.6)$$

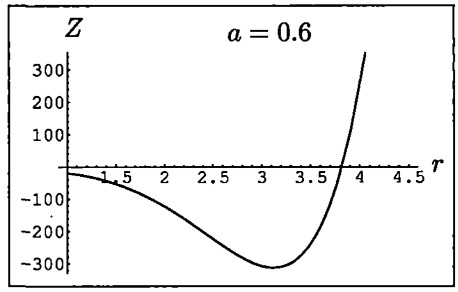
7.6 Arbitrarily Large Sagnac Phase-shift

As we have already noted in the context of Schwarzschild field that the Sagnac effect in the Kerr spacetime also displays singularities which may be obtained by putting Z of Eq. (7.5.5) equal to zero. Indeed one can readily verify from Eq. (7.5.5) that there is a singularity at $r = 3M$ for $a \to 0$ which is the Schwarzschild case. Incidentally the coordinate sphere $r = 3M$ indeed marks the onset of the pre-horizon regime [9] where the Fermi drag changes sign. We do not however notice any spectacular thing to happen in this region for Sagnac effect. Note that the Sagnac effect is not imaginary in the region $r < M$ as the Eq. (7.3.5) seems to suggest, since the equation is valid only for $r \geq 3M$. Within the pre-horizon regime ($2M < r < 3M$) the angular velocity can be handput and again it can be shown that for a wide range of its values the effect is regular showing singularity only when the observer speed is arbitrarily close to that of the coordinate speed of light. In order to see the occurence of arbitrarily large phase shifts it is instructive to plot Z (Eq. (7.5.6)) against r with different values of a and find the position of the singularity. Alternatively it is possible to solve the equation numerically and obtain the value of r of the expression (7.5.5) where the singularity occurs.

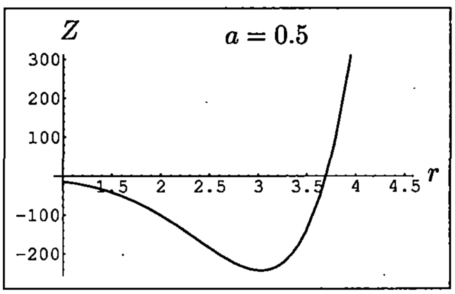
Fig. 7.1(a) through Fig. 7.1(h) represent such plots for retrograde ($\omega = \omega_+$) orbits for $a = 0.7$ to 0.06 ($M = 1$). The zero of Z occurs generally away from



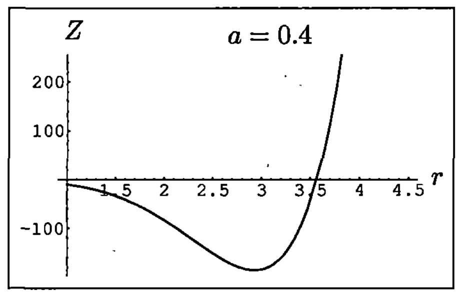
(a) Singularity with $a = 0.7$



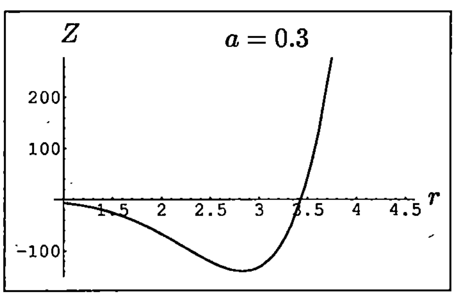
(b) Singularity with $a = 0.6$



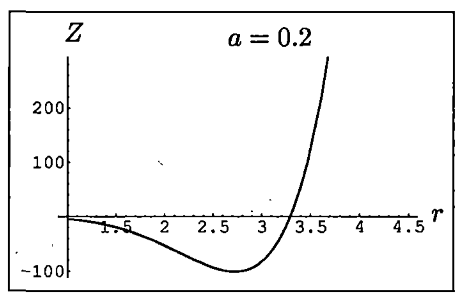
(c) Singularity with $a = 0.5$



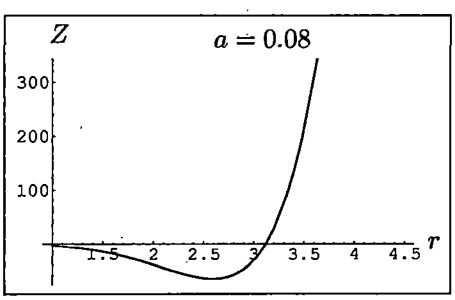
(d) Singularity with $a = 0.4$



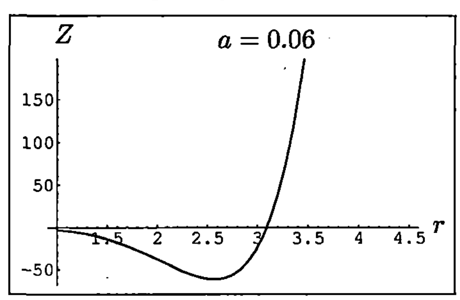
(e) Singularity with $a = 0.3$



(f) Singularity with $a = 0.2$

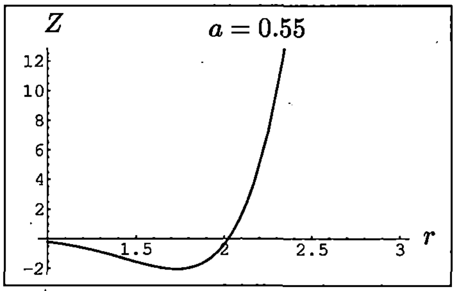


(g) Singularity with $a = 0.08$

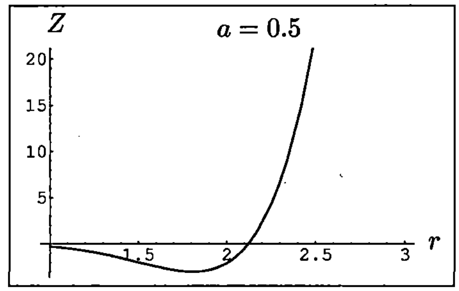


(h) Singularity with $a = 0.06$

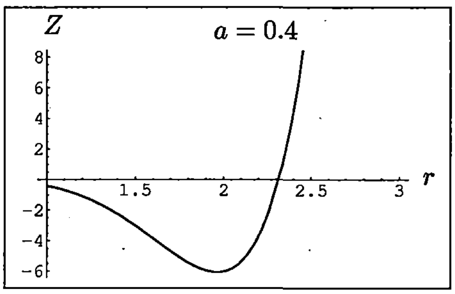
Figure 7.1: Plot of $Z(r) = 0$: Retrograde orbit ($M = 1$)



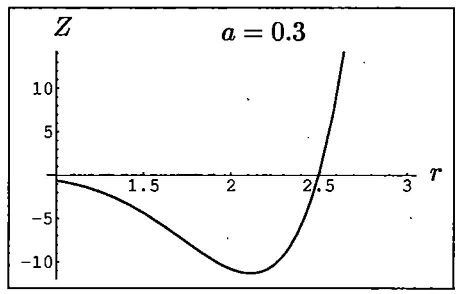
(a) Singularity with $a = 0.55$



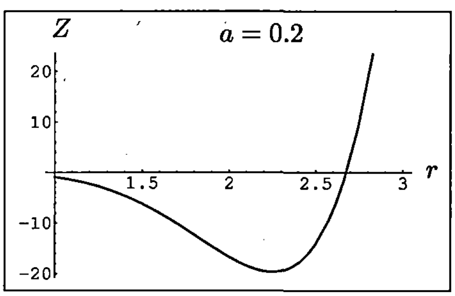
(b) Singularity with $a = 0.5$



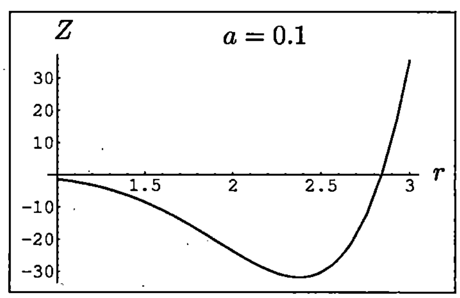
(c) Singularity with $a = 0.4$



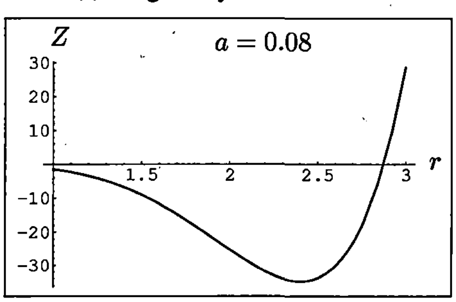
(d) Singularity with $a = 0.3$



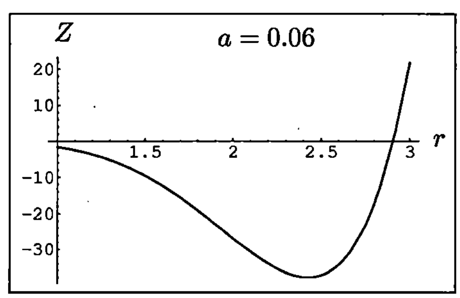
(e) Singularity with $a = 0.2$



(f) Singularity with $a = 0.1$



(g) Singularity with $a = 0.08$



(h) Singularity with $a = 0.06$

Figure 7.2: Plot of $Z(r) = 0$: Direct orbit ($M = 1$)

$r = 3M$ which is obtained only for $a = 0$. Recall that $r = 3M$ represents possible photon orbit in Schwarschild spacetime. For direct orbits ($\omega = \omega_-$) however the singularity occurs below $r = 3M$ and approaches it in zero angular momentum limit. This has been shown in Fig. 7.2(a) through Fig. 7.2(h)

In the literature plots of radii of circular equatorial orbits around a Kerr black hole as functions of the parameter a are available both for direct and retrograde orbits [6, pp.341]. It may be seen that the radii of the photon orbits approach to $r = 3M$ from lower values $r < 3M$ for direct orbits and from higher values $r > 3M$ for retrograde orbits. Our result therefore matches these results.

Reference

[1] A. Ashtekar and A. Magnon, *J. Math. Phys.*, **16**(2), (1975) pp. 341–344.

[2] J. Anandan, *Phys. Rev. D*, **24**, (1981) pp. 338–346.

[3] A. Tartaglia, *Phys. Rev. D*, **58**, (1998) p. 064009.

[4] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Relativistic sagnac effect and ehrenfest paradox", *Found. Phys.*, **33**(6), (2003) pp. 981–1001.

[5] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, "Rotating disc problem and sagnac phase-shift formula", in M. C. Duffy (Ed.), "Physical Interpretations of Relativity Theory, VIII", (P. D. Publication, Liverpool, 2000). Held at Imperial College London.

[6] S. Chandrasekhar, *The Mathematical Theory of Black Holes*, Oxford Classic Texts in the Physical Sciences (Clarendon, Oxford, 1985).

[7] S. K. Bose, *General Theory of Relativity* (Wiley Eastern, New Delhi).

[8] Charles W. Misner, Kip S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Fransisco, 1972).

[9] F. de Felice and S. Usseglio-Tomasset, *Class. Quant. Grav.*, **8**, (1991) pp. 1871–1880.

Appendix A

Conventionality of Simultaneity: A Review

A.1 Light speed and Synchronization

For unambiguous comparison of the temporal relations between two spatially separated arbitrary (unrelated) events in a **given** frame of reference, two synchronized clocks located at the points are necessary. In SR, synchronization of clocks is achieved using light signal whose velocity must be known a priori. But to measure the velocity of the signal two presynchronized clocks are necessary. And herein lies the logical circularity that the measurement of the velocity of light from one point to another (henceforth will be called the *one-way velocity/speed*(OWS)) needs two pre-synchronized clocks. The measurement of round-trip speed or the *two-way speed* (TWS) of light however poses no logical circularity since it needs only one clock. Einstein recognized this 'problem' of measurement of TWS of light and also identified the definitional character of simultaneity of two spatially separated events. In his 1905 paper [1, 2] he writes:

> *If there is a clock at point A in space, then an observer located at A can evaluate the time of events in the immediate vicinity of A by finding the positions of the hands of the clock that are simultaneous with these events. If there is another clock at point B that in all respects resembles the one at A, then the time of events in the immediate vicinity of B can be evaluated by an observer at B. But it is not possible to compare the time of an event at A with one at B without a further stipulation. So far we have defined only an "A-time" and a "B-time", but not a common "time" for A and B. The latter can now be determined by establishing by definition that the "time" required for light to travel from A to B is equal to the "time" it requires to travel from B to A. For, suppose a ray of light leaves*

from A for B at "A-time" t_A*, is reflected from B toward A at "B-time"* t_B*, and arrives back at A at "A-time"* t'_A*. The two clocks are synchronous by definition if*

$$t_B - t_A = t'_A - t_B.$$

We assume that it is possible for this definition of synchronism to be free of contradictions, and to be so for arbitrarily many points, and therefore that the following relations are generally valid:

1. *If the clock at B runs synchronously with the clock at A, the clock at A runs synchronously with the clock at B.*
2. *If the clock at A runs synchronously with the clock at B as well as with the clock at C, then the clocks at B and C also run synchronously relative to each other.*

By means of certain (imagined) physical experiments, we have established what is to be understood by synchronous clocks at rest relative to each other and located at diferent places, and thereby obviously arrived at definitions of "synchronous" and "time". The "time" of and event is the reading obtained simultaneously from a clock at rest that is located at the place of the event, which for all time determinations runs synchronously with a specified clock at rest, and indeed with the specified clock.

Also, in his popular exposition of relativity, Einstein wrote "that light requires the same time to traverse the path AM ... as the path BM [M being the midpoint of the line AB] is in reality *neither a supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own *free will*" [3].

It should now be quite clear that Einstein was well-aware of the *assumptions, definitions or stipulations* he had to make in regard to the OWS of light. It is interesting that textbooks considered these *stipulations* to be *truth.* But, truth must have some empirical content, which the measurement of OWS of light has not.

A.2 Reichenbach Formulation

It is therefore clear that in Einstein's formulation of SR, the synchronization procedure for spatially distant clocks in any inertial frame contains an element of convention (free stipulation) which is devoid of any empirical content. That the procedure for distant clock synchrony in SR has an element of convention other than that adopted by Einstein is known as the conventionality of simultaneity (CS) thesis. The possibility of using a synchronization other than that adopted by Einstein was first discussed by Reichenbach [4] in 1928. Reichenbach observes that the synchronization procedure adopted by Einstein fixes the simultaneity condition for two spatially separated events in a particular way. He comments that "this definition is *essential for the special theory of relativity but it is not epistemologically necessary*"[4, p127]. Einstein's definition is just one possible definition. One can follow an arbitrary rule restricted only to the form

$$t_B = t_A + \epsilon(t'_A - t_A) \qquad 0 < \epsilon < 1$$

and it would likewise be adequate and could not be termed false. Special relativity prefers the value of ϵ (often called "the *Reichenbach parameter*) to be $\frac{1}{2}$, then "*it does so on the ground that this definition leads to simpler relations*". This 'simplicity' is merely 'descriptive', and has no empirical content. The OWS of light is thus conventional. The arbitrariness in the choice of the value of ϵ and the choice of OWS of light is restricted through the following conditions.

1. ϵ must lie within 0 and 1 to preserve causality (light will not reach B before it is emitted from A)

2. The TWS of light is *empirically c.*

Note that the conventionality of synchronization leads to the conventionality in the OWS of any signal including that of light. Thus for any value for $\epsilon(\neq 1/2)$ the OWS' of light in the positive and negative $x-$direction, c_+ and c_- respectively (say) will be different. However the round-trip speed c will be given by the expression

$$\frac{2}{c} = \frac{1}{c_+} + \frac{1}{c_-}. \tag{A.2.1}$$

One is free to choose the OWS of light subject to *this* condition. At the same time, this condition sets the lower limit of OWS to be $c/2$. Note that for this choice, the OWS of return will be infinitely large. Thus, only the TWS of light c has objective status in relativity while the concept of the OWS of light is subjective – just a matter of *convention. Choosing a particular convention for the OWS of light means choosing a related definition of simultaneity of distant events.* To summarize, according to the CS thesis, therefore, there is a plethora of possibilities in assigning the value for the OWS of light. In appendix B, we shall show that the choice of different conventions for the OWS of light (*i.e.* choice of different values of ϵ) leads to different relativistic transformations. Although different synchronizations will be discussed therein, below we briefly mention source of the important synchronies used in the main chapters.

A.3 Einstein Synchrony and Relativity of Simultaneity

So far we have discussed the CS in a *given* inertial frame, without any reference to other inertial frames. Whatever may be the convention, if the *same* convention is adopted in different inertial frames, the spacetime coordinates of these frames will be connected by some transformation equation. According to Einstein's convention for synchrony, for light TWS=OWS in *any* frame. This stipulation is known as the *Einstein synchrony* or the *standard synchrony*, the adoption of which leads to the Lorentz transformation. a corollary of which is the so called "relativity of simultaneity".[1]

The assumption of isotropy of the OWS of light by Einstein made the transformation law (Lorentz transformation (LT)) symmetric and simpler. This alone made this synchronization procedure sacred to several authors. This simple and symmetric nature for the transformation equations however is only structural. According to the CS thesis the physics will remain the same if other synchronization conventions are used. Indeed "the relativity of simultaneity" which is often regarded as the new physical import is devoid of any empirical content in the light of the CS thesis. Unfortunately many authors fail to recognize this point. Some authors (see Ref. [5]) maintain that the relativity of simultaneity is the *actual* cause for the relativistic change of length and time. However others, for example Sjödin [6], contradicting this observes that "there are even physicists who maintain that there are no real length contraction and time dilation effects and that these effects are due solely to the process of synchronization and/or the properties of the space-time continuum ...." One may however favour the standard synchrony over the other because of its simplicity and symmetry but that

[1]The famous *train embankment experiment* qualitatively shows the role of standard synchrony in the train and the platform in arriving at the notion of relativity of simultaneity in relativity theory.

is altogether a different issue. Indeed if one considers synchronization on rotating frames one may give a value judgement (vide Sec. 5.6) in favour of absolute synchrony which we discuss in the next section.

A.4 Absolute synchrony

One may now ask if there exist a synchronization procedure which preserve simultaneity relation between two spatially separated events in all inertial frames, leading to different sets of transformation equations predicting relativistic effects such as Lorentz-Fitzerald contraction and time dilation. One such synchronization procedure is the so called *absolute synchronization.*[2] Here, first an inertial frame is *singled out arbitrarily* where the clocks are synchronized by Einstein method. In this sense, this is *preferred.* The arbitrariness in its choice is follows from the first relativity principle. The clocks in all other inertial frames are synchronized with those of the first frame when they fly past each of the latter ones (see Sec. B.4.2 below for details). As the same correspondence exits among the clocks in each inertial frame and the *preferred* frame, the spatially separated events simultaneous in the *preferred* frame will also be simultaneous in other inertial frames. The transformation arising from this synchrony procedure, the Tangherlini transformation (TT) (vide page 204) corroborates this results with the absence of space part in the time transformation. The Lorentz-Fitzerald contraction and time dilation effects are also predicted with reference to the *preferred* frame.[3] As this synchrony in an arbitrary frame is achieved with

[2]One must not be confused with the usage of the word *absolute*. Here it is meant that congruence of distant events between inertial frames related by this synchronization procedure is preserved.

[3]The simultaneity relation between events occurring in two arbitrary reference frames in this transformation set depends on the *addresses* of the two frames where *address of a frame* is certain function specifying the relation between an arbitrary reference frame and the preferred frame [7].

reference to the preferred frame, the synchrony is sometimes called *system external synchrony* [8]. The Einstein synchronization of clocks in each frame on the other hand is performed without drawing reference to any other reference frame. Hence the synchrony is called *system internal synchrony*. Further discussion in relation to the derivation of the TT is provided in Sec. B.4.2.

A.5 Synchronization in Rotating Ring

The problem of synchronization of clocks on a rotating frame was considered in connection with the Selleri paradox (vide Sec. 5.6). We therefore discuss this issue in somewhat more detail.

The Einstein method of synchrony works just fine in inertial frames. But the main disadvantage of the synchrony is that in accelerated frames the clocks go out of synchrony. Furthermore, in rotating frame, which is a special case of accelerated frame, a clock goes out synchrony with itself [9]![4] Absolute synchronization has the advantage that the clocks thus synchronized in a rotating frame (and, for that matter, in accelerated frames) never go out of synchrony. Cranor et. al. [10] suggested four synchronization schemes. In two of the methods, clocks on the given frame are synchronized before the frame is set into motion. In the other two, the clocks are synchronized when the frame is already in motion. We summarize the methods below.

Method 1. All other clocks on the ring are synchronized with an arbitrarily chosen clock on the ring in the *Einstein method of synchronization*, *i.e.* exchanging light signals, when the ring is at rest in the laboratory frame. The clocks now appear to be synchronized in an "incontrovertibly correct way" [10]. At this point the ring is

[4]This issue, in view of the present study has been discussed in Sec. 5.6.

uniformly (*i.e.* "all the points on the ring are treated identically") set into rotation.

Method 2. The clocks on the rings are synchronized with the laboratory clock stationed at the centre of the ring when the ring is at rest. The clocks on the ring are set to read $t = 0$ when they receive a signal from the lab clock at the centre. Note that this method is essentially equivalent to Einstein method of synchronization. After the synchronization process is complete the ring is gradually and uniformly put into rotation.

Method 3. All the clocks on the ring already in rotation are set to read $t = 0$ when they receive a flash of light signal from the lab clock stationed at the center of the ring, *i.e.* at rest in the laboratory frame. This method is essentially the same as described in the previous one with the difference that in this case at the time of the synchronization the ring is already in motion.

Method 4. The clocks are synchronized in the Einstein method (Method 1) but now the ring is in rotational motion.

Note that in *Method* 2 each clock on the rotating frame receives the light signal at the same time with the clock situated at the same space point on the inertial frame because in the later frame light wave is spherical. This amounts to synchronize the clock as described in Sec. A.4, *i.e.* this is a functional method of achieving *absolute synchronization in rotating frame.*

Is it possible to achieve absolute synchrony without any reference to the underlying inertial frame? A possible way is suggested in Chap. 5. We avoid repeating it and refer the reader to page 128.

A.6 Criticism of the CS Thesis: Arguments and Counter Arguments

Though the CS thesis stands on very firm logical ground of thesis of empiricism, there is no dearth of counter arguments. Textbooks, except a few (e.g. Ref. [11]) maintain a silence about the thesis and accept the standard synchrony as a proposition having a physical character (see, for example, Ref. [12]). The objections mainly circle around the following arguments:

CA1. Arbitrariness in the value of the OWS of light destroys the isotropy of space concept.

CA2. It is not necessary to put two clocks to measure the OWS of light and jump into the logical circularity. Maxwell's equations for electromagnetic waves set OWS of light at $(\epsilon_0\mu_0)^{-1/2}$.

CA3. It is possible to *arrive at* standard synchrony from the relation of symmetric causal connectibility structure of spacetime [13, 14]. Essentially the idea is first to show that standard Einstein synchrony is equivalent to the orthogonality of the time axis of the reference frame in Minkowski space and then to demonstrate that this orthogonality of Minkowski spacetime structure is definable from the causal structure of the spacetime, the lightcone structure. They claim that this approach does not adopt any convention about the OWS of light [15].

CA4. It is possible to bypass the *logical circularity* and thereby devise experiments to measure the OWS of light. Numerous such experiments over the years have been proposed by several authors [16, 17] which have been claimed to allow for an empirical test which might distinguish among the admissible synchrony conventions and thus refute the conventionalists' thesis that all admissible conventions are empirically equivalent. Also, it has been claimed that it is possible to synchronize clocks by other methods (without sending any signals to a distant point) unambiguously. One of the main contender is slow clock transport (SCT) synchrony.

CA5. The anisotropic effects arising from the adoption of nonstandard synchrony is associated with the loss of simplicity of the theory.

All of these counter arguments have been refuted by the conventionalists. We put them together below.

CC1. Isotropy of space is actually *defined* by one-way speed of light [18, 19] making the criticism incorrect. It is possible to define isotropy of space in terms of TWS of light (an empirically verifiable quantity). Fizeau's definition [18, p355:footnote 7] of isotropy of space contains time taken by light to perform round-trip journey in two opposite directions along the same path – equality rendering isotropy of the space. (Rotation destroys this isotropy as evident from the Sagnac effect [20]).

CC2. The simplistic point of view that the OWS of light may be "measured" using Maxwell's equation fails to appreciate the fact that Maxwell's equations use velocity of a charged particle. The adoption of the convention is included in the measurement procedure of velocity which needs two pre-synchronized clocks. Thus the magnitude of velocity of anything is *conventional.*

CC3. This approach, often referred to as Robb-Malament thesis, was considered to settle the issue of the CS thesis for the non-conventionalists [21]. But several authors refuted the claim (see, for example, Refs. [22–25]). We refrain from discussing the matter further. For a comprehensive review the reader is referred to Ref. [22].

CC4. There have been several attempts to devise experiments which can determine the OWS of light uniquely bypassing the *logical circularity*. Reichenbach discussed several experimental propositions (alongwith counter arguments) which *apparently* counters the CS thesis. Erlichson [26] discussed several possibilities and finally concludes that "distant absolute synchronization[5] is impossible and that distant synchronization must employ a convention". In response to the article by Erlichson, Brehme [12] tried to devise yet another method of synchronization without the use of light signals. However Ungar [27] rightly remarks that Brehme's method uses movement of clocks and does not "take into account anisotropic time dilation effects that ... affect the reading of moving clocks in such a way that anisotropy in one-way motion cannot be detected" thus upholding Erlichson's contention.

It was argued by many that it is possible to synchronize clocks by the method of slow transport of clocks without considering any convention whatsoever. A brief account of this debate is given in Sec. A.7 below.

CC5. This counter argument follows the "Occam's dictum that physical

[5]The term *absolute synchronization* used by Erlichson has a meaning different from that we have adopted throughout the present text. According to our definition of the term, a scheme of synchronization which gives 'non-relativity' of simultaneity between inertial frames moving with non-zero relative velocity is called absolute synchrony. The term *'absolute'* here is not to be confused with *'not conventional'*. But according to Erlichson the term *'absolute'* has been used to mean *'as opposed to convention'*. In other words when Erlichson says *absolute synchronization is impossible*, he means that *synchronization is conventional.*

descriptions ought to seek the simplest form" [12]. The description of physical theories becomes anisotropic with the adoption of nonstandard synchronies with anisotropic OWS of light. The transformation equations arising from nonstandard synchronies are also not as symmetric as the LT (see App. B). Brehme, in his response to Erlichson's article [12] commented that "it [nonstandard synchrony] can be done, but is so artificial as to jar our sense of fitness". He also commented there that "introducing an arbitrary convention is surely accompanied by the loss of simplicitly and sense". On this, Debs and Redhead rightly comment that "*... the conventionality thesis is an issue, not about simplicity, but about what is factual and what is conventional in the foundations of special relativity*" [15].

A.7 Slow Clock Transport

Poincaré is considered by many to be one of the first to discuss simultaneity, clock synchrony and their convention. He, in an essay in 1898 mentioned the distinction between the concept of the simultaneity of events that occurred at the same place and those that occurred at distant places [28]. The absence of access to a universal time to order distant events, according to him, compels one to decide on their simultaneity (or otherwise) on the basis of a convention.

In his 1891 essay Poincaré put forward the use of transported clocks to determine time differences of events occurring at distant places. These ideas were pre-relativistic and thus did not include the notion that the rate of a clock is affected by movement. Einstein remarks [1, 2] that where two separated clocks are synchronized with each other (using, obviously, standard synchronization), synchronization is lost if one of them is moved. But, according to a study by

Anderson, Bilger and Stedman [29, p.115], Einstein did not discuss, in that paper or elsewhere, the procedure of synchronization using the transport of clocks for the determination of simultaneity.

Eddington [30] suggested a different method of synchronization of clocks. In this method clocks are synchronized when they are at the same space point. Then they are moved away from each other to be placed at different space point of the reference frame. The fact of retardation of moving clocks are taken care of by limiting their velocities to zero. He notes that it leads to the same results as those obtained by the use of light, *i.e.* Einstein synchrony. He then comments

> *We can scarcely consider that either of these methods of comparing time at different places is an essential part of our primitive notion of time in the same way that measurement at one place by cyclic mechanism is; therefore they are best regarded as conventional.*

It is clear that his claim is that although these statements are conventions, they are empirically related, and are not independent. This all suggest that an empirical test of the equivalence would mean a test of "fundamental hypothesis" and of the directional independence (and observer independence) of the round trip speed of light [29].

One objection to the use of the SCT scheme of synchronization of clocks thus proposed is that there is no way of measuring the OWS of the transported clock until the clocks are synchronized – calling up again the logical circularity. Also, infinitesimal slowness of travel requires infinite arrival time which means that to have a set of synchronized clocks, one should start from infinite past. Bridgeman [31] suggests a way out. He modifies the Eddington synchronization in such a way that one need not start from infinite past. Instead of transporting single clock to a single point, he proposes to send a number of clocks (say from A to B) at various velocities. The readings of clocks at B differs and the one with minimum reading is singled out (the so-called self measured velocity). Comparison of readings of this clocks to that of other clocks will be plotted and then extrapolated to zero. Bridgeman remarks that "... Einstein's remark is by no means invalidated."

The method of synchronization of clocks described by Ellis and Bowman [32] is different but equivalent [23]. Two clocks C_A and C_B are placed at two spatially different points A and B. A third clock C_C is first synchronized with the clock C_A after placing it at A and then moved to B with "intervening 'velocity'" (their terminology) and the time interval is measured. This procedure is repeated with decreasing velocities. The result is extrapolated to find difference between the readings of C_B and C_C with the later's velocity limiting to zero. Finally, this difference is introduced in C_B.

A.8 SCT And The CS Thesis

Winnie [33, 34], in a series of two papers where he reformulated STR "without one-way velocity assumption" shows explicitly that synchrony by SCT agrees with Einstein synchrony when both are described in terms of an arbitrary value of ϵ (within the prescribed limit $0 < \epsilon < 1$). It is argued that in the arguments of EB, $\epsilon = 1/2$ is implicitly assumed. Winnie mentioned that "it is not possible that the method of slow-transport, or any other synchrony method, could, within the frameworks of the *non-conventional* ingredients of the Special Theory, result in fixing *any* particular value of ϵ to the exclusion of any other particular values."

Mansouri and Sexl [8, 35, 36] contends that standard synchrony *in general* differs from SCT synchrony. The equality, they claim, is "neither trivial nor logically cogent". Their findings supports the claim of Winnie that SCT synchrony and Einstein synchrony "agree if and only if the time dilation factor is given exactly by the special relativistic value $(1 - v^2)^{-1/2}$" $[c = 1]$. Both Winnie and Mansouri and Sexl accept SCT as just another convention.

The paper by Ellis and Bowman became prolegomenous to a panel discussion by Grünbaum, Salmon, Fraasen and Janis [37]. They criticized Ellis and Bowman from different angles. Grünbaum contends that "...in the STR, synchronism by slow clock transport neither refutes nor trivializes the ingredients of a convention in that theory's distant simultaneity." Fraasen concludes that "...Ellis and Bowman have not proved that the standard simultaneity relation is conventional, which it is not, but have succeded in exhibiting some *alternative conventions* which also yield that simultaneity relation."

There are other objections about SCT too. For example Erlichson remarks that to have a correct synchronization, clocks should be moved very slowly, and the synchronization will be acceptable in the limit $v \to 0$. In this limit the clock won't move at all and once it reaches the distant point it will show infinite time. Indeed for any arbitrary preassigned value for v one can choose the distance to be arbitrarily large. In practical sense this is unacceptable [26], although the present author believes that the question of practicality is beside the point.

A.9 Discussions

The basic idea behind the CS thesis is that the precise empirical determination of the OWS of light is impossible. It is the logical circularity that debars us to design any experiment. The logical circularity comes from the fact that to measure OWS of light one needs two spatially separated pre-synchronized clocks while to synchronize these clocks one needs to know the OWS of light. On the contrary, to measure the TWS or round trip speed of light, one needs only one clock. Thus the TWS of light is an empirically verifiable physical quantity. One can choose the OWS of light in one direction to be anything. But then the OWS of light in the opposite direction must be adjusted such that the round trip value (TWS) comes out to be c[6] thereby setting a limit to the choice of OWS.

Einstein first recognized this fact and chose light speed to be c in all directions at his 'free will' (as a convention). He then proposed to synchronize all the clocks in a frame with that *chosen* value of OWS of light with a clock at an arbitrarily chosen origin. In fact the "relativity of distant simultaneity' is an outcome of Einstein's convention which is often erroneously construed as a new philosophical import.

Reichenbach first recognized that it was possible to choose other values of the OWS of light still preserving the basic tenet of relativity. In that case the transformation equations will not be the Lorentz one. Tangherlini first showed that this is really possible. It was followed by Winnie, Mansouri and Sexl, Sjödin and Selleri, all of whom derived (general) relativistic transformation in some form. Ghosal, Mukhopadhyay and Chakraborty [39] derived a transformation, called Dolphin transformation, using nonluminal signals as synchronizing agent. General transformations that preserve the basics physics of SR will be discussed in appendix B.

Although many articles have been written on the CS thesis, most texts on relativity, except a few [11], do not discuss this topic at length. The fact that the CS thesis has not yet gained appropriate attention, among the physicists may be attributed to the fact that there is a tendency to regard the CS thesis as an antithesis of SR and anything that seems contrary to the standard formulation of relativity

[6]That this round trip value is independent of the inertial frame of reference chosen is the (reinterpreted) second relativity postulate [38]

is viewed with skepticism [40]. Several claims have been made about finding a method (thought experiment or performable experiment) to determine the OWS of light. Several round abouts have been suggested. To comment on these however, it is enough to say that every such test proposed can be shown to involve, in its analyses and assumptions, propositions logically equivalent to the adoption of the standard synchrony. And this amounts to a simple begging of the question rather than an independent empirical test [41]. In this context we quote a remark by Debs and Redhead [15]

> *...on the grounds that any method that establishes standard synchrony in amoving frame will automatically define nonstandard synchrony in a stationary frame, so the conventional element is restored in specifying simultaneity in the stationary frame ....*

Indeed in our opinion the CS thesis complements the SR and the understanding of the former helps clear out confusions that some times occur in the SR. As we have pointed out, the claim that the relativity of distant simultaneity is a new non-classical philosophical import is one example of various such confusions. In spite of the SR being one of the most simple physical theories, it is the most prolific in giving birth to fallacies, riddles, confusions and misconceptions. Overlooking of the CS thesis and misconstruing of the subtleties of the CS thesis are two of the major reasons for that. For a details analysis vide Ref. [38, 40, 42]. In this dissertation, the CS thesis is *used as a tool* precisely to understand some paradoxes in relativity theory.

Reference

[1] Albert Einstein, "On electrodynamics of moving body", *Annalen Der Physik*, **17**(891), (1905) pp. 1–21. The paper is now freely available at `http://www.fourmilab.ch/`.

[2] Albert Einstein, *Einstein's Miraculous year: Five Papers That Changed the Face of Physics* (Scientia, New Delhi, 1998). Edited and introduced by John Stachel, foreward by Roger Penrose.

[3] Albert Einstein, *The Meaning of Relativity* (Oxford University Press, New Delhi, 1951).

[4] H. Reichenbach, *The Philosophy of Space and Time* (Dover, NewYork, 1957), 2nd edition.

[5] R. Resnick, *Introduction to Special Relativity* (John Wiley and Sons (Asia), Singapore, 1998).

[6] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979) pp. 229–245.

[7] S.K. Ghosal and Minakshi Sarker, "Cosmologically preferred frame and sagnac effect", in M. C. Duffy (Ed.), "Physical Interpretation of Relativity Theory (VII)", volume Late Papers (1998). Held at Imperial College, London.

[8] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 497–513.

[9] G. Rizzi and A. Serafini, "Synchronization and desynchronization in rotating platforms", in G. Rizzi and M. L. Ruggiero (Eds.), "Relativity in Rotating

Frames", Foundations of Physics (Kluwer Academic, Dordrecht, 2004). Series Editor: A. van der Merwe.

[10] Maria B. Cranor, Elizabeth M. Heider and Richard H. Price, *Am. J. Phys.*, **68**(11), (2000) pp. 1016–1020.

[11] F. F. Taylor and J. A. Wheeler, *Space-time Physics* (Freeman, San Fransisco, 1963).

[12] R. W. Brehme, *Am. J. Phys.*, **53**(1), (1985) pp. 56–59.

[13] A. A. Robb, *A Theory of Time and Space* (Cambrige: The University Press, Cambridge, 1914).

[14] D. Malament, *Noûs*, **11**, (1977) p. 293.

[15] Talal A. Debs and Michael L. G. Redhead, *Am. J. Physics*, **64**(4), (1996) pp. 384–391.

[16] S. F. Fung and K. C. Hsieh, *Am. J. Phys.*, **48**, (1980) p. 654.

[17] C. Nissim-Sabat, *Am. J. Phys.*, **50**, (1982) p. 533.

[18] Adolf Grünbaum, *Philosophical Problems of Space and Time* (Alfred Knopf, New York, 1973), 2 edition.

[19] R. W. Brehme, *Am. J. Physics.*, **56**(9).

[20] G. Sagnac, *Comp. Rend. Acad. Sci.*, **157**, (1913) pp. 708–710.

[21] Roberto Torretti, *Relativity and Geometry* (Pergamon Press, Oxford, 183).

[22] R. Anderson, I. Vetharaniam and G. E. Stedman, "Conventionality of synchronisation, gauge depencence and test theories of relativity", *Physics*

Reports, **295**, (1998) pp. 93–180. This review article has an extensive list of references (240 in number).

[23] A. Janis, "Conventionality of simultaneity *in* stanford encyclopedia of philosophy", The Metaphysics Research Lab, Center for the Study of Language and Information, Stanford University. Available at http://plato.stanford.edu/entries/spacetime-convensimul.

[24] Adolf Grünbaum, "David malament and the conventionality of simultaneity: A reply", Available at philsci-archive.pitt.edu/archive/00000184/00/malament.pdf.

[25] Robert Rynasiewicz, "Is simultaneity conventional despite malament's result?", Available at http://philsci-archive.pitt.edu/archive/00000293.

[26] Herman Erlichson, *Am. J. Phys.*, **53**(1), (1985) pp. 53–55.

[27] A. Ungar, *Am. J. Phys.*, **56**(9), (1988) p. 814.

[28] H. Poincaré, *The Value of Science* (Dover, New York, 1958).

[29] R. Anderson, H. R. Bilger and G. E. Stedman, *Am. J. Phys.*, **62**(11), (1994) pp. 975–985.

[30] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, 1965).

[31] Bridgeman, *A Sophisticate's Primer to Relativity* (Wesleyan University, Middletown, 1962).

[32] B. Ellis and P. Bowman, *Phil. Sci.*, **34**, (1967) p. 116.

[33] A. Winnie, *Phil. Sc.*, **37**, (1970) pp. 81–99.

[34] A. Winnie, *Phil. Sc.*, **37**(2), (1970) p. 223.

[35] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 515–524.

[36] Reza Mansouri and Roman Sexl, *Gen. Rel. Grav.*, **8**, (1977) p. 809.

[37] A. Grünbaum, W. Salmon, B. van Fraassen and A. Janis, "A panel discussion of simultantiey by slow clock transport in the special and general theories of relativity.", *Phil. Sc.*, **36**, (1969) pp. 1–81.

[38] S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.*, **15**(4), (1991) pp. 369–374.

[39] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994) pp. 21–28.

[40] Papia Chakraborty, *On Some Foundational Questions in Relativity and Gravitation*, Unpublished, University of North Bengal (1993).

[41] B. Townsend, *Am. J. Phys.*, p. 1092.

[42] S.K. Ghosal, K. K. Nandi and P. Chakraborty, *Z. Naturforsch*, **46a**, (1991) pp. 256–258.

Appendix B

CS Thesis and General Transformation Equations

B.1 Introduction

We have already discussed that the CS thesis contends that there can be several possible choices on the OWS of light, of which Einstein convention is just one. In the relativistic world the set of transformation equations (TE) that arises out of this choice is the Lorentz transformation. Different choice of OWS of light will lead to different sets of transformation equations which, although different structurally, will predict the kinematical worlds. We have already seen in previous chapters that consideration of these TE's helps us in giving insights to many conceptual issues including some interesting paradoxes in SR. This appendix will supplement those chapters by discussing these TE's that can be obtained using different conventions regarding OWS of light in different worlds.

Winnie [1, 2] in 1970 first studied the consequences of SR when no assumption regarding the OWS of light was made and then developed a set of transformation equations, referred to in the literature as *ϵ-Lorentz transformation*, adopting non-Einstein OWS assumption or non-standard synchronization convention in general. In developing the ϵ-Lorentz transformation Winnie assumed a principle called the *Principle of equal Passage time*. This was used in addition to the *Linearity Principle* and the *Round-Trip light Principle*. These principles were then shown to be independent of OWS assumptions and thus may form the basis of a SR where no stipulation regarding distant simultaneity is made. In fact Winnie's theory was one dimensional. Ungar [3] extended Winnies's idea by considering a generalized Lorentz transformation group that does not embody Einstein's isotropy condition. The approach seems to be well suited for establishing the results of Winnie as well as some new results. However, these discussions were confined to one dimensional case only. But at least a two dimensional analysis is a must. Otherwise the isotropy of the TWS of light which follows from the reinterpreted second relativity postulate cannot be used and therefore some subtleties and richness of the relativistic physics have to be sacrificed[4, 5].

In a series of important papers Mansouri and Sexl [6, 7] developed a test theory of SR and investigated the role of convention in various definitions of clock synchronization and simultaneity in a given frame. They considered two principal methods of synchronization and called them *system internal* and *system external* synchronization. The *system internal synchronization* is the one where the clocks in a given reference frame are synchronized without any reference to any clock outside the frame. The *system external synchrony* on the contrary is done with reference to clocks residing outside the frame. Synchronization of clocks by the Einstein procedure (using the light signal) and that by slow clock transport (to correct all clocks at a given locality and then place them at all space points of a given reference frame) turn out to be equivalent if and only if the time dilation factor is given by the Einstein result $(1 - v^2/c^2)^{1/2}$. The authors also constructed an 'ether' theory with an external synchronization schemes that maintains absolute simultaneity and is shown to be kinematically equivalent to SR. This particular synchrony has already been discussed in the last appendix.

The next clarification came in 1979 from Sjödin. He developed and consolidated the CS thesis in an interesting paper by considering the whole issue more generally and also by assuming the role of synchronization in SR and some related theories. He presented all logically possible linear transformations between inertial frames depending on physical behaviour of scales and clocks in motion with respect to physical vacuum (ether) and then examined LT in the light of *true* length contraction and time dilation. In his article Sjödin tried to separate the *true effects* and the effects due to *synchronization convention.* For this the author considered two special cases:

- **The Newtonian world**, without any contraction of moving bodies and slowing down of moving clocks.
- **The Lorentzian world**, with longitudinal contraction of moving bodies and slowing down of clocks.

He then used standard synchrony (later called pseudo-standard synchrony by Ghosal, Mukhopadhyay and Chakraborti [8]) in Newtonian world and obtained the transformations derived by Zahar [9]. These transformations show some apparent relativistic effects which are only due to the choice of the special synchrony in this world. But when Sjödin used absolute synchronization in the Lorentzian world, the relevant transformations were due to Tangherlini [10] which shows the real effects. In this way Sjödin came to the conclusion that the confusion regarding the existence of the ether and the reality of the length contraction/time dilation effects is mainly due to the non-separation of the effects due to synchronization and the real contraction of moving bodies and retardation of moving clocks.

In this chapter, we present a derivation of a set of general transformation equations. We keep a free parameter (called the *synchronization parameter*) which depends on the method of synchronization (*i.e.* the adoption of the convention for the OWS of light). Choice of different OWS of light sets different values of the parameter giving different TEs. We use the matrix algebra because of its mathematical advantages in finding composite transformations, inversion, velocity addition foumula etc. We mention the relation of this TE, which relies heavily on the derivation by Sjödin [11] with those derived by Winnie and by Selleri. Then we discuss a TE due to Ghosal, Mukhopadhyay and Chakraborti [8] where the synchronization is performed, with nonluminal signal (assumed to propagate in a substratum) and show that all known relativistic (along with some non-relativistic) TEs can be obtained easily from this, choosing the appropriate synchronization convention. Indeed, in this set of TE, the role of light as a physical constant and as a synchronizing agent in SR are clearly separated.

B.2 The Transformation Matrix

We shall now find the transformation equations between two inertial frames. Let us choose an inertial frame S_0 arbitrarily (all inertial frames are equivalent). For definiteness we choose the convention that *in this frame* the OWS of light is the same in all directions and is equal to the TWS of light, *i.e.* c. (In the Galilean world however there is just one frame (ether frame) where the TWS of light will be isotropic). In other words in this frame we synchronize the clocks with the Einstein synchronization procedure. This frame will be a *preferred* one if the Einstein convention of isotropic OWS is chosen *only* in this frame. One should not confuse this notion of preferred frame with the pre-relativistic concept of an absolute space. Also, this choice does not violate the principle of relativity of SR because the frame is arbitrarily chosen, *any frame can be given the preferred status*. Following Sjödin we start from the assumptions below.

Assumption 1 *The dimensions of bodies and the time intervals measured by clocks depend on there velocities with respect to an inertial frame but do not depend on their position or acceleration. This is known as* Length and clock hypothesis.

Assumption 2 *The change of dimension of a body in the direction of motion is given by ϕ and that in the transverse direction by ψ. The change in the rate of* *clock is given by* Ω.[1] *According to assumption 5,* ϕ, ψ, Ω *are functions of velocity.*

Assumption 3 *Clocks are synchronized in the transverse direction in the same way as we do in the direction of motion.*

The last one is not an essential assumption, but inclusion of this simplifies calculation. The exclusion of the assumption will not give any new physical insight.

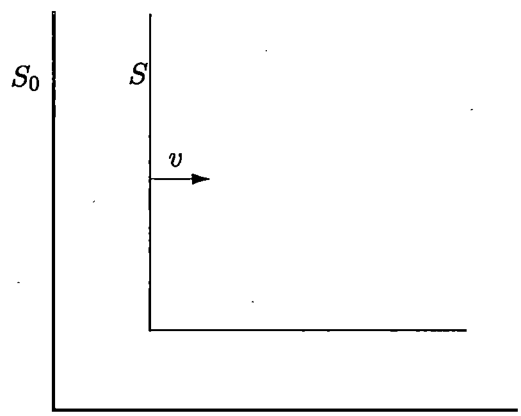


Figure B.1: Here S_0 is the preferred frame

Let us assume that an inertial frame S is moving with velocity[2] v, with respect to the preferred frame S_0. The motion of S in S_0 is along their common x-axis. Initially their origins coincided, *i.e.* ,

$$\text{at } t_0 = t = 0, \;\; x_0 = x = 0.$$

[1]We use the notations as in Ref. [11]

[2]There is, as we have mentioned earlier, an element of convention to measure velocity of any moving body. From now on, whenever we say *velocity*, it will mean that it is measured in the preferred frame *i.e.* $\epsilon = 1/2$, unless otherwise explicitly mentioned.

The homogeneity of spacetime allows us to choose the transformation equations to be linear.[3] Thus the most general transformation ($S_0 \to S$) is given by, in matrix form

$$\mathbf{X} = \mathbf{T}\mathbf{X}_0, \tag{B.2.1}$$

where (suppressing the z-coordinate)

$$\mathbf{X} = \begin{pmatrix} t \\ x \\ y \end{pmatrix}, \quad \mathbf{X}_0 = \begin{pmatrix} t_0 \\ x_0 \\ y_0 \end{pmatrix},$$

and

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & a_{02} \\ a_{10} & a_{11} & a_{12} \\ a_{20} & a_{21} & a_{22} \end{pmatrix}.$$

In a more compact notation

$$\mathbf{T} = \{a_{\mu\nu}\}, \quad (\mu, \nu = 0, 2).$$

Note that here the time co-ordinate is t and not ct, as found in usual four-vector formulations. $\mathbf{T}$ is the transformation matrix. Thus to determine transformation equations, $\mathbf{T}$ is to be found out. Inverse transformation is given by

$$\mathbf{X}_0 = \mathbf{T}^{-1}\mathbf{X}. \tag{B.2.2}$$

According to how we constructed the problem, $x-$axis always coincides with the x_0-axis. Consequently,

$$a_{2\nu} = 0 \qquad \nu = 0, 1. \tag{B.2.3}$$

[3]However very complicated synchronization schemes are clearly possible so that the homogeneity of space will not be explicit in the transformation equations.

Assuming the isotropy of space we accept that clocks placed symmetrically in y axis, about the $x-$axis should agree.This will be true provided

$$a_{02} = 0. \tag{B.2.4}$$

As S is moving along common $x-$axis with a velocity v

$$x = 0 \Longrightarrow x_0 = vt_0.$$

This gives

$$a_{10} = -va_{11},$$
$$a_{12} = 0.$$

Thus the transformation matrix becomes

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ -va_{11} & a_{11} & 0 \\ 0 & 0 & a_{22} \end{pmatrix}.$$

The parameter a_{22} must carry the information about the change of space in y direction respectively. If we write the transverse contraction factor (according to assumption 2)

$$a_{22} = \psi^{-1},$$

the transformation matrix becomes

$$\mathbf{T} = \begin{pmatrix} a_{00} & a_{01} & 0 \\ -va_{11} & a_{11} & 0 \\ 0 & 0 & \psi^{-1} \end{pmatrix}. \tag{B.2.5}$$

Let us consider now a rod of length L at rest on the $x-$axis. We may set $L = x_2 - x_1$. Also $t_{01} = t_{02}$ because to measure length, both the points should be measured simultaneously. This gives $L = a_{11}(x_{02} - x_{01})$. But $x_{02} - x_{01}$ is the length of the rod as measured in S_0, *i.e.* $x_{02} - x_{01} = \phi L$. Hence,

$$a_{11} = \phi^{-1}.$$

Let us now consider a clock at rest in S. Say its position is $x = x_1 = x_2$. Say two events occurred in S at time co-ordinates t_1 and t_2 $(t_2 > t_1)$.

In S_0 frame, the space co-ordinates of the clock is x_{01} and x_{02} and the times are t_{01} and t_{02} giving

$$x_{02} - x_{01} = v(t_{02} - t_{01}).$$

Thus we find,

$$t_2 - t_1 = (va_{01} + a_{00})(t_{02} - t_{01}).$$

Using *assumption* 1, we can write $t_2 - t_1 = \Omega(t_{02} - t_{01})$, and hence

$$a_{00} = \Omega - va_{01}.$$

Now inserting all the values in the transformation matrix (B.2.5) and writing $a = a_{01}$, we obtain

$$\mathbf{T} = \begin{pmatrix} \Omega - va & a & 0 \\ -v\phi^{-1} & \phi^{-1} & 0 \\ 0 & 0 & \psi^{-1} \end{pmatrix}. \tag{B.2.6}$$

In Eq. (B.2.6), 'a' will depend on the synchronization that we make. Let us call it the *synchronization parameter*. Our main task is to find the factor 'a'. Note that exclusion of *Assumption 2* invalidates Eq. (B.2.4). The term a_{02} will then be present in the transformation matrix (B.2.6) [12].

B.3 Determination of a

The value of 'a' carries the synchronization convention. This means that a must be a function of the OWS of light

To find this function, let us imagine a rod of length L at rest in S [11]. Say, the OWS of light at an angle θ (in S) is given by $c(\theta)$.

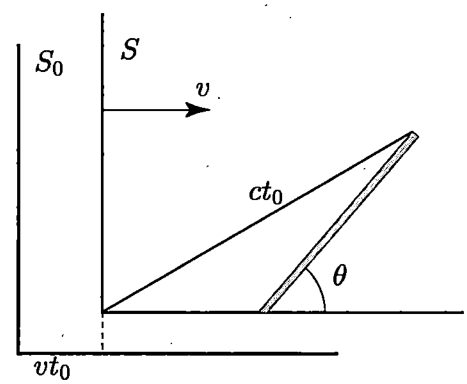


Figure B.2: Detivation of the OWS of light in an inertial frame, figure from Sjödin

In S_0, the light sphere relation is

$$x_0^2 + y_0^2 = c^2 t_0^2 \tag{B.3.1}$$

When measured in S_0, the length L suffers a change by a factor of ϕ in x direction and ψ in y and z directions. Using cylindrical co-ordinate system (r, θ, z) for L in S, we may write (see Fig. B.2)

$$\begin{aligned} x_0 &= vt_0 + \phi L \cos\theta \\ y_0 &= \psi L \sin\theta \end{aligned} \tag{B.3.2}$$

Putting all these in Eq. (B.3.1) and solving for t_0, we obtain

$$t_0 = \gamma^2 c^{-1} L\phi \left[\beta\cos\theta \pm \left\{1 - \left(1 - \gamma^{-2}\phi^{-2}\psi^2\right)\sin^2\theta\right\}^{\frac{1}{2}}\right].$$

In S, the OWS of light at an angle θ with respect to x axis is $c(\theta)$. Thus $L = c(\theta)\, t$. Using the transformation (B.2.6) we find

$$c(\theta) = \left[(1-\xi)\beta\cos\theta \pm \left\{1 - (1-\rho)\sin^2\theta +\right\}^{1/2}\right]^{-1} \chi, \tag{B.3.3}$$

where

$$\begin{aligned}
\xi &= -a\,\Omega^{-1}\gamma^{-2}\beta^{-1}c, \\
\rho &= \gamma^{-2}\phi^{-2}\psi^{2}, \\
\chi &= \gamma^{-2}\Omega^{-1}\phi^{-1}c, \\
\gamma &= \frac{1}{\sqrt{1-v^2/c^2}}.
\end{aligned}$$

Also,

$$a = (c(\theta)\phi\cos\theta)^{-1}\left(1 - \Omega c(\theta)\Sigma(\theta)\right), \tag{B.3.4}$$

where

$$\Sigma(\theta) = \gamma^{-2}c^{-1}\phi\left[\beta\cos\theta \pm \left\{1-(1-\rho)\sin^2\theta\right\}^{1/2}\right].$$

According to the CS thesis the OWS of light in a given reference frame can be fixed according to one's choice and convenience, while TWS of light is an empirically verifiable constant quantity. In our derivation of generalized relativistic transformation, we found that one parameter of the transformation matrix depends on the OWS. Surely this depends on the synchronization. The OWS of light is given by, as a function of the angle with $x-$axis,

$$c(\theta) = \left[(1-\xi)\beta\cos\theta \pm \left\{1-(1-\rho)\sin^2\theta\right\}^{1/2}\right]^{-1}\chi. \tag{B.3.3}$$

The $\pm$ sign denotes two possible values of $c(\theta)$. We can choose any one for our discussion and let us choose the positive sign.

$$c(\theta) = \left[(1-\xi)\beta\cos\theta + \left\{1-(1-\rho)\sin^2\theta\right\}^{1/2}\right]^{-1}\chi. \tag{B.3.5}$$

We avoided unnecessary complications choosing the same synchronization for x and y axes. This is evident here as we find, for $y-$axis,

$$c(\pi/2) = c(-\pi/2).$$

Also note that for $\xi = 1$, $c(\theta) = c(\theta+\pi)$ which means that OWS is same for the forward and reverse journey. This corresponds to the *Einstein convention.*

B.4 Transformation in Different Worlds

B.4.1 Classical World

In classical world, a signal with infinite speed is possible. Thus one can synchronize clocks with this signal and *empirically* find the OWS of light. Also since the readings of clocks do not change due to their motion, they can be synchronized in an absolute way. However different synchronization conventions can be chosen in this world too and hence different transformations are possible.

Zahar Transformation

In classical world, one may synchronize the clocks with light adopting the Einstein convention of OWS=TWS. Thus we obtain

- $\Omega = \phi = 1$ for classical world.
- $c(0) = c(\theta) = c(1 - \beta^2)$.
- $\psi = 1$, for classical world.

These will give the value of the synchronization parameter a.

$$a = -\gamma^2 \frac{v}{c^2}$$

Putting all this values in the transformation matrix (B.2.6) we obtain

$$\mathbf{T} = \begin{pmatrix} 1 + \beta^2\gamma^2 & -\frac{\beta}{c}\gamma^2 & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.1}$$

This is the transformation matrix for Zahar transformation. For a detailed derivation and analysis of this transformation reader is referred to the original work of E. Zahar [9]. In this dissertation, this transformation has been used as an important tool to resolve two paradoxes, namely the Tippe top paradox (Chap. 4) and the Selleri paradox (Chap. 5).

Approximate Zahar Transformation

Sufficiently small v/c will render $\beta^2 \to 0$ and hence $\gamma \simeq 1$. The TE is know as approximate Zahar transformation and is given by

$$\mathbf{T} = \begin{pmatrix} 1 & -\dfrac{\beta}{c} & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.2}$$

Note that this the same as the LT under the same approximation(discussed below)!

Galilean Transformation

In Zahar transformation, the synchronizing signal is light (with velocity c in the preferred frame). In classical world, we can synchronize with a signal whose velocity is infinite. So, we may put $c \to \infty$. This gives $a = 0$, and the transformation matrix (B.2.6) is given by

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.3}$$

B.4.2 Relativistic World

Lorentz Transformation

In relativistic world, the value of OWS=TWS=c in any reference frame if one uses the standard synchrony and it is the highest speed of signal transmission. This synchrony, by virtue of the second relativity postulate makes the OWS isotropic, that is, independent of θ. The value of ψ is also unity [13, pages 36-37].

Summarising

- $c(\theta) = c(\theta + \pi) = c$
- $\xi = 1$, as $c(\theta) = c(\theta + \pi)$
- $\psi = 1$

From these one may easily find out the values of all the parameters

$$\phi = \Omega = \gamma^{-1}$$
$$a = -\gamma\frac{\beta}{c}.$$

Putting these value in Eq. (B.2.6) we obtain the transformations matrix

$$\mathbf{T} = \begin{pmatrix} \gamma & -\gamma\frac{\beta}{c} & 0 \\ -\gamma v & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.4}$$

This is the Lorentz transformation matrix.

Approximate Lorentz Transformation

If the velocity becomes sufficiently small with respect to the velocity of light ($v/c \ll 1$) then $\gamma \simeq 1$ and the transformation is known as approximate Lorentz transformation [14, page 11]

$$\mathbf{T} = \begin{pmatrix} 1 & -\frac{\beta}{c} & 0 \\ -v & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.5}$$

Note that here $\mathbf{T}_{01}$ has not been put zero (that would correspond to GT). This term appears in the TE multiplied by the space coordinate x. For a sufficiently large

x, βx may be comparable to c^2 and thus will not vanish. Also observe that under the small velocity approximation LT does not go over to the GT since a mere approximation cannot alter the synchronization convention. For this reason, here AZT=ALT since both uses the standard synchrony. This has been first pointed out by Ghosal, Nandi and Chakraborty [15].

Tangherlini Transformation

In standard relativity the clocks in a given frame are synchronized by light signals having isotropic one way velocity. All the clocks in any of the frames are connected to each other by light signals. No reference of clocks in another frame is required here. This method of synchrony is some time referred to as *system internal synchronization* [6].

As the synchronization of clocks in a given reference frame in Einstein method makes no reference to clocks outside the frame, the concept of simultaneity of two distant events becomes frame specific – two distance events simultaneous in one frame are not simultaneous in any other frame.

Is it possible to synchronize clocks in such a way that simultaneity relation between two distant events in two frames within relativistic domain is preserved? One method is described below:

We choose one inertial frame arbitrarily and synchronize the clocks by Einstein procedure. This is our *preferred* frame S_0. All other systems moving past S_0 will synchronize their clocks by adjusting these clocks to $t = 0$ whenever they fly past a clock in S_0 which reads $t_0 = 0$. To find the transformation matrix in this case we have to determine a, the synchronization parameter.

Recall that a is a rename of a_{01} of the original general transformation matrix. a_{01} is related to the zeroth and first coordinate, *i.e.* t and x. Clearly we can infer that there should not be any inter-connection between t and x if the clocks are synchronized in this manner. Thus $a_{01} = a = 0$. This gives $\xi = 0$. Putting this value in $c(0)$ (or $c(\pi)$) we obtain $\Omega = \gamma^{-1}$. Also, $\rho = 1$, giving $\phi = \gamma^{-1}$.

Thus we obtain the transformation matrix as

$$\mathbf{T} = \begin{pmatrix} \gamma^{-1} & 0 & 0 \\ -v\gamma & \gamma & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{B.4.6}$$

$\{\mathbf{T}\}_{01} = 0$ manifests the fact that the time transformation does not contain a space component, as stated.

This transformation was first derived by F. Tangherlini [10] and thus referred to in the literature as Tangherlini transformation (TT).

B.4.3 Discussion on TT

A study of TT shows that while the space transformation equation is similar to the LT, the time transformation equation does not contain a space part. The basic features of the TT are:

1. $\Delta t_0 = 0 \Longrightarrow \Delta t = 0$. This in turns implies that the simultaneity is *absolute*. Two spatially separated events simultaneous in the preferred frame are simultaneous in other frames. One can also find transformation from any frame to *any* (not necessarily from the preferred frame) other frame (see Sec. B.6 below) and satisfy oneself that the statement is also true for this case also. That is why this synchronization is called *absolute synchrony*.

2. With respect to the preferred frame we obtain the usual length contraction and time dilation effect. Thus it predicts *relativistic effects* but the

simultaneity is absolute! One sometimes interprets these as effects which are *real*, inherent in relativity theory. The preferred frame is often identified with that of 'ether' or a cosmologically preferred one [11, 16]. However the physical status of the arbitrarily singled-out frame in unimportant for the present work.

Absoluteness in simultaneity is possible if we are in classical world with the GT where the synchronization is achieved by signals moving with infinite speed.[4] But no confusion should arise because though both GT and TT give absolute simultaneity, GT, unlike TT, does not predict time dilation (as well as length contraction) and thus is not a relativistic transformation. [5]

Let us now find the OWS of light in the frame where *external synchronization* (or here, *the absolute synchronization*) was achieved. OWS of light at an angle θ with respect to the X-axis ($c(0)$ is the positive x-direction) is given by (B.3.3)

$$c(\theta) = \left[(1-\xi)\beta\cos\theta - \{1-(1-\rho)\sin^2\theta\}^{1/2}\right]^{-1}\chi.$$

Putting the parameter expressions suitable for TT,we obtain,

$$c(\theta) = \frac{c}{1+\beta\cos\theta}. \tag{B.4.7}$$

Thus the OWS' are given by

$$\left.\begin{aligned} c(0) &= \frac{c}{1+\beta} \\ c(\pi) &= \frac{c}{1-\beta} \end{aligned}\right\}. \tag{B.4.8}$$

Thus the OWS' of light in frames where the clocks are synchronized in absolute way are not c . We have already made use of this result in a Chap. 5.

[4]Note, however, that using a finite speed signal as light and making use of Einstein-like (pseudo-standard) synchrony leads to Zahar transformation in classical world.

[5]ZT however predicts an apparent length contraction and time dilation effects which is nothing but an artefact of the pseudo-standard synchrony.

Another important feature of TT is that a moving frame of reference is always related to the "preferred" frame in some way, even if the TT represents transformation between any two arbitrary (see B.6) reference frames. Also this particular form of TT is valid only between frames one of which must be the preferred one. Thus the transformations are not reciprocal like the LT. Consequently while a rod at rest in the moving frame with respect to the preferred frame, the rod gets contracted with respect to an observer in the preferred frame. On the contrary, a rod at rest in the "preferred" frame is elongated with respect to the moving frame by the same relativistic factor. The reason lies in the fact that definition of measurement of length of a moving rod depends on the definition of simultaneity which in turn depends on the clock synchronization.

This non-reciprocity occurs for time measurement too. Clocks moving with respect to the preferred frame runs slower but a clock at rest in the preferred frame runs faster (by the usual relativistic factor) with respect to the *moving frame*. This feature is generally ascribed to the fact that we have singled out (though arbitrarily) one system with a unique convention of OWS of light.

The spacetime diagram in this case is given in Fig. B.3 [6].

X_0 and T_0 represents space and time axes in the preferred frame. In the moving frame, like that in SR, x and t are not orthogonal. However the space axis X remains parallel to X_0 in this case. The figure also shows the fact that in the moving frame OWS' of light are not equal.

It is observed that one can use TT to handle non-inertial frames straightforwardly [12]. This feature of TT has already been discussed and used in earlier chapters.

TT was arrived at by different authors by different methods [6, 8, 10, 11, 17]. Our treatment in finding the general transformation matrix closely follows that of

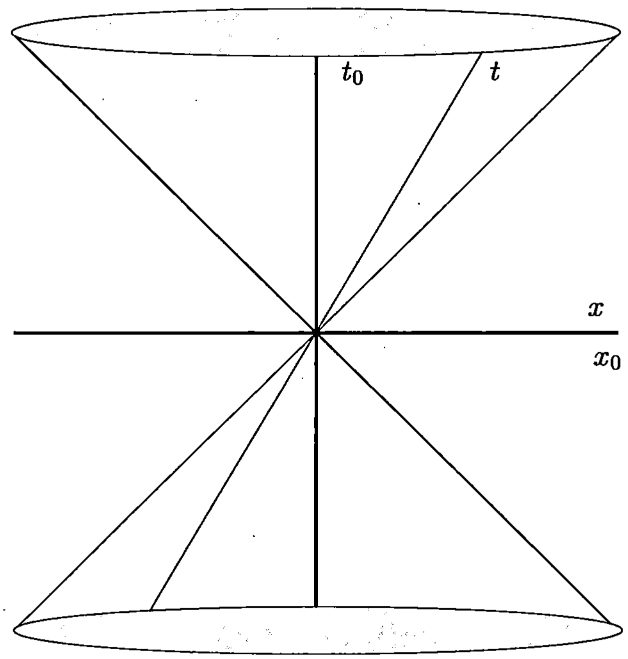


Figure B.3: Spacetime diagram for TT

Sjödin [11]. While Tangherlini himself obtained this by adjusting (among other things) the properties of line element, Ghosal, Mukhopdhyay and Chakraborty [8] obtained the same by studying the Dolphin transformations for arbitrarily large speed of the synchronizing signal.

Selleri [12] showed that while LT gives null result in the derivation of Sagnac effect from rotating frame perspective, TT gives the *correct* result. TT, however, has been obtained by him using his own inertial transformations. On commenting this, he termed TT (and *absolute synchronization*) as the *nature's choice* of synchrony.

B.4.4 OWS of light in Relativistic world

The OWS of light in any direction is given by Eq. (B.3.5)

$$c(\theta) = \left[(1 - \xi)\beta\cos\theta + \left\{1 - (1 - \rho)\sin^2\theta\right\}^{1/2}\right]^{-1} \chi. \tag{B.4.9}$$

Following our previous discussion we can set, for the relativistic world

$$\Omega = \phi = \gamma^{-1}.$$

This gives

$$\xi = c, \qquad \xi = -a\frac{c}{\beta}\gamma^{-1}.$$

Putting these values in Eq. (B.4.9), we obtain

$$\frac{1}{c(\theta)} = \frac{1}{c} + \left[\frac{\beta}{c} + a\gamma^{-1}\right]\cos\theta. \tag{B.4.10}$$

B.5 The Reichenbach Parameter ϵ

If we denote the OWS of light in positive x direction to be $\overrightarrow{c}$ then, $\theta = 0$ and we may write

$$\begin{aligned}\overrightarrow{c} &= [(1 - \xi) + 1]^{-1}\,\xi \\ &= (2 - \xi)^{-1}\gamma^{-2}\Omega^{-1}\phi^{-1}c.\end{aligned}$$

If one finds out $\overrightarrow{c}$ with the Reichenbach parameter then [1]

$$\overrightarrow{c} = \frac{c}{2\epsilon}$$

Thus

$$\epsilon = (1 - \xi/2)\gamma^2\Omega\phi. \tag{B.5.1}$$

This is the relation between ϵ and ξ. One may easily check that for relativistic world where $\Omega = \phi = \gamma^{-1}$, and $\xi = 1$, $\epsilon = 1/2$, the value of the Reichenbach parameter for the standard synchrony [18].

B.6 Composite Transformation Matrix

So far we have derived the transformation matrices from the 'preferred' frame S_0 to an inertial frame S. What will be the transformation matrix from one inertial to another inertial frame where none is chosen to be the 'preferred' one?

The answer is straightforward. Say we have two inertial frames S_1 and S_2 moving with velocities v_1 and v_2 respectively with respect to the ether frame. Then the transformation equations ($S_0 \to S_1$ and $S_0 \to S_2$) can be written as

$$\mathbf{X_1} = \mathbf{T_1 X_0}, \qquad \mathbf{X_2} = \mathbf{T_2 X_0}.$$

From this we find that the transformation $S_1 \to S_2$ is given by

$$\mathbf{X_2} = \mathbf{T_2 T_1^{-1} X_1} = \mathbf{T_{12} X_1}$$

where

$$\mathbf{T_{12}} = \mathbf{T_2 T_1^{-1}}.$$

The $\mathbf{T_i}$, $(i = 1, 2)$ matrices are characterized by adding a subscript i $(i = 1, 2)$ to all of the parameters which should be different for different frames, *i.e.* v_i, A_i, Ω_i, ϕ_i and ψ_i. The general $\mathbf{T_{12}}$ is given by

$$\mathbf{T_{12}} = \begin{pmatrix} P & Q & 0 \\ R & S & 0 \\ 0 & 0 & \psi_2^{-1}\psi_1 \end{pmatrix}, \tag{B.6.1}$$

where

$$\begin{aligned} P &= a_2\Omega_2^{-1}(v_1 - v_2) \\ Q &= \phi_1\Omega_2\left(a_2\Omega^{-1}a_1\Omega_1^{-1}\right) - (v_2 - v_1)a_1a_2\Omega_1^{-1}\Omega_2^{-1} \\ R &= \phi_2^{-1}\Omega_1^{-1}(v_1 - v_2) \\ S &= \phi_1\phi_2^{-1}(1 - a_1\Omega_1^{-1}(v_1 - v_2)). \end{aligned}$$

One must note here that a composite transformation is equivalent to a single transformation. Thus one can find the velocity transformation laws by comparing the equivalent terms in these two matrices [19].

B.7 Synchronization with Non-luminal Signal

In the earlier sections, different transformation have been discussed for different choices of the free parameter 'a'. For a given world, different choices of 'a' gives different OWS' of light, however the TWS remains the same since the latter is independent of the synchrony. Let us now take a look at the standard derivations of the LT. In the standard SR light has two roles to play. On the one hand it acts as a synchronizing agent, on the other hand it has invariant TWS in vacuum. (Indeed the relativistic world is defined by the existence of an invariant TWS). The second role has a basis in the empirically verifiable property, but the first one is purely prescriptive in origin. In the derivation of the LT in the standard SR, these two roles are mixed up. This inseparability contributes to several misconceptions and prejudices in relativity theory. In order to separate these roles one may introduce non-luminal signal to synchronize clocks and rederive transformation equations. This has been done by Ghosal, Mukhopadhyay and Chakraborty [8]. In their derivation of relativistic transformation equation, the authors considered reference frames submerged in a substrate. The clocks are synchronized by some signal mode characteristic of the substratum. Without any loss of generality one may assume it to be the acoustic signal (AS).

The authors first considered an acoustic wave generated at $t = 0$ at the common origin of the frames S_i and S_k. In all other frames except for the frame S_0 which is at rest relative to the substratum, the velocity of AS in the positive x-direction and negative x-direction will not be the same. However, according to the CS thesis, it is possible to define the synchronization of clocks so that these two velocities are equal in all the frames (although the values for these velocities are in general different in different frames). This synchrony is called (by the authors) the *pseudo-standard synchrony (PSS)* as opposed to the *Einstein (standard) synchrony.*

Now, according to pseudo-standard synchrony, the one dimensional wave front equation will be a pair of straight lines:

$$x_k^2 = a_{kx}^2\, t_k^2, \tag{B.7.1}$$

where x_k's are co-ordinates of a frame S_k which is moving with respect to S_0 frame which is fixed in the substrate and a_{kx} is the TWS of the AS in the x-direction.

The acoustic wave front will not be spherical in frames other than in S_0 frame, TWS of AS will not be the same in all direction. So, along the y-axis, say, one has to write

$$y_k^2 = a_{ky}^2\, t_k^2,$$

where, a_{ky} is the TWS of AS y-direction and may be different from a_{kx}.

B.7.1 The Derivation of Transformation Equation

In order to derive the TE, one first writes the TE in this usual linear form,

$$\begin{aligned} x_k &= \alpha(x_i - v_{ik}t_i), \\ y_k &= y_i, \\ t_k &= \xi_{ik}x_i + \beta_{ik}t_i. \end{aligned} \tag{B.7.2}$$

This is a set of transformation equations between two general inertial frame S_i and S_k. v_{ik} is the relative velocity between the two. α_{ik}, ξ_{ik} and β_{ik} are constant to be determined.

The observers within the substrate cannot use 3-sphere wave front of the AS wave as this will not be spherical in a general frame. But according to the chosen synchrony of the form (B.7.1) one can subject the TE to the condition

$$x_k^2 - a_{kx}^2 t_k^2 = \lambda_{ik}^2 (x_i^2 - a_{ix}^2 t_i^2), \tag{B.7.3}$$

where λ_{ik} is a scale factor and is independent of the space and time co-ordinates. One can now put Eq. (B.7.2) in Eq. (B.7.3) to obtain the transformation coefficients as

$$\begin{aligned} \alpha_{ik} &= \lambda_{ik}\gamma_{ik}, \\ \beta_{ik} &= \alpha_{ik}/\rho_{ik}, \\ \xi_{ik} &= -\frac{\alpha_{ik}/\rho_{ik}}{v_{ik}/a_{ix}^2}, \end{aligned} \tag{B.7.4}$$

where,

$$\begin{aligned} \rho_{ik} &= a_{kx}/a_{ix}, \\ \gamma_{ik} &= \left(1 - \frac{v_{ik}^2}{a_{ix}^2}\right)^{-1/2} \end{aligned} \tag{B.7.5}$$

and the TE for $S_i \to S_k$ can be written as

$$\begin{aligned} x_k &= \lambda_{ik}\gamma_{ik}\left(x_i - v_{ik}t_i\right), \\ t_k &= \frac{\lambda_{ik}}{\rho_{ik}}\gamma_{ik}\left(t_i - \frac{v_{ik}}{a_{ix}^2}x_i\right). \end{aligned} \tag{B.7.6}$$

The notable points about this set of equations are the following –

1. The TE contain TWS of synchronizing signal in the frames S_i and S_k.
2. The factor γ_{ik} resembles the relativistic γ factor with c replace by the velocity of the AS.
3. Simultaneity is relative.

All these are consequences of the PSS. Also, one can find

$$v_{ki} = -\rho_{ik} v_{ik}$$

which means that under this synchrony relative speeds are not symmetric as $\rho_{ik} \neq 1$ in general.

Note also that a mere change of subscripts give transformation between any two other inertial systems. Also one can easily work out the velocity transformation formula

$$v_{kl} = \frac{a_{kx}}{a_{ix}} \frac{v_{il} - v_{ik}}{1 - \dfrac{v_{ik} v_{il}}{a_{ix}^2}} \tag{B.7.7}$$

by comparing transformations $S_l \to S_k$ and $S_l \to S_i \to S_k$. This also gives a very important result

$$\lambda_{kl} = \frac{\lambda_{il}}{\lambda_{ik}}.$$

λ's are yet unknown quantities. Note that the TWS of AS is isotropic in the preferred frame S_o which is stationary with respect to the medium. But in a general frame S_k it will not be isotropic. If the isotropic signal speed is a_0, we may write

$$a_x^2 + a_y^2 = a_0^2.$$

The TWS in $x-$directions in S_k is given by

$$a_{kx} = \frac{\alpha_{0k} a_0 \left(1 - \dfrac{v_{0k}^2}{a_0^2}\right)}{\beta_{0k} + v_{0k}} \tag{B.7.8}$$

and,

$$a_{ky} = \frac{a_0 \left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\beta_{0k} + \xi_{0k} v_{0k}} \tag{B.7.9}$$

Also, the general TWS transformation laws for any *other* signal whose *isotropic*

TWS (equal to its OWS) in S_0 is a_0' (which may differ from a_0) can be written as

$$\text{TWS (longitudinal)} \quad a_{kx}' = \frac{\alpha_{0k} a_0' \left(1 - v_{0k}^2/a_0'^2\right)}{\beta_{0k} + \xi_{0k} v_{0k}},$$

$$\text{TWS (transverse)} \quad a_{ky}' = \frac{a_0' \left(1 - v_{0k}^2/a_0'^2\right)^{1/2}}{\beta_{0k} + \xi_{0k} v_{0k}}, \tag{B.7.10}$$

where a_{kx}' and a_{ky}' are the TWS's of the *other* signal as measure from in the longitudinal and the transverse directions respectively. It has been tacitly assumed here that in S_0 the PSS with AS and that with the *other* signal are equivalent.

From Eqs. (B.7.4), (B.7.9) and (B.7.8)

$$\lambda_{0k} = \frac{a_{kx}}{a_{ky}}. \tag{B.7.11}$$

Also

$$\lambda_{ik} = \frac{\lambda_{0k}}{\lambda_{0i}} = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \tag{B.7.12}$$

Putting the values of the unknown parameter λ_{ik} , the complete transformation equations between any two inertial frames $S_i \to S_k$ become

$$x_k = \frac{a_{kx}}{a_{ky}} \frac{a_{iy}}{a_{ix}} \frac{x_i - v_{ik} t_i}{\left(1 - v_{ik}^2/a_{ix}^2\right)^{1/2}},$$

$$t_k = \frac{a_{ik}}{a_{ky}} \frac{t_i - (v_{ik}/a_{ix}^2) x_i}{\left(1 - v_{ik}^2/a_{ik}^2\right)}. \tag{B.7.13}$$

In the preferred frame S_0, $a_{0x} = a_{0y} = a_0$. The TE from S_0 to any other inertial frame S_k is given by

$$x_k = \frac{a_{kx}}{a_{ky}} \frac{x_0 - v_{0k} t_0}{\left(1 - v_{0k}^2/a_{0k}^2\right)^{1/2}},$$

$$t_k = \frac{a_0}{a_{ky}} \frac{t_0 - (v_{0k}/a_0^2) x_0}{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}. \tag{B.7.14}$$

The authors named this set of transformation equations *Dolphin transformation* (DT) in a lighter vein because this will be the TE found by some intelligent dolphins who are assumed to reside within the substratum of sea water (say).

Note that DT in the form presented above (Eqs. (B.7.13) and (B.7.14)) is usable provided one knows the TWS of AS in the two frames concerned. If now one chooses to use light signal (in vacuum) instead of AS, by virtue of CVL postulate of SR one would obtain the familiar LT because now

$$a_{ix} = a_{iy} = a_{kx} = a_{ky} = c.$$

It looks rather surprising that here c, *apparently*, is not playing any important role in absence of any communication with the outside world even though the dolphins live in the relativistic world! But c has a fundamental role to play in relativity that its TWS is constant in vacuum. Indeed in the DT, c will appear as a *physical constant* through a_{kx} and a_{ky}.

To make use of DT, the Dolphins will have to know the TWS of AS in S_k as function of v_{0k} and one can anticipate that they will eventually find that

$$\begin{aligned} a_{kx} &= a_{kx}(v_{0k}, c), \\ a_{ky} &= a_{ky}(v_{0k}, c), \end{aligned} \tag{B.7.15}$$

where c appears as some *physical constant*. If now the dolphins are able to communicate with the outside world and discover that their world admits an invariant speed c, they would set, in Eq. (B.7.10),

$$a'_{kx} = a'_{ky} = a'_0 = c.$$

One can now find, using Eq. (B.7.4)

$$\rho_{0k} = \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)},$$

$$\lambda_{0k} = \frac{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\left(1 - v_{0k}^2/c^2\right)^{1/2}}. \tag{B.7.16}$$

Also, by using Eq. (B.7.5) and Eq. (B.7.11) one obtains

$$a_{kx} = a_0 \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)},$$

$$a_{ky} = a_0 \frac{\left(1 - v_{0k}^2/a_0^2\right)^{1/2}}{\left(1 - v_{0k}^2/c^2\right)^{1/2}}. \tag{B.7.17}$$

Eq. (B.7.17) conforms to our previous assertion as expressed in Eq. (B.7.15). Now inserting Eq. (B.7.17) in Eq. (B.7.14) we find the DT in frames $S_0 \to S_k$ in the relativistic world

$$x_k = \frac{x_0 - v_{0k} t_0}{\left(1 - v_{0k}^2/c^2\right)^{1/2}},$$

$$t_k = \frac{\left(1 - v_{0k}^2/c^2\right)^{1/2}}{\left(1 - v_{0k}^2/a_0^2\right)} \left[t_0 - \frac{v_{0k}}{a_0^2} x_0\right]. \tag{B.7.18}$$

From the above TE, the length contraction factor (LCF) and time dilation factor (TDF) come out to be

$$\text{LCF} = \frac{\left(1 - v_{0k}^2/a_0^2\right)}{\left(1 - v_{0k}^2/c^2\right)^{1/2}},$$

$$\text{TDF} = \frac{\left(1 - v_{0k}^2/c^2\right)^{1/2}}{\left(1 - v_{0k}^2/a_0^2\right)}. \tag{B.7.19}$$

Note that with respect to S_0, LCF and TDF will be the same as predicted by SR. This is not surprising because of the assertion that in S_0, the PSS and the Einstein synchrony coincide.

There are two very important consequences of DT and Eq. (B.7.19). These are the following –

1. There has been a considerable debate since the birth of relativity on reality of length contraction and time dilation. There are authors who believe that these effects are solely dependent on the *relativity of simultaneity* (for a discussion see also Ref. [11]). Earlier in this chapter we have shown that this is not so – different synchronization procedure may not have relativity of simultaneity but predict these effects. Eq. (B.7.19) further clarifies this. Observe that in Eq. (B.7.19), a_0 is the speed of AS, so it is *conventional.* c appears as a *physical constant* – the TWS of light – and is not based on any convention. The factor $\left(1 - v_{0k}^2/c^2\right)^{1/2}$ is due to real effects. The other factor, $\left(1 - v_{0k}^2/a_0^2\right)$ arises from the synchronization procedure which is evident from the presence of the term a_0.

2. As we have mentioned earlier that light has two roles to play in SR. One is that its TWS in vacuum is constant and the other is that it is the synchronization agent in SR. These two roles are mingled up beyond recognition in standard SR. In the derivation of DT we see that these two roles are clearly split up.

We shall show below that how some important transformation equations in relativistic and classical worlds can be obtained from DT by the choice and making use of the properties of the synchronization signal.

Relativistic world: Lorentz transformation

In standard synchrony the synchronization agent is light. Putting $a_0 = c$ in Eq. (B.7.18) one may obtain Lorentz transformation.

Relativistic world: Tangherlini transformation

If in the *preferred frame* the speed of synchonization signal $a_0 \to \infty$, then we obtain, (for $S_0 \to S_k$) the Tangherlini transformation.

Classical world: Zahar transformation

In classical world the velocity addition law is the Galilean one. Then the TWS of AS is obtained to be

$$a_{kx} = a_0 \left(1 - v_{0k}^2/a_0^2\right),$$
$$a_{ky} = a_0 \left(1 - v_{0k}^2/a_0^2\right)^{1/2}. \tag{B.7.20}$$

Inserting these in Eq. (B.7.14), we obtain DT ($S_0 \to S_k$) in the classical world.

$$x_k = x_0 - v_{0k}t_0,$$
$$t_k = \frac{t_0 - \left(v_{0k}/a_0^2\right)x_0}{1 - v_{0k}^2/a_0^2}. \tag{B.7.21}$$

This is Zahar transformation with PSS. If $a_0 = c$, *i.e.* standard synchrony is adopted, we obtain the Zahar transformation in its familiar form.

Classical world: Galilean transformation

If it is possible to synchronization with instantaneous signals, then we can put $a_0 \to \infty$ in Eq. (B.7.21) and the familiar from of GT is retrieved.

B.8 Epilogue

In his historic 1905 paper Einstein recognized the element of convention in the empirical measurement of the OWS of light. He chose the isotropy of OWS of light and considered a convenient value of OWS which equated the TWS of light, an experimentally measurable quantity. His synchronization procedure with light, which we have mentioned earlier as Einstein synchronization procedure, led to relativity of simultaneity. All this appeared so 'natural' that even today most of the textbook authors do not recognize the element of convention in the synchronization procedure. The first one who gave the conventionality its proper emphasis was Reichenbach in 1928. The whole CS thesis grew up to its present form due to work of Gruünbaum (and others) on the theory of Reichenbach.

Though a different transformation other than the LT for SR was first found out by Tangherlini in 1961, a general transformation which might incorporate all types of synchronization schemes was found first by Winnie in 1970. Later works were carried out by Mansouri and Sexl in 1977, Sjödin in 1979 and Selleri in 1995. They took different routes to reach the same goal. In 1977 Zahar did a remarkable job in our understanding of conventionality thesis by incorporating CS thesis in classical world to obtain a transformation where clocks are synchronized in Einstein procedure.

Our treatment of derivation of the TE (except DT) presented in the first few section of this appendix largely follows that of Sjödin. Though the approach is different, Selleri achieved the same transformation independently (inertial transformation) in a different form. It can be written as [12]

$$\begin{aligned}
x &= \frac{x_0 - \beta c t_0}{R(\beta)}, \\
y &= y_0, \\
z &= z_0, \\
t &= R(\beta) t_0 + \epsilon(x_0 - \beta c t_0) + e(y_0 + z_0).
\end{aligned} \tag{B.8.1}$$

Here ϵ, e are two underdetermined functions of velocity v and $\beta = v/c$, and $R(\beta) = \sqrt{1-\beta^2}$ (we have changed some notations for consistency). In its final form (with $e = 0$, for rotational invariance around x axis) Selleri's ϵ parameter and our 'a' parameter is the same.

The ϵ in ϵ-Lorentz transformation derived by Winnie is the Reichenbach parameter. The transformation equations are given by [2]

$$\begin{aligned}
x' &= \alpha^{-1}(x - \vec{v}_\epsilon t) \\
t' &= \alpha^{-1}\left[2\vec{v}_\epsilon c^{-1}(1 - \epsilon - \epsilon') + 1\right] - x c^{-2}\left[2c(\epsilon - \epsilon') + 4 v_\epsilon(\epsilon)(1 - \epsilon)\right],
\end{aligned} \tag{B.8.2}$$

where

$$\alpha = \frac{\sqrt{(c - \vec{v}_\epsilon(2\epsilon - 1))^2 - \vec{v}_\epsilon^{\,2}}}{c},$$

and ϵ and ϵ' are Reichenbach parameters in the two frames. The main difference between ϵ- LT and transformations derived by Sjödin and by Selleri is that ϵ-LT is derived in $(1+1)$ dimensions while the other two are derived in $(1+2)$ and $(1+3)$ dimensions respectively. Unless at least two space dimensions are taken into account, the concept of *isotropy* of TWS of light is devoid of any meaning and some subtleties of the relativistic physics have to be sacrificed [4, 5].

Ghosal Mukhopdhyay and Chakraborty first split up the two roles of light to find the dolphin transformation by introducing pseudo-standard synchrony. In this way they also identified the real and the synchrony dependent part in the length contraction and time dilation factors. Introduction of signal synchrony (luminal or non-luminal) give different transformation equation thus making this set of equations an interesting one by its ability to reveal true relativistic effects from apparent ones.

Reference

[1] A. Winnie, *Phil. Sc.*, **37**, (1970) pp. 81–99.

[2] A. Winnie, *Phil. Sc.*, **37**(2), (1970) p. 223.

[3] A. Ungar, *Phil. Sci.*, **53**, (1986) pp. 395–402.

[4] Papia Chakraborty, *On Some Foundational Questions in Relativity and Gravitation*, Unpublished, University of North Bengal (1993).

[5] S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.*, **15**(4), (1991) pp. 369–374.

[6] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 497–513.

[7] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977) pp. 515–524.

[8] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994) pp. 21–28.

[9] E. Zahar, *British Journal of Philosophy of Science*, **28**, (1977) pp. 195–213.

[10] F. Tangherlini, *Nuovo Cimento Suppl.*, **20**, (1961) pp. 1–86.

[11] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979) pp. 229–245.

[12] F. Selleri, *Found. Phys.*, **26**, pp. 641–664.

[13] C. Møller, *The Theory of Relativity* (Oxford University Press, New Delhi, 1972), second edition edition.

[14] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields* (Butterworth Heinemann, Oxford, 2002), 4th revised english edition edition.

[15] S.K. Ghosal, K. K. Nandi and P. Chakraborty, *Z. Naturforsch*, **46a**, (1991) pp. 256–258.

[16] G. Spinelli, *Il Nuovo Cimento B*, **75**, (1983) p. 11.

[17] F. Selleri, *Found. Phys. Lett.*, **10**, (1997) pp. 73–83.

[18] H. Reichenbach, *The Philosophy of Space and Time* (Dover, NewYork, 1957), 2nd edition.

[19] J. Aharoni, *The Special Theory of Relativity* (Clarendon, Oxford, 1965).

Bibliography

[1] J. Aharoni, *The Special Theory of Relativity* (Clarendon, Oxford, 1965).

[2] D. W. Allan *et al.*, *IEEE Trans. Instrum. Meas.*, **IM-34**(2), (1985), pp. 118.

[3] J. Anandan, *Phys. Rev. D*, **24**, (1981), pp. 338.

[4] R. Anderson, H. R. Bilger and G. E. Stedman, *Am. J. Phys.*, **62**(11), (1994), pp. 975.

[5] R. Anderson, I. Vetharaniam and G. E. Stedman, *Conventionality of Synchronisation, Gauge Depencence and Test Theories of Relativity, Physics Reports*, **295**, (1998), pp. 93, this review article has an extensive list of references (240 in number).

[6] Neil Ashby, *The Sagnac Effect in the Global Positioning System*, in G. Rizzi and M. L. Ruggiero, eds., *Relativity in Rotating Frame*, Fundamental Theories of Physics (Kluwer, Dordrecht, 2004).

[7] A. Ashtekar and A. Magnon, *J. Math. Phys.*, **16**(2), (1975), pp. 341.

[8] Aniket Basu, R. S. Saraswat, Kedar B. Khare, G. P. Sastry and Sougato Bose, *Eur. J. Phys.*, **23**, (2002), pp. 295.

[9] P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall of India, New Delhi, 1992).

[10] I. L. Bershtein, *Dokl. Akad. Nauk. SSSR*, **75**, (1950), p. 635.

[11] Max Born, *Ann. Physik*, **30**, (1909), p. 840.

[12] S. K. Bose, *General Theory of Relativity* (Wiley Eastern, New Delhi).

[13] R. W. Brehme, *Am. J. Phys.*, **53**(1), (1985), pp. 56.

[14] R. W. Brehme, *Am. J. Physics.*, **56**(9).

[15] Bridgeman, *A Sophisticate's Primer to Relativity* (Wesleyan University, Middletown, 1962).

[16] T. Budden, *Found. Phys. Lett.*, **11**, (1998), p. 4.

[17] G. Cavalleri, *Nuovo Cimento B*, **53**, (1968), pp. 415.

[18] Papia Chakraborty, *On Some Foundational Questions in Relativity and Gravitation*, Unpublished, University of North Bengal (1993).

[19] S. Chandrasekhar, *The Mathematical Theory of Black Holes*, Oxford Classic Texts in the Physical Sciences (Clarendon, Oxford, 1985).

[20] R. Colella, A. W. Overhauser, J. L. Staudenmann and S.A. Werner, *Phys. Rev. A*, **21**, (1980), pp. 1419.

[21] Maria B. Cranor, Elizabeth M. Heider and Richard H. Price, *Am. J. Phys.*, **68**(11), (2000), pp. 1016.

[22] Talal A. Debs and Michael L. G. Redhead, *Am. J. Physics*, **64**(4), (1996), pp. 384.

[23] P. Di Mauro, *La formula di diffusione Compton con al meccanica Newtoniana*, Atti del XVI Congresso Nazionale di Storia della Fisica e dell' Astronomia, Como 28-29 , 179-184 (Maggio 1999).

[24] D. Dieks, *Eur. J. Phys.*, **12**, (1991), pp. 253.

[25] D. Dieks and G. Nienhuis, *Am. J. Phys.*, **58**, (1990), pp. 650.

[26] A. Dufour and F. Prunier, *Compt. Rend.*, **204**, (1937), p. 1322.

[27] A. Dufour and F. Prunier, *Compt. Rend.*, **205**, (1937), p. 658.

[28] A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, 1965).

[29] Paul Ehrenfest, *Phys. Z.*, **10**, (1909), p. 918.

[30] A. Einstein, *Astron. Nach.*, **198**, (1914), p. 378.

[31] A. Einstein, *Relativity: the Special and the General Theory* (Methuen, London, 1960).

[32] Albert Einstein, *On Electrodynamics of Moving Body, Annalen Der Physik*, **17**(891), (1905), pp. 1, the paper is now freely available at `http://www.fourmilab.ch/`.

[33] Albert Einstein, *Ann. d. Phys.*, **23**, (1907), p. 371.

[34] Albert Einstein, *The Meaning of Relativity* (Oxford University Press, New Delhi, 1951).

[35] Albert Einstein, *Einstein's Miraculous year: Five Papers That Changed the Face of Physics* (Scientia, New Delhi, 1998), edited and introduced by John Stachel, foreward by Roger Penrose.

[36] B. Ellis and P. Bowman, *Phil. Sci.*, **34**, (1967), p. 116.

[37] Herman Erlichson, *Am. J. Phys.*, **53**(1), (1985), pp. 53.

[38] Aurthur Evett, *Understanding the Spacetime Concepts of Special Relativity* (Publishers Creative Services, New York, 1982).

[39] D. Fargion, L. Chiatti and A. Aiello, *Quantum Mach effect by Sagnac phase shift on Cooper pair in rf-SQUID*, preprint astro-ph/9606117, babbage.sissa.it.

[40] F. de Felice and S. Usseglio-Tomasset, *Class. Quant. Grav.*, **8**, (1991), pp. 1871.

[41] S. F. Fung and K. C. Hsieh, *Am. J. Phys.*, **48**, (1980), p. 654.

[42] S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.*, **15**(4), (1991), pp. 369.

[43] S. K. Ghosal, D. Mukhopadhyay and Papia Chakraborty, *Eur. J. Phys.*, **15**, (1994), pp. 21.

[44] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Rotating Disc Problem and Sagnac Phase-Shift formula*, in M. C. Duffy, ed., *Physical Interpretations of Relativity Theory, VIII* (P. D. Publication, Liverpool, 2000), held at Imperial College London.

[45] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Relativistic Sagnac Effect and Ehrenfest Paradox, Found. Phys.*, **33**(6), (2003), pp. 981.

[46] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys. Lett.*, **17**(5), (2004), pp. 457.

[47] S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhury and Minakshi Sarker, *Found. Phys. Lett.*, **16**(6), (2003), pp. 549.

[48] S. K. Ghosal, Biplab Raychaudhuri, Minakshi Sarker and Saroj Nepal, *On Synchronization and Desynchronization in Rotating Frames and a Recent*

Paradox, in M. C. Duffy, ed., *Physical Interpretation of Relativity Theory (IX)* (2004), held at Imperial College, London (To be Published).

[49] S.K. Ghosal, K. K. Nandi and P. Chakraborty, Z. *Naturforsch*, **46a**, (1991), pp. 256.

[50] S.K. Ghosal and Minakshi Sarker, *Cosmologically Preferred Frame and Sagnac Effect*, in M. C. Duffy, ed., *Physical Interpretation of Relativity Theory (VII)*, vol. Late Papers (1998), held at Imperial College, London.

[51] F. Goy, *Non-invariant Velocity of Light and Clock Synchronization in Accelerated Systems*, in M. C. Duffy, ed., *Physical Interpretation of Relativity Theory (V)*, pp. 129–138 (1996), held at Imperial College, London.

[52] D. M. Greenberger, *Rev. Mod. Phys.*, **55**, (1983), p. 875.

[53] Ø Grøn, *Am. J. Phys.*, **43**, (1975), pp. 869.

[54] Ø. Grøn, *Space Geometry in Rotating Reference Frames: A Historical Appraisal*, in G. Rizzi and M. L. Ruggiero, eds., *Relativity in Rotating Frames*, Fundamental Theories of Physics (Kluwer, New York, 2004), series Editor: A. van der Merwe.

[55] A. Grünbaum, W. Salmon, B. van Fraassen and A. Janis, *A Panel Discussion of Simultantiey by Slow Clock Transport in the Special and General Theories of Relativity.*, *Phil. Sc.*, **36**, (1969), pp. 1.

[56] Adolf Grünbaum, *David Malament and the Conventionality of Simultaneity: A Reply*, available at philsci-archive.pitt.edu/archive/00000184/00/malament.pdf.

[57] Adolf Grünbaum, *Philosophical Problems of Space and Time*, 2 edn. (Alfred Knopf, New York, 1973).

[58] J. C. Hafele and R. E. Keating, *Science*, **177**, (1972), p. 166.

[59] Francis Harress, Ph.D. thesis, Jena (1912).

[60] P. Harzer, *Astron. Nach.*, **198**, (1914), p. 378.

[61] H. Hasselbach and M. Nicklaus, *Phys. Rev. A*, **48**, (1993), pp. 143.

[62] A. Janis, *Conventionality of Simultaneity in Stanford Encyclopedia of Philosophy*, The Metaphysics Research Lab, Center for the Study of Language and Information, Stanford University, available at http://plato.stanford.edu/entries/spacetime-convensimul.

[63] R. Klauber, *Toward a Consistent Theory of Relativistic Rotation*, in G. Rizzi and M. L. Ruggiero, eds., *Relativity in Rotating Frames*, Foundations of Physics (Kluwer, Dordrecht, 2004), series Editor: A. van der Merwe.

[64] R. D. Klauber, *Found. Phys. Lett.*, **11**, (1998), pp. 405.

[65] R. D. Klauber, *Am. J. Phys.*, **67**, (1998), pp. 158.

[66] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th revised english edition edn. (Butterworth Heinemann, Oxford, 2002).

[67] P. Langevin, *Compt. Rend.*, **173**, (1921), p. 831.

[68] P. Langevin, *Compt. Rend.*, **205**, (1937), p. 51.

[69] Oliver Lodge, *Philos. Trans. R. Soc. (London)*, **184A**, (1893), p. 727.

[70] Oliver Lodge, *Philos. Trans. R. Soc. (London)*, **189A**, (1897), p. 149.

[71] A. A. Logunov and Yu. V. Chugree, *Sov. Phys. Uspekhi*, **31**, (1988), p. 861.

[72] W. M. Macek and D. T. M Davis., *Appl. Phys. Lett.*, **2**, (1963), p. 67.

[73] D. Malament, *Noûs*, **11**, (1977), p. 293.

[74] G. B. Malykin, *Sagnac Effect, Physics-Uspekhi*, **40**(3), (1997), pp. 317.

[75] Reza Mansouri and Roman Sexl, *Gen. Rel. Grav.*, **8**, (1977), p. 809.

[76] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977), pp. 497.

[77] Reza Mansouri and Roman U Sexl, *Gen. Rel. Grav.*, **8**(7), (1977), pp. 515.

[78] S. Merhab, *Aerospace Sensor Systems and Applications* (Springer, New York, 1996).

[79] A. A. Michelson, *Philos. Mag*, **8**(56), (1904), p. 716.

[80] E. Minguzzi, *Found. Phys. Lett.*, **15**(2), (2002), pp. 153, available at arXiv:gr-qc/0103049 v3.

[81] E. Minguzzi, *The Gauge Nature of Simultaneity*, `arXiv:gr-qc/0204063` (April 2002).

[82] Charles W. Misner, Kip S. Thorne and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Fransisco, 1972).

[83] C. Møller, *The Theory of Relativity*, second edition edn. (Oxford University Press, New Delhi, 1972).

[84] M. Yamamoto N. Saburi and K. Harada, *IEEE Trans. Instr. Meas.*, **IM-25**, (1976), pp. 473.

[85] C. Nissim-Sabat, *Am. J. Phys.*, **50**, (1982), p. 533.

[86] G. Petit and G. Wolf, *Astron. Astrophys.*, **286**, (1994), pp. 971.

[87] B. W. Petley, *Contemp. Phys.*, **21**, (1980), p. 607.

[88] Jr. T. E. Phipps, *NOLTR*, **47**, (1973), pp. 73.

[89] B. Pogany, *Ann. der Phys.*, **80**, (1926), p. 217.

[90] B. Pogany, *Naturwissenschaften*, **15**, (1927), p. 177.

[91] B. Pogany, *Ann. der Phys.*, **85**, (1928), p. 244.

[92] H. Poincaré, *The Value of Science* (Dover, New York, 1958).

[93] E. J. Post, *Sagnac Effect, Rev. Mod. Phys.*, **39**, (1967), p. 475.

[94] J. R. Pounder, *Comm. Dublin Institute for Advanced Studies A*, **11**.

[95] H. Reichenbach, *The Philosophy of Space and Time*, 2nd edn. (Dover, NewYork, 1957).

[96] R. Resnick, *Introduction to Special Relativity* (John Wiley and Sons (Asia), Singapore, 1998).

[97] F. Riehle, Th. Kister, A. Wittie, J. Helmcke and ch. J. Borde, *Phys. Rev. Lett.*, **67**, (1991), pp. 177.

[98] G. Rizzi and A. Serafini, *Synchronization and Desynchronization in Rotating Platforms*, in G. Rizzi and M. L. Ruggiero, eds., *Relativity in Rotating Frames*, Foundations of Physics (Kluwer Academic, Dordrecht, 2004), series Editor: A. van der Merwe.

[99] G. Rizzi and A. Tartaglia, *Found. Phys.*, **28**, (1998), pp. 1663.

[100] G. Rizzi and A. Tartaglia, *Found. Phys. Lett.*, **12**(2), (1999), pp. 179.

[101] A. A. Robb, *A Theory of Time and Space* (Cambrige: The University Press, Cambridge, 1914).

[102] Rodrigues and Sherif, *Found. Phys.*, **31**, (2001), pp. 1767, math-ph/0302008, 4 Feb 2003.

[103] A. H. Rosenthal, *J. Opt. Soc. Am.*, **52**, (1962), p. 1143.

[104] Robert Rynasiewicz, *Is Simultaneity Conventional Despite Malament's Result?*, Available at http://philsci-archive.pitt.edu/archive/00000293.

[105] G. Sagnac, *Compte. Rendus.*, **141**, (1905), p. 1220.

[106] G. Sagnac, *Comp. Rend. Acad. Sci.*, **157**, (1913), pp. 708.

[107] G. Sagnac, *Compt. Rend.*, **157**, (1913), p. 1410.

[108] G. Sagnac, *J. Phys. (Paris)*, **4**, (1914), p. 177.

[109] G. Sagnac, *J. Phys. Radium Ser.*, **4**, (1914), pp. 177.

[110] B. F. Schutz, *A first course of General Relativity* (Cambridge University Press, Cambridge, 1987).

[111] F. Selleri, *Found. Phys.*, **26**, , pp. 641.

[112] F. Selleri, *Found. Phys. Lett.*, **10**, (1997), pp. 73.

[113] F. Selleri, in F. Selleri, ed., *Open Question in Relativistic Physics*, pp. 69–80 (Apeiron, Montreal, 1998).

[114] F. Selleri, *Found. Phys. Lett.*, **17**, (2004), pp. 599.

[115] F. Selleri, *Sagnac Effect: End of a Mystery*, in G. Rizzi and M. L. Ruggiero, eds., *Relativity in Rotating Frame*, Fundamental Theories of Physics (Kluwer Academic, New York, 2004), series Editor: A. van der Merwe.

[116] C. W. Sherwin, *Three Theories of Relative Measurement* (Physical Interpretation of Relativity Theory Meeting (III), Imperial College, London, 1992).

[117] T. Sjödin, *Il Nuovo Cimento B*, **51**, (1979), pp. 229.

[118] Yu. I. Sokolvsky, *The Special Theory of Relativity* (Hindustan Publishing, New Delhi, 1962).

[119] G. Spinelli, *Il Nuovo Cimento B*, **75**, (1983), p. 11.

[120] J. Statchel, *Einstein and the Rigidly Rotating Disk*, in A. Held, ed., *General Relativity and Gravitation: One Hundred Years after the Birth of Alber Einstein*, vol. I, pp. 1–15 (Plenum, New York).

[121] G. E. Stedman, *Contemp. Phys.*, **26**, (1985), p. 311.

[122] J. L. Synge, *Studies in Mathematics and Mechanics: Presented to Richard von Mises by Friends, Colleagues and Pupils* (1954), new York.

[123] F. Tangherlini, *Nuovo Cimento Suppl.*, **20**, (1961), pp. 1.

[124] A. Tartaglia, *Phys. Rev. D*, **58**, (1998), p. 064009.

[125] A. Tartaglia, *Found. Phys. Lett.*, **12**(1), (1999), pp. 17.

[126] A. Tartaglia and M. L. Ruggiero, *Sagnac Effect and pure geometry*, `gr-qc/0401005v1` (January 2004).

[127] F. F. Taylor and J. A. Wheeler, *Space-time Physics* (Freeman, San Fransisco, 1963).

[128] Roberto Torretti, *Relativity and Geometry* (Pergamon Press, Oxford, 183).

[129] B. Townsend, *Am. J. Phys.*, p. 1092.

[130] V. A. Ugarov, *Special Theory of Relativity* (Mir Publisher, Moscow, 1979), translated from Russian by Yuri Atanov.

[131] A. Ungar, *Phil. Sci.*, **53**, (1986), pp. 395.

[132] A. Ungar, *Am. J. Phys.*, **56**(9), (1988), p. 814.

[133] V. Vali, R. W. Shorthill and M. F. Berg, *Appl. Opt.*, **16**, (1977), p. 2605.

[134] V. Varićak, *Phys. Z.*, **12**, (1911), p. 169.

[135] V. Vyschtskii *et al.*, *Phys. Usp.*, **62**, (1994), p. 289.

[136] G. Schiffner W. R. Leeb and E. Scheiterer, *Appl. Opt.*, **18**, (1979), p. 1293.

[137] R. Wang, Yi. Zheng, Aiping Yao and D. Langley, *Phys. Lett. A*, **312**, (2003), pp. 7.

[138] T. A. Weber, *Am. J. Phys.*, **65**, (1997), pp. 486.

[139] S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (John Wiley and Sons (Asia), Singapore, 2004), first Published in 1974.

[140] D. H. Weinstein, *Nature*, **232**, (1971), p. 548.

[141] S. A. Werner, *Phys. Today*, **33**, (1980), p. 24.

[142] J. R. Wilkinson, *Ring Lasers*, *Prog. Quant. Electr.*, **11**, (1987), pp. 1.

[143] A. Winnie, *Phil. Sc.*, **37**, (1970), pp. 81.

[144] A. Winnie, *Phil. Sc.*, **37**(2), (1970), p. 223.

[145] E. Zahar, *British Journal of Philosophy of Science*, **28**, (1977), pp. 195.

[146] J. E. Zimmerman, *Cryogenics*, **20**, (1980), p. 3.

[147] J. E. Zimmerman and J. E. Mercereau, *Phys. Rev. Lett.*, **14**, (1965), p. 887.

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Relativistic Sagnac Effect and Ehrenfest Paradox

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There seems to exist a dilemma in the literature as to the correct relativistic formula for the Sagnac phase-shift. The paper addresses this issue in the light of a novel, kinematically equivalent linear Sagnac-type thought experiment, which provides a vantage point from which the effect of rotation in the usual Sagnac effect can be analyzed. The question is shown to be related to the so-called rotating disc problem known as the Ehrenfest paradox. The relativistic formula for the Sagnac phase-shift seems to depend on the way the paradox is resolved. Kinematic resolution of the Ehrenfest paradox proposed by some authors predicts the usually quoted formula for the Sagnac delay but the resolution itself is shown to be based upon some implicit assumptions regarding the behaviour of solid bodies under acceleration. In order to have a greater insight into the problem, a second version of the thought experiment involving linear motion of a "special type" of a non-rigid frame of reference is discussed. It is shown by analogy that the usually quoted special relativistic formula for the Sagnac delay follows, provided the material of the disc matches the "special type."

KEY WORDS: special relativity; Sagnac effect; rotating frame; Ehrenfest paradox.

1. INTRODUCTION

There has been a great deal of interest in recent years in the Sagnac effect. This is not only for its practical importance in navigational application for sensing rotation but also for its rich theoretical ramifications. In 1913 Sagnac[(1)] in his life long quest for ether devised an experiment where he

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compared round-trip times of two light signals traveling in opposite directions along a closed path on a rotating disc. It was observed that the time required by a light signal to make a close circuit on the plane of the disc differed depending on the sense (direction) of the signal's round-trip with respect to the spin of the disc. The essentials of the experiment consist of a monochromatic source of light, an interferometer and a set of mirrors mounted on a turntable. Light from the source is split into two beams by a beam splitter (half-silvered mirror) allowing them to propagate in opposite directions. These beams then are constrained (by suitably placed mirrors) to make round-trips and are then reunited at the beam splitter to produce a fringe pattern. The difference in the round-trip times for these counterpropagating beams leads to a phase difference with a consequent shift in the fringe pattern when the turntable is put into rotation. This phenomenon is commonly known as the Sagnac effect (for reviews with historical perspectives, see, for example, Refs. 2–4). The effect is universal and is also manifested for matter waves.[4]

The optical Sagnac effect can be suitably analyzed by assuming the light circuits to be circular. This can be achieved by constraining the light beams to propagate tangentially to the internal surface of a cylindrical mirror. Within the framework of Newtonian physics it is quite straight forward to calculate the Sagnac phase-shift in terms of the difference of arrival times of the co-rotating and counter-rotating light signals when they are reunited at the beam splitter. Assuming that the light beams propagate along the periphery of the circular disc of radius R, the time difference can be shown to be given by

$$\Delta t = \frac{2Lv}{c^2}\,\gamma^2, \tag{1}$$

where $L = 2\pi R$ is the circumference of the disc, v is the linear speed of the disc at its periphery and $\gamma = (1-\beta^2)^{-1/2}$ with $\beta = v/c$. To a lowest order of v/c

$$\Delta t = \frac{2Lv}{c^2}. \tag{2}$$

[4] The effect has been observed in interferometers built for electrons,[5] neutrons,[6] atoms,[7] and superconducting Cooper pairs.[8] The accuracy of Sagnac type experiment is much improved by using lasers and in fibre optic interferometers.[3] Based on the Sagnac effect, ring laser and fibre optic gyros are being used as navigational tools.[9] Importance of this effect also lies in connection with the question of time keeping clocks of clock-stations around the earth.[10] Here the earth plays the role of the rotating platform from which clocks between clock stations are synchronized by sending light signals via satellites.[11, 12]

The result is often quoted as

$$\Delta t = \frac{4\pi A\omega}{c^2}, \tag{3}$$

where $A = \pi R^2$ is the area of the optical circuits and $\omega(= v/R)$ represents the angular speed of the disc. It may be noted that the presence of the square of the Lorentz factor γ in Eq. (1) has nothing to do with special relativity since the treatment is purely classical.

Calculation based on special relativity also gives identical result to the lowest order in v/c. However, the exact formula differs from Eq. (1). The special relativistic result $\Delta\tau$ for the Sagnac delay, usually found in the literature(2, 13, 14) in one form or the other, differs from (1) by a factor of γ:

$$\Delta\tau = \frac{2Lv}{c^2}\gamma. \tag{4}$$

The above result can be deduced in various ways, but a simple derivation of Eq. (4) due to Post(2) is as follows. Suppose a light pulse leaves the beam splitter at position B (as shown in Fig. 1) and propagates along the direction of rotation of the disc and meets the beam splitter again in time t_1. During this period the beam splitter has moved to a new position B'; hence light has to travel an extra distance (BB') $x = vt_1$ with respect to the inertial frame of the laboratory which is at rest with the axis of rotation. One therefore has

$$L + x = ct_1, \tag{5}$$

$$x = vt_1. \tag{6}$$

In order to write (5) it is implicitly assumed that the speed of light is c in the laboratory frame which, in accordance with special relativity, is independent of the motion of the source.[5] Eliminating x from Eqs. (5) and (6) one obtains

$$t_1 = \frac{L}{c - v}.$$

A similar argument for the counter-rotating beam gives its round-trip time t_2. In this time the beam splitter moves to its new position B'', so that

[5] Classically this is equivalent to the hypothesis of ether, through which light propagates and which is believed to be stationary with respect to the inertial frame of the laboratory.

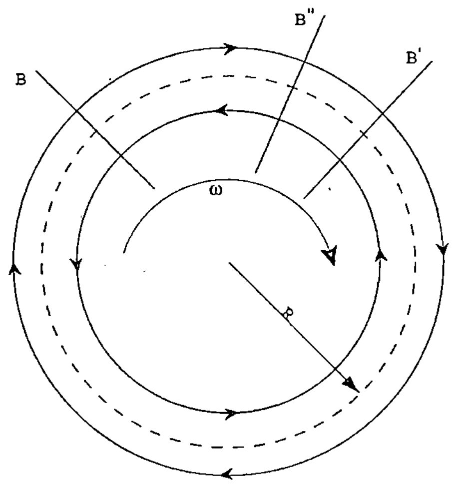


Fig. 1. B is the initial position of the beam splitter. As the disc (dotted circle) rotates clockwise, the beam-splitter moves towards the right. B' and B'' are the positions of the beam-splitter when the co-rotating (outer circle) and the counter-rotating (inner circle) light pulses respectively return to it. R and ω represent the radius and the angular velocity of the disc respectively.

with respect to the laboratory, the counter-rotating light pulse travels a path shorter than L by the amount $BB'' = vt_2$. For light propagation one may therefore write

$$l - vt_2 = ct_2,$$

i.e.,

$$t_2 = \frac{L}{c+v}.$$

The difference in these times is therefore given by

$$\Delta t = t_1 - t_2 = \frac{2Lv}{c^2 - v^2}. \tag{7}$$

The above expression for the time difference has already been quoted (Eq. (1)) as the classical expression for the Sagnac effect. The relativistic expression is obtained by recognizing that the above time difference refers to the observation from the stationary frame and is therefore dilated with respect to that measured (as the fringe shift) on board the disc. If $\Delta\tau$ corresponds to the time difference as observed from the rotating platform, clearly, because of this time dilatation effect

$$\Delta\tau = \gamma^{-1}\,\Delta t. \tag{8}$$

Therefore from Eqs. (7) and (8) one obtains the result as quoted earlier

$$\Delta\tau = \frac{2Lv}{c^2}\,\gamma. \tag{4}$$

Essentially the same expression for the effect has been derived by several authors in variety of ways.[6]

There are however a few exceptions. Selleri[(17)] for example, obtains a different result. He considers the length contraction effect in the periphery of the disc in addition to the time dilatation effect. If the rest circumference of the disc is denoted by L_0

$$L = \gamma^{-1} L_0, \tag{9}$$

since the periphery should suffer a Lorentz contraction. In this case, using Eqs. (8) and (9) in Eq. (7),

$$\Delta\tau = \frac{2L_0 v}{c^2}. \tag{10}$$

It may be noted that the above formula is the same as Eq. (2) which is an approximation of the classical or the relativistic expressions (Eqs. (1) and (4), respectively) to the lowest order of v/c. In this case however the formula is exact.

What therefore, is the correct relativistic formula for the Sagnac effect? Following Selleri, Goy[(18)] also quoted the same result pointing out explicitly the discrepancy between this result (Eq. (10)) and the special relativistic result that is generally believed to be the true one (Eq. (4)). The present day precision in measurement of the Sagnac effect may be unable

[6] For example, the formula can be derived from metrical considerations by introducing various types of transformation for rotating frames.[(2, 15)] This is also studied using the general relativity[(2, 13)] or by considering the relativistic Doppler effect at the mirrors.[(16)] An interesting derivation due to Dieks and Nienhuis[(14)] based on the direct use of the Lorentz tranformations will be followed here for the calculation of the effect in the next section.

to decide between the two formulae, nevertheless from the theoretical and pedagogical standpoint the question cannot be ignored.[7] We shall see below that the issue is intimately connected with the Ehrenfest paradox[(21, 22)] concerning the spatial geometry of a rotating disc. The paradox concerns a circular disc examined by an inertial observer at rest with the centre as the former passes from the rest to rotational motion. For such an observer the circumference but not the radius suffers Lorentz contraction. Hence the ratio of the circumference to the radius should be different from 2π thus violating Euclidean geometry in the *inertial frame*! However, for the moment, we refrain from discussing the paradox any further or from giving any verdict right away regarding the question we raised in the beginning of the paragraph concerning the correct relativistic formula, for the Sagnac effect. In order to appreciate the question, the real physics behind the Sagnac effect may be brought out first by delinking any other effect that may be present due to the rotation of the disc from some "pure" Sagnac effect.

Let us first recognize that the essential content of the Sagnac experiment lies in its ability to detect acceleration of the experimental platform by comparing the round-trip times for lights travelling parallel or antiparallel to the motion of the platform. It is therefore expected that the acceleration need not have to be due to rotation alone; a suitably modified Sagnac type experiment should as well be able to detect the change of the direction of motion of a platform which is allowed to move or shuttle along a straight line. In the next section we shall propose such a thought experiment which will mimic the optical Sagnac experiment in almost all respect but with the difference that the motion of the experimental platform will not involve rotation.[8] The outcome of the experiment will be called the

[7] To appreciate a current perspective of this pedagogical question consider the following: The Sagnac experiment is often regarded as fundamental as the Michelson–Morley experiment so much so that in some recent papers it is claimed that one may even rederive relativity theory from some new postulates based on the Sagnac effect.[(19, 20)] Unfortunately the Sagnac effect unlike the Michelson–Morley result is a verification of a first order effect. To hope to derive relativity theory from the Sagnac result therefore requires the exact Sagnac formula and one cannot remain content with the approximate one.

[8] One may object here that the Sagnac effect without rotation is a contradiction in terms. However this may not be so. This will be clear by the end of the next section. At the moment it is enough to say that the proposed linear (thought) experiment will include the following basic ingredients of the Sagnac experiment: (1) Light signals will be allowed to complete round-trips with the help of mirrors. (2) The experimental platform will have uniform *speed*. (3) Light signals will travel parallel (or antiparallel) to the direction of motion of the platform throughout during their round-trips. (4) The difference of the round-trip times (Sagnac delay) for the parallel and the antiparallel light signals will be measured on board the platform.

"pure" Sagnac effect. This will give us a perspective which will enable us to appreciate the connection between the Sagnac effect and the Ehrenfest paradox. We shall see below that the formula obtained for the "pure" Sagnac effect may or may not be modified when rotation is introduced. This modification or the lack of it will depend on the way the Ehrenfest paradox is resolved.[22, 23]

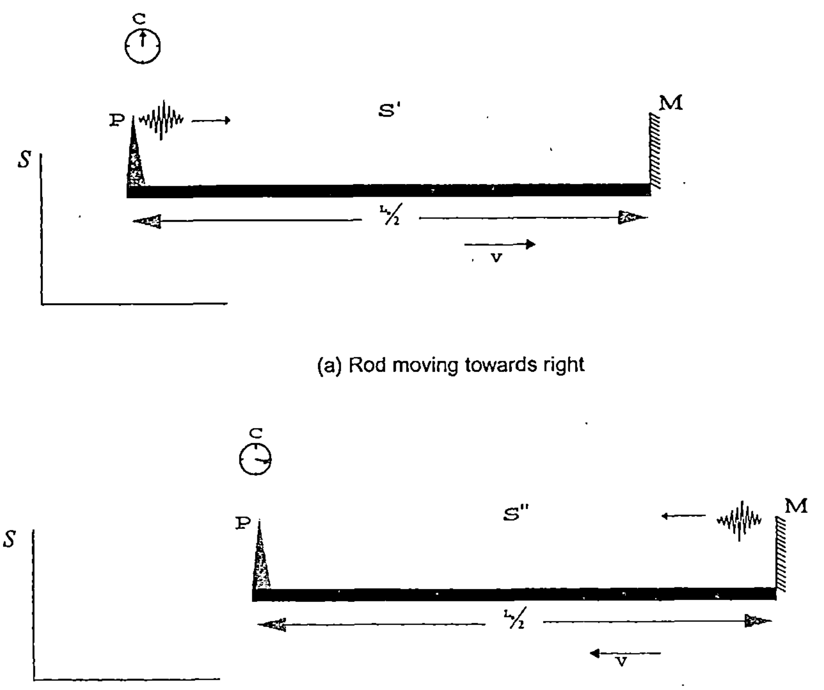
It is interesting to note that no author has ever explicitly mentioned any role of the Ehrenfest paradox in the derivation of the Sagnac delay. In Sec. 5 we shall argue that the so-called kinematic resolution of the Ehrenfest paradox is based on some implicit assumptions regarding the behaviour of the solid discs when set into rotation. In order to prepare the background of this argument, in Secs. 3 and 4 we shall consider another version of the linear Sagnac experiment and analysis involving a non-rigid frame of reference of a special kind. We shall see that the Sagnac type experiment performed on such platforms gives the usually quoted formula for the Sagnac delay. Significance of these observations among other things will be discussed in Sec. 5 and finally will be summarized in the concluding section.

2. LINEAR SAGNAC EFFECT–I

As discussed in the last section, in optical Sagnac experiment one essentially compares the round-trip times of two light signals, one of which propagates parallel and the other travels antiparallel to the direction of motion of the edge of the rotating disc on which all the measurements are carried out. Let us now present a linear version of the experiment.[24]

Suppose a pulse of light is emitted from a source placed at one end of a linear platform and travels towards a mirror (facing the light source) placed at the other end of the platform which can move with constant speed in either directions along its length. Light after being emitted makes a round-trip after it is reflected back by the mirror. To ensure that the motion of the light pulse remains parallel (antiparallel) throughout its round-trip, it may be assumed that by some mechanism the platform reverses its direction of motion (without changing its speed) as soon as light falls on the mirror.

The linear version of the Sagnac experiment will therefore consist of two separate experiments, one for parallel and the other for antiparallel light propagation. For definiteness, in the first experiment (Figs. 2(a) and (b)) we consider an inertial frame S with respect to which a rigid rod (A) of rest length $L_0/2$ is moving along the positive x-direction with a relative velocity v. Assume that a light pulse sent from a light source (P) placed at



(a) Rod moving towards right

(b) Rod moving towards left

Fig. 2. Parallel Propagation Experiment.

the left end of the rod propagates towards the right (Fig. 2(a)) and in due course, falls on a mirror placed on the right end of the rod. As soon as the light falls on the mirror, the rod reverses its direction of motion without changing its speed and at the same time the light signal after reflection, proceeds to the source (Fig. 2(b)) where a clock (C) records the round-trip time of the light pulse. Clearly the aforesaid arrangement ensures that propagation of light remains parallel (to the direction of motion of the rod) throughout its round-trip. We call it the parallel propagation experiment. The signal completing the journey in this way is analogous to the beam of light *co-rotating* with the disc in ordinary idealized Sagnac experiment.

One needs a similar arrangement for antiparallel propagation of light which may correspond to the *counter-rotating* beam of the Sagnac experiment. For this anti-parallel propagation experiment, one repeats the same sequence of events as the first one except now the motion of the rod is to be along the negative x-direction to start with, before its direction of

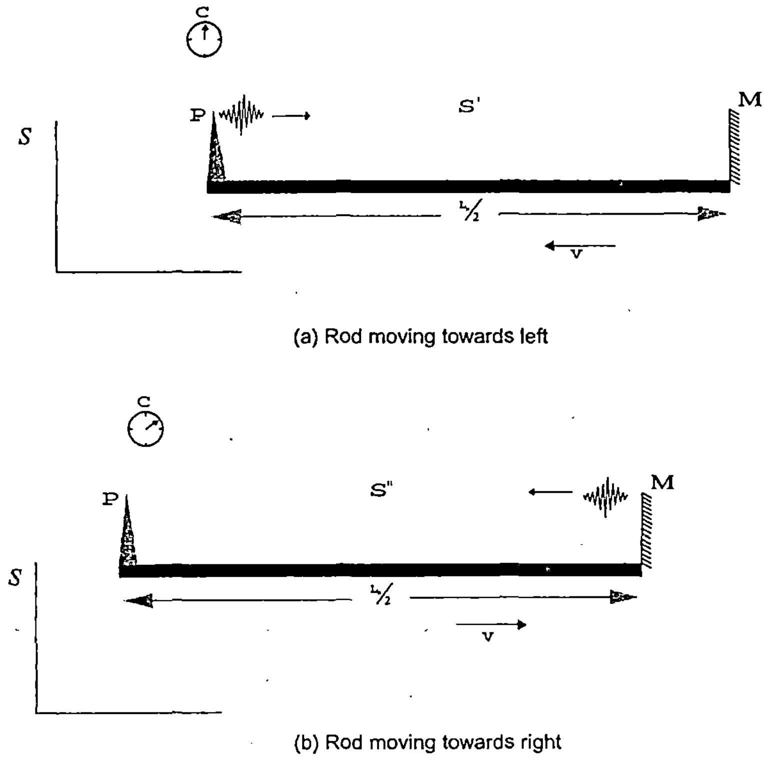


Fig. 3. Antiparallel Propagation Experiment.

motion is reversed on falling the light signal on the mirror (Figs. 3(a) and (b)).

Note that a frame of reference K attached to the rod in these experiments is non-inertial because of the reversal of its direction of motion during the experiment. However one may consider two inertial frames S' and S'' which are co-moving with the rod for its forward and reverse journeys respectively. On account of Lorentz transformation one may therefore write for the motion of the light pulse

$$dt_{\text{lab}} = \gamma\left(dt \pm \frac{v\,dx}{c^2}\right), \tag{11}$$

where t_{lab} is the co-ordinate time of the global inertial frame of the laboratory and t and x refer to the local time and spatial co-ordinate of the instantaneous inertial frames S' and S'' and the plus and minus signs apply for the forward and reverse journeys of the rod respectively.

For the first experiment when the light signal moves in the direction same as that of the moving rod, we obtain (by integrating Eq. (11)) for the time $\Delta t_{\text{lab}}(1)$ needed for the round-trip

$$\gamma^{-1}\,\Delta t_{\text{lab}}(1) = \int dt + \int_0^{\frac{L_0}{2}} \frac{|v|\,dx}{c^2} - \int_{\frac{L_0}{2}}^{0} \frac{|v|\,dx}{c^2} = \int dt + \frac{L_0\,|v|}{c^2}. \qquad (12)$$

Similarly, for the second experiment,

$$\gamma^{-1}\,\Delta t_{\text{lab}}(2) = \int dt - \int_0^{\frac{L_0}{2}} \frac{|v|\,dx}{c^2} + \int_{\frac{L_0}{2}}^{0} \frac{|v|\,dx}{c^2} = \int dt - \frac{L_0\,|v|}{c^2}. \qquad (13)$$

The integrals $\int dt$ in the above equations are the same and is the measure of the "total" time as interpreted from two successive inertial frames, S' and S'' for each experiment. This equals to L_0/c since according to each of these inertial observers speed of light is c and the distance covered by light in either of its forward and reverse journeys is $L_0/2$, i.e., the distance between the source and the mirror.

This time $\int dt$ ought not to be confused with the time measured by a co-moving clock placed at the position of the light source which undergoes direction reversal. If the later is denoted by $d\tau$, we have by virtue of the time dilatation effect,

$$\Delta\tau = \gamma^{-1}\,\Delta t_{\text{lab}}.$$

Hence we may write Eqs. (12) and (13) as

$$\Delta\tau(1) = \int dt + \frac{L_0\,|v|}{c^2}, \qquad (14)$$

$$\Delta\tau(2) = \int dt - \frac{L_0\,|v|}{c^2}. \qquad (15)$$

The difference of these times gives for the Sagnac delay for this linear version of the Sagnac experiment,

$$\delta\tau = \frac{2L_0\,|v|}{c^2}. \qquad (16)$$

Clearly the expression is linear in v since the proper length L_0 of the rod is a constant. Note that all the mathematical steps that are involved in this derivation smoothly go over to that leading to the usual Sagnac Effect. Except there, instead of two inertial frames S' and S'', one has to consider an infinite number of momentary inertial frames for the calculations.[14, 25] Since no other effect (that might be present because of rotational motion of the disc in the usual Sagnac experiment) is involved in this linear arrangement the time delay expressed by the above relation may be said to be the result of the "pure" Sagnac effect.

3. LINEAR SAGNAC EFFECT–II

In the previous experiment the light source and the mirror were attached to the two ends of a rigid rod of proper length $L_0/2$. Let us now consider a slightly different arrangement. Consider two aircrafts (unbonded, i.e., not tied together) separated by a distance $L/2$ initially at rest with the laboratory.

Suppose that these aircrafts are programmed in such a way that they can move in any direction always preserving a constant separation $L/2$ with respect to the laboratory. To a casual observer in the laboratory there will appear to be a bond between the objects because of the programmed constant separation of the two, but in reality these are unbonded. One may term this apparent bond between the aircrafts as a "software bond" as distinguished from the bond that exists between any molecules in a solid body.[26]

The twin experiment as described in Sec. 2 can be repeated by attaching the light source to one aircraft and the reflector to the other. For the first experiment assume that the aircrafts are accelerated from rest and finally the system moves with constant velocity v along the positive x-direction. Suppose now a light pulse from a source attached to the first aircraft travels towards the second one on the right and falls on the mirror attached to it. As soon as the light falls on the mirror, the "software bonded" system starts moving in the negative x-direction. The reflected pulse of light also travels in this direction and the time of transit for the light pulse for parallel propagation is recorded as it returns to the source.

In the second experiment almost the whole programme is repeated except now, at the time of emission of the light pulse the motion of the aircraft system is along the negative x-direction although the light pulse travels towards the right that subsequently falls on the mirror attached to the second aircraft. The direction of motion of the unbonded system is reversed as light falls on the mirror and travels to the left to be recorded at the position of the source again. In this arrangement, propagation of light remains antiparallel throughout its entire journey.

The round-trip time (measured in the laboratory) for light for parallel and anti-parallel propagation experiments may easily be calculated following the procedure as discussed in Sec. 1.[9] They are respectively given by

$$t_1 = \frac{L}{c-v}$$

and

$$t_2 = \frac{L}{c+v}$$

and their difference can be written as

$$\Delta t = t_1 - t_2 = \frac{2Lv}{c^2}\gamma^2.$$

If $\Delta\tau$ denotes the corresponding time difference by an observer in the first aircraft, one gets on account of time dilatation,

$$\Delta\tau = \gamma^{-1}\Delta t = \frac{2Lv}{c^2}\gamma. \tag{17}$$

Note that $\Delta\tau$ now is *not* linearly related to v.

Clearly the formula (17) differs from the earlier expression for $\Delta\tau$ (Eq. (16)) obtained for the "pure" Sagnac effect which was linear in v. One may however argue that in Eq. (16) L_0 is the proper length of the rod and in contrast L in Eq. (17) represents the distance of the aircrafts as measured from the laboratory frame. If the Eq. (16) were expressed in terms of L, the formulae (16) and (17) would have agreed. However, there is a subtle point here. If the issue is the question of dependence of the Sagnac delay on the speed of the platform, the formulae (16) and (17) predict different results. Although one is at liberty to quote the expression for the Sagnac delay in terms either of the proper distance L_0 or of the coordinate distance L between the source and the reflector, one must know which of these lengths is independent of v.

In the case of the experiment using rigid platform, proper length $L_0/2$ of the rigid rod is an invariant. On the other hand, for the experiment

[9] Although here the experiment is linear, the kinematical considerations leading to the relations (5) and (6) still remain valid. For example, for the parallel propagation experiment, in order to complete the round-trip in time t_1, the light pulse, as viewed from the laboratory, has to cover in addition to the distance L, an extra distance $x = vt_1$ because of the to and fro motion (with speed v) of the aircrafts. This means $ct_1 = L + vt_1$, i.e., $t_1 = L/(c-v)$.

performed in the frame of reference of the unbonded aircrafts, the distance $L/2$ between them as measured by an observer in the laboratory is held constant by a common program controlling the aircrafts as the system is brought up to different speeds from their initial state of rest. Special relativity then demands that the proper separation between the aircrafts will increase as the speed of the unbonded system increases. The next section will provide more clarifications in this regard.

4. COORDINATE SYSTEM OF THE UNBONDED FRAME

Consider an one-dimensional array of some software bonded particles that constitutes a frame of reference K and suppose from its state of rest at $t = 0$, the system is set in motion. If the space-time co-ordinates of the laboratory frame S are denoted by x and t, the equation of motion of the particle at the origin of K may be expressed as

$$x = f(t),$$

where $f(t)$ is some function of time which is zero at $t = 0$.

For any particle of the array this may be written as

$$x = x' + f(t),$$

where x' is the spatial coordinate of the particle with respect to S, when it was at rest with the laboratory (at $t = 0$). The variable x' may be used to label the array of points and these may act as spatial coordinates in K. Taking the coordinate time t' of K same as t one may write the following transformations between S and K in terms of the coordinate differentials

$$dx' = dx - \dot{f}(t)\, dt, \qquad dt' = dt. \tag{18}$$

where $\dot{f}(t) = v = df(t)/dt$ is the instantaneous velocity of the aircraft.

The line-element in natural unit ($c = 1$) of the 2-dimensional Minkowskian space in the coordinate system S

$$ds^2 = dt^2 - dx^2,$$

may be transformed accordingly so that with respect to x' and t' one may write

$$ds^2 = \gamma^{-2}\, dt'^2 - 2v\, dx'\, dt' - dx'^2, \tag{19}$$

where $\gamma = (1 - v^2)^{-1/2}$.

Again, for any line-element

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu,$$

the proper distance dl between two points x^i and $x^i + dx^i$ is given by (see Ref. 27, for instance)

$$dl = \sqrt{\left(\frac{g_{0i}\, g_{0k}}{g_{00}} - g_{ik}\right) dx^i\, dx^k}, \tag{20}$$

where $i, k = 1, 2, 3$. For the line element (19), this is given by

$$dl = \frac{dx'}{\sqrt{1 - v^2}}. \tag{21}$$

If the points x' and $x' + dx'$ refer to the co-ordinates of two neighbouring particles of K, by definition dx' is an invariant. In that case the proper distance should stretch according to Eq. (21) honouring the well-known special relativistic effect of length contraction as the unbounded array of particles is set into motion. Note that expression (19) represents a perfectly legitimate co-ordinate description of the 2-dimensional Minkowski space and although the transformations (18) have Galilean structure, special relativity is taken care of when we used Eq. (20) to obtain the proper distance.

5. DISCUSSION

Let us now return to the original question posed in Sec. 1 as to the correct relativistic formula for the usual Sagnac effect. As pointed out earlier there are two contesting claims for Sagnac delay:

Claim 1: $\Delta\tau \propto \frac{\beta}{\sqrt{1-\beta^2}}$.

Claim 2: $\Delta\tau \propto \beta$.

If the special relativistic correction due to time dilatation is incorporated in the classical expression (Eq. (1)) one obtains Eq. (4) which corresponds to claim 1. On the other hand if not only time dilatation but also Lorentz contraction of the circumference of the rotating disc is taken into account one gets Eq. (15) and claim 2 appears to be the true one. Barring a few exceptions[17, 18] most authors adhere to claim 1 without stating explicitly the reason behind not considering the length contraction effect.

However as we have seen there was no ambiguity as to the correct formula for time delay for the linear Sagnac effect (thought) experiment with rigid rod. There it was evident that the Lorentz contraction of the rigid platform ought to be taken into account in addition to the time dilatation effect.

For the usual Sagnac experiment on a rotating disc, the Lorentz contraction of the disc's circumference is not generally considered perhaps to avoid the Ehrenfest paradox (see Sec. 1). On the contrary, favouring claim 2, Selleri[17] and Goy[18] assume relativistic contraction of the edge of the disc without addressing any possible paradox that may arise due to such an assumption. Indeed the result is inconsistent unless it is explicitly assumed that the disc does not remain a disc and becomes a non-flat object.

It is therefore amply clear that in order to decide between claims 1 and 2 it is necessary to understand how the Ehrenfest problem is resolved. Ehrenfest's problem concerns the mechanical behaviour of a material disc set in rotation from rest. The paradox remains a paradox as long as one implicitly assumes that the disc is Born rigid.[28, 29] By definition Born rigid motion of a body leaves the proper lengths of the body unchanged. Grøn[22] showed that the transition of the disc from rest to rotational motion in a Born rigid way is a kinematic impossibility. It is the recognition of this fact which is known as the kinematic resolution of the Ehrenfest paradox.[22, 23]

Cavalleri[29] on the contrary observes that the Ehrenfest paradox cannot be solved from a purely kinematical point of view and the solution of the paradox is intrinsically dynamical. This was refuted by Grøn[22] who rather endorsed a remark by Phipps[30] that to think that dynamics can exist "without the foundation of logically consistent kinematics" is an absurdity.

The present authors believe that both the viewpoints are correct in the present context. To recognize that Born rigid rotation is an impossibility and an implicit assumption on the contrary is the source of the paradox, may follow from pure kinematics; but if it is asked—"what exactly will happen to the solid disc?" the answer will lie in the realm of dynamics. It appears that there is no unanimity in the literature as to this precise question. Synge[31] and Pounder[32] introduce the concept of superficial rigidity[29] according to which the circumference and radius of the disc when put into rotation, undergo change in accordance with special relativity but suggest that the flat disc changes to a surface of revolution symmetric about the axis of rotation. In this way the possible violation of Euclidean geometry in the inertial system is avoided. In the case of uniform rotation this allows radial contraction without any change of meridian arc-length. Some specific prescriptions were also suggested by a few earlier authors who proposed bending of the surface of the rotating disc in the form of a paraboloid of revolution (vide[29] for references). The rotating disc or wheel taking the shape of spherical segment when in rotation was suggested by Sokolovsky[33] as a resolution of the "wheel paradox."

Eddington[34] also investigated the problem of the rotating disc. He studied the question of alteration of the radius of a disc made of homogeneous incompressible material when caused to rotate with angular velocity ω. He showed that the radius of the disc is a function of the angular velocity ω and is approximately given by

$$a' = a(1 - \tfrac{1}{8}\omega^2 a^2),$$

where a is the rest radius of the disc. A similar view has also been expressed by Weinstein[35] who holds that the disc under rotation will be in torsion with a consequent reduction of both the radius and the circumference.

Of recent interest is the so called kinematic resolution of the Ehrenfest paradox as discussed by Grøn[22] and Weber.[23] According to the authors, it follows from purely kinematic considerations, that the radius of the disc remains unaltered but the proper measure of the circumference is increased in such an extent that the Lorentz contraction effect just gets compensated. In other word although there is a Lorentz contraction of the periphery with respect to the laboratory frame it is not visible because of the stretching of the periphery.[10] However, the stretching of the disc's circumference in its proper frame is a dynamical effect (related to the property of the solid material of the disc). How can one hope to get this dynamical effect purely from the kinematic considerations? Clearly the result must have been assumed implicitly. To clarify this let us consider a rotating co-ordinate system which is often discussed in connection with the Ehrenfest paradox. Suppressing one spatial dimension Grøn considers the following transformation:

$$r' = r, \qquad \theta' = \theta - \omega t, \qquad t' = t, \tag{22}$$

where r and θ refer to the radial and angular co-ordinates and t refers to the time co-ordinate of laboratory frame and the primed quantities refer the same in the rotating system.

The rotating frame of reference is equated with that of the rotating disc. It is precisely this equation where lies the implicit assumption that the

[10] Recently Klauber[19, 20] and Tartaglia[36] based upon different arguments also conclude that there will be no contraction of the circumference of the disc. The authors believe that relativistic contraction effect will not at all take place for rotating discs.

disc's periphery is stretched due to rotation. Based on these transformations the line-element may be written as

$$ds^2 = dr'^2 + r'^2\, d\theta'^2 + 2\omega r'^2\, d\theta'\, dt' - \left(1 - \frac{\omega^2 r'^2}{c^2}\right) c^2\, dt'^2. \tag{23}$$

Using the formula (20) for proper spatial distance for the line element (23) the tangential proper spatial line-element is obtained as

$$dl = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} r\, d\theta'. \tag{24}$$

Note that while using Eq. (20) a minus sign under the radical is required since now the metric (23) has a different signature.

Integrating Eq. (24) along the whole element, one obtains the proper length L_0

$$L_0 = L\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}, \tag{25}$$

where $L = 2\pi R$ is the circumference of the disc as observed from the inertial frame of the laboratory. However it will be wrong to assume, from the above argument, that the proper circumference of the disc has increased by a Lorentz factor. To conclude from (25) that the periphery of the disc is stretched by a γ-factor due to rotation is to assume that L was also the circumference of the disc when it was at rest and remains the same, as it is brought up to rotation from its state of rest. Any transformation (representing rotation) must reflect the validity of this assumption. Clearly Eq. (22) does not guarantee this.

The transition of the disc to its rotational motion from its state of rest can be expressed by writing the transformations (22) in a slightly modified form

$$r' = r, \qquad \theta' = \theta - f(t), \qquad t' = t, \tag{26}$$

where the function $f(t)$ is assumed to have the following properties: $f(t)$ and $df/dt = \omega(t) = 0$ at $t = 0$ and $\omega(t)$ thereafter increases and finally approaches a constant value. Obviously the transformations (26) then represent a rotating coordinate system with constant angular velocity after a period of angular acceleration from its state of rest, which in terms of the differentials read

$$dr' = dr, \qquad d\theta' = d\theta - \omega(t)\, dt, \qquad dt' = dt. \tag{27}$$

Note that relations (23) and (24) still remain valid. Now, if we recall our discussions in Secs. 3 and 4, and the transformations (18), we see by analogy that the transformations (27) represent the motion of the disc which is composed of an unbonded arrangement of particles that are programmed to move in a particular way so that their mutual separations with respect to the laboratory frame of reference remain constant. Therefore the constancy of L (and not L_0) in other words, is the outcome of the assumed programme of motion of the particles of the disc governed by Eq. (26).

6. SUMMARY

We are now in a position to summarize our findings. We have seen that there is a scope for confusion regarding the correct relativistic expression for the Sagnac delay. Although the oft-quoted result is that given by Eq. (4), no role of the so-called Ehrenfest paradox in arriving at the result is usually discussed. It is expected that the special relativistic result for the Sagnac effect will differ from its classical counterpart, usually due to two kinematic effects of special relativity—the length contraction and the time dilatation. The inclusion of the length contraction effect in the circumference (and not in the radius) of a rotating disc invites a paradox that there is an apparent violation of the Euclidean geometry in an inertial frame. On the other hand, the non-inclusion of the Lorentz contraction effect will violate special relativity. To understand and clarify these issues a Sagnac-type thought experiment (without rotation) performed on a linear rigid platform has been presented. Since no paradox is associated with this arrangement although kinematically all aspects of the usual Sagnac experiment are incorporated in it, the linear experiment sets the right kind of perspective against which the role of the Ehrenfest paradox in the rotating disc experiment can be discussed.

Although the resolution of the Ehrenfest paradox lies in appreciating the fact that "Born rigid" rotation of the disc from its state of rest is a kinematic impossibility, people differ when trying to be specific about the exact deformation of the disc brought about by rotation. We give below just two opposite viewpoints that are found in the literature.

According to the so-called kinematic resolution of the paradox there should not be any contraction of the circumference as observed from the inertial frame of the laboratory that is at rest with the axis but, the periphery should stretch in terms of proper measure so that the Lorentz contraction effect of special relativity is automatically taken care of. The conclusion apparently follows from the widely discussed line element representing a rotating co-ordinate system.[22, 27, 37] It has been however shown, by drawing analogy with the version II linear Sagnac type (thought) experiment, that the kinematic resolution presupposes that the disc material is composed of "unbonded" particles that are programmed to rotate in such a way that the distances between the particles remain fixed with respect to the laboratory as the system passes to a rotational motion from its state of rest. If this happens, the formula for the Sagnac delay will be given by Eq. (4).

The other view point is to suppose that the disc material obeys the Synge–Pounder criterion of superficial rigidity. In this case the disc should bend and take a shape of a paraboloid so that at any radial point, the circumference is Lorentz contracted but there is no contraction of the meridian. However the distance of the periphery from the centre will be shortened. Therefore the paradox does not exist. In this case too the resolution of the paradox is based on a specific postulate regarding the behaviour of the material of the disc undergoing rotation. As a consequence, the Sagnac delay should be given by Eq. (10) that corresponds to the result obtained for the linear Sagnac effect of the first form (Eq. (16)). Some authors[17,18] have quoted this result too however not addressing any role of the Ehrenfest paradox in their derivation. If instead of a disc, the rotating platform is assumed to be a massive solid cylinder, the deformations of the kind just mentioned are perhaps excluded and the usual formula (Eq. (4)) pertains to this case. However, in this case, the constraint imposed on the particles of the cylinder by the form of the solid body would work in such a way, that the particles of the body can be thought of as "unbonded" as the cylinder is set into rotation (vide Sec. 5). Indeed for a disc there cannot be one right formula; for example, the deformation of the kind considered by Eddington[34] as mentioned in Sec. 5, would give a result different from Eq. (4) or Eq. (10).

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REFERENCES

1. M. G. Sagnac, *C. R. Acad. Sci.* **157**, 708–710 (1913).
2. E. J. Post, *Rev. Modern Phys.* **39**, 475–493 (1967).
3. R. Anderson, H. R. Bilger, and G. E. Stedman, *Amer. J. Phys.* **62**, 975–985 (1994).
4. G. B. Malykin, *Phys.-Usp.* **40**, 317–321 (1997).
5. H. Hasselbach and M. Nicklaus, *Phys. Rev. A* **48**, 143–151 (1993).
6. R. Colella, A. W. Overhauser, J. L. Staudenmann, and S. A. Werner, *Phys. Rev. A* **21**, 1419–1438 (1980).
7. F. Riehle, Th. Kister, A. Wittie, J. Helmcke, and Ch. J. Borde, *Phys. Rev. Lett.* **67**, 177–180 (1991).
8. D. Fargion, L. Chiatti, and A. Aiello, "Quantum Mach effect by Sagnac phase shift on Cooper pair in rf-SQUID," preprint astro-ph/9606117 (1996), 19 pages.
9. S. Merhav, *Aerospace Sensor Systems and Applications* (Springer, New York, 1996).
10. D. W. Allan *et al., IEEE Trans. Instrum. Meas.* **IM-34**(2), 118–125 (1985).
11. N. Saburi, M. Yamamoto, and K. Harada, *IEEE Trans. Instrum. Meas.* **IM-25**, 473–477 (1976).
12. G. Petit and G. Wolf, *Astronom. Astrophys.* **286**, 971–977 (1994).
13. J. Anandan, *Phys. Rev. D.* **24**, 338–346 (1981).
14. D. Dieks and G. Nienhuis, *Amer. J. Phys.* **58**, 650–655 (1990).
15. P. Langevin, *C. R. Acad. Sci. (Paris)* **173**, 831–834, (1921).
16. M. Dresden and C. N. Yang, *Phys. Rev. D* **20**, 1846–1848 (1979).
17. F. Selleri, *Found. Phys.* **26**, 641–664 (1996).
18. F. Goy, "Non-invariant velocity of light and clock synchronization in accelerated systems," *Proc. Physical Interpretation of Relativity Theory (V)*, held at Imperial College, London, pp. 129–138 (1996).
19. R. D. Klauber, *Found. Phys. Lett.* **11**, 405–444 (1998).
20. R. D. Klauber, *Amer. J. Phys.* **67**, 158–159 (1998).
21. P. Ehrenfest, *Phys. Z.* **10**, 918 (1909).
22. Ø. Grøn, *Amer. J. Phys.* **43**, 869–876 (1975).
23. T. A. Weber, *Amer. J. Phys.* **65**, 486–487 (1997).
24. S. K. Ghosal and M. Sarker, "Cosmologically preferred frame, linear Sagnac effect and absolute synchrony," *Proc. Physical Interpretation of Relativity Theory Meeting (VI) (Supplementary papers)*, held at Imperial College, London, pp. 52–64 (1998).
25. D. Dieks, *Eur. J. Phys.* **12**, 253–259 (1991).
26. C. W. Sherwin, "Three theories of relative measurement," Paper presented at the Physical Interpretation of Relativity Theory Meeting (III), held at Imperial College, London (1992).
27. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th revised English edition (Pergamon, Oxford, 1975).
28. M. Born, *Ann. Physik* **30**, 840 (1909).
29. G. Cavalleri, *Nuovo Cimento B* **53**, 415–431 (1968).
30. T. E. Phipps, Jr., "Experiment on relativistic rigidity of a rotating disc," NOLTR 73-9 (Naval Ordinance Laboratory, 1973), p. 47.

31. J. L. Synge, *Studies in Mathematics and Mechanics: Presented to Richard von Mises by Friends, Colleagues and Pupils* (New York, 1954).
32. J. R. Pounder, *Comm. Dublin Institute for Advance Studies A* **11** (1954).
33. Yu. I. Sokolovsky, *The Special Theory of Relativity* (Hindustan Publishing, Delhi, 1962).
34. A. S. Eddington, *The Mathematical Theory of Relativity* (Cambridge University Press, Cambridge, 1965).
35. D. H. Weinstein, *Nature* **232**, 548 (1971).
36. A. Tartaglia, *Found. Phys. Lett.* **12**(1), 17–28 (1999).
37. C. Møller, *The Theory of Relativity* (Oxford University Press, Oxford, 1952).

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CONVENTIONALITY OF SIMULTANEITY AND THE TIPPE TOP PARADOX IN RELATIVITY

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In this paper we critically examine a recently posed paradox (tippe top paradox in relativity) and its suggested resolution. A tippe top when spun on a table, tips over after a few rotations and eventually stands spinning on its stem. The ability of the top to demonstrate this charming feat depends on its geometry (all tops are not tippe tops). To a rocket-bound observer the top geometry should change because of the Lorentz contraction. This gives rise to the possibility that for a sufficiently fast observer the geometry of the top may get altered to such an extent that the top may not tip over! This is certainly paradoxical since a mere change of the observer cannot alter the fact that the top tips over on the table. In an effort to resolve the issue the authors of the paradox compare the equations of motion of the particles of the top from the perspective of the inertial frames of the rocket and the table and observe among other things that (1) the relativity of simultaneity plays an essential role in resolving the paradox and (2) the puzzle in some way is connected with one of the corrolaries of special relativity that the notion of rigidity is inconsistent with the theory. We show here that the question of the incompatibility of the notion of rigidity with special relativity has nothing to do with the current paradox and the role of the lack of synchronization of clocks in the context of the paradox is grossly over-emphasized. The conventionality of simultaneity of special relativity and the notion of the standard

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(Einstein) synchrony in the Galilean world have been used to throw light on some subtle issues concerning the paradox.

Key words: special relativity, conventionality of simultaneity, Zahar transformation, Tangherlini transformation, lack of synchronization of clocks, rigidity.

1. INTRODUCTION

Since the advent of the special theory of relativity paradoxes concerning the theory have always been very common. Even today in the literature newer paradoxes continue to pour in, a latest item in the list being the tippe top paradox. In an interesting paper, Basu *et al.* [1] have posed the paradox concerning a fascinating toy known as tippe top. The top has the remarkable property that after giving it an initial vertical spin on a table about its symmetry axis, the top turns itself (after a few rotations) upside down and stands spinning on its stem. The statement of the paradox goes like this: Not all tops demonstrate this charming feat. In order for this to happen, the ratios of its principal moments of inertia should fall in a certain regime. If now such a top is observed from a rocket frame in which the spinning top appears to recede with a uniform velocity, the cross-section of the top will appear elliptical due to the relativistic length contraction effect along the boost direction. For a sufficiently fast rocket, therefore, it is possible that the ratios of the principal moments of inertia go out of the regime required for the top to tip over. This is certainly paradoxical since a mere change of reference frame cannot alter the fact that the top tips over.

The paper claims to have resolved the paradox first by noting that the paradoxes in relativity often arise because one tends to focus only on one relativistic effect (in this case the length contraction effect) while losing sight of the other kinematical consequences of special relativity (SR). The authors then demonstrate, by applying the Lorentz transformations (LT) to the equations of motion of the particles of the top, that the whole body of relativistic effects such as the length contraction, the time dilation, the relativity of simultaneity and the likes find their appearances in the description of the spinning motion of the particles of the top from the reference frame of the rocket. The particle orbits have been studied both analytically and by computer simulation to demonstrate how the classical notion of the rigid body fails in relativity. The origin of the paradox has then been attributed, in different words, to our adherence to the classical notion of rigidity in the relativistic domain.

Although it is interesting as well as instructive to view on the computer screen the motion of the particles of the spinning top as observed from the rocket frame (but referring it to the moving point of contact of the top with the table), we consider the paper to suffer from certain drawbacks which we would like to address. It appears, in accordance with the paper's claim, that the resolution of the paradox relies heavily on the lack of synchrony aspect of SR. This, in our opinion, is a weak point of an otherwise interesting paper. Indeed we will argue (in Sec. 3) that the lack of synchrony aspect may well exist in the Galilean world where it is well known that no such paradox should truly exist. The other drawback is the authors' attempt to connect the paradox with one of the corollaries of SR, that the notion of rigid body is inconsistent with it. In Sec. 4 below it will be shown that this aspect of SR has no bearing with the current paradox. It is indeed true that many paradoxes can arise if one inadvertantly carries the classical notion of rigidity over into relativistic situation [2]. The classical rigid body by definition must move as one entity when it is pushed at one end, i.e., the disturbance at one end of the body would be propagated with infinite velocity through the body. This is in contradiction to the relativity principle that there is a finite upper limit to the speed of transmission of a signal. The analysis of Basu *et al.* considers only uniform rotation of the particles of the top with respect to the table frame; no transients are involved. It is therefore surprising how the rigidity issue should be connected to the problem. The aim of the present paper is to deliberate on these and related issues.

In recent years a new approach based on conventionality of simultaneity (CS), to understanding paradoxes in relativity has been found fruitful. For example, Redhead and Debs [3] have shown that the CS approach provides a means to put an end to the question concerning the notorious twin paradox as to where and when the differential ageing takes place. As another example Selleri [4] has shown that a particular simultaneity convention compared to that of Einstein seems to be more appropriate in explaining the Sagnac effect from the point of view of the rotating turn-table. The present paper will also follow the CS approach to critically examine the work of Basu *et al.* A brief introduction to the CS-thesis of relativity, therefore, is in order. This will be done in Sec. 3. The main arguments will be presented in Sec. 4-6 before we summarise all this in Sec. 7. However, in order to set the stage we will briefly reproduce in Sec. 2 the basic arguments used in [1] to clarify the paradox.

2. EQUATION OF MOTION AND COORDINATE SYSTEM

For simplicity consider a vertical top such that a typical particle P of the top in the table frame Σ^0 executes a horizontal (in the X-Y plane) circular motion. The equation of motion of the particle in the coordinate system of Σ^0 is given by

$$x^0 = R\cos\omega t^0, \quad y^0 = R\sin\omega t^0, \quad x^{0^2} + y^{0^2} = R^2, \tag{1}$$

where ω is the angular speed of the top and R represents the distance of P from the axis.

Consider now a frame of reference Σ of the rocket with respect to which the table and the top moves along the positive x-direction common to both Σ and Σ^0. Suppose that the x and t coordinates of the rocket frame are linearly related to the corresponding x^0 and t^0 of Σ^0 through the following transformations:

$$x^0 = ax + bt, \quad t^0 = gx + ht, \tag{2}$$

and also suppose $y^0 = y$, where a, b, g and h are independent of x and t. In Ref. [1], however, they have written LT, but here we wish to keep it a bit more general for reasons which will be apparent soon. Clearly, from the above transformation equations it follows that the origin of Σ^0 satisfies the following equation of motion with respect to Σ:

$$ax_{\text{orig}} + bt = 0, \tag{3}$$

where the suffix 'orig' refers to the origin. In other words the translational velocity of the origin, as observed from the rocket frame is given by

$$u = -b/a. \tag{4}$$

Since this translatory motion is irrelevant to the spinning of the top about its axis, it may be subtracted out from the *apparent* equation of motion of the particles of the top as seen from Σ. We may thus define the spinning coordinates of P as

$$\begin{aligned} x_s &= x - x_{\text{orig}} = x + (b/a)\ t, \\ y_s &= y, \quad t_s = t. \end{aligned} \tag{5}$$

Inserting Eqs. (2) and (5) into Eq. (1), one obtains

$$x_s = (R/a)\cos\omega\left[(h - gb/a)\,t + gx_s\right], \tag{6}$$

$$y_s = R\sin\omega\left[(h - gb/a)\,t + gx_s\right]. \tag{7}$$

The trajectory of the particle P in the rocket frame Σ is obtained by eliminating t from the above equations, and one thus has

$$a^2{x_s}^2 + {y_s}^2 = R^2. \tag{8}$$

Now in SR the transformation Eq. (2) is nothing but LT:

$$x^0 = \gamma(x - \beta ct), \quad y^0 = y, \quad t^0 = \gamma\left(t - \beta c^{-1}x\right), \tag{9}$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, with $\beta c = v$ representing the speed of the rocket with respect to the table. In other words, for LT the transformation matrix representing Eq. (2) is given by

$$\mathrm{T} = \begin{pmatrix} a\, b \\ g\, h \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta c \\ -\beta c^{-1} & 1 \end{pmatrix}. \tag{10}$$

Inserting these values of a, b, g and h into (6), (7), and (8), we obtain the equation of motion and that of the trajectory of the particle P as

$$x_s = \gamma^{-1} R \cos\omega \left(\gamma^{-1} t - \gamma\beta c^{-1} x_s\right), \tag{11}$$

$$y_s = R \sin\omega \left(\gamma^{-1} t - \gamma\beta c^{-1} x_s\right), \tag{12}$$

$$\gamma^2 x_s^2 + y_s^2 = R^2. \tag{13}$$

Authors of Ref. [1] rightly point out that the above equations display all the well-known effects of relativity. For example, Eq. (13) shows that what is a circle with respect to the table frame is now an ellipse with respect to the rocket frame (in the spinning coordinates) because of the length contraction effect. Equations (11) and (12) show that the angular frequency ω of the top is reduced to $\gamma^{-1}\omega$ because of the time dilation. Finally the relativity of simultaneity (i.e., the lack of synchronization of clocks) is manifested in the presence of the spatial coordinate x_s in the phase factors of the sinusoidal functions in Eqs. (11) and (12). The authors claim that this feature plays an essential role in the resolution of the paradox. We will however show that this is not quite correct.

The arguments of Basu *et al.* [1] are based on the apparent non-rigid behaviour of the top as seen from the rocket. However, before we go into these in detail let us now study this non-rigid character a bit closely. Equations (11), (12), and (13) display qualitatively two distinct types of non-rigidity viz. type I and type II.

Type I. This type of non-rigidity is manifested in the periodical change of the distance of any particle of the top from the center, as the particle travels along the elliptical path according to Eq. (13).

Type II. From Eq. (11) and (12) it will be evident that a chain of particles which lie along the radius of the circle (see Eq. (1)) parallel to the x-axis of Σ^0 at $t^0 = 0$ will appear to take a shape of a bow below the semi-major axis at $t = 0$ with respect to the rocket frame (see Fig. 4 of Ref. [1] for a schematic diagram of the positions of the particles in the radial chain at $t = 0$). A particle in the chain which is farther away from the center will lie farther below the x-axis. This progressive increase of initial phases of the particles is clearly the consequence of the phase factor in Eqs. (11) and (12). As time passes, the bow-like chain rotates, straightens itself, again bends down and straightens up and continues like this. (For the stills of the computer movie of the chain of particles in the table and the rocket frames see Fig. 5 of Ref. [1].) This corresponds to another type of non-rigid behaviour of the material of the top as distinct from the type I. We call it type II. Note that this type II non-rigidity is clearly the result of the relativity of simultaneity.

The authors of Ref. [1] observe that the spinning top is "more like a visco-elastic fluid in a weird centrifuge subjected to anisotropic external stresses." However, although it is correct that the concept of rigid body dynamics and moments of inertia appears to be no longer valid in the spinning coordinate system, the type II non-rigidity at least has nothing to do with the paradox. In fact this apparent fluid-like motion of the top particles has no connection with the question of the inconsistency of the notion of rigidity in relativity. In the next section we will show, among other things, that the fluid-like motion of the particles of the top can also be seen in the non-relativistic world although it is well known that the notion of rigid body is quite consistent in classical mechanics.

3. CONVENTIONALITY OF SIMULTANEITY AND TRANSFORMATION EQUATIONS

3.1. Relativistic World

In special relativity spatially separated clocks in a given inertial frame are synchronized by light signals. This synchronization is possible provided one knows beforehand the one-way speed (OWS) of the signal. But the measurement of OWS requires pre-synchronized clocks and therefore one ends up in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the synchronizing signal within certain bounds. Einstein assumed the OWS of light to be equal to c which is the same as the two-way speed (TWS) of light. The TWS of a signal is an empirically verifiable quantity, as this can be measured by a single clock without requiring any distant clock synchrony. Note that this stipulation (the equality of OWS and TWS) of Einstein has nothing to do with his "constancy of velocity of light" postulate [5, 6]. The assertion that the procedure for distant clock synchrony in SR has an element of convention is known as the CS-thesis, first discussed by Reichenbach [7] and Grünbaum [8]. The synchronization convention adopted by Einstein is commonly known as the Einstein synchrony or the standard synchrony. The possibility of using a synchronization convention other than that adopted by Einstein and consequent transformation equations between inertial frames are much discussed by various authors [4–6, 9–11]. For example it is known that the relativistic world can well be described by the so called Tangherlini transformations (TT) by adopting absolute synchrony [4–6, 11–13]:

$$x = \gamma\left(x^0 + \beta c t^0\right), \qquad y = y^0, \qquad t = \gamma^{-1} t^0. \tag{14}$$

Notice that the absence of spatial coordinate in the time transformation above, means the distant simultaneity is absolute (since $\Delta t^0 = 0 \Longrightarrow \Delta t = 0$). Since, according to the CS thesis, the question of simultaneity of any two spatially separated events depends on the synchronization convention, the issue of relativity of simultaneity which is often considered as one of the most fundamental imports of SR, has little significance [5]. We will get back to this issue and the transformations (14) in particular in section 5 in the context of the paradox.

3.2. Galilean World

A less well-known fact, however, is that the CS thesis can also be imported in the classical (Galilean) world. Consider as a fiction that we live in the Galilean world and suppose light travels through ether, stationary with respect to Σ^0. The spacetime coordinates of an arbitrary inertial frame Σ moving with speed $v = -\beta c$ with respect to Σ^0 are related to those in Σ^0 by the so-called Galilean transformations (GT)

$$x^0 = x - \beta c t, \qquad y^0 = y, \qquad t^0 = t. \tag{15}$$

In the Galilean world synchronization issue usually does not come in, since in principle, all the spatially separated clocks can be synchronized instantaneously by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However, ingredients of Einstein synchrony can be incorporated even in this world. Say in a somewhat playful spirit one chooses to synchronize an arbitrarily located clock in any frame Σ with one placed at its origin by sending a light signal from the origin to the clock in such a way that the OWS of light *along any line* passing through the origin is independent of the direction of propagation and is equal to the TWS of the signal along the line. In this case, one obtains the so-called Zahar transformation (ZT) [6, 11, 14]

$$x = x^0 + \beta c t^0, \qquad y = y^0, \qquad t = \gamma^2\left(t^0 + \beta c^{-1} x^0\right), \tag{16}$$

and its inverse

$$x^0 = \gamma^2\left(x - \gamma^{-2}\beta c t\right), \qquad y^0 = y, \qquad t^0 = t - \gamma^2 \beta c^{-1} x. \tag{17}$$

One may verify from the transformations (16) and (17) that the TWS of light along any direction measured in an arbitrary reference frame is given by the same expression as one would have obtained using GT.

For example one may verify that the TWS of light along the x-axis and y-axis in Σ are given by the Galilean results, $c\,(1-\beta^2)$ and $c\,(1-\beta^2)^{\frac{1}{2}}$, respectively [5, 13].[3] Clearly the presence of the spatial coordinate in the time transformations of (16) and (17) is the result of the adopted synchrony. The properties of rods and clocks also do not change due to their motions with respect to Σ^0. However, there is an apparent length contraction and time dilation effect with respect to Σ because of different simultaneity criterion used in this frame.

It may also be noted that even in the Galilean world the transformation Eqs. (16) and (17) depend on the speed of light c in ether, since light has been chosen as the synchronizing agent. If instead of light the clocks are synchronized by any other signal with speed c' in Σ^0 the transformation equations would have been

$$x = x^0 + \beta' c' t^0, \qquad y = y^0, \qquad t = \gamma'^2 \left(t^0 + \beta' c'^{-1} x^0\right), \tag{18}$$

where β' and γ' are the same as β and γ except where c is replaced by c'.[4] The synchronization in this case may be called *pseudo-standard synchrony* since the synchronization agent here is not the light signal.

4. TIPPE TOP PARADOX IN THE GALILEAN WORLD

The paradox can now be posed even in the Galilean world. With respect to the observer in the rocket frame Σ, the geometry of the top will appear to have altered because of the length contraction effect which is the outcome of the distant clock synchrony adopted in the rocket frame. This may give rise to the possibility that the ratios of the principal moments of inertia go out of the regime required for the top to tip over! We now proceed to "resolve" the paradox by following the line of arguments used in Ref. [1]. The matrix of ZT (Eqs. (16)

[3]The Galilean world or classical world is thus defined to be a world where the TWS of any signal obeys the transformation law that one would have obtained by using the Galilean velocity addition rule. On the other hand, the world is said to be relativistic if the space-time admits an invariant TWS [6]. It may be noted, by virtue of the CS thesis, that kinematically different transformations and OWS' may correspond to a same kinematical "world" [13].

[4]Operationally one may consider that Σ^0 is a frame of reference stationary with respect to some fluid which supports an acoustic mode with isotropic speed c'. Clocks in any frame are assumed to be synchronized following Einstein's convention using the signal. Equation (18) was called Dolphin transformations (DT) in the Galilean world in Ref. [6], where it was first derived.

and (17)) representing the coordinate transformations for x^0 and t^0 is

$$\mathbf{T} = \begin{pmatrix} \gamma^2 & -\beta c \\ -\gamma^2\beta c^{-1} & 1 \end{pmatrix}. \tag{19}$$

The equations of motion and the equation of the trajectory of the particle in the spinning coordinates can be obtained by inserting the elements of $\mathbf{T}$, *i.e.*, a, b, g and h in Eqs. (6), (7), and (8) as

$$x_s = \gamma^{-2} R \cos\omega(\gamma^{-2}t - \gamma^2\beta c^{-1}x_s), \tag{20}$$

$$y_s = R\sin\omega(\gamma^{-2}t - \gamma^2\beta c^{-1}x_s), \tag{21}$$

$$\gamma^4 x_s^2 + y_s^2 = R^2. \tag{22}$$

As before, the time dilation, the length contraction and the lack of synchronization of clocks seem to be present in these equations. Only quantitatively these effects differ from those obtained earlier using the LT.

These equations suggest that qualitatively the top material displays the same form of non-rigidity (of both type I and II) as has been observed in the relativistic world as the particles of the top moves in accordance with Eqs. (20) and (21) with respect to the observer in the rocket frame. Indeed one only needs to slightly modify the programming codes developed in Ref. [1] to simulate the motion of the particles of the top and see for oneself the picturesque output on the computer screen displaying as before, the non-rigid character of the body.

The idea of Einstein synchrony in the Galilean world may at a first sight seem to be a bit weird. However, the idea is not as strange as it appears. Consider LT in the non-relativistic regime where $\beta^2 \ll 1$ so that the approximation $\gamma \cong 1$ holds. In this approximation, contrary to common belief, LT does not go over to GT, instead one obtains the so-called approximate Lorentz transformation (ALT) [13, 15, 16]

$$x^0 = x - \beta ct, \qquad t^0 = t - \beta c^{-1}x. \tag{23}$$

As is expected the transformations (23) do not exhibit length contraction and time dilation. However, by virtue of the presence of the spatial coordinate in the time transformation the simultaneity is not absolute. It can also be verified [13] that ALT represents Einstein synchrony. It is therefore not surprising that ZT also reduces to Eq. (23) under the same approximation. ALT or approximate Zahar transformation (AZT) therefore represents the Galilean world with Einstein synchrony. Inserting the coefficients of ALT/AZT in Eqs. (6), (7), and (8) one obtains the equations of motion and the trajectory of the spinning particle as

$$x_s = R\cos\omega\left(t - \beta c^{-1}x_s\right), \tag{24}$$

$$y_s = R\sin\omega\left(t - \beta c^{-1}x_s\right), \tag{25}$$

$$x_s^2 + y_s^2 = R^2. \tag{26}$$

Clearly according to these equations the material of the top still displays non-rigidity of type II.[5]

In line with Ref. [1], one may now try to say that the concept of rigid body does not fit in the classical world. One may even be tempted to explain away the paradox by saying that nothing is rigid and all bodies are compressible and failure to comprehend this, leads to the paradox! Clearly this is absurd in the Galilean world or in the non-relativistic regime.

In order to understand the situation more clearly, let us examine the meaning of the spinning co-ordinate system. From the transformation Eqs. (2) and (5) one may connect the vectors

$$\mathbf{x}^0 = \begin{pmatrix} x^0 \\ t^0 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_s = \begin{pmatrix} x_s \\ t_s \end{pmatrix}$$

as

$$\mathbf{x}^0 = \mathbf{B}\mathbf{x}_s, \tag{27}$$

with

$$\mathbf{B} = \begin{pmatrix} a & 0 \\ g\,h - gb/a \end{pmatrix}, \tag{28}$$

where, as before, the y-coordinate has been suppressed, although we keep in mind that $y_s = y = y^0$. For Galilean transformation (see Eq. (15))

$$\mathbf{T} = \begin{pmatrix} 1 & -\beta c \\ 0 & 1 \end{pmatrix}, \tag{29}$$

$\mathbf{B}$ turns out to be the identity matrix.

In this case the equations of motion and trajectory of any particle represented by Eq. (1) will not lead to a fluid-like motion of the particles in the spinning coordinate system. However for $\mathbf{B} = \mathbf{1}$, these coordinates are the same as that used in Σ^0!

[5]Equations (24), (25), and (26) could have been obtained using the approximation $\gamma \cong 1$ directly in Eqs. (11), (12), and (13) or, alternatively, in (20), (21), and (22); however, the present approach is more instructive since it clearly shows the role of the lack of synchrony of clocks in the type II non-rigidity of the top.

This only demonstrates that the rigidly rotating top will continue to display its rigid character only in the coördinate system defined in the frame of reference at rest with the table. Therefore, it is obvious that the concept of rigid body dynamics or moments of inertia are applicable only in this unique frame and this knowledge therefore surely resolves the paradox. However, this fact does not depend on whether the world is classical or relativistic. Therefore, in accordance with our earlier assertion, there is no connection of the apparent fluid-like behaviour of the top material in the spinning coordinate system with the issue of incompatibility of the notion of rigidity in SR.

5. LACK OF SYNCHRONIZATION – IS IT CRUCIAL?

Basu *et al.* [1] claimed that the lack of synchronization of clocks i.e., the relativity of simultaneity aspect of SR plays an essential role in the resolution of the paradox. But we have already observed in section 3, that the question of relativity of simultaneity is purely conventional and therefore is devoid of any empirical content. Kinematically the relativistic world can be described by TT (14) representing absolute simultaneity. The transformation matrix representing the inverse of TT in the $x - t$ plane may be written as

$$\mathrm{T} = \begin{pmatrix} \gamma^{-1} & -\gamma\beta c \\ 0 & \gamma \end{pmatrix}. \tag{30}$$

Inserting the elements of T in (6), (7), and (8), we get

$$x_s = \gamma R \cos \gamma\omega t, \tag{31}$$

$$y_s = R \sin \gamma\omega t, \tag{32}$$

$$\gamma^{-2} x_s^2 + y_s^2 = R^2. \tag{33}$$

Notice the absence of the phase terms in the sinusoidal functions. This means that the top does not display type II non-rigidity with respect to the rocket frame. However, the trajectory (33) of P is still an ellipse (this time the semi-major axis is along the x-direction) manifesting type I non-rigidity of the top. This only reiterates that the rigid rotation has to be defined in the table frame, but for this conclusion to hold the lack of synchronization aspect of SR does not play any role.

6. RIGIDITY AND TRANSCENDENTAL EQUATION

It has been noted in [1] that the transcendental Eq. (6) is of the form

$$x_s = f(x_s), \tag{34}$$

which can be solved by iteration provided

$$|f'(x_s)| < 1. \tag{35}$$

It is claimed that the condition (35) when applied to Eq. (18) leads to

$$\omega R < c, \tag{36}$$

which only says that no particle of the top can exceed the speed of light. This result, although fascinating, seems to be fortuitous, since instead of Eq. (6), if Eq. (20) (which pertains to the Galilean world) is used in the inequality (35), one obtains the same constraint condition (36) on the speed of a particle of the top. This is surprising, since in the Galilean world, there is no such speed limit intrinsically.

For the DT in the Galilean world (see Eq. (18)), the condition (35) leads to

$$\omega R < c', \tag{37}$$

which is more surprising.

On the other hand, we have seen that TT which represents the absolute synchrony in the relativistic world does not lead to any transcendental equation (see Eq. (31)) and hence no constraint on the speed of the particle is visible.

7. SUMMARY

The present paper discusses the tippe top paradox and different aspects of its resolution proposed in Ref. [1]. Clock synchronization issues in the relativistic and the Galilean world figured in course of our discussion. A few transformation equations in addition to LT were discussed in this connection. It is therefore worthwhile to summarise different properties of the transformations in the context of the paradox. This we do in Table 1 so that one is able to get the whole picture at a glance. The table is self-explanatory, however explanations of a few entries may be in order.

Table 1

World	Transformation	Synchrony Type	Type I Non-rigidity	Type II Non-rigidity	Length Contraction	Time Dilation Effect	Paradox exists? (prima facie)
Relativistic	LT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
Relativistic	TT	Absolute	Yes	No	Yes (w.r.t. Σ^0)	Yes (w.r.t Σ^0)	Yes
Relativistic /classical	ALT/AZT	Standard (Einstein)	No	Yes	No	No	No
Galilean (Classical)	GT	Absolute	No	No	No	No	No
	ZT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
	DT	Pseudo Standard	Yes	Yes	Yes	Yes	Yes

Basically we discussed two worlds – relativistic and classical, but an overlapping world "Relativistic/Classical" is included in column 1 as a separate entry. This corresponds to the transformations ALT and AZT (see column 2) which are the forms of LT and ZT respectively under the approximation $\gamma \simeq 1$. For both the transformations, the synchrony type (as shown in column 3) is standard. These transformations do not predict the length contraction and the time dilation effects (see entries in column 6 and 7). The paradox in this case does not exist prima facie (see the entry in the last column) in this regime since there is no length contraction effect. However the observer in Σ will find the top material to exhibit non-rigidity of type II.

Note that the entries in the 1st, 5th, and 6th rows from column 3 onwards corresponding to the transformations LT, ZT, and DT, respectively, are exactly the same. This means that the paradox and the resolution as suggested in Ref. [1] completely fits in the classical world too. It therefore dismisses the claim that the paradox has its origin in the incompatibility of the notion of rigidity with SR.

The entries against TT shows that in the relativistic world non-rigidity of Type I of the top exists, although the synchrony here is absolute. It therefore follows that "lack of synchronization of clocks" cannot play an essential role in resolving the paradox.

REFERENCES

1. Aniket Basu, R. S. Saraswat, Kedar B. Khare, G. P. Sastry, and Sougato Bose, *Eur. J. Phys.* **23** 295-305 (2002).
2. Aurthur Evett, *Understanding the Spacetime concepts of Special Relativity* (Publishers Creative Services, New York, 1982).
3. M. Redhead and T. A. Debs, *Am. J. Phys.*, **64**(4) 384-391 (1996).
4. F. Selleri, *Found. Phys.* **26**, 641-664 (1996).
5. S. K. Ghosal, P. Chakraborty and D. Mukhopadhyay, *Europhys. Lett.* 15(4), 369-374 (1991).
6. S. K. Ghosal, D. Mukhopadhyay, and Papia Chakraborty, *Eur. J. Phys.* **15**, 21-28 (1994).
7. H. Reichenbach, *The Philosophy of Space and Time* (Dover, New York, 1958).
8. A. Grünbaum, *Philosophical Problems of Space and Time*, 1st edn. (Knopf, New York, 1963).
9. A. Winnie, *Phil. Sci.* **37**, 81-99 (1970).
10. R. Mansouri and R. U. Sexl, *Gen. Rel. Grav.* **8**, 497-513 (1977).
11. T. Sjödin, *Nuovo Cimento B* **51**, 229-245 (1979).
12. F. R. Tangherlini, *Nuovo Cimento Suppl.* **20**, 1-86 (1961).
13. S. K. Ghosal, K. K. Nandi, and P. Chakraborty, *Z. Naturforsch.* **46a**, 256-258 (1991).
14. E. Zahar, *Brit. J. Phil. Sci.* **28**, 195-213 (1977).

15. L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th revised English edition (Pergamon, Oxford, 1975).
16. P. Di Mauro "La formula di diffusione Compton con al meccanica Newtoniana," *Atti del XVI Congresso Nazionale di Storia della Fisica e dell'Astronomia, Como 28-29 Maggio*, 179-184 (1999).

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ON THE ANISTROPY OF THE SPEED OF LIGHT ON A ROTATING PLATFORM

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The paper discusses a recently posed paradox in relativity concerning the speed of light as measured by an observer on board a rotating turn-table. The counter-intuitive problem put forward by F. Selleri concerns the theoretical prediction of an anisotropy in the speed of light in a reference frame comoving with the edge of a rotating disc even in the limit of zero acceleration. The paradox not only challenges the internal consistency of the special relativity theory but also undermines the basic tenet of the conventionality of simultaneity thesis of relativity. The present paper resolves the issue in a novel way by recasting the original paradox in the Galilean world and thereby revealing, in a subtle way, the weak points of the reasonings leading to the fallacy. As a background the standard and the non-standard synchronies in the relativistic as well as in the Galilean world are discussed. In passing, this novel approach also clarifies (contrary to often made assertions in the literature) that the so-called "desynchronization" of clocks cannot be regarded as the root cause of the Sagnac effect. Finally in spite of the flaw in the reasonings leading to the paradox Selleri's observation regarding the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

Key words: special relativity, Selleri paradox, Sagnac effect, rotating frame, speed of light, conventionality of simultaneity.

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1. INTRODUCTION

The term "inertial frame of reference" in physics refers to an idealised concept. Our knowledge of physics in inertial frames has always been obtained in frames having small but non-zero acceleration. Indeed it is well-known that no perfectly inertial frame can be identified in practice. It is therefore expected that physics in non-inertial frames will go over smoothly to that in inertial frames in the mathematical limit of zero acceleration. In some recent papers [1, 2] Selleri observes that the existing relativity theory fails our expectations on that count. In this connection Selleri poses a paradox concerning the speed of light as measured by an observer on board a rotating turn-table. If two light beams from a common source are sent along the rim of a rotating disc in opposite directions and the round-trip speeds (c_+ for counter-rotating and c_- for co-rotating beams) for these two light beams are measured, it will be found from simple kinematics, that the ratio of these speeds $\rho = c_+/c_-$ is only a function of the linear speed v of disc at its edge and it differs from unity if $v \neq 0$. This observation finds its support in the well known Sagnac effect [3, 4] which is manifested in the experimentally observed asymmetry in the round-trip times of light signals co-rotating and counter-rotating with the interferometer. However, since the rotating turn-table is not an inertial frame, one might initially not be inclined to consider the observed anisotropy of light propagation with respect to this frame a startling result as such. But Selleri then considers a situation where one gradually increases the radius of the disc and at the same time allows the angular velocity ω of the same to get decreased proportionately in such a way that the linear speed $v = \omega R$ of the periphery remains constant. The edge of the disc can then be thought of approaching (locally) an inertial frame since, in the limit the centripetal acceleration $a = v^2/R$, tends to zero. [3] In an inertial frame ρ must strictly be unity since, light propagation is considered to be isotropic in such a frame according to the special relativity (SR). However Selleri shows that the ratio ρ on board the rotating disc does not change in the limit process provided v remains constant and therefore will continue to differ from unity as long as v is finite. A discontinuity in the behaviour of ρ as a function of acceleration is thus predicted. This is certainly paradoxical in the light of the observations made in the beginning of this section. We hereafter

[3] There is a scope for confusion here. Although an element of the disc will have zero acceleration in the limit considered, an observer on the turn table would be able to detect its rotation since the latter is an absolute concept. In an article Klauber [5] even claimed that there would be, for example a change of mass of a particle on the disc due to a general relativistic effect which can be seen to depend only on the circumferential velocity and not on the acceleration. This effect would even enable one to determine in principle the rotational motion of the platform from local measurements alone!

refer to it as the Selleri paradox (SP).

SP has so far met with evolving but inadequate responses. For example in one paper Rizzi and Tartaglia [6] observe that the "calculations of Selleri are quite careful" and the "paradox cannot be avoided if it is maintained that the round-trip on the turn-table corresponds to a well defined circumference whose length is univocally defined".

It appears that the authors of Ref. [6] cannot accept the global anisotropy of light speed in the frame of reference of the rotating disc and hold that because of the "impossibility of a symmetrical and transitive synchronization at large", the notion of whole physical space on the platform at a given instant is conventional. Their final conclusion is that the counter-rotating and the co-rotating light beams travel different distances with respect to the *frame of the disc* in such a way that the *global* ratio ρ remains unity.

The view point towards the resolution of SP also finds its endorsement in a later paper by Tartaglia [7]; although in a subsequent paper Rizzi and Tartaglia [8] somewhat retract from the past position and allow an observer at rest on the disc to consider a notion of its unique circumference in the "relative space of the disc" and hence endorse the view that light propagation can be anisotropic in the reference frame of the rotating turn-table. In conjunction with Budden's observation [9] the authors then correctly identify the root of the paradox and hold that the basic weak point of Selleri's arguments lies in equating the global ratio ρ of the speeds of light propagating in opposite directions along the rim with the local ratio ρ' of the same at any point on the edge of the disc. The latter ratio is always equal to unity if Einstein synchrony is used in any local inertial frame instantaneously comoving with the element of the rim at the point concerned (such frames will hereafter be referred to as momentarily comoving inertial frames (MCIF))and therefore SP does not pose any harm to SR.

However, Selleri's argument regarding the equality of two ratios ρ and ρ' is based on a symmetry argument (rotational invariance) but the authors of Ref. [8] do not clearly state what is precisely wrong with Selleri's symmetry reasoning. Further the arguments by the authors although correct, are blurred by their ambivalent observations (in the same paper) that the global ratio ρ itself comes out to be unity (a) if the time measuring clock is suitably corrected to "account for the desynchronization effect" or (b)if the space is suitably defined according to "geometry of Minkowskian spacetime". Note that (b) is the reiteration of their earlier stand in this regard [6, 7].

In Ref. [1, 2] Selleri raises another matter in connection with SP. In light of conventionality of (distant) simultaneity (CS) thesis of SR, the author discusses the conventionality issue on a rotating turn-table and argues that not the Lorentz transformation (LT) but the relativistic transformation with absolute synchrony (which is one of the many possible synchronization conventions for which light propagation is anisotropic) only correspond to the correct expression for ρ

(see Eq. (16) later). In a recent paper Minguzzi [10], whose view we share, briefly addresses the issue. The author agrees that isotropic convention (standard synchrony) can be unsuitable in certain situations but maintains that the possibility of anisotropic conventions does not imply any inconsistency of SR. However SP has not been discussed therein in its entirety.

To sum up it may be said that the responses to SP so far available in the literature are not fully satisfactory. We therefore hold that the paradox which poses a challenge to the very foundations of SR by questioning its self consistency, deserves to be given a fuller treatment. Indeed there are many subtle issues concerning SP. For example it will be seen in Sec. 3 that the paradox not only undermines the standard relativity theory but also denies the basic tenet of the CS thesis. The purpose of the present paper is to re-examine Selleri's arguments in the light of the CS thesis and provide a resolution of SP in a novel way by recasting the paradox in the classical world (see Sec. 4). It will however be argued that while both the self-consistency of SR and the CS thesis remain unchallenged, SP has a merit in that if properly interpreted in the light of reasonings presented in this paper, the whole issue will throw new light on various related issues like the question of time on rotating platform, desynchronization and its debated role in the explanation of Sagnac effect [11, 12].

We organize the paper as follows. Before we present our main arguments in Sec. 4 and onwards, the CS thesis will be discussed (in Sec. 3) in the context of the paradox. However in order to set the stage we will briefly reproduce in Sec. 2 the arguments of Selleri leading to SP. In Sec. 5 the issue of desynchronization vis-a-vis the Sagnac effect will be addressed and finally in Sec. 6 the standard synchrony and absolute synchrony will be compared upholding Selleri's point of view in this regard.

2. THE PARADOX

Suppose a light source is placed at some fixed position Σ on the rim of the turn-table and two light signals start from Σ at the laboratory time t_{01}, and are constrained (by allowing them for example, to graze a suitably placed cylindrical mirror on the rim) to travel in opposite directions in a circular path along the periphery of the disc. Let that, after making the round trips, the counter-rotating and co-rotating light flashes reach Σ at times t_{02} and t_{03} respectively.

As seen from the laboratory, the counter-rotating light signal travels a distance shorter than the circumference L_0 by the amount

$$x = v(t_{02} - t_{01}), \tag{1}$$

where $v = \omega R$ is the linear speed of the disc at its periphery. Similarly the co-rotating light beam has to travel a distance larger than L_0 by the amount

$$y = v(t_{03} - t_{01}). \tag{2}$$

From simple kinematics it therefore follows that

$$L_0 - x = c(t_{02} - t_{01}), \tag{3}$$

$$L_0 + y = c(t_{03} - t_{01}), \tag{4}$$

where L_0 is the disc's circumference as seen from the laboratory. From equations (3) and (4) and using equations (1) and (2) one readily obtains the round-trip times for counter-rotating and co-rotating signals, respectively, as

$$t_{02} - t_{01} = L_0/c(1+\beta), \tag{5}$$

and

$$t_{03} - t_{01} = L_0/c(1-\beta). \tag{6}$$

By taking the difference of these times, one may note here that the delay between the arrival of the two light signals at the point Σ is obtained as

$$\Delta t_s = t_{03} - t_{02} = (2/c)L_0\beta\gamma^2, \tag{7}$$

where $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$. As an aside remark, it may be noted that (7) is nothing but the well-known delay time of classical Sagnac Effect. The relativistic formula for Sagnac delay can easily be obtained by noting that Δt_s in Eq. (7) is not the time as measured on board the platform and hence time dilation effect has to be considered. By multiplying both sides of the equation by $\gamma^{-1} = (1-\beta^2)^{\frac{1}{2}}$, one obtains the relativistic formula for Sagnac delay as

$$\Delta\tau_s = (2/c)L_0\beta\gamma, \tag{8}$$

where $\Delta\tau_s = \gamma^{-1}\Delta t_s$ denotes the delay time as measured on board the turn-table.[4]

Suppose now that a clock C_Σ is placed on the disc's rim at Σ so that it co-rotates with the platform and also let t denotes the time of C_Σ. When the disc is in motion, according to Selleri, the laboratory time t_0 and t may be assumed to be related most generally as

$$t_0 = tF_1(v, a). \tag{9}$$

Similarly for the circumference also Selleri assumes a relation between L_0 and the proper circumference L as

$$L_0 = LF_2(v, a), \tag{10}$$

[4]There is a mild controversy however as to the correct special-relativistic formula for the Sagnac delay. However, Eq. (8) is the most widely quoted one. For a detailed discussion on the issue *vis-à-vis* the Ehrenfest paradox, see Ref. [13].

where F_1 and F_2 are some functions of the linear velocity $v = \omega R$ and the acceleration $a = v^2/R$ of the edge of the disc.

Although, from the widely accepted hypothesis of locality [6, 12] it is evident that these functions are nothing but the usual time dilation and length contraction factors

$$F_1 = F_2^{-1} = \gamma, \tag{11}$$

however, Selleri keeps open the possibility that F_1 and F_2 may depend on the acceleration as well.

Inserting equations (9) and (10) in equation (5) and (6) one gets the following times of flight of the counter-rotating and co-rotating light signals as measured on board the disc,

$$t_2 - t_1 = \frac{L}{c(1+\beta)} \frac{F_2}{F_1}, \tag{12}$$

$$t_3 - t_1 = \frac{L}{c(1-\beta)} \frac{F_2}{F_1}. \tag{13}$$

The round-trip speeds for these beams are therefore given by

$$c_+ = \frac{L}{t_2 - t_1} = c(1+\beta)\frac{F_1}{F_2}, \tag{14}$$

$$c_- = \frac{L}{t_3 - t_1} = c(1-\beta)\frac{F_1}{F_2}. \tag{15}$$

Consequently the ratio of these light speeds turns out to be

$$\rho = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{16}$$

Selleri now argues that since no point on the rim is preferred, the instantaneous velocities of either signals at any point of the rim must be the same, and therefore, the above ratio ρ is true not only for the global light velocities but also for the instantaneous velocities at any point on the rim. Now, as pointed out in Sec.1, if we consider that $R \to \infty$ and $\omega \to 0$ in such a way that v, the linear speed of any element of the circumference remains constant, so that the centripetal acceleration $a \to 0$, any short part of the circumference can be thought of as an inertial frame of reference in the limit. However, the ratio ρ does not change as long as v is kept constant. Hence, a discontinuity results in the behaviour of ρ as a function of acceleration ($\rho = \rho(a)$), since as $a \to 0$, but not equal to zero, ρ continues to differ from unity, but if $a = 0$, SR predicts that ρ must be equal to unity!

It may be argued that the above gedanken experiment with infinitely sized disc is impossible to perform since the times of flight of the co-rotating and counter-rotating light beams whose ratio we are currently interested in, would become infinite and therefore unmeasurable [14]. However it is enough to note that if the radius of the disc is increased arbitrarily from a finite value and at the same time v is kept constant by suitably adjusting the angular velocity, no tendency for the ratio ρ getting diminished would be seen although the acceleration of a point on the circumference gets reduced arbitrarily in the process.

It is worthwhile to mention in this context that recently Wang *et al.* [15] has obtained a travel time difference $\Delta t = 2vL/c^2$ between two counter-propagating light beams (indicating $\rho \neq 1$) in a uniformly moving fibre where v is the speed of the light source or the detector (comoving with the fibre) with respect to the laboratory and L in the length of the fibre. The experiment has been performed using a fibre optic conveyor(FOC) where two light beams leaving a source travel in opposite directions through an optical fibre loop which is made to move with uniform speed like a conveyor belt by a couple of rotating wheels separated by a distance. The interesting feature of the FOC arrangement is that here the observer (*i.e.,* the source or the detector) is attached to one of the straight-fibre segments and therefore moves with *uniform velocity* along a straight line. Experimental observation together with a symmetry argument (similar to that used by Selleri in the rotating disc context) may lead one to infer that the statement $\rho \neq 1$ is also valid *locally* in a segment of uniformly moving fibre indicating *local* anisotropy in the speed of light in vacuum [5] with respect to an inertial observer! Such an outcome which apparently follows from a symmetry argument is also paradoxical if one believes in SR. Although the purported scope of the present paper restricts us to deliberating on SP in its original form and consequent issues following a few responses it has received, the arguments that will be used in the following sections will equally apply to the paradox in the FOC context as well.

Indeed all Selleri wanted to achieve was to obtain an inertial observer with respect to whom $\rho = (1+\beta)/(1-\beta)$. In the FOC arrangement this comes naturally dispensing with the trick of letting the radius of the disc go to infinity and the angular speed to zero while the peripheral velocity is kept constant.

Before we leave this section, we write explicitly the expressions for c_+ and c_- in the full-relativistic context:

$$c_+ = c/(1-\beta) \ , \tag{17}$$

$$c_- = c/(1+\beta) \ , \tag{18}$$

[5]As suggested by Wang et-al, here we have assumed that experiment using FOC with a hollow core would give the same result. Indeed the result $\rho = (1+\beta)/(1-\beta)$ remains valid in this case since the simple minded analysis presented in this section leading to the equation also applies to this situation.

which follow from Eqs. (14) and (15) where the expressions for F_1 and F_2 as given in Eq. (11) have been substituted. Here we may point out that the genesis of SP relates to these equations since in the limit of infinite radius and zero angular velocity, the above results do not change indicating (as if) the violation of second relativity postulate (isotropy and constancy of light speed). As we have mentioned earlier, the above results although correct, are so counter-intuitive that the authors of Refs. [6, 7] in their initial reactions discarded the results altogether only to retract from their position later in a sort of a rejoinder [8].

3. C-S THESIS AND ABSOLUTE SYNCHRONY

In the relativity theory distant simultaneity is conventional. In order to synchronize spatially separated clocks in a given inertial frame one should know the one-way speed (OWS) of the synchronizing signal, however to know OWS one needs pre-synchronized clocks. One therefore is caught in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the light signal within certain bounds. The C-S thesis, first discussed by Reichenbach and Grünbaum [16, 17],[6] is the assertion that the procedure for distant clock synchrony is conventional. Einstein therefore assumes as a convention that the OWS of light is isotropic and is equal to the two-way speed (TWS) c of the signal. Note that the latter is an empirically verifiable quantity since it does not depend on the convention regarding the synchronization of spatially separated clocks since the TWS can be measured by a single clock. The synchrony is commonly known as the Einstein synchrony or the standard synchrony. However since the clock synchronization procedure is conventional, conventions other than the standard one may equally be chosen [19–22]. Selleri [1, 2] has shown that the space-time transformation between a preferred inertial frame S_0 (where clocks are standard-synchronized so that OWS is isotropic in the frame) and any other frame S may generally be written as

$$\begin{aligned} x &= \gamma(x_0 - \beta c t_0), \\ y &= y_0, \\ t &= \gamma t_0 + \epsilon(x_0 - \beta c t_0), \end{aligned} \tag{19}$$

which represents a set of theories *equivalent* to SR. The free parameter ϵ which can at most be a function of the relative velocity of S with respect to S_0, depends on the simultaneity criterion adopted in S. For

[6]For a comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman [18].

the standard synchrony however,

$$\epsilon = -\beta\gamma/c. \tag{20}$$

For this value of ϵ, Eq. (19) reduces to a Lorentz transformation. The OWS' of light in S, c'_+ and c'_- along the negative and positive x-directions respectively may easily be obtained from the transformation (19) as

$$\frac{1}{c'_+} = \frac{1}{c} - \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right] \tag{21}$$

and

$$\frac{1}{c'_-} = \frac{1}{c} + \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right]. \tag{22}$$

If S_0 is assumed to be the inertial frame of reference at rest with the axis of the rotating disc and S be an MCIF, c'_+ and c'_- would then mean the local speeds of light counter-rotating and co-rotating with the disc respectively as measured on board the rotating platform. From Eqs. (21) and (22) one may thus obtain the ratio for these local speeds of light ρ' in terms of the ϵ-parameter

$$\rho' = \frac{c'_+}{c'_-} = \frac{1+\beta+\epsilon c\gamma^{-1}}{1-\beta-\epsilon c\gamma^{-1}}, \tag{23}$$

which agrees with the ratio ρ given by Eq. (16) provided $\epsilon = 0$. But as mentioned in the last section, the equality of ρ and ρ' according to Selleri as if follows from the symmetry of the situation. Therefore, in the rotating disc context, $\epsilon = 0$ appears to be the only allowed convention according to which the speed of light is anisotropic. Note that for this value of ϵ only the local speeds of light as given by Eqs. (21) and (22) reduce to the expressions (17) and (18). The transformation (19) with $\epsilon = 0$ is known as the Tangherlini transformation(TT) [23]

$$\begin{aligned} x &= \gamma\left(x_0 - \beta c t_0\right), \\ y &= y_0, \\ t &= \gamma^{-1} t_0. \end{aligned} \tag{24}$$

The transformation represents the relativistic world with absolute synchrony [20, 22].[7]

[7]Notice that in view of the absence of the spatial coordinate x in the above transformation for time, the simultaneity is independent of the frame of reference considered and is therefore absolute.

We now have a ramification of the original paradox. The value of ρ (and hence ρ') represented by equation (16), which implies anisotropic propagation of light in the rotating frame, is obtained theoretically from the perspective of the inertial frame S_0. The result also finds its empirical support in the Sagnac effect. It therefore appears that as if a particular (non-standard) synchrony is dictated both by theory and experiment. This is absurd since, if it were true it not only would reject the Lorentz transformation but also would contradict the basic tenet of the CS thesis that clock synchronization is conventional.

Before offering a resolution of the SP we ask ourselves if such a paradox could exist in the classical (Galilean) world too. The initial reaction would be to answer in the negative since nothing seems to be mysterious or enigmatic in this world and as is well-known, counter intuitive problems by contrast exist in the relativity theory probably because of its new philosophical imports. However we answer the question in the affirmative. One of the so-called new philosophical imports of SR is the notion of relativity of simultaneity. It can be shown that this notion can also be introduced in the Galilean world. Indeed in the next section it will be shown that by doing so the paradox can be artificially created even in this world where normally one would not expect it to exist.[8] The perspective of the paradox will hopefully provide deeper understanding of the problem and other related issues.

4. SELLERI PARADOX IN THE GALILEAN WORLD

Let us consider a fiction that we live in the Galilean (classical) world and suppose light travels through ether stationary with respect to an inertial frame S_0. The space-time coordinates of an arbitrary inertial frame S moves with respect to S_0 are related to those in S_0 by the so-called Galilean transformation(GT):

$$x = x_0 - \beta t_0, \qquad y = y_0, \qquad t = t_0. \tag{25}$$

In the Galilean world, synchronization issues usually do not figure in, since in principle all the clocks in any given inertial frame can be synchronized by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However the ingredients of the CS thesis can also be incorporated in this world. For example, one may employ the Einstein synchrony to describe the kinematics in this world. Suppose one sends out a light signal from the origin of S outwards along a line which makes an angle θ with the x-axis and the signal comes back to the origin along the same line after being reflected by a suitably placed mirror, the TWS can be obtained by measuring the

[8]Such an approach has been found fruitful elsewhere in understanding a recent paradox in relativity [24].

time of flight of the round-trip by a clock placed at the origin. The expression for this TWS can be obtained from Eq. (25) and is given by [20]

$$\overleftrightarrow{c}(\theta) = \frac{c(1-\beta^2)}{(1-\beta\sin^2\theta)^{1/2}}. \tag{26}$$

Now in a somewhat playful spirit one may choose to synchronize arbitrarily located clocks with one placed at the origin by sending light by *stipulating* the OWS of light to be equal to the TWS (in fact none can prevent one in doing so), the relevant transformation that would honour such a stipulation would be given by

$$\begin{aligned} x &= x_0 - \beta c t_0, \\ t &= \gamma^2 (t_0 - \beta x_0/c), \end{aligned} \tag{27}$$

which was originally obtained by E. Zahar in 1977 [25] and is now commonly known as the Zahar transformation(ZT). For a quick check one may readily verify that the TWS of light along the x-axis and y-axis in S,that follow from Eq. (27) are given by the well-known classical results, $c(1-\beta^2)$ and $c(1-\beta^2)^{1/2}$, respectively [21, 26].

In the context of the rotating disc, x and t denote the coordinate and time of an event in an MCIF at any point on the edge of the disc, while x_0 and t_0 refer to the same in the inertial frame S_0 which is stationary with the axis of rotation. Let us now write the inverse of ZT (Eq. (27)) for time in the differential form as

$$dt_0 = dt \pm \gamma^2\beta \; dx/c, \tag{28}$$

where dx refer to the length of the infinitesimal element of the disc which is covered by the light signal in time dt when the signal is co-rotating (+ sign) or counter-rotating (− sign) with the disc. Note that the phase term (space dependent term) in (28) was absent in the GT. Clearly the term is an artefact of the Einstein synchrony. For the complete revolution for the counter-rotating light signal, the round-trip time in the laboratory is thus obtained by integrating (28) as

$$\Delta t_{0+} = \oint dt - \oint \frac{\gamma^2\beta}{c}\, dx \tag{29}$$

or

$$\Delta t_{0+} = \oint dt - \frac{\gamma^2\beta L_0}{c}; \tag{30}$$

and, similarly for the co-rotating signal,

$$\Delta t_{0-} = \oint dt + \frac{\gamma^2\beta L_0}{c}. \tag{31}$$

Notice that $\oint dt$ in Eqs. (30) and (31) are the same because of the adopted synchrony which is given by

$$\oint dt = \frac{L_0}{c(1-\beta^2)}, \tag{32}$$

since (from Eq. (26)), for $\theta = 0$,

$$\overleftrightarrow{c}\,(0) = c(1-\beta^2), \tag{33}$$

which has been assumed to be the same as the OWS following the synchrony. That the Zahar transformation and hence Eqs. (30) and (31) are consistent with the classical world can be checked by calculating $c_\pm\,(= L_0/\Delta t_{0\pm})$ from Eqs. (30) and (31) and by making use of Eq. (32). They are obtained as

$$c_\pm = c(1 \pm \beta), \tag{34}$$

which could have been obtained from elementary kinematics using GT. This agreement is expected since the global round-trip speeds are observables independent of the synchrony gauge. Further by taking the difference of Eqs. (30) and (31), by virtue of the cancellation of the $\oint dt$ terms one obtains the usual classical expression for the Sagnac delay quoted earlier

$$\Delta t_s = (2/c)L_0\beta\gamma^2. \tag{35}$$

From Eq. (34) it is evident that in the classical world, too,

$$\rho_{\text{classical}} = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{36}$$

Clearly we are confronted with the same apparent paradox that the ratio of the round-trip speeds of the two counter-propagating light signals differ from unity ($\rho \neq 1$) although locally the one-way speeds of light in opposite directions have been assumed to be the same ($\rho' = 1$). (This is manifested in the cancellation of $\oint dt$ terms while taking the difference of (30) and (31) in arriving at the classical Sagnac effect formula (35)).The rather tortuous way of deriving the Eqs.(34), (35) and (36) serves two things. It demonstrates how the Sagnac effect can be construed as an effect of "desynchronization" of clocks (due to the contribution of the phase terms in Eq. (28)) on the rotating platform even in the classical world. This effect is usually regarded as a 'real' physical phenomenon in the context of the relativistic Sagnac effect [27]. But the present derivation demonstrates that the desynchronization cannot be an objective phenomenon since here we clearly see it as an artifact of standard synchrony which is nothing but a stipulation. The other utility of this scheme of the derivation is that it allows us to understand clearly that the two apparently contradictory results ($\rho = 1$ and $\rho \neq 1$) follow from the same transformation (27). The contradiction is therefore a logical one. It means that the trouble lies in the arguments (and not in the physical theory — in this case it is the classical kinematics) leading to the paradoxical conclusions.

Further, not only Zahar transformation, the Galilean world can also be represented by the following transformation [20]

$$\begin{aligned} x &= x_0 - \beta c t_0, \\ y &= y_0, \\ t &= t_0 + \epsilon\,(x_0 - \beta c t_0)\,, \end{aligned} \tag{37}$$

where, as before, ϵ is a free parameter which depends on the choice of synchrony. GT and ZT are recovered for $\epsilon = 0$ and $\epsilon = -\gamma^2\beta/c$ respectively. Note that these are the classical analogues of Selleri's transformation (19). The OWS' of light that follow from (37) are given by

$$\frac{1}{c'_+} = \frac{1}{c(1-\beta)} + \epsilon, \tag{38}$$

$$\frac{1}{c'_-} = \frac{1}{c(1-\beta)} - \epsilon; \tag{39}$$

and the corresponding ratio of these velocities is given by

$$\rho'_{\text{classical}} = \frac{c'_+}{c'_-} = \frac{1-\beta}{1+\beta}.\frac{1-c\epsilon(1+\beta)}{1+c\epsilon(1-\beta)}. \tag{40}$$

As before, here also we see that $\rho'_{\text{classical}}$ corresponds to $\rho_{\text{classical}}$, provided $\epsilon = 0$. For ZT ($\epsilon = -\gamma^2\beta/c$), $\rho'_{\text{classical}} = 1$ which agrees with the stipulation of standard synchrony used to derive the transformation. But now $\rho'_{\text{classical}} \neq \rho_{\text{classical}}$, although the latter ratio also has been obtained using the same transformation *i.e.* ZT. We thus see that Selleri's arguments, if carried over into the classical world, also lead to the paradox.

As remarked earlier, in order to address the paradox one needs to look into the reasonings leading to it rather than expecting any flaw in the theories (relativistic or classical). One may ask why Selleri expects that ρ should be equal to ρ' (or equivalently why $\rho_{\text{classical}}$ should be equal to $\rho'_{\text{classical}}$)? The primed ratios are measured in the MCIF whereas the unprimed ratios are global, *i.e.* they are based on the measurements of the average speeds of light signals when they make complete roundtrips. Selleri's argument goes somewhat like this: Since the stationary inertial reference frame at rest with the centre of the disc is isotropic in every sense, the isotropy of space should ensure that the instantaneous velocities of light are the same at all points on the rim of the disc and therefore the average velocities should coincide with the instantaneous ones.

It is interesting that there is nothing wrong even in Selleri's observation regarding the symmetry of the situation, however the conclusion that the two ratios (ρ and ρ') are equal does not necessarily follow from the symmetry arguments. Below we give an example and explain why and how the local speeds of light may differ from their global values in spite of the symmetry.

Consider the motion of a rigid rod AB of length $L_0/2$ with respect to the inertial frame S_0. Suppose that the rod initially moves with uniform velocity βc towards the right parallel to the x-axis of S_0. The left end A of the rod is assumed to coincide with the origin of S_0 at the laboratory time $t_0 = 0$, when an observer at A on board the rod who carries a clock C_A sends out a light pulse towards B where another observer sitting on the rod holds a mirror facing A. As soon as the light pulse reaches the observer at B and is reflected back and starts to travel towards A, the rod is also made to change its direction of motion and travel towards the left with the same uniform speed βc.[9]

Suppose now that the observers in the laboratory record the times of the following three events:

Event 1: The light pulse sent out from A at the laboratory time $t_0 = t_{01} = 0$.
Event 2: The light pulse received at B at the laboratory time $t_0 = t_{02}$.
Event 3: The reflected light pulse received at A at the laboratory time $t_0 = t_{03}$.

From simple kinematics one obtains

$$t_{02} = \frac{L_0}{2c(1-\beta)}, \quad t_{03} = \frac{L_0}{c(1-\beta)}. \tag{41}$$

If Galilean transformation is used for any event, there is no distinction between the laboratory times and the corresponding times measured by observers on board the rod.

However if the observers wish to adopt the Einstein synchrony (*i.e.*, for light TWS= OWS) in the inertial frames of the moving rod (we label them S_1 for the rod moving towards the right and S_2 when

[9]The present analysis of this thought experiment, which essentially corresponds to a linear Sagnac effect discussed elsewhere [13, 28] by the present authors can be seen to fit well (with minor adjustment in the reasonings) with the FOC experiment [15] in the limit when the size of the wheels at the two ends tend to zero.

it moves towards the left say), they may do it by correcting the times for the event 2 in the respective frames. Let us denote these corrected times by t_{12} for S_1 and t_{22} for S_2.[10] The corrected times will be given by

$$t_{12} = t_{01} + \frac{L_0}{2\overleftrightarrow{c}(0)} = \frac{L_0}{2c}\gamma^2 \tag{42}$$

and

$$t_{22} = t_{03} - \frac{L_0}{2\overleftrightarrow{c}(0)} = \frac{L_0(1+2\beta)}{2c}\gamma^2, \tag{43}$$

where we have made use of Eq. (41) and inserted Eq. (33).

Note that for the derivation of Eq. (42) and (43), it has been implicitly stipulated that the times recorded on rod-observer's clock, t_{11} and t_{23} (which are the times recorded on C_A) are the same as the laboratory times t_{01} and t_{03} respectively. In the classical situation this is possible because no rate-correction is necessary. In the relativistic situation this stipulation is also possible by making the rate correction to the moving clocks by an appropriate Lorentz factor.

For event 2 the total disagreement of times between observers in S_1 and S_2 is therefore given by

$$\delta t_{\text{gap}} = t_{22} - t_{12} = L_0\beta\gamma^2/c. \tag{44}$$

Now, in this example, physics is the same whether light propagates forward or backward with respect to S_0, but still the global speed $L_0/(t_{03} - t_{01}) = c(1-\beta)$ is different from the local speeds

$$\frac{L_0/2}{(t_{12} - t_{01})} = \frac{L_0/2}{(t_{03} - t_{22})} = c\left(1 - \beta^2\right), \tag{45}$$

since the total discrepancy in synchrony given by Eq. (44) remains unaccounted for in such a comparison if Einstein synchrony is used. Thus we see that in spite of the symmetric situation the global speed of light ought to be different from its local counterpart in this synchrony.

It is interesting to note that in the rotating disc situation this discrepancy in synchronization between any two adjacent MCIFs can be evenly distributed throughout its circumference by honouring the symmetry of this situation. It is therefore evident that the global ratio ρ is in general not the same as the local ratio ρ'. In fact it should be amply clear by now that while the former is an empirically verifiable quantity (based on the measurements of times of flight of light by a *single* clock) the latter quantity depends only on one's own choice of

[10]The symbol t_{ik} refers to the time of the k-th event according to an observer of the inertial frame S_i.

synchrony (see Eq. (42) or (43) to understand how the times of the event 2 in S_1 and S_2 are required to be adjusted in order to synchronize the clocks in the Einstein way). Note that in this respect the classical kinematics is no different from its relativistic counterpart.

5. DESYNCHRONIZATION

From the above analysis it is evident that if in order to calculate the round-trip time for light in the (non-inertial) frame of the rod, one adds up the times of flight of the same in the inertial frames S_1 and S_2, where the Einstein synchrony has been employed, the result will be wrong by the amount δt_{gap}. This happens since $t_{22} \neq t_{12}$. It only means that S_1 and S_2 cannot be meshed together. However in seeking to dovetail these frames one may set $t_{22} = t_{12} = L_0\gamma^2/2c$. But in that case t_{23} has to be altered by the amount δt_{gap} to preserve the Einstein synchrony in S_2. However since according to our stipulation t_{23} is the time measured by C_A, any possibility of alteration in the value of t_{23} would mean C_A is desynchronized with itself.

In the literature this phenomenon is known as the "desynchronization" in the context of synchronization of clocks in a rotating platform. It is not difficult to show that the measure of this desynchronization in the case of a rotating disc, which is often termed as the "time lag" [11, 29] (for the corotating light signal) is the same as δt_{gap} obtained in the shuttling rod example above. Note that this δt_{gap} is just half of the classical Sagnac delay (see Eq. (7)). If the same effect is calculated for the counter propagating beam, the total time lag $\Delta\tau_{\mathrm{lag}}$ comes out to be $2\delta t_{\mathrm{gap}}$. As mentioned earlier, people tend to regard this desynchronization ($\Delta\tau_{\mathrm{lag}}$) as the real cause of the Sagnac effect in the relativistic context [6, 7, 11, 29]. For example in Ref. [7], Tartaglia observes that the "simplest explanation for this effect attributes it to the time lag accumulated along any round trip ...." Earlier, Rizzi and Tartaglia [6] expressed a similar view in order to give the "true" relativistic explanation for the Sagnac time difference by ascribing it to the non-uniformity of time on the rotating platform and to the "time lag" arising in synchronizing clocks along the rim of the disc. Selleri also remarks (while not sharing this view) that "an "orthodox" approach to dealing with the rotating platform problem is to consider a position dependent desynchronization ... as an objective phenomenon."

The present analysis of the classical Sagnac effect using Einstein synchrony reveals that this desynchronization is only an artefact of the Einstein synchrony and hence is devoid of any empirical content. Since, if instead of ZT, one uses the Galilean transformation, there is no "desynchronization" but still there is Sagnac effect. Therefore "desynchronization" is conventional in nature and hence cannot be considered an "objective phenomenon." For future reference we call this desynchronization desync1.

In a recent paper Rizzi and Serafini [11] acknowledges Selleri and Klauber (see footnote on p. 4 of Ref. [11]) who have brought to their attention this fact that the much talked about "desynchronization" is merely a "theoretical artefact." However the present paper reveals this in a more convincing way by explicitly showing how this "desynchronization" can be manufactured in the classical world too.

The authors of Ref. [11] however somewhat supporting the orthodox view regarding the connection of the Sagnac effect and the "desynchronization", redefines the latter in the following way: Starting from any point Σ on the rim of a rotating disc if two synchronized clocks are slowly transported in opposite directions along the periphery and are brought back to the same position, they will be found to be out of synchrony by the amount which is equal to that obtained for desync1, *i.e.*, $\Delta\tau_{\text{lag}}$. This desynchronization will hereafter be referred to as desync2.

The desynchronization, thus defined, is the result of the comparison of two clocks at the same space point and hence is independent of the distant synchrony convention. The authors therefore claim that they have revealed the "deep physical" and "non-conventional nature" of the time lag. However it is enough to point out the fallacy of this claim by mentioning that these two desynchronizations (desync1 and desync2) are two different things altogether, since if something is conventional, it can be changed or removed by altering the convention, but the "time lag" or time difference in the readings of the two slowly transported clocks after their round trips cannot be altered by redefining the synchrony on the rotating disc.

The equality of these time lags (*i.e.*, desync1 and desync2), therefore, is itself conventional and is true accidentally (as opposed to logically) in the relativistic situation if the Einstein synchrony is used in the rotating frame. If instead, the absolute synchrony is used desync1 $= 0$ while desync2 still remains non-zero. In the classical case the situation is reversed, since in this case desync2 is always zero since there is no time dilation of clocks with respect to the laboratory frame; however for the Einstein synchrony in the disc (which corresponds to ZT) desync1 $\neq 0$. These are however equal in the absolute synchrony (which corresponds to GT). Rizzi and Serafini also claim that desync2 brings to light the "dark physical root of the Sagnac effect." However this claim is also in error too. It is obvious that desync2 cannot be regarded as the physical cause of the Sagnac effect, since we observe that in the classical world desync2 is always zero but still the Sagnac effect exists. This reveals that desync2 and Sagnac effect are unconnected entities. The equality of these two different entities in the relativistic world is at best fortuitous.

6. SYNCHRONY – A VALUE JUDGEMENT

One is now in a position to inquire if it is possible to consistently synchronize clocks on the rim of the turn-table so that no gap in synchrony arises. Let us call such a synchrony as "good synchrony". To answer this consider the following scheme for synchronization due to Cranor *et. al.* [30]. In this scheme before the disc is set into motion with respect to S_0 all observers on the rim of the disc and those in the laboratory set their clocks according to the Einstein synchrony. The disc is then set into rotation uniformly (here 'uniformly' means all the points of the rim are treated identically [30]) which after some time may be assumed to attain a constant angular velocity. Alternatively one may set all the clocks on the rim (as well as those adjacent to them in S_0) a common time (say $t = 0$) as soon as the observers on the rim receive a flash of light sent out from a light source at the center of the disc.

Clearly the symmetry of the problem demands that the observers in the laboratory as well as those on the rim of the disc should continue to agree on the question of simultaneity as the synchronization process "favours no particular observer" [30]. This symmetry argument is evidently true in the classical as well as in the relativistic world. Only in the latter case although the observers in the laboratory frame and in the rotating frame agree on simultaneity, the clock *rates* in these frames differ due to the time dilation effect of relativity.

It is evident that there will be no gap in the synchrony between two successive MCIFs (in the linear example between S_1 and S_2) if the observers in these frames agree on simultaneity with those in S_0. Again if there is no synchrony gap the global ratio ρ should be equal to the local ratio ρ'. The agreement on simultaneity between the frames in turn requires ϵ to be equal to zero in Eqs. (19) and (37). In the classical world this implies GT, on the other hand in the relativistic world this corresponds to TT (Eq. (24)).

It means if the clocks of the disc were synchronized according to the scheme discussed above when the latter was at rest with respect to S_0, nothing has to be done further to synchronize them in order to have consistent synchrony throughout the rim when the disc picks up its uniform angular speed. The synchrony is thus "automatic". Any other synchrony (which corresponds to $\epsilon \neq 0$) including the Einstein Synchrony is to be achieved through human intervention. Selleri [31] therefore singled out the absolute synchrony by calling it as "nature's choice."

One may however ask at this point if it is at all possible to synchronize the clocks on the rim in the absolute way (so that $\epsilon = 0$) without referring to the underlying inertial frame *i.e.* by means attached to the turn table itself. Indeed this can be done in practice. For instance an observer with a clock on the rim at a point Σ can start the process by sending a light pulse to an adjacent clock in the anticlockwise direction and synchronize the latter with his own clock first by assuming the OWS of light to be equal to c. In the same way the third clock adjacent to the second one in the same direction can be synchronized with the latter and the synchronization procedure may continue in this way until finally one arrives at the first clock. The observer then discovers that the clock at Σ is not synchronized with itself. The desynchronization, *i.e.*, the defect in synchrony will again be different if checked clockwise rather than counter-clockwise. By trial however the observer will be able to discover that the defect in synchrony disappears if the one-way speeds in the two different directions correspond to two different numerical values c_1 and c_2 (say). With these obtained empirical values for the OWS of light, not only the clocks on the rim are synchronized in the absolute way but also the linear speed of the rim βc and hence the angular velocity $\omega = \beta c/R$ of the rotating disc are determined if c_1 and c_2 are substituted for c_+ and c_- in Eqs. (17) and (18) (or alternatively in Eq. (34) if one considers the Galilean world). All this however refers to the question of synchronization in the large and does not mean that in MCIFs it is mandatory to adopt the absolute (non-isotropic) synchronization.

One may now question our nomenclature "good synchrony" for the one for which light propagation is anisotropic (remember that for $\epsilon = 0$ light propagation is anisotropic in the classical as well as in the relativistic world). Let us clarify this: In the classical world people would be immediately happy to know that the demand for consistent synchronization in the large requires $\epsilon = 0$, which recovers GT. They would say, "After all we get back our old time tested transformation, the Einstein synchrony (leading to ZT) is a bad one, since it is not automatic and natural and it leads to inconsistent synchronization in the large." What should be our reaction who live in the relativistic world? If one carries on the same sort of arguments in the relativistic world, one may give a value judgment in favour of the absolute synchrony ($\epsilon = 0$) hence may call it the "good synchrony" by contrast, unless one seeks to indulge in double standard.

7. CONCLUSION

SP refers to a theoretical prediction regarding the OWS of light grazing the circumference of a rotating disc. The essential content of SP is that simple kinematics together with some appropriate symmetry arguments predict an anisotropy in the speed of light with respect to an "inertial observer." The claim apparently is substantiated by the Sagnac effect. (In the recent FOC experiment [15], the "inertial observer" is obtained automatically (see Sec. 2), while in the original rotating disc context one needs to take the limit $R \to \infty$ and $\omega \to 0$, while preserving the linear speed of the rim of the disc so that any point on the rim can be thought of as an inertial observer.)

Some earlier responses to the issue are either incomplete or they suffer from certain drawbacks. Here we have shown that by adopting the Einstein synchrony SP can be recast in the Galilean world (see Sec. 4). This facilitates in understanding the weak point of the reasoning leading to the fallacy.

It has been argued that SP hinges on the assumption that the (global) ratio of the round-trip speeds of the light beams co-rotating and counter-rotating with the disc (as if) ought to be the same as the local ratio of the OWS in the MCIFs since no point on the rim is preferred. The present analysis in the classical world reveals how in spite of the symmetry of the situation the two ratios can be different.

The issue of the "desynchronisation of clocks" which is often regarded as the physical cause of the Sagnac effect has been put under the scanner. It is held that of the two types of desynchronisation discussed here, desync1 is a theoretical artifact while desync2, although a convention-free entity, is also unable to qualify itself as the root cause of the effect. Finally, in spite of the lacunae in the reasonings leading to SP, the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

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REFERENCES

1. F. Selleri, *Found. Phys. Lett.* **10**, 73-83 (1997).
2. F. Selleri, *Open questions in Relativistic Physics*, F. Selleri, ed. (Apeiron, Montreal, 1998), pp. 69-80.
3. M. G. Sagnac, *C. R. Acad. Sci.* **157**, 708-710 (1913).
4. E. J. Post, *Rev. Mod. Phys.* **39**, 475-493 (1967).
5. R. D. Klauber, *Found. Phys. Lett.* **11**, 405-444 (1998).
6. G. Rizzi and A. Tartaglia, *Found. Phys.* **28**, 1663-1683 (1998).
7. A. Tartaglia, *Found. Phys. Lett.* **12**(1), 17-28 (1999).
8. G. Rizzi and A. Tartglia, *Found. Phys. Lett.* **12**(2), 179-186 (1999).
9. T. Budden, *Found. Phys. Lett.* **11**, 4 (1998).
10. E. Minguzzi *Found. Phys. Lett.* **15**(2), 153-169, April 2002.
11. G. Rizzi and A. Serafini, in *Relativity in Rotating Frames*, G. Rizzi and M. L. Ruggiero, eds. (Kluwer Academic, Dordrecht, 2004).
12. R. Klauber, in *Relativity in Rotating Frames*, G. Rizzi and M. L. Ruggiero, eds. (Kluwer Academic, Dordrecht, 2004).
13. S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri and Minakshi Sarker, *Found. Phys.* **33**(6), 981-1001 (2003).

14. A. Tartaglia and M. L. Ruggiero, "Sagnac effect and pure geometry," gr–qc/0401005v1 5 Jan 2004.
15. R. Wang, Yi Zheng, Aiping Yao, and D. Langley, *Phys. Lett. A* **312**, 7-10 (2003).
16. H. Reichenbach, *The Philosophy of Space and Time* (Dover, New York, 1958).
17. A. Grünbaum, *Philosophical Problems of Space and Time*, 1st edn. (Knopf, New York, 1963).
18. R. Anderson, I. Vetharaniam, and G. E. Stedman, *Phys. Rep.* **295**, 93-180 (1998).
19. A. Winnie, *Phil. Sci.* **37**, 81-99 (1970).
20. T. Sjödin, *Il Nuovo Cimento B* **51**, 229-245 (1979).
21. S. K. Ghosal, P. Chakraborty, and D. Mukhopadhyay, *Europhys. Lett.* **15**(4), 369-374 (1991).
22. S. K. Ghosal, D. Mukhopadhyay, and Papia Chakraborty, *Eur. J. Phys.* **15**, 21-28 (1994).
23. F. R. Tangherlini, *Nuovo Cimento Suppl.* **20**, 1-86 (1961).
24. S. K. Ghosal, Biplab Raychaudhuri, Anjan Kumar Chowdhuri, and Minakshi Sarker, *Found. Phys. Lett.* **16**(6), 549-563 (2003).
25. E. Zahar, *Brit. J. Phil. Sci.* **28**, 195-213 (1977).
26. S. K. Ghosal, K. K. Nandi, and P. Chakraborty, *Z. Naturforsch.* **46a**, 256-258 (1991).
27. D. Dieks *Eur. J. Phys.* **12**, 253-259 (1991).
28. S. K. Ghosal and Minakshi Sarker, "Cosmologically preferred frame, linear Sagnac effect and absolute synchrony," in M. C. Duffy, ed., *Proceedings, Physical Interpretation of Relativity Theory VI (Supplementary Papers)* (1998), pp. 52-64.
29. J. Anandan, *Phys. Rev. D.* **24**, 338–346 (1981).
30. Maria B. Cranor, Elizabeth M. Heider, and Richard H. Price, *Am. J. Phys.* **68**(11), 1016-1020(2000).
31. F. Selleri, *Found. Phys.* **26**, 641-664 (1996).