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Relativistic Sagnac Effect and Ehrenfest Paradox

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There seems to exist a dilemma in the literature as to the correct relativistic formula for the Sagnac phase-shift. The paper addresses this issue in the light of a novel, kinematically equivalent linear Sagnac-type thought experiment, which provides a vantage point from which the effect of rotation in the usual Sagnac effect can be analyzed. The question is shown to be related to the so-called rotating disc problem known as the Ehrenfest paradox. The relativistic formula for the Sagnac phase-shift seems to depend on the way the paradox is resolved. Kinematic resolution of the Ehrenfest paradox proposed by some authors predicts the usually quoted formula for the Sagnac delay but the resolution itself is shown to be based upon some implicit assumptions regarding the behaviour of solid bodies under acceleration. In order to have a greater insight into the problem, a second version of the thought experiment involving linear motion of a "special type" of a non-rigid frame of reference is discussed. It is shown by analogy that the usually quoted special relativistic formula for the Sagnac delay follows, provided the material of the disc matches the "special type."

KEY WORDS: special relativity; Sagnac effect; rotating frame; Ehrenfest paradox.

1. INTRODUCTION

There has been a great deal of interest in recent years in the Sagnac effect. This is not only for its practical importance in navigational application for sensing rotation but also for its rich theoretical ramifications. In 1913 Sagnac[(1)] in his life long quest for ether devised an experiment where he

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compared round-trip times of two light signals traveling in opposite directions along a closed path on a rotating disc. It was observed that the time required by a light signal to make a close circuit on the plane of the disc differed depending on the sense (direction) of the signal's round-trip with respect to the spin of the disc. The essentials of the experiment consist of a monochromatic source of light, an interferometer and a set of mirrors mounted on a turntable. Light from the source is split into two beams by a beam splitter (half-silvered mirror) allowing them to propagate in opposite directions. These beams then are constrained (by suitably placed mirrors) to make round-trips and are then reunited at the beam splitter to produce a fringe pattern. The difference in the round-trip times for these counterpropagating beams leads to a phase difference with a consequent shift in the fringe pattern when the turntable is put into rotation. This phenomenon is commonly known as the Sagnac effect (for reviews with historical perspectives, see, for example, Refs. 2–4). The effect is universal and is also manifested for matter waves.[4]

The optical Sagnac effect can be suitably analyzed by assuming the light circuits to be circular. This can be achieved by constraining the light beams to propagate tangentially to the internal surface of a cylindrical mirror. Within the framework of Newtonian physics it is quite straight forward to calculate the Sagnac phase-shift in terms of the difference of arrival times of the co-rotating and counter-rotating light signals when they are reunited at the beam splitter. Assuming that the light beams propagate along the periphery of the circular disc of radius R, the time difference can be shown to be given by

$$\Delta t = \frac{2Lv}{c^2}\,\gamma^2, \tag{1}$$

where $L = 2\pi R$ is the circumference of the disc, v is the linear speed of the disc at its periphery and $\gamma = (1-\beta^2)^{-1/2}$ with $\beta = v/c$. To a lowest order of v/c

$$\Delta t = \frac{2Lv}{c^2}. \tag{2}$$

[4] The effect has been observed in interferometers built for electrons,[(5)] neutrons,[(6)] atoms,[(7)] and superconducting Cooper pairs.[(8)] The accuracy of Sagnac type experiment is much improved by using lasers and in fibre optic interferometers.[(3)] Based on the Sagnac effect, ring laser and fibre optic gyros are being used as navigational tools.[(9)] Importance of this effect also lies in connection with the question of time keeping clocks of clock-stations around the earth.[(10)] Here the earth plays the role of the rotating platform from which clocks between clock stations are synchronized by sending light signals via satellites.[(11, 12)]

The result is often quoted as

$$\Delta t = \frac{4\pi A\omega}{c^2}, \tag{3}$$

where $A = \pi R^2$ is the area of the optical circuits and $\omega(= v/R)$ represents the angular speed of the disc. It may be noted that the presence of the square of the Lorentz factor γ in Eq. (1) has nothing to do with special relativity since the treatment is purely classical.

Calculation based on special relativity also gives identical result to the lowest order in v/c. However, the exact formula differs from Eq. (1). The special relativistic result $\Delta\tau$ for the Sagnac delay, usually found in the literature[2, 13, 14] in one form or the other, differs from (1) by a factor of γ:

$$\Delta\tau = \frac{2Lv}{c^2}\gamma. \tag{4}$$

The above result can be deduced in various ways, but a simple derivation of Eq. (4) due to Post[2] is as follows. Suppose a light pulse leaves the beam splitter at position B (as shown in Fig. 1) and propagates along the direction of rotation of the disc and meets the beam splitter again in time t_1. During this period the beam splitter has moved to a new position B'; hence light has to travel an extra distance (BB') $x = vt_1$ with respect to the inertial frame of the laboratory which is at rest with the axis of rotation. One therefore has

$$L + x = ct_1, \tag{5}$$

$$x = vt_1. \tag{6}$$

In order to write (5) it is implicitly assumed that the speed of light is c in the laboratory frame which, in accordance with special relativity, is independent of the motion of the source.[5] Eliminating x from Eqs. (5) and (6) one obtains

$$t_1 = \frac{L}{c - v}.$$

A similar argument for the counter-rotating beam gives its round-trip time t_2. In this time the beam splitter moves to its new position B'', so that

[5] Classically this is equivalent to the hypothesis of ether, through which light propagates and which is believed to be stationary with respect to the inertial frame of the laboratory.

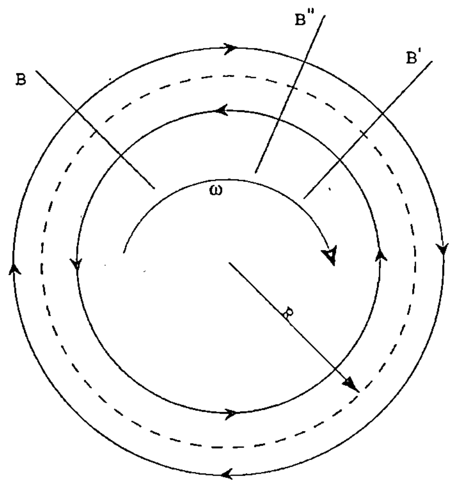


Fig. 1. B is the initial position of the beam splitter. As the disc (dotted circle) rotates clockwise, the beam-splitter moves towards the right. B' and B'' are the positions of the beam-splitter when the co-rotating (outer circle) and the counter-rotating (inner circle) light pulses respectively return to it. R and ω represent the radius and the angular velocity of the disc respectively.

with respect to the laboratory, the counter-rotating light pulse travels a path shorter than L by the amount $BB'' = vt_2$. For light propagation one may therefore write

$$l - vt_2 = ct_2,$$

i.e.,

$$t_2 = \frac{L}{c+v}.$$

The difference in these times is therefore given by

$$\Delta t = t_1 - t_2 = \frac{2Lv}{c^2 - v^2}. \tag{7}$$

The above expression for the time difference has already been quoted (Eq. (1)) as the classical expression for the Sagnac effect. The relativistic expression is obtained by recognizing that the above time difference refers to the observation from the stationary frame and is therefore dilated with respect to that measured (as the fringe shift) on board the disc. If $\Delta\tau$ corresponds to the time difference as observed from the rotating platform, clearly, because of this time dilatation effect

$$\Delta\tau = \gamma^{-1}\,\Delta t. \tag{8}$$

Therefore from Eqs. (7) and (8) one obtains the result as quoted earlier

$$\Delta\tau = \frac{2Lv}{c^2}\,\gamma. \tag{4}$$

Essentially the same expression for the effect has been derived by several authors in variety of ways.[6]

There are however a few exceptions. Selleri[(17)] for example, obtains a different result. He considers the length contraction effect in the periphery of the disc in addition to the time dilatation effect. If the rest circumference of the disc is denoted by L_0

$$L = \gamma^{-1} L_0, \tag{9}$$

since the periphery should suffer a Lorentz contraction. In this case, using Eqs. (8) and (9) in Eq. (7),

$$\Delta\tau = \frac{2L_0 v}{c^2}. \tag{10}$$

It may be noted that the above formula is the same as Eq. (2) which is an approximation of the classical or the relativistic expressions (Eqs. (1) and (4), respectively) to the lowest order of v/c. In this case however the formula is exact.

What therefore, is the correct relativistic formula for the Sagnac effect? Following Selleri, Goy[(18)] also quoted the same result pointing out explicitly the discrepancy between this result (Eq. (10)) and the special relativistic result that is generally believed to be the true one (Eq. (4)). The present day precision in measurement of the Sagnac effect may be unable

[6] For example, the formula can be derived from metrical considerations by introducing various types of transformation for rotating frames.[(2, 15)] This is also studied using the general relativity[(2, 13)] or by considering the relativistic Doppler effect at the mirrors.[(16)] An interesting derivation due to Dieks and Nienhuis[(14)] based on the direct use of the Lorentz tranformations will be followed here for the calculation of the effect in the next section.

to decide between the two formulae, nevertheless from the theoretical and pedagogical standpoint the question cannot be ignored.[7] We shall see below that the issue is intimately connected with the Ehrenfest paradox[(21, 22)] concerning the spatial geometry of a rotating disc. The paradox concerns a circular disc examined by an inertial observer at rest with the centre as the former passes from the rest to rotational motion. For such an observer the circumference but not the radius suffers Lorentz contraction. Hence the ratio of the circumference to the radius should be different from 2π thus violating Euclidean geometry in the *inertial frame*! However, for the moment, we refrain from discussing the paradox any further or from giving any verdict right away regarding the question we raised in the beginning of the paragraph concerning the correct relativistic formula, for the Sagnac effect. In order to appreciate the question, the real physics behind the Sagnac effect may be brought out first by delinking any other effect that may be present due to the rotation of the disc from some "pure" Sagnac effect.

Let us first recognize that the essential content of the Sagnac experiment lies in its ability to detect acceleration of the experimental platform by comparing the round-trip times for lights travelling parallel or antiparallel to the motion of the platform. It is therefore expected that the acceleration need not have to be due to rotation alone; a suitably modified Sagnac type experiment should as well be able to detect the change of the direction of motion of a platform which is allowed to move or shuttle along a straight line. In the next section we shall propose such a thought experiment which will mimic the optical Sagnac experiment in almost all respect but with the difference that the motion of the experimental platform will not involve rotation.[8] The outcome of the experiment will be called the

[7] To appreciate a current perspective of this pedagogical question consider the following: The Sagnac experiment is often regarded as fundamental as the Michelson–Morley experiment so much so that in some recent papers it is claimed that one may even rederive relativity theory from some new postulates based on the Sagnac effect.[(19, 20)] Unfortunately the Sagnac effect unlike the Michelson–Morley result is a verification of a first order effect. To hope to derive relativity theory from the Sagnac result therefore requires the exact Sagnac formula and one cannot remain content with the approximate one.

[8] One may object here that the Sagnac effect without rotation is a contradiction in terms. However this may not be so. This will be clear by the end of the next section. At the moment it is enough to say that the proposed linear (thought) experiment will include the following basic ingredients of the Sagnac experiment: (1) Light signals will be allowed to complete round-trips with the help of mirrors. (2) The experimental platform will have uniform *speed*. (3) Light signals will travel parallel (or antiparallel) to the direction of motion of the platform throughout during their round-trips. (4) The difference of the round-trip times (Sagnac delay) for the parallel and the antiparallel light signals will be measured on board the platform.

"pure" Sagnac effect. This will give us a perspective which will enable us to appreciate the connection between the Sagnac effect and the Ehrenfest paradox. We shall see below that the formula obtained for the "pure" Sagnac effect may or may not be modified when rotation is introduced. This modification or the lack of it will depend on the way the Ehrenfest paradox is resolved.[22,23]

It is interesting to note that no author has ever explicitly mentioned any role of the Ehrenfest paradox in the derivation of the Sagnac delay. In Sec. 5 we shall argue that the so-called kinematic resolution of the Ehrenfest paradox is based on some implicit assumptions regarding the behaviour of the solid discs when set into rotation. In order to prepare the background of this argument, in Secs. 3 and 4 we shall consider another version of the linear Sagnac experiment and analysis involving a non-rigid frame of reference of a special kind. We shall see that the Sagnac type experiment performed on such platforms gives the usually quoted formula for the Sagnac delay. Significance of these observations among other things will be discussed in Sec. 5 and finally will be summarized in the concluding section.

2. LINEAR SAGNAC EFFECT–I

As discussed in the last section, in optical Sagnac experiment one essentially compares the round-trip times of two light signals, one of which propagates parallel and the other travels antiparallel to the direction of motion of the edge of the rotating disc on which all the measurements are carried out. Let us now present a linear version of the experiment.[24]

Suppose a pulse of light is emitted from a source placed at one end of a linear platform and travels towards a mirror (facing the light source) placed at the other end of the platform which can move with constant speed in either directions along its length. Light after being emitted makes a round-trip after it is reflected back by the mirror. To ensure that the motion of the light pulse remains parallel (antiparallel) throughout its round-trip, it may be assumed that by some mechanism the platform reverses its direction of motion (without changing its speed) as soon as light falls on the mirror.

The linear version of the Sagnac experiment will therefore consist of two separate experiments, one for parallel and the other for antiparallel light propagation. For definiteness, in the first experiment (Figs. 2(a) and (b)) we consider an inertial frame S with respect to which a rigid rod (A) of rest length $L_0/2$ is moving along the positive x-direction with a relative velocity v. Assume that a light pulse sent from a light source (P) placed at

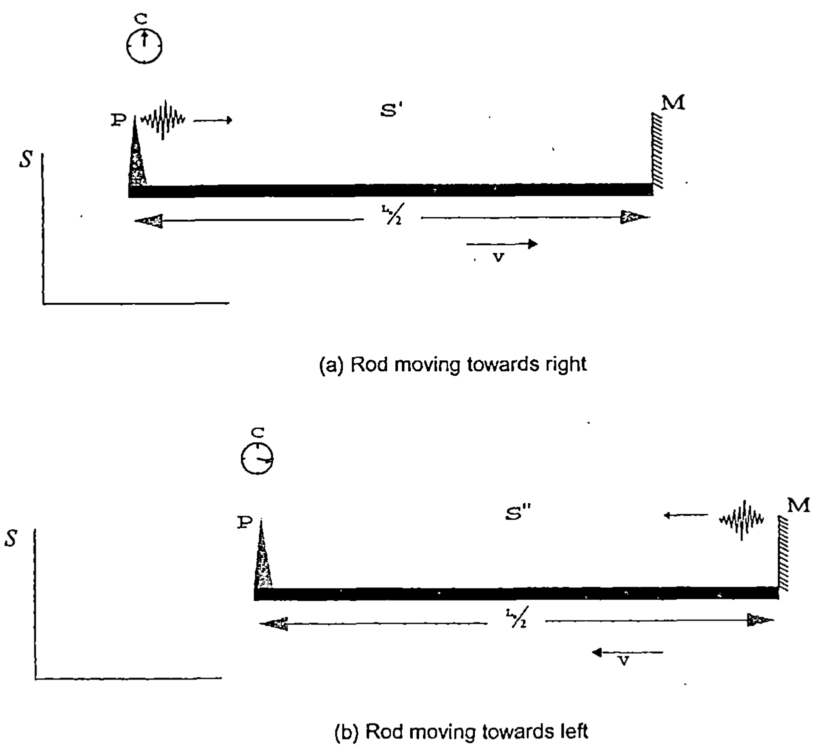
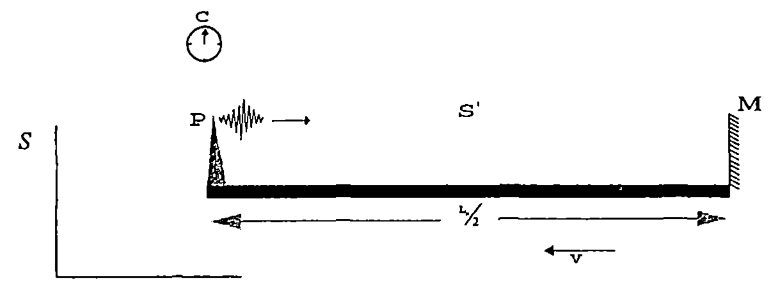


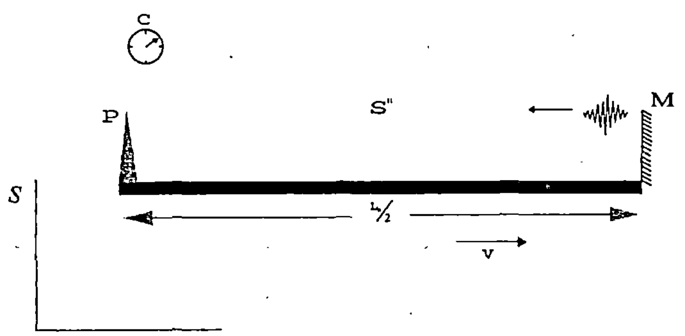
Fig. 2. Parallel Propagation Experiment.

the left end of the rod propagates towards the right (Fig. 2(a)) and in due course, falls on a mirror placed on the right end of the rod. As soon as the light falls on the mirror, the rod reverses its direction of motion without changing its speed and at the same time the light signal after reflection, proceeds to the source (Fig. 2(b)) where a clock (C) records the round-trip time of the light pulse. Clearly the aforesaid arrangement ensures that propagation of light remains parallel (to the direction of motion of the rod) throughout its round-trip. We call it the parallel propagation experiment. The signal completing the journey in this way is analogous to the beam of light *co-rotating* with the disc in ordinary idealized Sagnac experiment.

One needs a similar arrangement for antiparallel propagation of light which may correspond to the *counter-rotating* beam of the Sagnac experiment. For this anti-parallel propagation experiment, one repeats the same sequence of events as the first one except now the motion of the rod is to be along the negative x-direction to start with, before its direction of



(a) Rod moving towards left



(b) Rod moving towards right

Fig. 3. Antiparallel Propagation Experiment.

motion is reversed on falling the light signal on the mirror (Figs. 3(a) and (b)).

Note that a frame of reference K attached to the rod in these experiments is non-inertial because of the reversal of its direction of motion during the experiment. However one may consider two inertial frames S' and S'' which are co-moving with the rod for its forward and reverse journeys respectively. On account of Lorentz transformation one may therefore write for the motion of the light pulse

$$dt_{\text{lab}} = \gamma \left(dt \pm \frac{v\,dx}{c^2} \right), \tag{11}$$

where t_{lab} is the co-ordinate time of the global inertial frame of the laboratory and t and x refer to the local time and spatial co-ordinate of the instantaneous inertial frames S' and S'' and the plus and minus signs apply for the forward and reverse journeys of the rod respectively.

For the first experiment when the light signal moves in the direction same as that of the moving rod, we obtain (by integrating Eq. (11)) for the time $\Delta t_{\rm lab}(1)$ needed for the round-trip

$$\gamma^{-1}\,\Delta t_{\rm lab}(1)=\int dt+\int_0^{\frac{L_0}{2}}\frac{|v|\,dx}{c^2}-\int_{\frac{L_0}{2}}^{0}\frac{|v|\,dx}{c^2}=\int dt+\frac{L_0\,|v|}{c^2}. \tag{12}$$

Similarly, for the second experiment,

$$\gamma^{-1}\,\Delta t_{\rm lab}(2)=\int dt-\int_0^{\frac{L_0}{2}}\frac{|v|\,dx}{c^2}+\int_{\frac{L_0}{2}}^{0}\frac{|v|\,dx}{c^2}=\int dt-\frac{L_0\,|v|}{c^2}. \tag{13}$$

The integrals $\int dt$ in the above equations are the same and is the measure of the "total" time as interpreted from two successive inertial frames, S' and S'' for each experiment. This equals to L_0/c since according to each of these inertial observers speed of light is c and the distance covered by light in either of its forward and reverse journeys is $L_0/2$, i.e., the distance between the source and the mirror.

This time $\int dt$ ought not to be confused with the time measured by a co-moving clock placed at the position of the light source which undergoes direction reversal. If the later is denoted by $d\tau$, we have by virtue of the time dilatation effect,

$$\Delta\tau=\gamma^{-1}\,\Delta t_{\rm lab}.$$

Hence we may write Eqs. (12) and (13) as

$$\Delta\tau(1)=\int dt+\frac{L_0\,|v|}{c^2}, \tag{14}$$

$$\Delta\tau(2)=\int dt-\frac{L_0\,|v|}{c^2}. \tag{15}$$

The difference of these times gives for the Sagnac delay for this linear version of the Sagnac experiment,

$$\delta\tau=\frac{2L_0\,|v|}{c^2}. \tag{16}$$

Clearly the expression is linear in v since the proper length L_0 of the rod is a constant. Note that all the mathematical steps that are involved in this derivation smoothly go over to that leading to the usual Sagnac Effect. Except there, instead of two inertial frames S' and S'', one has to consider an infinite number of momentary inertial frames for the calculations.[14, 25] Since no other effect (that might be present because of rotational motion of the disc in the usual Sagnac experiment) is involved in this linear arrangement the time delay expressed by the above relation may be said to be the result of the "pure" Sagnac effect.

3. LINEAR SAGNAC EFFECT–II

In the previous experiment the light source and the mirror were attached to the two ends of a rigid rod of proper length $L_0/2$. Let us now consider a slightly different arrangement. Consider two aircrafts (unbonded, i.e., not tied together) separated by a distance $L/2$ initially at rest with the laboratory.

Suppose that these aircrafts are programmed in such a way that they can move in any direction always preserving a constant separation $L/2$ with respect to the laboratory. To a casual observer in the laboratory there will appear to be a bond between the objects because of the programmed constant separation of the two, but in reality these are unbonded. One may term this apparent bond between the aircrafts as a "software bond" as distinguished from the bond that exists between any molecules in a solid body.[26]

The twin experiment as described in Sec. 2 can be repeated by attaching the light source to one aircraft and the reflector to the other. For the first experiment assume that the aircrafts are accelerated from rest and finally the system moves with constant velocity v along the positive x-direction. Suppose now a light pulse from a source attached to the first aircraft travels towards the second one on the right and falls on the mirror attached to it. As soon as the light falls on the mirror, the "software bonded" system starts moving in the negative x-direction. The reflected pulse of light also travels in this direction and the time of transit for the light pulse for parallel propagation is recorded as it returns to the source.

In the second experiment almost the whole programme is repeated except now, at the time of emission of the light pulse the motion of the aircraft system is along the negative x-direction although the light pulse travels towards the right that subsequently falls on the mirror attached to the second aircraft. The direction of motion of the unbonded system is reversed as light falls on the mirror and travels to the left to be recorded at the position of the source again. In this arrangement, propagation of light remains antiparallel throughout its entire journey.

The round-trip time (measured in the laboratory) for light for parallel and anti-parallel propagation experiments may easily be calculated following the procedure as discussed in Sec. 1.[9] They are respectively given by

$$t_1 = \frac{L}{c - v}$$

and

$$t_2 = \frac{L}{c + v}$$

and their difference can be written as

$$\Delta t = t_1 - t_2 = \frac{2Lv}{c^2}\,\gamma^2.$$

If $\Delta\tau$ denotes the corresponding time difference by an observer in the first aircraft, one gets on account of time dilatation,

$$\Delta\tau = \gamma^{-1}\,\Delta t = \frac{2Lv}{c^2}\,\gamma. \tag{17}$$

Note that $\Delta\tau$ now is *not* linearly related to v.

Clearly the formula (17) differs from the earlier expression for $\Delta\tau$ (Eq. (16)) obtained for the "pure" Sagnac effect which was linear in v. One may however argue that in Eq. (16) L_0 is the proper length of the rod and in contrast L in Eq. (17) represents the distance of the aircrafts as measured from the laboratory frame. If the Eq. (16) were expressed in terms of L, the formulae (16) and (17) would have agreed. However, there is a subtle point here. If the issue is the question of dependence of the Sagnac delay on the speed of the platform, the formulae (16) and (17) predict different results. Although one is at liberty to quote the expression for the Sagnac delay in terms either of the proper distance L_0 or of the coordinate distance L between the source and the reflector, one must know which of these lengths is independent of v.

In the case of the experiment using rigid platform, proper length $L_0/2$ of the rigid rod is an invariant. On the other hand, for the experiment

[9] Although here the experiment is linear, the kinematical considerations leading to the relations (5) and (6) still remain valid. For example, for the parallel propagation experiment, in order to complete the round-trip in time t_1, the light pulse, as viewed from the laboratory, has to cover in addition to the distance L, an extra distance $x = vt_1$ because of the to and fro motion (with speed v) of the aircrafts. This means $ct_1 = L + vt_1$, i.e., $t_1 = L/(c - v)$.

performed in the frame of reference of the unbonded aircrafts, the distance $L/2$ between them as measured by an observer in the laboratory is held constant by a common program controlling the aircrafts as the system is brought up to different speeds from their initial state of rest. Special relativity then demands that the proper separation between the aircrafts will increase as the speed of the unbonded system increases. The next section will provide more clarifications in this regard.

4. COORDINATE SYSTEM OF THE UNBONDED FRAME

Consider an one-dimensional array of some software bonded particles that constitutes a frame of reference K and suppose from its state of rest at $t=0$, the system is set in motion. If the space-time co-ordinates of the laboratory frame S are denoted by x and t, the equation of motion of the particle at the origin of K may be expressed as

$$x = f(t),$$

where $f(t)$ is some function of time which is zero at $t=0$.

For any particle of the array this may be written as

$$x = x' + f(t),$$

where x' is the spatial coordinate of the particle with respect to S, when it was at rest with the laboratory (at $t=0$). The variable x' may be used to label the array of points and these may act as spatial coordinates in K. Taking the coordinate time t' of K same as t one may write the following transformations between S and K in terms of the coordinate differentials

$$dx' = dx - \dot{f}(t)\, dt, \qquad dt' = dt. \tag{18}$$

where $\dot{f}(t) = v = df(t)/dt$ is the instantaneous velocity of the aircraft.

The line-element in natural unit $(c=1)$ of the 2-dimensional Minkowskian space in the coordinate system S

$$ds^2 = dt^2 - dx^2,$$

may be transformed accordingly so that with respect to x' and t' one may write

$$ds^2 = \gamma^{-2}\, dt'^2 - 2v\, dx'\, dt' - dx'^2, \tag{19}$$

where $\gamma = (1-v^2)^{-1/2}$.

Again, for any line-element

$$ds^2 = g_{\mu\nu}\, dx^\mu\, dx^\nu,$$

the proper distance dl between two points x^i and $x^i + dx^i$ is given by (see Ref. 27, for instance)

$$dl = \sqrt{\left(\frac{g_{0i}\, g_{0k}}{g_{00}} - g_{ik}\right) dx^i\, dx^k}, \tag{20}$$

where $i, k = 1, 2, 3$. For the line element (19), this is given by

$$dl = \frac{dx'}{\sqrt{1-v^2}}. \tag{21}$$

If the points x' and $x' + dx'$ refer to the co-ordinates of two neighbouring particles of K, by definition dx' is an invariant. In that case the proper distance should stretch according to Eq. (21) honouring the well-known special relativistic effect of length contraction as the unbounded array of particles is set into motion. Note that expression (19) represents a perfectly legitimate co-ordinate description of the 2-dimensional Minkowski space and although the transformations (18) have Galilean structure, special relativity is taken care of when we used Eq. (20) to obtain the proper distance.

5. DISCUSSION

Let us now return to the original question posed in Sec. 1 as to the correct relativistic formula for the usual Sagnac effect. As pointed out earlier there are two contesting claims for Sagnac delay:

Claim 1: $\Delta\tau \propto \frac{\beta}{\sqrt{1-\beta^2}}$.

Claim 2: $\Delta\tau \propto \beta$.

If the special relativistic correction due to time dilatation is incorporated in the classical expression (Eq. (1)) one obtains Eq. (4) which corresponds to claim 1. On the other hand if not only time dilatation but also Lorentz contraction of the circumference of the rotating disc is taken into account one gets Eq. (15) and claim 2 appears to be the true one. Barring a few exceptions[17, 18] most authors adhere to claim 1 without stating explicitly the reason behind not considering the length contraction effect.

However as we have seen there was no ambiguity as to the correct formula for time delay for the linear Sagnac effect (thought) experiment with rigid rod. There it was evident that the Lorentz contraction of the rigid platform ought to be taken into account in addition to the time dilatation effect.

For the usual Sagnac experiment on a rotating disc, the Lorentz contraction of the disc's circumference is not generally considered perhaps to avoid the Ehrenfest paradox (see Sec. 1). On the contrary, favouring claim 2, Selleri[(17)] and Goy[(18)] assume relativistic contraction of the edge of the disc without addressing any possible paradox that may arise due to such an assumption. Indeed the result is inconsistent unless it is explicitly assumed that the disc does not remain a disc and becomes a non-flat object.

It is therefore amply clear that in order to decide between claims 1 and 2 it is necessary to understand how the Ehrenfest problem is resolved. Ehrenfest's problem concerns the mechanical behaviour of a material disc set in rotation from rest. The paradox remains a paradox as long as one implicitly assumes that the disc is Born rigid.[(28, 29)] By definition Born rigid motion of a body leaves the proper lengths of the body unchanged. Grøn[(22)] showed that the transition of the disc from rest to rotational motion in a Born rigid way is a kinematic impossibility. It is the recognition of this fact which is known as the kinematic resolution of the Ehrenfest paradox.[(22, 23)]

Cavalleri[(29)] on the contrary observes that the Ehrenfest paradox cannot be solved from a purely kinematical point of view and the solution of the paradox is intrinsically dynamical. This was refuted by Grøn[(22)] who rather endorsed a remark by Phipps[(30)] that to think that dynamics can exist "without the foundation of logically consistent kinematics" is an absurdity.

The present authors believe that both the viewpoints are correct in the present context. To recognize that Born rigid rotation is an impossibility and an implicit assumption on the contrary is the source of the paradox, may follow from pure kinematics; but if it is asked—"what exactly will happen to the solid disc?" the answer will lie in the realm of dynamics. It appears that there is no unanimity in the literature as to this precise question. Synge[(31)] and Pounder[(32)] introduce the concept of superficial rigidity[(29)] according to which the circumference and radius of the disc when put into rotation, undergo change in accordance with special relativity but suggest that the flat disc changes to a surface of revolution symmetric about the axis of rotation. In this way the possible violation of Euclidean geometry in the inertial system is avoided. In the case of uniform rotation this allows radial contraction without any change of meridian arc-length. Some specific prescriptions were also suggested by a few earlier authors who proposed bending of the surface of the rotating disc in the form of a paraboloid of revolution (vide[29] for references). The rotating disc or wheel taking the shape of spherical segment when in rotation was suggested by Sokolovsky[33] as a resolution of the "wheel paradox."

Eddington[34] also investigated the problem of the rotating disc. He studied the question of alteration of the radius of a disc made of homogeneous incompressible material when caused to rotate with angular velocity ω. He showed that the radius of the disc is a function of the angular velocity ω and is approximately given by

$$a' = a(1 - \tfrac{1}{8}\omega^2 a^2),$$

where a is the rest radius of the disc. A similar view has also been expressed by Weinstein[35] who holds that the disc under rotation will be in torsion with a consequent reduction of both the radius and the circumference.

Of recent interest is the so called kinematic resolution of the Ehrenfest paradox as discussed by Grøn[22] and Weber.[23] According to the authors, it follows from purely kinematic considerations, that the radius of the disc remains unaltered but the proper measure of the circumference is increased in such an extent that the Lorentz contraction effect just gets compensated. In other word although there is a Lorentz contraction of the periphery with respect to the laboratory frame it is not visible because of the stretching of the periphery.[10] However, the stretching of the disc's circumference in its proper frame is a dynamical effect (related to the property of the solid material of the disc). How can one hope to get this dynamical effect purely from the kinematic considerations? Clearly the result must have been assumed implicitly. To clarify this let us consider a rotating co-ordinate system which is often discussed in connection with the Ehrenfest paradox. Suppressing one spatial dimension Grøn considers the following transformation:

$$r' = r, \qquad \theta' = \theta - \omega t, \qquad t' = t, \tag{22}$$

where r and θ refer to the radial and angular co-ordinates and t refers to the time co-ordinate of laboratory frame and the primed quantities refer the same in the rotating system.

The rotating frame of reference is equated with that of the rotating disc. It is precisely this equation where lies the implicit assumption that the

[10] Recently Klauber[19, 20] and Tartaglia[36] based upon different arguments also conclude that there will be no contraction of the circumference of the disc. The authors believe that relativistic contraction effect will not at all take place for rotating discs.

disc's periphery is stretched due to rotation. Based on these transformations the line-element may be written as

$$ds^2 = dr'^2 + r'^2\, d\theta'^2 + 2\omega r'^2\, d\theta'\, dt' - \left(1 - \frac{\omega^2 r'^2}{c^2}\right) c^2\, dt'^2. \tag{23}$$

Using the formula (20) for proper spatial distance for the line element (23) the tangential proper spatial line-element is obtained as

$$dl = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} r\, d\theta'. \tag{24}$$

Note that while using Eq. (20) a minus sign under the radical is required since now the metric (23) has a different signature.

Integrating Eq. (24) along the whole element, one obtains the proper length L_0

$$L_0 = L\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}, \tag{25}$$

where $L = 2\pi R$ is the circumference of the disc as observed from the inertial frame of the laboratory. However it will be wrong to assume, from the above argument, that the proper circumference of the disc has increased by a Lorentz factor. To conclude from (25) that the periphery of the disc is stretched by a γ-factor due to rotation is to assume that L was also the circumference of the disc when it was at rest and remains the same, as it is brought up to rotation from its state of rest. Any transformation (representing rotation) must reflect the validity of this assumption. Clearly Eq. (22) does not guarantee this.

The transition of the disc to its rotational motion from its state of rest can be expressed by writing the transformations (22) in a slightly modified form

$$r' = r, \qquad \theta' = \theta - f(t), \qquad t' = t, \tag{26}$$

where the function $f(t)$ is assumed to have the following properties: $f(t)$ and $df/dt = \omega(t) = 0$ at $t = 0$ and $\omega(t)$ thereafter increases and finally approaches a constant value. Obviously the transformations (26) then represent a rotating coordinate system with constant angular velocity after a period of angular acceleration from its state of rest, which in terms of the differentials read

$$dr' = dr, \qquad d\theta' = d\theta - \omega(t)\, dt, \qquad dt' = dt. \tag{27}$$

Note that relations (23) and (24) still remain valid. Now, if we recall our discussions in Secs. 3 and 4, and the transformations (18), we see by analogy that the transformations (27) represent the motion of the disc which is composed of an unbonded arrangement of particles that are programmed to move in a particular way so that their mutual separations with respect to the laboratory frame of reference remain constant. Therefore the constancy of L (and not L_0) in other words, is the outcome of the assumed programme of motion of the particles of the disc governed by Eq. (26).

6. SUMMARY

We are now in a position to summarize our findings. We have seen that there is a scope for confusion regarding the correct relativistic expression for the Sagnac delay. Although the oft-quoted result is that given by Eq. (4), no role of the so-called Ehrenfest paradox in arriving at the result is usually discussed. It is expected that the special relativistic result for the Sagnac effect will differ from its classical counterpart, usually due to two kinematic effects of special relativity—the length contraction and the time dilatation. The inclusion of the length contraction effect in the circumference (and not in the radius) of a rotating disc invites a paradox that there is an apparent violation of the Euclidean geometry in an inertial frame. On the other hand, the non-inclusion of the Lorentz contraction effect will violate special relativity. To understand and clarify these issues a Sagnac-type thought experiment (without rotation) performed on a linear rigid platform has been presented. Since no paradox is associated with this arrangement although kinematically all aspects of the usual Sagnac experiment are incorporated in it, the linear experiment sets the right kind of perspective against which the role of the Ehrenfest paradox in the rotating disc experiment can be discussed.

Although the resolution of the Ehrenfest paradox lies in appreciating the fact that "Born rigid" rotation of the disc from its state of rest is a kinematic impossibility, people differ when trying to be specific about the exact deformation of the disc brought about by rotation. We give below just two opposite viewpoints that are found in the literature.

According to the so-called kinematic resolution of the paradox there should not be any contraction of the circumference as observed from the inertial frame of the laboratory that is at rest with the axis but, the periphery should stretch in terms of proper measure so that the Lorentz contraction effect of special relativity is automatically taken care of. The conclusion apparently follows from the widely discussed line element representing a rotating co-ordinate system.[22,27,37] It has been however shown, by drawing analogy with the version II linear Sagnac type (thought) experiment, that the kinematic resolution presupposes that the disc material is composed of "unbonded" particles that are programmed to rotate in such a way that the distances between the particles remain fixed with respect to the laboratory as the system passes to a rotational motion from its state of rest. If this happens, the formula for the Sagnac delay will be given by Eq. (4).

The other view point is to suppose that the disc material obeys the Synge–Pounder criterion of superficial rigidity. In this case the disc should bend and take a shape of a paraboloid so that at any radial point, the circumference is Lorentz contracted but there is no contraction of the meridian. However the distance of the periphery from the centre will be shortened. Therefore the paradox does not exist. In this case too the resolution of the paradox is based on a specific postulate regarding the behaviour of the material of the disc undergoing rotation. As a consequence, the Sagnac delay should be given by Eq. (10) that corresponds to the result obtained for the linear Sagnac effect of the first form (Eq. (16)). Some authors(17, 18) have quoted this result too however not addressing any role of the Ehrenfest paradox in their derivation. If instead of a disc, the rotating platform is assumed to be a massive solid cylinder, the deformations of the kind just mentioned are perhaps excluded and the usual formula (Eq. (4)) pertains to this case. However, in this case, the constraint imposed on the particles of the cylinder by the form of the solid body would work in such a way, that the particles of the body can be thought of as "unbonded" as the cylinder is set into rotation (vide Sec. 5). Indeed for a disc there cannot be one right formula; for example, the deformation of the kind considered by Eddington(34) as mentioned in Sec. 5, would give a result different from Eq. (4) or Eq. (10).

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CONVENTIONALITY OF SIMULTANEITY AND THE TIPPE TOP PARADOX IN RELATIVITY

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In this paper we critically examine a recently posed paradox (tippe top paradox in relativity) and its suggested resolution. A tippe top when spun on a table, tips over after a few rotations and eventually stands spinning on its stem. The ability of the top to demonstrate this charming feat depends on its geometry (all tops are not tippe tops). To a rocket-bound observer the top geometry should change because of the Lorentz contraction. This gives rise to the possibility that for a sufficiently fast observer the geometry of the top may get altered to such an extent that the top may not tip over! This is certainly paradoxical since a mere change of the observer cannot alter the fact that the top tips over on the table. In an effort to resolve the issue the authors of the paradox compare the equations of motion of the particles of the top from the perspective of the inertial frames of the rocket and the table and observe among other things that (1) the relativity of simultaneity plays an essential role in resolving the paradox and (2) the puzzle in some way is connected with one of the corrolaries of special relativity that the notion of rigidity is inconsistent with the theory. We show here that the question of the incompatibility of the notion of rigidity with special relativity has nothing to do with the current paradox and the role of the lack of synchronization of clocks in the context of the paradox is grossly over-emphasized. The conventionality of simultaneity of special relativity and the notion of the standard

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(Einstein) synchrony in the Galilean world have been used to throw light on some subtle issues concerning the paradox.

Key words: special relativity, conventionality of simultaneity, Zahar transformation, Tangherlini transformation, lack of synchronization of clocks, rigidity.

1. INTRODUCTION

Since the advent of the special theory of relativity paradoxes concerning the theory have always been very common. Even today in the literature newer paradoxes continue to pour in, a latest item in the list being the tippe top paradox. In an interesting paper, Basu *et al.* [1] have posed the paradox concerning a fascinating toy known as tippe top. The top has the remarkable property that after giving it an initial vertical spin on a table about its symmetry axis, the top turns itself (after a few rotations) upside down and stands spinning on its stem. The statement of the paradox goes like this: Not all tops demonstrate this charming feat. In order for this to happen, the ratios of its principal moments of inertia should fall in a certain regime. If now such a top is observed from a rocket frame in which the spinning top appears to recede with a uniform velocity, the cross-section of the top will appear elliptical due to the relativistic length contraction effect along the boost direction. For a sufficiently fast rocket, therefore, it is possible that the ratios of the principal moments of inertia go out of the regime required for the top to tip over. This is certainly paradoxical since a mere change of reference frame cannot alter the fact that the top tips over.

The paper claims to have resolved the paradox first by noting that the paradoxes in relativity often arise because one tends to focus only on one relativistic effect (in this case the length contraction effect) while losing sight of the other kinematical consequences of special relativity (SR). The authors then demonstrate, by applying the Lorentz transformations (LT) to the equations of motion of the particles of the top, that the whole body of relativistic effects such as the length contraction, the time dilation, the relativity of simultaneity and the likes find their appearances in the description of the spinning motion of the particles of the top from the reference frame of the rocket. The particle orbits have been studied both analytically and by computer simulation to demonstrate how the classical notion of the rigid body fails in relativity. The origin of the paradox has then been attributed, in different words, to our adherence to the classical notion of rigidity in the relativistic domain.

Although it is interesting as well as instructive to view on the computer screen the motion of the particles of the spinning top as observed from the rocket frame (but referring it to the moving point of contact of the top with the table), we consider the paper to suffer from certain drawbacks which we would like to address. It appears, in accordance with the paper's claim, that the resolution of the paradox relies heavily on the lack of synchrony aspect of SR. This, in our opinion, is a weak point of an otherwise interesting paper. Indeed we will argue (in Sec. 3) that the lack of synchrony aspect may well exist in the Galilean world where it is well known that no such paradox should truly exist. The other drawback is the authors' attempt to connect the paradox with one of the corollaries of SR, that the notion of rigid body is inconsistent with it. In Sec. 4 below it will be shown that this aspect of SR has no bearing with the current paradox. It is indeed true that many paradoxes can arise if one inadvertantly carries the classical notion of rigidity over into relativistic situation [2]. The classical rigid body by definition must move as one entity when it is pushed at one end, i.e., the disturbance at one end of the body would be propagated with infinite velocity through the body. This is in contradiction to the relativity principle that there is a finite upper limit to the speed of transmission of a signal. The analysis of Basu *et al.* considers only uniform rotation of the particles of the top with respect to the table frame; no transients are involved. It is therefore surprising how the rigidity issue should be connected to the problem. The aim of the present paper is to deliberate on these and related issues.

In recent years a new approach based on conventionality of simultaneity (CS), to understanding paradoxes in relativity has been found fruitful. For example, Redhead and Debs [3] have shown that the CS approach provides a means to put an end to the question concerning the notorious twin paradox as to where and when the differential ageing takes place. As another example Selleri [4] has shown that a particular simultaneity convention compared to that of Einstein seems to be more appropriate in explaining the Sagnac effect from the point of view of the rotating turn-table. The present paper will also follow the CS approach to critically examine the work of Basu *et al.* A brief introduction to the CS-thesis of relativity, therefore, is in order. This will be done in Sec. 3. The main arguments will be presented in Sec. 4-6 before we summarise all this in Sec. 7. However, in order to set the stage we will briefly reproduce in Sec. 2 the basic arguments used in [1] to clarify the paradox.

2. EQUATION OF MOTION AND COORDINATE SYSTEM

For simplicity consider a vertical top such that a typical particle P of the top in the table frame Σ^0 executes a horizontal (in the X-Y plane) circular motion. The equation of motion of the particle in the coordinate system of Σ^0 is given by

$$x^0 = R\cos\omega t^0, \quad y^0 = R\sin\omega t^0, \quad x^{0^2} + y^{0^2} = R^2, \tag{1}$$

where ω is the angular speed of the top and R represents the distance of P from the axis.

Consider now a frame of reference Σ of the rocket with respect to which the table and the top moves along the positive x-direction common to both Σ and Σ^0. Suppose that the x and t coordinates of the rocket frame are linearly related to the corresponding x^0 and t^0 of Σ^0 through the following transformations:

$$x^0 = ax + bt, \quad t^0 = gx + ht, \tag{2}$$

and also suppose $y^0 = y$, where a, b, g and h are independent of x and t. In Ref. [1], however, they have written LT, but here we wish to keep it a bit more general for reasons which will be apparent soon. Clearly, from the above transformation equations it follows that the origin of Σ^0 satisfies the following equation of motion with respect to Σ:

$$ax_{\text{orig}} + bt = 0, \tag{3}$$

where the suffix 'orig' refers to the origin. In other words the translational velocity of the origin, as observed from the rocket frame is given by

$$u = -b/a. \tag{4}$$

Since this translatory motion is irrelevant to the spinning of the top about its axis, it may be subtracted out from the *apparent* equation of motion of the particles of the top as seen from Σ. We may thus define the spinning coordinates of P as

$$\begin{aligned} x_s &= x - x_{\text{orig}} = x + (b/a)\ t, \\ y_s &= y, \quad t_s = t. \end{aligned} \tag{5}$$

Inserting Eqs. (2) and (5) into Eq. (1), one obtains

$$x_s = (R/a)\cos\omega\left[(h - gb/a)\,t + gx_s\right], \tag{6}$$

$$y_s = R\sin\omega\left[(h - gb/a)\,t + gx_s\right]. \tag{7}$$

The trajectory of the particle P in the rocket frame Σ is obtained by eliminating t from the above equations, and one thus has

$$a^2{x_s}^2 + {y_s}^2 = R^2. \tag{8}$$

Now in SR the transformation Eq. (2) is nothing but LT:

$$x^0 = \gamma(x - \beta ct), \quad y^0 = y, \quad t^0 = \gamma\left(t - \beta c^{-1}x\right), \tag{9}$$

where $\gamma = (1-\beta^2)^{-\frac{1}{2}}$, with $\beta c = v$ representing the speed of the rocket with respect to the table. In other words, for LT the transformation matrix representing Eq. (2) is given by

$$\mathrm{T} = \begin{pmatrix} a\, b \\ g\, h \end{pmatrix} = \gamma \begin{pmatrix} 1 & -\beta c \\ -\beta c^{-1} & 1 \end{pmatrix}. \tag{10}$$

Inserting these values of a, b, g and h into (6), (7), and (8), we obtain the equation of motion and that of the trajectory of the particle P as

$$x_s = \gamma^{-1} R \cos\omega \left(\gamma^{-1} t - \gamma\beta c^{-1} x_s\right), \tag{11}$$

$$y_s = R \sin\omega \left(\gamma^{-1} t - \gamma\beta c^{-1} x_s\right), \tag{12}$$

$$\gamma^2 x_s^2 + y_s^2 = R^2. \tag{13}$$

Authors of Ref. [1] rightly point out that the above equations display all the well-known effects of relativity. For example, Eq. (13) shows that what is a circle with respect to the table frame is now an ellipse with respect to the rocket frame (in the spinning coordinates) because of the length contraction effect. Equations (11) and (12) show that the angular frequency ω of the top is reduced to $\gamma^{-1}\omega$ because of the time dilation. Finally the relativity of simultaneity (i.e., the lack of synchronization of clocks) is manifested in the presence of the spatial coordinate x_s in the phase factors of the sinusoidal functions in Eqs. (11) and (12). The authors claim that this feature plays an essential role in the resolution of the paradox. We will however show that this is not quite correct.

The arguments of Basu *et al.* [1] are based on the apparent non-rigid behaviour of the top as seen from the rocket. However, before we go into these in detail let us now study this non-rigid character a bit closely. Equations (11), (12), and (13) display qualitatively two distinct types of non-rigidity viz. type I and type II.

Type I. This type of non-rigidity is manifested in the periodical change of the distance of any particle of the top from the center, as the particle travels along the elliptical path according to Eq. (13).

Type II. From Eq. (11) and (12) it will be evident that a chain of particles which lie along the radius of the circle (see Eq. (1)) parallel to the x-axis of Σ^0 at $t^0 = 0$ will appear to take a shape of a bow below the semi-major axis at $t = 0$ with respect to the rocket frame (see Fig. 4 of Ref. [1] for a schematic diagram of the positions of the particles in the radial chain at $t = 0$). A particle in the chain which is farther away from the center will lie farther below the x-axis. This progressive increase of initial phases of the particles is clearly the consequence of the phase factor in Eqs. (11) and (12). As time passes, the bow-like chain rotates, straightens itself, again bends down and straightens up and continues like this. (For the stills of the computer movie of the chain of particles in the table and the rocket frames see Fig. 5 of Ref. [1].) This corresponds to another type of non-rigid behaviour of the material of the top as distinct from the type I. We call it type II. Note that this type II non-rigidity is clearly the result of the relativity of simultaneity.

The authors of Ref. [1] observe that the spinning top is "more like a visco-elastic fluid in a weird centrifuge subjected to anisotropic external stresses." However, although it is correct that the concept of rigid body dynamics and moments of inertia appears to be no longer valid in the spinning coordinate system, the type II non-rigidity at least has nothing to do with the paradox. In fact this apparent fluid-like motion of the top particles has no connection with the question of the inconsistency of the notion of rigidity in relativity. In the next section we will show, among other things, that the fluid-like motion of the particles of the top can also be seen in the non-relativistic world although it is well known that the notion of rigid body is quite consistent in classical mechanics.

3. CONVENTIONALITY OF SIMULTANEITY AND TRANSFORMATION EQUATIONS

3.1. Relativistic World

In special relativity spatially separated clocks in a given inertial frame are synchronized by light signals. This synchronization is possible provided one knows beforehand the one-way speed (OWS) of the signal. But the measurement of OWS requires pre-synchronized clocks and therefore one ends up in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the synchronizing signal within certain bounds. Einstein assumed the OWS of light to be equal to c which is the same as the two-way speed (TWS) of light. The TWS of a signal is an empirically verifiable quantity, as this can be measured by a single clock without requiring any distant clock synchrony. Note that this stipulation (the equality of OWS and TWS) of Einstein has nothing to do with his "constancy of velocity of light" postulate [5, 6]. The assertion that the procedure for distant clock synchrony in SR has an element of convention is known as the CS-thesis, first discussed by Reichenbach [7] and Grünbaum [8]. The synchronization convention adopted by Einstein is commonly known as the Einstein synchrony or the standard synchrony. The possibility of using a synchronization convention other than that adopted by Einstein and consequent transformation equations between inertial frames are much discussed by various authors [4–6, 9–11]. For example it is known that the relativistic world can well be described by the so called Tangherlini transformations (TT) by adopting absolute synchrony [4–6, 11–13]:

$$x = \gamma\left(x^0 + \beta c t^0\right), \qquad y = y^0, \qquad t = \gamma^{-1} t^0. \tag{14}$$

Notice that the absence of spatial coordinate in the time transformation above, means the distant simultaneity is absolute (since $\Delta t^0 = 0 \Longrightarrow \Delta t = 0$). Since, according to the CS thesis, the question of simultaneity of any two spatially separated events depends on the synchronization convention, the issue of relativity of simultaneity which is often considered as one of the most fundamental imports of SR, has little significance [5]. We will get back to this issue and the transformations (14) in particular in section 5 in the context of the paradox.

3.2. Galilean World

A less well-known fact, however, is that the CS thesis can also be imported in the classical (Galilean) world. Consider as a fiction that we live in the Galilean world and suppose light travels through ether, stationary with respect to Σ^0. The spacetime coordinates of an arbitrary inertial frame Σ moving with speed $v = -\beta c$ with respect to Σ^0 are related to those in Σ^0 by the so-called Galilean transformations (GT)

$$x^0 = x - \beta c t, \qquad y^0 = y, \qquad t^0 = t. \tag{15}$$

In the Galilean world synchronization issue usually does not come in, since in principle, all the spatially separated clocks can be synchronized instantaneously by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However, ingredients of Einstein synchrony can be incorporated even in this world. Say in a somewhat playful spirit one chooses to synchronize an arbitrarily located clock in any frame Σ with one placed at its origin by sending a light signal from the origin to the clock in such a way that the OWS of light *along any line* passing through the origin is independent of the direction of propagation and is equal to the TWS of the signal along the line. In this case, one obtains the so-called Zahar transformation (ZT) [6, 11, 14]

$$x = x^0 + \beta c t^0, \qquad y = y^0, \qquad t = \gamma^2\left(t^0 + \beta c^{-1} x^0\right), \tag{16}$$

and its inverse

$$x^0 = \gamma^2\left(x - \gamma^{-2}\beta c t\right), \qquad y^0 = y, \qquad t^0 = t - \gamma^2 \beta c^{-1} x. \tag{17}$$

One may verify from the transformations (16) and (17) that the TWS of light along any direction measured in an arbitrary reference frame is given by the same expression as one would have obtained using GT.

For example one may verify that the TWS of light along the x-axis and y-axis in Σ are given by the Galilean results, $c\,(1-\beta^2)$ and $c\,(1-\beta^2)^{\frac{1}{2}}$, respectively [5, 13].[3] Clearly the presence of the spatial coordinate in the time transformations of (16) and (17) is the result of the adopted synchrony. The properties of rods and clocks also do not change due to their motions with respect to Σ^0. However, there is an apparent length contraction and time dilation effect with respect to Σ because of different simultaneity criterion used in this frame.

It may also be noted that even in the Galilean world the transformation Eqs. (16) and (17) depend on the speed of light c in ether, since light has been chosen as the synchronizing agent. If instead of light the clocks are synchronized by any other signal with speed c' in Σ^0 the transformation equations would have been

$$x = x^0 + \beta' c' t^0, \qquad y = y^0, \qquad t = \gamma'^2 \left(t^0 + \beta' c'^{-1} x^0\right), \tag{18}$$

where β' and γ' are the same as β and γ except where c is replaced by c'.[4] The synchronization in this case may be called *pseudo-standard synchrony* since the synchronization agent here is not the light signal.

4. TIPPE TOP PARADOX IN THE GALILEAN WORLD

The paradox can now be posed even in the Galilean world. With respect to the observer in the rocket frame Σ, the geometry of the top will appear to have altered because of the length contraction effect which is the outcome of the distant clock synchrony adopted in the rocket frame. This may give rise to the possibility that the ratios of the principal moments of inertia go out of the regime required for the top to tip over! We now proceed to "resolve" the paradox by following the line of arguments used in Ref. [1]. The matrix of ZT (Eqs. (16)

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[3]The Galilean world or classical world is thus defined to be a world where the TWS of any signal obeys the transformation law that one would have obtained by using the Galilean velocity addition rule. On the other hand, the world is said to be relativistic if the space-time admits an invariant TWS [6]. It may be noted, by virtue of the CS thesis, that kinematically different transformations and OWS' may correspond to a same kinematical "world" [13].

[4]Operationally one may consider that Σ^0 is a frame of reference stationary with respect to some fluid which supports an acoustic mode with isotropic speed c'. Clocks in any frame are assumed to be synchronized following Einstein's convention using the signal. Equation (18) was called Dolphin transformations (DT) in the Galilean world in Ref. [6], where it was first derived.

and (17)) representing the coordinate transformations for x^0 and t^0 is

$$\mathrm{T} = \begin{pmatrix} \gamma^2 & -\beta c \\ -\gamma^2\beta c^{-1} & 1 \end{pmatrix}. \tag{19}$$

The equations of motion and the equation of the trajectory of the particle in the spinning coordinates can be obtained by inserting the elements of T, *i.e.*, a, b, g and h in Eqs. (6), (7), and (8) as

$$x_s = \gamma^{-2} R \cos\omega(\gamma^{-2}t - \gamma^2\beta c^{-1}x_s), \tag{20}$$

$$y_s = R \sin\omega(\gamma^{-2}t - \gamma^2\beta c^{-1}x_s), \tag{21}$$

$$\gamma^4 x_s^2 + y_s^2 = R^2. \tag{22}$$

As before, the time dilation, the length contraction and the lack of synchronization of clocks seem to be present in these equations. Only quantitatively these effects differ from those obtained earlier using the LT.

These equations suggest that qualitatively the top material displays the same form of non-rigidity (of both type I and II) as has been observed in the relativistic world as the particles of the top moves in accordance with Eqs. (20) and (21) with respect to the observer in the rocket frame. Indeed one only needs to slightly modify the programming codes developed in Ref. [1] to simulate the motion of the particles of the top and see for oneself the picturesque output on the computer screen displaying as before, the non-rigid character of the body.

The idea of Einstein synchrony in the Galilean world may at a first sight seem to be a bit weird. However, the idea is not as strange as it appears. Consider LT in the non-relativistic regime where $\beta^2 \ll 1$ so that the approximation $\gamma \cong 1$ holds. In this approximation, contrary to common belief, LT does not go over to GT, instead one obtains the so-called approximate Lorentz transformation (ALT) [13, 15, 16]

$$x^0 = x - \beta ct, \qquad t^0 = t - \beta c^{-1}x. \tag{23}$$

As is expected the transformations (23) do not exhibit length contraction and time dilation. However, by virtue of the presence of the spatial coordinate in the time transformation the simultaneity is not absolute. It can also be verified [13] that ALT represents Einstein synchrony. It is therefore not surprising that ZT also reduces to Eq. (23) under the same approximation. ALT or approximate Zahar transformation (AZT) therefore represents the Galilean world with Einstein synchrony. Inserting the coefficients of ALT/AZT in Eqs. (6), (7), and (8) one obtains the equations of motion and the trajectory of the spinning particle as

$$x_s = R\cos\omega\left(t - \beta c^{-1}x_s\right), \tag{24}$$

$$y_s = R\sin\omega\left(t - \beta c^{-1}x_s\right), \tag{25}$$

$$x_s^2 + y_s^2 = R^2. \tag{26}$$

Clearly according to these equations the material of the top still displays non-rigidity of type II.[5]

In line with Ref. [1], one may now try to say that the concept of rigid body does not fit in the classical world. One may even be tempted to explain away the paradox by saying that nothing is rigid and all bodies are compressible and failure to comprehend this, leads to the paradox! Clearly this is absurd in the Galilean world or in the non-relativistic regime.

In order to understand the situation more clearly, let us examine the meaning of the spinning co-ordinate system. From the transformation Eqs. (2) and (5) one may connect the vectors

$$\mathbf{x}^0 = \begin{pmatrix} x^0 \\ t^0 \end{pmatrix} \quad \text{and} \quad \mathbf{x}_s = \begin{pmatrix} x_s \\ t_s \end{pmatrix}$$

as

$$\mathbf{x}^0 = \mathbf{B}\mathbf{x}_s, \tag{27}$$

with

$$\mathbf{B} = \begin{pmatrix} a & 0 \\ g\,h - gb/a \end{pmatrix}, \tag{28}$$

where, as before, the y-coordinate has been suppressed, although we keep in mind that $y_s = y = y^0$. For Galilean transformation (see Eq. (15))

$$\mathbf{T} = \begin{pmatrix} 1 & -\beta c \\ 0 & 1 \end{pmatrix}, \tag{29}$$

$\mathbf{B}$ turns out to be the identity matrix.

In this case the equations of motion and trajectory of any particle represented by Eq. (1) will not lead to a fluid-like motion of the particles in the spinning coordinate system. However for $\mathbf{B} = \mathbf{1}$, these coordinates are the same as that used in Σ^0!

[5]Equations (24), (25), and (26) could have been obtained using the approximation $\gamma \cong 1$ directly in Eqs. (11), (12), and (13) or, alternatively, in (20), (21), and (22); however, the present approach is more instructive since it clearly shows the role of the lack of synchrony of clocks in the type II non-rigidity of the top.

This only demonstrates that the rigidly rotating top will continue to display its rigid character only in the coordinate system defined in the frame of reference at rest with the table. Therefore, it is obvious that the concept of rigid body dynamics or moments of inertia are applicable only in this unique frame and this knowledge therefore surely resolves the paradox. However, this fact does not depend on whether the world is classical or relativistic. Therefore, in accordance with our earlier assertion, there is no connection of the apparent fluid-like behaviour of the top material in the spinning coordinate system with the issue of incompatibility of the notion of rigidity in SR.

5. LACK OF SYNCHRONIZATION – IS IT CRUCIAL?

Basu *et al.* [1] claimed that the lack of synchronization of clocks i.e., the relativity of simultaneity aspect of SR plays an essential role in the resolution of the paradox. But we have already observed in section 3, that the question of relativity of simultaneity is purely conventional and therefore is devoid of any empirical content. Kinematically the relativistic world can be described by TT (14) representing absolute simultaneity. The transformation matrix representing the inverse of TT in the $x - t$ plane may be written as

$$\mathrm{T} = \begin{pmatrix} \gamma^{-1} & -\gamma\beta c \\ 0 & \gamma \end{pmatrix}. \tag{30}$$

Inserting the elements of T in (6), (7), and (8), we get

$$x_s = \gamma R \cos \gamma\omega t, \tag{31}$$

$$y_s = R \sin \gamma\omega t, \tag{32}$$

$$\gamma^{-2} x_s^2 + y_s^2 = R^2. \tag{33}$$

Notice the absence of the phase terms in the sinusoidal functions. This means that the top does not display type II non-rigidity with respect to the rocket frame. However, the trajectory (33) of P is still an ellipse (this time the semi-major axis is along the x-direction) manifesting type I non-rigidity of the top. This only reiterates that the rigid rotation has to be defined in the table frame, but for this conclusion to hold the lack of synchronization aspect of SR does not play any role.

6. RIGIDITY AND TRANSCENDENTAL EQUATION

It has been noted in [1] that the transcendental Eq. (6) is of the form

$$x_s = f(x_s), \tag{34}$$

which can be solved by iteration provided

$$|f'(x_s)| < 1. \tag{35}$$

It is claimed that the condition (35) when applied to Eq. (18) leads to

$$\omega R < c, \tag{36}$$

which only says that no particle of the top can exceed the speed of light. This result, although fascinating, seems to be fortuitous, since instead of Eq. (6), if Eq. (20) (which pertains to the Galilean world) is used in the inequality (35), one obtains the same constraint condition (36) on the speed of a particle of the top. This is surprising, since in the Galilean world, there is no such speed limit intrinsically.

For the DT in the Galilean world (see Eq. (18)), the condition (35) leads to

$$\omega R < c', \tag{37}$$

which is more surprising.

On the other hand, we have seen that TT which represents the absolute synchrony in the relativistic world does not lead to any transcendental equation (see Eq. (31)) and hence no constraint on the speed of the particle is visible.

7. SUMMARY

The present paper discusses the tippe top paradox and different aspects of its resolution proposed in Ref. [1]. Clock synchronization issues in the relativistic and the Galilean world figured in course of our discussion. A few transformation equations in addition to LT were discussed in this connection. It is therefore worthwhile to summarise different properties of the transformations in the context of the paradox. This we do in Table 1 so that one is able to get the whole picture at a glance. The table is self-explanatory, however explanations of a few entries may be in order.

Table 1

World	Transformation	Synchrony Type	Type I Non-rigidity	Type II Non-rigidity	Length Contraction	Time Dilation Effect	Paradox exists? (prima facie)
Relativistic	LT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
Relativistic	TT	Absolute	Yes	No	Yes (w.r.t. Σ^0)	Yes (w.r.t Σ^0)	Yes
Relativistic /classical	ALT/AZT	Standard (Einstein)	No	Yes	No	No	No
	GT	Absolute	No	No	No	No	No
Galilean (Classical)	ZT	Standard (Einstein)	Yes	Yes	Yes	Yes	Yes
	DT	Pseudo Standard	Yes	Yes	Yes	Yes	Yes

Basically we discussed two worlds – relativistic and classical, but an overlapping world "Relativistic/Classical" is included in column 1 as a separate entry. This corresponds to the transformations ALT and AZT (see column 2) which are the forms of LT and ZT respectively under the approximation $\gamma \simeq 1$. For both the transformations, the synchrony type (as shown in column 3) is standard. These transformations do not predict the length contraction and the time dilation effects (see entries in column 6 and 7). The paradox in this case does not exist prima facie (see the entry in the last column) in this regime since there is no length contraction effect. However the observer in Σ will find the top material to exhibit non-rigidity of type II.

Note that the entries in the 1st, 5th, and 6th rows from column 3 onwards corresponding to the transformations LT, ZT, and DT, respectively, are exactly the same. This means that the paradox and the resolution as suggested in Ref. [1] completely fits in the classical world too. It therefore dismisses the claim that the paradox has its origin in the incompatibility of the notion of rigidity with SR.

The entries against TT shows that in the relativistic world non-rigidity of Type I of the top exists, although the synchrony here is absolute. It therefore follows that "lack of synchronization of clocks" cannot play an essential role in resolving the paradox.

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ON THE ANISTROPY OF THE SPEED OF LIGHT ON A ROTATING PLATFORM

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The paper discusses a recently posed paradox in relativity concerning the speed of light as measured by an observer on board a rotating turn-table. The counter-intuitive problem put forward by F. Selleri concerns the theoretical prediction of an anisotropy in the speed of light in a reference frame comoving with the edge of a rotating disc even in the limit of zero acceleration. The paradox not only challenges the internal consistency of the special relativity theory but also undermines the basic tenet of the conventionality of simultaneity thesis of relativity. The present paper resolves the issue in a novel way by recasting the original paradox in the Galilean world and thereby revealing, in a subtle way, the weak points of the reasonings leading to the fallacy. As a background the standard and the non-standard synchronies in the relativistic as well as in the Galilean world are discussed. In passing, this novel approach also clarifies (contrary to often made assertions in the literature) that the so-called "desynchronization" of clocks cannot be regarded as the root cause of the Sagnac effect. Finally in spite of the flaw in the reasonings leading to the paradox Selleri's observation regarding the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

Key words: special relativity, Selleri paradox, Sagnac effect, rotating frame, speed of light, conventionality of simultaneity.

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1. INTRODUCTION

The term "inertial frame of reference" in physics refers to an idealised concept. Our knowledge of physics in inertial frames has always been obtained in frames having small but non-zero acceleration. Indeed it is well-known that no perfectly inertial frame can be identified in practice. It is therefore expected that physics in non-inertial frames will go over smoothly to that in inertial frames in the mathematical limit of zero acceleration. In some recent papers [1, 2] Selleri observes that the existing relativity theory fails our expectations on that count. In this connection Selleri poses a paradox concerning the speed of light as measured by an observer on board a rotating turn-table. If two light beams from a common source are sent along the rim of a rotating disc in opposite directions and the round-trip speeds (c_+ for counter-rotating and c_- for co-rotating beams) for these two light beams are measured, it will be found from simple kinematics, that the ratio of these speeds $\rho = c_+/c_-$ is only a function of the linear speed v of disc at its edge and it differs from unity if $v \neq 0$. This observation finds its support in the well known Sagnac effect [3, 4] which is manifested in the experimentally observed asymmetry in the round-trip times of light signals co-rotating and counter-rotating with the interferometer. However, since the rotating turn-table is not an inertial frame, one might initially not be inclined to consider the observed anisotropy of light propagation with respect to this frame a startling result as such. But Selleri then considers a situation where one gradually increases the radius of the disc and at the same time allows the angular velocity ω of the same to get decreased proportionately in such a way that the linear speed $v = \omega R$ of the periphery remains constant. The edge of the disc can then be thought of approaching (locally) an inertial frame since, in the limit the centripetal acceleration $a = v^2/R$, tends to zero. [3] In an inertial frame ρ must strictly be unity since, light propagation is considered to be isotropic in such a frame according to the special relativity (SR). However Selleri shows that the ratio ρ on board the rotating disc does not change in the limit process provided v remains constant and therefore will continue to differ from unity as long as v is finite. A discontinuity in the behaviour of ρ as a function of acceleration is thus predicted. This is certainly paradoxical in the light of the observations made in the beginning of this section. We hereafter

[3] There is a scope for confusion here. Although an element of the disc will have zero acceleration in the limit considered, an observer on the turn table would be able to detect its rotation since the latter is an absolute concept. In an article Klauber [5] even claimed that there would be, for example a change of mass of a particle on the disc due to a general relativistic effect which can be seen to depend only on the circumferential velocity and not on the acceleration. This effect would even enable one to determine in principle the rotational motion of the platform from local measurements alone!

refer to it as the Selleri paradox (SP).

SP has so far met with evolving but inadequate responses. For example in one paper Rizzi and Tartaglia [6] observe that the "calculations of Selleri are quite careful" and the "paradox cannot be avoided if it is maintained that the round-trip on the turn-table corresponds to a well defined circumference whose length is univocally defined".

It appears that the authors of Ref. [6] cannot accept the global anisotropy of light speed in the frame of reference of the rotating disc and hold that because of the "impossibility of a symmetrical and transitive synchronization at large", the notion of whole physical space on the platform at a given instant is conventional. Their final conclusion is that the counter-rotating and the co-rotating light beams travel different distances with respect to the *frame of the disc* in such a way that the *global* ratio ρ remains unity.

The view point towards the resolution of SP also finds its endorsement in a later paper by Tartaglia [7]; although in a subsequent paper Rizzi and Tartaglia [8] somewhat retract from the past position and allow an observer at rest on the disc to consider a notion of its unique circumference in the "relative space of the disc" and hence endorse the view that light propagation can be anisotropic in the reference frame of the rotating turn-table. In conjunction with Budden's observation [9] the authors then correctly identify the root of the paradox and hold that the basic weak point of Selleri's arguments lies in equating the global ratio ρ of the speeds of light propagating in opposite directions along the rim with the local ratio ρ' of the same at any point on the edge of the disc. The latter ratio is always equal to unity if Einstein synchrony is used in any local inertial frame instantaneously comoving with the element of the rim at the point concerned (such frames will hereafter be referred to as momentarily comoving inertial frames (MCIF))and therefore SP does not pose any harm to SR.

However, Selleri's argument regarding the equality of two ratios ρ and ρ' is based on a symmetry argument (rotational invariance) but the authors of Ref. [8] do not clearly state what is precisely wrong with Selleri's symmetry reasoning. Further the arguments by the authors although correct, are blurred by their ambivalent observations (in the same paper) that the global ratio ρ itself comes out to be unity (a) if the time measuring clock is suitably corrected to "account for the desynchronization effect" or (b)if the space is suitably defined according to "geometry of Minkowskian spacetime". Note that (b) is the reiteration of their earlier stand in this regard [6, 7].

In Ref. [1, 2] Selleri raises another matter in connection with SP. In light of conventionality of (distant) simultaneity (CS) thesis of SR, the author discusses the conventionality issue on a rotating turn-table and argues that not the Lorentz transformation (LT) but the relativistic transformation with absolute synchrony (which is one of the many possible synchronization conventions for which light propagation is anisotropic) only correspond to the correct expression for ρ

(see Eq. (16) later). In a recent paper Minguzzi [10], whose view we share, briefly addresses the issue. The author agrees that isotropic convention (standard synchrony) can be unsuitable in certain situations but maintains that the possibility of anisotropic conventions does not imply any inconsistency of SR. However SP has not been discussed therein in its entirety.

To sum up it may be said that the responses to SP so far available in the literature are not fully satisfactory. We therefore hold that the paradox which poses a challenge to the very foundations of SR by questioning its self consistency, deserves to be given a fuller treatment. Indeed there are many subtle issues concerning SP. For example it will be seen in Sec. 3 that the paradox not only undermines the standard relativity theory but also denies the basic tenet of the CS thesis. The purpose of the present paper is to re-examine Selleri's arguments in the light of the CS thesis and provide a resolution of SP in a novel way by recasting the paradox in the classical world (see Sec. 4). It will however be argued that while both the self-consistency of SR and the CS thesis remain unchallenged, SP has a merit in that if properly interpreted in the light of reasonings presented in this paper, the whole issue will throw new light on various related issues like the question of time on rotating platform, desynchronization and its debated role in the explanation of Sagnac effect [11, 12].

We organize the paper as follows. Before we present our main arguments in Sec. 4 and onwards, the CS thesis will be discussed (in Sec. 3) in the context of the paradox. However in order to set the stage we will briefly reproduce in Sec. 2 the arguments of Selleri leading to SP. In Sec. 5 the issue of desynchronization vis-a-vis the Sagnac effect will be addressed and finally in Sec. 6 the standard synchrony and absolute synchrony will be compared upholding Selleri's point of view in this regard.

2. THE PARADOX

Suppose a light source is placed at some fixed position Σ on the rim of the turn-table and two light signals start from Σ at the laboratory time t_{01}, and are constrained (by allowing them for example, to graze a suitably placed cylindrical mirror on the rim) to travel in opposite directions in a circular path along the periphery of the disc. Let that, after making the round trips, the counter-rotating and co-rotating light flashes reach Σ at times t_{02} and t_{03} respectively.

As seen from the laboratory, the counter-rotating light signal travels a distance shorter than the circumference L_0 by the amount

$$x = v(t_{02} - t_{01}), \tag{1}$$

where $v = \omega R$ is the linear speed of the disc at its periphery. Similarly the co-rotating light beam has to travel a distance larger than L_0 by the amount

$$y = v(t_{03} - t_{01}). \tag{2}$$

From simple kinematics it therefore follows that

$$L_0 - x = c(t_{02} - t_{01}), \tag{3}$$

$$L_0 + y = c(t_{03} - t_{01}), \tag{4}$$

where L_0 is the disc's circumference as seen from the laboratory. From equations (3) and (4) and using equations (1) and (2) one readily obtains the round-trip times for counter-rotating and co-rotating signals, respectively, as

$$t_{02} - t_{01} = L_0/c(1 + \beta), \tag{5}$$

and

$$t_{03} - t_{01} = L_0/c(1 - \beta). \tag{6}$$

By taking the difference of these times, one may note here that the delay between the arrival of the two light signals at the point Σ is obtained as

$$\Delta t_s = t_{03} - t_{02} = (2/c)L_0\beta\gamma^2, \tag{7}$$

where $\gamma = (1 - \beta^2)^{-1/2}$, $\beta = v/c$. As an aside remark, it may be noted that (7) is nothing but the well-known delay time of classical Sagnac Effect. The relativistic formula for Sagnac delay can easily be obtained by noting that Δt_s in Eq. (7) is not the time as measured on board the platform and hence time dilation effect has to be considered. By multiplying both sides of the equation by $\gamma^{-1} = (1 - \beta^2)^{\frac{1}{2}}$, one obtains the relativistic formula for Sagnac delay as

$$\Delta\tau_s = (2/c)L_0\beta\gamma, \tag{8}$$

where $\Delta\tau_s = \gamma^{-1}\Delta t_s$ denotes the delay time as measured on board the turn-table.[4]

Suppose now that a clock C_Σ is placed on the disc's rim at Σ so that it co-rotates with the platform and also let t denotes the time of C_Σ. When the disc is in motion, according to Selleri, the laboratory time t_0 and t may be assumed to be related most generally as

$$t_0 = tF_1(v, a). \tag{9}$$

Similarly for the circumference also Selleri assumes a relation between L_0 and the proper circumference L as

$$L_0 = LF_2(v, a), \tag{10}$$

[4]There is a mild controversy however as to the correct special-relativistic formula for the Sagnac delay. However, Eq. (8) is the most widely quoted one. For a detailed discussion on the issue *vis-à-vis* the Ehrenfest paradox, see Ref. [13].

where F_1 and F_2 are some functions of the linear velocity $v = \omega R$ and the acceleration $a = v^2/R$ of the edge of the disc.

Although, from the widely accepted hypothesis of locality [6, 12] it is evident that these functions are nothing but the usual time dilation and length contraction factors

$$F_1 = F_2^{-1} = \gamma, \tag{11}$$

however, Selleri keeps open the possibility that F_1 and F_2 may depend on the acceleration as well.

Inserting equations (9) and (10) in equation (5) and (6) one gets the following times of flight of the counter-rotating and co-rotating light signals as measured on board the disc,

$$t_2 - t_1 = \frac{L}{c(1+\beta)} \frac{F_2}{F_1}, \tag{12}$$

$$t_3 - t_1 = \frac{L}{c(1-\beta)} \frac{F_2}{F_1}. \tag{13}$$

The round-trip speeds for these beams are therefore given by

$$c_+ = \frac{L}{t_2 - t_1} = c(1+\beta)\frac{F_1}{F_2}, \tag{14}$$

$$c_- = \frac{L}{t_3 - t_1} = c(1-\beta)\frac{F_1}{F_2}. \tag{15}$$

Consequently the ratio of these light speeds turns out to be

$$\rho = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{16}$$

Selleri now argues that since no point on the rim is preferred, the instantaneous velocities of either signals at any point of the rim must be the same, and therefore, the above ratio ρ is true not only for the global light velocities but also for the instantaneous velocities at any point on the rim. Now, as pointed out in Sec.1, if we consider that $R \to \infty$ and $\omega \to 0$ in such a way that v, the linear speed of any element of the circumference remains constant, so that the centripetal acceleration $a \to 0$, any short part of the circumference can be thought of as an inertial frame of reference in the limit. However, the ratio ρ does not change as long as v is kept constant. Hence, a discontinuity results in the behaviour of ρ as a function of acceleration ($\rho = \rho(a)$), since as $a \to 0$, but not equal to zero, ρ continues to differ from unity, but if $a = 0$, SR predicts that ρ must be equal to unity!

It may be argued that the above gedanken experiment with infinitely sized disc is impossible to perform since the times of flight of the co-rotating and counter-rotating light beams whose ratio we are currently interested in, would become infinite and therefore unmeasurable [14]. However it is enough to note that if the radius of the disc is increased arbitrarily from a finite value and at the same time v is kept constant by suitably adjusting the angular velocity, no tendency for the ratio ρ getting diminished would be seen although the acceleration of a point on the circumference gets reduced arbitrarily in the process.

It is worthwhile to mention in this context that recently Wang *et al.* [15] has obtained a travel time difference $\Delta t = 2vL/c^2$ between two counter-propagating light beams (indicating $\rho \neq 1$) in a uniformly moving fibre where v is the speed of the light source or the detector (comoving with the fibre) with respect to the laboratory and L in the length of the fibre. The experiment has been performed using a fibre optic conveyor(FOC) where two light beams leaving a source travel in opposite directions through an optical fibre loop which is made to move with uniform speed like a conveyor belt by a couple of rotating wheels separated by a distance. The interesting feature of the FOC arrangement is that here the observer (*i.e.*, the source or the detector) is attached to one of the straight-fibre segments and therefore moves with *uniform velocity* along a straight line. Experimental observation together with a symmetry argument (similar to that used by Selleri in the rotating disc context) may lead one to infer that the statement $\rho \neq 1$ is also valid *locally* in a segment of uniformly moving fibre indicating *local* anisotropy in the speed of light in vacuum [5] with respect to an inertial observer! Such an outcome which apparently follows from a symmetry argument is also paradoxical if one believes in SR. Although the purported scope of the present paper restricts us to deliberating on SP in its original form and consequent issues following a few responses it has received, the arguments that will be used in the following sections will equally apply to the paradox in the FOC context as well.

Indeed all Selleri wanted to achieve was to obtain an inertial observer with respect to whom $\rho = (1+\beta)/(1-\beta)$. In the FOC arrangement this comes naturally dispensing with the trick of letting the radius of the disc go to infinity and the angular speed to zero while the peripheral velocity is kept constant.

Before we leave this section, we write explicitly the expressions for c_+ and c_- in the full-relativistic context:

$$c_+ = c/(1-\beta) \,, \tag{17}$$

$$c_- = c/(1+\beta) \,, \tag{18}$$

[5] As suggested by Wang et-al, here we have assumed that experiment using FOC with a hollow core would give the same result. Indeed the result $\rho = (1+\beta)/(1-\beta)$ remains valid in this case since the simple minded analysis presented in this section leading to the equation also applies to this situation.

which follow from Eqs. (14) and (15) where the expressions for F_1 and F_2 as given in Eq. (11) have been substituted. Here we may point out that the genesis of SP relates to these equations since in the limit of infinite radius and zero angular velocity, the above results do not change indicating (as if) the violation of second relativity postulate (isotropy and constancy of light speed). As we have mentioned earlier, the above results although correct, are so counter-intuitive that the authors of Refs. [6, 7] in their initial reactions discarded the results altogether only to retract from their position later in a sort of a rejoinder [8].

3. C-S THESIS AND ABSOLUTE SYNCHRONY

In the relativity theory distant simultaneity is conventional. In order to synchronize spatially separated clocks in a given inertial frame one should know the one-way speed (OWS) of the synchronizing signal, however to know OWS one needs pre-synchronized clocks. One therefore is caught in a logical circularity. In order to break the circularity one has to *assume*, as a convention, a value for the OWS of the light signal within certain bounds. The C-S thesis, first discussed by Reichenbach and Grünbaum [16, 17],[6] is the assertion that the procedure for distant clock synchrony is conventional. Einstein therefore assumes as a convention that the OWS of light is isotropic and is equal to the two-way speed (TWS) c of the signal. Note that the latter is an empirically verifiable quantity since it does not depend on the convention regarding the synchronization of spatially separated clocks since the TWS can be measured by a single clock. The synchrony is commonly known as the Einstein synchrony or the standard synchrony. However since the clock synchronization procedure is conventional, conventions other than the standard one may equally be chosen [19–22]. Selleri [1, 2] has shown that the space-time transformation between a preferred inertial frame S_0 (where clocks are standard-synchronized so that OWS is isotropic in the frame) and any other frame S may generally be written as

$$\begin{aligned} x &= \gamma(x_0 - \beta c t_0), \\ y &= y_0, \\ t &= \gamma t_0 + \epsilon(x_0 - \beta c t_0), \end{aligned} \tag{19}$$

which represents a set of theories *equivalent* to SR. The free parameter ϵ which can at most be a function of the relative velocity of S with respect to S_0, depends on the simultaneity criterion adopted in S. For

[6]For a comprehensive review of the thesis see a recent paper by Anderson, Vetharaniam and Stedman [18].

the standard synchrony however,

$$\epsilon = -\beta\gamma/c. \tag{20}$$

For this value of ϵ, Eq. (19) reduces to a Lorentz transformation. The OWS' of light in S, c'_+ and c'_- along the negative and positive x-directions respectively may easily be obtained from the transformation (19) as

$$\frac{1}{c'_+} = \frac{1}{c} - \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right] \tag{21}$$

and

$$\frac{1}{c'_-} = \frac{1}{c} + \left[\frac{\beta}{c} + \epsilon\gamma^{-1}\right]. \tag{22}$$

If S_0 is assumed to be the inertial frame of reference at rest with the axis of the rotating disc and S be an MCIF, c'_+ and c'_- would then mean the local speeds of light counter-rotating and co-rotating with the disc respectively as measured on board the rotating platform. From Eqs. (21) and (22) one may thus obtain the ratio for these local speeds of light ρ' in terms of the ϵ-parameter

$$\rho' = \frac{c'_+}{c'_-} = \frac{1+\beta+\epsilon c\gamma^{-1}}{1-\beta-\epsilon c\gamma^{-1}}, \tag{23}$$

which agrees with the ratio ρ given by Eq. (16) provided $\epsilon = 0$. But as mentioned in the last section, the equality of ρ and ρ' according to Selleri as if follows from the symmetry of the situation. Therefore, in the rotating disc context, $\epsilon = 0$ appears to be the only allowed convention according to which the speed of light is anisotropic. Note that for this value of ϵ only the local speeds of light as given by Eqs. (21) and (22) reduce to the expressions (17) and (18). The transformation (19) with $\epsilon = 0$ is known as the Tangherlini transformation(TT) [23]

$$\begin{aligned} x &= \gamma\left(x_0 - \beta c t_0\right), \\ y &= y_0, \\ t &= \gamma^{-1} t_0. \end{aligned} \tag{24}$$

The transformation represents the relativistic world with absolute synchrony [20, 22].[7]

[7]Notice that in view of the absence of the spatial coordinate x in the above transformation for time, the simultaneity is independent of the frame of reference considered and is therefore absolute.

We now have a ramification of the original paradox. The value of ρ (and hence ρ') represented by equation (16), which implies anisotropic propagation of light in the rotating frame, is obtained theoretically from the perspective of the inertial frame S_0. The result also finds its empirical support in the Sagnac effect. It therefore appears that as if a particular (non-standard) synchrony is dictated both by theory and experiment. This is absurd since, if it were true it not only would reject the Lorentz transformation but also would contradict the basic tenet of the CS thesis that clock synchronization is conventional.

Before offering a resolution of the SP we ask ourselves if such a paradox could exist in the classical (Galilean) world too. The initial reaction would be to answer in the negative since nothing seems to be mysterious or enigmatic in this world and as is well-known, counter intuitive problems by contrast exist in the relativity theory probably because of its new philosophical imports. However we answer the question in the affirmative. One of the so-called new philosophical imports of SR is the notion of relativity of simultaneity. It can be shown that this notion can also be introduced in the Galilean world. Indeed in the next section it will be shown that by doing so the paradox can be artificially created even in this world where normally one would not expect it to exist.[8] The perspective of the paradox will hopefully provide deeper understanding of the problem and other related issues.

4. SELLERI PARADOX IN THE GALILEAN WORLD

Let us consider a fiction that we live in the Galilean (classical) world and suppose light travels through ether stationary with respect to an inertial frame S_0. The space-time coordinates of an arbitrary inertial frame S moves with respect to S_0 are related to those in S_0 by the so-called Galilean transformation(GT):

$$x = x_0 - \beta t_0, \qquad y = y_0, \qquad t = t_0. \tag{25}$$

In the Galilean world, synchronization issues usually do not figure in, since in principle all the clocks in any given inertial frame can be synchronized by sending signals with arbitrarily large velocities. Note that there is no speed limit in this world. However the ingredients of the CS thesis can also be incorporated in this world. For example, one may employ the Einstein synchrony to describe the kinematics in this world. Suppose one sends out a light signal from the origin of S outwards along a line which makes an angle θ with the x-axis and the signal comes back to the origin along the same line after being reflected by a suitably placed mirror, the TWS can be obtained by measuring the

[8] Such an approach has been found fruitful elsewhere in understanding a recent paradox in relativity [24].

time of flight of the round-trip by a clock placed at the origin. The expression for this TWS can be obtained from Eq. (25) and is given by [20]

$$\overleftrightarrow{c}(\theta) = \frac{c(1-\beta^2)}{(1-\beta\sin^2\theta)^{1/2}}. \tag{26}$$

Now in a somewhat playful spirit one may choose to synchronize arbitrarily located clocks with one placed at the origin by sending light by *stipulating* the OWS of light to be equal to the TWS (in fact none can prevent one in doing so), the relevant transformation that would honour such a stipulation would be given by

$$\begin{aligned} x &= x_0 - \beta c t_0, \\ t &= \gamma^2 \left(t_0 - \beta x_0/c\right), \end{aligned} \tag{27}$$

which was originally obtained by E. Zahar in 1977 [25] and is now commonly known as the Zahar transformation(ZT). For a quick check one may readily verify that the TWS of light along the x-axis and y-axis in S,that follow from Eq. (27) are given by the well-known classical results, $c(1-\beta^2)$ and $c(1-\beta^2)^{1/2}$, respectively [21, 26].

In the context of the rotating disc, x and t denote the coordinate and time of an event in an MCIF at any point on the edge of the disc, while x_0 and t_0 refer to the same in the inertial frame S_0 which is stationary with the axis of rotation. Let us now write the inverse of ZT (Eq. (27)) for time in the differential form as

$$dt_0 = dt \pm \gamma^2\beta \; dx/c, \tag{28}$$

where dx refer to the length of the infinitesimal element of the disc which is covered by the light signal in time dt when the signal is co-rotating (+ sign) or counter-rotating (− sign) with the disc. Note that the phase term (space dependent term) in (28) was absent in the GT. Clearly the term is an artefact of the Einstein synchrony. For the complete revolution for the counter-rotating light signal, the round-trip time in the laboratory is thus obtained by integrating (28) as

$$\Delta t_{0+} = \oint dt - \oint \frac{\gamma^2\beta}{c}\, dx \tag{29}$$

or

$$\Delta t_{0+} = \oint dt - \frac{\gamma^2\beta L_0}{c}; \tag{30}$$

and, similarly for the co-rotating signal,

$$\Delta t_{0-} = \oint dt + \frac{\gamma^2\beta L_0}{c}. \tag{31}$$

Notice that $\oint dt$ in Eqs. (30) and (31) are the same because of the adopted synchrony which is given by

$$\oint dt = \frac{L_0}{c(1-\beta^2)}, \tag{32}$$

since (from Eq. (26)), for $\theta = 0$,

$$\overleftrightarrow{c}\,(0) = c(1-\beta^2), \tag{33}$$

which has been assumed to be the same as the OWS following the synchrony. That the Zahar transformation and hence Eqs. (30) and (31) are consistent with the classical world can be checked by calculating $c_\pm\,(= L_0/\Delta t_{0\pm})$ from Eqs. (30) and (31) and by making use of Eq. (32). They are obtained as

$$c_\pm = c(1 \pm \beta), \tag{34}$$

which could have been obtained from elementary kinematics using GT. This agreement is expected since the global round-trip speeds are observables independent of the synchrony gauge. Further by taking the difference of Eqs. (30) and (31), by virtue of the cancellation of the $\oint dt$ terms one obtains the usual classical expression for the Sagnac delay quoted earlier

$$\Delta t_s = (2/c)L_0\beta\gamma^2. \tag{35}$$

From Eq. (34) it is evident that in the classical world, too,

$$\rho_{\text{classical}} = \frac{c_+}{c_-} = \frac{1+\beta}{1-\beta}. \tag{36}$$

Clearly we are confronted with the same apparent paradox that the ratio of the round-trip speeds of the two counter-propagating light signals differ from unity ($\rho \neq 1$) although locally the one-way speeds of light in opposite directions have been assumed to be the same ($\rho' = 1$). (This is manifested in the cancellation of $\oint dt$ terms while taking the difference of (30) and (31) in arriving at the classical Sagnac effect formula (35)).The rather tortuous way of deriving the Eqs.(34), (35) and (36) serves two things. It demonstrates how the Sagnac effect can be construed as an effect of "desynchronization" of clocks (due to the contribution of the phase terms in Eq. (28)) on the rotating platform even in the classical world. This effect is usually regarded as a 'real' physical phenomenon in the context of the relativistic Sagnac effect [27]. But the present derivation demonstrates that the desynchronization cannot be an objective phenomenon since here we clearly see it as an artifact of standard synchrony which is nothing but a stipulation. The other utility of this scheme of the derivation is that it allows us to understand clearly that the two apparently contradictory results ($\rho = 1$ and $\rho \neq 1$) follow from the same transformation (27). The contradiction is therefore a logical one. It means that the trouble lies in the arguments (and not in the physical theory — in this case it is the classical kinematics) leading to the paradoxical conclusions.

Further, not only Zahar transformation, the Galilean world can also be represented by the following transformation [20]

$$\begin{aligned} x &= x_0 - \beta c t_0, \\ y &= y_0, \\ t &= t_0 + \epsilon\left(x_0 - \beta c t_0\right), \end{aligned} \tag{37}$$

where, as before, ϵ is a free parameter which depends on the choice of synchrony. GT and ZT are recovered for $\epsilon = 0$ and $\epsilon = -\gamma^2\beta/c$ respectively. Note that these are the classical analogues of Selleri's transformation (19). The OWS' of light that follow from (37) are given by

$$\frac{1}{c'_+} = \frac{1}{c(1-\beta)} + \epsilon, \tag{38}$$

$$\frac{1}{c'_-} = \frac{1}{c(1-\beta)} - \epsilon; \tag{39}$$

and the corresponding ratio of these velocities is given by

$$\rho'_{\text{classical}} = \frac{c'_+}{c'_-} = \frac{1-\beta}{1+\beta} \cdot \frac{1 - c\epsilon(1+\beta)}{1 + c\epsilon(1-\beta)}. \tag{40}$$

As before, here also we see that $\rho'_{\text{classical}}$ corresponds to $\rho_{\text{classical}}$, provided $\epsilon = 0$. For ZT ($\epsilon = -\gamma^2\beta/c$), $\rho'_{\text{classical}} = 1$ which agrees with the stipulation of standard synchrony used to derive the transformation. But now $\rho'_{\text{classical}} \neq \rho_{\text{classical}}$, although the latter ratio also has been obtained using the same transformation *i.e.* ZT. We thus see that Selleri's arguments, if carried over into the classical world, also lead to the paradox.

As remarked earlier, in order to address the paradox one needs to look into the reasonings leading to it rather than expecting any flaw in the theories (relativistic or classical). One may ask why Selleri expects that ρ should be equal to ρ' (or equivalently why $\rho_{\text{classical}}$ should be equal to $\rho'_{\text{classical}}$)? The primed ratios are measured in the MCIF whereas the unprimed ratios are global, *i.e.* they are based on the measurements of the average speeds of light signals when they make complete roundtrips. Selleri's argument goes somewhat like this: Since the stationary inertial reference frame at rest with the centre of the disc is isotropic in every sense, the isotropy of space should ensure that the instantaneous velocities of light are the same at all points on the rim of the disc and therefore the average velocities should coincide with the instantaneous ones.

It is interesting that there is nothing wrong even in Selleri's observation regarding the symmetry of the situation, however the conclusion that the two ratios (ρ and ρ') are equal does not necessarily follow from the symmetry arguments. Below we give an example and explain why and how the local speeds of light may differ from their global values in spite of the symmetry.

Consider the motion of a rigid rod AB of length $L_0/2$ with respect to the inertial frame S_0. Suppose that the rod initially moves with uniform velocity βc towards the right parallel to the x-axis of S_0. The left end A of the rod is assumed to coincide with the origin of S_0 at the laboratory time $t_0 = 0$, when an observer at A on board the rod who carries a clock C_A sends out a light pulse towards B where another observer sitting on the rod holds a mirror facing A. As soon as the light pulse reaches the observer at B and is reflected back and starts to travel towards A, the rod is also made to change its direction of motion and travel towards the left with the same uniform speed βc.[9]

Suppose now that the observers in the laboratory record the times of the following three events:

Event 1: The light pulse sent out from A at the laboratory time $t_0 = t_{01} = 0$.
Event 2: The light pulse received at B at the laboratory time $t_0 = t_{02}$.
Event 3: The reflected light pulse received at A at the laboratory time $t_0 = t_{03}$.

From simple kinematics one obtains

$$t_{02} = \frac{L_0}{2c(1-\beta)}, \quad t_{03} = \frac{L_0}{c(1-\beta)}. \tag{41}$$

If Galilean transformation is used for any event, there is no distinction between the laboratory times and the corresponding times measured by observers on board the rod.

However if the observers wish to adopt the Einstein synchrony (*i.e.*, for light TWS= OWS) in the inertial frames of the moving rod (we label them S_1 for the rod moving towards the right and S_2 when

[9]The present analysis of this thought experiment, which essentially corresponds to a linear Sagnac effect discussed elsewhere [13, 28] by the present authors can be seen to fit well (with minor adjustment in the reasonings) with the FOC experiment [15] in the limit when the size of the wheels at the two ends tend to zero.

it moves towards the left say), they may do it by correcting the times for the event 2 in the respective frames. Let us denote these corrected times by t_{12} for S_1 and t_{22} for S_2.[10] The corrected times will be given by

$$t_{12} = t_{01} + \frac{L_0}{2 \overset{\leftrightarrow}{c}(0)} = \frac{L_0}{2c}\gamma^2 \tag{42}$$

and

$$t_{22} = t_{03} - \frac{L_0}{2 \overset{\leftrightarrow}{c}(0)} = \frac{L_0(1+2\beta)}{2c}\gamma^2, \tag{43}$$

where we have made use of Eq. (41) and inserted Eq. (33).

Note that for the derivation of Eq. (42) and (43), it has been implicitly stipulated that the times recorded on rod-observer's clock, t_{11} and t_{23} (which are the times recorded on C_A) are the same as the laboratory times t_{01} and t_{03} respectively. In the classical situation this is possible because no rate-correction is necessary. In the relativistic situation this stipulation is also possible by making the rate correction to the moving clocks by an appropriate Lorentz factor.

For event 2 the total disagreement of times between observers in S_1 and S_2 is therefore given by

$$\delta t_{\rm gap} = t_{22} - t_{12} = L_0\beta\gamma^2/c. \tag{44}$$

Now, in this example, physics is the same whether light propagates forward or backward with respect to S_0, but still the global speed $L_0/(t_{03} - t_{01}) = c(1-\beta)$ is different from the local speeds

$$\frac{L_0/2}{(t_{12} - t_{01})} = \frac{L_0/2}{(t_{03} - t_{22})} = c\left(1 - \beta^2\right), \tag{45}$$

since the total discrepancy in synchrony given by Eq. (44) remains unaccounted for in such a comparison if Einstein synchrony is used. Thus we see that in spite of the symmetric situation the global speed of light ought to be different from its local counterpart in this synchrony.

It is interesting to note that in the rotating disc situation this discrepancy in synchronization between any two adjacent MCIFs can be evenly distributed throughout its circumference by honouring the symmetry of this situation. It is therefore evident that the global ratio ρ is in general not the same as the local ratio ρ'. In fact it should be amply clear by now that while the former is an empirically verifiable quantity (based on the measurements of times of flight of light by a *single* clock) the latter quantity depends only on one's own choice of

[10]The symbol t_{ik} refers to the time of the k-th event according to an observer of the inertial frame S_i.

synchrony (see Eq. (42) or (43) to understand how the times of the event 2 in S_1 and S_2 are required to be adjusted in order to synchronize the clocks in the Einstein way). Note that in this respect the classical kinematics is no different from its relativistic counterpart.

5. DESYNCHRONIZATION

From the above analysis it is evident that if in order to calculate the round-trip time for light in the (non-inertial) frame of the rod, one adds up the times of flight of the same in the inertial frames S_1 and S_2, where the Einstein synchrony has been employed, the result will be wrong by the amount δt_{gap}. This happens since $t_{22} \neq t_{12}$. It only means that S_1 and S_2 cannot be meshed together. However in seeking to dovetail these frames one may set $t_{22} = t_{12} = L_0\gamma^2/2c$. But in that case t_{23} has to be altered by the amount δt_{gap} to preserve the Einstein synchrony in S_2. However since according to our stipulation t_{23} is the time measured by C_A, any possibility of alteration in the value of t_{23} would mean C_A is desynchronized with itself.

In the literature this phenomenon is known as the "desynchronization" in the context of synchronization of clocks in a rotating platform. It is not difficult to show that the measure of this desynchronization in the case of a rotating disc, which is often termed as the "time lag" [11, 29] (for the corotating light signal) is the same as δt_{gap} obtained in the shuttling rod example above. Note that this δt_{gap} is just half of the classical Sagnac delay (see Eq. (7)). If the same effect is calculated for the counter propagating beam, the total time lag $\Delta\tau_{\text{lag}}$ comes out to be $2\delta t_{\text{gap}}$. As mentioned earlier, people tend to regard this desynchronization $(\Delta\tau_{\text{lag}})$ as the real cause of the Sagnac effect in the relativistic context [6, 7, 11, 29]. For example in Ref. [7], Tartaglia observes that the "simplest explanation for this effect attributes it to the time lag accumulated along any round trip ...." Earlier, Rizzi and Tartaglia [6] expressed a similar view in order to give the "true" relativistic explanation for the Sagnac time difference by ascribing it to the non-uniformity of time on the rotating platform and to the "time lag" arising in synchronizing clocks along the rim of the disc. Selleri also remarks (while not sharing this view) that "an "orthodox" approach to dealing with the rotating platform problem is to consider a position dependent desynchronization ... as an objective phenomenon."

The present analysis of the classical Sagnac effect using Einstein synchrony reveals that this desynchronization is only an artefact of the Einstein synchrony and hence is devoid of any empirical content. Since, if instead of ZT, one uses the Galilean transformation, there is no "desynchronization" but still there is Sagnac effect. Therefore "desynchronization" is conventional in nature and hence cannot be considered an "objective phenomenon." For future reference we call this desynchronization desync1.

In a recent paper Rizzi and Serafini [11] acknowledges Selleri and Klauber (see footnote on p. 4 of Ref. [11]) who have brought to their attention this fact that the much talked about "desynchronization" is merely a "theoretical artefact." However the present paper reveals this in a more convincing way by explicitly showing how this "desynchronization" can be manufactured in the classical world too.

The authors of Ref. [11] however somewhat supporting the orthodox view regarding the connection of the Sagnac effect and the "desynchronization", redefines the latter in the following way: Starting from any point Σ on the rim of a rotating disc if two synchronized clocks are slowly transported in opposite directions along the periphery and are brought back to the same position, they will be found to be out of synchrony by the amount which is equal to that obtained for desync1, *i.e.*, $\Delta\tau_{\text{lag}}$. This desynchronization will hereafter be referred to as desync2.

The desynchronization, thus defined, is the result of the comparison of two clocks at the same space point and hence is independent of the distant synchrony convention. The authors therefore claim that they have revealed the "deep physical" and "non-conventional nature" of the time lag. However it is enough to point out the fallacy of this claim by mentioning that these two desynchronizations (desync1 and desync2) are two different things altogether, since if something is conventional, it can be changed or removed by altering the convention, but the "time lag" or time difference in the readings of the two slowly transported clocks after their round trips cannot be altered by redefining the synchrony on the rotating disc.

The equality of these time lags (*i.e.*, desync1 and desync2), therefore, is itself conventional and is true accidentally (as opposed to logically) in the relativistic situation if the Einstein synchrony is used in the rotating frame. If instead, the absolute synchrony is used desync1 $= 0$ while desync2 still remains non-zero. In the classical case the situation is reversed, since in this case desync2 is always zero since there is no time dilation of clocks with respect to the laboratory frame; however for the Einstein synchrony in the disc (which corresponds to ZT) desync1 $\neq 0$. These are however equal in the absolute synchrony (which corresponds to GT). Rizzi and Serafini also claim that desync2 brings to light the "dark physical root of the Sagnac effect." However this claim is also in error too. It is obvious that desync2 cannot be regarded as the physical cause of the Sagnac effect, since we observe that in the classical world desync2 is always zero but still the Sagnac effect exists. This reveals that desync2 and Sagnac effect are unconnected entities. The equality of these two different entities in the relativistic world is at best fortuitous.

6. SYNCHRONY – A VALUE JUDGEMENT

One is now in a position to inquire if it is possible to consistently synchronize clocks on the rim of the turn-table so that no gap in synchrony arises. Let us call such a synchrony as "good synchrony". To answer this consider the following scheme for synchronization due to Cranor *et. al.* [30]. In this scheme before the disc is set into motion with respect to S_0 all observers on the rim of the disc and those in the laboratory set their clocks according to the Einstein synchrony. The disc is then set into rotation uniformly (here 'uniformly' means all the points of the rim are treated identically [30]) which after some time may be assumed to attain a constant angular velocity. Alternatively one may set all the clocks on the rim (as well as those adjacent to them in S_0) a common time (say $t = 0$) as soon as the observers on the rim receive a flash of light sent out from a light source at the center of the disc.

Clearly the symmetry of the problem demands that the observers in the laboratory as well as those on the rim of the disc should continue to agree on the question of simultaneity as the synchronization process "favours no particular observer" [30]. This symmetry argument is evidently true in the classical as well as in the relativistic world. Only in the latter case although the observers in the laboratory frame and in the rotating frame agree on simultaneity, the clock *rates* in these frames differ due to the time dilation effect of relativity.

It is evident that there will be no gap in the synchrony between two successive MCIFs (in the linear example between S_1 and S_2) if the observers in these frames agree on simultaneity with those in S_0. Again if there is no synchrony gap the global ratio ρ should be equal to the local ratio ρ'. The agreement on simultaneity between the frames in turn requires ϵ to be equal to zero in Eqs. (19) and (37). In the classical world this implies GT, on the other hand in the relativistic world this corresponds to TT (Eq. (24)).

It means if the clocks of the disc were synchronized according to the scheme discussed above when the latter was at rest with respect to S_0, nothing has to be done further to synchronize them in order to have consistent synchrony throughout the rim when the disc picks up its uniform angular speed. The synchrony is thus "automatic". Any other synchrony (which corresponds to $\epsilon \neq 0$) including the Einstein Synchrony is to be achieved through human intervention. Selleri [31] therefore singled out the absolute synchrony by calling it as "nature's choice."

One may however ask at this point if it is at all possible to synchronize the clocks on the rim in the absolute way (so that $\epsilon = 0$) without referring to the underlying inertial frame *i.e.* by means attached to the turn table itself. Indeed this can be done in practice. For instance an observer with a clock on the rim at a point Σ can start the process by sending a light pulse to an adjacent clock in the anticlockwise direction and synchronize the latter with his own clock first by assuming the OWS of light to be equal to c. In the same way the third clock adjacent to the second one in the same direction can be synchronized with the latter and the synchronization procedure may continue in this way until finally one arrives at the first clock. The observer then discovers that the clock at Σ is not synchronized with itself. The desynchronization, *i.e.*, the defect in synchrony will again be different if checked clockwise rather than counter-clockwise. By trial however the observer will be able to discover that the defect in synchrony disappears if the one-way speeds in the two different directions correspond to two different numerical values c_1 and c_2 (say). With these obtained empirical values for the OWS of light, not only the clocks on the rim are synchronized in the absolute way but also the linear speed of the rim βc and hence the angular velocity $\omega = \beta c/R$ of the rotating disc are determined if c_1 and c_2 are substituted for c_+ and c_- in Eqs. (17) and (18) (or alternatively in Eq. (34) if one considers the Galilean world). All this however refers to the question of synchronization in the large and does not mean that in MCIFs it is mandatory to adopt the absolute (non-isotropic) synchronization.

One may now question our nomenclature "good synchrony" for the one for which light propagation is anisotropic (remember that for $\epsilon = 0$ light propagation is anisotropic in the classical as well as in the relativistic world). Let us clarify this: In the classical world people would be immediately happy to know that the demand for consistent synchronization in the large requires $\epsilon = 0$, which recovers GT. They would say, "After all we get back our old time tested transformation, the Einstein synchrony (leading to ZT) is a bad one, since it is not automatic and natural and it leads to inconsistent synchronization in the large." What should be our reaction who live in the relativistic world? If one carries on the same sort of arguments in the relativistic world, one may give a value judgment in favour of the absolute synchrony ($\epsilon = 0$) hence may call it the "good synchrony" by contrast, unless one seeks to indulge in double standard.

7. CONCLUSION

SP refers to a theoretical prediction regarding the OWS of light grazing the circumference of a rotating disc. The essential content of SP is that simple kinematics together with some appropriate symmetry arguments predict an anisotropy in the speed of light with respect to an "inertial observer." The claim apparently is substantiated by the Sagnac effect. (In the recent FOC experiment [15], the "inertial observer" is obtained automatically (see Sec. 2), while in the original rotating disc context one needs to take the limit $R \to \infty$ and $\omega \to 0$, while preserving the linear speed of the rim of the disc so that any point on the rim can be thought of as an inertial observer.)

Some earlier responses to the issue are either incomplete or they suffer from certain drawbacks. Here we have shown that by adopting the Einstein synchrony SP can be recast in the Galilean world (see Sec. 4). This facilitates in understanding the weak point of the reasoning leading to the fallacy.

It has been argued that SP hinges on the assumption that the (global) ratio of the round-trip speeds of the light beams co-rotating and counter-rotating with the disc (as if) ought to be the same as the local ratio of the OWS in the MCIFs since no point on the rim is preferred. The present analysis in the classical world reveals how in spite of the symmetry of the situation the two ratios can be different.

The issue of the "desynchronisation of clocks" which is often regarded as the physical cause of the Sagnac effect has been put under the scanner. It is held that of the two types of desynchronisation discussed here, desync1 is a theoretical artifact while desync2, although a convention-free entity, is also unable to qualify itself as the root cause of the effect. Finally, in spite of the lacunae in the reasonings leading to SP, the superiority of the absolute synchrony over the standard one for a rotating observer has been upheld.

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